Binary black hole coalescence: Analysis of the plunge

Sourabh Nampalliwar[1,2]

with Richard Price[2,3] and Gaurav Khanna[4]

[1]Fudan University, Shanghai

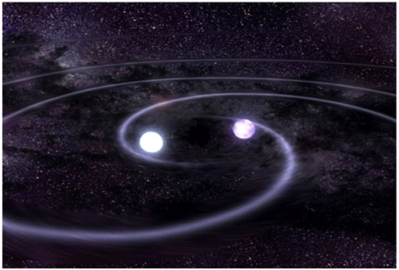
[2]U. Texas, Brownsville

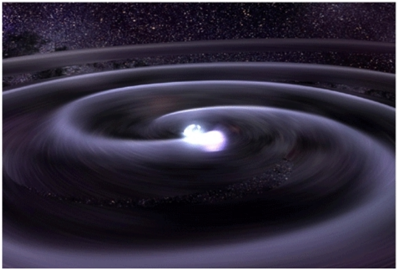
[3]Massachusetts Institute of Technology

[4]U. Mass., Dartmouth

August 30, 2016

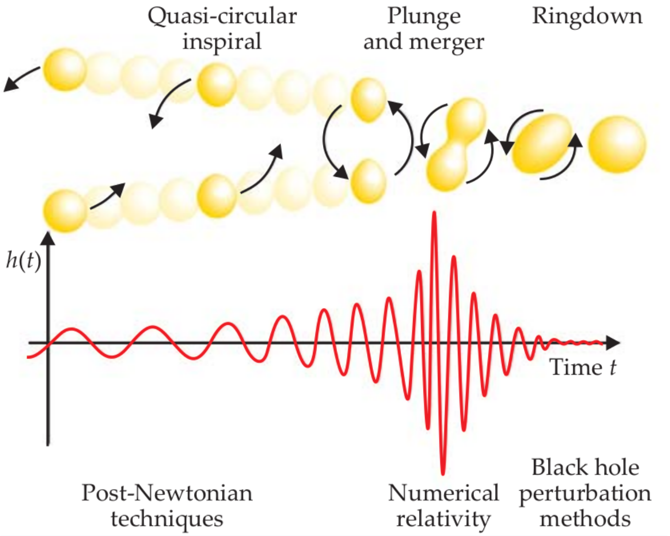
Gravitational wave sources





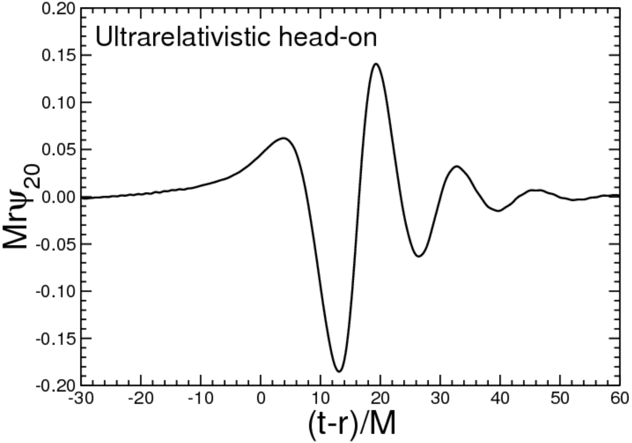
via LIGO

Binary black hole coalescence - the standard picture



via T. W. Baumgarte, S. L. Shapiro, Numerical Relativity, Cambridge U. Press, New York (2010)

Quasinormal ringing during the ringdown



via Berti et al., CQG 26 (2009) 163001

Binary black hole coalescence

- Goal: Understand plunge, in particular the excitation of QNR.
- Approach: Get insights, test insights.
- Principle: Simplify.

Modelling

Complex frequencies → FDGF → Linearized Einstein equations

Modelling

Complex frequencies → FDGF → Linearized Einstein equations

$$\Phi = \frac{1}{r} \sum_{l,m} \Psi_{lm}(r,t)\, Y_{lm}(\theta, \phi) \tag{1}$$

Modelling

Complex frequencies → FDGF → Linearized Einstein equations

$$\Phi = \frac{1}{r} \sum_{l,m} \Psi_{lm}(r,t)\, Y_{lm}(\theta, \phi) \tag{1}$$

$$-\frac{\partial^2 \Psi_{\ell m}}{\partial t^2} + \frac{\partial^2 \Psi_{\ell m}}{\partial x^2} - V_\ell(x)\Psi_{\ell m} = S_{\ell m}(x,t) \tag{2}$$

$$r \to r^* \to x \tag{3}$$

Modelling

Complex frequencies → FDGF → Linearized Einstein equations

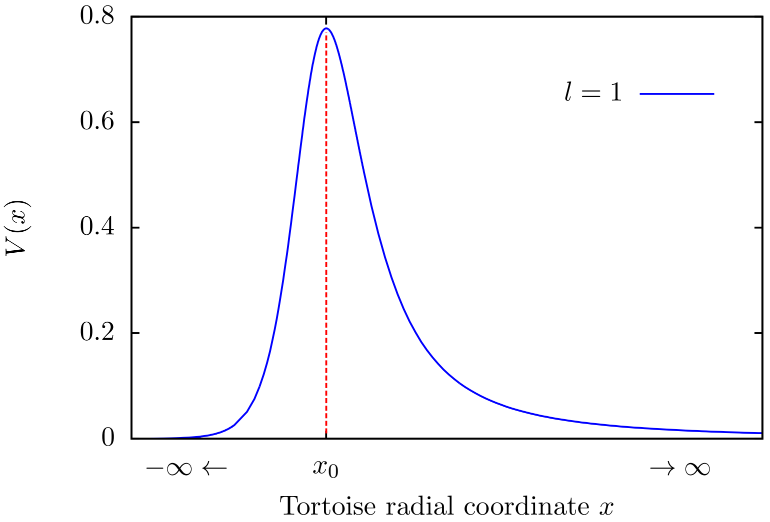
$$\Phi = \frac{1}{r} \sum_{l,m} \Psi_{lm}(r,t)\, Y_{lm}(\theta, \phi) \tag{1}$$

$$-\frac{\partial^2 \Psi_{\ell m}}{\partial t^2} + \frac{\partial^2 \Psi_{\ell m}}{\partial x^2} - V_\ell(x)\Psi_{\ell m} = S_{\ell m}(x,t) \tag{2}$$

$$r \to r^* \to x \tag{3}$$

$$\Psi(t,x) = \text{Integral over Green function} \tag{4}$$

Comparison - TDP vs Schwarzschild



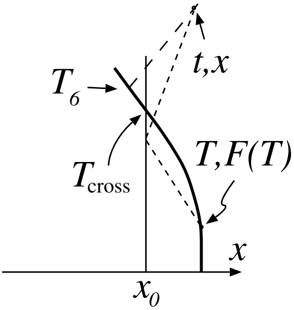
Analysis - QNR from radial infall in TDP

$$\Psi(t-x) = \frac{x_0}{2}\int_{T_{\rm cross}}^{T_6} e^{-\omega_d\theta}\left[\frac{1}{x_0}\left(\sin\omega_o\theta - \cos\omega_o\theta\right) - \frac{2}{x}\sin\omega_o\theta\right] dT$$
$$- \frac{1}{2}\int_{-\infty}^{T_{\rm cross}} e^{-\omega_d\xi}\left[\left(\frac{2x_0}{F(T)} - 1\right)\cos\omega_o\xi - \sin\omega_o\xi\right.$$
$$\left. - \frac{2x_0^2}{xF(T)}\left(\cos\omega_o\xi - \sin\omega_o\xi\right)\right] dT\,. \quad (5)$$

where

$$\theta = u - T + F(T), \qquad \xi = u - T - F(T) + 2x_0.$$

Analysis - QNR from radial infall in TDP



Analysis - QNR from radial infall in TDP

$$\Psi(u) = \frac{x_0}{2} \int_{T_{\rm cross}}^{T_6} e^{-\omega_d \theta} \left[\frac{1}{x_0} (\sin \omega_o \theta - \cos \omega_o \theta) - \frac{2}{x} \sin \omega_o \theta \right] dT$$
$$- \frac{1}{2} \int_{-\infty}^{T_{\rm cross}} e^{-\omega_d \xi} \left[\left(\frac{2x_0}{F(T)} - 1 \right) \cos \omega_o \xi - \sin \omega_o \xi - \frac{2x_0^2}{xF(T)} (\cos \omega_o \xi - \sin \omega_o \xi) \right] dT \,. \tag{6}$$

where

$$\theta = u - T + F(T), \qquad \xi = u - T - F(T) + 2x_0$$

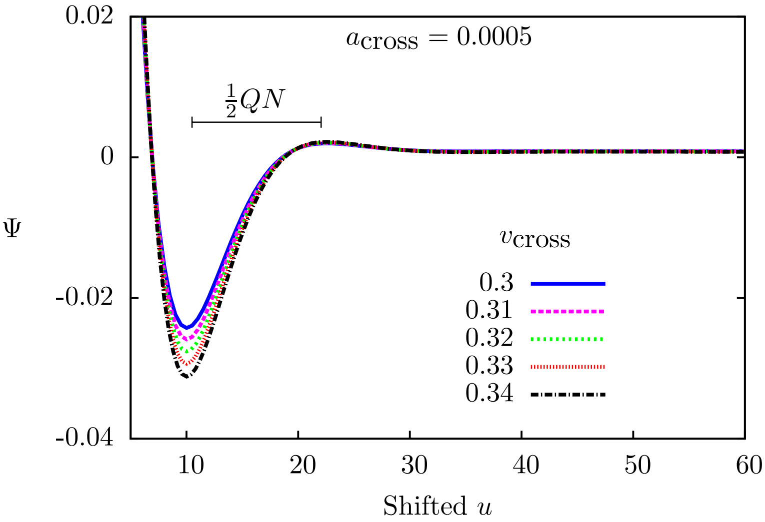
Analysis - QNR from radial infall in TDP

$$\Psi(u) = \frac{x_0}{2}\int_{T_{\rm cross}}^{T_6} e^{-\omega_d\theta}\left[\frac{1}{x_0}\left(\sin\omega_o\theta - \cos\omega_o\theta\right) - \frac{2}{x}\sin\omega_o\theta\right] dT$$
$$- \frac{1}{2}\int_{-\infty}^{T_{\rm cross}} e^{-\omega_d\xi}\left[\left(\frac{2x_0}{F(T)} - 1\right)\cos\omega_o\xi - \sin\omega_o\xi - \frac{2x_0^2}{xF(T)}\left(\cos\omega_o\xi - \sin\omega_o\xi\right)\right] dT\,. \qquad (7)$$

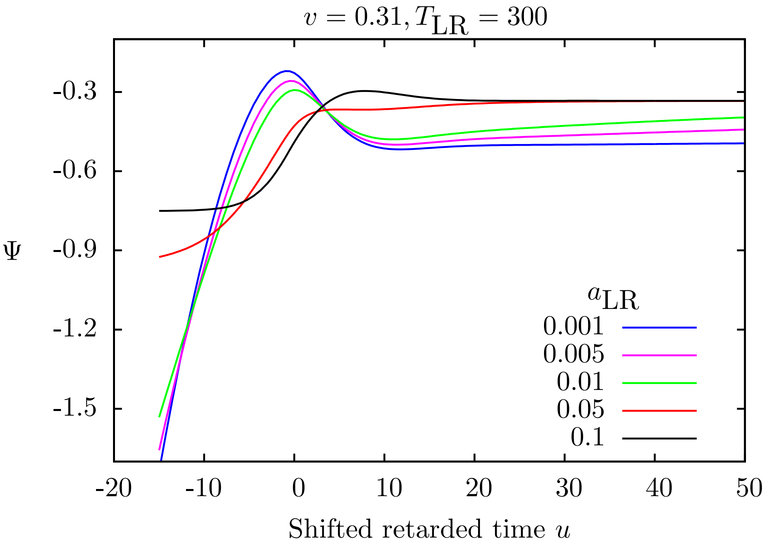
where

$$\theta = u - T + F(T), \qquad \xi = u - T - F(T) + 2x_0$$

Analysis - QNR from radial infall in TDP

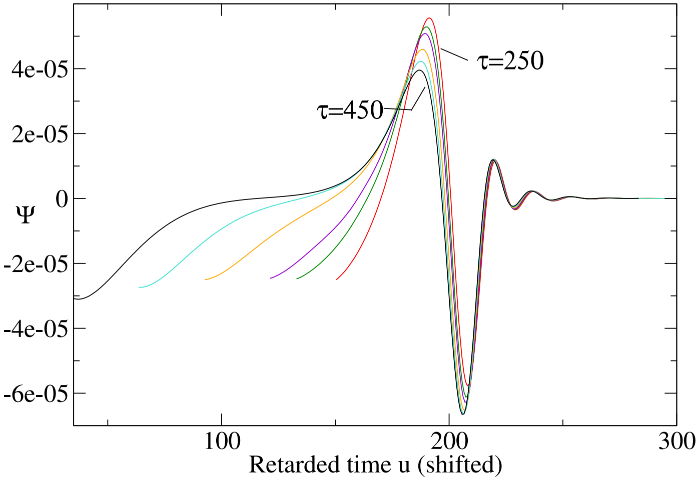


Analysis - QNR from radial infall in TDP



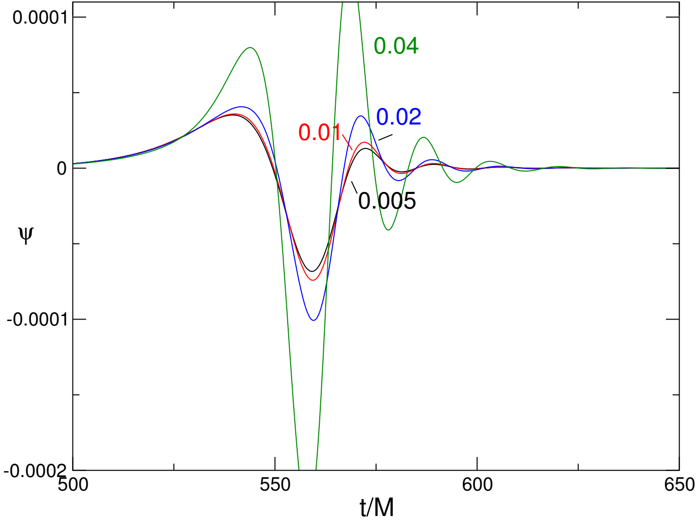
Analysis - QNR from radial infall in Schwarzschild

$$-\frac{\partial^2 \Psi_{\ell m}}{\partial t^2} + \frac{\partial^2 \Psi_{\ell m}}{\partial x^2} - V_\ell(x)\Psi_{\ell m} = S_{\ell m}(x,t) \tag{8}$$

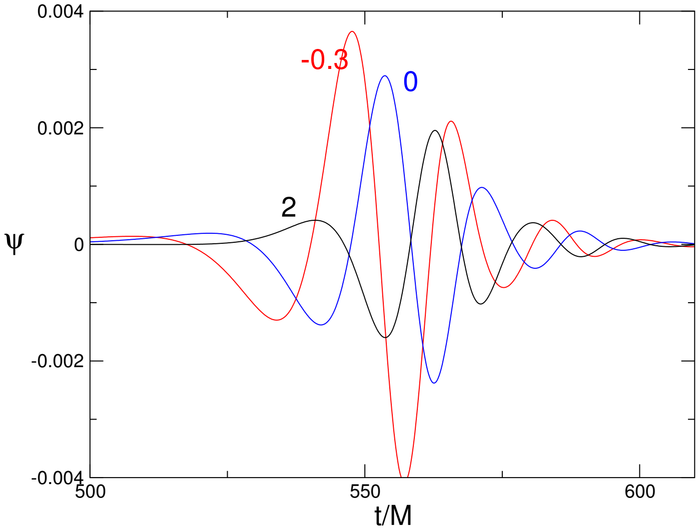


Data courtesy Gaurav Khanna

Analysis - QNR from orbital infall in Schwarzschild



Data courtesy Gaurav Khanna



Data courtesy Gaurav Khanna

Conclusion

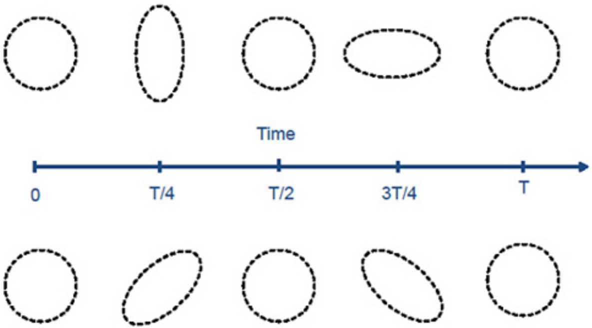
During a binary black hole coalescence, the transition from the inspiral phase to the ringdown phase is completely determined by the local properties at the photon orbit.

Thank you!

BONUS!

Gravitational waves

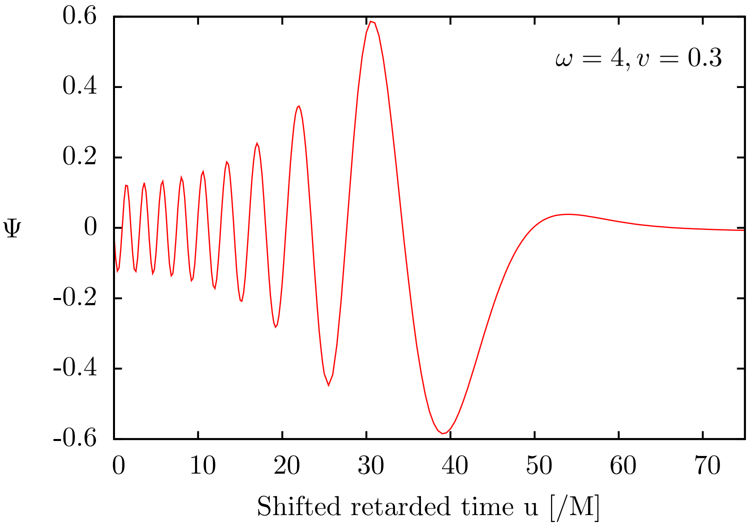
- Travel at the speed of light.
- Carry energy.



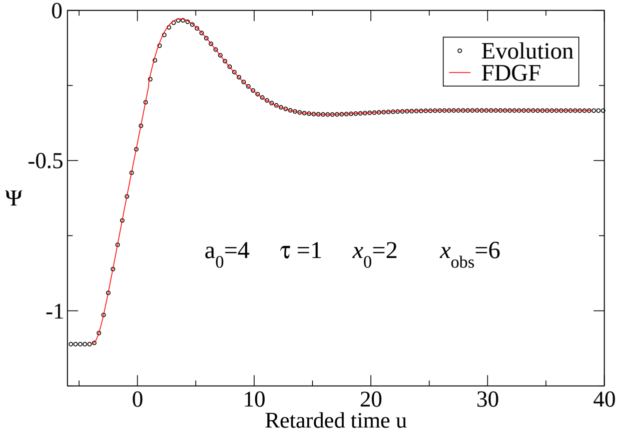
via learner.org

Quasinormal ringing

Analysis - QNR from orbital infall in TDP



Comparison - Evolution vs FDGF



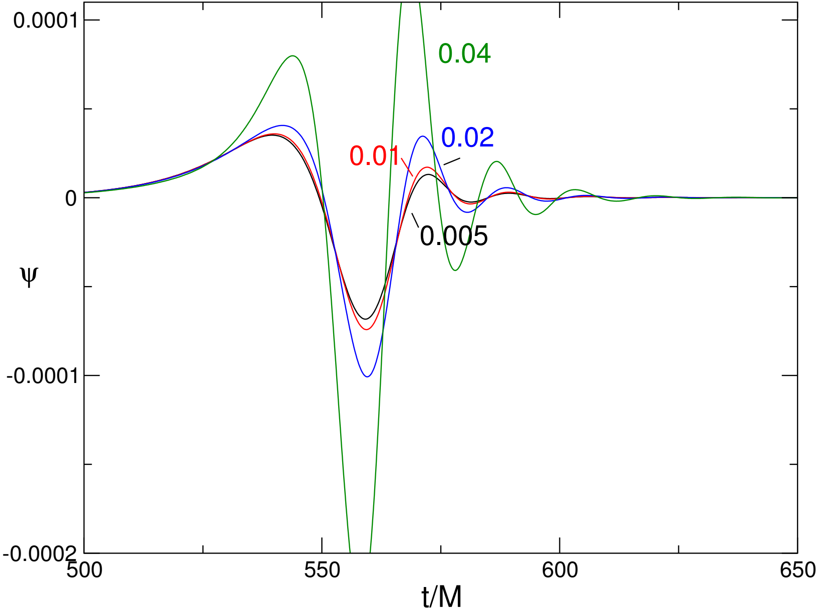
Evolution data courtesy Gaurav Khanna

Assumption - Trajectories: Orbital

- Redshift in angular velocity.
- $d\omega/dt = 0$ at the light ring.
- Speed of light should not be breached.

$$\omega(T) = \omega_{\rm LR}\frac{27M^2}{(1+\sigma)r(T)^2}\left(1-\frac{2M}{r(T)}\right)\left(1+\frac{3\sigma M}{r(T)}\right) \tag{9}$$

Analysis - QNR from orbital motion in Schwarzschild



Interesting features of orbital trajectories

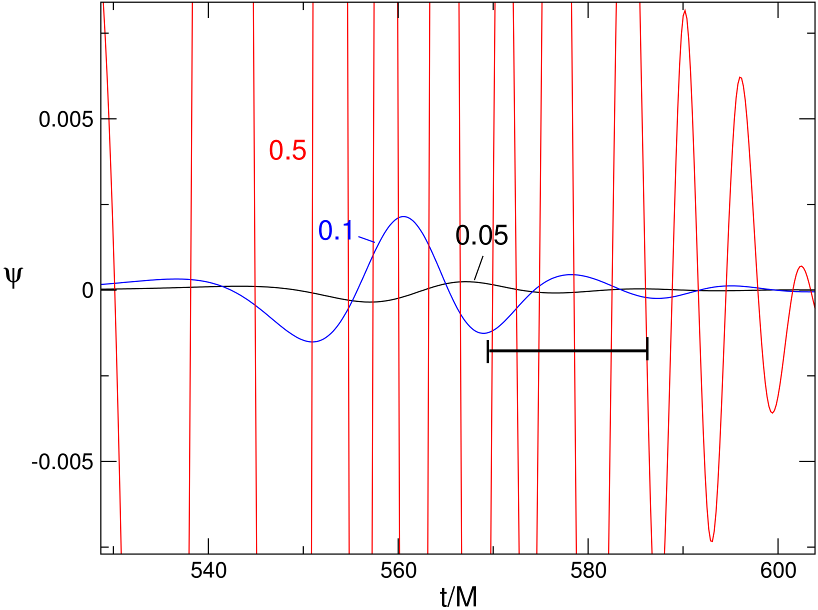
- Speed of light constraint:

$$\omega_{\rm LR}^2 M^2 < (1 - v_{\rm LR}^2)/27. \tag{10}$$

- Here this implies

$$\omega_{\rm LR} < 0.1836/M. \tag{11}$$

Direct radiation



Green's function

$$\frac{\partial^2 G}{\partial x^2} - \frac{\partial^2 G}{\partial t^2} - V(x)G = \delta(x-a)\delta(t-T) \tag{12}$$

$$\Psi(t,x) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(x,a;t-T)f(T)\delta(a-F[T])\,da\,dT \tag{13}$$

$$G(x,a;t-T) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\omega(t-T)}\mathcal{G}(x,a;\omega)d\omega\,, \tag{14}$$

$$\Psi(t,x) = \frac{1}{2\pi}\int\int\int_{-\infty}^{\infty} e^{-i\omega(t-T)}\mathcal{G}(x,a;\omega)\delta(a-F[T])\,d\omega\,da\,dT\,. \tag{15}$$

Assumption - Trajectories: Radial

- $dx/dt \to -1$ near the horizon.
- At least two parameters.
- Static initial data.

$$F(T) = \begin{cases} a_0 + \tau - \left(T^3 + \tau^3\right)^{1/3} & T \geq 0 \\ a_0 & T < 0\,. \end{cases} \tag{16}$$

Approximation - Truncated Multipole Potential

- Features of *curvature potential**

$$V_\ell = \begin{cases} \frac{\ell(\ell+1)}{x^2} & \text{for } r \gg 2M \\ \left(1 - \frac{2M}{r}\right) & \text{for } r \sim 2M \end{cases} \qquad (17)$$

*Cunningham, Price & Moncrief, ApJ, 224, 643-667 (1978)

Approximation - Truncated Multipole Potential

- Features of *curvature potential**

$$V_\ell = \begin{cases} \frac{\ell(\ell+1)}{x^2} & \text{for } r \gg 2M \\ \\ (1 - \frac{2M}{r}) & \text{for } r \sim 2M \end{cases} \tag{17}$$

- Truncated Dipole Potential (TDP)

$$V = \begin{cases} 2/x^2 & \text{for } x \geq x_0 \\ 0 & \text{for } x < x_0 \end{cases} \tag{18}$$

where x_0 is the light ring *aka* photon orbit *aka* UCO.

*Cunningham, Price & Moncrief, ApJ, 224, 643-667 (1978)