

# Lensing and cosmological tests of general relativity

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Paris, 4 July 2007

## Bernard:

- *Do you think black-holes can couple differently to gravity?*
  - *Do you think dark matter can be a fermionic condensate?*
  - *MOND vs DM (recurent and oscillating topic!)*
- 
- Why shall we test general relativity on astrophysical and cosmological scales
  - What should we test?
  - Dark matter and lensing
  - Cosmological tests
  - Conclusions

# INTRODUCTION

**Goal:** remind the hypothesis used in the interpretation of the cosmological data

See JPU, [astro-ph/0605313](#)

The interpretation of the dynamics of the universe and its large scale structure relies on the hypothesis that gravity is well described by General Relativity

## Galaxy rotation curves

Introduction of *Dark Matter*

Einsteinian interpretation

Most of the time Newtonian interpretation

## Acceleration of the cosmic expansion

Introduction of *Dark Energy*

Einsteinian interpretation

But more important Friedmanian interpretation

The standard cosmological model lies on 3 hypothesis:

**H1-** Gravity is well described by general relativity

**H2-** Copernican Principle

*On large scales the universe is homogeneous and isotropic*

## Consequences:

- 1- The dynamics of the universe reduces to the one of the scale factor
- 2- It is dictated by the Friedmann equations

$$3 \left( H^2 + \frac{K}{a^2} \right) = 8\pi G \rho$$
$$\frac{\ddot{a}}{a} = - \frac{4\pi G}{3} (\rho + 3P)$$

**H3-** Ordinary matter (standard model fields)

## Consequences:

- 3- On cosmological scales: pressureless + radiation
- 4- The dynamics of the expansion is dictated by

$$\Omega \equiv \frac{8\pi G \rho}{3H^2}$$

$$H^2(z)/H_0^2 = \Omega_m^0 (1+z)^3 + \Omega_r^0 (1+z)^4 + \Omega_K^0 (1+z)^2$$

Independently of any theory (**H1, H3**), the Copernican principle implies that the geometry of the universe reduces to  $a(t)$ .

**Consequences:**

•  $1 + z = \frac{E_{rec}}{E_{em}} \stackrel{H2}{=} \frac{a_0}{a(t)}$

•  $a(t) = a_0 \left[ 1 + H_0(t - t_0) - \frac{1}{2}q_0 H_0^2 (t - t_0)^2 + \dots \right]$

so that

$H^2(z)/H_0^2 = 1 + (q_0 + 1)z + \mathcal{O}(z^2)$

$q_0 = \Omega_{m0}/2$

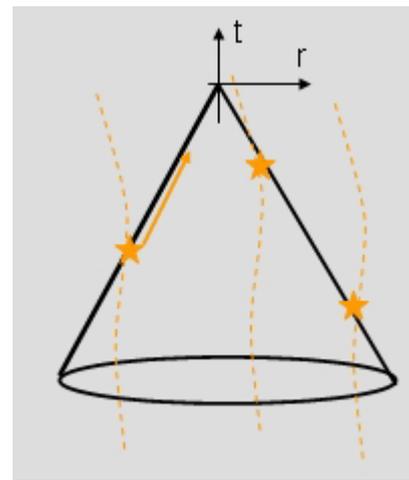
- **Hubble diagram** gives
  - $H_0$  at small  $z$
  - $q_0$

Supernovae data (1998+) show

$q_0 < 0$



The expansion is now **accelerating**

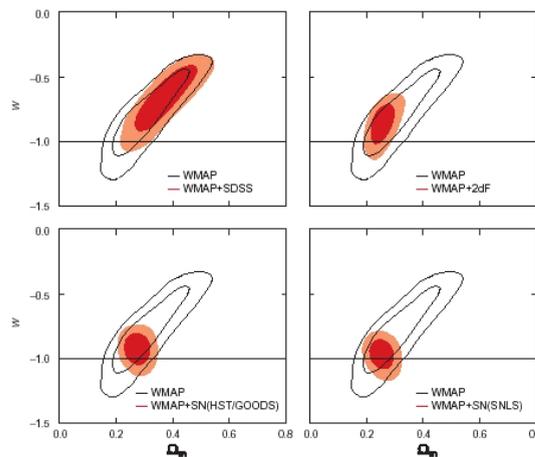
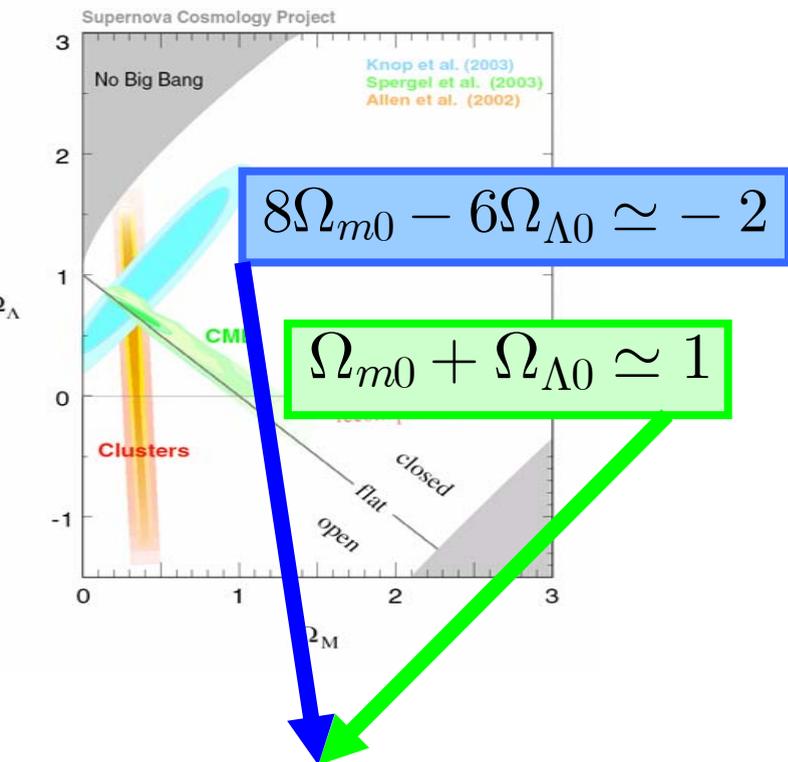


# $\Lambda$ CDM (REFERENCE) MODEL

The simplest extension consists in introducing a cosmological constant

- constant energy density
- well defined model and completely predictive

$$\rho_{\Lambda} = \frac{\Lambda}{8\pi G} = -P_{\Lambda}$$



Spergel et al., astro-ph/0603449

$$P_{de} = w\rho_{de}$$

$\Lambda$ CDM consistent with all current data

Observationally, very good  
Phenomenologically, very simple  
But: cosmological constant problem

$$\Omega_{m0} \sim 0.3, \quad \Omega_{\Lambda0} \sim 0.7$$

The dark sector reflects the fact that the current understanding of the cosmological data drives us to introduce new degrees of freedom.

## Dark matter

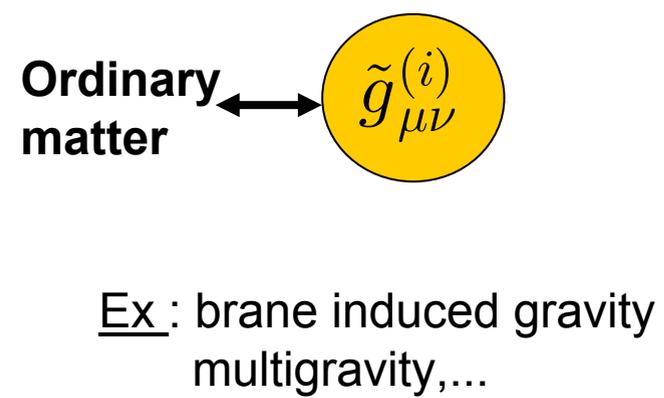
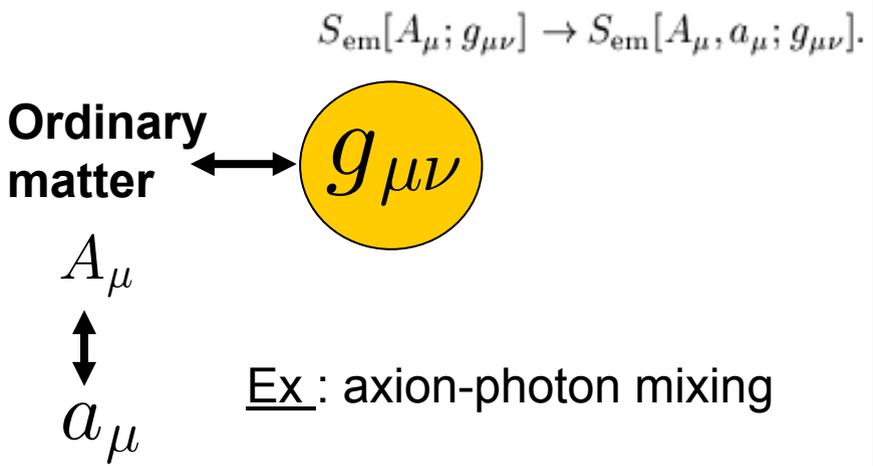
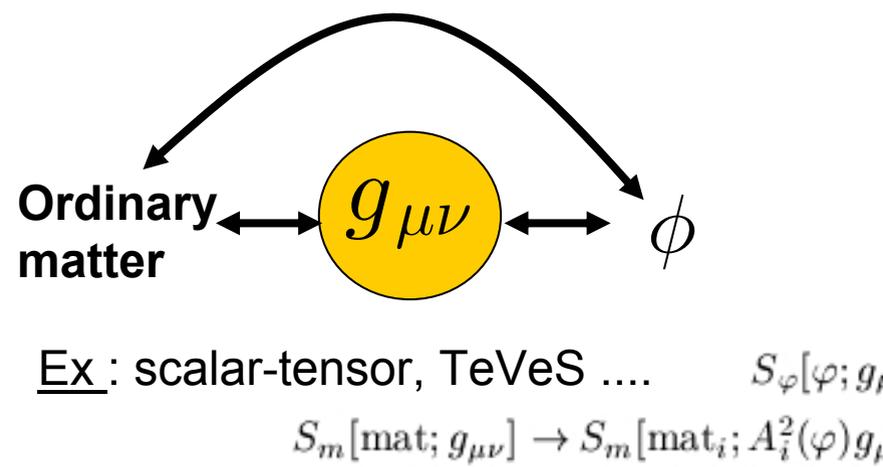
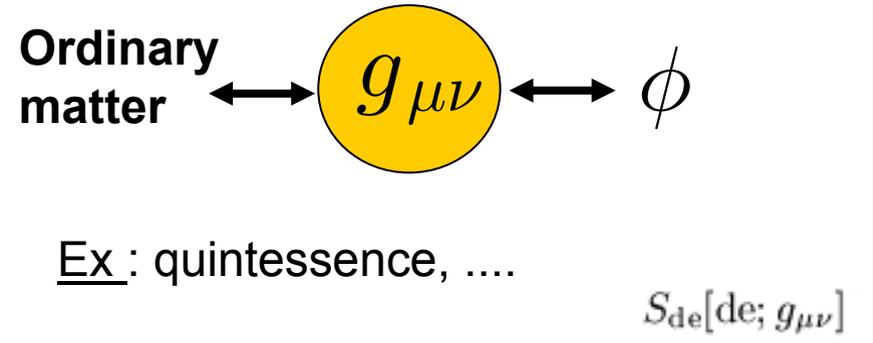
MOND and TeVeS alternative

## Dark energy

- 1- The Copernican principle does not hold
- 2- There exists matter such that  $\rho + 3P < 0$
- 3- Gravity is not well described by GR on large scales

	Measurement	Scale	$\Omega_m$
1	peculiar velocities: relative rms	20 kpc $\lesssim r \lesssim$ 1 Mpc	$0.20e^{\pm 0.4}$
2	redshift space anisotropy	10 Mpc $\lesssim r \lesssim$ 30 Mpc	$0.30 \pm 0.08$
3	mean relative velocities	10 Mpc $\lesssim r \lesssim$ 30 Mpc	$0.30^{+0.17}_{-0.07}$
4	numerical action solutions	$r \sim$ 1 Mpc	$0.15 \pm 0.08$
5	virgocentric flow	$r \sim$ 20 Mpc	$0.20^{+0.22}_{-0.15}$
6	weak lensing: galaxy-mass	100 kpc $\lesssim r \lesssim$ 1 Mpc	$0.20^{+0.06}_{-0.05}$
7	mass-mass	300 kpc $\lesssim r \lesssim$ 3 Mpc	$0.31 \pm 0.08$

**Gravitation = any long range force that cannot be screened**



**Always need NEW fields**

# CLASSICAL TESTS OF GR

**Goal:** remind the tests in the Solar system  
understand those that can be generalized

## Einstein equivalence principle

*universality of free fall*

*local Lorentz invariance*

*local position invariance*

## Metric theories of gravity

*spacetime is endowed with a symmetric metric*

*trajectories of free-falling test bodies are geodesic of that metric*

*in a freely reference frame, the laws of non-gravitational physics are those written in the language of special relativity*

## General relativity is a metric theory of gravity

$$S = \frac{1}{16\pi G} \int \sqrt{-g} R dx + \int L_m(\psi, g_{\mu\nu}) \sqrt{-g} dx$$

General relativity is well tested in the Solar system  
is our *reference* theory of gravity

## Universality of free fall

$$\eta = 2 \frac{|a_1 - a_2|}{|a_1 + a_2|}$$

## Local Lorentz invariance

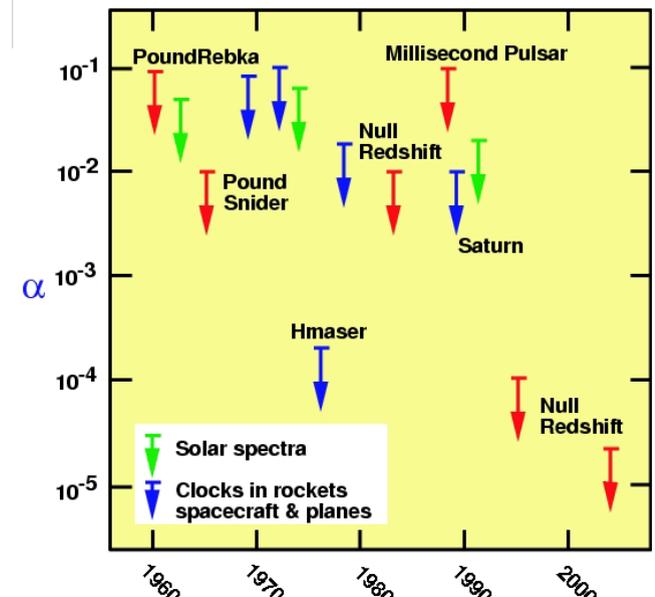
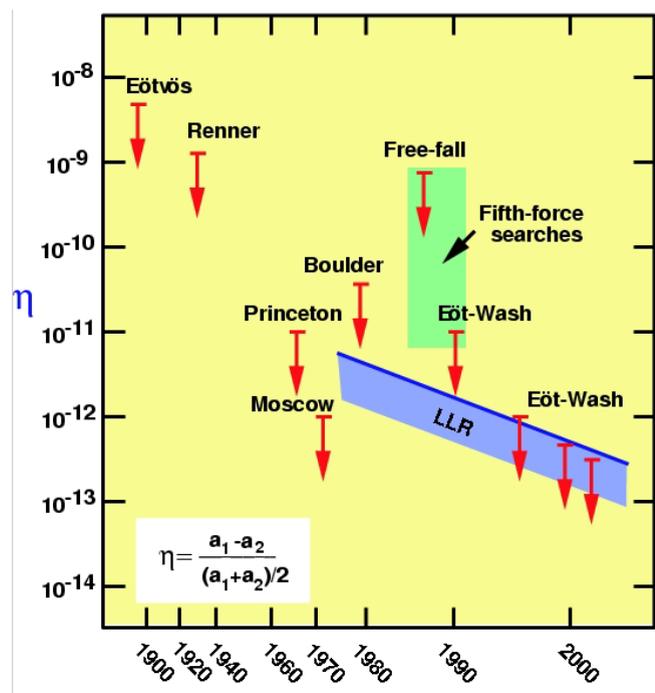
*Michelson-Morley experiments,  
isotropy of the speed of light  
independence of the speed of light on  
velocity of the source*

## Local position invariance

*gravitational redshift*

$$Z = \frac{\delta\nu}{\nu} = (1 + \alpha) \frac{\Delta U_{\text{newt}}}{c^2}$$

*constants*



Metric theories are usually tested in the PPN formalism

In its simplest form

$$ds^2 = (-1 + 2U + 2(\beta - \gamma)U^2)dt^2 + (1 + 2\gamma U)dr^2 + r^2d\Omega^2$$

$$U = \frac{GM}{rc^2}$$

If gravity is described by GR then  $\beta = \gamma = 1$

These parameters can be constrained, independently of a precise theory, from Solar system observations

## Light deflection

$$\Delta\theta = 2(1 + \gamma)\frac{GM}{bc^2}$$

## Perihelion shift of Mercury

$$\Delta\varphi = \frac{2\pi GM}{a(1-e^2)}(2 + 2\gamma - \beta)$$

## Shapiro time delay

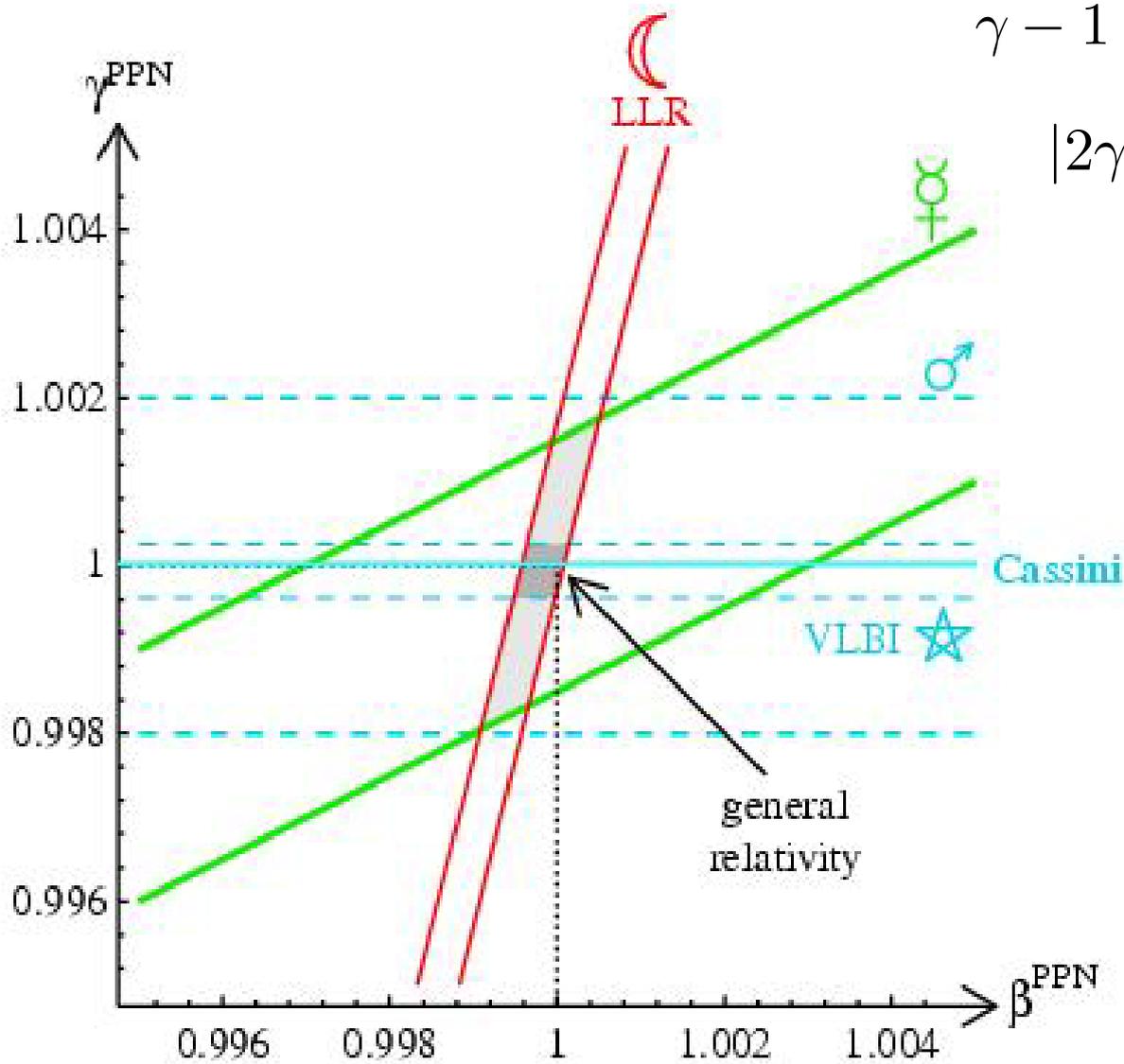
$$\delta t \propto (1 + \gamma)$$

## Nordtvedt effect

$$\delta r \sim 13.1(4\beta - \gamma - 3) \cos(\omega_0 - \omega_s)t \quad (\text{m})$$

# TESTS OF GR IN THE SOLAR SYSTEM

Courtesy of G. Esposito-Farèse



$$\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$$

$$|2\gamma - \beta - 1| < 3 \times 10^{-3}$$

Among the previous tests, it seems possible to generalize

- **Light deflection**

need to determine independently mass and deflection  
*cosmology*: - we do not measure the deflection but the  
distortion of light bundles  
- energy of the photons

- **Motion of test-bodies**

growth of structures / velocity fields

- **Constants**

But:

- time evolution (growth of structure):

*information on the dynamics*

*evolution effects*

- statistical interpretation and dependence on the initial conditions

- super-Hubble modes

# DARK MATTER AND LENSING

For any spherically symmetric metric of the form

$$ds^2 = - B(r)c^2 dt^2 + A(r)dr^2 + r^2 d\Omega^2$$

the deflection angle is

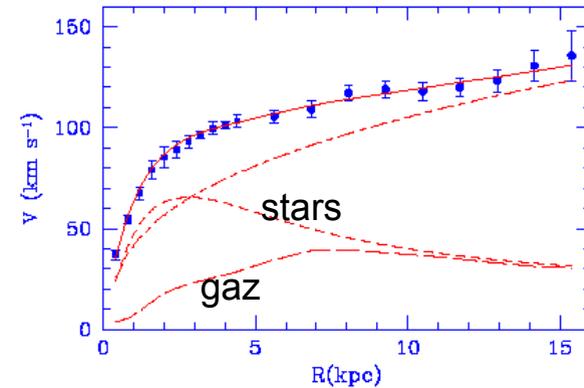
$$\delta\theta = -\pi + 2 \int_b^\infty \frac{dr}{r^2} \sqrt{\frac{A(r)B(r)}{B(r_0)/r_0^2 - B(r)/r^2}}$$

## Rotation curves

$$v^2(r) \rightarrow v_\infty^2 \equiv \sqrt{GMa_0}$$

If this dynamics is due to the existence of dark matter, then

$$\delta\theta_{GR} \rightarrow \frac{2\pi\sqrt{GMa_0}}{c^2}$$



## MOND alternative

$a_0$ : limit acceleration

$$a < a_0 : \quad a = \sqrt{a_N a_0} = \sqrt{GMa_0} / r$$

Equivalent to have an effective potential

$$\Phi = -\frac{GM}{r} + \sqrt{GMa_0} \ln r$$

$$r > \sqrt{\frac{GM}{a_0}}, \quad \delta\theta_{MOND} = \frac{2\pi\sqrt{GMa_0}}{c^2}$$

new fields

$$\text{Gravity} + \rho_{\text{dark}}(r) + \rho_b(r)$$

$$\text{Newton} + \rho_{\text{DM}}(r) + \rho_b(r)$$

$$\text{MOND} + 0 + \rho_b(r)$$

$$T_{\mu\nu}^{\text{dark}} \gg T_{\mu\nu}^{\text{b}}$$

$$v^2(r)$$

$$T_{\mu\nu}^{\text{dark}} \ll T_{\mu\nu}^{\text{b}}$$

**Dark matter**

**Gravity**

**Constraint DM**

In the Solar system, we can determine the mass of the Sun and the deflection angle independently

This is why we have a test of GR

Now, one has (at least) 3 notions of mass:

- Baryonic mass,  $M_b$ ,  
assumed to be proportional to the luminous mass
- Dynamical mass,  $M_{rot}$ ,  
evaluated from rotation curves
- Deflecting mass,  $M_{lens}$ ,  
evaluated from lensing

In the standard DM interpretation

$$M_b < M_{DM} \simeq M_{rot} \simeq M_{lens}$$

Let us consider lensing in a large family of gravity theories including General Relativity

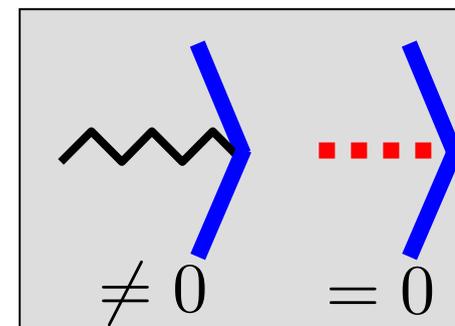
$$S = \frac{c^3}{16\pi G} \int \sqrt{-g} \{ R - 2(\partial_\mu \phi)^2 - V(\phi) \} + S_m \{ \text{matter}, \tilde{g}_{\mu\nu} = A^2(\phi) g_{\mu\nu} \}$$

RAQUAL version  $(\partial_\mu \phi)^2 \rightarrow f[(\partial_\mu \phi)^2, \phi]$

**Maxwell electromagnetism is conformally invariant in d=4**

$$S_{em} = \frac{1}{4} \int \sqrt{-\tilde{g}} \tilde{g}^{ab} \tilde{g}^{cd} F_{ac} F_{bd} d^d x$$

$$= \frac{1}{4} \int \sqrt{-g} g^{ab} g^{cd} F_{ac} F_{bd} A^{d-4}(\phi) d^d x$$

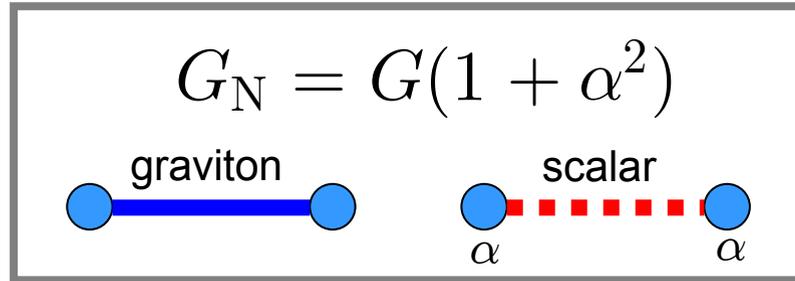


Light deflection is given as in GR

$$\delta\theta = \frac{4GM}{bc^2}$$

# WHAT IS THE DIFFERENCE?

The difference with GR comes from the fact that massive matter feels the scalar field



$$\alpha = d \ln A / d\phi$$

Motion of massive bodies determines  $G_N M$  **not**  $GM$

Thus, in terms of observable quantities, light deflection is given by

$$\delta\theta = \frac{4G_N M}{(1+\alpha^2)bc^2} \leq \frac{4GM}{bc^2}$$

Which means

$$M_{\text{lens}} \leq M_{\text{rot}}$$

A nice trick allows to increase light deflection in scalar-tensor theories

$$\tilde{g}_{\mu\nu} = A^2(\varphi) [g_{\mu\nu} + B(\varphi) \partial_\mu \varphi \partial_\nu \varphi]$$

Bekenstein, gr-qc/921101

Bekenstein, Sanders,  
gr-qc/931106

Preferred direction  
(radial for spherical system)

The only difference with GR is in the radial component and thus

$$\delta\theta = \delta\theta_{GR} + \int_b^\infty \frac{dr}{r\sqrt{r^2/b^2-1}} B(\partial_r\phi)^2$$

Now, assume that

$$B(\phi)(\partial_r\phi)^2 = 4\sqrt{GMa_0}/c^2$$

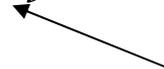
then

$$\delta\theta = \delta\theta_{GR}^b + \frac{2\pi\sqrt{GMa_0}}{c^2} \simeq \delta\theta_{GR+DM}$$

The former trick was extended by Bekenstein (TeVes theory...)

$$\tilde{g}_{\mu\nu} = A^2(\varphi)g_{\mu\nu} + B(\varphi)V_\mu V_\nu$$

Dynamical unit timelike vector



This is at the basis of the construction of TeVeS theories

When dealing with a specific theory, before determining how well it fits the data, one should investigate if it does not have any pathologies

See Bruneton & Esposito-Farèse, [arXiv:0705.4043](https://arxiv.org/abs/0705.4043)

In conclusion, all we are doing is to test the **compatibility** of the mass distribution measured by different methods.

Early studies:

- Comparison of X-ray and strong lensing

Miralda-Escude & Babul, ApJ **449** (1995) 18

- add weak lensing

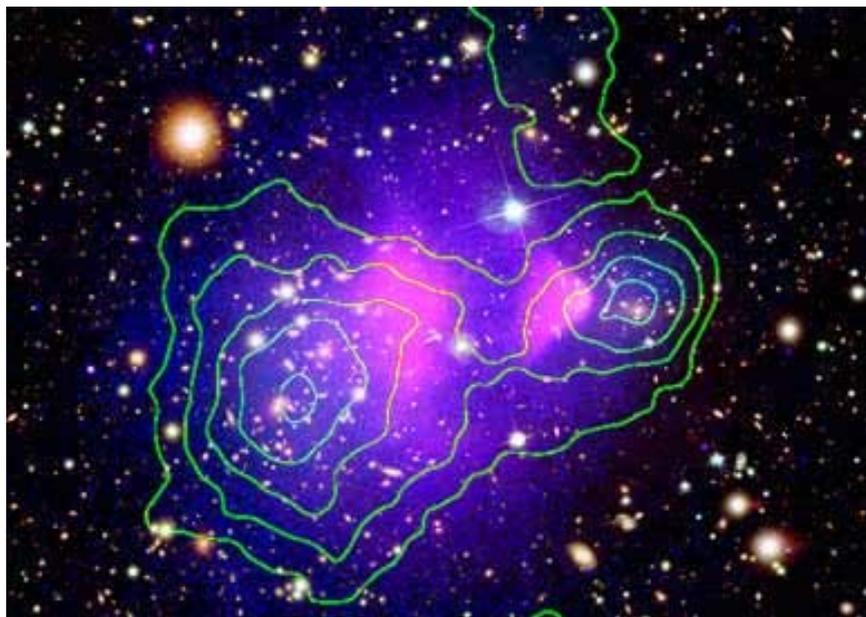
Squires et al., ApJ **461** (1996) 572

- Cluster scale (2 Mpc): X-ray vs lensing.

Allen et al. MNRAS **324** (2001) 877

- Use of SZ

Recent data allow to go beyond the spherically symmetric case



Cluster merger at  $z=0.296$   
Spatial segregation of collisionless matter/plasma  
Lensing reconstruction does not follow the plasma distribution

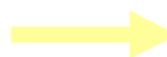
Proof of the existence of DM (...)

Mond in non-spherical geometry (dependence on the version of the theory and on fitting function)

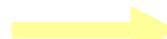
Angus et al., astro-ph/06062

Necessity for 2 eV neutrinos

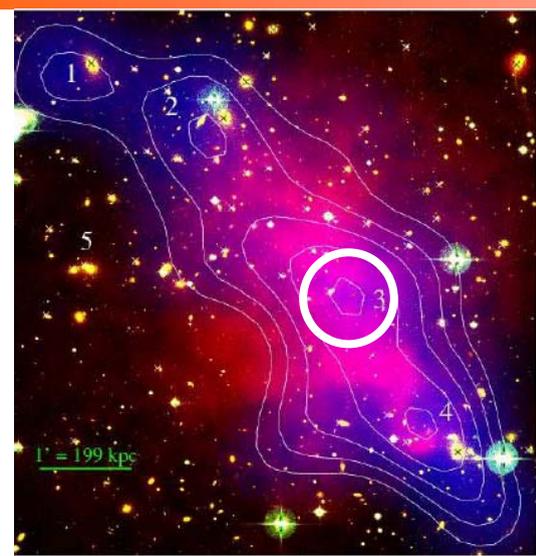
Angus et al., astro-ph/060912



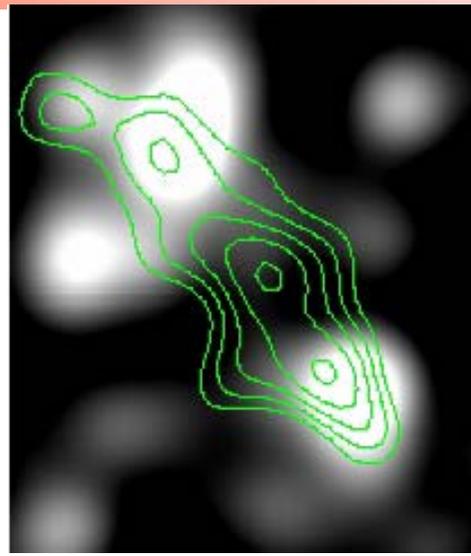
See Robert Sanders talk for more



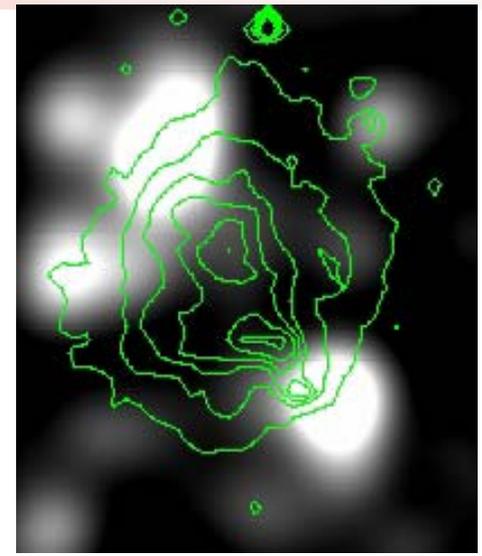
See Douglas Clowe talk for more



X vs lensing



red light vs lensing



red light vs X

Existence of a dark core that coincides with the peak of X-emission

Bernard: MOND regime at 90 kpc....

It is always possible to design coupling to reproduce the deflection angle by DM+GR

We have mostly considered spherically symmetric solutions

The most important issue is how well we can measure the profiles  $M_b(r)$ ,  $M_{\text{rot}}(r)$  and  $M_{\text{lens}}(r)$

Recent observations drive to go beyond spherical symmetry

Then, conclusions are not straightforward:

- depend on the *version of MOND*
- depend on the *choice of the fitting functions*

See also discussion CL0024+17 this morning

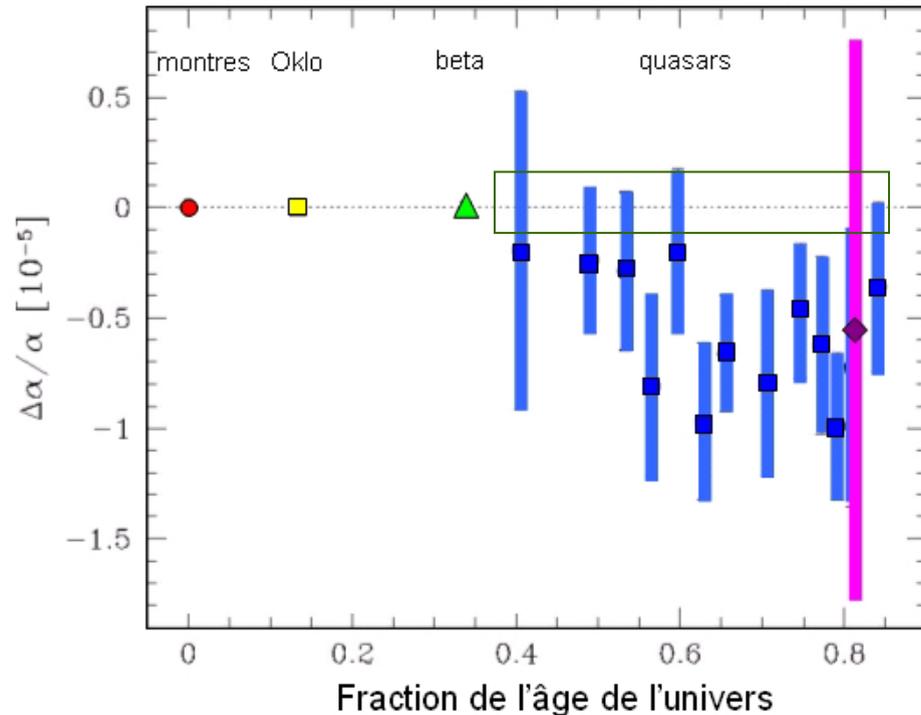
# **COSMOLOGICAL TESTS**

# CONSTANTS (LOCAL POSITION INVARIANCE)

Many tests concerning various constants ( $\alpha$ ,  $\mu$ ,  $G$  mainly).

Tests on different time scales:

<b>local</b>	( $z=0$ )	atomic clocks, Solar System
<b>geophysical</b>	( $z=0.1..0.4$ )	Oklo, meteorites
<b>astrophysical</b>	( $z=0.2-3.5$ )	quasars
<b>cosmological</b>	( $z=10^3, 10^8$ )	CMB, BBN.



General investigation of the link of these constraints and gravity theories

JPU, RMP **75** (2003) 425  
astro-ph/0409424

Most observations involve only low- $z$  and sub-Hubble regime  
(but CMB and BBN)

$$ds^2 = a^2(\eta) [ - (1 + 2\Phi) d\eta^2 + (1 - 2\Psi) \gamma_{ij} dx^i dx^j ]$$

## Background

$$H^2/H_0^2 = \Omega_m^0 (1+z)^3 + (1 - \Omega_m^0 - \Omega_\Lambda^0) (1+z)^2 + \Omega_\Lambda^0$$

## Sub-Hubble perturbations

$$\Phi = \Psi$$

$$\Delta\Psi = 4\pi G\rho a^2\delta$$

$$\delta' + \theta = 0$$

$$\theta' + \mathcal{H}\theta = -\Delta\Phi$$

In the linear regime, the growth of density perturbation is then dictated by

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho_{\text{mat}}\delta = 0$$

This implies a *rigidity* between the growth rate and the expansion history

Bertschinger, astro-ph/0604485,  
JPU, astro-ph/0605313

It can be considered as an equation for  $H(a)$

Chiba & Takahashi, astro-ph/070334

$$(H^2)' + 2 \left( \frac{3}{a} + \frac{\delta''}{\delta'} \right) H^2 = 3 \frac{\Omega_0 H_0^2 \delta}{a^5 \delta'}$$

$$\frac{H^2}{H_0^2} = 3\Omega_{m0} \frac{(1+z)^2}{\delta'(z)^2} \int_z \frac{\delta}{1+z} (-\delta') dz$$

*Proposal:*  $D(z)$  from galaxy cluster survey

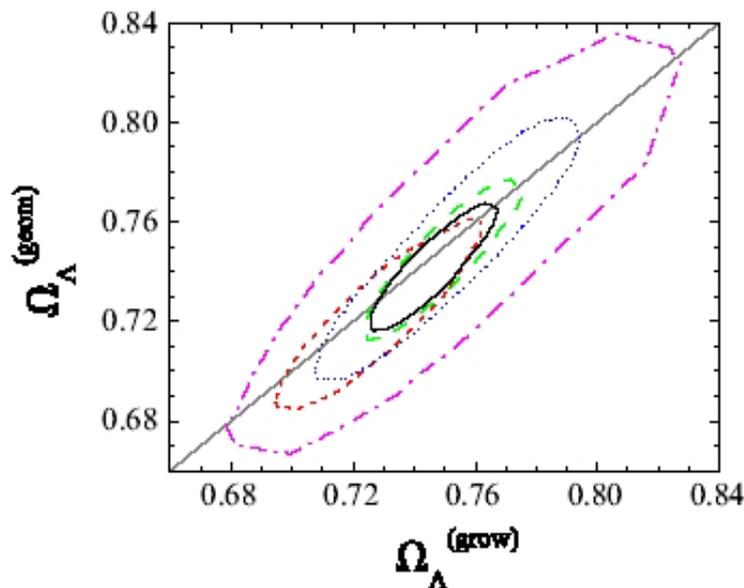
Tang et al, astro-ph/0609028

$H(a)$  from the background (geometry) and growth of perturbation have to agree.

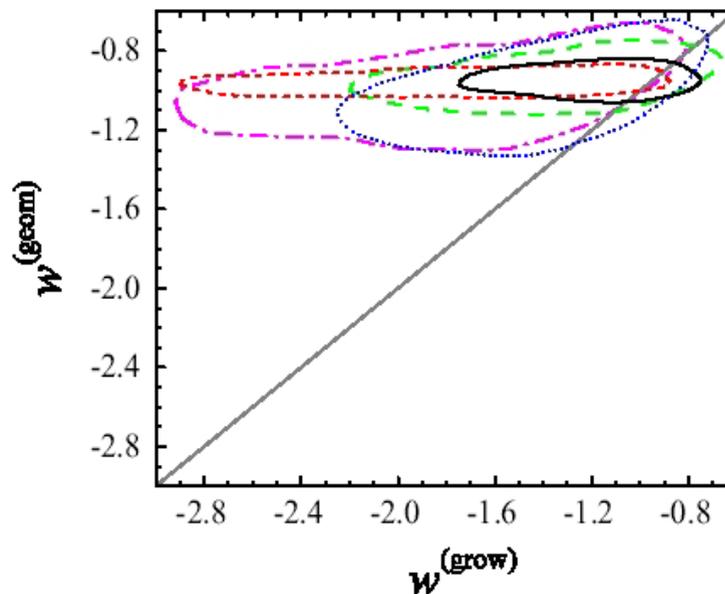
SNLS – WL from 75 deg<sup>2</sup> CTIO – 2dfGRS – SDSS (luminous red gal)  
 CMB (WMAP/ACBAR/BOOMERanG/CBI)

Wang et al., arViv:0705.0165

**Flat  $\Lambda$ CDM model**



**Flat  $w = \text{constant}$**



Consistency check of any DE model within GR with non clustering DE  
 Assume Friedmannian symmetries! (see e.g. [Dunsby and JPU](#))

To go beyond we need a parameterization of the possible deviations

Restricting to low- $z$  and sub-Hubble regime

$$ds^2 = a^2(\eta) [ - (1 + 2\Phi)d\eta^2 + (1 - 2\Psi)\gamma_{ij}dx^i dx^j ]$$

### Background

$$H^2/H_0^2 = \Omega_m^0(1+z)^3 + (1 - \Omega_m^0 - \Omega_\Lambda^0)(1+z)^2 + \Omega_{de}(z)$$

### Sub-Hubble perturbations

$$\Delta(\Phi - \Psi) = \pi_{de}$$

$$-k^2\Phi = 4\pi G_N F(k, H) \rho a^2 \delta + \Delta_{de}$$

$$\delta' + \theta = 0$$

$$\theta' + \mathcal{H}\theta = -\Delta\Phi + S_{de}$$

JPU, astro-ph/06053

$\Lambda$ CDM  $(F, \pi_{de}, \Delta_{de}, S_{de}) = (1, 0, 0, 0)$

**DATA****OBSERVABLE**

Weak lensing

$$\kappa \propto \Delta(\Phi + \Psi)$$

Galaxy map

$$\delta_g = b \delta$$

Velocity field

$$\theta = \beta \delta$$

Integrated Sachs-Wolfe

$$\Theta_{SW} \propto \dot{\Phi} + \dot{\Psi}$$

Various combinations of these variables have been considered

# PART OF THE POISSON EQUATION

On sub-Hubble scales, the gravitational potential and density contrast are related by

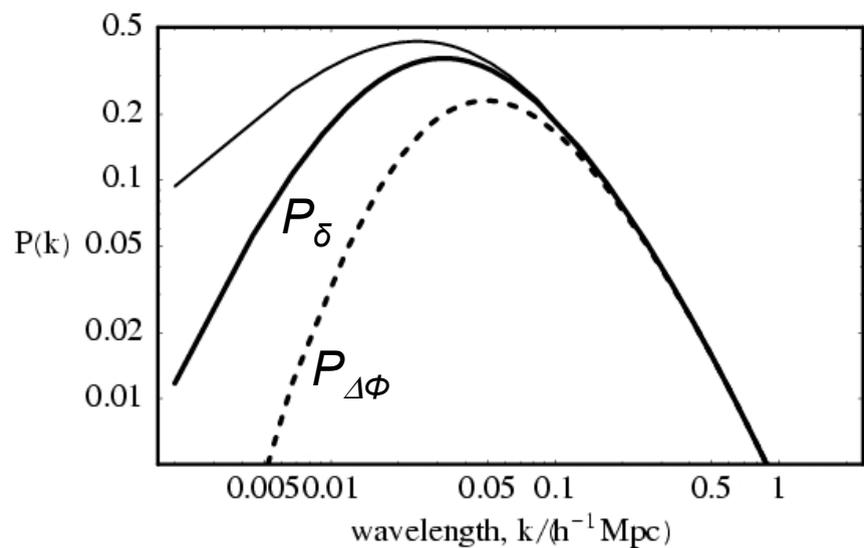
$$\Delta\Phi = 4\pi G\rho a^2\delta$$

**Galaxy catalogs** (SDSS, 2dF...)

measurement of  $\xi(r)$  up to  $500h^{-1}\text{Mpc}$

**Weak lensing**

will be measured up to  $100h^{-1}\text{Mpc}$



**Toy model:** 4D-5D gravity (brane induced)

perturbations freeze on large scales (idem as effect of  $\Lambda$ )

power spectra of  $\Phi$  and  $\delta$  are not identical

velocity map

$$\langle \delta_g \dot{\theta} \rangle = b\beta \langle \delta^2 \rangle$$

Galaxy map

$$\langle \delta_g \kappa \rangle \propto b \langle \delta \Delta (\Phi + \Psi) \rangle \stackrel{\Lambda\text{CDM}}{\propto} 8\pi G \rho a^2 b \langle \delta^2 \rangle$$

weak lensing

The ratio of these 2 quantities is independent of the bias

Zhang et al, arXiv:0704.1932

- Assume - no velocity bias      ( $S_{DE}=0$ )
- no clustering of DE    ( $\Delta_{DE}=0$ )

# CORRELATIONS

Correlations	Dependence	Limit	Case
$\langle \delta_g \delta_g \rangle - \langle \kappa \kappa \rangle$ JPU-Bernardeau	$(F, \pi_{\text{de}}, \Delta_{\text{de}})$	bias	
$\langle \delta_g \theta \rangle - \langle \delta_g \kappa \rangle$ Zhang et al, arXiv:0704.1932	$(F, \pi_{\text{de}}, \Delta_{\text{de}})$	velocity bias	
$\langle \delta_g \Theta_{SW} \rangle$ Schmidt et al, arXiv:0706.1775		bias	TeVes

A Full study of all the correlations needs to be performed

No test alone can bring a proof of deviation from GR and most studies assume  $\Delta_{\text{DE}}=0$

Possible to constrain the cases where  $S_{\text{DE}}=\Delta_{\text{DE}}=0$ . Quite general.

Null tests for deviation from  $\Lambda\text{CDM}$

At **linear order**, growth factor entangles  $H(a)$  and Poisson equation.

$$\delta^{(1)} = D(t)\varepsilon(x)$$

At **second order**

$$\ddot{\delta}^{(2)} + 2H\dot{\delta}^{(2)} = 4\pi G\rho(\delta^{(1)})^2 + a^{-2}\nabla\Phi\cdot\nabla\delta^{(1)} + a^{(-2)}\partial_{ij}u_i^{(1)}u_j^{(1)}$$

$$\langle\delta^3\rangle = \langle(\delta^{(1)})^3\rangle + \langle(\delta^{(1)})^2\delta^{(2)}\rangle$$

$S^3 = \langle\delta^3\rangle/\langle\delta^2\rangle^2$  is independent of  $D(t)$ . It depends slightly on the cosmological

parameters – dependence on spectral index - Gaussianity

# COSMIC SHEAR 3-POINT FUNCTION

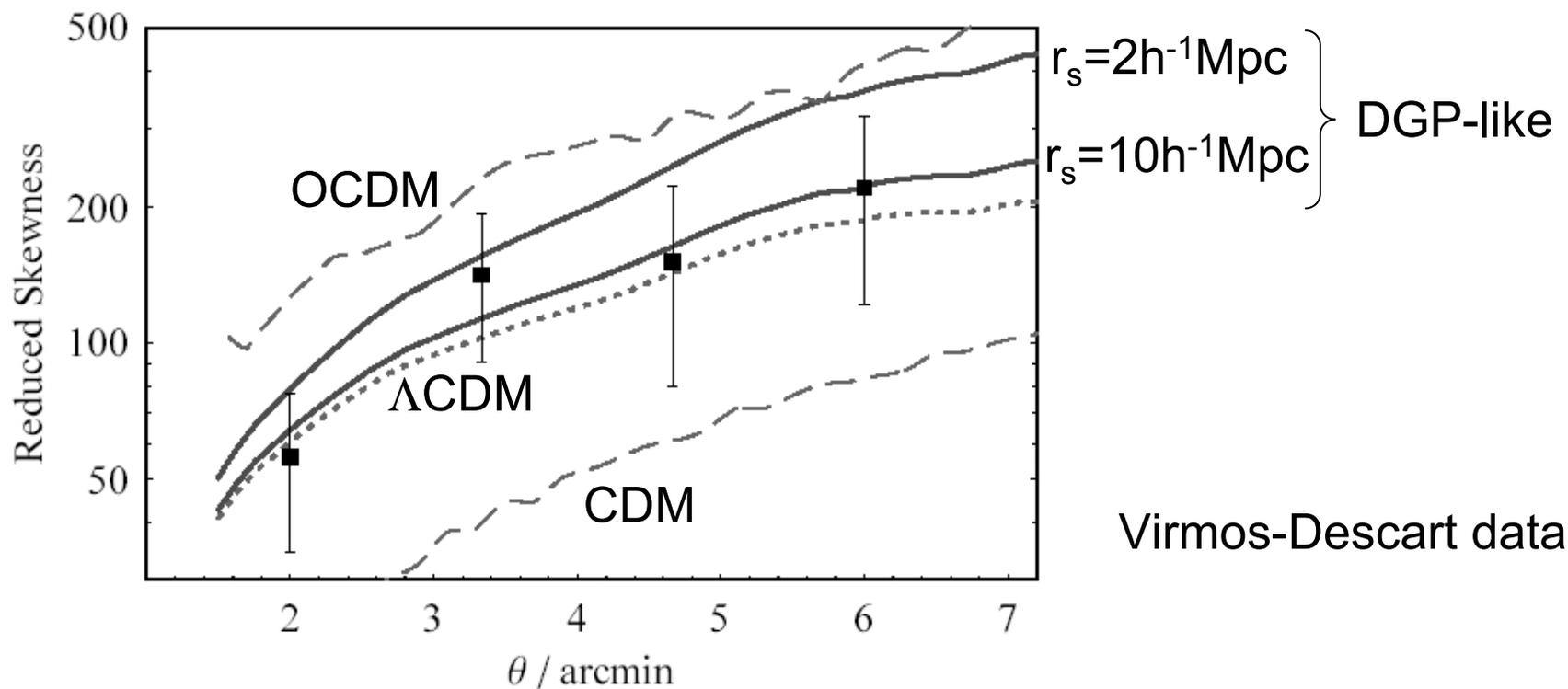
Assume a modified (scale dependent) Poisson equation

Compute the reduced third moment.

Use 3-point correlation of the shear field

Bernardeau et al, A.A.Lett.**389**(2002)2

Pen et al., ApJ**592**(2003)664

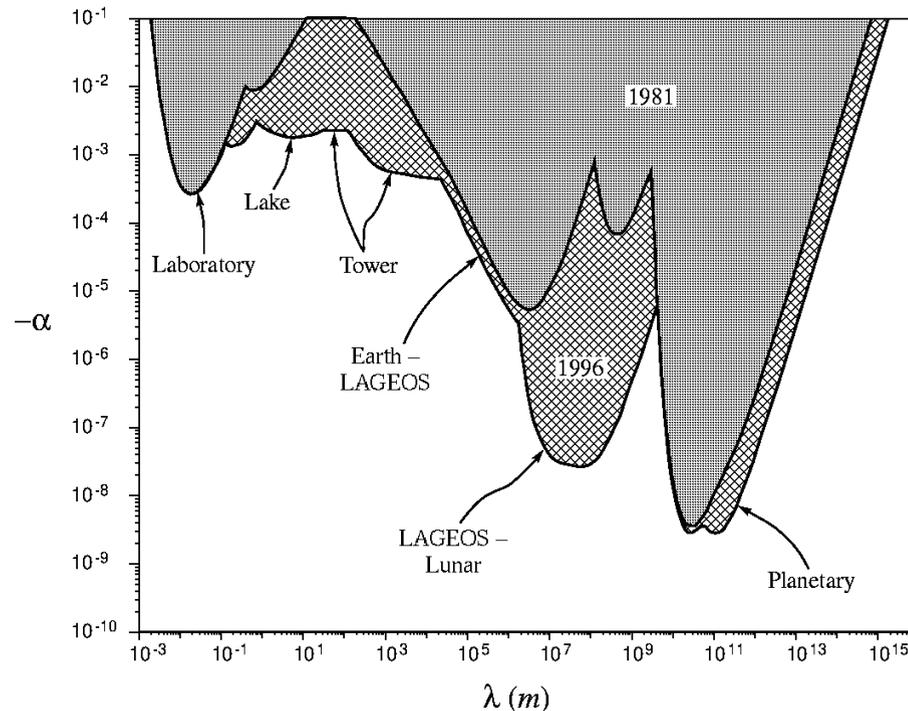


Disfavor  $r_s < 2h^{-1} \text{Mpc}$

Various studies have focused on a Yukawa modification of GR

$$U = \frac{Gm}{r} \left( 1 + \alpha e^{-r/\lambda} \right)$$

Such a deviation is well constrained in the Solar system

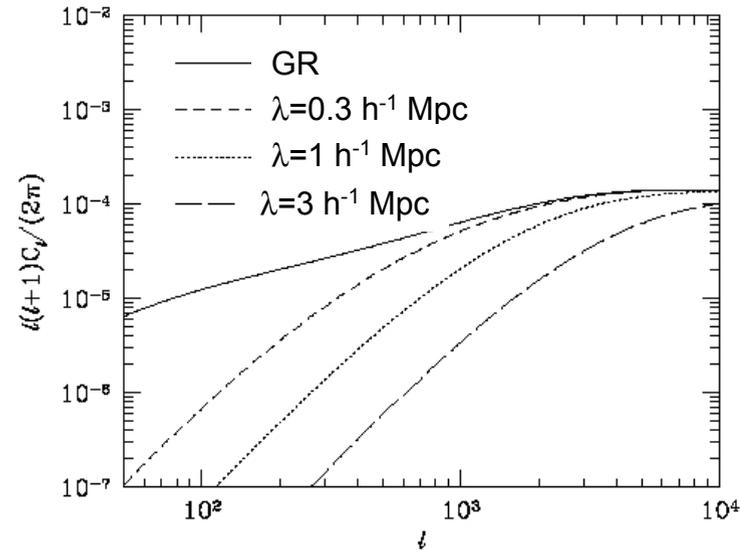


Concerning the growth of structure, it reduces to assuming

$$-k^2\Phi = 4\pi G(1 + f_Y(k\lambda))\rho a^2\delta \quad \Phi = \Psi$$

White & Kochanek, astro-ph/0105227

Weak lensing computed from propagation  
of rays through a known density distribution.  
No consistent analysis of the growth of structures



Sealfon et al., astro-ph/0404111

Compute power spectrum and bispectrum of LSS

$$\alpha = 0.025 \pm 1.7 \text{ (2dF)} \quad \alpha = -0.35 \pm 0.9 \text{ (SDSS)}$$

on a scale  $\lambda \sim 6h^{-1}\text{Mpc}$

## Shirata et al., astro-ph/0501366

Linear evolution + Peacock&Dodds for NL

Comparison with SDSS

$$-0.5 < \alpha < 0.6 \quad (\lambda = 5h^{-1}\text{Mpc})$$

$$-0.8 < \alpha < 0.9 \quad (\lambda = 10h^{-1}\text{Mpc})$$

Exclusion plot in  $(\alpha, \lambda)$  less obvious than in Solar system  
(dependence on cosmological parameters...)

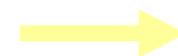
## Stabenau & Jain, astro-ph/0604038

N-body simulations on scales 1-100 Mpc

The scale dependence modification of the growth factor in linear regime is enhanced by NL

Peacock&Dodds approach can be extended

Lensing power spectra



See Bhuvnesh Jain

## Sereno & Peacock, astro-ph/0605498

Effect is almost degenerate on power spectrum shape with effect of massive neutrinos.

In models involving 2 metrics (scalar-tensor, TeVeS,...), gravitons and standard matter are coupled to different metrics.

## In GR:

photons and gravitons are massless and follow geodesics of the same spacetime

$$\delta T_{\gamma g} = T_{\gamma} - T_g = 0$$

## In bi-metric:

photons and gravitons follow geodesics of two spacetimes

$$\delta T_{\gamma g} \neq 0$$

## Example:

TeVSe model. Observable=SN1987a

$$\delta T_{\gamma g} = - 5.3 \text{ days}$$

# DISTANCE DUALITY RELATION

Photons travel on null geodesics  
Geodesic deviation equation holds

Etherington, *Phil. Mag.* **15** (1933) 761; Ellis, 1971

**Reciprocity relation:**  $r_s = r_o(1+z)$

If number of photons is conserved

$D_{lum}(z) = (1+z)^2 D_A(z)$

SN Ia data+radio galaxies

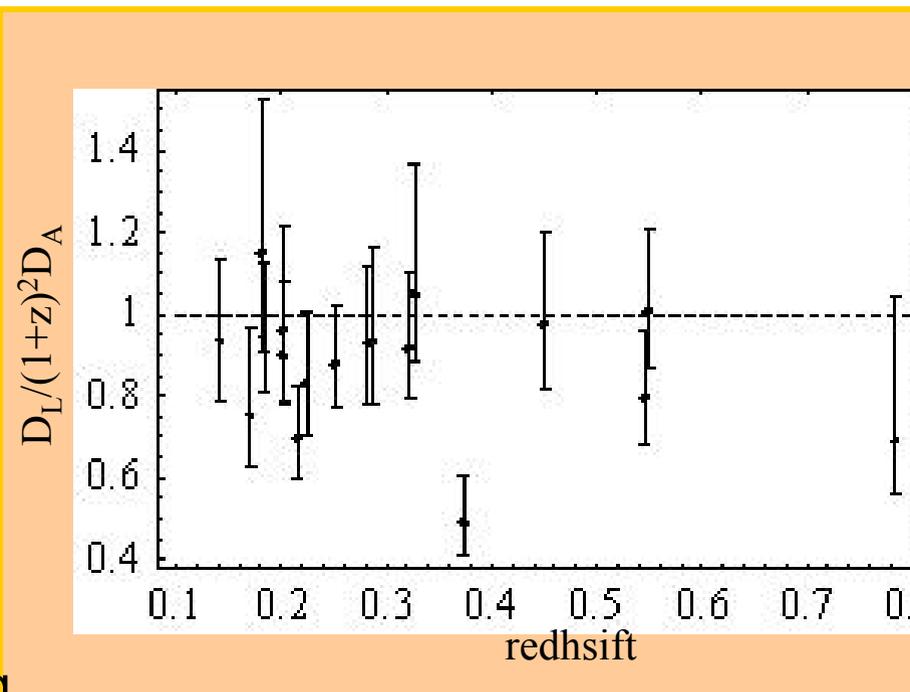
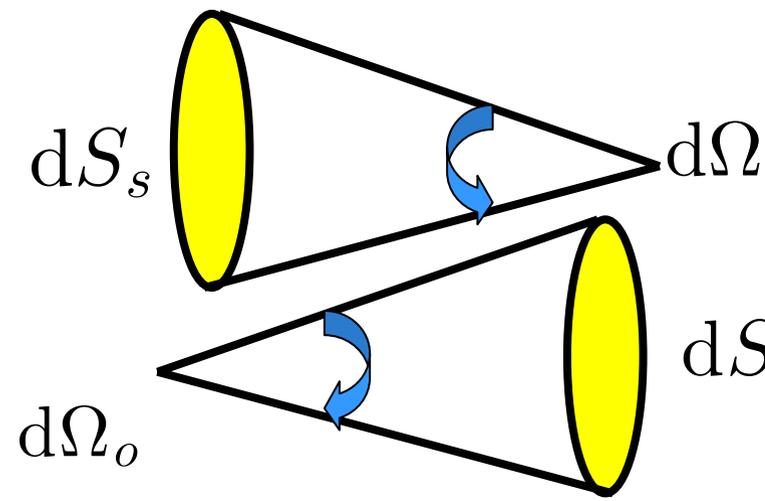
2σ violation

Basset and Kunz, *PRD***69** (2004)101305

X-ray + SZ observation of clusters

no indication of violation

Set constraints on photon-axion mixing



I will not detail the numerous studies in which one given model (TeVeS, DGP, scalar-tensor,...) is compared to combined set of data.

e.g. Amendola et al., arXiv:0704.242  
Song, astro-ph/0602598,  
Knox et al., astro-ph/0503644,...

## *General limits:*

- **Non-linear regime:** mappings are determined from numerical simulations assuming Newtonian gravity.
- **Effect of massive neutrinos:** can induce scale dependent modification of the power spectrum

## *Lifting degeneracies:*

- background: 1 function  $H(a)$
- low  $z$  – sub-Hubble:  $D(a)$
- one can construct several models reproducing the same subset of data
- needs to include local constraints

# **CONCLUSIONS**

Good motivations to test GR on astrophysical scales

*important to understand the parameters we are measuring in  $\Lambda$ CDM.*

Are there reasons to extend the  $\Lambda$ CDM framework

- *post- $\Lambda$ CDM formalism (?)*

- *importance null-tests vs fitting models*

Would allow to design parameterizations adapted to each class of models

Many tests have been proposed but yet no systematic investigation

Dependence on initial conditions and other limitations

*Statistical analysis-initial conditions*

*massive neutrinos*

*NL regimes*

*Theoretical limitations*

Importance to consider background/perturbation/local tests

Galactic scales / cosmological scales