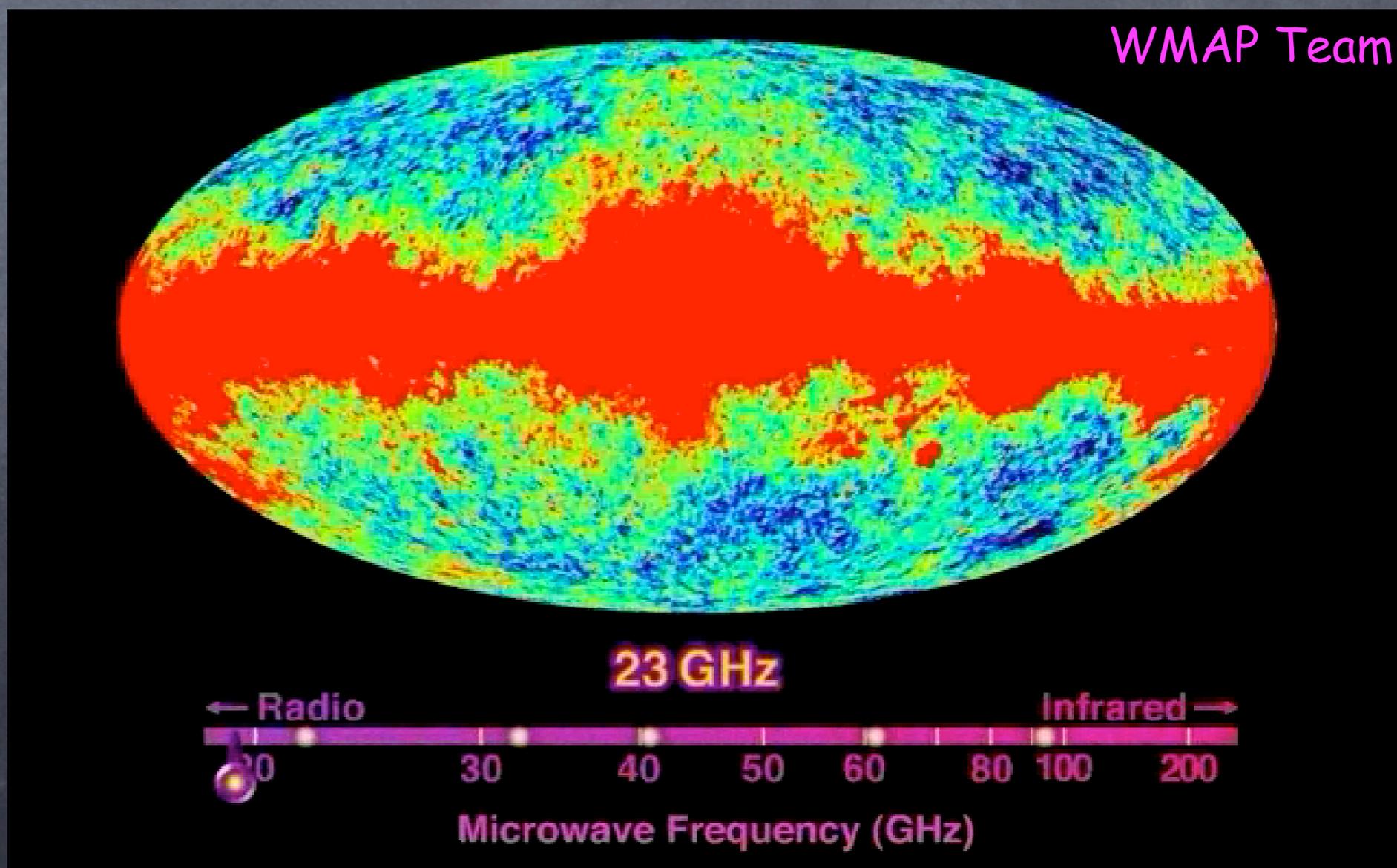


Inflation and the Cosmic Microwave Background

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WMAP Team



Precision cosmology!!

Old Universe – *New* Numbers

$$\Omega_{\text{tot}} = 1.02^{+0.02}_{-0.02}$$

$$w < -0.78 \text{ (95\% CL)}$$

$$\Omega_{\Lambda} = 0.73^{+0.04}_{-0.04}$$

$$\Omega_b h^2 = 0.0224^{+0.0009}_{-0.0009}$$

$$\Omega_b = 0.044^{+0.004}_{-0.004}$$

$$n_b = 2.5 \times 10^{-7} \text{ cm}^{-3} \text{ }^{+0.1 \times 10^{-7}}_{-0.1 \times 10^{-7}}$$

$$\Omega_m h^2 = 0.135^{+0.008}_{-0.009}$$

$$\Omega_m = 0.27^{+0.04}_{-0.04}$$

$$\Omega_\nu h^2 < 0.0076 \text{ (95\% CL)}$$

$$m_\nu < 0.23 \text{ eV (95\% CL)}$$

$$T_{\text{cmb}} = 2.725^{+0.002}_{-0.002} \text{ K}$$

$$n_\gamma = 410.4^{+0.9}_{-0.9} \text{ cm}^{-3}$$

$$\eta = 6.1 \times 10^{-10} \text{ }^{+0.3 \times 10^{-10}}_{-0.2 \times 10^{-10}}$$

$$\Omega_b \Omega_m^{-1} = 0.17^{+0.01}_{-0.01}$$

$$\sigma_8 = 0.84^{+0.04}_{-0.04} \text{ Mpc}$$

$$\sigma_8 \Omega_m^{0.5} = 0.44^{+0.04}_{-0.05}$$

$$A = 0.833^{+0.086}_{-0.083}$$

$$n_s = 0.93^{+0.03}_{-0.03}$$

$$dn_s/d \ln k = -0.031^{+0.016}_{-0.018}$$

$$r < 0.71 \text{ (95\% CL)}$$

$$z_{\text{dec}} = 1089^{+1}_{-1}$$

$$\Delta z_{\text{dec}} = 195^{+2}_{-2}$$

$$h = 0.71^{+0.04}_{-0.03}$$

$$t_0 = 13.7^{+0.2}_{-0.2} \text{ Gyr}$$

$$t_{\text{dec}} = 379^{+8}_{-7} \text{ kyr}$$

$$t_r = 180^{+220}_{-80} \text{ Myr (95\% CL)}$$

$$\Delta t_{\text{dec}} = 118^{+3}_{-2} \text{ kyr}$$

$$z_{\text{eq}} = 3233^{+194}_{-210}$$

$$\tau = 0.17^{+0.04}_{-0.04}$$

$$z_r = 20^{+10}_{-9} \text{ (95\% CL)}$$

$$\theta_A = 0.598^{+0.002}_{-0.002}$$

$$d_A = 14.0^{+0.2}_{-0.3} \text{ Gpc}$$

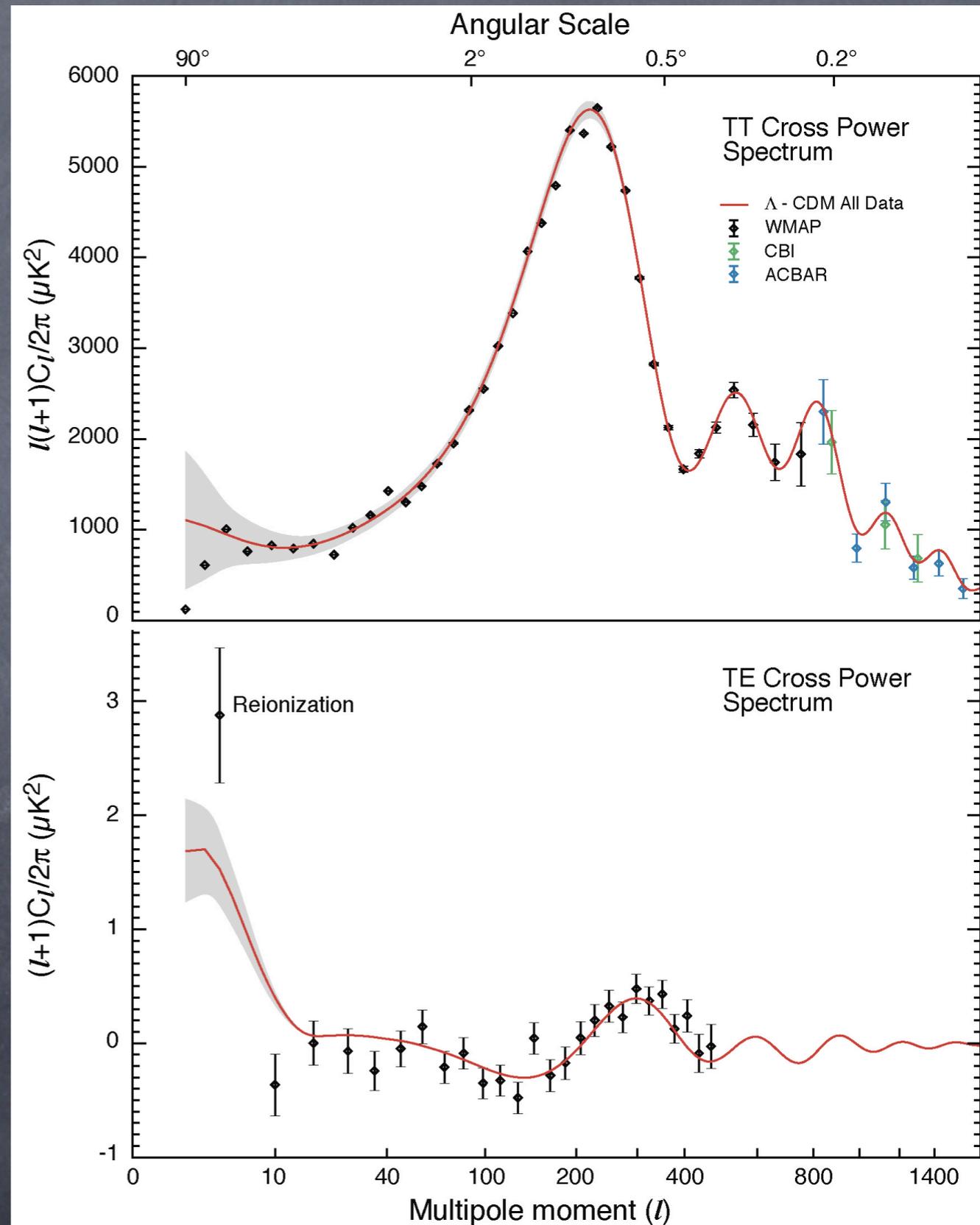
$$l_A = 301^{+1}_{-1}$$

$$r_s = 147^{+2}_{-2} \text{ Mpc}$$

Conclusions from WMAP:

If you want to explain this data, your best bet is ...

- A spatially-flat Universe
- Dark matter and dark energy
- Initial perturbations which are gaussian, adiabatic and nearly scale-invariant, e.g. as given by inflation.



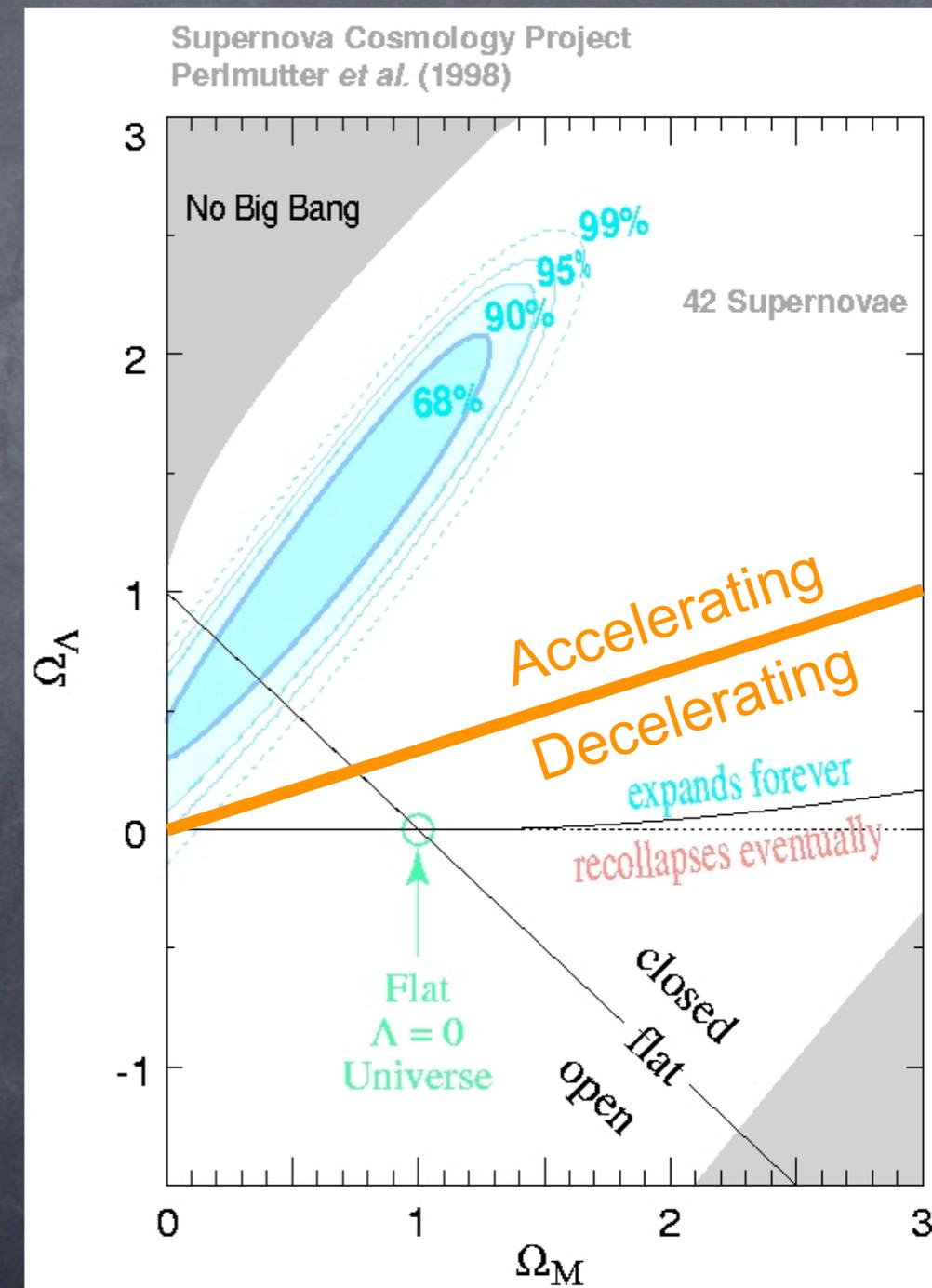
What is inflation?

Inflation is any period of the Universe's evolution during which the Universe is accelerating $\ddot{a} > 0$

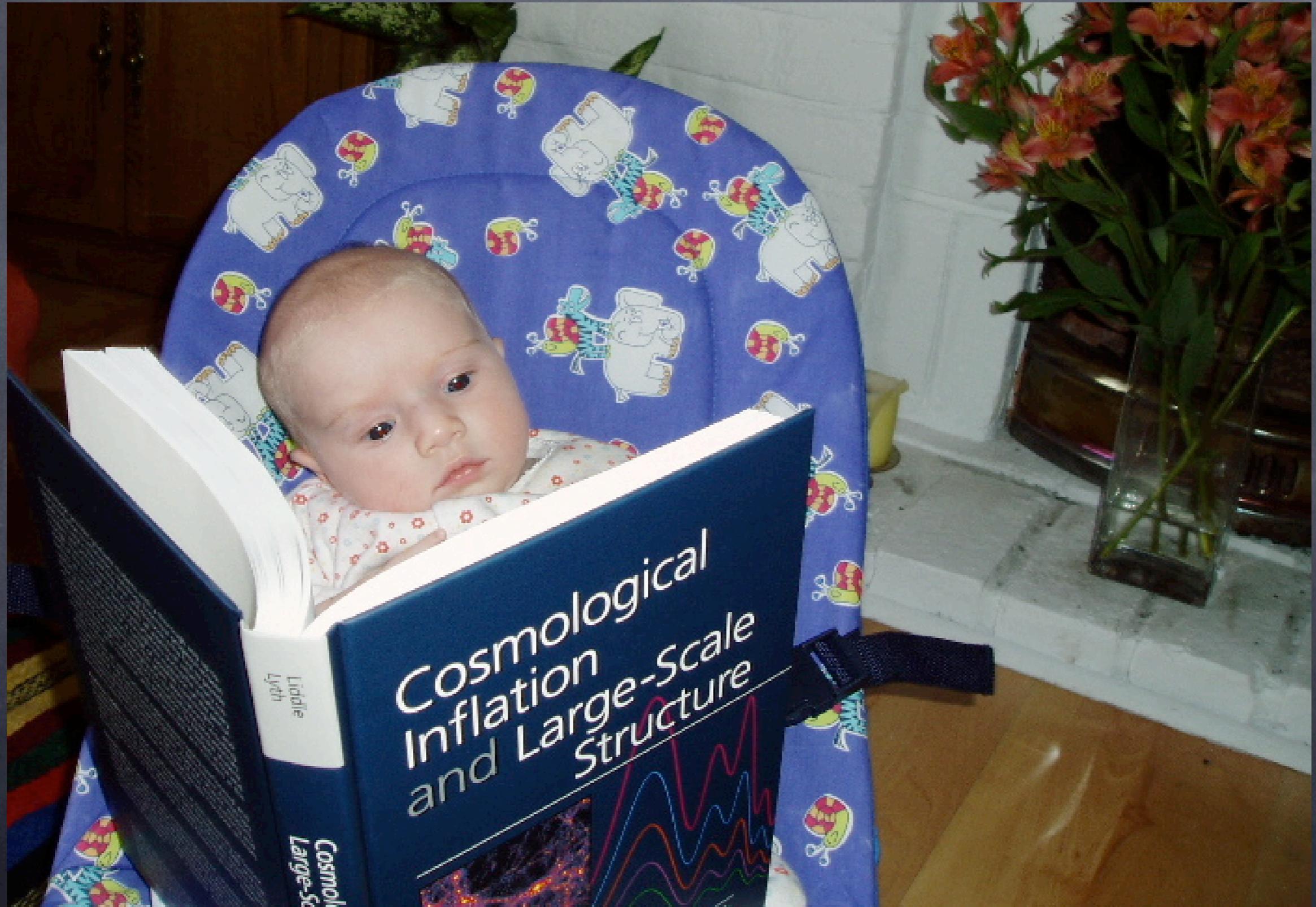
This can also be written in terms of the comoving Hubble length as

$$\frac{d(H^{-1}/a)}{dt} < 0$$

For this talk, we are interested in possible accelerated expansion in the early Universe.



Inflation is by now regarded as a standard ingredient of cosmological models.



Inflation and perturbations

The main motivation for being interested in inflation is that it leads to a perturbed Universe. During inflation, quantum fluctuations are imprinted on the Universe.

• Scalar field fluctuations

- Lead to scalar metric perturbations
- Cause gravitational collapse to form structures

• Gravitational wave perturbations

- Not associated with gravitational collapse but may influence the CMB.

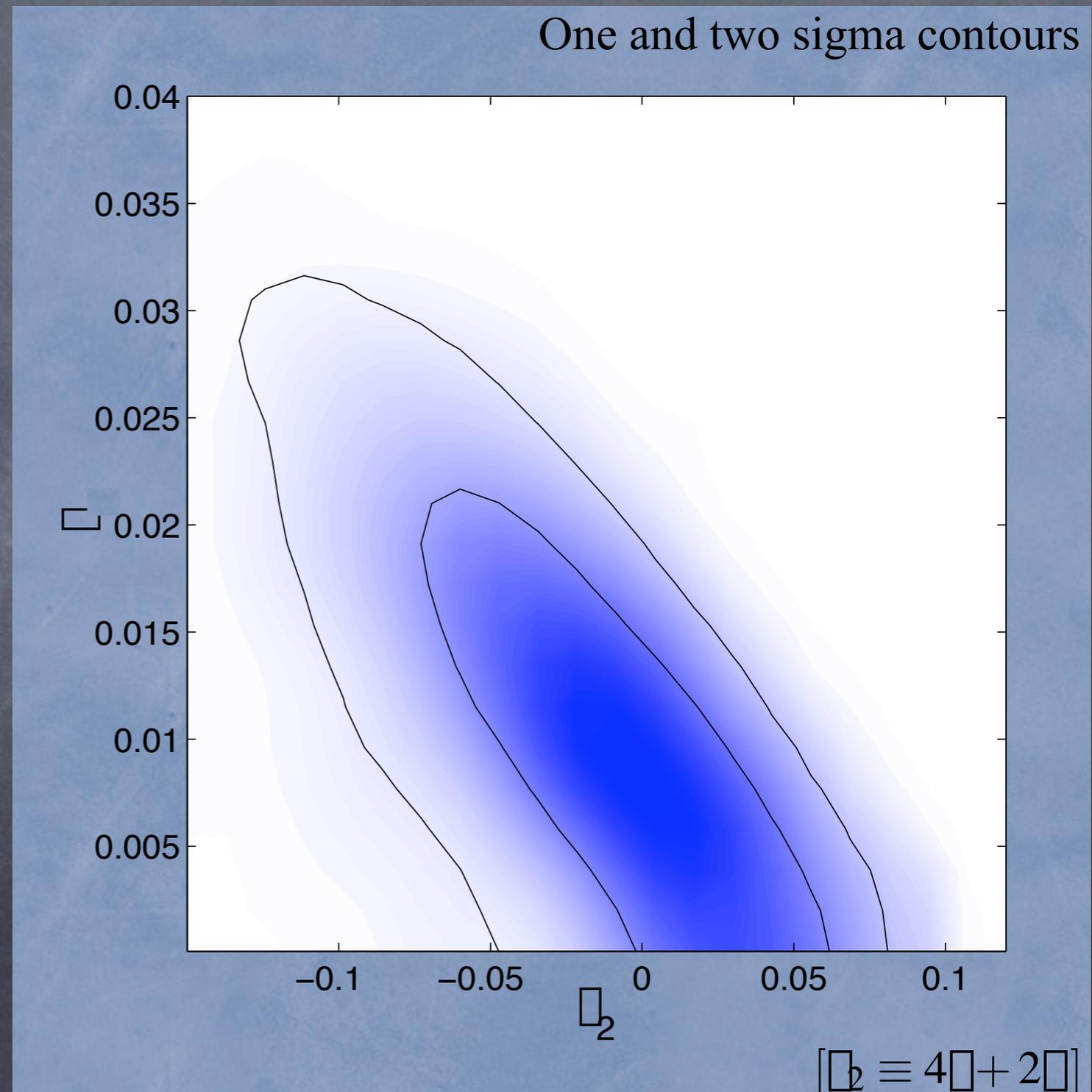


Current constraints on single-field models

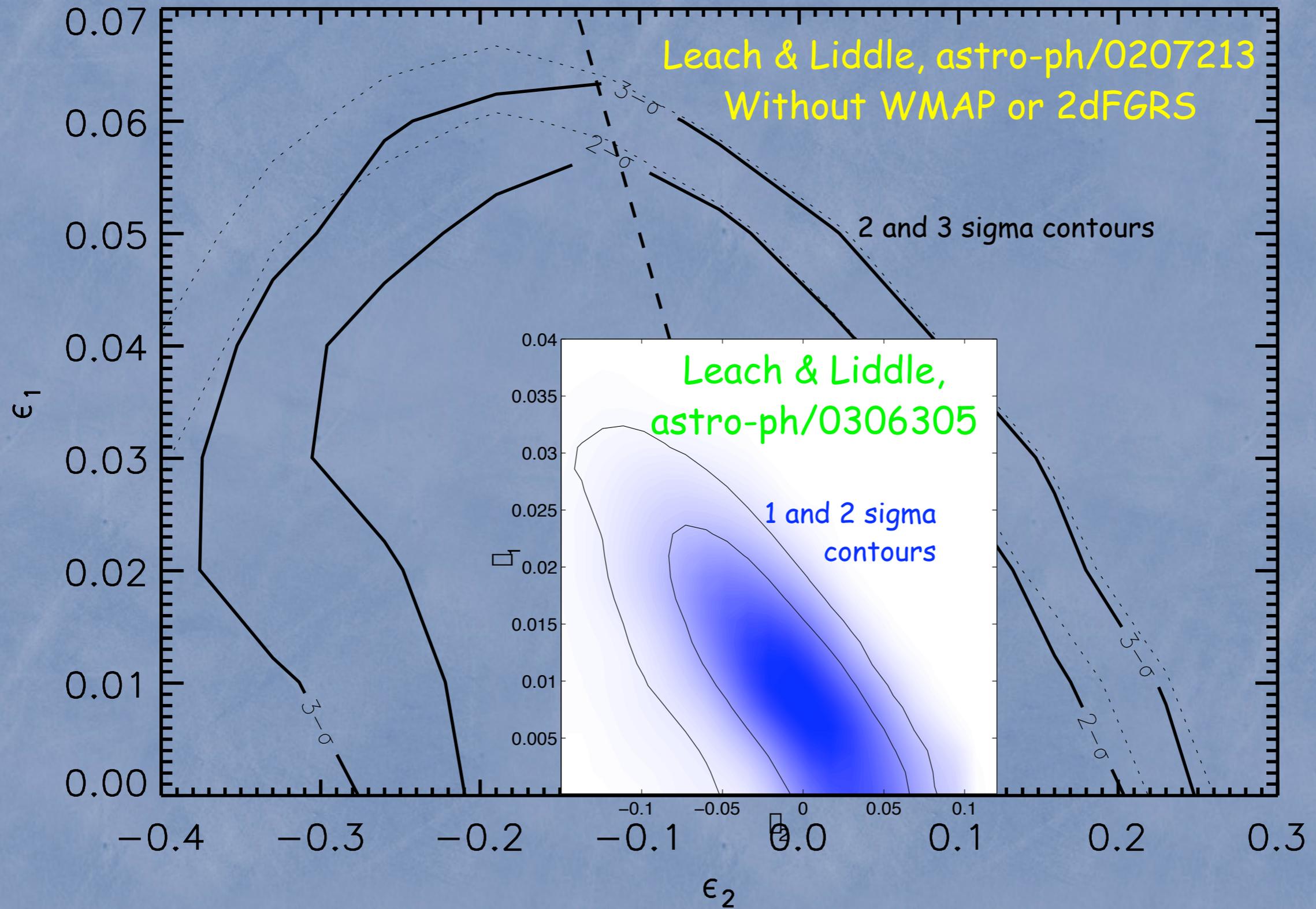
Leach & Liddle, PRD, astro-ph/0306305

Comparison with observations:

- Fit to data compilation of WMAP, other CMB experiments (VSA, CBI and ACBAR), and 2dF galaxy survey.
- Use CAMB plus CosmoMC plus WMAP likelihood code plus slow-roll inflation module.



What changed with WMAP?

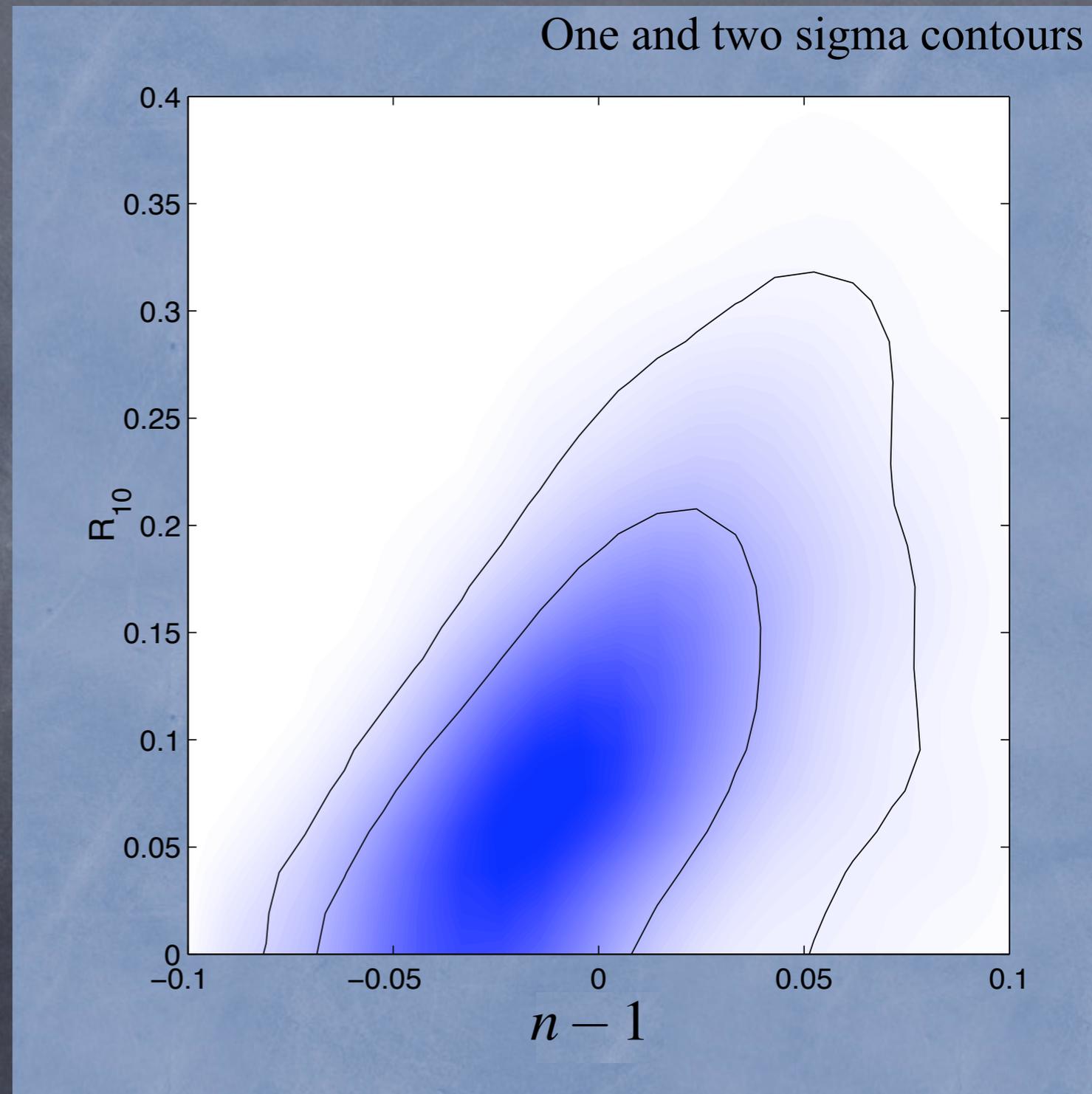


Current constraints

Leach & Liddle, PRD, astro-ph/0306305

Comparison with observations:

- Fit to data compilation of WMAP, other CMB experiments (VSA, CBI and ACBAR), and 2dF galaxy survey.
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Predictions of the simplest models

The simplest models of inflation predict **nearly scale-invariant** spectra of **adiabatic gaussian** density perturbations and **gravitational waves**, in their **growing mode**, in a **spatially-flat** Universe.

WMAP does not provide any evidence against any of these, and gives support to all but the gravitational waves. As such, it gives strong general support to the inflationary paradigm (but not uniquely to inflation).

Nearly scale-invariant spectrum

$$n = 0.99 \pm 0.04$$

(for a power-law fit to the data)

Adiabatic perturbations

Good fit to data assuming these dominate.

Gaussian perturbations

No unambiguous evidence of primordial non-gaussianity.

Gravitational waves

Not detected: $r < 0.43$

(for a power-law fit to the data)

Growing mode

Good fit to data assuming no decaying mode.
Temperature-polarization anti-correlation.

Spatial flatness

$$\Omega_{\text{tot}} = 1.02 \pm 0.02$$

Conclusion: the simplest inflation models are doing very well!!

More complicated models

There is presently no observational need to consider more complicated models, but here's some possibilities:

- **Multi-field models.** These allow isocurvature perturbations, which may be correlated with the usual adiabatic ones.
- **The Curvaton.** In this model, negligible adiabatic perturbations are produced during inflation, being later generated from isocurvature perturbations generated during inflation. This is a natural way to introduce some non-gaussianity.
- **Braneworld Inflation.** In standard braneworld inflation, the form of perturbations generated from a given potential changes, though the general predictions are unharmed.

Even if effects from these more complex models are never seen, they introduce degeneracies in interpreting observations.

With so many ideas around, can we really draw robust conclusions from the observational data?

What is the Standard Cosmological Model?

While there is broad consensus that the standard cosmological model gives an excellent description of the observed data, there isn't actually agreement on what the standard cosmological model is!

The precise constraints obtained depend on

- The observational datasets used.
- The set of cosmological parameters used to define the cosmological model.

There have been a variety of choices made for both of these.

Table 7. Best Fit Parameters: Power Law Λ CDM

	WMAP	WMAPext ^{16a}	WMAPext+2dFGRS	WMAPext+ 2dFGRS+ Lyman α
A	0.9 ± 0.1	0.8 ± 0.1	0.8 ± 0.1	$0.75^{+0.08}_{-0.07}$
n_s	0.99 ± 0.04	0.97 ± 0.03	0.97 ± 0.03	0.96 ± 0.02
τ	$0.166^{+0.076}_{-0.071}$	$0.143^{+0.071}_{-0.062}$	$0.148^{+0.073}_{-0.071}$	$0.117^{+0.057}_{-0.053}$
h	0.72 ± 0.05	0.73 ± 0.05	0.73 ± 0.03	0.72 ± 0.03
$\Omega_m h^2$	0.14 ± 0.02	0.13 ± 0.01	0.134 ± 0.006	0.133 ± 0.006
χ^2_{eff}/ν	1429/1341	1440/1352	1468/1381	... ^b

Table 8. Best Fit Parameters for the Running Spectral Index Λ CDM Model

	WMAP	WMAPext	WMAPext+2dFGRS	WMAPext+ 2dFGRS+ Lyman α
A	0.92 ± 0.12	0.9 ± 0.1	0.84 ± 0.09	$0.83^{+0.09}_{-0.08}$
n_s	$0.93^{+0.07}_{-0.07}$	0.91 ± 0.06	$0.93^{+0.04}_{-0.05}$	0.93 ± 0.03
$dn_s/d \ln k$	-0.047 ± 0.04	-0.055 ± 0.038	$-0.031^{+0.023}_{-0.025}$	$-0.031^{+0.016}_{-0.017}$
τ	0.20 ± 0.07	0.20 ± 0.07	0.17 ± 0.06	0.17 ± 0.06
h	0.70 ± 0.05	0.71 ± 0.06	0.71 ± 0.04	$0.71^{+0.04}_{-0.03}$
$\Omega_m h^2$	0.14 ± 0.02	0.14 ± 0.01	0.136 ± 0.009	$0.135^{+0.008}_{-0.009}$
χ^2_{eff}/ν	1431/1342	1437/1350	1465/1380	* ^a

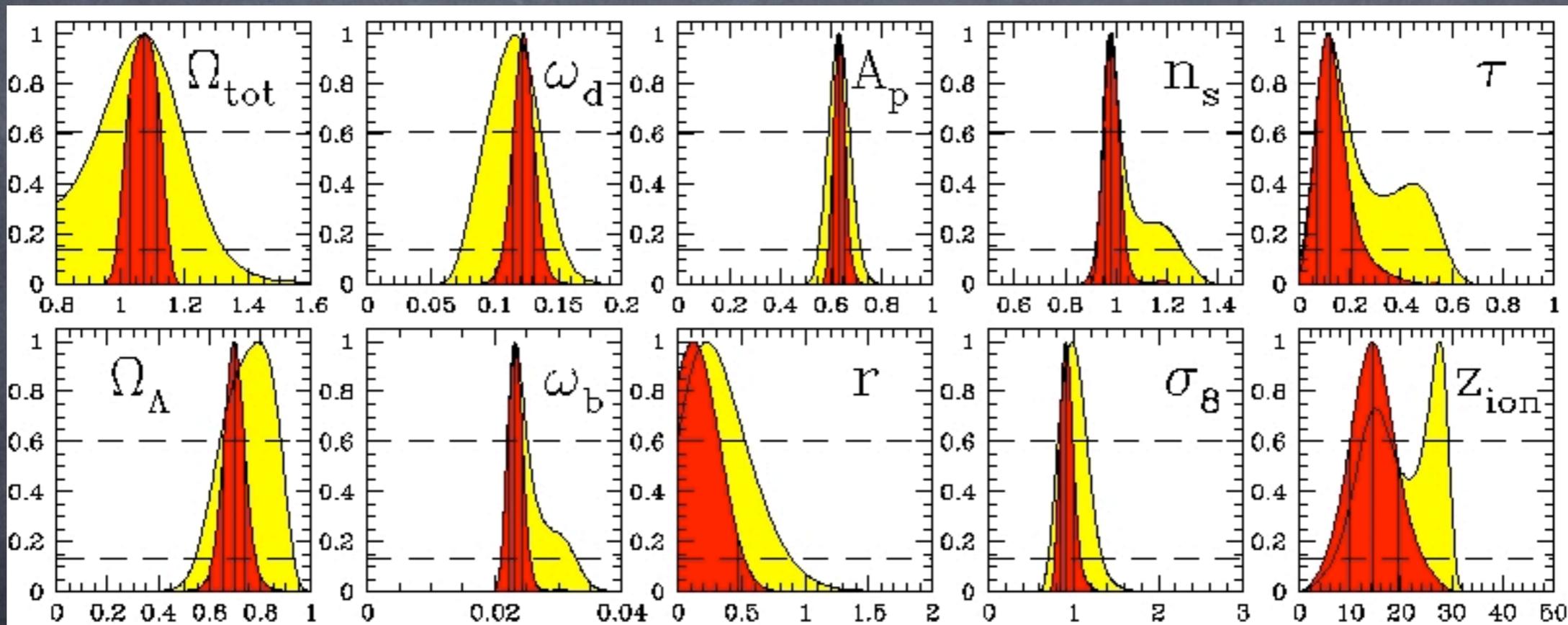
Can we use the data itself to decide which parameters should be used in the fit?

This is the statistical problem of *model selection*. It arises across many science disciplines.

e.g. in medical trials, one may have many factors that might contribute to disease susceptibility, and want to know which are effective in predicting disease.

Having decided the set of cosmological parameters defining the model, the standard method of data fitting is to compute the likelihood function

$\mathcal{L}(\theta_i)$ $\theta_1, \dots, \theta_k$ are the cosmological parameters



Tegmark et al. (2003)

The maximum likelihood gives the best values for the parameters, and the neighbouring behaviour gives the confidence limits.

How do we compare different cosmological models (i.e. different choices of fundamental parameters)?
Can we say which model is best?

Problem 1: if we add extra parameters, typically the maximum likelihood will increase, even if the new parameter actually has no physical relevance.

Problem 2: as we add extra parameters, the uncertainties on existing parameters increase, and eventually we learn nothing useful about anything.

We need a way of penalizing use of extra parameters - an implementation of Ockham's razor.

The Information Criteria

Liddle, astro-ph/0401198

Since the mid-seventies, statisticians have explored the connection between statistical inference and information theory to analyze this situation.

■ Akaike information criterion (Akaike 1974)

$$\text{AIC} = -2 \ln \mathcal{L}_{\max} + 2k \quad k = \text{number of parameters}$$

■ Bayesian information criterion (Schwarz 1978)

$$\text{BIC} = -2 \ln \mathcal{L}_{\max} + k \ln N \quad N = \text{number of datapoints}$$

The preferred model is the one which minimizes the information criterion.

[Other model selection techniques: **likelihood ratio test** and **Bayesian evidence**.]

The **Akaike Information Criterion** was derived using information theory techniques. It gives an approximate minimization of the so-called **Kullback-Leibler information entropy**, which is a measure of the difference between two probability distributions.

The **Bayesian Information Criterion** was derived using Bayesian statistics. It is an approximation to the **Bayesian evidence**, which gives the posterior odds of one of two models being correct (the BIC making the assumption that they were equally likely before the comparison to data). It is related to the integrated likelihood.

There is some dispute in the statistics literature as to which is the better, which seems to depend on circumstances.

The AIC tends to be preferred if the model complexity increases with the size of the dataset, not thought to be true in cosmology.

The AIC has the problem that it is 'dimensionally inconsistent', meaning that it can favour the wrong model even with an infinite dataset.

My reading is that the BIC is preferred for cosmology.

Application to present data

[WMAPext+SDSS, Tegmark et al. (2003)]

Start with a `base' set of cosmological parameters, the simplest set known to give an acceptable fit to the data.

A spatially-flat cosmology with radiation, baryons, dark matter and dark energy, with scale-invariant adiabatic gaussian density perturbations.

Ω_m	matter density
Ω_b	baryon density
Ω_r	radiation density
h	hubble parameter
A	adiabatic density perturbation amplitude
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τ	reionization optical depth
b	bias parameter (or parameters)

There are many, many ways in which this base cosmological model can be extended.

Table 2. Candidate parameters: those which might be relevant for cosmological observations, but for which there is presently no convincing evidence requiring them. They are listed so as to take the value zero in the base cosmological model. Those above the line are parameters of the background homogeneous cosmology, and those below describe the perturbations.

Ω_k	spatial curvature
$N_\nu - 3.04$	effective number of neutrino species (CMBFAST definition)
m_{ν_i}	neutrino mass for species ‘ i ’ [or more complex neutrino properties]
m_{dm}	(warm) dark matter mass
$w + 1$	dark energy equation of state
dw/dz	redshift dependence of w [or more complex parametrization of dark energy evolution]
$c_s^2 - 1$	effects of dark energy sound speed
$1/r_{\text{top}}$	topological identification scale [or more complex parametrization of non-trivial topology]
$d\alpha/dz$	redshift dependence of the fine structure constant
dG/dz	redshift dependence of the gravitational constant
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$n - 1$	scalar spectral index
$dn/d \ln k$	running of the scalar spectral index
r	tensor-to-scalar ratio
$r + 8n_T$	violation of the inflationary consistency equation
$dn_T/d \ln k$	running of the tensor spectral index
k_{cut}	large-scale cut-off in the spectrum
A_{feature}	amplitude of spectral feature (peak, dip or step) ...
k_{feature}	... and its scale [or adiabatic power spectrum amplitude parametrized in N bins]
f_{NL}	quadratic contribution to primordial non-gaussianity [or more complex parametrization of non-gaussianity]
\mathcal{P}_S	CDM isocurvature perturbation ...
n_S	... and its spectral index ...
$\mathcal{P}_{S\mathcal{R}}$... and its correlation with adiabatic perturbations ...
$n_{S\mathcal{R}} - n_S$... and the spectral index of that correlation [or more complicated multi-component isocurvature perturbation]
$G\mu$	cosmic string component of perturbations

Here are the information criteria for various cosmological models analyzed by Tegmark et al.

$$N = 1367 \implies \ln N \simeq 7$$

Model	parameters	$-2 \ln \mathcal{L}$	AIC	BIC
Base model	6	1447.9	1459.9	1491.2
Base + n	7	1447.2	1461.2	1497.7

The absolute values of the criteria are not of interest, only the relative values. For the BIC, a difference of 2 is regarded as significant evidence, and of 6 as strong evidence, against the model with the higher value.

Both information criteria favour the simplest model, with the AIC and BIC increasing with model complexity.

Implications for inflation

As far as the present data are concerned, even the simplest models of inflation are too complicated a description. We just need one parameter, the adiabatic density perturbation amplitude.

Hopefully future data will not only be of higher quality, but will also require the introduction of extra parameters. Otherwise the physics we can extract will be very limited.



Conclusions

- Precision cosmology has arrived, with several cosmological parameters known to high accuracy, and good agreement amongst different datasets.
- Inflationary models are commonly taken as part of the standard cosmological model, and indeed give excellent fits to the data.
- However, as far as the data alone are concerned, the preferred fit uses a Harrison-Zel'dovich spectrum, and does not need inflationary physics.
- Hopefully, future data will require the inclusion of more parameters, allowing us to access more physics.
- There should be less focus on parameter estimation, and more on model selection.

