# Recovering WMAP quadrupole and octopole

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# 1 – Introduction

Quadrupole and Ocutpole:

- Both have seemingly low amplitude
- Various claimed alignments between:
  - quadrupole and octopole
  - with ecliptic plane
  - with galactic plane
- Most tests require full-sky map
- The ILC/TOH map not ideal - note WMAP team warning
- Since there is no beliveable full-sky map, need *realisations* of the sky compatible with data
- Tests can then be performed on these realisations to infer confidence limits.

# 2 – Introduction II

- A full-sky map can be inverted to give $a_{\ell m}$s.
- Sky cuts and foregrounds result in a *probability distribution* $p(a_{\ell m})$.
- How do we calculate it?

# 3 – Calculating $p(a_{\ell m})$

- A model map can be constructed $\mathbf{d}_t = \sum_{\ell,m} a_{\ell,m} Y_{\ell m}$
- Probability $p(a_{\ell m})$ is the probability that a $\mathbf{d} - \mathbf{d}_t$ is a possible noise realisation.
- Therefore

$$\log p(a_{\ell m}) \propto (\mathbf{d} - \mathbf{d}_t)^{\mathrm{T}} \mathbf{C}^{-1} (\mathbf{d} - \mathbf{d}_t) \tag{1}$$

- The covariance matrix $\mathbf{C}$ contains:
  - "Noise" due fluctuations in higher multipoles - assume fiducial PS
  - Instrumental noise (diagonal)
  - Marginalisation over known foregrounds

# 4 – Maps

- Need to invert covariance matrix (just once!)
- Infeasible (and pointless) on full-resolution maps
- We do it on reduced resolution maps ($\mathrm{nside} : 512 \longrightarrow 16$).
- We do not take foreground-corrected maps, because we correct for them.

# 5 – Foregrounds

Three known foregrounds: Synchrotron, Free-Free, Dust

There are three options:

- Subtract them - need confidence that your model is correct
- Marginalise over them :

$$C_{\rm fg} = \lambda \mathbf{t}\mathbf{t}^{\rm T}, \tag{2}$$

  where $\lambda \to \infty$ and $\mathbf{t}$ is a template vector.xs
- Set $\lambda = 1$
- We opted for two options:
  - Not-that-conservative: Take W channel, KP2 mask, subtract Free-Free, marginalise over dust — `WDUST`
  - Conservative: Take V channel, KP2 mask and marginalise over everything — `VKP2`

# 6 – Exploring $p(a_{\ell m})$

Two methods to explore $p(a_{\ell m})$:

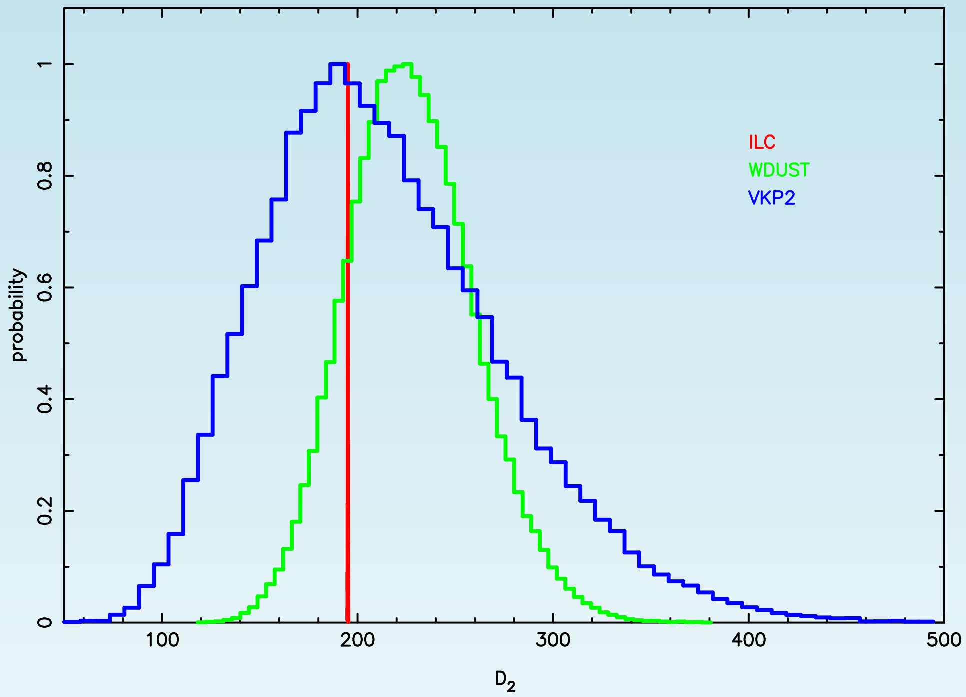
- Use MCMC algorithm (astro-ph/0404567)
- Realise that $p(a_{\ell m})$ must be a multivariate Gaussian:
  - Need only a certain number of evaluations of likelihoood to constrain parabolla $\longrightarrow$ dramatic increase in speed
  - Under some simplifying assumptions can do it directly from the data, but requires an inversion of $n \times n$ matrix.

# 7 – Why?

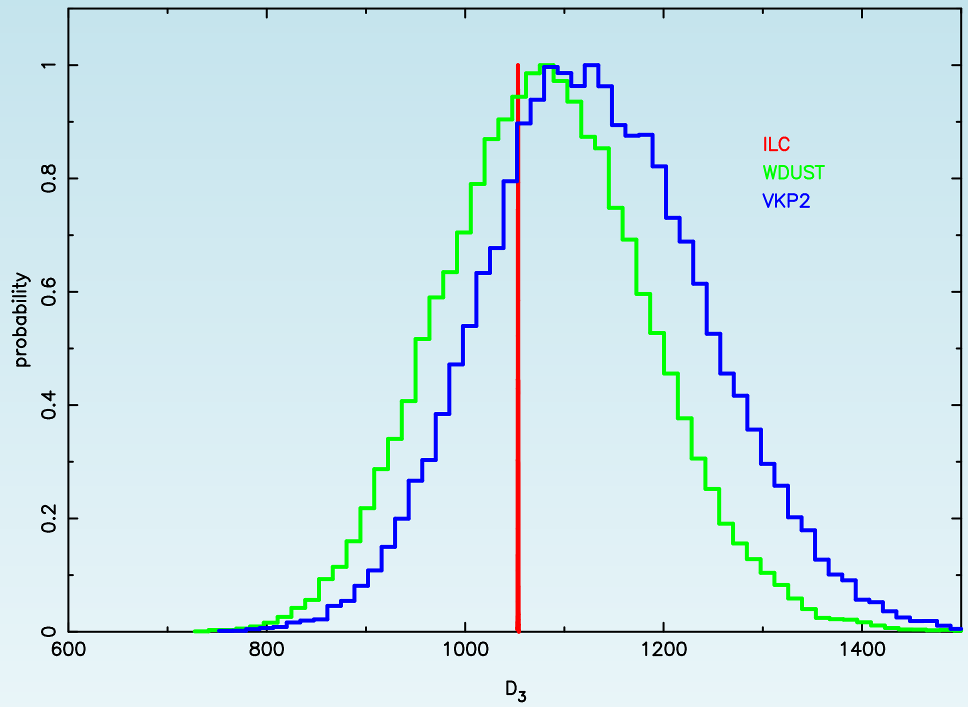
Once we have distribution of $p(a_{\ell m})$ we can play many games:

- Calculate $D_\ell = \langle a_{\ell m} a^\star_{\ell m} \rangle$ to decouple cosmic variance from foreground / sky cuts effects
- Impose confidence limits on various statistics claiming alignment, etc.

# 8 – $D_2$ and $D_3$

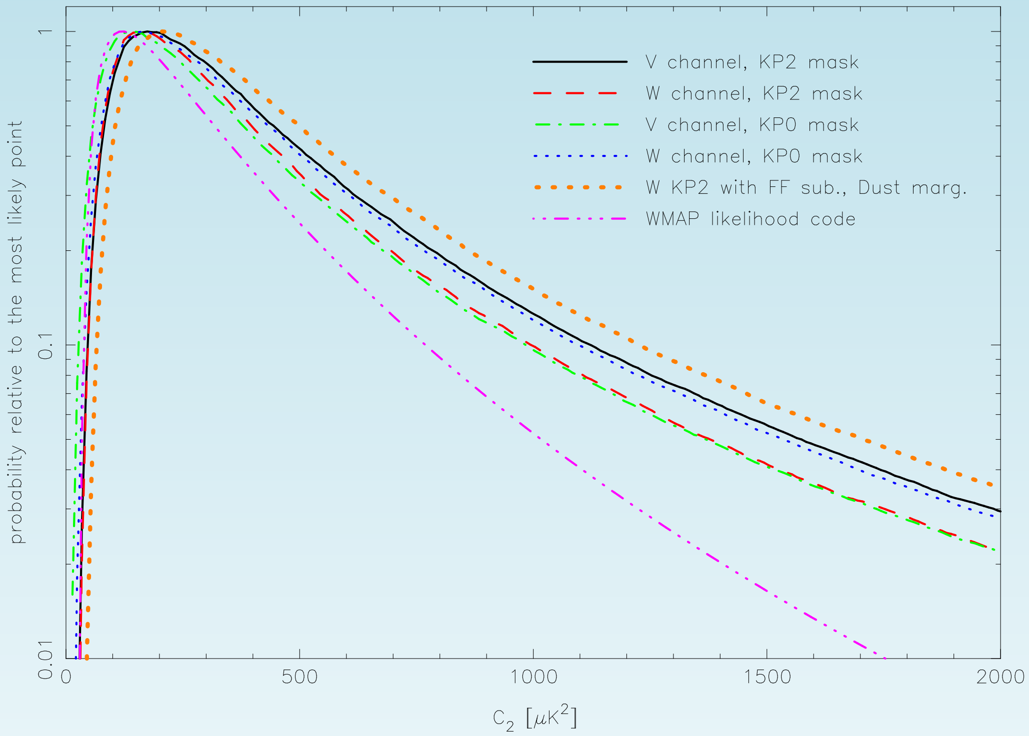


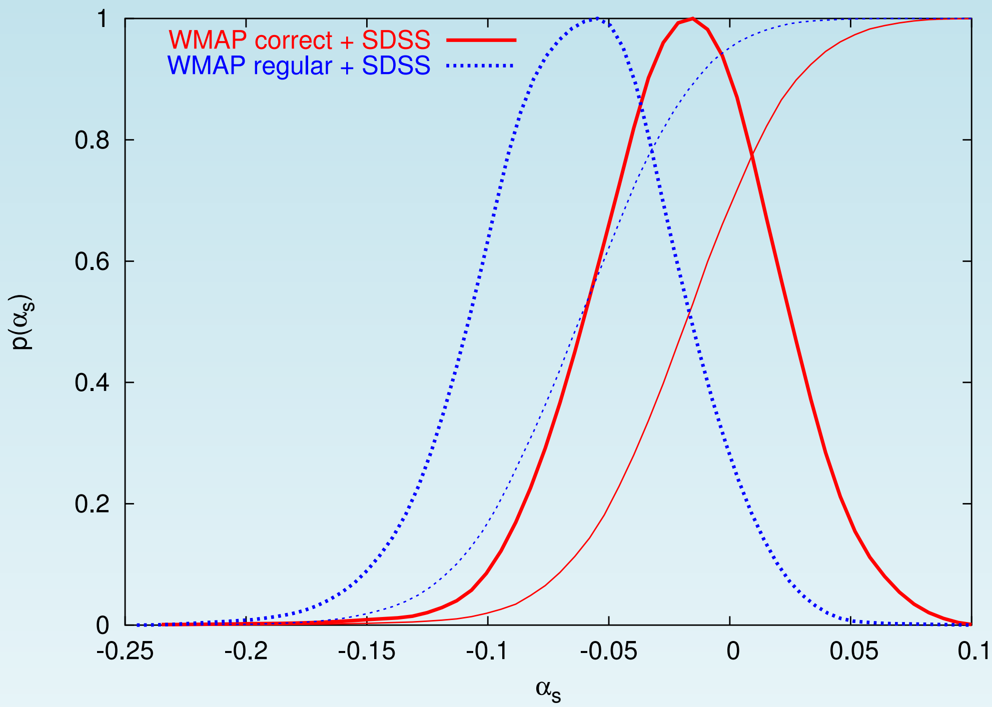
probability
0
0.2
0.4
0.6
0.8
1
100
200
300
400
500
D2
ILC
WDUST
VKP2

probability
0
0.2
0.4
0.6
0.8
1
600
800
1000
1200
1400
D3
ILC
WDUST
VKP2

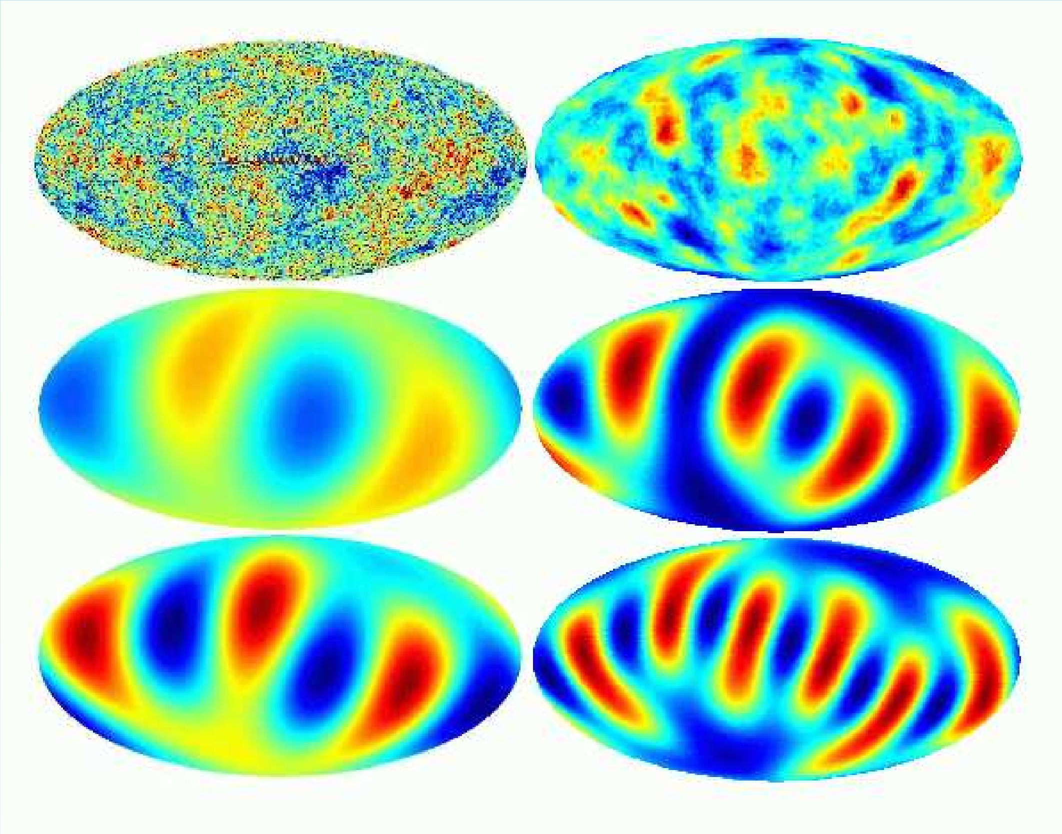
# 9 – Exact $C_\ell$ likelihood




(astro-ph/0403073)

# 10 – Quadrupole and Ocutpole Alignment



(Taken from do Oliveira-Costa et al)

- Visually aligned
- One would like to quantify this.

# 11 – de Oliveira-Costa vectors

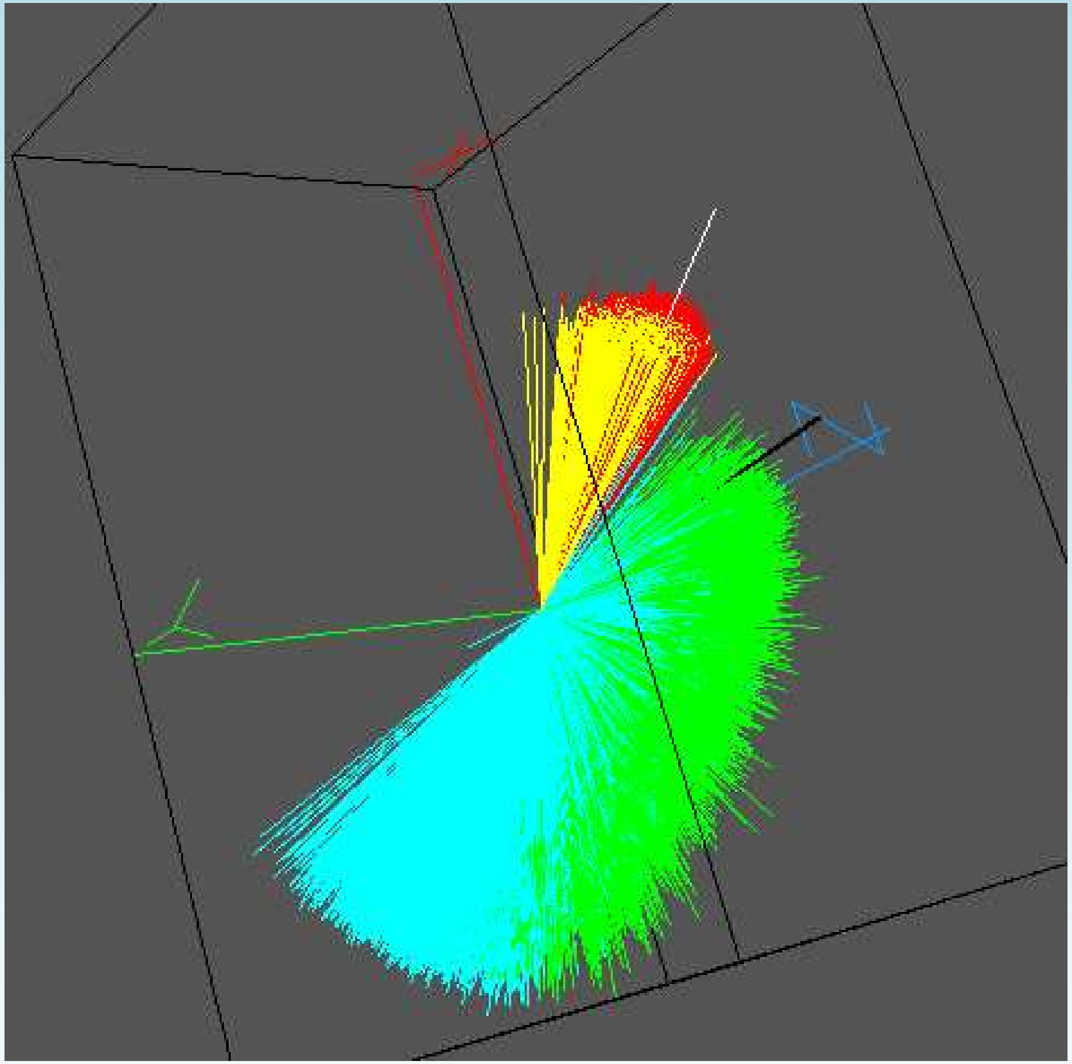
De Oliveira et all introduce an axis assigned to each multipole.

This axis maximises the angular momentum dispersion

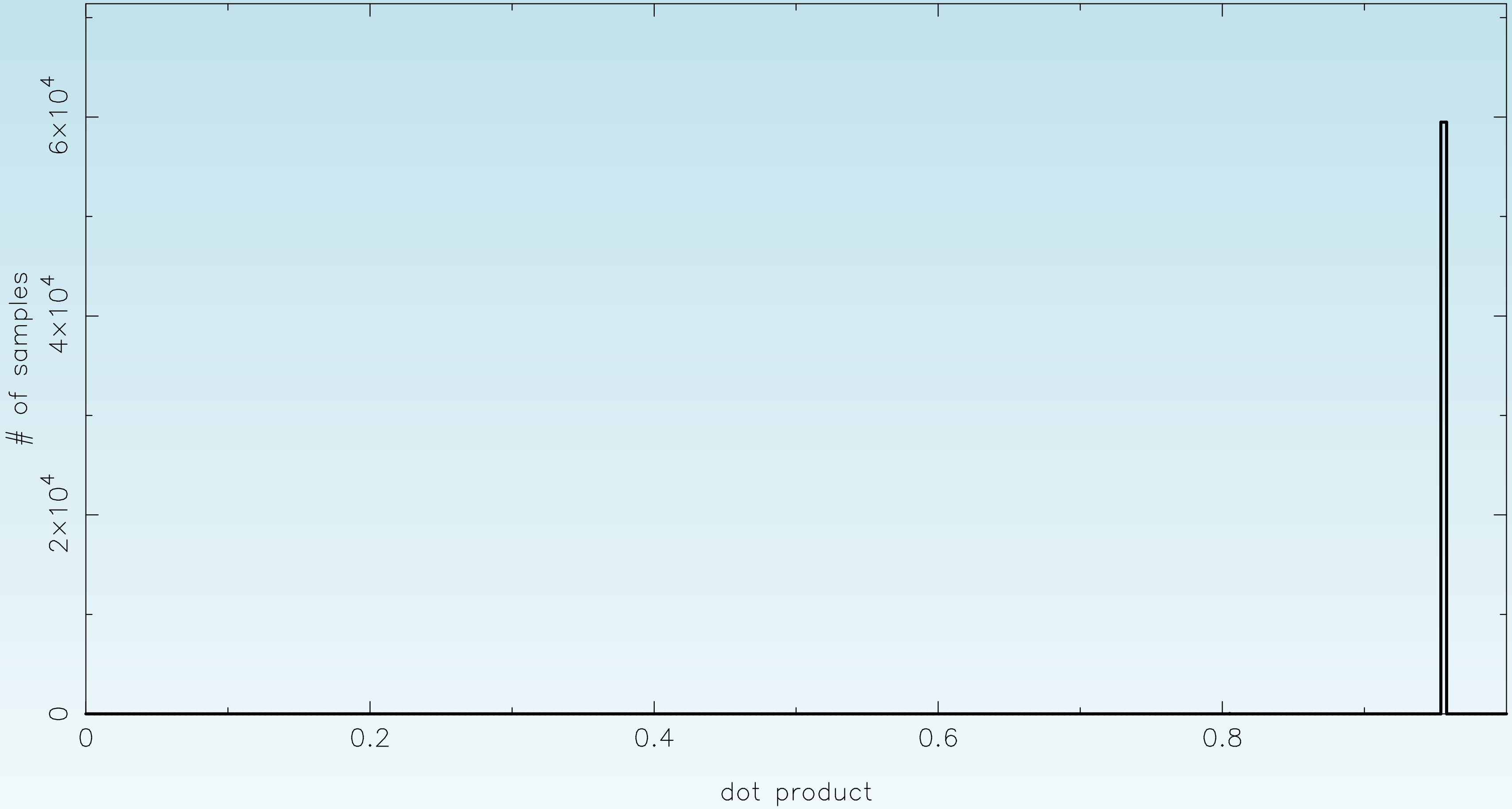
$$K = \sum_m a_{\ell m} a^*_{\ell m} m^2 \tag{3}$$

Using TOH version ILC map, the dot product for quadrupole and octopole 0.98.

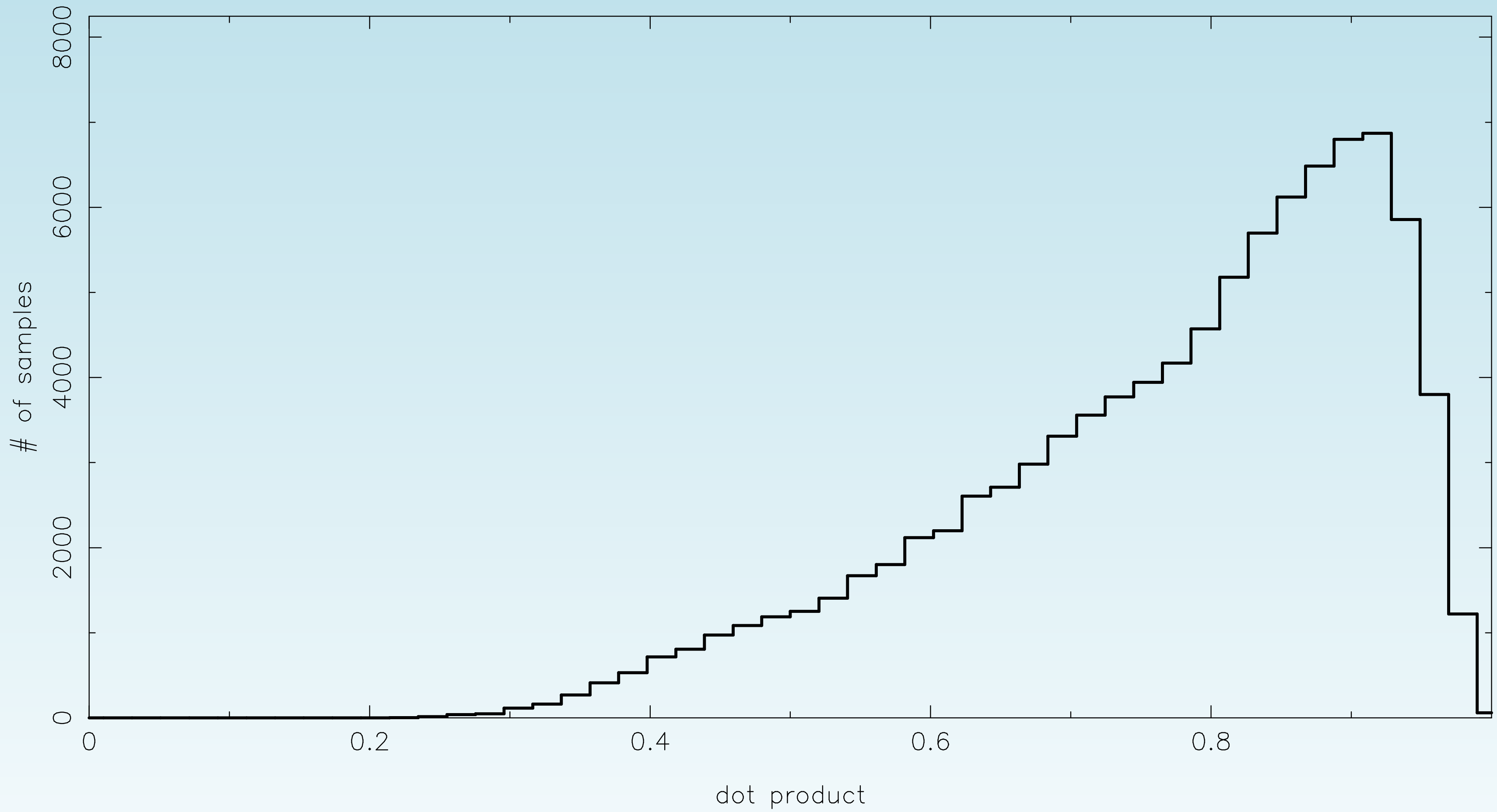
# 12 – 3D vectors



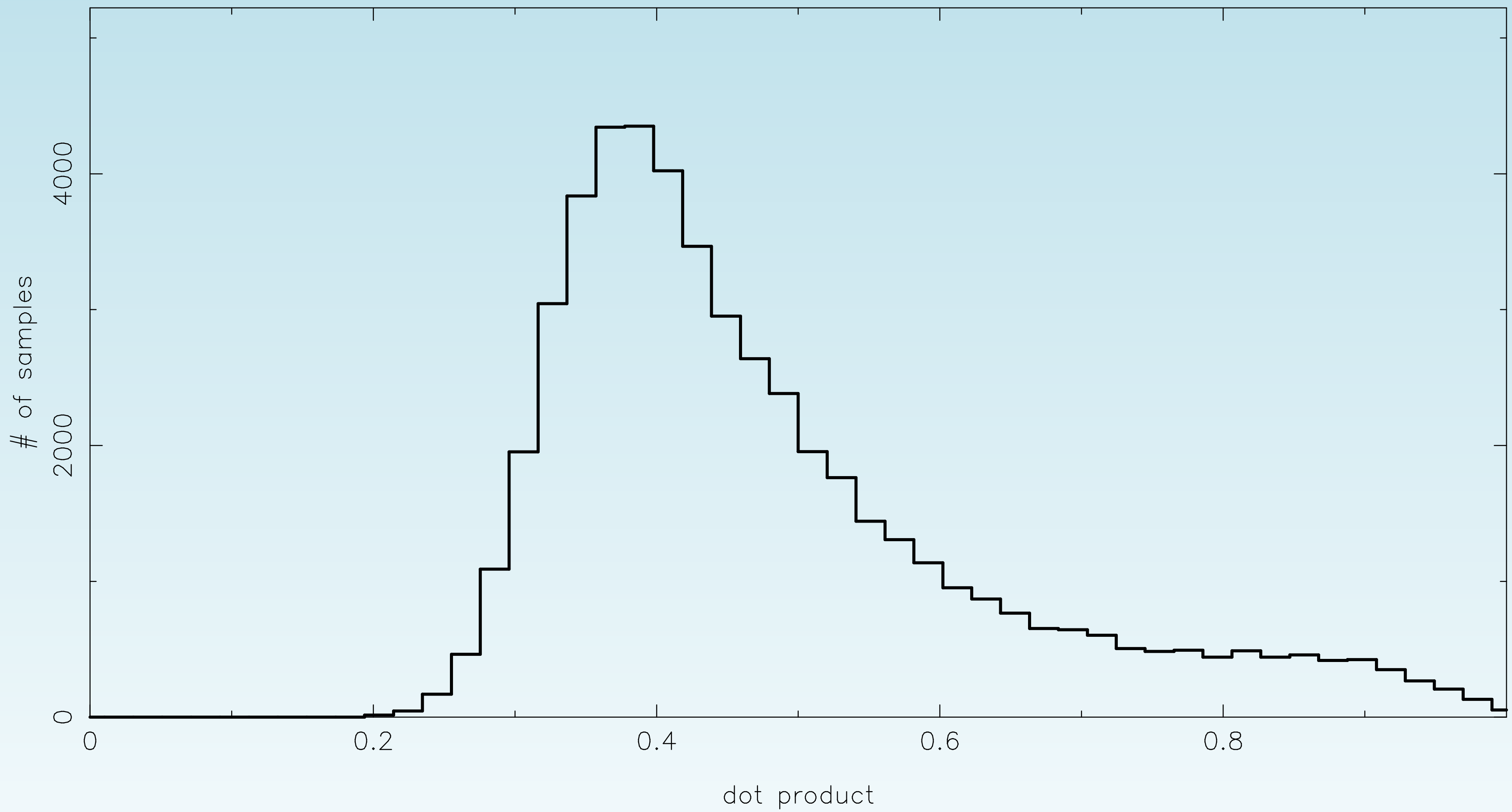
# 13 – Vector alignment: ILC



# 14 – Vector alignment: WDUST



# 15 – Vector alignment: VKP2



# 16 – Multipole vectors

An alternative ideas are the multipole vectors (Copi et al.)

It is based on the idea that every multipole of the order $\ell$ is fully determined by $\ell$ headless vectors $\hat{\mathbf{v}}^{\ell,i}$ such that

$$\sum_m Y_{\ell m}(\hat{e})\, a_{\ell m} = A^{(\ell)} \prod_{i=1}^{\ell} \left(\hat{\mathbf{v}}^{\ell,i} \cdot \hat{\mathbf{e}}\right) . \quad (4)$$

# 17 – Multipole vectors and alignment

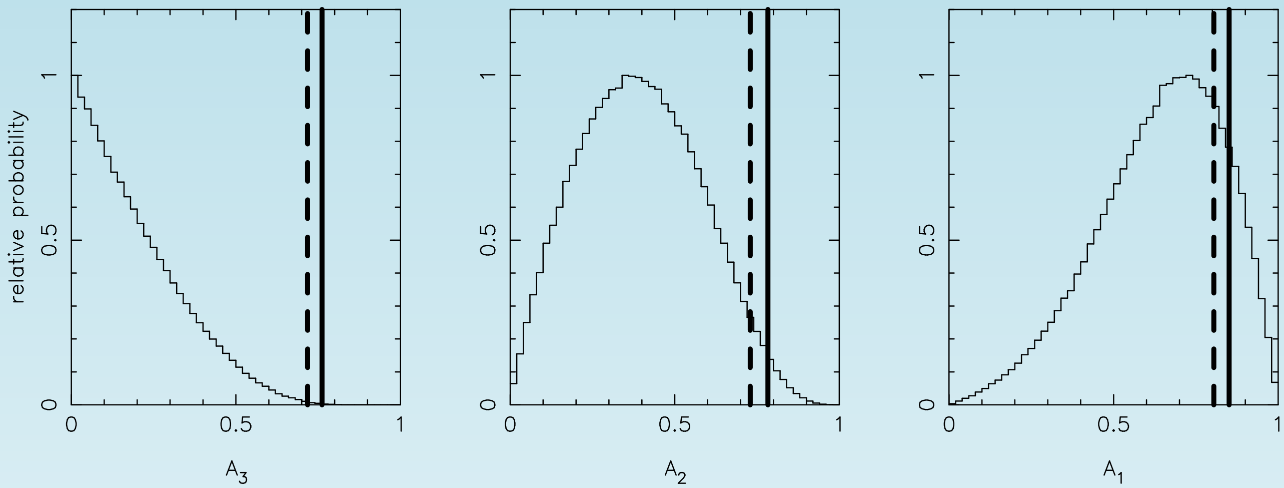
Pairs of these vectors can be used to form oriented areas:

$$\mathbf{w}^{\ell,i,j} = \hat{\mathbf{v}}^{\ell,i} \times \hat{\mathbf{v}}^{\ell,j}. \tag{5}$$

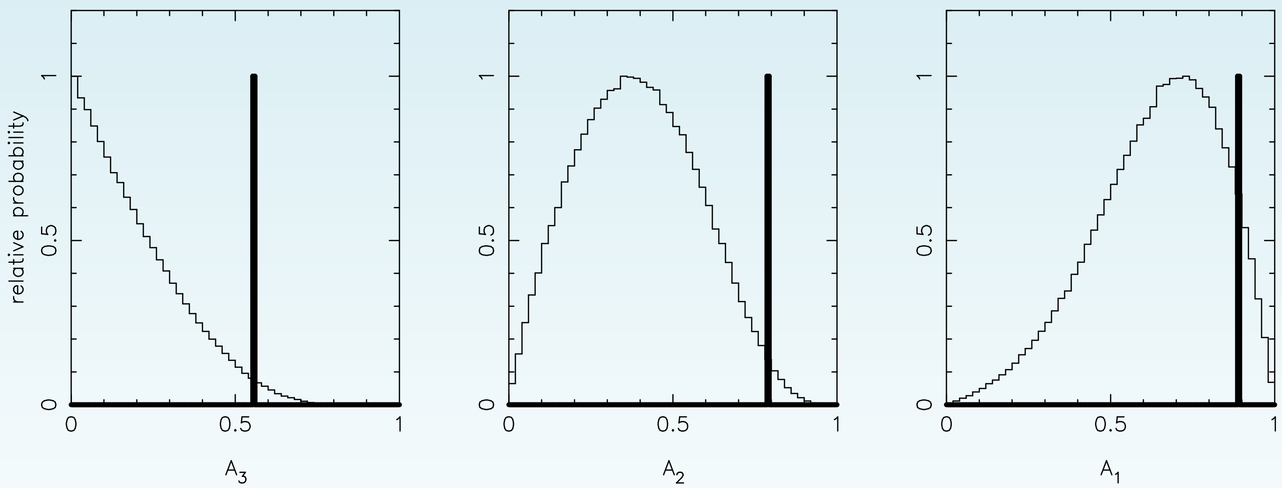
- There is one such “area” vector for the quadrupole and three for the octopole.
- If one takes the dot-products between $\mathbf{w}^{2,1,2}$ (quadrupole) and $\mathbf{w}^{3,i,j}$ (three octopole vectors) and orders them in decreasing magnitude one obtains three numbers denoted $A_1$, $A_2$ and $A_3$.
- $A_{1,2,3}$ unusually high (Schwarz et al.)
- **This procedure is highly a-posteriori.**

# 18 – Multipole vectors

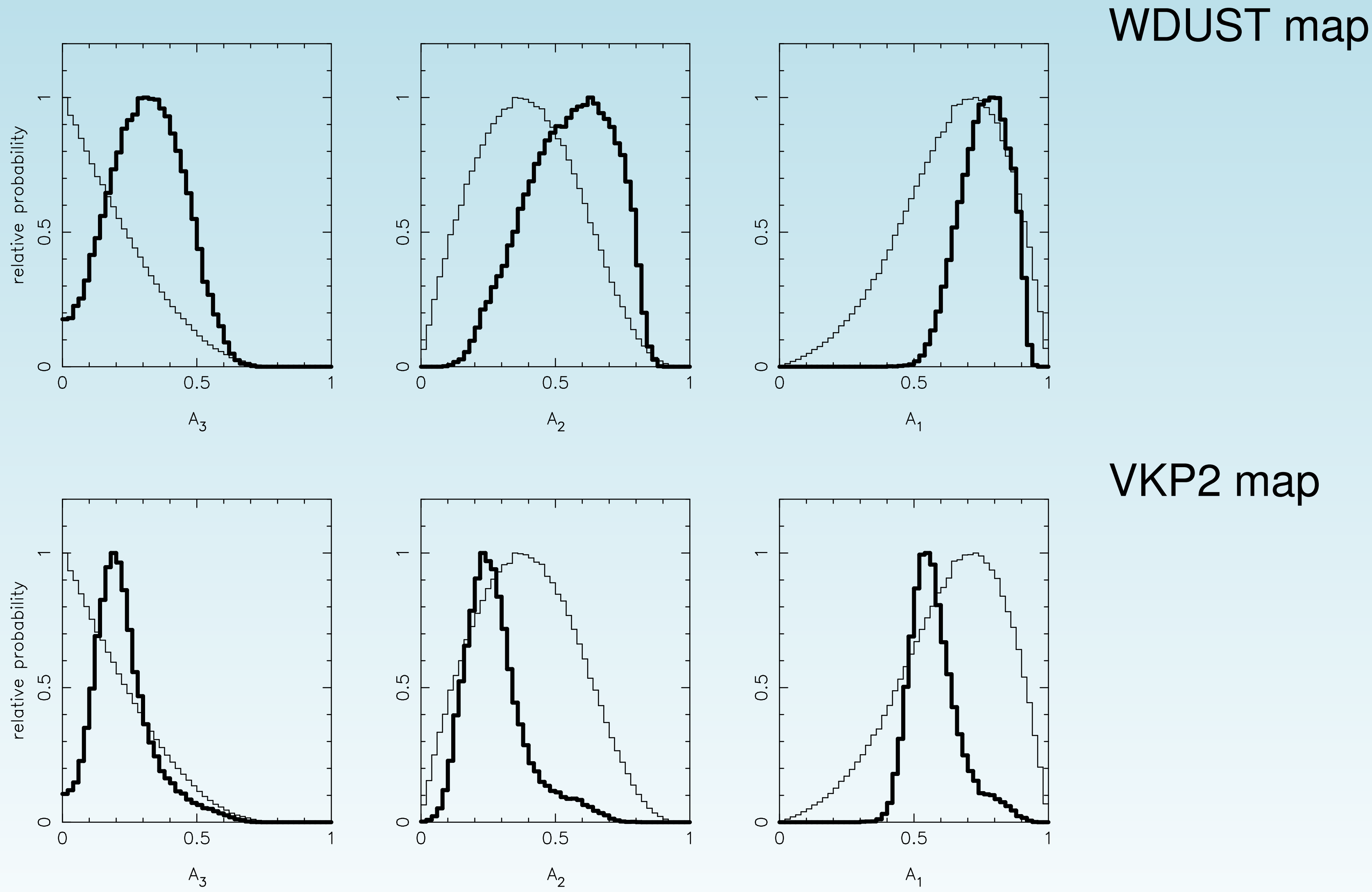
TOH map




ILC map



# 19 – Multipole vectors

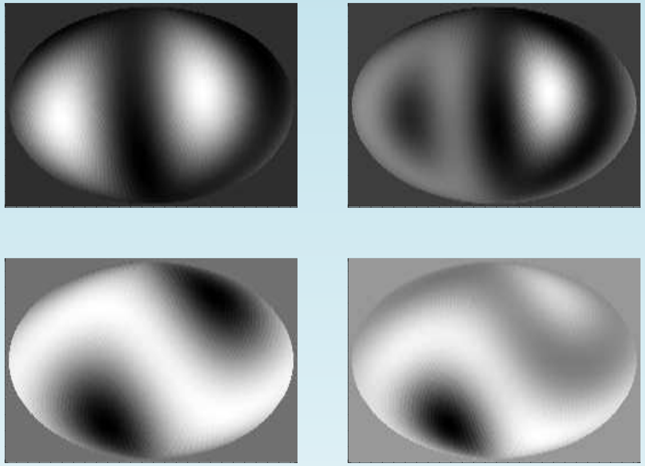


# 20 – My two cents

Feature matching:

Multiply quadrupole and octopole:

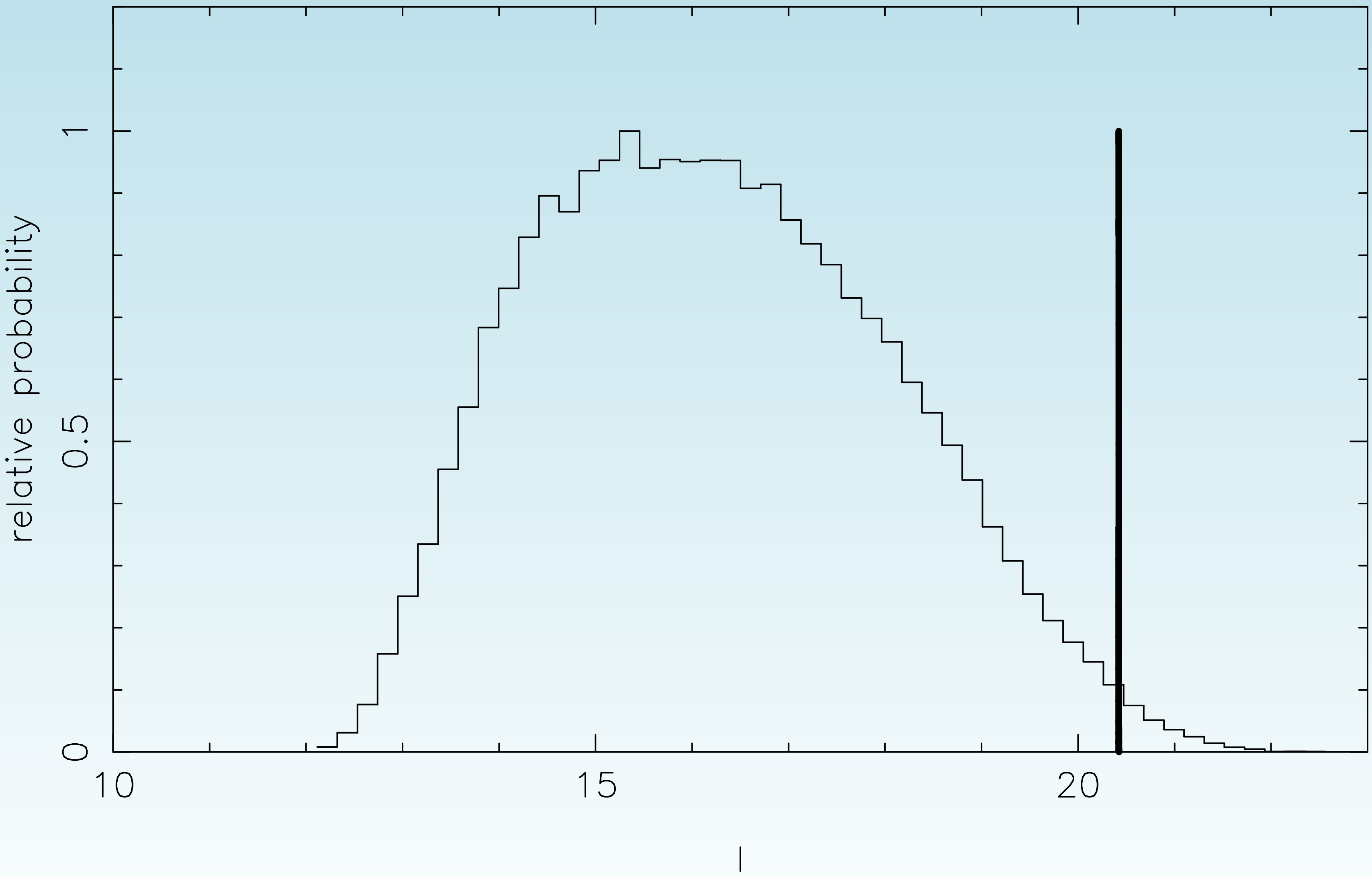
$$T_\times = \hat{T}_{\ell=2} \times \hat{T}_{\ell=3}. \tag{6}$$



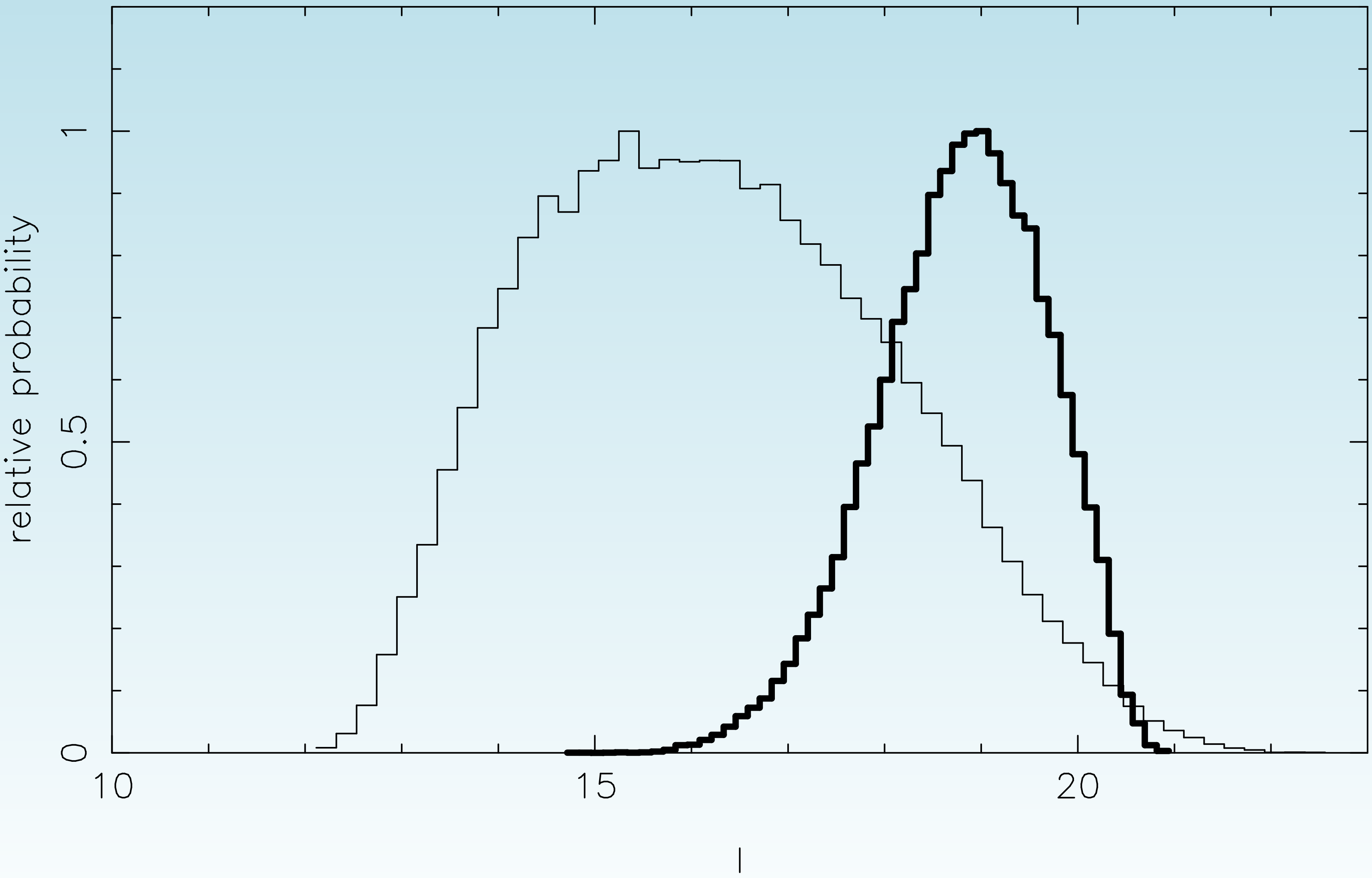
Form a quantity that peaks if features are matched:

$$I = \int \left(T_\times^2\right) \mathrm{d}A, \tag{7}$$

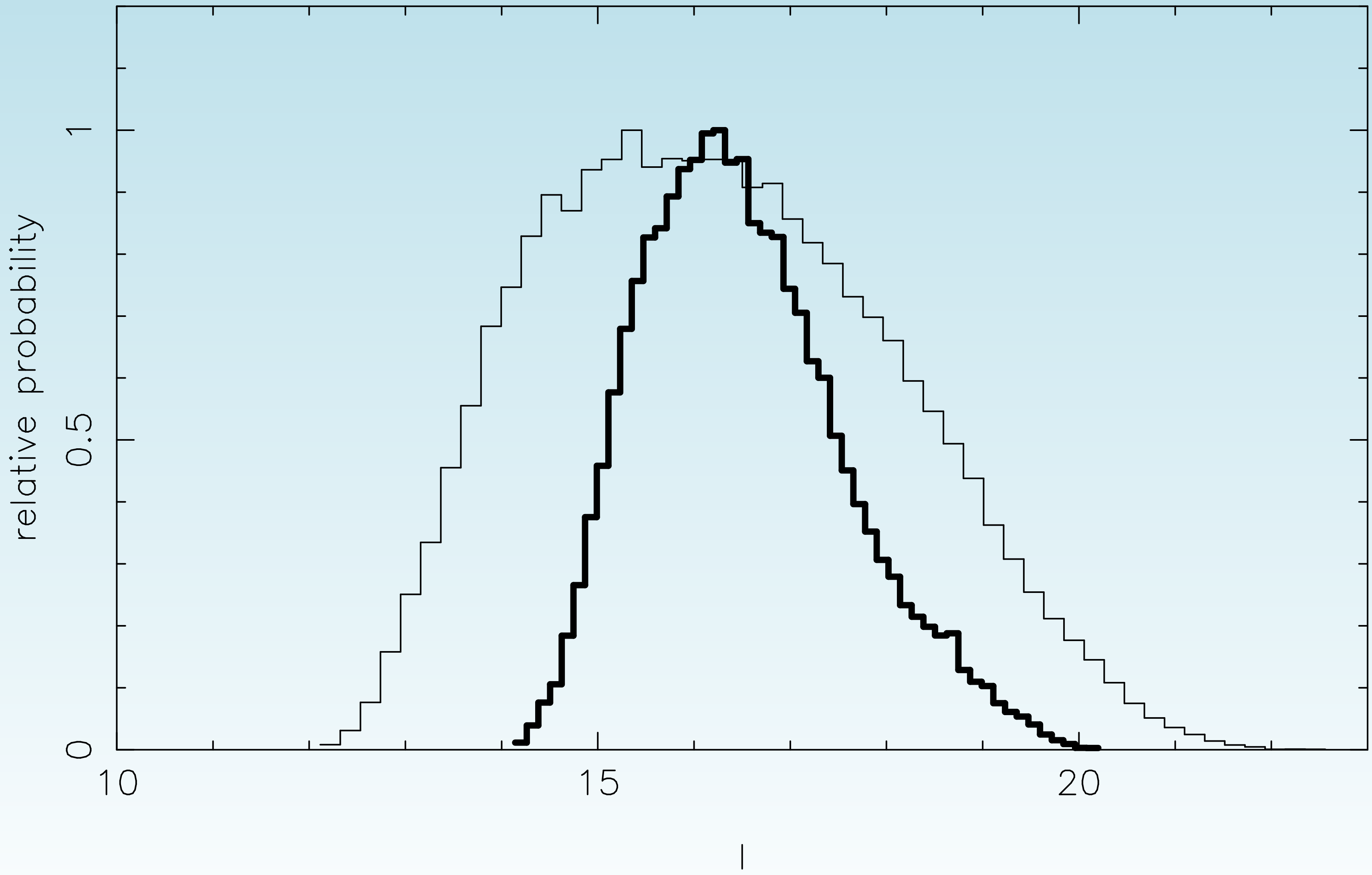
# 21 – Feature matching: ILC



# 22 – Feature matching: WDUST



# 23 – Feature matching: VKP2



# 24 – Alignment with ecliptic

Finally, there were claims that quadrupole and octopole are aligned with ecliptic. Here we test these claims using multipole vectors $\mathbf{w}^{\ell,i,j}$ (Schwartz et al).

Again we form quantities:

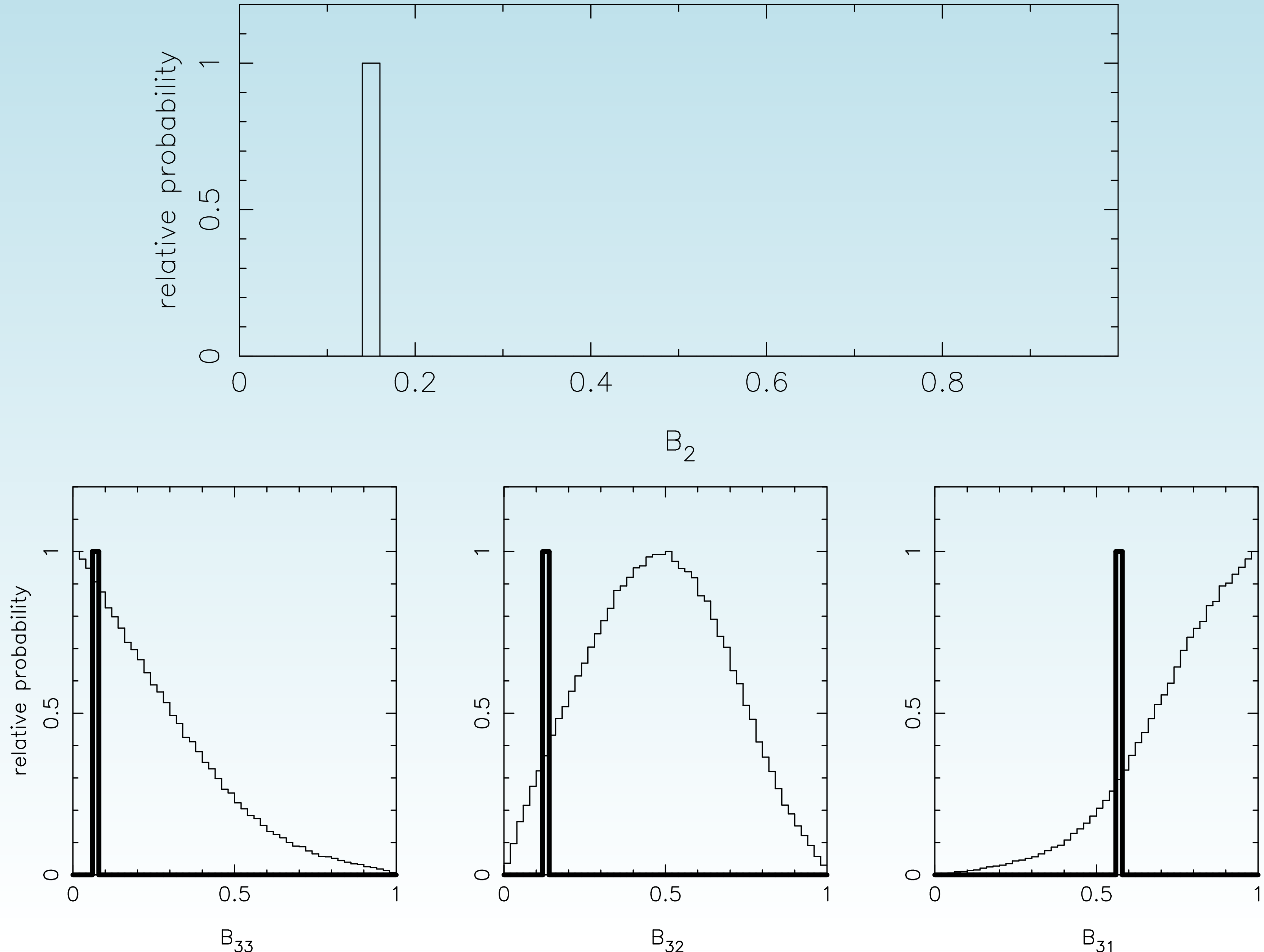
$$B_2 = \mathbf{w}^{2,0,1} \cdot \hat{n}_{\rm ecl} \tag{8}$$

$$B_{31} = \mathbf{w}^{3,0,1} \cdot \hat{n}_{\rm ecl} \tag{9}$$

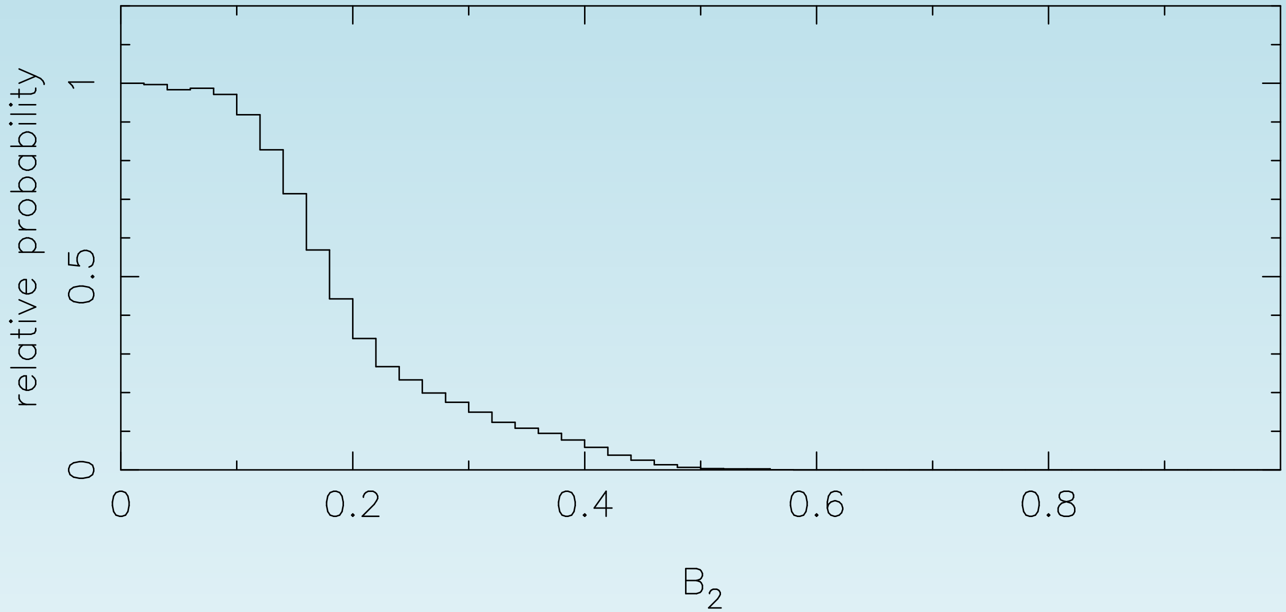
$$B_{32} = \mathbf{w}^{3,0,2} \cdot \hat{n}_{\rm ecl} \tag{10}$$

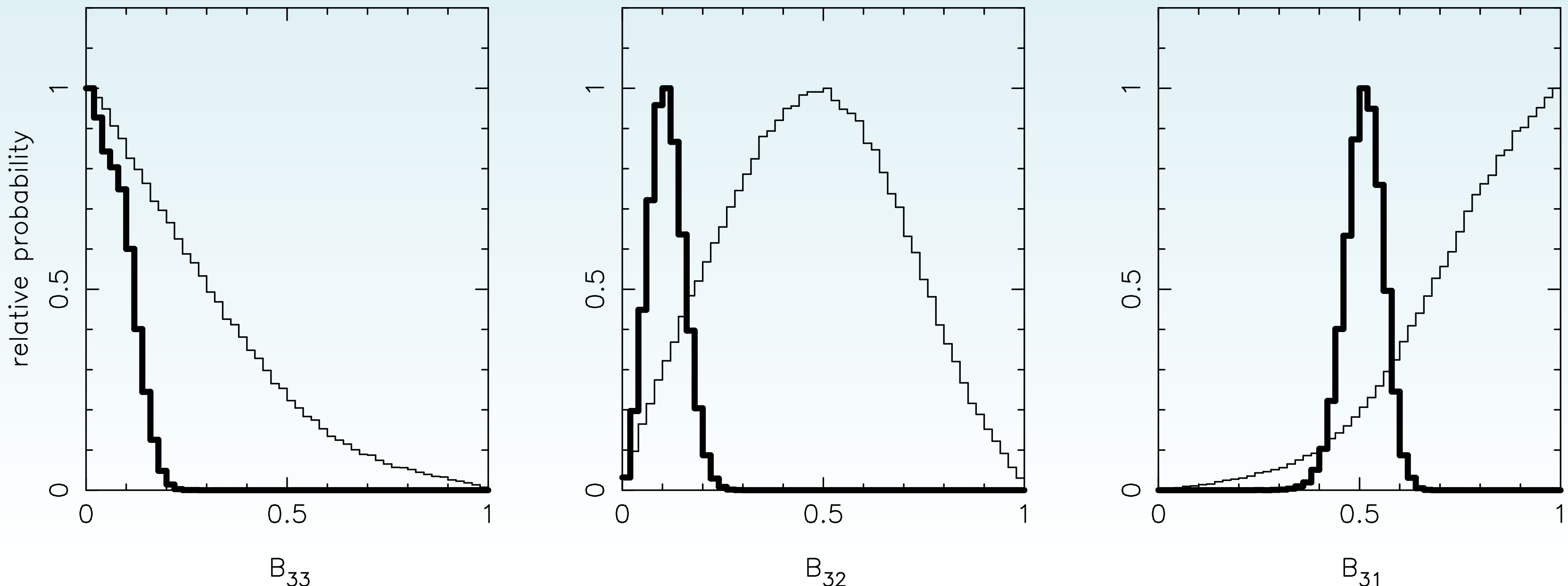
$$B_{33} = \mathbf{w}^{3,1,2} \cdot \hat{n}_{\rm ecl} \tag{11}$$

# 25 – Ecliptic alignment - ILC

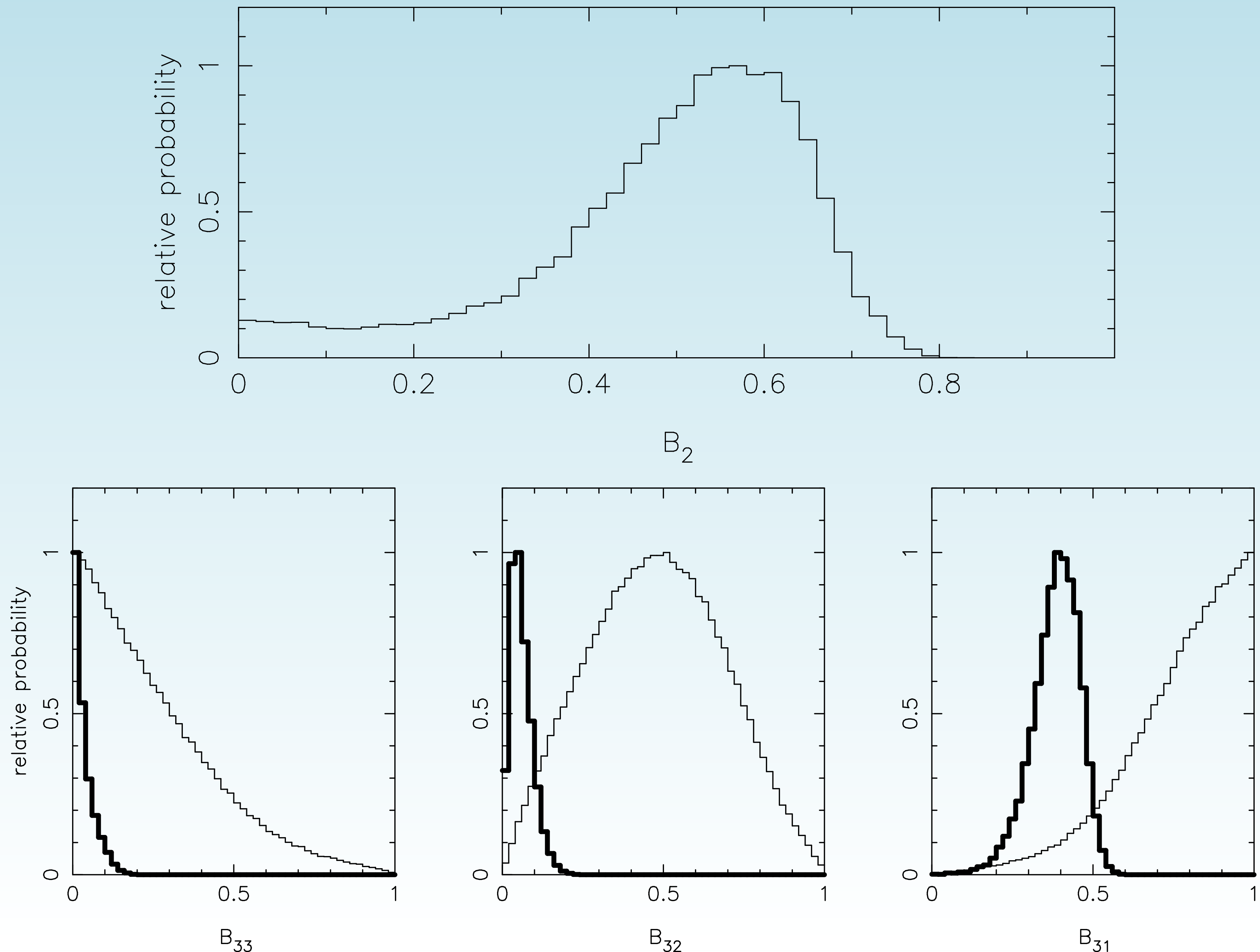


# 26 – Ecliptic alignment - WDUST



# 27 – Ecliptic alignment - VKP2



# 28 – Is it statistically significant?

- Taking evidence ratio between models with $B_{32} = B_{33}(= B_2) = 0$ and isotropic favours the former at quite high confidence (1 in $\sim 40$).
- However, these models very a-posteriori
- Taking in account a number of models one can “invent”, it drops to $1\sigma$.
- Schwarz et al disagree.

## 29 – Conclusions

- MCMC chains in $a_{2m}$ and $a_{3m}$ allow a novel study of low multipoles
- Using better computational techniques, one can go up to $\ell \sim 30$.
- Alignment between quadrupole and octopole seems to vanish, regardless of the statistic used.
- Alignment between quadrupole / octopole and ecliptic is to some extend subjective.