Dynamics of non-spinning compact binary systems at the fourth post-Newtonian order

Laura Bernard

based on Phys.Rev.D93 084037 (arXiv: 1512.02876) + in prep.,
in collaboration with L. Blanchet, A. Bohé, G. Faye, S. Marsat

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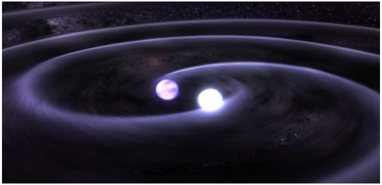
Plan

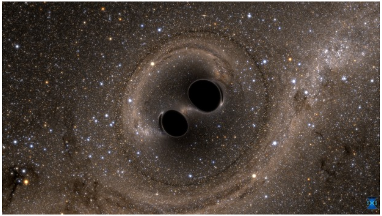
Coalescing compact binary systems

Binary neutron stars



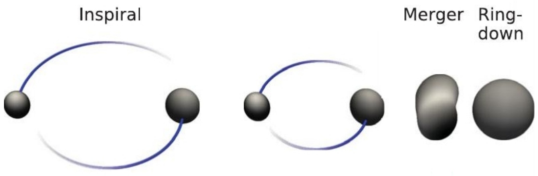
Credits : NASA

Binary black holes

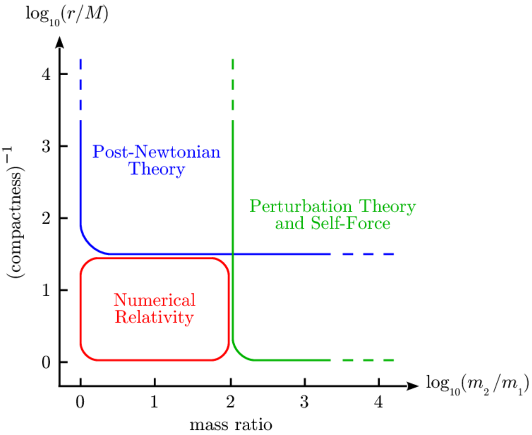


Credits : the Simulating eXtreme Spacetimes (SXS) project

The coalescence



A compact binary system



Post-Newtonian formalism

For **weak gravitational field** and **slow motion**, we can develop perturbatively the dynamics in $\varepsilon \sim \frac{v^2}{c^2} \sim \frac{GM}{rc^2}$. Post-Newtonian order : $1\text{PN} = \mathcal{O}\left(\frac{1}{c^2}\right)$.

Principle of the Fokker action

1. We start from the classical action

$$S_{\rm tot}\left[g_{\mu\nu},\mathbf{y}_A,\mathbf{v}_A\right]=S_{\rm grav}\left[g_{\mu\nu}(x)\right]+S_{\rm mat}\left[\mathbf{v}_A;g_{\mu\nu}(\mathbf{y}_A,\mathbf{v}_A)\right]\ ,$$

   $A=1,2.$

2. solve the Einstein equation $\frac{\delta S_{\rm tot}}{\delta g_{\mu\nu}}=0\rightarrow\overline{g}_{\mu\nu}\left[\mathbf{y}_A(t),\mathbf{v}_A(t),\cdots\right],$

3. and construct the Fokker action

$$S_{\rm Fokker}\left[\mathbf{y}_A,\mathbf{v}_A,\cdots\right]=\ S_{\rm tot}\left[\overline{g}_{\mu\nu}\left(\mathbf{y}_A,\mathbf{v}_A,\cdots\right),\mathbf{y}_A,\mathbf{v}_A\right].$$

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$$S_{\rm Fokker}\left[\mathbf{y}_A,\mathbf{v}_A,\cdots\right]=\ S_{\rm tot}\left[\overline{g}_{\mu\nu}\left(\mathbf{y}_A,\mathbf{v}_A,\cdots\right),\mathbf{y}_A,\mathbf{v}_A\right].$$

▷ The dynamics for the particles is unchanged

$$\frac{\delta S_{\rm Fokker}}{\delta y_A}=\underbrace{\left.\frac{\delta S_{\rm tot}}{\delta g_{\mu\nu}}\right|_{g=\overline{g}}}_{=0}\cdot\frac{\delta g_{\mu\nu}}{\delta y_A}+\left.\frac{\delta S_{\rm mat}}{\delta y_A}\right|_{g=\overline{g}}$$

$$=\left.\frac{\delta S_{\rm mat}}{\delta y_A}\right|_{g=\overline{g}}.$$

Our Fokker action

$$S_{\rm grav} = \frac{c^3}{16\pi G}\int \mathrm{d}^4x\,\sqrt{-g}\left[g^{\mu\nu}\left(\Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\rho} - \Gamma^{\rho}_{\mu\nu}\Gamma^{\lambda}_{\rho\lambda}\right) - \underbrace{\frac{1}{2}g_{\mu\nu}\Gamma^{\mu}\Gamma^{\nu}}_{\text{gauge fixing term}}\right],$$

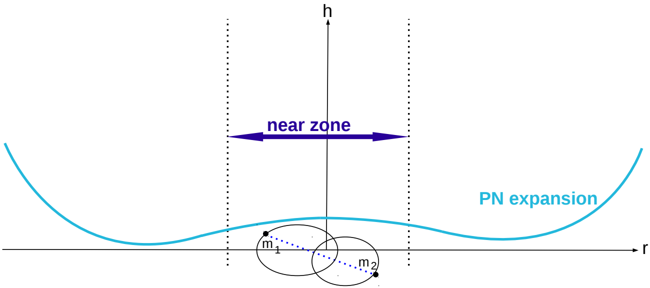
$$S_{\rm mat} = -\sum_{A=1,2} m_A c^2 \int \mathrm{d}t\sqrt{-\left(g_{\mu\nu}\right)_A \frac{v_A^{\mu}v_A^{\nu}}{c^2}}\,.$$

Relaxed Einstein equations

$$\Box h^{\mu\nu} = \frac{16\pi G}{c^4}|g|T^{\mu\nu} + \Lambda^{\mu\nu}\left[h, \partial h, \partial^2 h\right]$$

- with $h^{\mu\nu} = \sqrt{|g|}g^{\mu\nu} - \eta^{\mu\nu}$ the metric perturbation variable.
- We don't impose the harmonicity condition $\partial_\nu h^{\mu\nu} = 0$.
- $\Lambda^{\mu\nu}$ encodes the non-linearities, with supplementary harmonicity terms containing $H^\mu = \partial_\nu h^{\mu\nu}$.

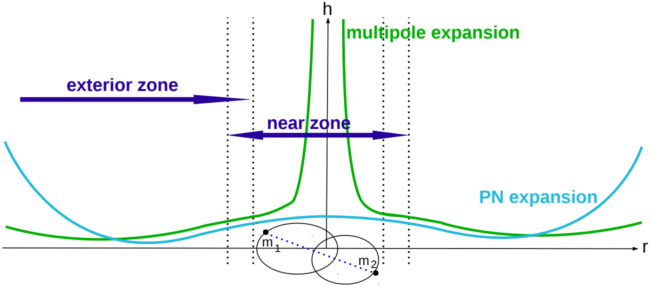
The multipolar post-Newtonian formalism



Near zone / Wave zone

- **Near zone** : post-Newtonian expansion $h = \overline{h}$,

The multipolar post-Newtonian formalism



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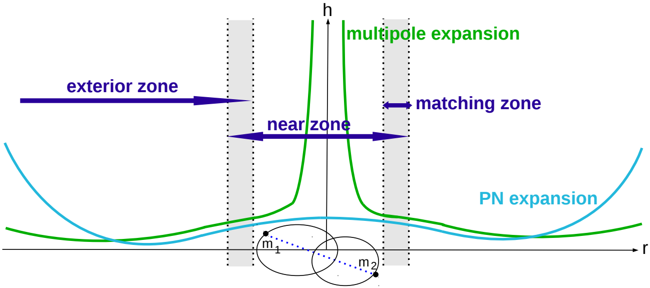

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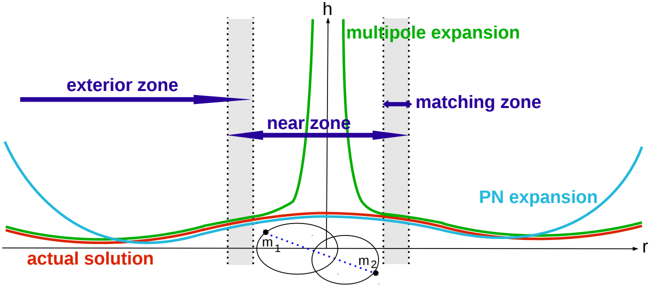
The multipolar post-Newtonian formalism



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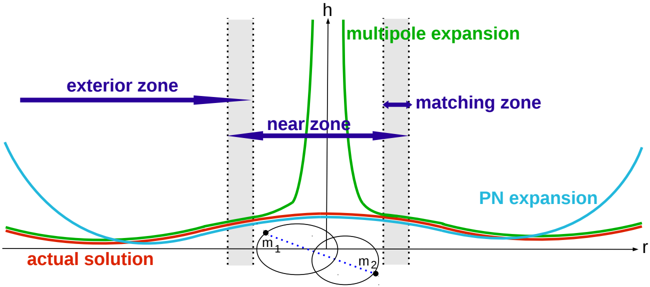
$$S_g = \underset{B=0}{\mathrm{FP}} \int \mathrm{d}t \int \mathrm{d}^3\mathbf{x} \left(\frac{r}{r_0}\right)^B \overline{\mathcal{L}}_g + \underset{B=0}{\mathrm{FP}} \int \mathrm{d}t \int \mathrm{d}^3\mathbf{x} \left(\frac{r}{r_0}\right)^B \mathcal{M}\left(\mathcal{L}_g\right)$$

Near zone / Wave zone



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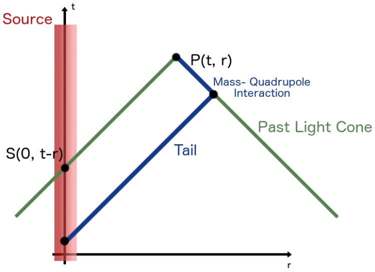
The tail effects at 4PN



- At 4PN we have to insert some tail effects,

$$\overline{h}^{\mu\nu} = \overline{h}^{\mu\nu}_{\text{part}} - \frac{2G}{c^4} \sum_{l=0}^{+\infty} \frac{(-1)^l}{l!} \partial_L \left\{ \frac{\mathcal{A}_L^{\mu\nu}(t-r/c) - \mathcal{A}_L^{\mu\nu}(t+r/c)}{r} \right\}$$

- When inserted into the Fokker action it gives in the following contribution

$$S_{\text{tail}} = \frac{G^2(m_1+m_2)}{5c^8} \underset{\frac{2s_0}{c}}{\text{Pf}} \int\int \frac{\mathrm{d}t\,\mathrm{d}t'}{|t-t'|} I_{ij}^{(3)}(t)\, I_{ij}^{(3)}(t')$$

Different regularization schemes

IR regularization

- IR singularity of the PN expansion at infinity : r_0 (Hadamard regularization),
- From the tail contribution : s_0,
- These two constants of regularization are linked through $s_0 = r_0\, \mathrm{e}^{-\alpha}$.
- The constant α will be determined by comparison with self-force results.

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UV singularity at the location of the point particles

- Dimensional regularization,
  1. We calculate the Lagrangian in $d = 3 + \varepsilon$ dimensions.
  2. We expand the results when $\varepsilon \to 0$: appearance of a pole $1/\varepsilon$.
  3. We renormalize the pole through a redefinition of the trajectories of particles.
- **The physical result should not depend on ε.**

The particular solution - post-Newtonian counting in a Fokker action

Cancellations between gravitational and matter terms

We decompose $\overline{h}^{\mu\nu}_n \longrightarrow \left(\overline{h}^{00ii} \equiv \overline{h}^{00} + \overline{h}^{ii}, \overline{h}^{0i}, \overline{h}^{ij}\right) = \mathcal{O}(n+2, n+1, n+2)$, ($n$ pair).

We define the rests $\overline{r}_{n+2} = (\overline{r}^{00ii}_{n+4}, \overline{r}^{0i}_{n+3}, \overline{r}^{ij}_{n+4}) = \mathcal{O}(n+4, n+3, n+4)$, and expand the action

$$S_{\mathsf{F}}\left[\overline{h}\right] = S_{\mathsf{F}}\left[\overline{h}_n\right] + \int \left[\frac{\delta S_{\mathsf{F}}}{\delta \overline{h}^{00ii}}\left[\overline{h}_n\right]\overline{r}^{00ii}_{n+4} + \frac{\delta S_{\mathsf{F}}}{\delta \overline{h}^{0i}}\left[\overline{h}_n\right]\overline{r}^{0i}_{n+3} + \frac{\delta S_{\mathsf{F}}}{\delta \overline{h}^{ij}}\left[\overline{h}_n\right]\overline{r}^{ij}_{n+4} + \cdots\right]$$

Using that $\frac{\delta S_{\mathsf{F}}}{\delta \overline{h}^{00ii}}\left[\overline{h}_n\right] = \mathcal{O}\big(n\big)$, $\quad \frac{\delta S_{\mathsf{F}}}{\delta \overline{h}^{0i}}\left[\overline{h}_n\right] = \mathcal{O}\big(n-1\big)$, $\quad \frac{\delta S_{\mathsf{F}}}{\delta \overline{h}^{ij}}\left[\overline{h}_n\right] = \mathcal{O}\big(n\big)$, we get

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▷ to have the Lagrangian at nPN *i.e.* $\mathcal{O}\left(\frac{1}{c^{2n}}\right)$, we need to know the metric at :

$$\left(h^{00ii}, h^{0i}, h^{ij}\right) = \mathcal{O}(n+2, n+1, n+2) = \mathcal{O}\left(\frac{1}{c^{n+2}}\right).$$

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▷ **For 4 PN :** $\left(h^{00ii}, h^{0i}, h^{ij}\right) = \mathcal{O}\left(\frac{1}{c^6}, \frac{1}{c^5}, \frac{1}{c^6}\right)$

The particular solution

Metric decomposition (in d dimensions)

$$\overline{h}^{00ii} = -\frac{4}{c^2}V - \frac{4}{c^4}\left[\frac{d-1}{d-2}V^2 - 2\frac{d-3}{d-2}K\right]$$
$$- \frac{8}{c^6}\left[2\hat{X} + V\hat{W} + \frac{1}{3}\left(\frac{d-1}{d-2}\right)^2 V^3 - 2\frac{d-3}{d-1}V_iV_i - 2\frac{(d-1)(d-3)}{(d-2)^2}KV\right] + \mathcal{O}(8) ,$$
$$\overline{h}^{0i} = -\frac{4}{c^3}V_i - \frac{4}{c^5}\left(2\hat{R}_i + \frac{d-1}{d-2}VV_i\right) + \mathcal{O}(7) ,$$
$$\overline{h}^{ij} = -\frac{4}{c^4}\left(\hat{W}_{ij} - \frac{1}{2}\delta_{ij}\hat{W}\right) - \frac{16}{c^6}\left(\hat{Z}_{ij} - \frac{1}{2}\delta_{ij}\hat{Z}\right) + \mathcal{O}(8) .$$

Each potential obeys a flat space-time wave equation :

$$\Box V = -4\pi G\,\sigma ,$$
$$\Box V_i = -4\pi G\,\sigma_i ,$$
$$\Box \hat{W}_{ij} = -4\pi G\left(\sigma_{ij} - \delta_{ij}\,\frac{\sigma_{kk}}{d-2}\right) - \frac{d-1}{2(d-2)}\partial_i V\partial_j V .$$

with $\sigma = \frac{T^{00}+T^{ii}}{c^2}$, $\sigma_i = \frac{T^{0i}}{c}$ and $\sigma_{ij} = T^{ij}$.

The conservative dynamics at 4PN

The generalized Fokker Lagrangian at 4PN

$$L_{4\mathrm{PN}} = \frac{Gm_1m_2}{r_{12}} + \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + L_{1\mathrm{pn}} + L_{2\mathrm{pn}} + L_{3\mathrm{pn}} + L_{4\mathrm{pn}}[y_A(t), v_A(t), a_A(t), \partial a_A(t), \cdots] + L_{4\mathrm{pn}}^{\mathrm{tail}}$$

The 4PN equations of motion

$$a^i_{1,4\mathrm{PN}} = -\frac{Gm_2}{r_{12}^2}n_{12}^i + a^i_{1,1\mathrm{pn}} + a^i_{1,2\mathrm{pn}} + a^i_{1,3\mathrm{pn}} + a^i_{1,4\mathrm{pn}}[\alpha] + a^{\mathrm{tail}\,i}_{1,4\mathrm{pn}}$$

3PN :

- ADM Hamiltonian (Damour, Jaranowski & Schäfer, 1999, 2001),
- Harmonic coordinates (Blanchet, Faye & de Andrade, 2000, 2001),
- Surface integrals (Itoh, Futamase & Asada 2001-2003),
- Effective field theory (Foffa & Sturani 2011).

4PN :

- Partial result from EFT (Foffa & Sturani 2012),
- ADM Hamiltonian formalism (Damour, Jaranowski & Schäfer 2013, 2014),
- **Harmonic coordinates (Bernard, Blanchet, Bohé, Faye & Marsat 2015).**

Some consistency checks

We have checked that

- the result does not depend on the regularisaton scheme : **no r_0** and **no pole $1/\varepsilon$**,
- in the test mass limit we recover the **Schwarzschild geodesics**,
- the equations of motion are manifestly **Lorentz invariant**.
- we recover the **conserved energy for circular orbits** (known from self force calculations).

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Conserved quantities in harmonic coordinates

- Energy $E = E_{\text{inst}} + E_{\text{tail}}$, with $\frac{\mathrm{d}E}{\mathrm{d}t} = 0$,
- Angular momentum $J^i = J^i_{\text{inst}} + J^i_{\text{tail}}$, with $\frac{\mathrm{d}J^i}{\mathrm{d}t} = 0$,
- Linear momentum $P^i = P^i_{\text{inst}} + P^i_{\text{tail}}$, with $\frac{\mathrm{d}P^i}{\mathrm{d}t} = 0$,
- Center of mass $G^i = G^i_{\text{inst}} + G^i_{\text{tail}}$, with $\frac{\mathrm{d}G^i}{\mathrm{d}t} = P^i$,

Energy in harmonic coordinates

Barycentric coordinates $G^i = 0$

$$y^i_{1,\mathrm{CM}} = \frac{m_2}{m_1+m_2}x^i + \cdots + y^i_{\mathrm{CM,tail}} + \mathcal{O}(10), \quad y^i_2 = \cdots + \mathcal{O}(10)\,,$$

$$a^i_{\mathrm{CM}} = -\frac{G(m_1+m_2)}{r^3}x^i + \cdots + a^i_{\mathrm{CM,tail}} + \mathcal{O}(10)$$

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Reduction to circular orbits

In polar coordinates (r, φ), we define $\omega = \dot{\varphi}$. For circular orbits $\dot{r} = 0$, $\dot{\varphi} = 0$ and $v^2 = r^2\omega^2$.

$$\begin{aligned} E(x;\nu) = -\frac{\mu c^2 x}{2}\Bigg[1 &- \left(\frac{3}{4}+\frac{\nu}{12}\right)x + \left(-\frac{27}{8}+\frac{19\nu}{8}-\frac{\nu^2}{24}\right)x^2 \\ &+ \left(-\frac{675}{64} + \left(\frac{34445}{576}-\frac{205\pi^2}{96}\right)\nu - \frac{155\nu^2}{96} - \frac{35\nu^3}{5184}\right)x^3 \\ &+ \left(-\frac{3969}{128} + \left(\frac{9037\pi^2}{1536} - \frac{123671}{5760} + \frac{448}{15}\left(2\gamma + \ln(16x)\right)\right)\nu \right. \\ &\left. - \left(\frac{3157\pi^2}{576} - \frac{198449}{3456}\right)\nu^2 + \frac{301\nu^3}{1728} + \frac{77\nu^4}{31104}\right)x^4\Bigg], \end{aligned}$$

where $x = \left(\frac{G(m_1+m_2)\Omega}{c^3}\right)^{2/3}$ and $\nu = \frac{m_1 m_2}{(m_1+m_2)^2}$ is the symmetric mass ratio.

Periastron advance

Inverting the equations giving the constant of motion E and $J = |\boldsymbol{J}|$, we get

$$\dot{r}^2 = \mathcal{R}\left[r; E, J\right],$$
$$\dot{\varphi} = \mathcal{S}\left[r; E, J\right]$$

Orbital period P and fractional angle K

$$P = 2\int_{r_\mathrm{p}}^{r_\mathrm{a}} \frac{\mathrm{d}r}{\sqrt{\mathcal{R}[r]}} \quad \text{and} \quad K = \frac{1}{\pi}\int_{r_\mathrm{p}}^{r_\mathrm{a}} \mathrm{d}r \frac{\mathcal{S}[r]}{\sqrt{\mathcal{R}[r]}}$$

The precession of the periastron per orbital period is $\Delta\Phi = 2\pi(K-1)$.

For circular orbits

$$K^{\text{circ}} = 1 + 3x + (\cdots)x^2 + (\cdots)x^3 + (K^{(4)}_{\text{inst}} + K^{(4)}_{\text{tail}})x^4 + \mathcal{O}(x^5)$$

Comparison with the Hamiltonian formalism [DJS 2014,2015]

Comparison of the EOM at 4PN

- We find a disagreement with the ADM result at 4PN

$$a_1^i - (a_1^i)_{\rm DJS} = \frac{2}{15}\frac{G^4 m m_1 m_2^2}{c^8 r_{12}^5}\left[\frac{272}{9} v_{12}^i (n_{12}v_{12}) + n_{12}^i\left(-\frac{238}{3}(n_{12}v_{12})^2 + \frac{34}{3}v_{12}^2\right)\right],$$

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- No more discrepancy on the contribution of the non local part of the action to the energy for circular orbits,

$$E_{\rm tail} = -\frac{224}{15}(m_1+m_2)\nu^2 c^2\left[\ln(16x) + 2\gamma_{\rm E} + \frac{2}{7}\right]$$

- Still a small discrepancy $\Longrightarrow$ Can be solved by adding a second ambiguity parameter [Bernard et al., in preparation]

A second ambiguity parameter

A second ambiguity parameter β

- Introduce it in the acceleration,

$$\Delta a_1^i = \frac{2}{15} \frac{G^4 m m_1 m_2^2}{c^8 r_{12}^5} \Big[n_1(\alpha,\beta)\, v_{12}^i (n_{12} v_{12}) + n_{12}^i \Big(n_2(\alpha,\beta)\, (n_{12} v_{12})^2 + n_3(\alpha,\beta)\, v_{12}^2 \Big) \Big],$$

- The two ambiguity parameters α and β are fixed by comparison with the energy and periastron advance obtained by self-force calculations.

A second ambiguity parameter

A second ambiguity parameter β

- Introduce it in the acceleration,

$$\Delta a_1^i = \frac{2}{15}\frac{G^4 m m_1 m_2^2}{c^8 r_{12}^5}\Big[n_1(\alpha,\beta)\, v_{12}^i (n_{12}v_{12}) + n_{12}^i\Big(n_2(\alpha,\beta)\,(n_{12}v_{12})^2 + n_3(\alpha,\beta)\, v_{12}^2\Big)\Big],$$

- The two ambiguity parameters α and β are fixed by comparison with the energy and periastron advance obtained by self-force calculations.

Improve the IR regularization scheme

- So far : Hadamard regularization,
- Work in progress : dimensional regularization,
  - Origin of the second ambiguity parameter β,
  - Test of robustness of the regularisation process.

Summary & prospects

The 4PN dynamics

- We obtained the dynamics of compact binaries at 4PN using a Fokker Lagrangian, adapted to the post-Newtonian formalism.
- We compared it to previous results in the Hamiltonian formalism and found a discrepancy.
  - The introduction of a second ambiguity parameter can solve this discrepancy.
  - Our result gives the correct the energy and periastron advance for circular orbits (obtained by self force results).
- Computation of all the conserved quantities in harmonic coordinates from the Fokker action.

Prospects

- Use dimensional regularization for the IR divergences $\Longrightarrow$ meaning of the second ambiguity parameter.
- Complete the dynamics at 4.5PN including radiation reaction effect and determine the gravitational waveform at 4.5 PN.