Binned bispectrum results and isocurvature constraints

Bartjan van Tent

Laboratoire de Physique Théorique, Orsay (Paris-Sud)

On behalf of the Planck collaboration

Binned bispectrum estimator: 1) f_{NL}

[M.Bucher, BvT, C.Carvalho, arXiv:0911.1642]

f_{NL} for a shape = inner product of the bispectrum template for that shape and the bispectrum of the map, weighted by the inverse covariance matrix:

Binned bispectrum estimator: 1) $f_{\rm NL}$

[M.Bucher, BvT, C.Carvalho, arXiv:0911.1642]

$f_{\rm NL}$ for a shape = inner product of the bispectrum template for that shape and the bispectrum of the map, weighted by the inverse covariance matrix:

$$\hat{f}_{\rm NL} = \frac{1}{F} \sum_{\ell_1 \le \ell_2 \le \ell_3} \sum_{\substack{p_1,\dots,p_6 \\ \in \{T,E\}}} (B^{\rm th}_{\vec{\ell}})^{\vec{p}_A} ({\rm Cov}_{\vec{\ell}})^{-1}_{\vec{p}_A \vec{p}_B} (B^{\rm obs}_{\vec{\ell}})^{\vec{p}_B}$$

where $\vec{\ell} = \ell_1 \ell_2 \ell_3$,
$\vec{p}_A = p_1 p_2 p_3$,
$\vec{p}_B = p_4 p_5 p_6$;

$$({\rm Cov}_{\vec{\ell}})_{\vec{p}_A \vec{p}_B} \sim \begin{pmatrix} (b^T_{\ell_1})^2 C^{TT}_{\ell_1} + N^T_{\ell_1} & b^T_{\ell_1} b^E_{\ell_1} C^{TE}_{\ell_1} \\ b^T_{\ell_1} b^E_{\ell_1} C^{TE}_{\ell_1} & (b^E_{\ell_1})^2 C^{EE}_{\ell_1} + N^E_{\ell_1} \end{pmatrix}_{p_1 p_4} \begin{pmatrix} \ell_1 \\ \updownarrow \\ \ell_2 \end{pmatrix}_{p_2 p_5} \begin{pmatrix} \ell_1 \\ \updownarrow \\ \ell_3 \end{pmatrix}_{p_3 p_6}, \quad \begin{matrix} b_\ell = \text{beam}, \\ N_\ell = \text{noise}; \end{matrix}$$

$B^{\rm obs}_{\vec{\ell}} \to B^{\rm obs}_{\vec{\ell}} - B^{\rm lin}_{\vec{\ell}}$ (linear term reduces variance when rotational invariance broken).

Binned bispectrum estimator: 1) $f_{\rm NL}$

[M.Bucher, BvT, C.Carvalho, arXiv:0911.1642]

$f_{\rm NL}$ for a shape = inner product of the bispectrum template for that shape and the bispectrum of the map, weighted by the inverse covariance matrix:

$$\hat{f}_{\rm NL} = \frac{1}{F} \sum_{\ell_1\le\ell_2\le\ell_3} \sum_{\substack{p_1,\ldots,p_6\\ \in\{T,E\}}} (B^{\rm th}_{\vec{\ell}})^{\vec{p}_A} ({\rm Cov}_{\vec{\ell}})^{-1}_{\vec{p}_A\vec{p}_B} (B^{\rm obs}_{\vec{\ell}})^{\vec{p}_B}$$

where $\vec{\ell} = \ell_1\ell_2\ell_3$,
$\vec{p}_A = p_1p_2p_3$,
$\vec{p}_B = p_4p_5p_6$;

$$({\rm Cov}_{\vec{\ell}})_{\vec{p}_A\vec{p}_B} \sim \begin{pmatrix} (b^T_{\ell_1})^2 C^{TT}_{\ell_1} + N^T_{\ell_1} & b^T_{\ell_1} b^E_{\ell_1} C^{TE}_{\ell_1} \\ b^T_{\ell_1} b^E_{\ell_1} C^{TE}_{\ell_1} & (b^E_{\ell_1})^2 C^{EE}_{\ell_1} + N^E_{\ell_1} \end{pmatrix}_{p_1p_4} \begin{pmatrix} \ell_1 \\ \updownarrow \\ \ell_2 \end{pmatrix}_{p_2p_5} \begin{pmatrix} \ell_1 \\ \updownarrow \\ \ell_3 \end{pmatrix}_{p_3p_6},$$

b_ℓ = beam, N_ℓ = noise;

$B^{\rm obs}_{\vec{\ell}} \to B^{\rm obs}_{\vec{\ell}} - B^{\rm lin}_{\vec{\ell}}$ (linear term reduces variance when rotational invariance broken).

Binning allows us to compute this expression in practice:

$$\hat{f}_{\rm NL} \approx \frac{1}{F^{\rm binned}} \sum_{\substack{\rm bins\\ i_1\le i_2\le i_3}} \sum_{\substack{p_1,\ldots,p_6\\ \in\{T,E\}}} \Big(\sum_{\substack{\ell_1,\ell_2,\ell_3\\ \in{\rm bin}}} B^{\rm th}_{\vec{\ell}}\Big)^{\vec{p}_A} \Big(\sum_{\substack{\ell_1,\ell_2,\ell_3\\ \in{\rm bin}}} {\rm Cov}_{\vec{\ell}}\Big)^{-1}_{\vec{p}_A\vec{p}_B} \Big(\sum_{\substack{\ell_1,\ell_2,\ell_3\\ \in{\rm bin}}} B^{\rm obs}_{\vec{\ell}}\Big)^{\vec{p}_B}$$

with $\sum_{\ell_1,\ell_2,\ell_3\in{\rm bin}} B^{{\rm obs}\,\vec{p}_B}_{\vec{\ell}} = \int d\Omega\, M^{p_4}_{\Delta\ell_1} M^{p_5}_{\Delta\ell_2} M^{p_6}_{\Delta\ell_3}$ where $M^p_{\Delta\ell}(\Omega) = \sum_{\ell\in{\rm bin}} \sum_m a^p_{\ell m} Y_{\ell m}(\Omega)$.

One determines the optimal binning by maximizing the correlation between the binned and the exact template. We use 57 bins for the Planck 2014 results.

Binned bispectrum estimator

$$\hat{f}_{\rm NL} \approx \frac{1}{F^{\rm binned}} \sum_{\substack{\rm bins \\ i_1 \le i_2 \le i_3}} \sum_{\substack{p_1,\ldots,p_6 \\ \in\{T,E\}}} \Big(\sum_{\substack{\ell_1,\ell_2,\ell_3 \\ \in {\rm bin}}} B^{\rm th}_{\vec{\ell}} \Big)^{\vec{p}_A} \Big(\sum_{\substack{\ell_1,\ell_2,\ell_3 \\ \in {\rm bin}}} {\rm Cov}_{\vec{\ell}} \Big)^{-1}_{\vec{p}_A \vec{p}_B} \Big(\sum_{\substack{\ell_1,\ell_2,\ell_3 \\ \in {\rm bin}}} B^{\rm obs}_{\vec{\ell}} \Big)^{\vec{p}_B}$$

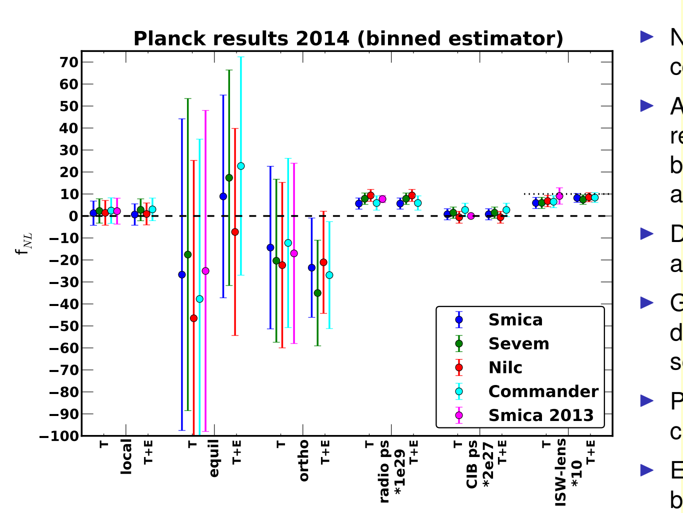
Advantages:

- Fast on a single map.
- Theoretical template does not need to be separable.
- Theoretical and observational part computed and saved separately, only combined in final sum over bins (which takes just seconds to compute) $\Rightarrow$
  - No need to rerun maps to determine e.g. $f_{\rm NL}$ for an additional template.
  - Full (binned) bispectrum is direct output of code.
- Easy to investigate dependence on ℓ by leaving out bins from final sum.

Disadvantages:

- Theoretical template must not change too much over a bin (OK for local, equilateral, orthogonal, point sources; a bit less for ISW-lensing).
- (Current implementation) Linear term cannot be precomputed, so computation time scales linearly with number of maps.

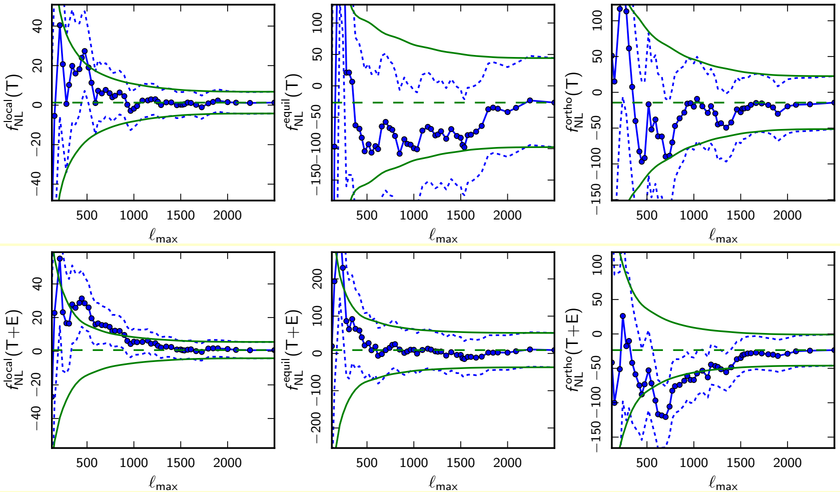
Independent (joint for ps) f_{NL} results for T and T+E, corrected for ISW-lensing:



- No (leo) primordial NG, consistent with 2013
- Addition of polarization: results consistent, error bars smaller (esp. equil and ortho)
- Detection ISW-lensing at correct level
- Good agreement different component separation methods
- Point sources remain in cleaned maps
- Excellent agreement between bispectrum estimators

PRELIMINARY results

Dependence on $\ell_{\max}$ of local, equil, ortho $f_{\rm NL}$ for T and T+E (Smica):

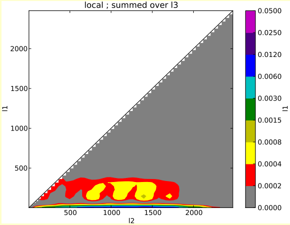


green solid: error bar as function of $\ell_{\max}$ around $f_{\rm NL}$ value for $\ell_{\max} = 2500$; blue dashed: error bars around individual points.

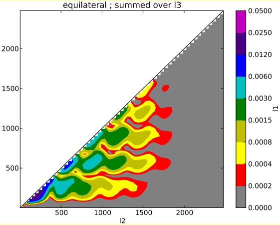
Note consistency with WMAP local result at $\ell_{\max} \sim 500$ **PRELIMINARY results**

Relative weight of the different templates in ℓ_1-ℓ_2 space, summed over ℓ_3 ($\ell_1 \leq \ell_2 \leq \ell_3$), for **T-only**. The colour scale is logarithmic.

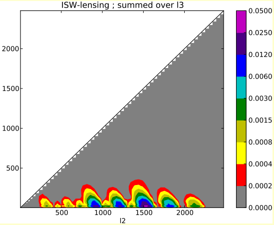
local

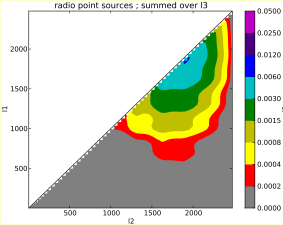


equilateral

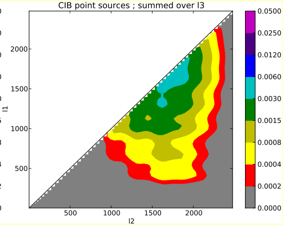


ISW-lensing





radio point sources (unclustered)



CIB point sources (clustered)

Correlation coefficients:

	equ	ort	isw-l	r-ps	CIB
loc	0.21	-0.44	0.28	0.002	0.006
equ		-0.05	0.003	0.008	0.027
ort			-0.15	-0.003	-0.001
isw-l				-0.005	-0.026
r-ps					0.93

Point sources uncorrelated with other shapes ⇒ Contamination no problem!

Binned bispectrum estimator: 2) smoothed bispectrum

Since the binned bispectrum of the map is a direct output of the code, it can be studied explicitly, without any theoretical assumptions ("blind" / non-parametric).

To investigate if there is any significant non-Gaussianity in the maps, we consider the bispectrum divided by its expected standard deviation:

$$\mathcal{B}^{XYZ}_{i_1 i_2 i_3} = \frac{B^{\mathrm{obs}\,XYZ}_{i_1 i_2 i_3}}{\sqrt{\mathrm{Var}(B^{\mathrm{obs}\,XYZ}_{i_1 i_2 i_3})}}$$

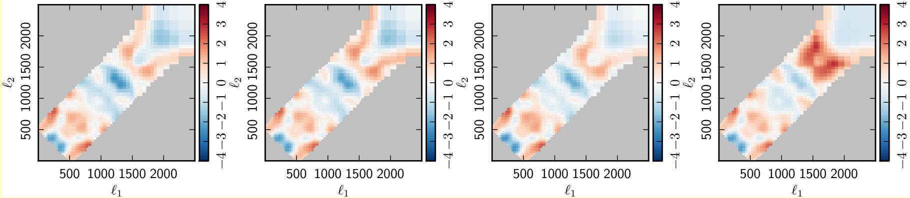
where *XYZ* is one of the four: TTT, TTE (including permutations), TEE (idem), EEE.

To bring out coherent features, $\mathcal{B}$ is smoothed with a Gaussian kernel with $\sigma = 2$ in bin units.

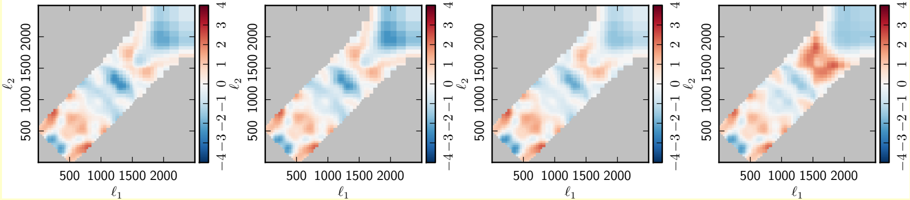
In the next slides $\mathcal{B}$ is shown as a function of ℓ_1 and ℓ_2, for a given bin in ℓ_3. Very red or blue regions indicate significant non-Gaussianity.

Result for TTT with $\ell_3 \in [518, 548]$: **PRELIMINARY results**

Smica Sevem Nilc Commander



Subtracting radio and CIB point source templates with observed amplitude:

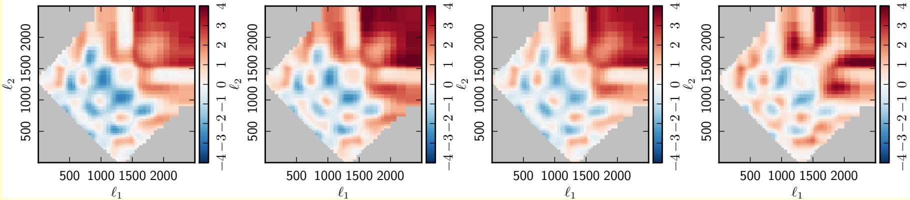


Good agreement between all methods, in particular after subtracting point sources.
No significant NG visible (after subtraction).

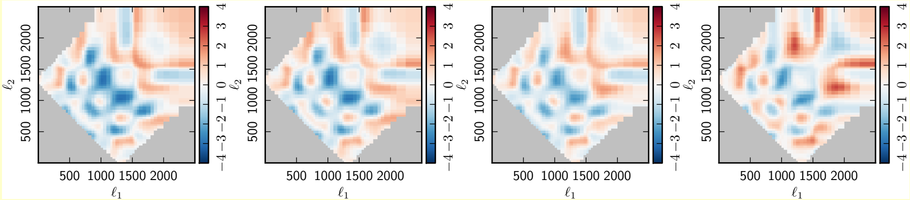
[Commander includes different frequencies above and below $\ell = 1000$, which is not taken into account in our subtraction.]

Result for TTT with $\ell_3 \in [1291, 1345]$: **PRELIMINARY results**

Smica Sevem Nilc Commander



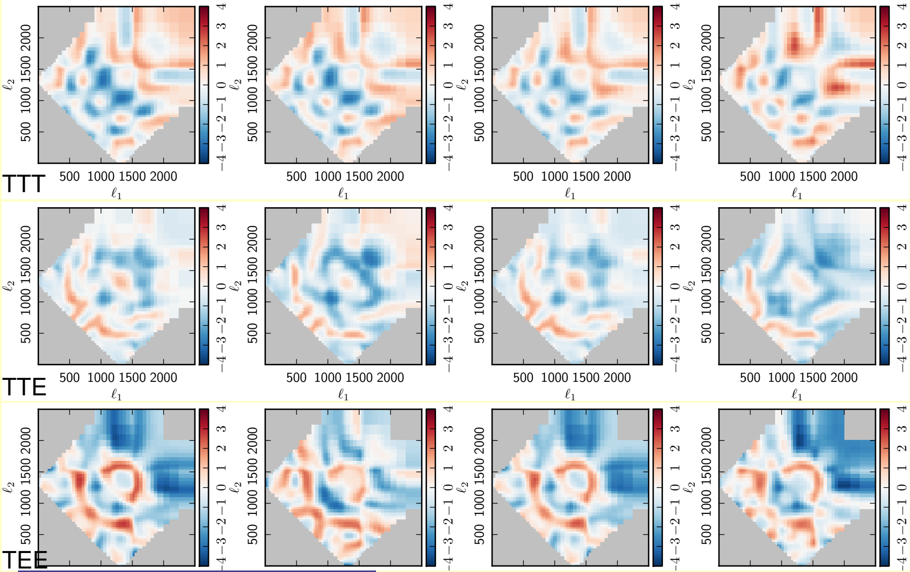
Subtracting radio and CIB point source templates with observed amplitude:



Again good agreement between all methods, after subtracting the very significant point source contribution. No significant NG visible (after subtraction).

Results for TTT (minus point sources), TTE, and TEE, with $\ell_3 \in [1291, 1345]$: **PRELIMINARY**

Smica Sevem Nilc Commander



Isocurvature

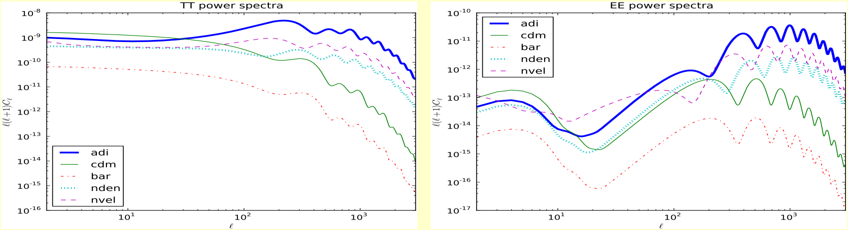
- The most common type of perturbation is the **adiabatic** mode δ, with:

$$\frac{\delta n_c}{n_c} = \frac{\delta n_b}{n_b} = \frac{\delta n_\nu}{n_\nu} = \frac{\delta n_\gamma}{n_\gamma} \quad \Leftrightarrow \quad \delta_c = \delta_b = \tfrac{3}{4}\delta_\nu = \tfrac{3}{4}\delta_\gamma$$

with $\delta \equiv \delta\rho/\rho$ and c=cold dark matter (CDM), b=baryons, ν=neutrinos, γ=photons.

- If different species were created from different primordial degrees of freedom (e.g. multiple-field inflation), we can have additional **isocurvature** modes S:
  - CDM density isocurv. (**CDI**): $\delta_c = S_c + \tfrac{3}{4}\delta_\gamma, \quad \delta_b = \tfrac{3}{4}\delta_\nu = \tfrac{3}{4}\delta_\gamma$
  - Neutrino density isocurv. (**NDI**): $\tfrac{3}{4}\delta_\nu = S_{\nu d} + \tfrac{3}{4}\delta_\gamma, \quad \delta_b = \delta_c = \tfrac{3}{4}\delta_\gamma$
  - Neutrino velocity isocurv. (**NVI**): $V_\nu = S_{\nu v}, \quad V_{\gamma b} = -\tfrac{7}{8}N_\nu(\tfrac{4}{11})^{4/3}S_{\nu v}$

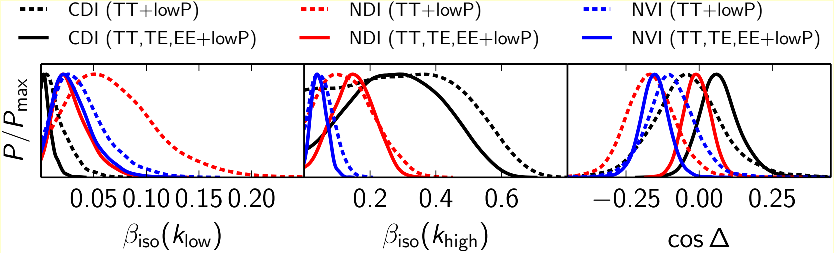
  (V=velocity, N_ν=number of species of massless neutrinos)



Isocurvature constraints from the power spectrum

- Assume 1 **adiabatic** ($\mathcal{R}$) + 1 **isocurvature** ($\mathcal{I}$) mode (cold dark matter (**CDI**), neutrino density (**NDI**), or neutrino velocity (**NVI**)).
- For the power spectrum this adds 3 new params, which effectively describe: $\mathcal{P}_{\mathcal{II}}(k_0)$, $\mathcal{P}_{\mathcal{RI}}(k_0)$, $n_{\mathcal{II}}$ (in addition to the usual $\mathcal{P}_{\mathcal{RR}}(k_0)$ and $n_{\mathcal{RR}}$).
- We define the primordial isocurvature ($\beta_{\rm iso}$) and correlation ($\cos\Delta$) fraction:

$$\beta_{\rm iso}(k) = \frac{\mathcal{P}_{\mathcal{II}}(k)}{\mathcal{P}_{\mathcal{RR}}(k)+\mathcal{P}_{\mathcal{II}}(k)} \qquad \cos\Delta = \frac{\mathcal{P}_{\mathcal{RI}}(k)}{\sqrt{\mathcal{P}_{\mathcal{RR}}(k)\mathcal{P}_{\mathcal{II}}(k)}}$$

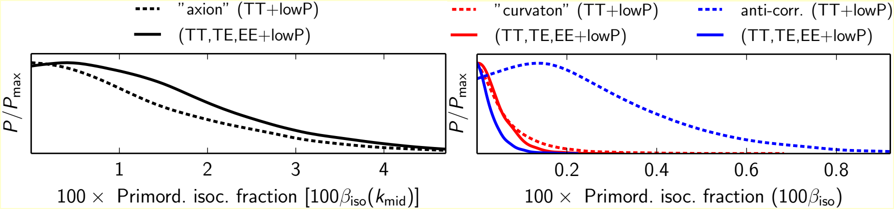


$k_{\rm low} = 0.002\ {\rm Mpc}^{-1}$ ($\ell \sim 30$), $k_{\rm high} = 0.1\ {\rm Mpc}^{-1}$ ($\ell \sim 1500$)

PRELIMINARY results

Additional 1-parameter CDI models (where $\mathcal{P}_{\mathcal{RI}}$ and $n_{\mathcal{II}}$ are fixed):

- Uncorrelated ("axion"): $\cos\Delta = 0$, $n_{\mathcal{II}} = 1$
- Fully correlated ("curvaton"): $\cos\Delta = 1$, $n_{\mathcal{II}} = n_{\mathcal{RR}}$
- Fully anti-correlated: $\cos\Delta = -1$, $n_{\mathcal{II}} = n_{\mathcal{RR}}$



PRELIMINARY results

Isocurvature: preliminary conclusions from power spectrum [from slide by J. Väliviita]

3-parameter extensions to the adiabatic ΛCDM model were studied, allowing for a (correlated) mixture of adiabatic and one isocurvature mode (CDI, NDI, or NVI):

- **No evidence of isocurvature** in the Planck high-ℓ temperature and low-ℓ temperature and polarization data within Planck's accuracy.
- Adding the **high-ℓ polarization** data leads to much **stronger constraints**.
  - High-ℓ TE/EE data pull CDI and NDI towards (slightly) positive correlation, while (high-ℓ) TT allows for a larger negative correlation.

  Polarization results reported here are very **preliminary**, because we do not yet have confidence that all systematic and foreground uncertainties have been properly characterized, and results may therefore be subject to revision.
- **Determination of the standard cosmological parameters is robust** against the more general initial conditions.
- In addition, **determination of primordial tensor-to-scalar ratio** from the Planck data alone **is robust** against allowing for CDI.

1-parameter extensions to the adiabatic ΛCDM model were also studied. These correspond to axion or curvaton motivated models:

- With Planck TT+lowP, generally stronger constraints than in 2013.
- High-ℓ polarization data strengthen the constraints significantly, except in the axion case.

Isocurvature non-Gaussianity

[Langlois, BvT, arXiv:1104.2567, 1204.5042]

[see also earlier works by Kawasaki, Nakayama, Sekiguchi, Suyama, Takahashi, Hikage]

Assume local primordial bispectrum (I,J,K labels **adiabatic** and **isocurvature** modes):

$$B^{IJK}(k_1, k_2, k_3) = f_{\rm NL}^{I,JK} \mathcal{P}_{\mathcal{RR}}(k_2) \mathcal{P}_{\mathcal{RR}}(k_3) + f_{\rm NL}^{J,KI} \mathcal{P}_{\mathcal{RR}}(k_1) \mathcal{P}_{\mathcal{RR}}(k_3) + f_{\rm NL}^{K,IJ} \mathcal{P}_{\mathcal{RR}}(k_1) \mathcal{P}_{\mathcal{RR}}(k_2)$$

[Produced for example in multiple-field inflation where primordial adiabatic and isocurvature perturbations X^I can be expressed as $X^I = N_a^I \delta\phi^a + \frac{1}{2} N_{ab}^I \delta\phi^a \delta\phi^b + \ldots$
Negligible scale dependence of N_a^I and N_{ab}^I $\Rightarrow$ all power spectra same spectral index.]

Due to symmetries $f_{\rm NL}^{I,JK} = f_{\rm NL}^{I,KJ}$ $\Rightarrow$ **6 independent $f_{\rm NL}$ parameters** in the case of 1 adiabatic + 1 isocurvature mode: $f_{\rm NL}^{a,aa}, f_{\rm NL}^{a,ai}, f_{\rm NL}^{a,ii}, f_{\rm NL}^{i,aa}, f_{\rm NL}^{i,ai}, f_{\rm NL}^{i,ii}$.

Note: some inflation/curvaton models *[Langlois, Lepidi, arXiv:1007.5498]* predict a larger isocurvature than adiabatic bispectrum, and at the same time a negligible isocurvature power spectrum.

Isocurvature non-Gaussianity results for T and T+E (Smica):

(**joint analysis**, ISW-lensing subtracted)

Allowing for correlations between the primordial modes:

	CDI	NDI	NVI
T a,aa	21 ± 13	-27 ± 52	-32 ± 48
T a,ai	-39 ± 26	140 ± 210	370 ± 350
T a,ii	17000 ± 8200	-4500 ± 4500	-1300 ± 3800
T i,aa	96 ± 120	40 ± 99	-27 ± 51
T i,ai	-2100 ± 1000	220 ± 630	75 ± 170
T i,ii	4200 ± 2000	-750 ± 2400	-970 ± 1400
T+E a,aa	5 ± 10	-35 ± 27	2 ± 24
T+E a,ai	-12 ± 20	74 ± 94	330 ± 130
T+E a,ii	-1800 ± 1300	-3000 ± 1400	-3200 ± 1200
T+E i,aa	53 ± 47	51 ± 45	-44 ± 24
T+E i,ai	140 ± 170	170 ± 210	20 ± 74
T+E i,ii	-280 ± 390	-390 ± 860	480 ± 430

- No evidence for any isocurvature non-Gaussianity
- Many error bars tighten significantly with the inclusion of polarization.

Assuming primordial modes to be completely uncorrelated:

	CDI	NDI	NVI
T a,aa	1.0 ± 5.3	19 ± 12	-0.2 ± 5.4
T i,ii	65 ± 280	-840 ± 540	440 ± 230
T+E a,aa	0.5 ± 5.0	3.0 ± 7.9	-0.3 ± 4.9
T+E i,ii	35 ± 170	-120 ± 290	87 ± 130

PRELIMINARY results

Conclusions

PRELIMINARY results

- The **binned bispectrum estimator** is fast, gives optimal results and has a convenient modular setup.
- Allows both parametric ($f_{\rm NL}$) and non-parametric bispectrum estimation.
- **Planck $f_{\rm NL}$ results**: no (leo) primordial NG, with inclusion polarization leading to smaller error bars but consistent results; detection ISW-lensing.
- **Planck bispectrum reconstruction**: blind tests see point source bispectrum; no obvious indication of other NG.
- **Good agreement** between different estimators and component separation methods.
- **No evidence for isocurvature** in the Planck data, neither in the power spectrum nor in the bispectrum. Inclusion of polarization tightens the constraints significantly.

The scientific results that we present today are a product of the Planck Collaboration, including individuals from more than 100 scientific institutes in Europe, the USA and Canada

Planck is a project of the European Space Agency, with instruments provided by two scientific Consortia funded by ESA member states (in particular the lead countries: France and Italy) with contributions from NASA (USA), and telescope reflectors provided in a collaboration between ESA and a scientific Consortium led and funded by Denmark.