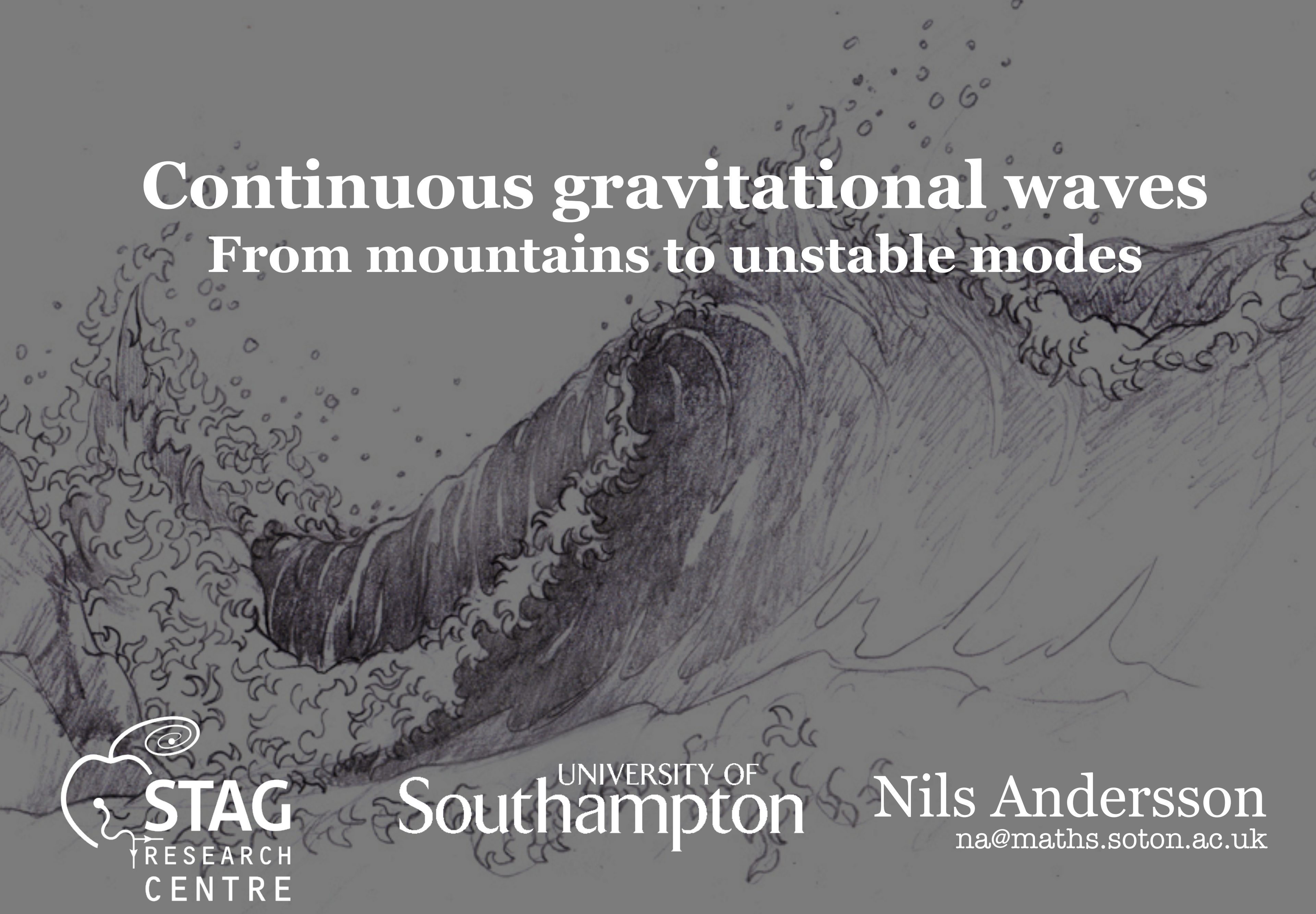
Continuous gravitational waves

From mountains to unstable modes



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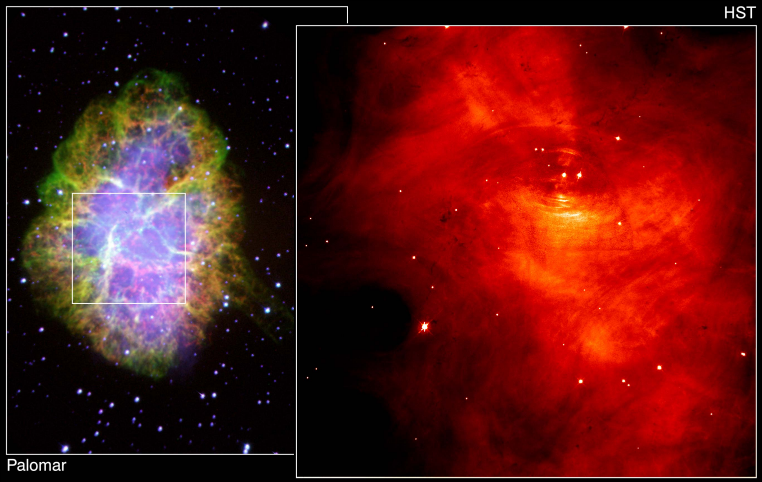
a star is born

A neutron star is born when a medium-weight star runs out of nuclear fuel and collapses under its own weight.

Once the dust from the supernova explosion has settled the remnant can usually be seen as a radio pulsar.

In addition, there are accreting neutron stars in LMXBs, strongly magnetized magnetars, isolated neutron stars...



Crab Nebula
Hubble Space Telescope • Wide Field Planetary Camera 2

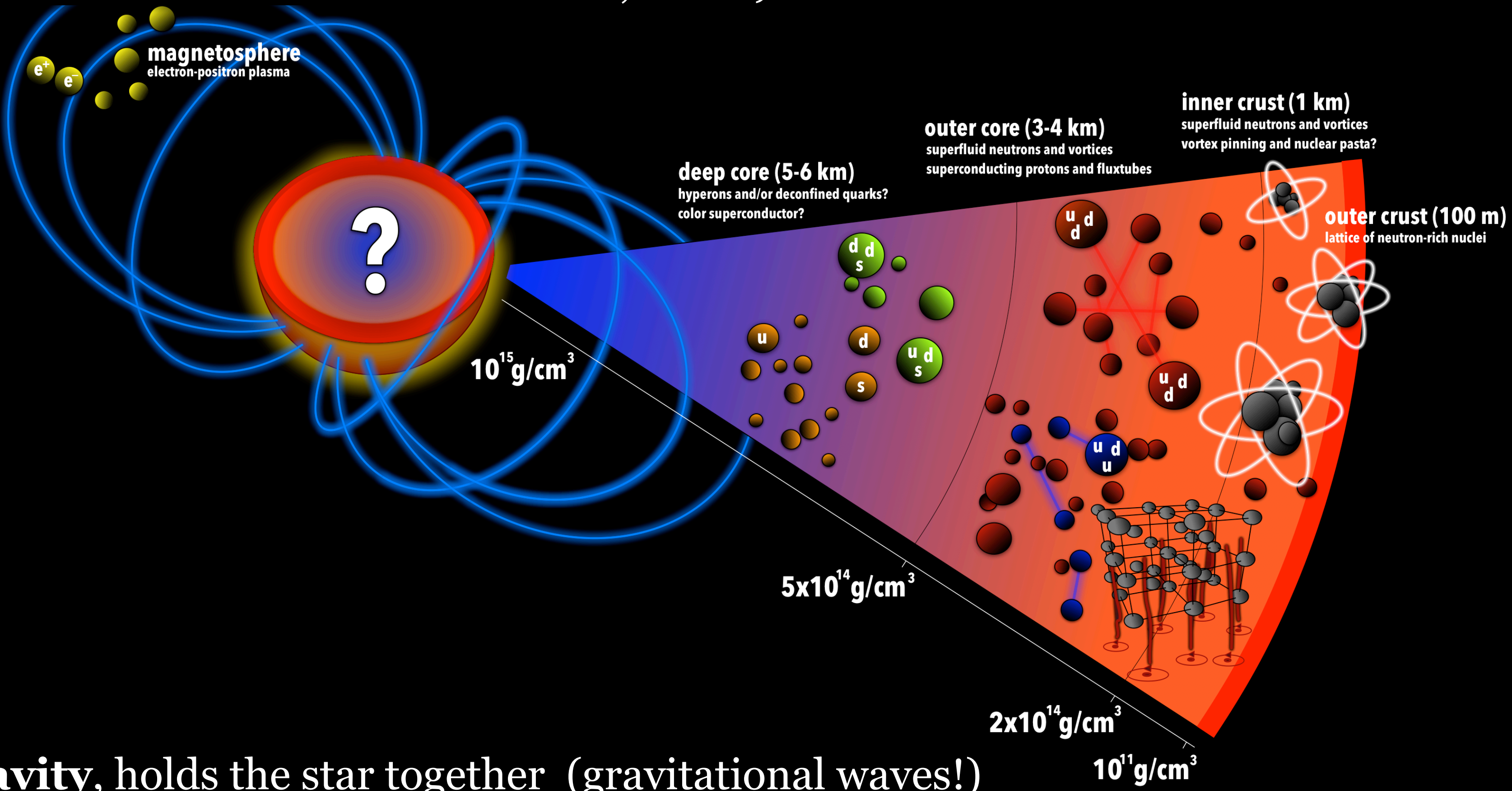
These systems exhibit a rich phenomenology.

With a solar mass compressed inside a radius of about 10 km, a neutron star represents extreme physics that cannot be tested in the laboratory.

In this talk I will discuss (some of) the ways gravitational-wave observations may help us probe this physics.

The problem is difficult because;

i) these are “hands-off” laboratories, that ii) involve all four fundamental forces



Gravity, holds the star together (gravitational waves!)

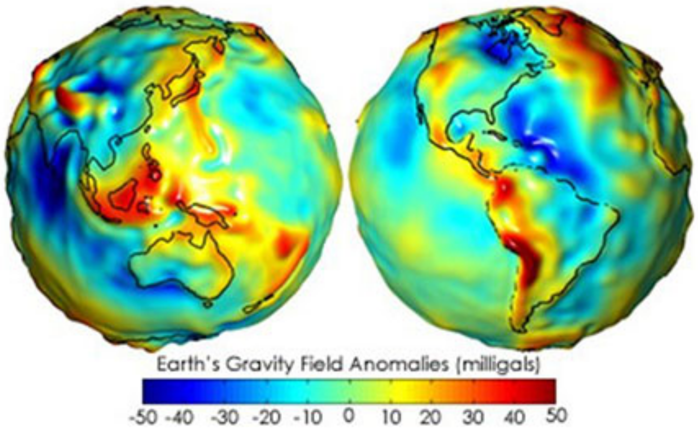
Electromagnetism, makes pulsars pulse and magnetars flare

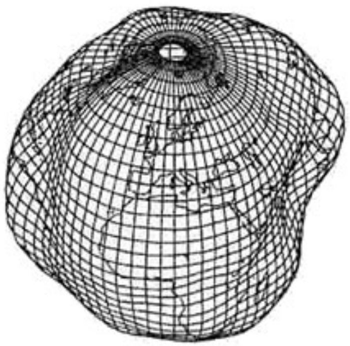
Strong interaction, determines the internal composition

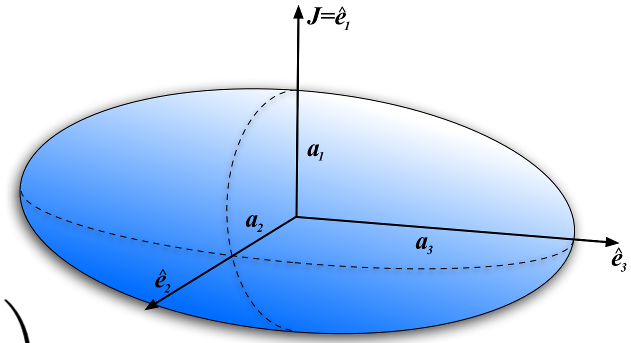
Weak interaction, affects reaction rates - cooling and internal viscosity

“mountains”

In principle, any rotating deformed body (like the Earth!) radiates gravitational waves.







For a triaxial star rotating steadily we have;

$$h = 3 \times 10^{-28} \left(\frac{\epsilon}{10^{-6}}\right) \left(\frac{f_{\rm spin}}{10\,{\rm Hz}}\right)^2 \left(\frac{1\,{\rm kpc}}{r}\right)$$

where ε represents the (dimensionless) asymmetry in the moment of inertia tensor.

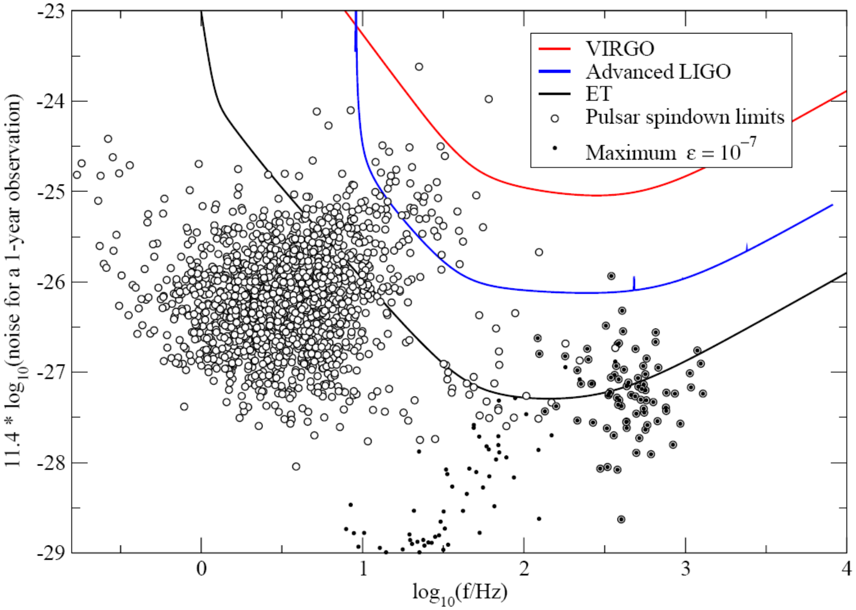
The signal is weak, but the effective amplitude improves as the square-root of the observation time. Requires long observation time, but many objects have known frequency and position.

Optimist:

Assume all observed pulsar spin-down is due to gravitational waves.

Realist:

Worry about the physics (braking index!). What level of asymmetry can neutron stars sustain?



[from DI Jones]

Asymmetry may be due to:

- Elastic strain in solid phase(s);

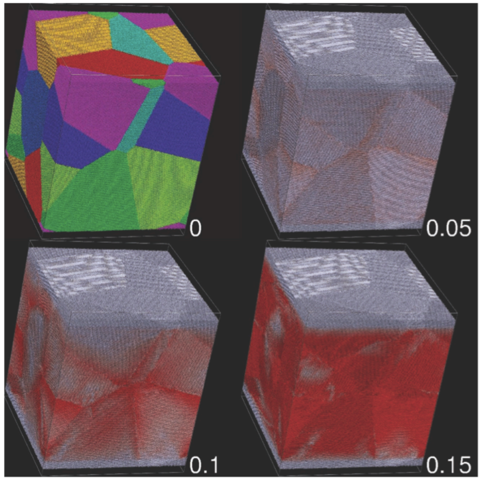
$$\epsilon \approx \frac{\mu V_{\rm crust}}{GM^2/R} \times u_{\rm break} \approx 10^{-6}\left(\frac{u_{\rm break}}{10^{-1}}\right)$$

- Magnetic stresses in the star’s interior;

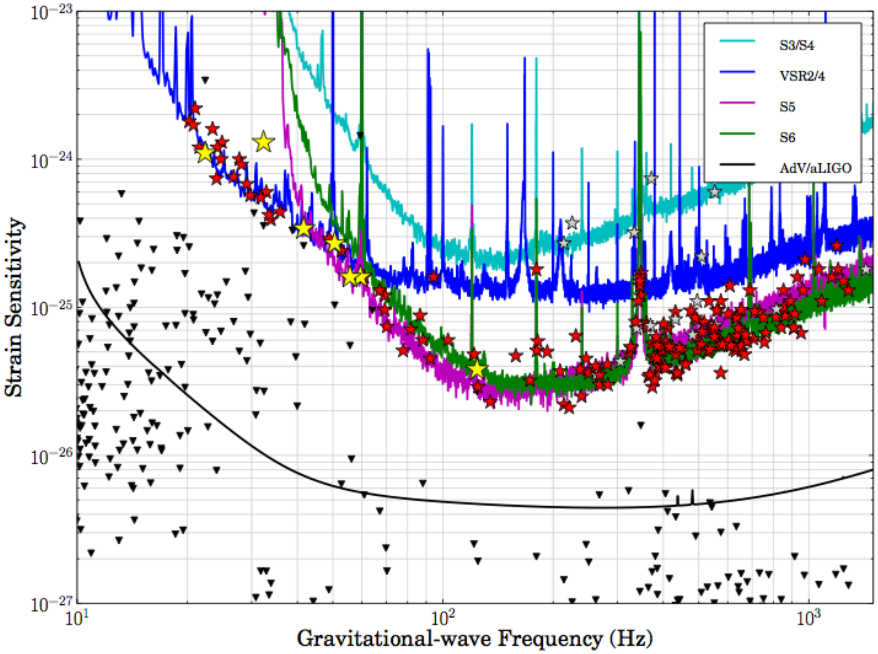
$$\epsilon \sim \frac{\int B^2\,dV}{GM^2/R} \sim 10^{-12}\left(\frac{B}{10^{12}\,{\rm G}}\right)^2$$

Molecular dynamics simulations suggest that the breaking strain u_{break} is larger than expected, around 0.1. The crust is super-strong!

Key questions: Do real neutron star mountains reach breaking strain or are stresses released gradually through plastic flow? How does the crust “yield”?



[Horowitz and Kadau 2009]



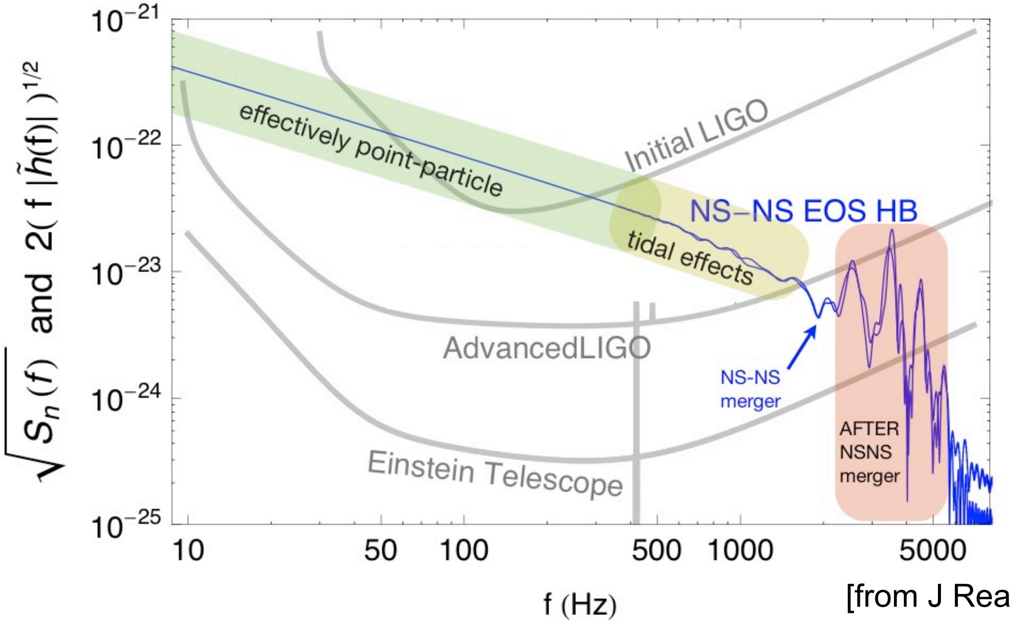
[Aasi et al 2014]

Upper limits (targeted observations):

- Crab pulsar: $\varepsilon < 10^{-4}$, less than 1% of energy loss going into GWs
- Vela pulsar: $\varepsilon < 6 \times 10^{-4}$, less than 10% of energy loss going into GWs
- Strongest constraint $\varepsilon < 6.7 \times 10^{-8}$ for the 200Hz pulsar J2124-3358 (astonishing symmetry?)

tidal deformation

The deviation from point-mass dynamics becomes important at the late stages of binary inspiral.



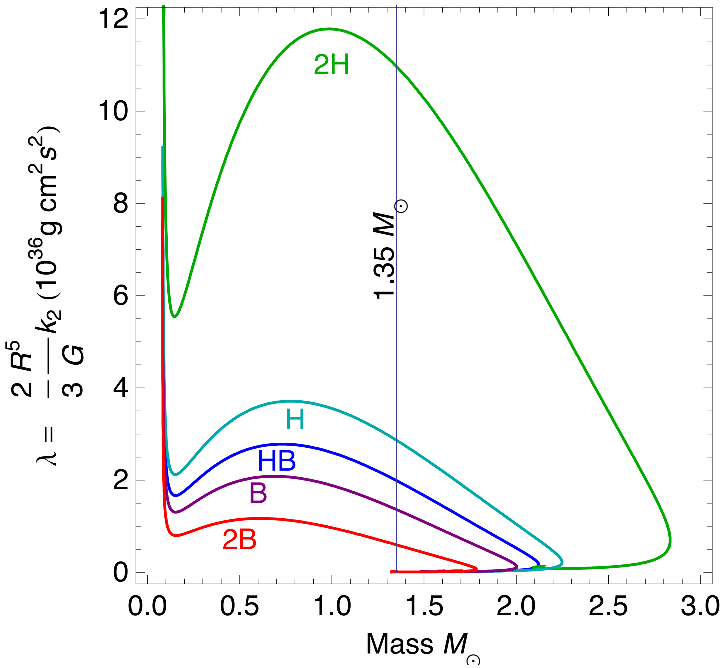
[from J Read]

Difficult to alter GW phasing (e.g. 10^{46} erg at 100 Hz leads to shift of 10^{-3} radians), but the star's deformability, encoded in the so-called Love number, may lead to a distinguishable secular effect;

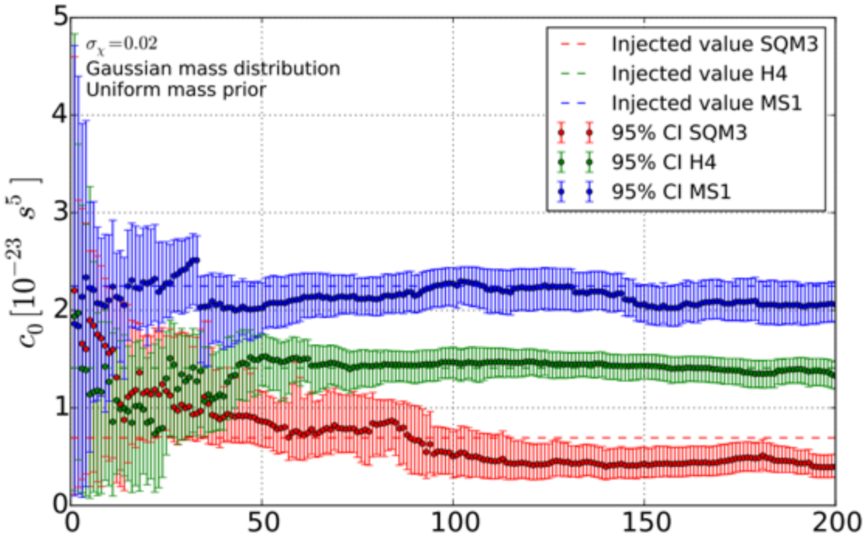
$$\lambda = \frac{2}{3}k_2R^5 = \frac{Q}{E} = \frac{\text{quadrupole deformation}}{\text{strength of tidal field}}$$

Given that the masses can be extracted from the chirp, observations may allow us to constrain the compressibility of the equation of state.

Note: If the radius can be inferred with an accuracy <5%, we would do better than any upcoming nuclear physics experiments.



[Read et al 2013]



[Agathos et al 2015]

However, detecting the tidal effect from individual events may be difficult (unless we are very lucky).

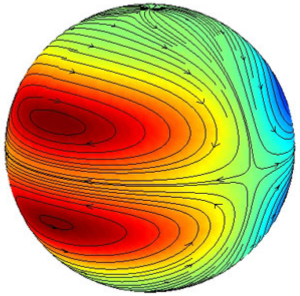
Will likely need a number of detections to distinguish different equations of state.

seismology

Neutron stars have rich oscillation spectra, with families of modes more or less directly associated with different core physics (cf. Helioseismology).

f-mode:	scales with average density, and is the most effective GW emitter.
p-modes:	acoustic modes, depend on sound speed.
g-modes:	depend on thermal/composition gradients. Instability in hot star may trigger convection.
w-modes:	pure spacetime oscillations.
r-modes:	inertial mode restored by the Coriolis force. Radiates mainly through current multipoles. Driven unstable by GW emission!

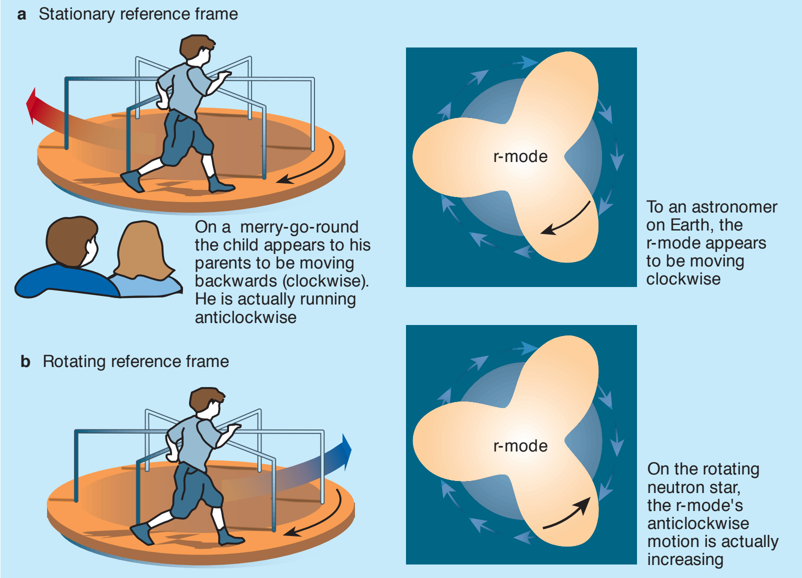
Observations **would** constrain the theory.



the CFS instability

Gravitational waves may drive an instability in rotating relativistic stars.

Interesting because the mechanism may limit the spin of neutron stars at the same time as it generates a detectable signal.



Cartoon explanation: A given mode is unstable if the star is losing "negative energy".

A "neutral" mode of oscillation signals the onset of instability.

The modes that are thought to be the most important are the "acoustic" f-modes, and the "Coriolis driven" r-modes.

Instability windows depend sensitively on uncertain physics. Simplest models involve shear- and bulk viscosity.

Key point: This problem probes non-equilibrium properties of matter.

the r-modes

The *l*=*m*=2 r-mode grows (due to current multipole radiation) on a timescale

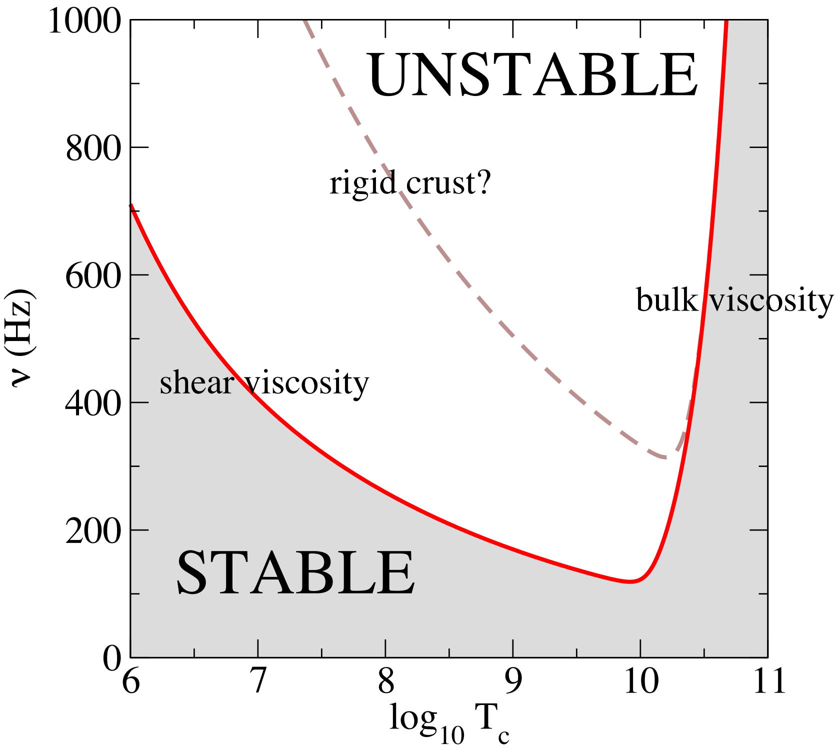
$$t_{\rm gw} \approx 50 M_{1.4}^{-1} R_{10}^{-4} P_{-3}^{6} \ {\rm s}$$

Viscosity may stabilise the star. At low temperature, shear viscosity is expected to dominate. For nn scattering we have

$$t_{sv} \approx 7\times 10^{7} M_{1.4}^{-5/4} R_{10}^{23/4} T_{9}^{2} \ {\rm s}$$

Bulk viscosity is important at high temperatures (requires density perturbation which arises at second order in Ω)

$$t_{bv} \approx 3\times 10^{11} M_{1.4} R_{10}^{-1} P_{-3}^{2} T_{9}^{-6} \ {\rm s}$$



In principle, we should not find any (normal) pulsars inside the instability window. Best constraint (?): The x-ray pulsar J0537-6910 would have been born with a period in the range 6-9 ms.

Phenomenology:

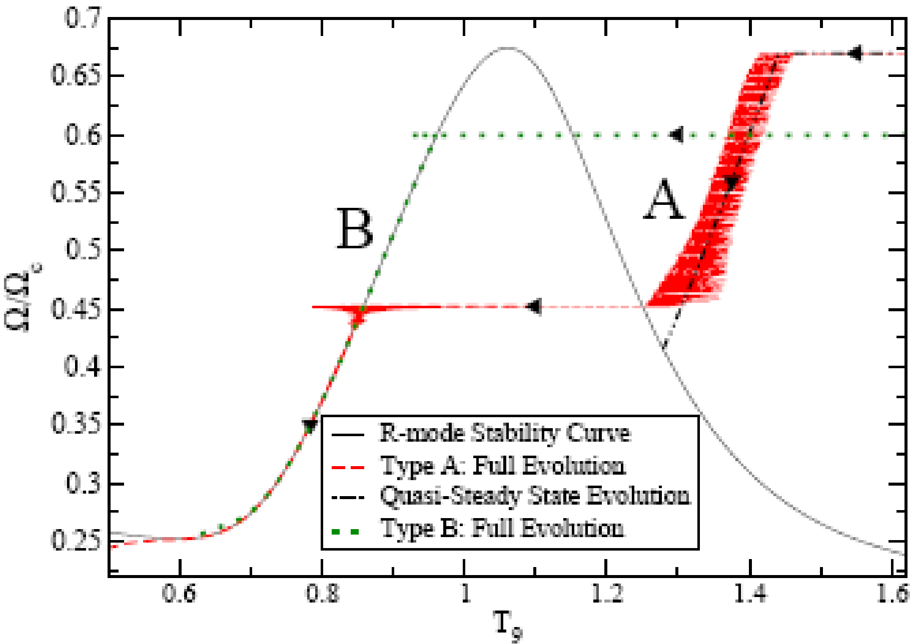
At some point the r-mode “saturates” and, as angular momentum is radiated away, the star spins down.

Model using Ω and α and use evolution equations from the conservation of energy and angular momentum:

$$\delta v \propto \alpha \Omega R$$

Nonlinear mode-coupling limits the r-mode amplitude to

$$\alpha < 10^{-3}$$



[Bondarescu et al 2009]

Good news: In many cases, a relatively simple “three-mode network” provides a good description.

Also, the general evolution can be understood from stationary solutions (cooling balance heating etc).

Bad news: The amplitude may not be slowly varying, so it could be difficult to detect the gravitational waves.

Cassiopeia A

Estimates suggest that the r-mode signal from newly born neutron stars may only be detectable out to 1 kpc-1 Mpc, meaning that the event rate is likely to be low.

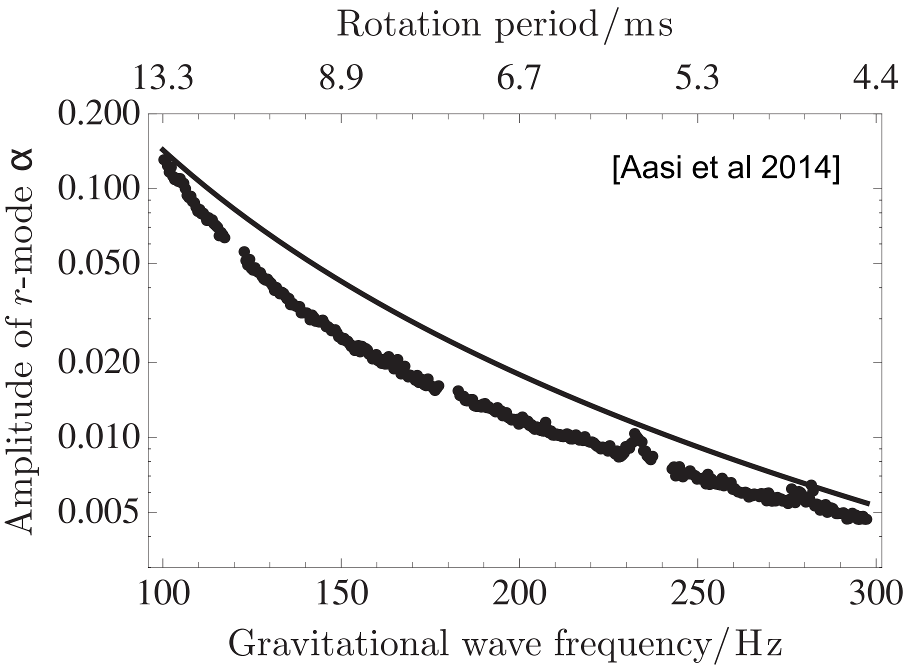
However, if the evolution is slow, then the instability may be active for a long time.

The Cassiopeia A remnant hosts the youngest neutron star in the galaxy (roughly 300 yrs).

LIGO searched 12 days of S5 data for periodic gravitational waves from this neutron star.

Searches are complicated by the fact that the spin rate of the star is unknown.

Still, the results provide the first observational upper limit for the r-mode amplitude.



With a 1 yr aLIGO observation these constraints may improve by a factor of 50 or so. This would begin to probe the "interesting" regime.

LMXBs

Accreting neutron stars in LMXBs may be more “promising”. Observations suggest these systems rotate well below the break-up limit, so some kind of speed-limit may be enforced.

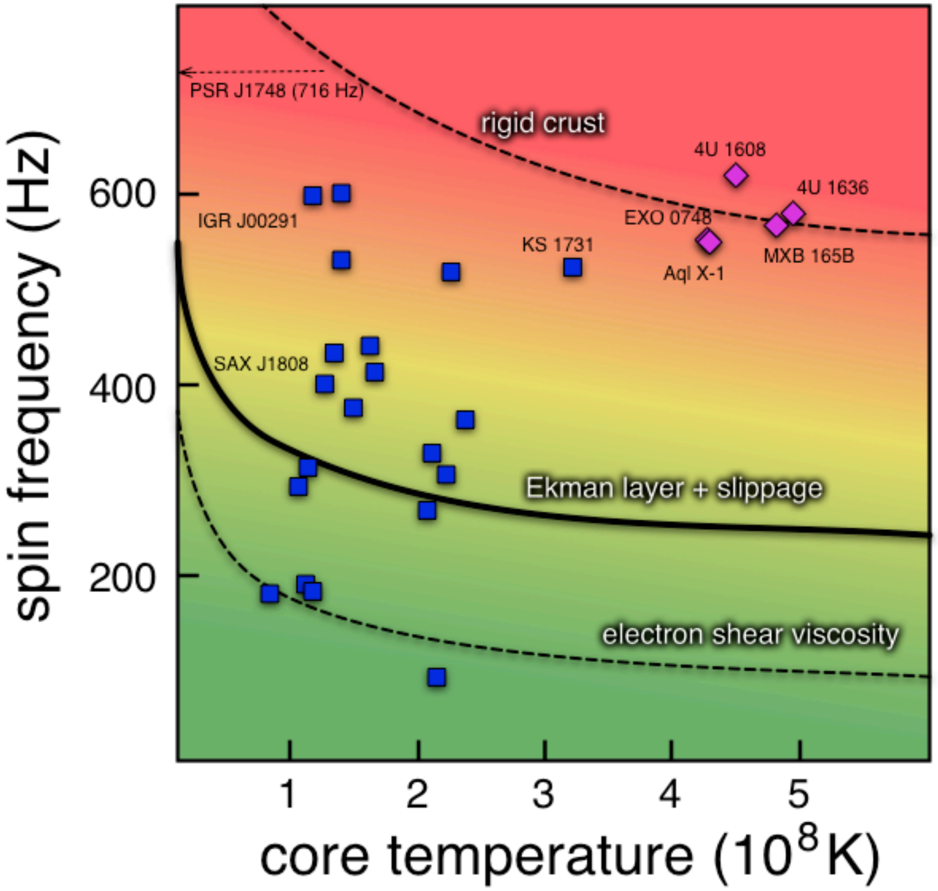
The r-modes could provide an explanation. Many systems lie inside the “conservative” instability window.

Rigid crust with viscous (Ekman) boundary layer would lead to sufficient damping…

…but the crust is more like jelly, so the effect is reduced (“slippage”).

Saturation amplitude due to mode-coupling is too large to allow evolution far into instability region.

Note: No evidence in the data for “clustering” near an instability threshold.



[Ho, NA & Haskell 2011]

issues

Neutron stars are promising GW sources, but the generation mechanisms tend to involve unknown physics aspects.

1. How do (quadrupole) deformations form/evolve? Is plastic flow important?
2. Accreting systems may seem promising, but how do we track the long-term spin-evolution?
3. Are issues like composition (e.g. leading to g-modes) relevant for the tidal problem?

The r-mode remains a "viable" GW source and may be the mechanism that limits neutron star spin. Instability window depends on core physics (composition/state of matter/transport coefficients).

1. Are the r-modes unstable in a realistic neutron star model (magnetic field)?
2. Why does the growth of an unstable mode saturate and what is the achieved amplitude?
3. How does a star with an active instability actually evolve (differential rotation)?