The 13th Microlensing Workshop

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Estimating the Extended-Source Effect for an Arbitrary Lens and Source

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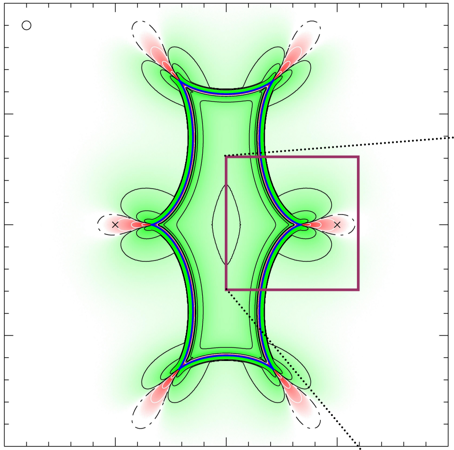
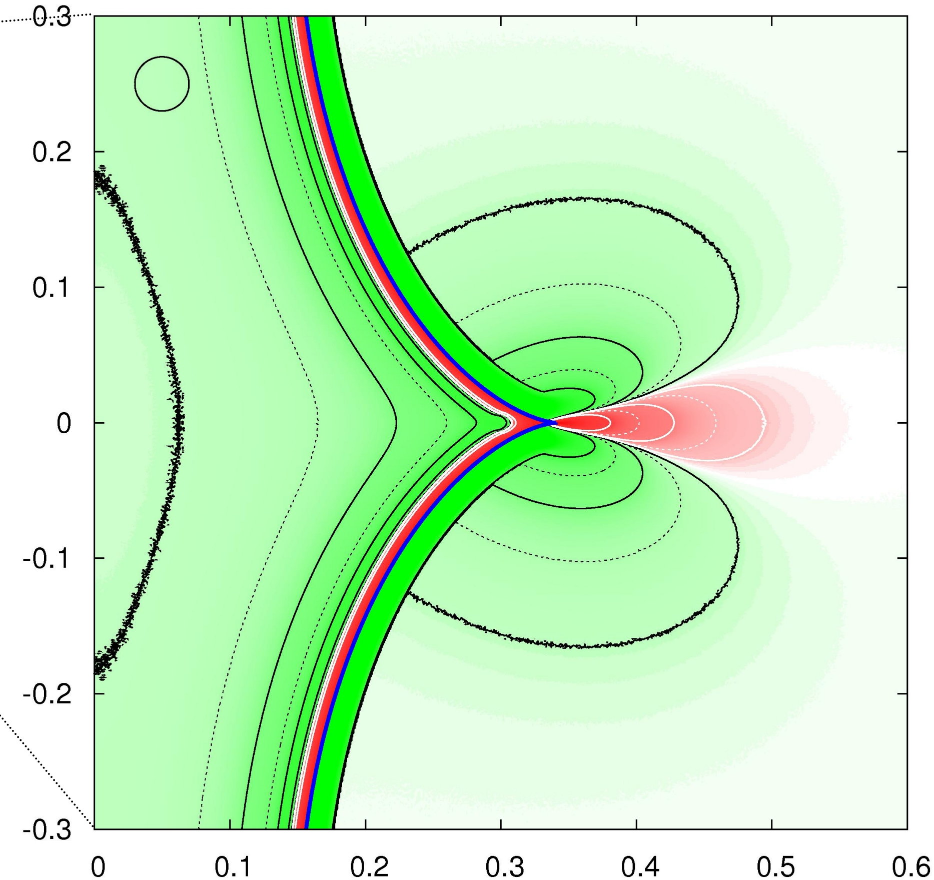
Extended-source effect

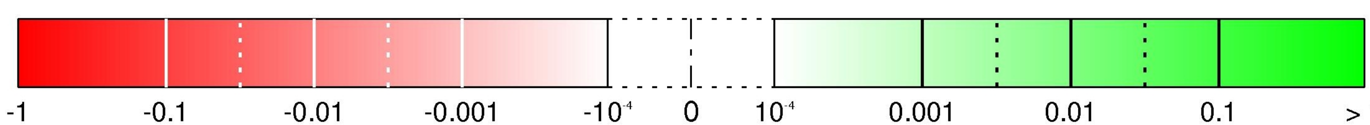
maps of $\delta_{EX}(\vec{y}_C) = \frac{A_*(\vec{y}_C)}{A_0(\vec{y}_C)} - 1$ *(Pejcha & Heyrovský 2009)*





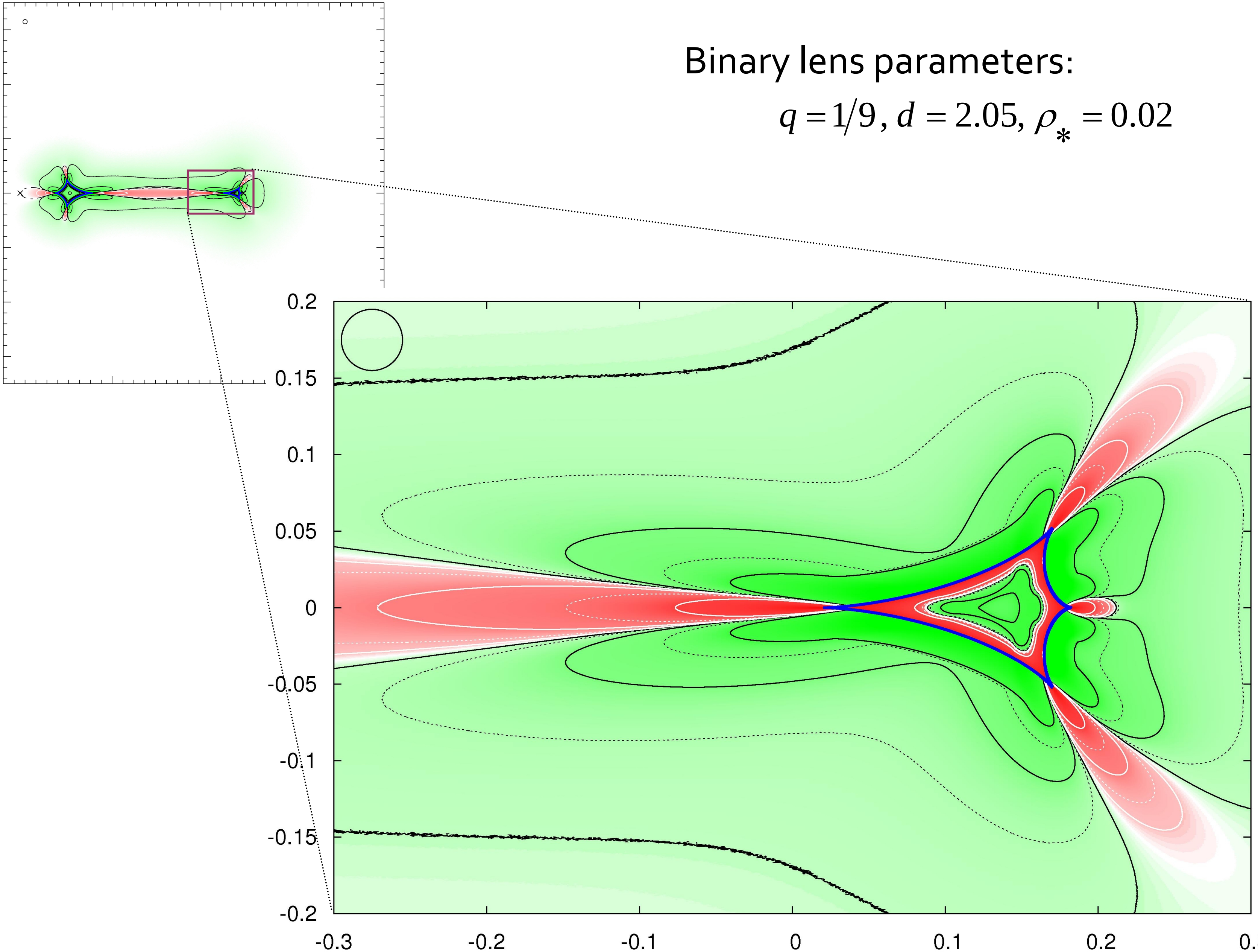
Binary lens parameters:

$q = 1,\ d = 1,\ \rho_* = 0.02$



Binary lens parameters:

$$q = 1/9,\ d = 2.05,\ \rho_* = 0.02$$



Outline

- A_* approximation: source off caustic
  - circularly symmetric source
  - elliptically symmetric source
  - asymmetric source
- A_* approximation: source crossing caustic
- Summary

Amplification: source off caustic

Extended-source amplification $A_*(\vec{y}_C) = \dfrac{\int A_0(\vec{y}_C + \vec{y}')\, B(\vec{y}')\, d^2\vec{y}'}{\int B(\vec{y}')\, d^2\vec{y}'}$

with source dimension ρ_* : $\vec{y}' = \rho_* \vec{r}$ $B(\vec{y}') = I(\vec{r})$

$$A_*(\vec{y}_C) = \frac{\int A_0(\vec{y}_C + \rho_* \vec{r})\, I(\vec{r})\, d^2\vec{r}}{\int I(\vec{r})\, d^2\vec{r}}$$

source off caustic $\Longrightarrow$ expand in powers of ρ_* :

$$A_*(\vec{y}_C) = \sum_{n=0}^{\infty} \frac{1}{n!} \rho_*^n \sum_{k=0}^{n} \binom{n}{k} \frac{\partial^n A_0(\vec{y}_C)}{\partial y_1^k \partial y_2^{n-k}} \frac{\int r_1^k r_2^{n-k} I(\vec{r})\, d^2\vec{r}}{\int I(\vec{r})\, d^2\vec{r}}$$

GEOMETRY

Circularly symmetric source $I(\vec{r}) = I(r)$

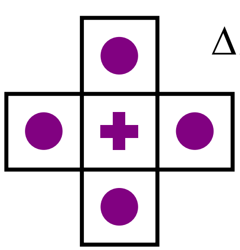
$$A_*(\vec{y}_C) = \sum_{p=0}^{\infty} \frac{1}{(p!)^2} \left(\frac{\rho_*}{2}\right)^{2p} \Delta^p A_0(\vec{y}_C) \frac{\int_0^1 I(r) r^{2p+1} dr}{\int_0^1 I(r) r\, dr} \approx \underbrace{A_0(\vec{y}_C) + \frac{1}{4} \Delta A_0(\vec{y}_C)\, \rho_*^2 \frac{\int_0^1 I(r) r^3 dr}{\int_0^1 I(r) r\, dr}}_{A_{APP}(\vec{y}_C)} + O\left(\rho_*^4\right)$$

estimate of $\delta_{EX}(\vec{y}_C) \approx \frac{\Delta A_0(\vec{y}_C)}{4 A_0(\vec{y}_C)} \underbrace{\rho_*^2 \frac{\int_0^1 I(r) r^3 dr}{\int_0^1 I(r) r\, dr}}_{\gamma_{SRC}} + O\left(\rho_*^4\right)$

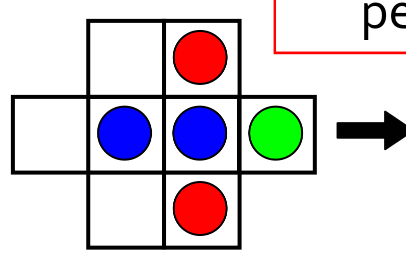
for stellar broadband limb darkening $\frac{\int_0^1 I(r) r^3 dr}{\int_0^1 I(r) r\, dr} \in [0.4;0.5]$

$\Longrightarrow$ source-size estimate $\rho_* \approx \gamma_{SRC}^{1/2} \sqrt{\frac{\int_0^1 I(r) r dr}{\int_0^1 I(r) r^3\, dr}} \approx 1.5\, \gamma_{SRC}^{1/2} (1 \pm 0.07)$

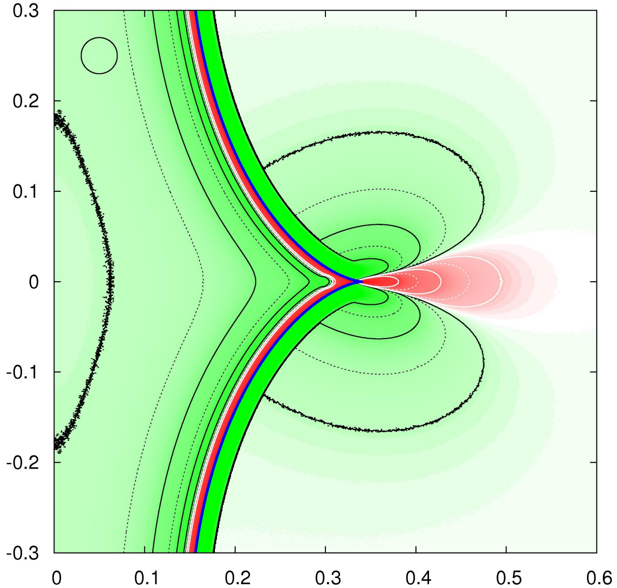
Numerical evaluation of $\Delta A_0(+)$:



$\Delta A_0(\,) \approx \left[\sum A_0(\bullet) - 4 A_0(+)\right] \delta^{-2}$



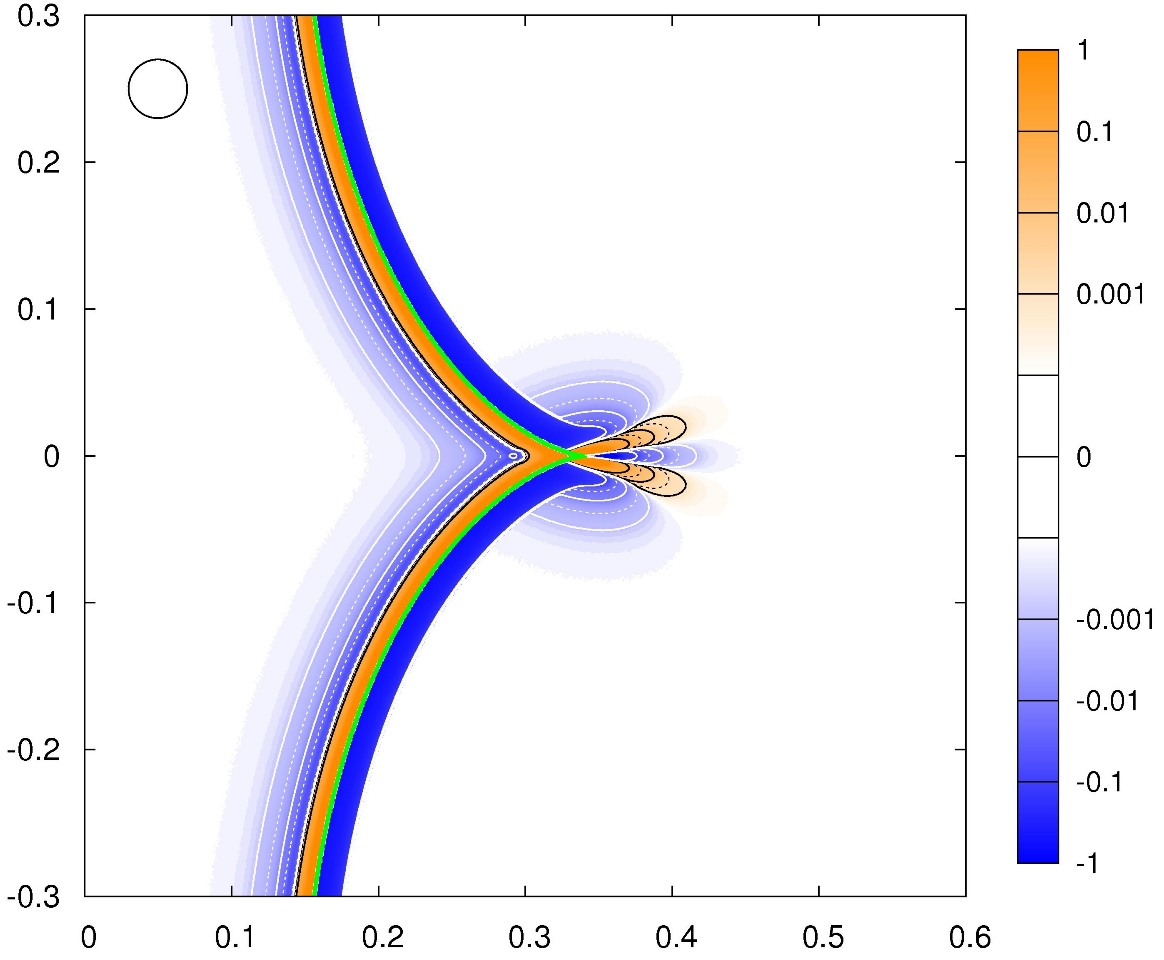
only 2 extra evaluations per light-curve point



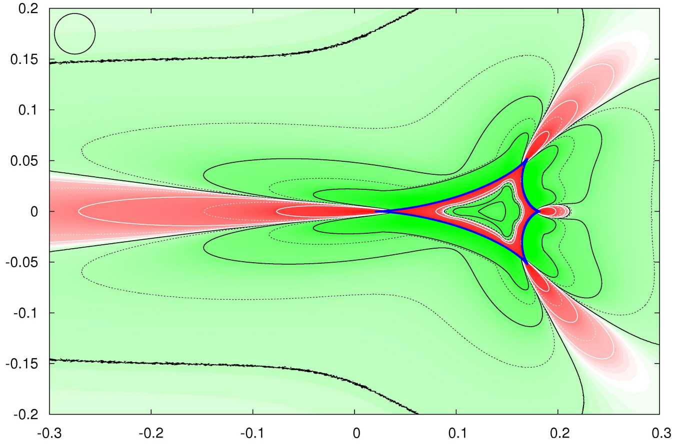
$\delta_{EX}(\vec{y}_C)$

Accuracy of 2nd order approximation

$$\delta_{APP}(\vec{y}_C) = A_{APP}(\vec{y}_C) / A_*(\vec{y}_C) - 1$$

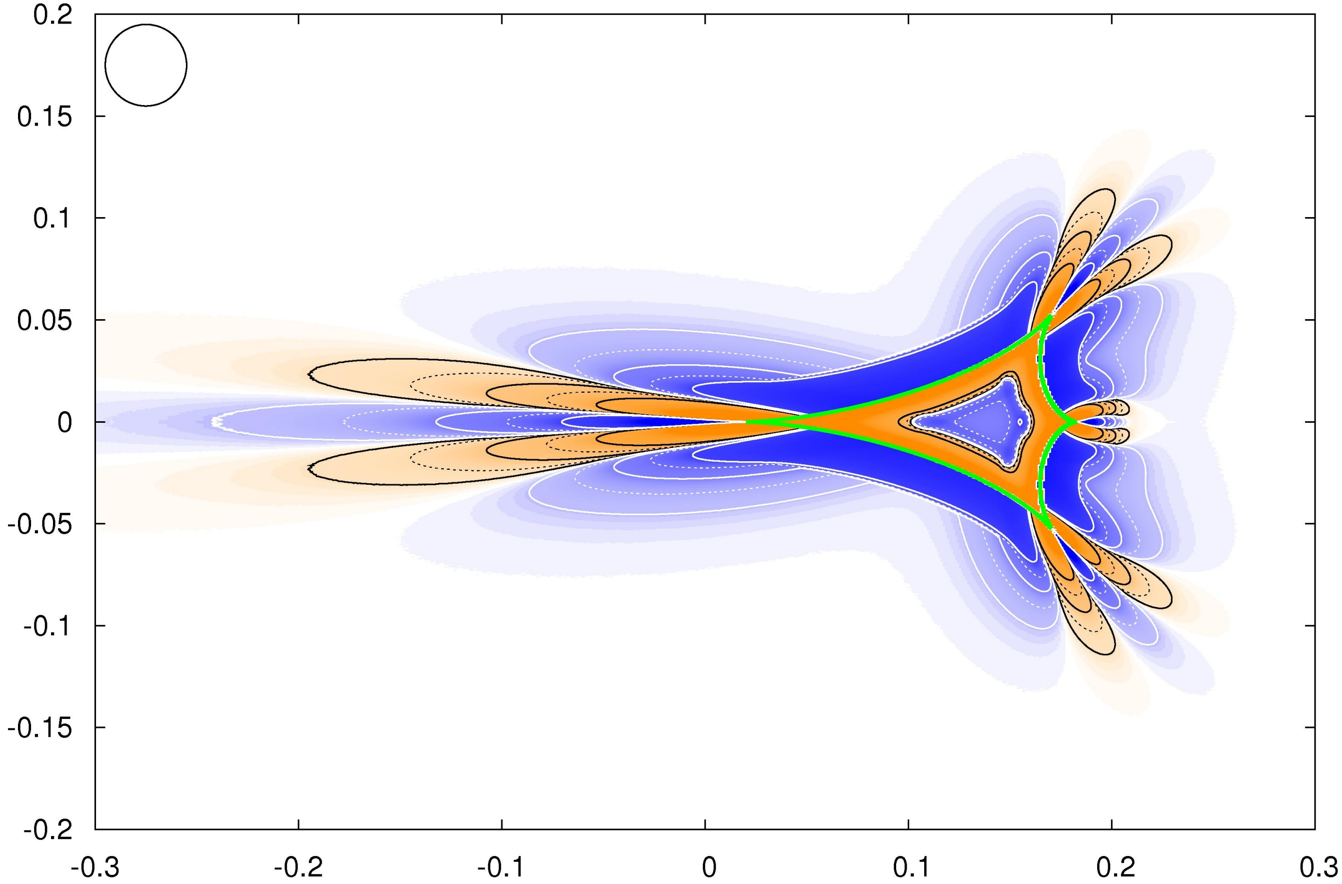


Accuracy of 2nd order approximation



$$\delta_{EX}(\vec{y}_C)$$

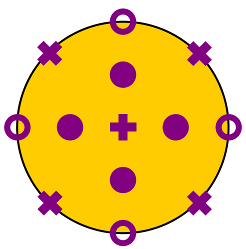
$$\delta_{APP}(\vec{y}_C) = A_{APP}(\vec{y}_C) / A_*(\vec{y}_C) - 1$$



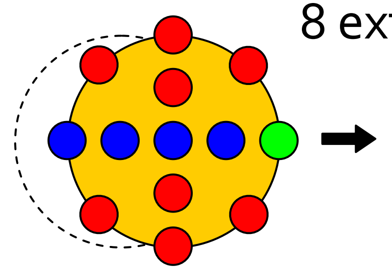
4^{th} order approximation

$$\overbrace{A_*(\vec{y}_C) \approx A_0(\vec{y}_C) + \frac{1}{4}\Delta A_0(\vec{y}_C)\,\rho_*^2 \frac{\int_0^1 I(r) r^3 dr}{\int_0^1 I(r) r\, dr} + \frac{1}{64}\Delta^2 A_0(\vec{y}_C)\,\rho_*^4 \frac{\int_0^1 I(r) r^5 dr}{\int_0^1 I(r) r\, dr}}^{A_{APP}^{(4)}(\vec{y}_C)} + O\left(\rho_*^6\right)$$

Numerical evaluation of $\Delta^2 A_0(+)$:

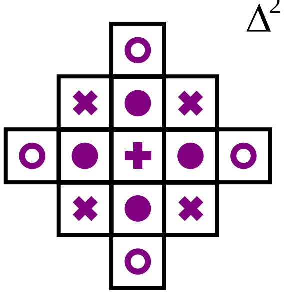


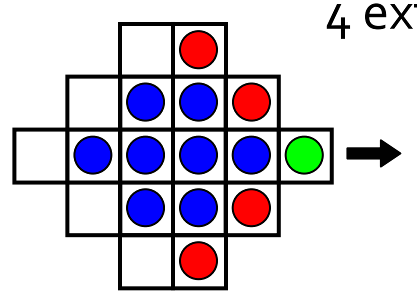
Gould (2008)



8 extra evaluations / l-c pt.
if $\Delta t = \rho_*/2$

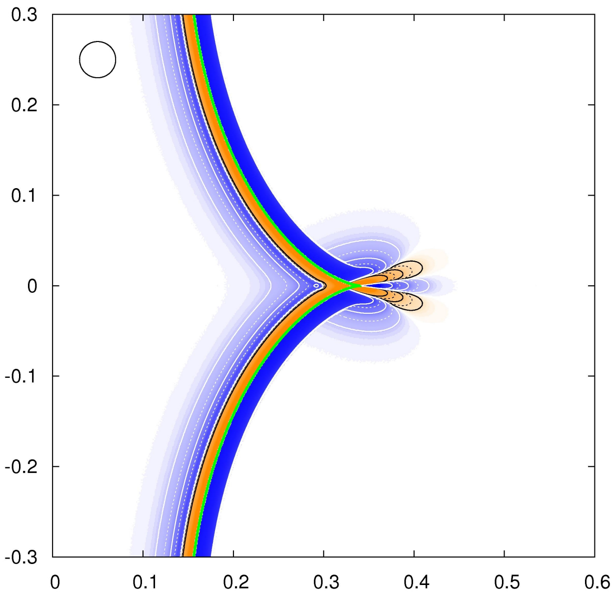
$$\Delta^2 A_0(\;) \approx \left[\sum \Delta A_0(\bullet) - 4\Delta A_0(+)\right]\delta^{-2}$$





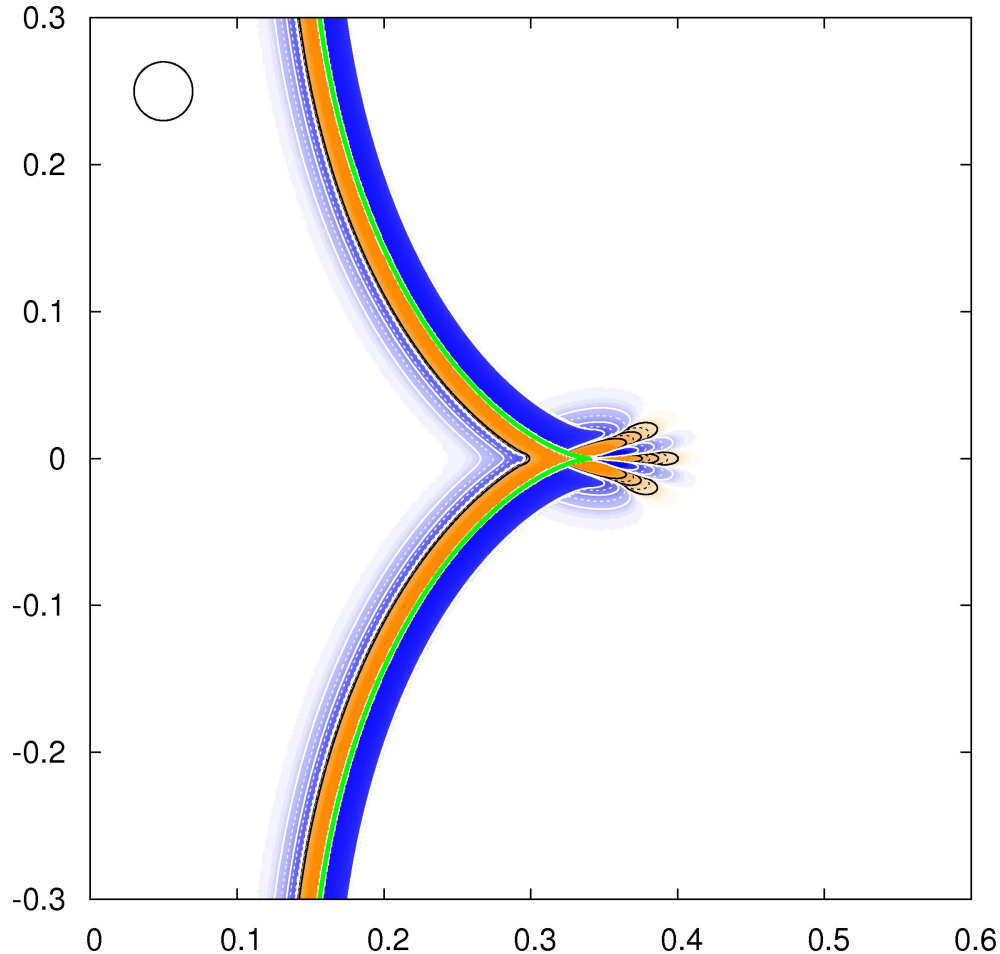
4 extra evaluations / l-c pt.

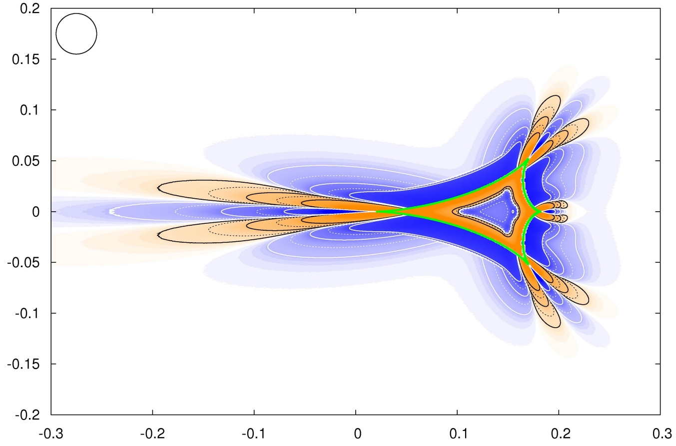
Accuracy of 4th order approximation



$$\delta_{APP}(\vec{y}_C)$$

$$\delta^{(4)}_{APP}(\vec{y}_C) = A^{(4)}_{APP}(\vec{y}_C)\,/\,A_{*}(\vec{y}_C) - 1$$

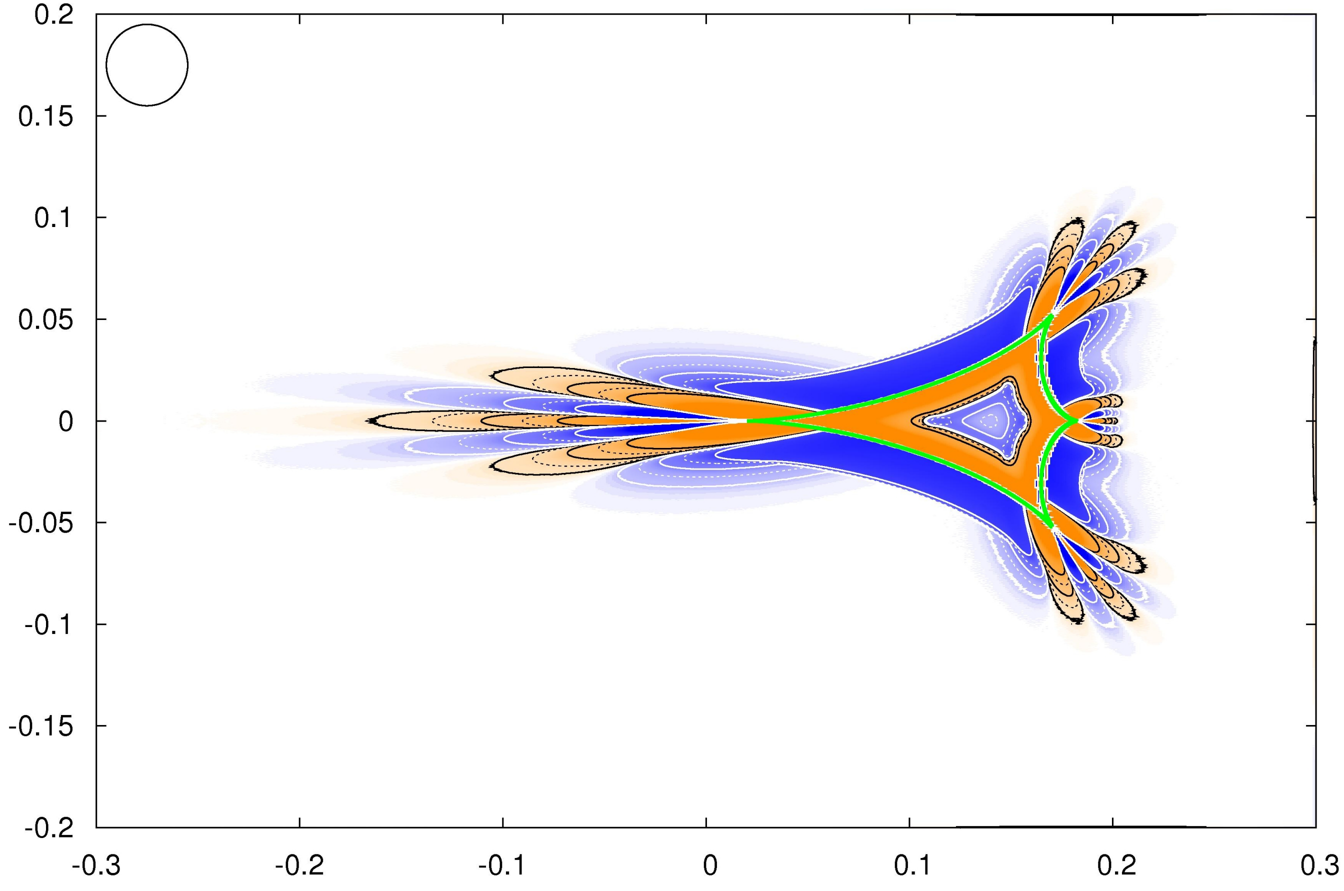




Accuracy of 4th order approximation

$$\delta^{(4)}_{APP}(\vec{y}_C) = A^{(4)}_{APP}(\vec{y}_C) / A_*(\vec{y}_C) - 1$$

$\delta_{APP}(\vec{y}_C)$



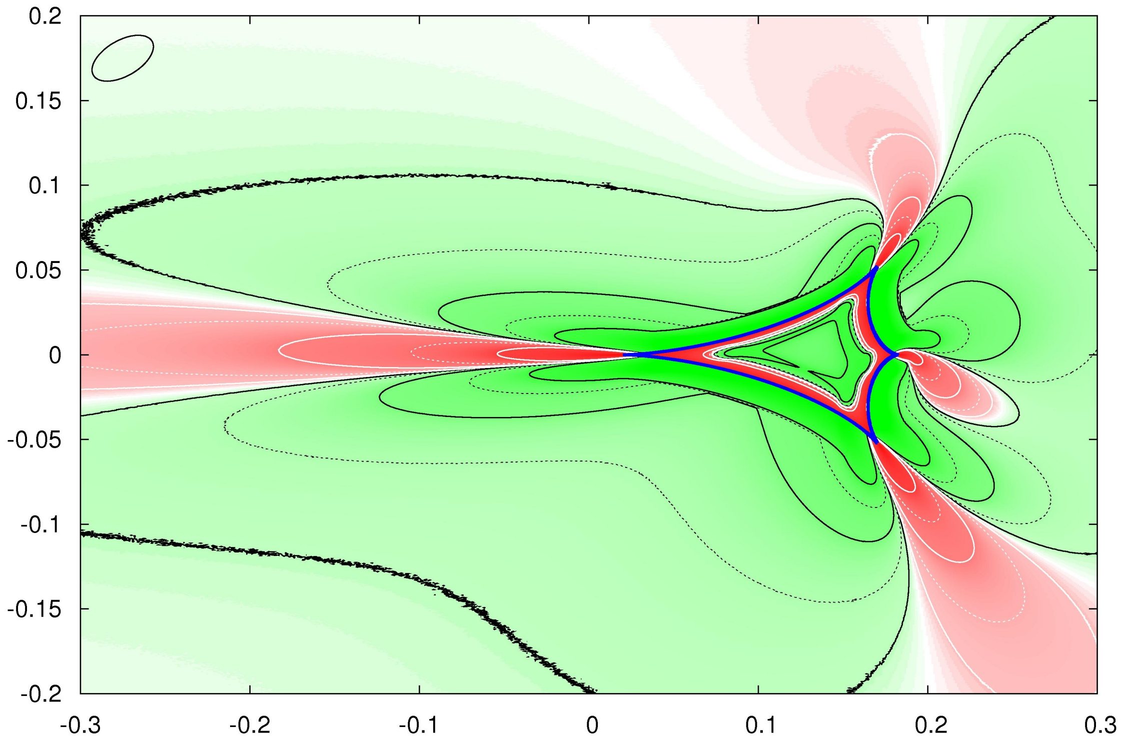
Elliptically symmetric source

parameters:

semi-major axis	ρ_*
eccentricity	e
tilt of axis	α_0

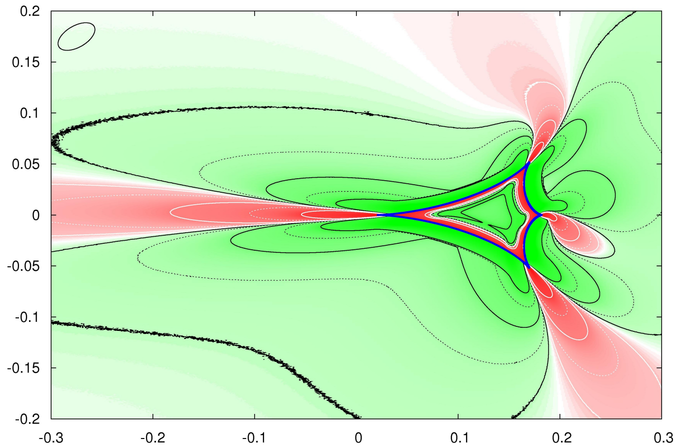
$$A_*(\vec{y}_C) = \sum_{p=0}^{\infty} \frac{1}{(p!)^2} \left(\frac{\rho_*}{2}\right)^{2p} \left[(1-e^2\sin^2\alpha_0)\frac{\partial^2}{\partial y_1^2} + (1-e^2\cos^2\alpha_0)\frac{\partial^2}{\partial y_2^2} + e^2\sin(2\alpha_0)\frac{\partial^2}{\partial y_1 \partial y_2} \right]^p A_0(\vec{y}_C)\, \frac{\int_0^1 I(r) r^{2p+1} dr}{\int_0^1 I(r) r\, dr}$$

$\delta_{EX}(\vec{y}_C)$



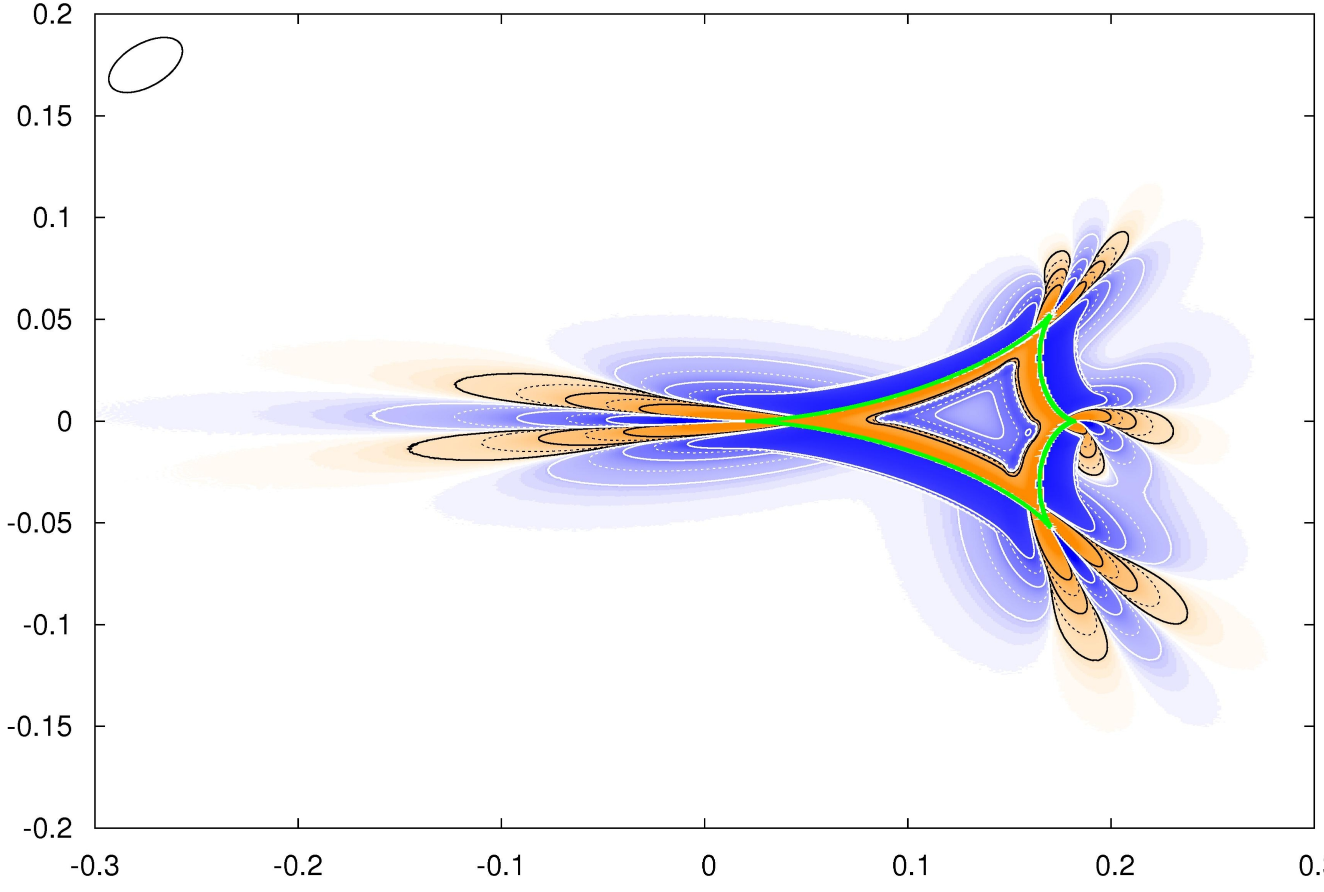
$\rho_* = 0.02, e = 0.85, \alpha_0 = 30°$

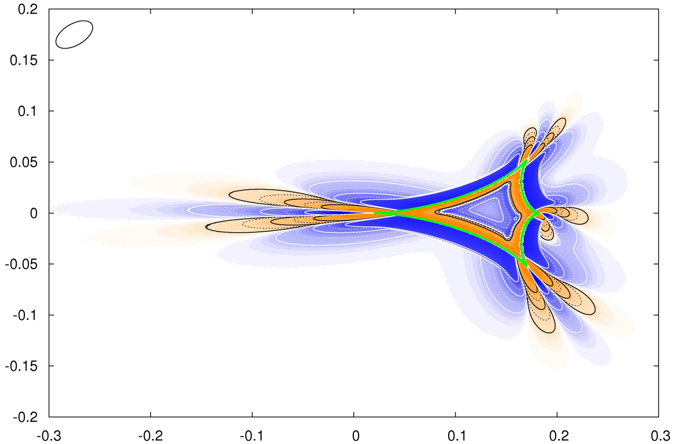
Accuracy of 2nd order approximation



$$\delta_{EX}(\vec{y}_C)$$

$$\delta_{APP}(\vec{y}_C) = A_{APP}(\vec{y}_C) / A_{*}(\vec{y}_C) - 1$$

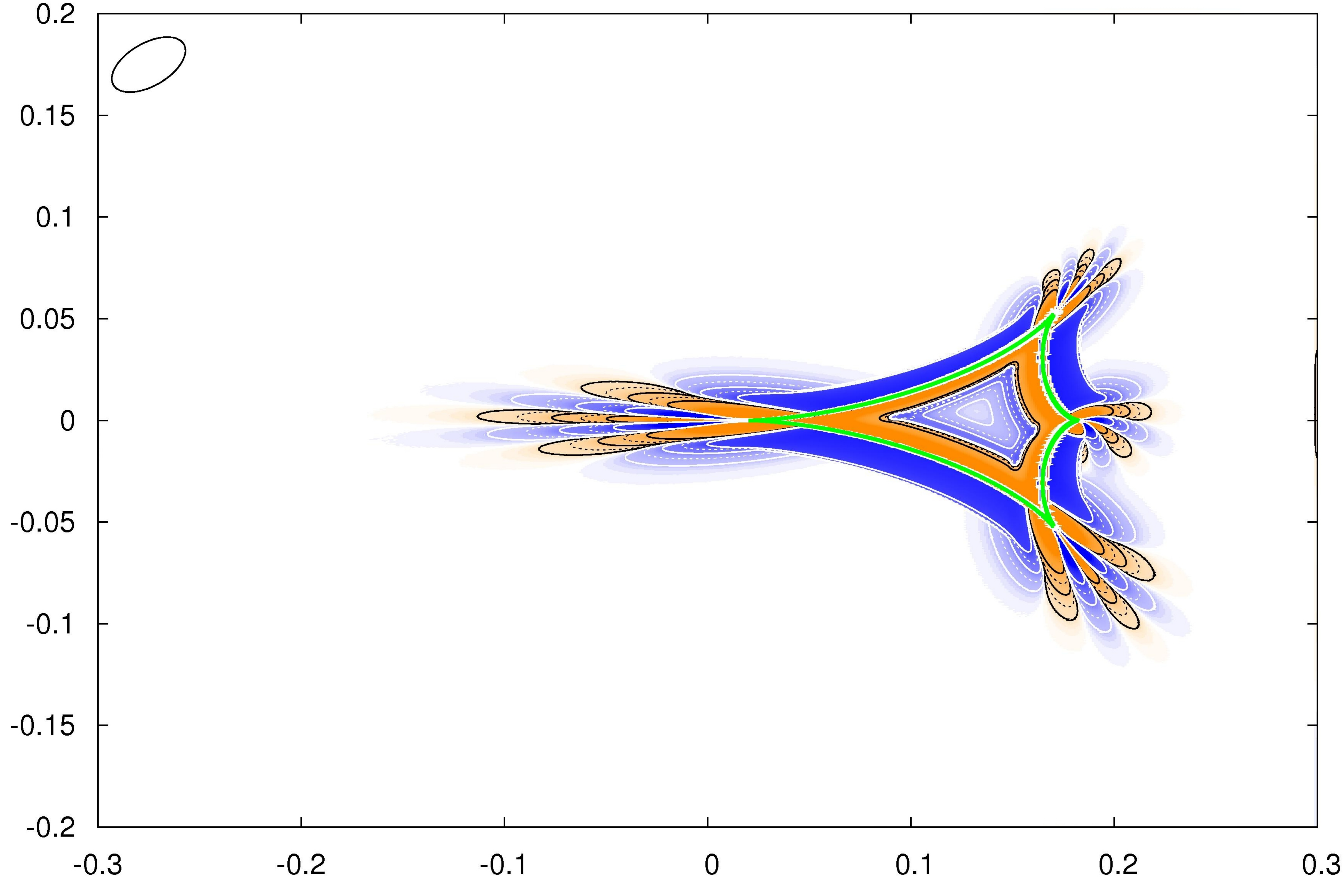




Accuracy of 4th order approximation

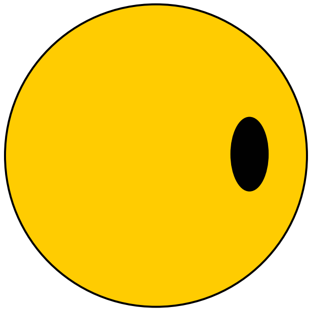
$$\delta_{APP}^{(4)}(\vec{y}_C) = A_{APP}^{(4)}(\vec{y}_C) / A_*(\vec{y}_C) - 1$$

$\delta_{APP}(\vec{y}_C)$



Asymmetric source

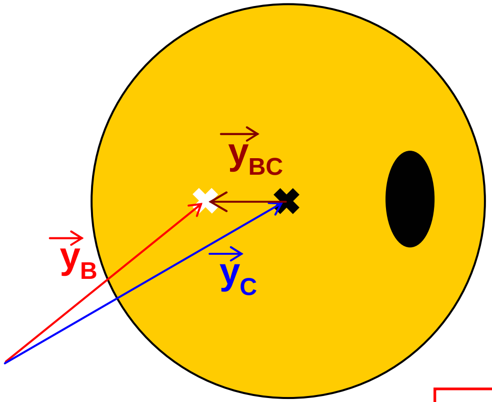
asymmetry in shape or in surface brightness



general expansion:

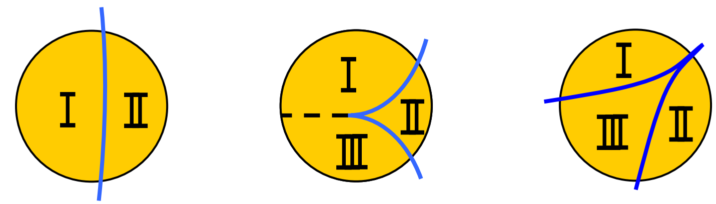
$$A_*(\vec{y}_C) \approx A_0(\vec{y}_C) + \vec{\nabla}A_0(\vec{y}_C)\cdot\vec{y}_{BC} + O(\rho_*^2)$$

can be eliminated by expanding around brightness center $\vec{y}_B$



$$A_*(\vec{y}_C) \approx A_0(\vec{y}_B) + \frac{1}{2}\sum_{k=0}^{2}\binom{2}{k}\frac{\partial^2 A_0(\vec{y}_B)}{\partial y_1^k \partial y_2^{2-k}}\left[\frac{\int {y'_1}^k {y'_2}^{2-k} B(\vec{y}')\, d^2\vec{y}'}{\int B(\vec{y}')\, d^2\vec{y}'} - {y_{BC1}}^k {y_{BC2}}^{2-k}\right] + O(\rho_*^3)$$

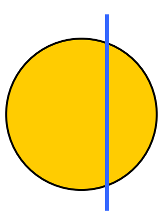
Source crossing a caustic

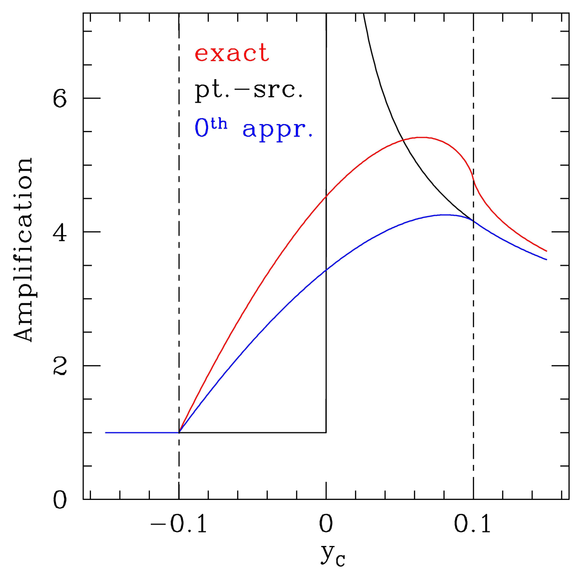


$A_0(\vec{y})$ can be expanded separately in each section S_j , $j = I \dots M \geq N$

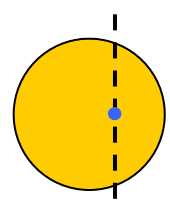
$$A_*(\vec{y}_C) = \frac{\sum_{j=I}^{M} \int_{S_j} A_0(\vec{y}_C + \vec{y}')\, B(\vec{y}')\, d^2\vec{y}'}{\int_S B(\vec{y}')\, d^2\vec{y}'} \approx \sum_{j=I}^{M} \frac{\int_{S_j} B(\vec{y}')\, d^2\vec{y}'}{\int_S B(\vec{y}')\, d^2\vec{y}'} A_0(\vec{y}_{B_j}) + O(\rho_*^2)$$

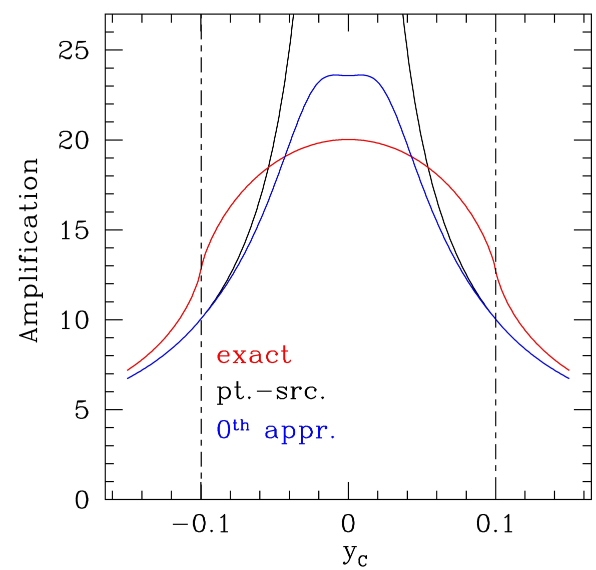
FOLD





POINT LENS





Summary

- detect extended-source effects by 2^{nd} order approximation
  (at the cost of computing 3 p-s light curves instead of 1)

- when detected, measure radius, check 4^{th} order
  (at the cost of computing 5 p-s light curves instead of 1)

- when 4^{th} order significant, time to consider full integration