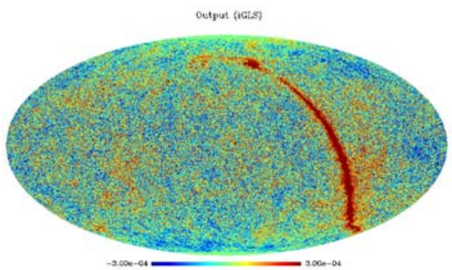
CMB MapMaking



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Most of our knowledge of the CMB anisotropy is mediated by a map

- Power spectrum estimation (and, hence, cosmological parameters probed by it)
- Constraints on non Gaussianity
- Maps widely used for non CMB analysis

Size of dataset is an issue…

- Need to observe large fractions of the sky to increase statistical significance
- Resolve features up to few arcmin, to gather the core cosmological information

→ Maps with many resolution elements (pixel) typically in the 100K – few M range
(more for Planck)

Noise is a powerful enemy…

- Good CMB detectors have noise sensitivities in the range 100 to 300 $\mu K\sqrt{s}$
- Experiments must spin (or chop) quickly to modulate the signal (exp. low modes, dipole often used for calibration) in the frequency range were amplifiers are stable enough.

cosmological signal is largely subdominant in the timestream, have to resort to non trivial statistical techniques to dig it out

Going to high precision is going to complicate your life

- Polarization is the typical case: requires very careful analysis.
- There is a plethora of systematic effects likely to project on the sky.
- Map Making must be used as a tool to assess the quality of the final maps.

In short:

- Large size problem
  - Up to several Gb/channel/day of data to reduce
  - 10^5 to 10^7 "pixels" to be estimated
- Detectors inject noise
- Noise is often (almost always) correlated (long memory, or "*1/f*")
- Systematics (non statistical noise) play an increasingly important role in high accuracy experiments.

Constraints and Requirement

- Given size, can hardly afford superlinear behavior
- Look for linear model/estimator
- Must be used in Monte Carlo pipelines (i.e. fast)

A minimal data model

$$d_t = A_{tp} m_p + n_t$$

$$< n_t A_{tp} m_p > = 0$$

The structure of A is kept as simple as possible!

But can account for many things…

$$\mathcal{S}_t = \frac{1}{2} A_{tp} \left(I_p + Q_p \cos 2\phi_t + U_p \sin 2\phi_t \right) + n_t$$

$$\mathcal{D}_t = \mathbf{A}_{tp} \mathbf{S}_p + \mathbf{n}$$

✓Polarization
✓Multidector

$$\mathcal{D}_t \equiv \begin{pmatrix} \mathcal{D}_t{}^1 \\ \vdots \\ \mathcal{D}_t{}^n \end{pmatrix}$$

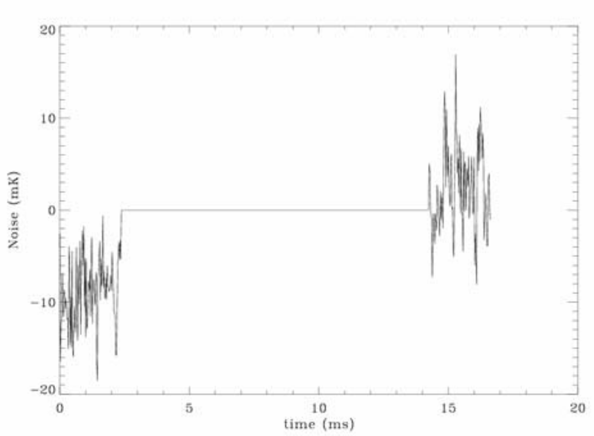
$$\mathbf{S}_p \equiv \begin{pmatrix} I_p \\ Q_p \\ U_p \end{pmatrix}$$

$$\mathbf{n}_t \equiv \begin{pmatrix} n_t^1 \\ \vdots \\ n_t^n \end{pmatrix}$$

$$\mathbf{A}_{tp} \equiv \frac{1}{2} \begin{pmatrix} A_{tp}^1 & \cos 2\phi_t A_{tp}^1 & \sin 2\phi_t A_{tp}^1 \\ \vdots & \vdots & \vdots \\ A_{tp}^n & \cos 2\phi_t A_{tp}^n & \sin 2\phi_t A_{tp}^n \end{pmatrix}$$

What about "real world" issues?

- Can be taken in account if they fall into the linear model
- Example: timeline gaps



$$d_t = A_{tp} m_p + n_t + m' h_t$$

$$h_t = \begin{cases} 1 & t \in \text{gap} \\ 0 & t \notin \text{gap} \end{cases}$$

m' is just an extra (fake) pixel

✓Constrain noise inside gaps (e.g. Hoffman Ribak)

✓ Also works for other effects:

- TOD offsets
- Parasitic signals
- **As long as the template is known**

Linear model calls for a linear solution

$$\tilde{m}_p = W_{pt} d_t$$

With *W* matrix of some form…

Historical solutions

$$W = (A^T A)^{-1} A^T d$$

$$W = (A^T A)^{-1} A^T K d$$

$$W = (A^T V^{-1} A)^{-1} A^T V^{-1} d$$

Historical solutions

$$W = (A^T A)^{-1} A^T d$$

Plain binning (OLS)

- Only good if noise is white (flat Fourier PSD)
- Trivial for a one horned experiment
- Not so trivial if A has a more complex structure (e.g. differential experiments like the WMAP)
- Unbiased

$$W = (A^T A)^{-1} A^T K d$$

$$W = (A^T V^{-1} A)^{-1} A^T V^{-1} d$$

Historical solutions

$$W = (A^T A)^{-1} A^T d$$

Plain binning on "altered" data

$$W = (A^T A)^{-1} A^T K d$$

- "Naïve" map making (filtering + binning):
  - ✓ Biased (information loss)
  - ✓ Quick and dirty
- Destriping methods
  - ✓ Linear regression to estimate low frequency features (e.g. offsets), which are the removed
  - ✓ Unbiased
  - ✓ Developed in the context of Planck (scanning is through rings)
  - ✓ Fast, but not always feasible (depending on scanning strategy)

$$W = (A^T V^{-1} A)^{-1} A^T V^{-1} d$$

Historical solutions

$$W = (A^T A)^{-1} A^T d$$

"Weighted" binning

$$W = (A^T A)^{-1} A^T K d$$

$$W = (A^T V^{-1} A)^{-1} A^T V^{-1} d$$

- Unbiased (obviously V non singular)
- Usually weighting is by noise (GLS)
  - ✓ $V = \langle nn^T \rangle$
  - ✓ Minimum variance (among other LE)
  - ✓ Maximum likelihood if Gaussian noise
  - ✓ Sometimes called "optimal" map making
  - ✓ Is highly computationally intensive if implemented brute force (requires inversion of a rank N_{pix} matrix, totally unfeasible
  - ✓ Need to estimate the noise (e.g. subtract signal iteratively, performing many map making stages)

GLS solution…

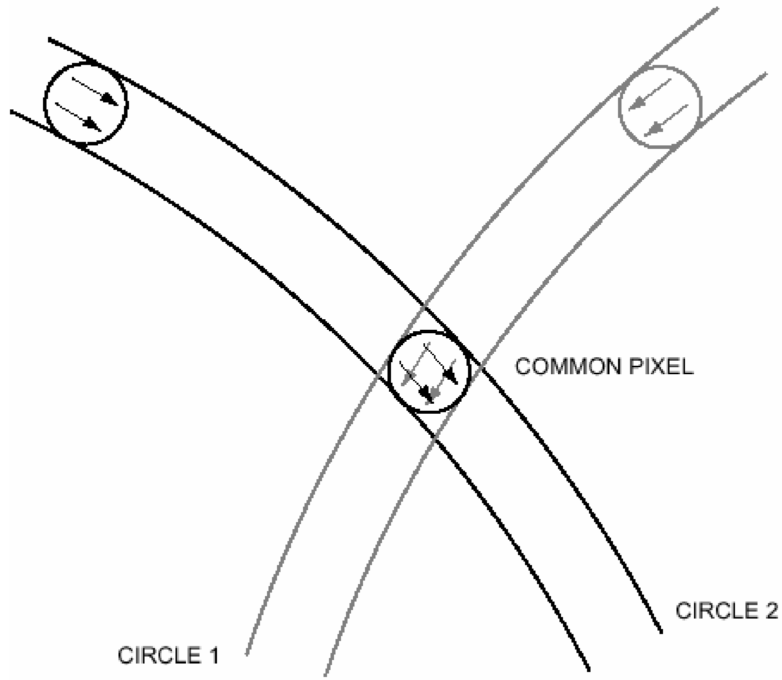
$$\widetilde{\mathbf{S}}_{\mathbf{p}} = \left(\mathbf{A}^t\mathbf{N}^{-1}\mathbf{A}\right)^{-1}\mathbf{A}^t\mathbf{N}^{-1}\mathcal{D},$$

$$\mathbf{N} \equiv \langle \mathbf{n}_t \mathbf{n}_{t'} \rangle = \begin{pmatrix} \langle n_t^1 n_{t'}^1 \rangle & \cdots & \langle n_t^1 n_{t'}^n \rangle \\ \vdots & \ddots & \vdots \\ \langle n_t^n n_{t'}^1 \rangle & \cdots & \langle n_t^n n_{t'}^n \rangle \end{pmatrix}$$

$$\left\langle n_t^i n_{t'}^j \right\rangle = \left\langle n_t^j n_{t'}^i \right\rangle = \mathbf{0} \ .$$

Destriping

- Observe many time same path (e.g. ring) on the sky
- Coadd samples observing same pixel → creates offsets
- Solve for offsets minimizing χ^2
  - Delabrouille 1998
  - Maino et al 1999
- Can be extended to polarization
  - Revenu 1999
- Can even do without ring coaddition (helps when pointing is not perfect)
  - Kehihanen 2003



“Generalized” Least Squares

$$\tilde{m} = (P^T N^{-1} P)^{-1} P^T N^{-1} d$$

A very old idea, rediscovered within CMB literature

$$N \equiv \langle nn^T \rangle$$

Tegmark, 1997
Delabrouille, 1998
Borrill, 1998 (MADCAP)

Circulant if
noise is stationary

Wright, 1996 ⟶ FFT based iterative solver

Must solve iteratively for large datasets:

$$P^T N^{-1} P \tilde{m} = P^T N^{-1} d$$

Unroll, Convolve & Bin

- ✓ Need a good solver
- ✓ Preconditioned (by pixel hits) CG is OK
- ✓ Usually fast (~ 100 iterations)
- ✓ Parallelization straightforward

Need to estimate N !

Other than a computational price, GLS asks for an information toll: TOD are sum of noise and signal

- Get noise properties from a combination of noise and signal
- The better you know the timeline noise, the lower its variance in the map
- Very important for low signal to noise (e.g. polarization)
- Usually done iteratively

Ferreira & Jaffe 1998
Prunet et al. 2000
Dorè et al. 2001
Stompor et al. 2001

Compute map : $\tilde{m} = (A^T N^{-1} A)^{-1} A^T N^{-1} d$

Derive noise estimate : $\tilde{n} = d - A\tilde{m}$

Estimate noise properties

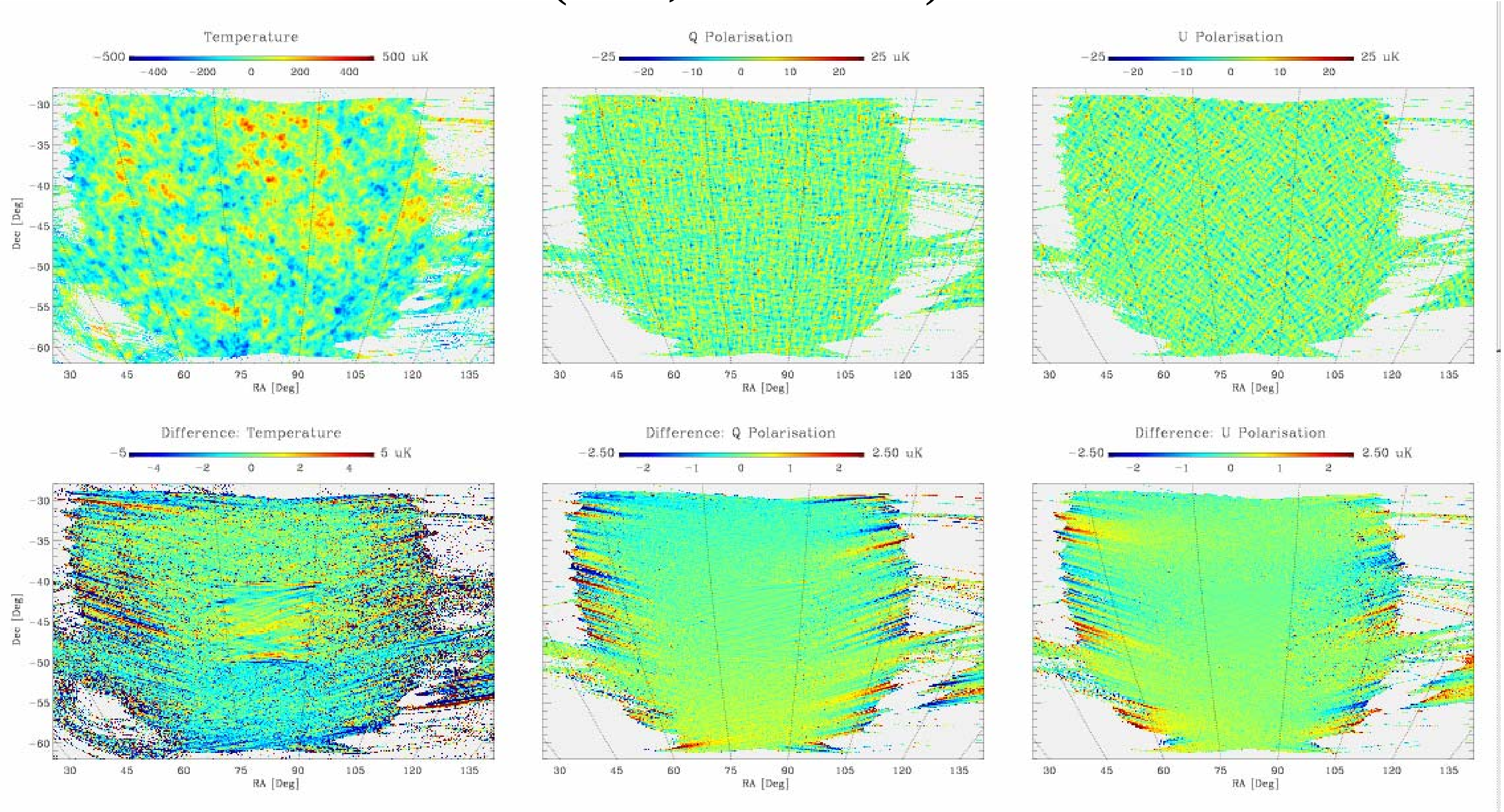
Restart over again

An alternative approach: Harmonic Space Map Making

- Solves for the a_{lm}'s rather than for pixels.
- Obviously interesting only for high coverage surveys (e.g. satellites).
- Hard job: the "pointing" and noise matrices in l space are in general dense and non trivial, scanning symmetries are needed.
- This is the correct space to handle the beam.
- Current best implementations (Planck) use iterative PCG and scale roughly as $\sim l^4$ per iteration (Challinor et al 2004).

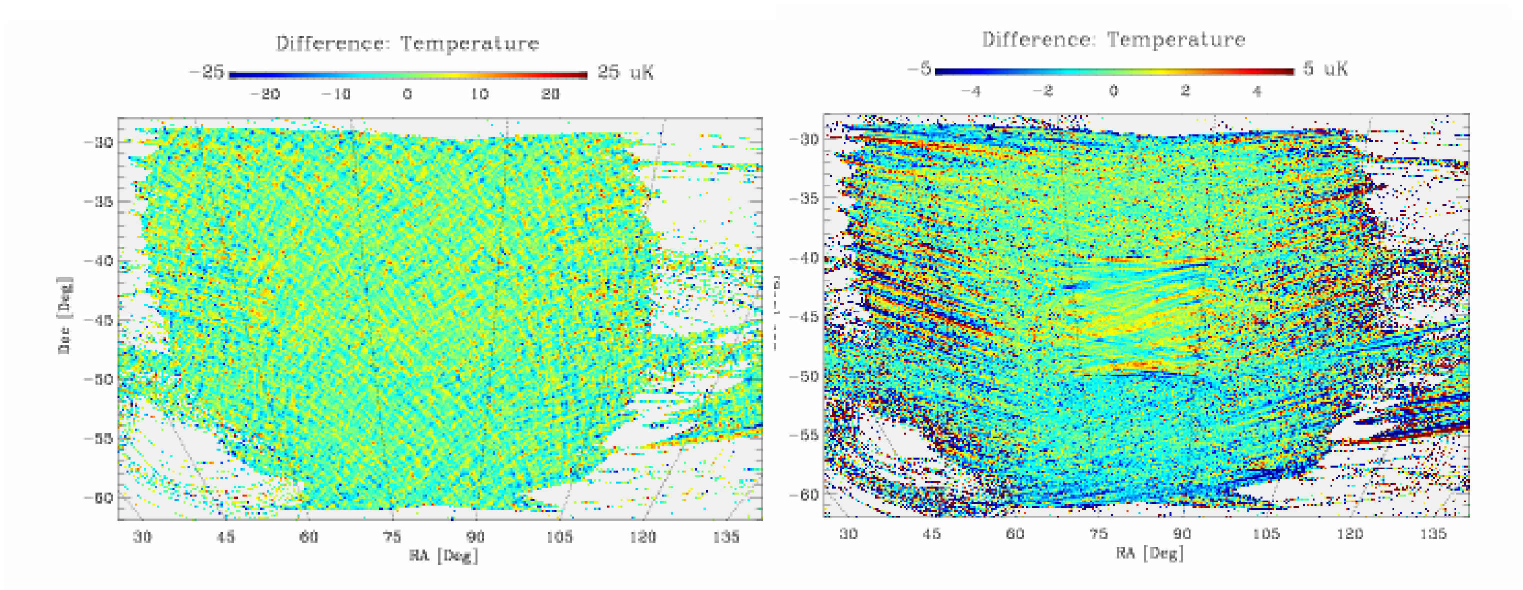
A few results from pixel space GLS mapmaking

T+P maps for a highly realistic balloon simulation (B2K, 8 detectors)



de Gasperis et al. 2004

This is why solving jointly for T & P is important



Remarks on GLS map making

- Fast: scales linearly with *d* (have to look at each sample *at least* once!)
- Accurate, with proper solver (PCG), even at low *l* modes
- Include polarization and main “real world” complication
- Trivially parallelized

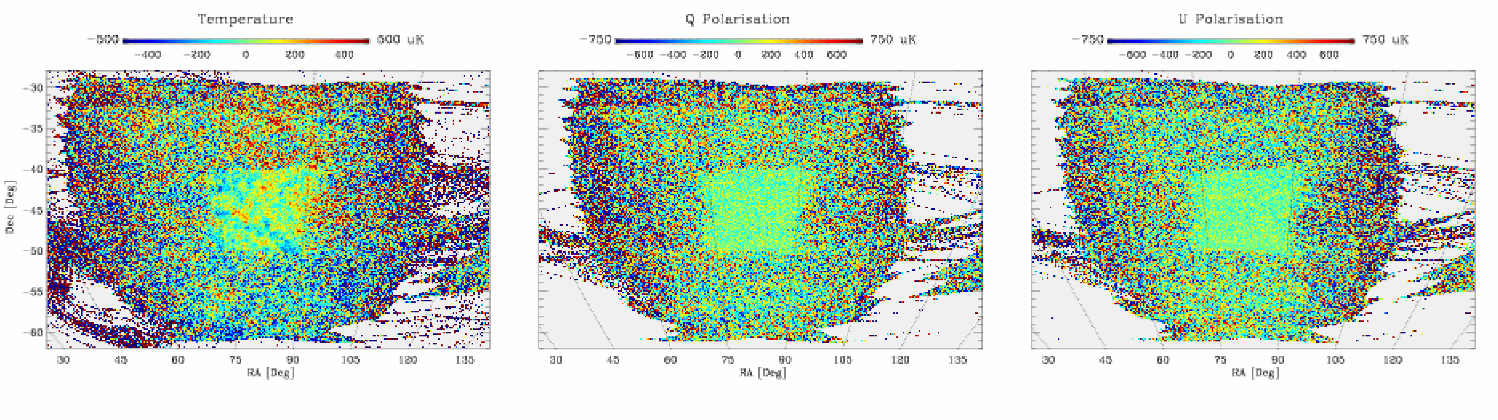
- Must estimate noise
- Noise must be (piecewise) stationary for method to be feasible (Circulant/Toeplitz noise matrix)
- Need parallel hardware for large dateset (and Monte Carlo)
- Somewhat delicate, convergence slower on real data w.r.t. sim data
- Does not provide covariance matrix (would be unmanageable anyway)

What is on the market

- MADCAP (Borrill 1998)
  - Matrix inversion (Parallel: SCALapack)
  - Maxima Boomerang
  - Downloadable
- Destripers (Planck Sim data)
  - Matrix inversion/ iterative
  - Planck/LFI
    - Kehihanen/Maino code
    - Polarization
  - Planck/HFI (not really public!)
    - Eldestino/Moka (Ansari et al.)
    - Desperado (Larquere et al.)
- MAPCUMBA (Dore et al. 2001)
  - Improved by Hivon & Prunet
  - Iterative (PCG)
  - Archeops/Boomerang/Planck
  - Downloadable (old version?)
- ROMA iGLS (Natoli,de Gasperis 2001)
  - Boomerang/Planck
  - Iterative (PCG)
  - Polarization
- MADMap + MADness (Cantalupo, Borrill, Stompor, 2002)
  - Boomerang/Maxipol/Planck
  - Iterative (PCG)
  - Polarization
  - Downloadable
- MIRAGE (Yvon, Mayet 2004)
  - Archeops
  - Iterative (+ filtering)
  - Only temperature

Final remarks

- Map Making is by no means a closed problem, but significant steps ahead have been made in the last few years. Large and complex datasets successfully tackled.
- There are many avenues left for future research. These will be explored when it will be realized that the data need finer treatment.
  - beam (de)convolution.
  - estimation of pixel-pixel (co)variances.
  - Planck simulated data analysis may well suggest that destriping is a good compromise between accuracy, robustness and speed.
- If the future is in detector arrays, hope that single elements have negligible cross-talks (otherwise analysis will get very complex).


Temperature
-500
500 uK
Q Polarisation
-750
750 uK
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-750
750 uK
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Overview:

- The basic facts
- How to complicate your life…
- Even more complicated: real world
- Coping with high precision
- The problem of noise estimation
- Map Making as a tool