Introductory remarks on inflation

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Present status of the inflationary scenario

Potential reconstruction from the power scalar spectrum

Physical scales related to inflation

Visualizing small differences in the duration of inflation

Gravitational waves from inflation and other effects

Inflation in $f(R)$ gravity

Conclusions

Main epochs of the Universe evolution

$H \equiv \frac{\dot{a}}{a}$ where $a(t)$ is a scale factor of an isotropic homogeneous spatially flat universe (a Friedmann-Lemaître-Robertson-Walker background):

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2) + small\ perturbations$$

The history of the Universe in one line: four main epochs

$$? \longrightarrow DS \Longrightarrow FLRWRD \Longrightarrow FLRWMD \Longrightarrow \overline{DS} \longrightarrow ?$$

Geometry

$$|\dot{H}| << H^2 \Longrightarrow H = \frac{1}{2t} \Longrightarrow H = \frac{2}{3t} \Longrightarrow |\dot{H}| << H^2$$

Physics

$$p \approx -\rho \Longrightarrow p = \rho/3 \Longrightarrow p \ll \rho \Longrightarrow p \approx -\rho$$

Duration in terms of the number of e-folds $\ln(a_{fin}/a_{in})$

$$> 60 \qquad \sim 55 \qquad 8 \qquad 0.3$$

Main advantages of inflation

1. Aesthetic elegance
Inflation – hypothesis about an almost maximally symmetric (quasi-de Sitter) stage of the evolution of our Universe in the past, before the hot Big Bang. If so, preferred initial conditions for (quantum) inhomogeneities with sufficiently short wavelengths exist – the adiabatic in-vacuum ones. In addition, these initial conditions represent an attractor for a much larger compact open set of initial conditions having a non-zero measure in the space of all initial conditions.

2. Predictability, proof and/or falsification
Given equations, this gives a possibility to calculate all subsequent evolution of the Universe up to the present time and even further to the future. Thus, any concrete inflationary model can be proved or disproved by observational data.

3. Naturalness of the hypothesis

Remarkable qualitative similarity between primordial and present dark energy.

4. Relates quantum gravity and quantum cosmology to astronomical observations

Makes quantum gravity effects observable at the present time and at very large – cosmological – scales.

5. Produces (non-universal) arrow of time for our Universe

Origin – initial quasi-vacuum fluctuation with a fantastically large correlation radius.

Present status of inflation

Now we have numbers.
P. A. R. Ade et al., arXiv:1303.5082

The primordial spectrum of scalar perturbations has been measured and its deviation from the flat spectrum $n_s = 1$ in the first order in $|n_s - 1| \sim N^{-1}$ has been discovered (using the multipole range $\ell > 40$):

$$P_\zeta(k) = \int \frac{\Delta_\zeta^2(k)}{k} dk, \quad \Delta_\zeta^2 = \left(2.20^{+0.05}_{-0.06}\right) 10^{-9} \left(\frac{k}{k_0}\right)^{n_s - 1}$$

$$k_0 = 0.05 \mathrm{Mpc}^{-1}, \quad n_s - 1 = -0.040 \pm 0.007$$

N.B.: The value is obtained under some natural assumptions, the most critical of them is $N_\nu = 3$, for $N_\nu = 4$ many things have to be reconsidered and $n_s \approx 1$ is not excluded.

Comparison with some simple models

Model	Parameter	*Planck*+WP	*Planck*+WP+lensing	*Planck* + WP+high-ℓ	*Planck*+WP+BAO
ΛCDM + tensor	n_s	0.9624 ± 0.0075	0.9653 ± 0.0069	0.9600 ± 0.0071	0.9643 ± 0.0059
	$r_{0.002}$	< 0.12	< 0.13	< 0.11	< 0.12
	$-2\Delta \ln \mathcal{L}_{max}$	0	0	0	-0.31

Table 4. Constraints on the primordial perturbation parameters in the ΛCDM+r model from *Planck* combined with other data sets. The constraints are given at the pivot scale $k_* = 0.002$ Mpc^{-1}.

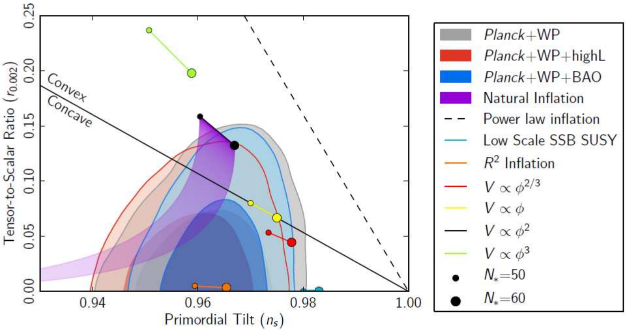


Fig. 1. Marginalized joint 68% and 95% CL regions for n_s and $r_{0.002}$ from *Planck* in combination with other data sets compared to the theoretical predictions of selected inflationary models.

From "proving" inflation to using it as a tool

Simple (one-parameter, in particular) models may be good in the first approximation (indeed so), but it is difficult to expect them to be absolutely exact, small corrections due to new physics should exist (indeed so).

Present status of inflation: transition from "proving" it in general and testing some of its simplest models to applying the inflationary paradigm to investigate particle physics at super-high energies and the actual history of the Universe in the remote past using real observational data on $n_s(k) - 1$ and $r(k)$.

The reconstruction approach – determining curvature and inflaton potential from observational data.

The most important quantities:
1) for classical gravity – $H, \dot{H}$
2) for super-high energy particle physics – m^2_{infl}.

Generation of scalar and tensor perturbations during inflation

A genuine quantum-gravitational effect: a particular case of the effect of particle-antiparticle creation by an external gravitational field. Requires quantization of a space-time metric. Similar to electron-positron creation by an electric field. From the diagrammatic point of view: an imaginary part of a one-loop correction to the propagator of a gravitational field from all quantum matter fields including the gravitational field itself, too.

The effect can be understood from the behaviour of a light scalar field in the de Sitter space-time.

For scales of astronomical and cosmological interest, the effect of creation of metric perturbations occurs at the primordial de Sitter (inflationary) stage when $k \sim a(t)H(t)$ where $k \equiv |\mathbf{k}|$ (the first Hubble radius crossing).

After that, for a very long period when $k \ll aH$ until the second Hubble radius crossing (which occurs rather recently at the radiation or matter dominated stages), there exist one mode of scalar (adiabatic, density) perturbations and two modes of tensor perturbations (primordial gravitational waves) for which metric perturbations are constant (in some gauge) and independent of (unknown) local microphysics due to the causality principle.

N.B. This has to be rechecked if the local Lorentz invariance is abandoned.

Quantum-to-classical transition

In the super-Hubble regime in the coordinate representation:

$$ds^2 = dt^2 - a^2(t)(\delta_{lm} + h_{lm})dx^l dx^m, \quad l, m = 1, 2, 3$$

$$h_{lm} = 2\zeta(\mathbf{r})\delta_{lm} + \sum_{a=1}^{2} g^{(a)}(\mathbf{r})\, e_{lm}^{(a)}$$

$$e_l^{l(a)} = 0, \;\; g^{(a)}_{,l}\, e_m^{l(a)} = 0, \;\; e_{lm}^{(a)}\, e^{lm(a)} = 1$$

ζ describes primordial scalar perturbations, g – primordial tensor perturbations (primordial gravitational waves (GW)).

This behaviour makes an effective quantum-to-classical transition possible: in fact, metric perturbations h_{lm} are quantum (operators in the Heisenberg representation) and remain quantum up to the present time. But, after omitting of a very small part, decaying with time, they become commuting and, thus, equivalent to classical (c-number) stochastic quantities with the Gaussian statistics (up to small terms quadratic in $\zeta,\ g$).

Remaining quantum coherence: deterministic correlation between $\mathbf{k}$ and $-\mathbf{k}$ modes - shows itself in the appearance of acoustic oscillations (primordial oscillations in case of GW).

FLRW dynamics with a scalar field

In the absence of spatial curvature and other matter:

$$H^2 = \frac{\kappa^2}{3}\left(\frac{\dot{\phi}^2}{2} + V(\phi)\right)$$

$$\dot{H} = -\frac{\kappa^2}{2}\dot{\phi}^2$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

where $\kappa^2 = 8\pi G$ $(\hbar = c = 1)$.

Inflationary slow-roll dynamics

Slow-roll occurs if: $|\ddot{\phi}| \ll H|\dot{\phi}|$, $\dot{\phi}^2 \ll V$, and then $|\dot{H}| \ll H^2$.

Necessary conditions: $|V'| \ll \kappa V$, $|V''| \ll \kappa^2 V$. Then

$$H^2 \approx \frac{\kappa^2 V}{3}, \quad \dot{\phi} \approx -\frac{V'}{3H}, \quad N \equiv \ln \frac{a_f}{a} \approx \kappa^2 \int_{\phi_f}^{\phi} \frac{V}{V'}\, d\phi$$

First obtained in A. A. Starobinsky, Sov. Astron. Lett. 4, 82 (1978) in the $V = \frac{m^2\phi^2}{2}$ case and for a bouncing model.

Spectral predictions of the one-field inflationary scenario in GR

Scalar (adiabatic) perturbations:

$$P_\zeta(k) = \frac{H_k^4}{4\pi^2\dot{\phi}^2} = \frac{GH_k^4}{\pi|\dot{H}|_k} = \frac{128\pi G^3 V_k^3}{3V_k'^2}$$

where the index k means that the quantity is taken at the moment $t = t_k$ of the Hubble radius crossing during inflation for each spatial Fourier mode $k = a(t_k)H(t_k)$. Through this relation, the number of e-folds from the end of inflation back in time $N(t)$ transforms to $N(k) = \ln \frac{k_f}{k}$ where $k_f = a(t_f)H(t_f)$, t_f denotes the end of inflation.
The spectral slope

$$n_s(k) - 1 \equiv \frac{d \ln P_\zeta(k)}{d \ln k} = \frac{1}{\kappa^2}\left(2\frac{V_k''}{V_k} - 3\left(\frac{V_k'}{V_k}\right)^2\right)$$

Tensor perturbations (A. A. Starobinsky, JETP Lett. 50, 844 (1979)):

$$P_g(k) = \frac{16GH_k^2}{\pi}; \quad n_g(k) \equiv \frac{d \ln P_g(k)}{d \ln k} = -\frac{1}{\kappa^2}\left(\frac{V_k'}{V_k}\right)^2$$

The consistency relation:

$$r(k) \equiv \frac{P_g}{P_\zeta} = \frac{16|\dot{H}_k|}{H_k^2} = 8|n_g(k)|$$

Tensor perturbations are always suppressed by at least the factor $\sim 8/N(k)$ compared to scalar ones. For the present Hubble scale, $N(k_H) = (50 - 60)$.

Potential reconstruction from scalar power spectrum

In the slow-roll approximation:

$$\frac{V^3}{V'^2} = CP_\zeta(k(t(\phi))), \quad C = \frac{12\pi^2}{\kappa^6}$$

Changing variables for ϕ to $N(\phi)$ and integrating, we get:

$$\frac{1}{V(N)} = -\frac{\kappa^4}{12\pi^2}\int \frac{dN}{P_\zeta(N)}$$

$$\kappa\phi = \int dN\sqrt{\frac{d\ln V}{dN}}$$

An ambiguity in the form of $V(\phi)$ because of an integration constant in the first equation. Information about $P_g(k)$ helps to remove this ambiguity.

In particular, if primordial GW are not discovered in the order $n_s - 1$:

$$r \ll 8|n_s - 1| \approx 0.3 \ ,$$

then $\left(\frac{V'}{V}\right)^2 \ll \left|\frac{V''}{V}\right|, \ |n_g| = \frac{r}{8} \ll |n_s - 1|, \ |n_g|N \ll 1 \ .$

This is possible only if $V = V_0 + \delta V, \ |\delta V| \ll V_0$ – a plateau-like potential. Then

$$\delta V(N) = \frac{\kappa^4 V_0^2}{12\pi^2} \int \frac{dN}{P_\zeta(N)}$$

$$\kappa\phi = \int \frac{dN}{\sqrt{V_0}} \sqrt{\frac{d(\delta V(N))}{dN}}$$

Here, integration constants renormalize V_0 and shift ϕ. Thus, the unambiguous determination of the form of $V(\phi)$ without knowledge of $P_g(k)$ becomes possible.

In particular, if $n_s - 1 = -\frac{2}{N} \approx -0.04$ for all $N = 1 - 60$ and $r \ll 8|n_s - 1|$, then

$$V(\phi) = V_0\left(1 - \exp(-\alpha\kappa\phi)\right)$$

with $\alpha\kappa\phi \gg 1$ but α not very small, and

$$r = \frac{8}{\alpha^2 N^2}$$

The simplest models producing the observed scalar slope

I. In the Einstein gravity:

$$V(\phi) = \frac{m^2\phi^2}{2}$$

$$m \approx 1.8 \times 10^{-6} \left(\frac{55}{N}\right) M_{Pl} \approx 2 \times 10^{13}\,\mathrm{GeV}$$

$$n_s - 1 = -\frac{2}{N} \approx -0.036, \quad r = \frac{8}{N} \approx 0.15$$

$$H_{dS}(N = 55) = 1.4 \times 10^{14}\,\mathrm{GeV}$$

II. In the modified, scalar-tensor gravity:

$$f(R) = R + \frac{R^2}{6M^2}$$

$$M = 2.6 \times 10^{-6} \left(\frac{55}{N}\right) M_{Pl} \approx 3 \times 10^{13}\,\mathrm{GeV}$$

$$n_s - 1 = -\frac{2}{N} \approx -0.036, \quad r = \frac{12}{N^2} \approx 0.004$$

$$H_{dS}(N = 55) = 1.4 \times 10^{14}\,\mathrm{GeV}$$

The same prediction from a scalar field model with $V(\phi) = \frac{\lambda\phi^4}{4}$ at large ϕ and strong non-minimal coupling to gravity $\xi R\phi^2$ with $\xi < 0, \ |\xi| \gg 1$, including the Brout-Englert-Higgs inflationary model.

Note similar predictions for inflaton masses and essentially the same prediction for H_{dS}.

Inflation + observations suggest the existence of curvatures in the past $H \sim 10^{14}\,\mathrm{GeV} \sim 10^{-28}\,\mathrm{cm}^{-1}$ and the inflaton mass $m \sim 10^{13}\,\mathrm{GeV}$, significantly less than the GUT scale $10^{15} - 10^{16}\,\mathrm{GeV}$.

Often another energy scale $E = (\hbar^3 c^3 V)^{1/4} \sim \sqrt{HM_{Pl}}$ is introduced which is indeed of the order of the GUT scale. But is this quantity physical?

Let us apply the same method to water and discover

the characteristic energy scale of water:

$E = (1 \frac{\mathrm{g}}{\mathrm{cm}^3} \times c^2)^{1/4} = 45\,\mathrm{keV}$.

Compete nonsense!

So, if this method leads to a totally misleading result for water, could it be better for primordial dark energy driving inflation?

Visualizing small differences in the number of e-folds

Local duration of inflation in terms of $N_{tot} = \ln\left(\frac{a(t_{fin})}{a(t_{in})}\right)$ is different is different point of space: $N_{tot} = N_{tot}(\mathbf{r})$. Then

$$\zeta(\mathbf{r}) = \delta N_{tot}(\mathbf{r})$$

Correct generalization to the non-linear case: the space-time metric after the end of inflation at super-Hubble scales

$$ds^2 = dt^2 - a^2(t)e^{2N_{tot}(\mathbf{r})}(dx^2 + dy^2 + dz^2)$$

First derived in A. A. Starobinsky, Phys. Lett. B **117**, 175 (1982) in the case of one-field inflation.

$$T_\gamma = (2.72548 \pm 0.00057)\mathrm{K}$$

$$\Delta T(\theta, \phi) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\theta, \phi)$$

$$< a_{\ell m} a_{\ell' m'} >= C_\ell \delta_{\ell \ell'} \delta_{m m'}$$

Theory: averaging over realizations.
Observations: averaging over the sky for a fixed ℓ.

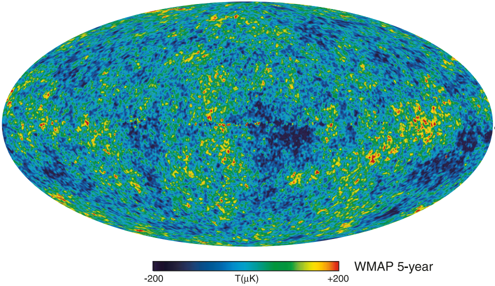
For scalar perturbations, generated mainly at the last scattering surface (the surface or recombination) at $z_{LSS} \approx 1090$ (the Sachs-Wolfe, Silk and Doppler effects), but also after it (the integrated Sachs-Wolfe effect).
For GW – only the ISW works.

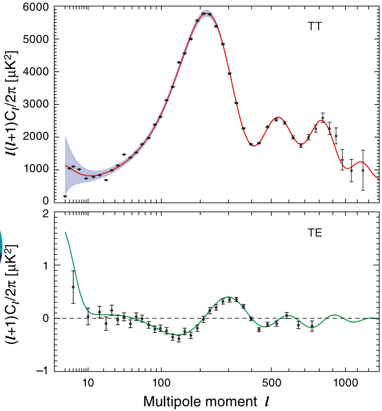
For $\ell \lesssim 50$, neglecting the Silk and Doppler effects, as well as the ISW effect due the presence of dark energy,

$$\frac{\Delta T(\theta,\phi)}{T_\gamma} = -\frac{1}{5}\zeta(r_{LSS},\theta,\phi) = -\frac{1}{5}\delta N_{tot}(r_{LSS},\theta,\phi)$$

For $n_s = 1$,

$$\ell(\ell+1)C_{\ell,s} = \frac{2\pi}{25}P_\zeta$$





Accuracy: with $\frac{\Delta T}{T} \sim 10^{-6}$, $\delta N \sim 5 \times 10^{-6}$, and for $H \sim 10^{14}\,\mathrm{GeV}$, $\delta t \sim t_{Pl}$!

Where is the primordial GW contribution to CMB temperature anisotropy?

For $1 \ll \ell \lesssim 50$, the Sachs-Wolfe plateau occurs for the contribution from GW, too:

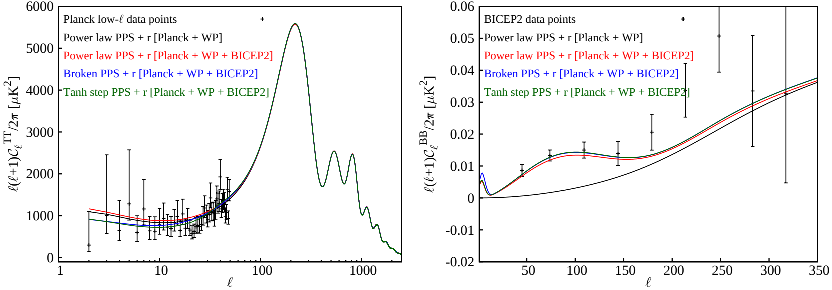
$$\ell(\ell+1)C_{\ell,g} = \frac{\pi}{36}\left(1 + \frac{48\pi^2}{385}\right) P_g$$

assuming $n_t = 1$ (A. A. Starobinsky, Sov. Astron. Lett. 11, 133 (1985)). So,

$$C_\ell = C_{\ell,s} + C_{\ell,g} = (1 + 0.775r)C_{\ell,s}$$

.

For larger $\ell > 50$, $\ell(\ell+1)C_{\ell,s}$ grows and the first acoustic peak forms at $\ell \approx 200$, while $\ell(\ell+1)C_{\ell,g}$ decreases quickly. Thus, the presence of GW should lead to a step-like enhancement of $\ell(\ell+1)C_\ell$ for $\ell \lesssim 50$.



Planck low-ℓ data points
Power law PPS + r [Planck + WP]
Power law PPS + r [Planck + WP + BICEP2]
Broken PPS + r [Planck + WP + BICEP2]
Tanh step PPS + r [Planck + WP + BICEP2]
$\ell(\ell+1)C_\ell^{TT}/2\pi\ [\mu K^2]$
6000
5000
4000
3000
2000
1000
0
1
10
100
1000
ℓ
BICEP2 data points
Power law PPS + r [Planck + WP]
Power law PPS + r [Planck + WP + BICEP2]
Broken PPS + r [Planck + WP + BICEP2]
Tanh step PPS + r [Planck + WP + BICEP2]
$\ell(\ell+1)C_\ell^{BB}/2\pi\ [\mu K^2]$
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
50
100
150
200
250
300
350
ℓ

The most critical discordance between WMAP and Planck results from one side and the BISEP2 ones from the other: no sign of GW in the CMB temperature anisotropy power spectrum.

Instead of the $\sim 10\%$ increase of $\ell(\ell+1)C_\ell$ over the multipole range $2 \ll \ell < 50$, a $\sim 10\%$ depression is seen for $20 \lesssim \ell \lesssim 40$ (see e.g. Fig. 39 of arXiv:1303.5076).

The feature exists even if $r \ll N^{-1}$ but the presence of $r \sim 0.1$ makes it larger.

More detailed analysis in D. K. Hazra, A. Shafieloo, G. F. Smoot and A. A. Starobinsky, JCAP 1406, 061 (2014), arXiv:1403.7786 :
the power-law form of $P_\zeta(k)$ is excluded at more than 3σ CL.

Other local features in the same range

The effect of at least the same order: an upward wiggle at $\ell \approx 40$ (the Archeops feature) and a downward one at $\ell \approx 22$.

Lesson: irrespective of an analysis of foreground contamination in the BISEP2 result, features in the anisotropy spectrum for $20 \lesssim \ell \lesssim 40$ confirmed by WMAP and Planck should be taken into account and studied seriously.

A more elaborated class of model suggested by previous studies of sharp features in the inflaton potential caused, e.g. by a fast phase transition occurred in another field coupled to the inflaton during inflation:
D. K. Hazra, A. Shafieloo, G. F. Smoot and A. A. Starobinsky, JCAP 1408, 048 (2014); arXiv:1405.2012

In particular, the potential with a sudden change of its first derivative:

$$V(\phi) = \gamma\phi^2 + \lambda\phi^p(\phi - \phi_0)\,\theta(\phi - \phi_0)$$

which generalizes the exactly soluble model considered in A. A. Starobinsky, JETP Lett. **55**, 489 (1992) produces $-2\Delta \ln \mathcal{L} = -11.8$ compared to the best-fitted power law scalar spectrum, partly due to the better description of wiggles at both $\ell \approx 40$ and $\ell \approx 22$.

$f(R)$ gravity

The simplest model of modified gravity (= geometrical dark energy) considered as a phenomenological macroscopic theory in the fully non-linear regime and non-perturbative regime.

$$S = \frac{1}{16\pi G}\int f(R)\sqrt{-g}\,d^4x + S_m$$

$$f(R) = R + F(R), \quad R \equiv R_\mu^\mu$$

Here $f''(R)$ is not identically zero. Usual matter described by the action S_m is minimally coupled to gravity.

Vacuum one-loop corrections depending on R only (not on its derivatives) are assumed to be included into $f(R)$. The normalization point: at laboratory values of R where the scalaron mass (see below) $m_s \approx \mathrm{const}$.

Metric variation is assumed everywhere. Palatini variation leads to a different theory with a different number of degrees of freedom.

Field equations

$$\frac{1}{8\pi G}\left(R_{\mu}^{\nu}-\frac{1}{2}\,\delta_{\mu}^{\nu}R\right)=-\left(T_{\mu\,(vis)}^{\nu}+T_{\mu\,(DM)}^{\nu}+T_{\mu\,(DE)}^{\nu}\right)\ ,$$

where $G = G_0 = const$ is the Newton gravitational constant measured in laboratory and the effective energy-momentum tensor of DE is

$$8\pi G T_{\mu\,(DE)}^{\nu}=F'(R)\,R_{\mu}^{\nu}-\frac{1}{2}\,F(R)\delta_{\mu}^{\nu}+\left(\nabla_{\mu}\nabla^{\nu}-\delta_{\mu}^{\nu}\nabla_{\gamma}\nabla^{\gamma}\right)F'(R)\,.$$

Because of the need to describe DE, de Sitter solutions in the absence of matter are of special interest. They are given by the roots $R = R_{dS}$ of the algebraic equation

$$Rf'(R)=2f(R)\ .$$

Transformation to the Einstein frame and back

In the Einstein frame, free particles of usual matter do not follow geodesics and atomic clocks do not measure proper time.

From the Jordan (physical) frame to the Einstein one:

$$g^{E}_{\mu\nu} = f' g^{J}_{\mu\nu}, \quad \kappa\phi = \sqrt{\frac{3}{2}} \ln f', \quad V(\phi) = \frac{f'R - f}{2\kappa^2 f'^2}$$

where $\kappa^2 = 8\pi G$.

Inverse transformation:

$$R = \left(\sqrt{6}\kappa \frac{dV(\phi)}{d\phi} + 4\kappa^2 V(\phi)\right) \exp\left(\sqrt{\frac{2}{3}}\kappa\phi\right)$$

$$f(R) = \left(\sqrt{6}\kappa \frac{dV(\phi)}{d\phi} + 2\kappa^2 V(\phi)\right) \exp\left(2\sqrt{\frac{2}{3}}\kappa\phi\right)$$

$V(\phi)$ should be at least C^1.

Analogues of large-field (chaotic) inflation: $F(R) \approx R^2 A(R)$ for $R \to \infty$ with $A(R)$ being a slowly varying function of R, namely

$$|A'(R)| \ll \frac{A(R)}{R} \; , \; |A''(R)| \ll \frac{A(R)}{R^2} \; .$$

In particular,

$$f(R) \approx \frac{R^2}{6m^2 \ln^2(R/m^2)}$$

for $R \gg m^2$ to have the same n_s, r as for $V = m^2\phi^2/2$.

Analogues of small-field (new) inflation, $R \approx R_1$:

$$F'(R_1) = \frac{2F(R_1)}{R_1} \; , \; F''(R_1) \approx \frac{2F(R_1)}{R_1^2} \; .$$

Thus, all inflationary models in $f(R)$ gravity are close to the simplest one over some range of R.

Conclusions

- Inflation is being transformed into a normal physical theory, based on some natural assumptions confirmed by observations and used to obtain new theoretical knowledge from them.
- First quantitative observational evidence for small quantities of the first order in the slow-roll parameters: $n_s(k) - 1$ and $r(k)$.
- The quantitative theoretical prediction of these quantities is based on gravity (space-time metric) quantization and requires very large space-time curvature in the past of our Universe with a characteristic length only five orders of magnitude larger than the Planck one.

- Regarding CMB temperature anisotropy, small features in the multipole range $20 \lesssim \ell \lesssim 40$ at the accuracy level $\sim 1\ \mu\mathrm{K}$ which mask the GW contribution to CMB temperature anisotropy have to be investigated and understood. They may reflect some fine structure of inflation (i.e. fast phase transitions in other quantum fields coupled to an inflaton during inflation).
- Though the Einstein gravity plus a minimally coupled inflaton remains sufficient for description of inflation with existing observational data, modified (in particular, scalar-tensor or $f(R)$) gravity can do it as well.