THE EFFECT OF MATTER PERTURBATIONS ON THE CHIRP SIGNAL

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Outline

- GW emitted by a binary propagate in the FLRW universe to the observer

- the expansion of the universe and the distribution of matter structure (galaxies, clusters) affect the GW propagation

- we identify three redshift-dependent effects on the chirp signal:
  - *time variation of the background expansion of the universe*
  - *time variation of the gravitational potential at the GW source*
  - <u>*time variation of the peculiar velocity of the GW source*</u>

- Earth based interferometers are not sensitive to these effects because they do not follow the GW source for enough time

- but these effects are <u>relevant for eLISA</u>: binaries with

low chirp-mass
low redshift (higher SNR)
that enter the detector at few mHz

Outline

- the peculiar acceleration of the binary introduces a term in the phase of the chirp signal depending on
    - *the amplitude of the acceleration*
    - *the redshift of the source*
    - *the frequency (effective -4PN term)*

- this introduces a bias on the parameter estimation of the binary:
    - *on the individual masses*
    - *on the time to coalescence*

- the latter can be important for binaries visible first in the eLISA band and then coalescing in the LIGO-Virgo band

- a new parameter representing the amplitude of this effect should be inserted in the parameter estimation analysis

Binary at cosmological distance: unperturbed redshift (usual case)

$$h_+(t_S) = \frac{4}{a(t_S)\,r} (GM_c)^{\frac{5}{3}} (\pi f_S)^{\frac{2}{3}} \frac{1+\cos^2 \imath}{2} \cos[\Phi_S(t_S)]$$

time at the source

scale factor at the source

chirp mass

frequency at the source

phase at the source

FLRW metric $$ds^2 = -dt^2 + a^2 \delta_{ij} dx^i dx^j$$

Binary at cosmological distance: unperturbed redshift (usual case)

$$h_+(t_S) = \frac{4}{a(t_S)\, r} (GM_c)^{\frac{5}{3}} (\pi f_S)^{\frac{2}{3}} \frac{1+\cos^2 \imath}{2} \cos[\Phi_S(t_S)]$$

time at the source

scale factor at the source

chirp mass

frequency at the source

phase at the source

if the redshift is constant during the time of observation of the GW signal

$$f_S = (1+z) f_O \qquad 1+z = \frac{a_O}{a_S} \qquad \tau_O = (1+z)\tau_S$$

Binary at cosmological distance: unperturbed redshift (usual case)

$$h_+(t_O) = \frac{4(G\mathcal{M}_c(z))^{\frac{5}{3}}}{d_L}(\pi f_O)^{\frac{2}{3}}\,\frac{1+\cos^2\imath}{2}\,\cos(\Phi_O)$$

time at the observer

luminosity distance

redshifted chirp mass

frequency at the observer

phase at the observer

frequency and phase at the observer:

$$f_O(\tau_O) = \frac{1}{\pi}\left(\frac{5}{256\tau_O}\right)^{3/8}(G\mathcal{M}_c)^{-5/8}$$

$$\Phi_O(\tau_O) = -2\left(\frac{\tau_O}{5G\mathcal{M}_c}\right)^{\frac{5}{8}} + \Phi_i$$

Binary at cosmological distance: varying redshift

relax the assumption that the redshift is constant
during the time of observation of the GW signal

1. the <u>background expansion</u> of the universe varies during the time of observation of the binary

2. the <u>redshift perturbations</u> due to the distribution of matter between the GW source and the observer vary in time during the time of observation of the binary

Binary at cosmological distance: varying redshift

relax the assumption that the redshift is constant
during the time of observation of the GW signal

first order scalar perturbations of the FLRW metric

$$ds^2 = -(1+2\psi)dt^2 + a^2(1-2\phi)\delta_{ij}dx^i dx^j$$

definition of the redshift $$1 + z = \frac{f_S}{f_O} = \frac{E_S}{E_O} = \frac{(k^\mu u_\mu)_S}{(k^\mu u_\mu)_O}$$

$$\frac{dk^\mu}{d\lambda} + \Gamma^\mu_{\alpha\beta}k^\alpha k^\beta = 0$$

$$u^\mu = \frac{1}{a}(1-\psi, \mathbf{v})$$

GW wave-vector

four velocity at source and observer

Binary at cosmological distance: varying redshift

relax the assumption that the redshift is constant
during the time of observation of the GW signal

first order scalar perturbations of the FLRW metric

$$ds^2 = -(1+2\psi)dt^2 + a^2(1-2\phi)\delta_{ij}dx^i dx^j$$

definition of the redshift $\quad 1+z = \frac{f_S}{f_O} = \frac{E_S}{E_O} = \frac{\left(k^\mu u_\mu\right)_S}{\left(k^\mu u_\mu\right)_O}$

$$1+z = \underbrace{\frac{a_O}{a_S}}_{\text{background expansion}}\left[1 + \underbrace{(\mathbf{v}_S - \mathbf{v}_O)\cdot\mathbf{n}}_{\text{Doppler term}} + \underbrace{\psi_O - \psi_S}_{\text{gravitational redshift}} - \underbrace{\int_{t_S}^{t_O} dt(\dot\phi + \dot\psi)}_{\text{integrated Sachs Wolfe}}\right]$$

Binary at cosmological distance: varying redshift

relax the assumption that the redshift is constant
during the time of observation of the GW signal

first order scalar perturbations of the FLRW metric

$$ds^2 = -(1+2\psi)dt^2 + a^2(1-2\phi)\delta_{ij}dx^i dx^j$$

definition of the redshift $\quad 1+z = \frac{f_S}{f_O} = \frac{E_S}{E_O} = \frac{\left(k^\mu u_\mu\right)_S}{\left(k^\mu u_\mu\right)_O}$

$$1+z = \frac{a_O}{a_S}\left[1 + (\mathbf{v}_S - \mathbf{v}_O)\cdot\mathbf{n} + \psi_O - \psi_S - \int_{t_S}^{t_O} dt(\dot{\phi}+\dot{\psi})\right]$$

$$= (1+\bar{z})(1+\delta z)$$

Binary at cosmological distance: varying redshift

the variation of the redshift perturbations during the time of observation of the binary introduces additional contributions in the frequency and the phase of the chirp signal with new time dependence

$$f_S = (1+z) f_O \qquad \tau_S = \frac{\tau_O}{1+\bar{z}} \left[1 - Y(z)\tau_O\right]$$

$$Y(z) = \frac{1}{2}\left(H_0 - \frac{H_S}{1+\bar{z}}\right) + \frac{1}{2}\left[\frac{\dot{\mathbf{v}}_S \cdot \mathbf{n}}{1+\bar{z}} - \dot{\mathbf{v}}_O \cdot \mathbf{n} + \frac{\dot{\phi}_S}{1+\bar{z}} - \dot{\phi}_O\right]$$

variation of the cosmological expansion during observation time

acceleration of the binary and the observer during observation time

time variation of the potentials during observation time

Binary at cosmological distance: varying redshift

the variation of the redshift perturbations during the time of observation of the binary introduces additional contributions in the frequency and the phase of the chirp signal with new time dependence

$$f(\tau_O) = \frac{1}{\pi}\left(\frac{5}{256\,\tau_O}\right)^{3/8} (G\mathcal{M}_c)^{-5/8} \left(1 + \frac{3}{8} Y(z)\tau_O\right)$$

$$\Phi_O(\tau_O) = -2\left(\frac{\tau_O}{5G\mathcal{M}_c}\right)^{5/8} \left(1 - \frac{5}{8}\, Y(z)\tau_O\right) + \Phi_i$$

$$h_c(\tau_O) = \frac{(G\mathcal{M}_c(z))^{5/4}}{d_L(z)}\left(\frac{5}{\tau_O}\right)^{1/4} \left(1 + \frac{1}{4}\, Y(z)\tau_O\right)$$

Modification of the waveform as a function of frequency

$$\tilde{h}_+(f) = A(f)\frac{1+\cos^2 \imath}{2}\,\exp(i\Psi(f))$$

$$\Psi(f) = \phi_c + 2\pi f t_c - \frac{\pi}{4} - \Phi_i + \frac{3}{128}\,(\pi G\mathcal{M}_c)^{-5/3}\,\frac{1}{f^{5/3}}$$

$$- \frac{25}{32768\,\pi}\,(\pi G\mathcal{M}_c)^{-10/3}\,\frac{Y(z)}{f^{13/3}} \longrightarrow$$ effectively -4PN

$$A(f) = \sqrt{\frac{5}{24\pi^{4/3}}}\,\frac{(G\mathcal{M}_c)^{5/6}}{d_L(z)}\,\frac{1}{f^{7/6}}\,\left[1 - \frac{5\,(G\mathcal{M}_c)^{-5/3}}{384\pi^{8/3}}\,\frac{Y(z)}{f^{8/3}}\right]$$

Estimate of the amplitude of Y(z)

$$Y(z) = \frac{1}{2}\left(H_0 - \frac{H_S}{1+\bar{z}}\right) + \frac{1}{2}\left[\frac{\dot{\mathbf{v}}_S\cdot\mathbf{n}}{1+\bar{z}} - \dot{\mathbf{v}}_O\cdot\mathbf{n} + \frac{\dot{\phi}_S}{1+\bar{z}} - \dot{\phi}_O\right]$$

variation of the cosmological expansion: depends only on the cosmology

peculiar acceleration of the binary: dominates over the effect due to cosmological expansion for realistic situations

subdominant

$\dot{\phi} \sim H_0 \phi \sim 10^{-5} H_0$

accounted for by eLISA motion

Estimate of the amplitude of Y(z)

$$Y(z) = \frac{1}{2}\left(H_0 - \frac{H_S}{1+\bar{z}}\right) + \frac{1}{2}\left[\frac{\dot{\mathbf{v}}_S \cdot \mathbf{n}}{1+\bar{z}} - \cancel{\dot{\mathbf{v}}_O \cdot \mathbf{n}} + \cancel{\frac{\dot{\phi}_S}{1+\bar{z}}} - \cancel{\dot{\phi}_O}\right]$$

variation of the cosmological expansion: depends only on the cosmology

$$2.4 \times 10^{-2}\,\epsilon\,\frac{H_0}{1+\bar{z}}$$

BINARY IN A GALAXY :

$$\epsilon = \left(\frac{v_S}{100\,\mathrm{km/s}}\right)^2 \frac{10\,\mathrm{kpc}}{r}\,\mathbf{e}\cdot\mathbf{n}$$

projected on the line of sight

rotation around the galactic centre

distance from the galactic centre

$\mathcal{O}(200)\,\mathrm{km/s}$

Estimate of the amplitude of Y(z)

$$Y(z) = \frac{1}{2}\left(H_0 - \frac{H_S}{1+\bar{z}}\right) + \frac{1}{2}\left[\frac{\dot{\mathbf{v}}_S \cdot \mathbf{n}}{1+\bar{z}} - \cancel{\dot{\mathbf{v}}_O \cdot \mathbf{n}} + \cancel{\frac{\dot{\phi}_S}{1+\bar{z}}} - \cancel{\dot{\phi}_O}\right]$$

variation of the cosmological expansion: depends only on the cosmology

$$2.4 \times 10^{-2}\, \epsilon\, \frac{H_0}{1+\bar{z}}$$

GALAXY IN A VIRIALISED CLUSTER :

$$\epsilon = \frac{3}{10}\left(\frac{v_S}{100\,\mathrm{km/s}}\right)^2 \frac{10\,\mathrm{kpc}}{r}\, \mathbf{e}\cdot\mathbf{n}$$

projected on the line of sight

velocity of the galaxy

distance from the cluster centre

$\mathcal{O}(2000)\,\mathrm{km/s}$

Estimate of the amplitude of the phase shift

$$\Phi_O(\tau_O) = -2\left(\frac{\tau_O}{5G\mathcal{M}_c}\right)^{5/8}\left(1-\frac{5}{8}Y(z)\tau_O\right)+\Phi_i$$

- for a typical eLISA binary observed up to coalescence

$$\Delta\Phi_{\mathrm{coal}} \simeq 3.96\cdot 10^{-5}\,\frac{Y(z)}{H_0}\left(\frac{5\cdot 10^3 M_\odot}{\mathcal{M}_c(z)}\right)^{\frac{10}{3}}\left(\frac{10^{-3}\mathrm{Hz}}{f_O}\right)^{\frac{13}{3}}$$

- for a typical eLISA binary observed for a finite time interval (smaller than the mission lifetime)

$$\Delta\Phi_{\Delta\mathrm{t}} \simeq 0.1\,\frac{Y(z)}{H_0}\left(\frac{50 M_\odot}{\mathcal{M}_c(z)}\right)^{\frac{5}{3}}\left(\frac{10^{-3}\mathrm{Hz}}{f_O}\right)^{\frac{5}{3}}\frac{\Delta t}{\mathrm{year}}$$

Estimate of the effect on GW detection

suppose we try to recover the binary parameters with a template not containing the acceleration effect (a waveform with Y=0)

$$\langle h_1 | h_2 \rangle \equiv 2 \int_0^\infty \frac{\tilde{h}_1(f)\tilde{h}_2^*(f) + \tilde{h}_2(f)\tilde{h}_1^*(f)}{S_n(f)}\, df$$

true signal with acceleration effect (simulated injections)

template at 3.5PN without acceleration effect

eLISA noise
N2A2M5L6

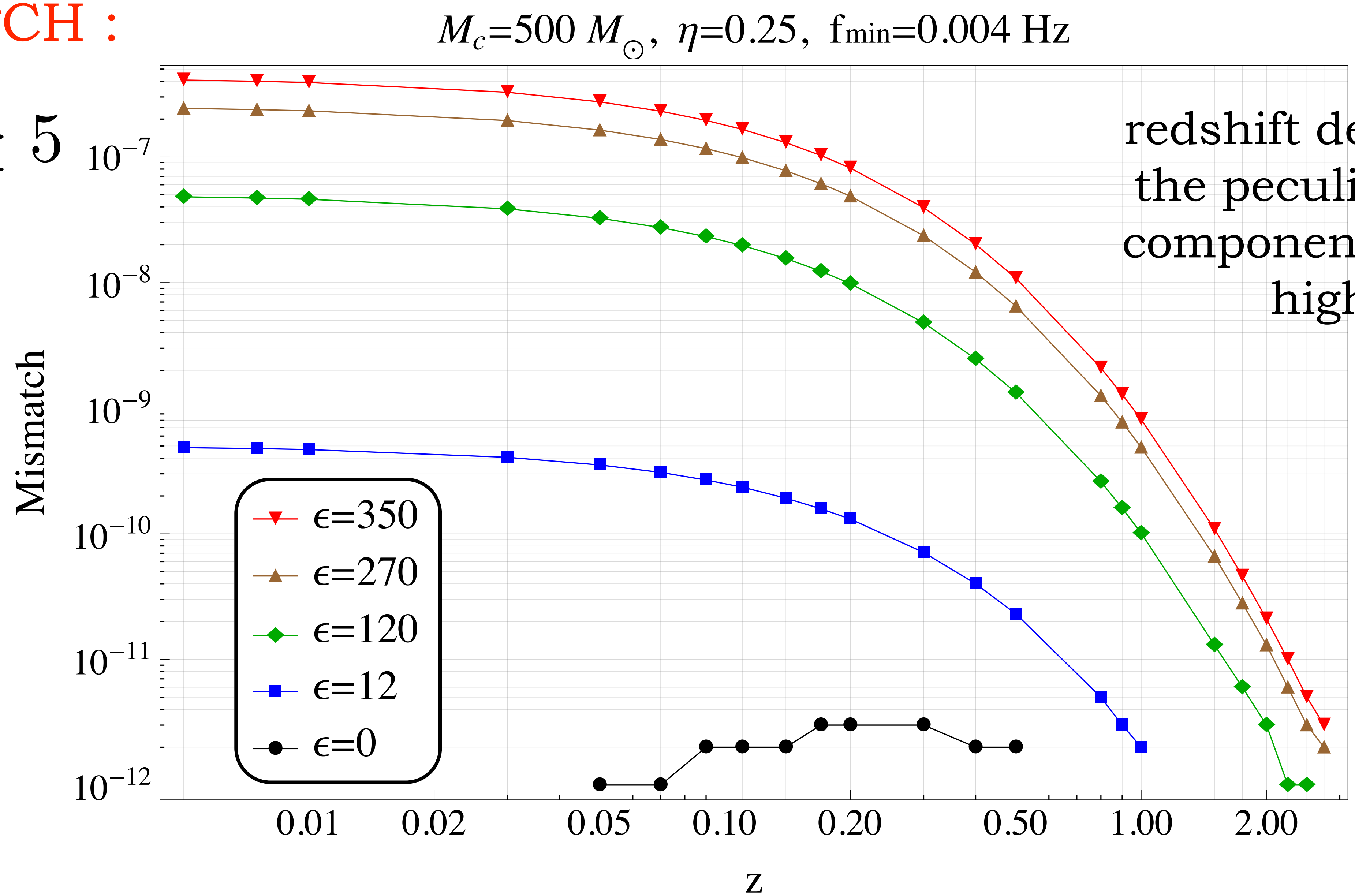
MISMATCH : $$m = 1 - \underset{\Delta t_c, \Delta\phi_c, \Delta M_c, \Delta\eta}{\mathrm{Max}} \frac{\langle h_1 | h_2 \rangle}{||h_1||\,||h_2||}$$

determine how many GW events will be lost and/or biased

Estimate of the effect on GW detection

MISMATCH :

$\mathrm{SNR} \geq 5$



redshift dependence from the peculiar acceleration component : power law at high redshift

$\epsilon = 12$

$\epsilon = 350$

$v_S = 2000\,\mathrm{km/s}$

$r = 100\,\mathrm{kpc}$

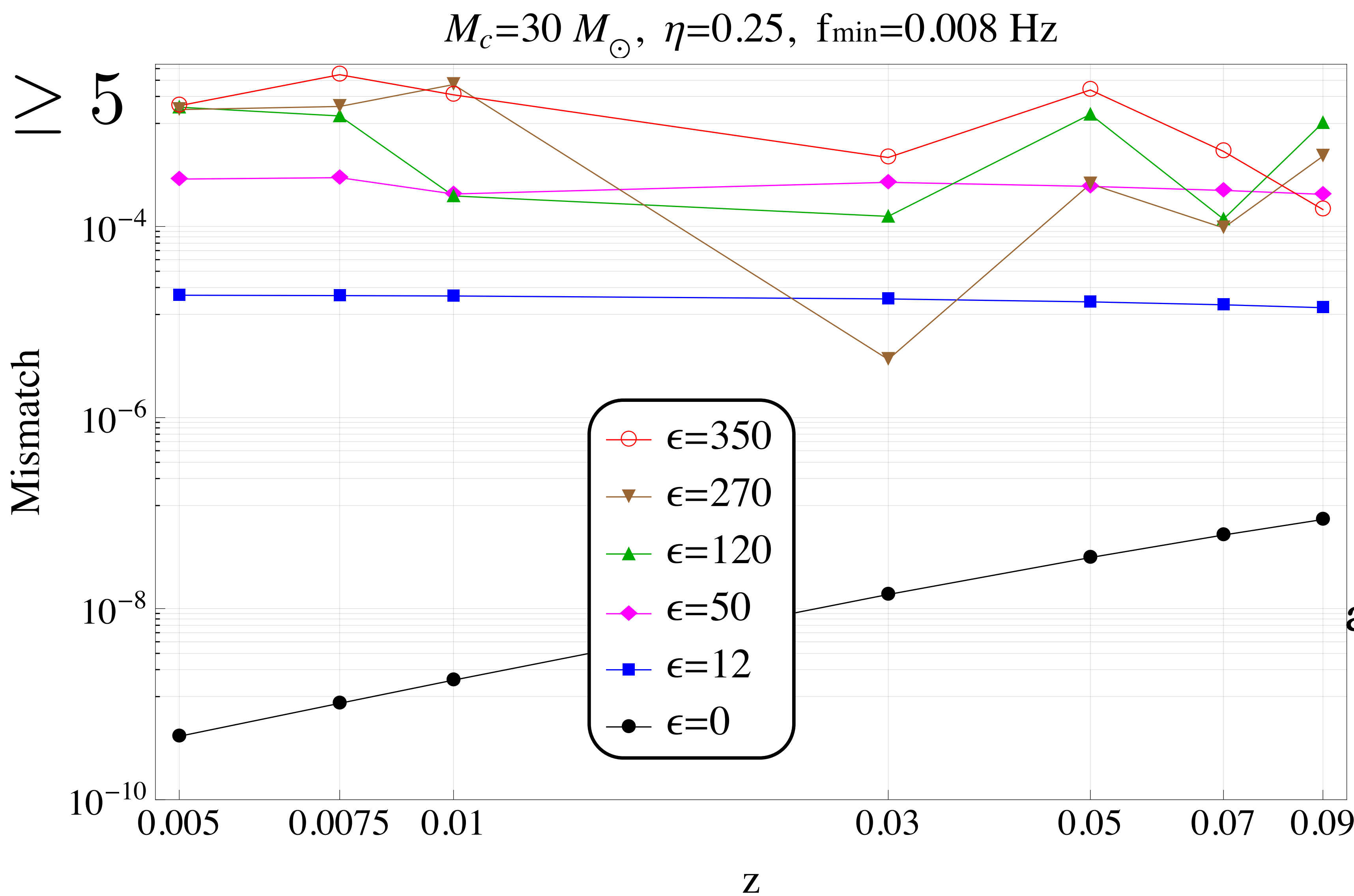
$v_S = 3000\,\mathrm{km/s}$ $+$ $v_S = 310\,\mathrm{km/s}$

$r = 10\,\mathrm{kpc}$ $\quad$ $r = 1.2\,\mathrm{kpc}$

Estimate of the effect on GW detection

MISMATCH :

$\mathrm{SNR} \geq 5$



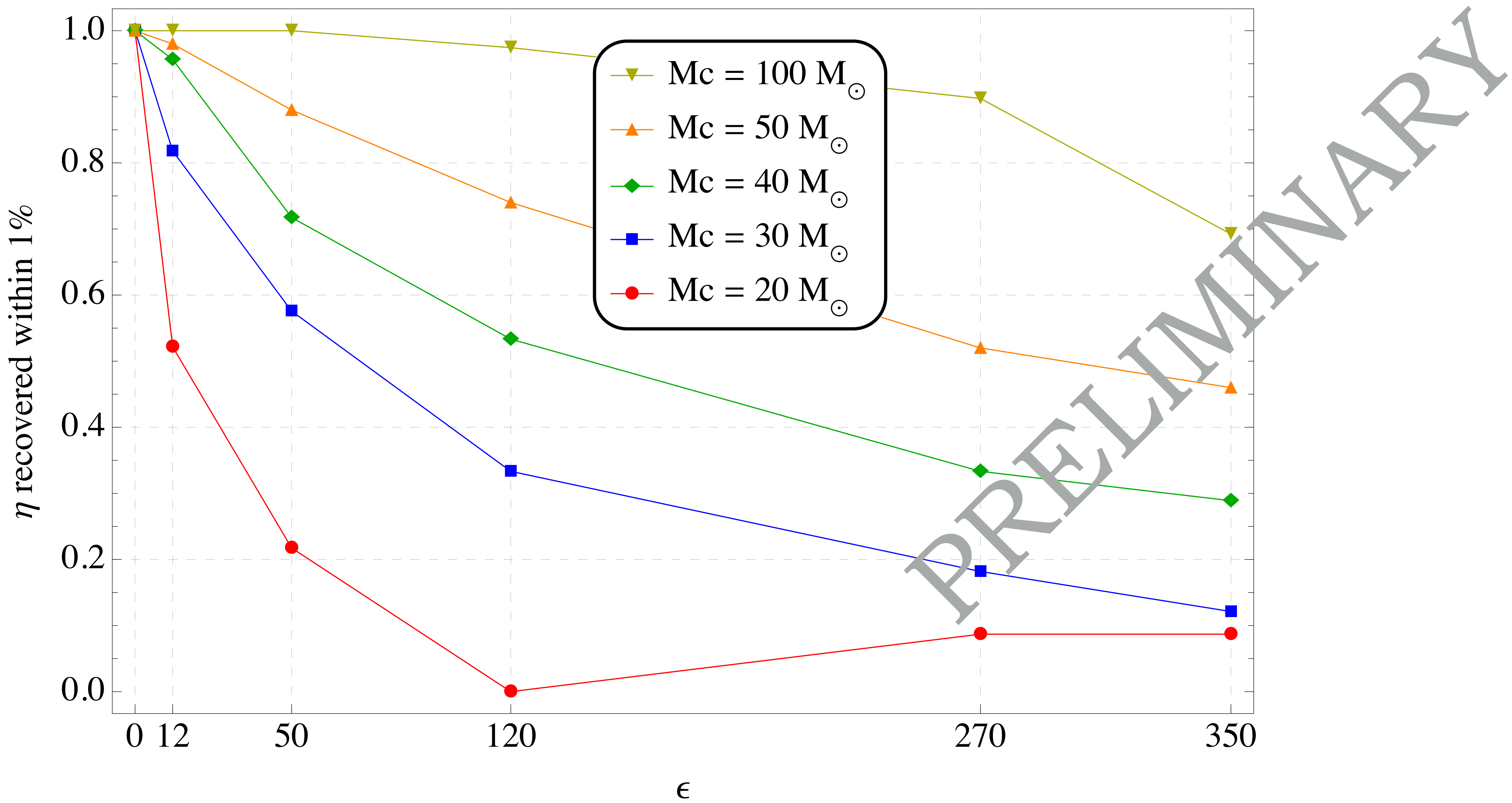
flat at low redshift (scatter at high ϵ due to numerics)

redshift dependence from the universe acceleration component

- the mismatch is small so there will be no loss of detections if one uses a usual 3.5PN template without the effect
- how much bias on the parameters is generated by ignoring the acceleration effect?

Estimate of the effect on GW detection

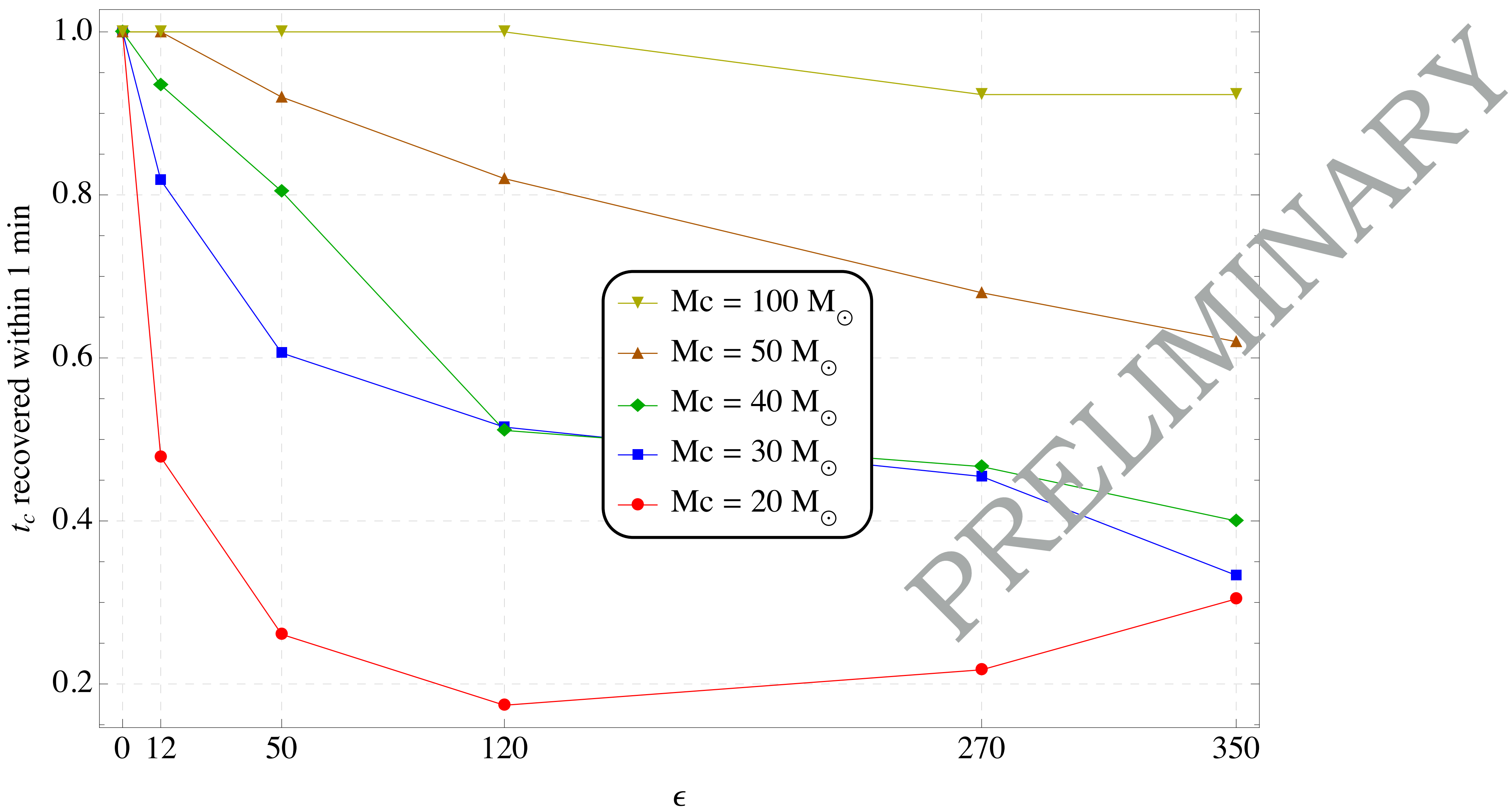
bias on the symmetric mass ratio



at low masses, a consistent fraction of events will be recovered with η wrong by more than 1%

Estimate of the effect on GW detection

bias on the time to coalescence



at low masses, a consistent fraction of events will be recovered with t_c wrong by more than 1min

Conclusions

- the chirp signal is affected by the fact that GW propagate in a dynamical, inhomogeneous universe
- in particular this is due to the evolution of the redshift perturbations during the time of observation of the binary
- the effect is not relevant for Earth-based detectors but for typical eLISA sources that stay in band for enough time: those with low masses
- in the phase of the waveform the frequency dependence of the effect corresponds to -4PN, with unknown amplitude
- the dominant among the components entering the amplitude is the peculiar acceleration of the binary

Conclusions

- we have analyzed the error one would do by ignoring the peculiar acceleration effect in the analysis of GW signals in eLISA with configuration N2A2M5L6
- the new term in the phase due to the peculiar acceleration effect does not cause significant loss of detections
- however, the recovered parameters of the GW signals are biased by more than the forecasted eLISA precision
- in particular the time to coalescence
  (maybe relevant for those binaries that are first visible in eLISA and then in LIGO/Virgo)
- we believe this effect should in principle be included in the search templates for eLISA for low mass binaries