# Towards a new model of atmospheric tides: from Venus to super-Earths

Pierre Auclair-Desrotour, Jacques Laskar (IMCCE), Stéphane Mathis (CEA)

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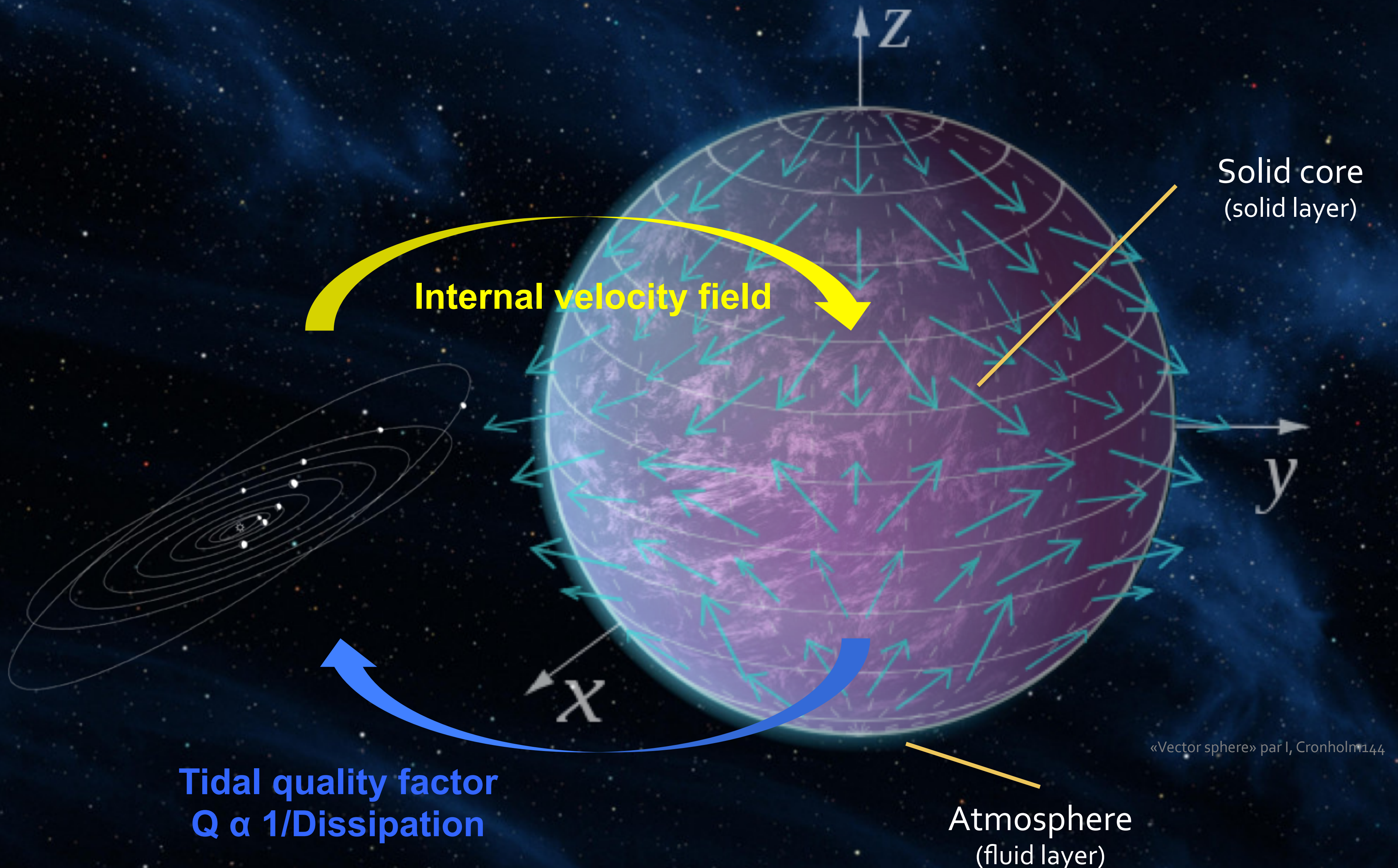
# Introduction



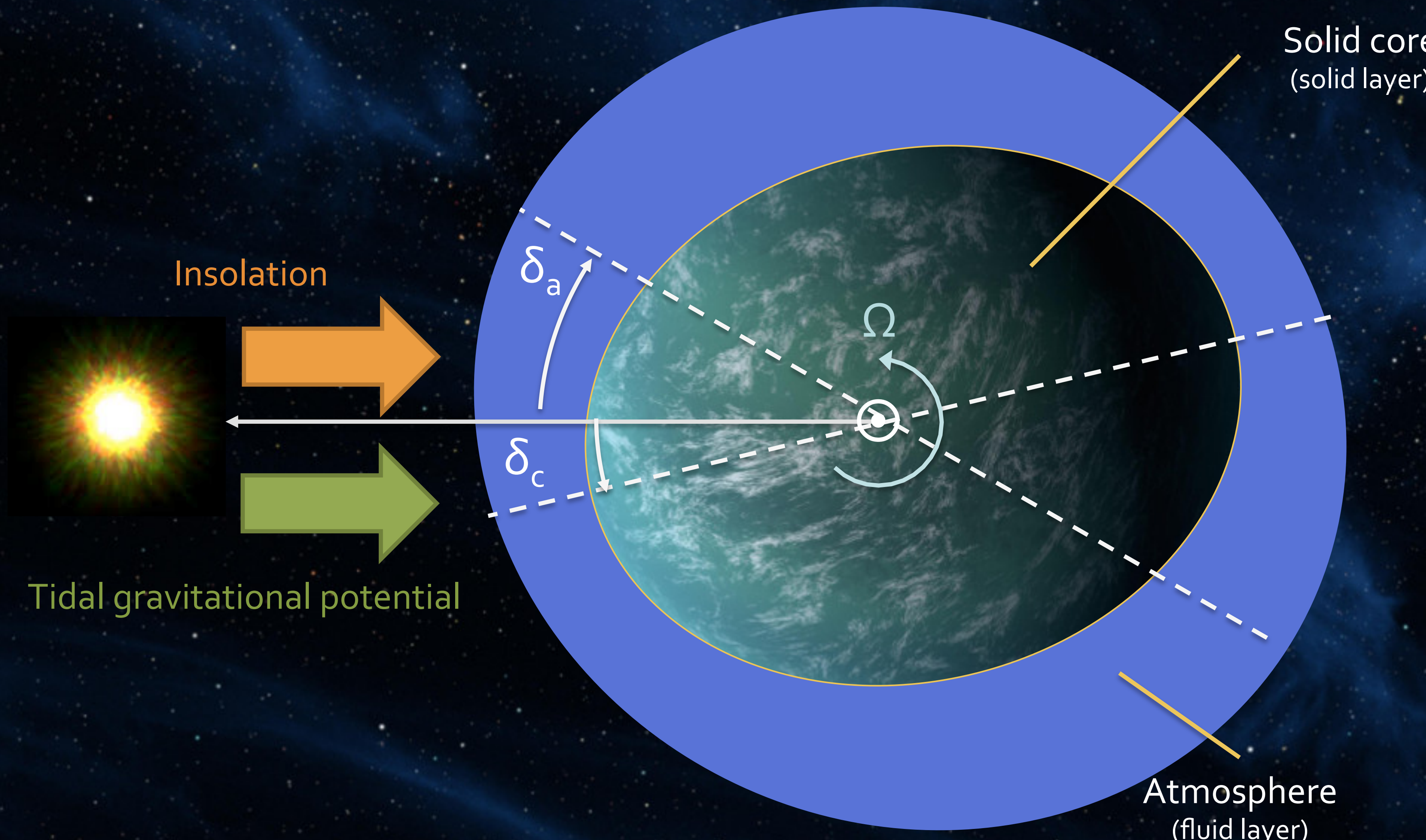
# State of the art

**Atmospheric tidal dissipation little understood and poorly quantified!**

**A non-exhaustive history of theoretical works dealing with atmospheric tides:**

- → **Atmospheric tides of the Earth**
  - ➢ Chapman & Lindzen (1970)
- → **Spin equilibrium**
  - ➢ Gold & Soter (1969), Correia, Laskar, Néron de Surgy (2001), Correia & Laskar (2003), Correia, Levrard, Laskar (2008)
- → **Atmosphere of super-Earths**
  - ➢ Forget & Leconte (2014)

# Tidal effects in super-Earths



# Equilibrium states: a torques balance

Spin equation:

$$\frac{dL}{dt} = -\frac{\partial U_g}{\partial \vartheta} - \frac{\partial U_a}{\partial \vartheta}$$

Spin

Tidal potential

$$U_g = -k_2 \frac{G M_*^2}{R} \left(\frac{R}{r_*}\right)^3 \left(\frac{R}{r}\right)^3 P_2(\cos S)$$

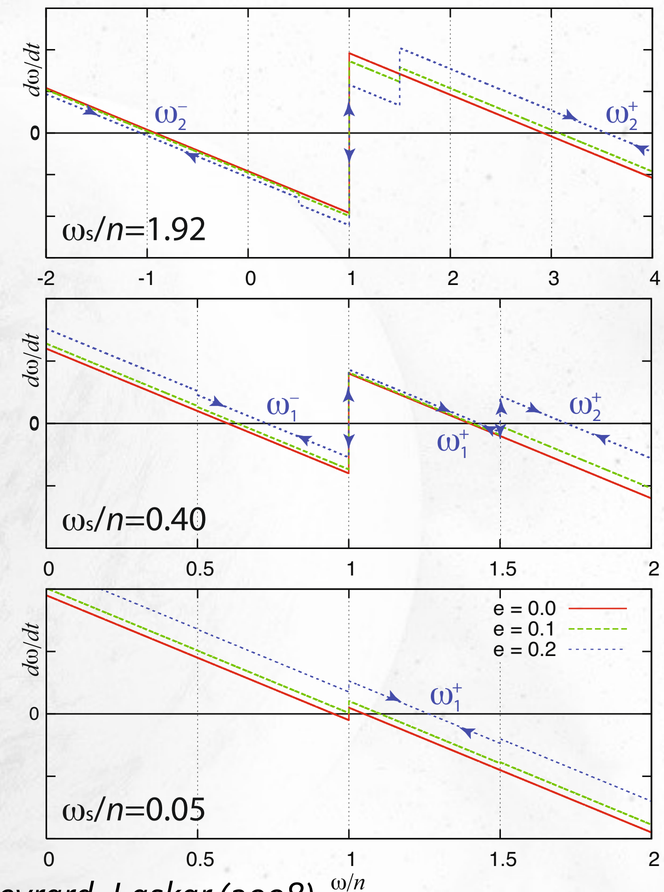
Tidal gravitational potential

$$U_a = -\frac{3}{5} \frac{\tilde{p}_2}{\rho} \left(\frac{R}{r}\right)^3 P_2(\cos S)$$

Tidal atmospheric potential

Pressure oscillations at the ground !

$\tilde{p}_2$ ?

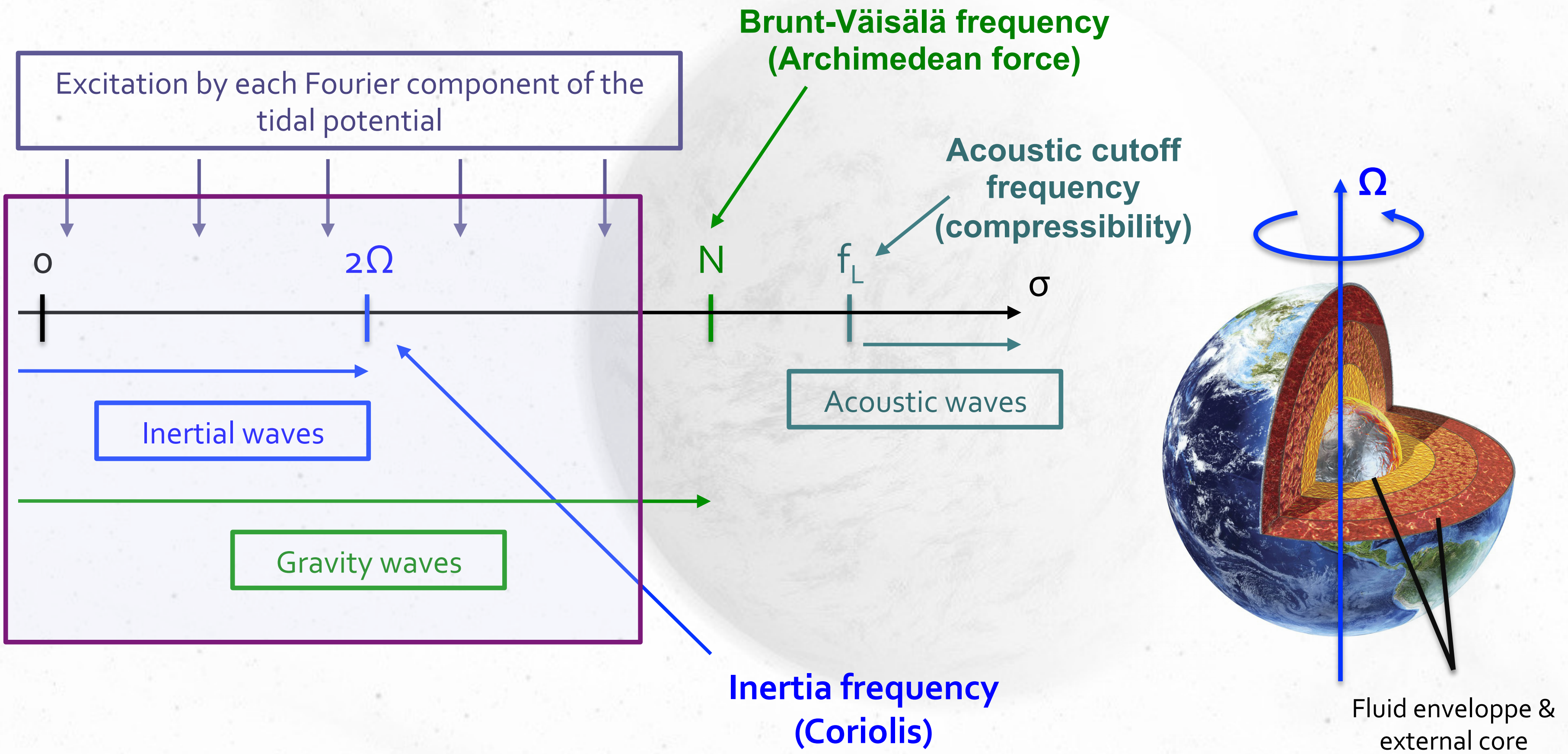



*Correia, Levrard, Laskar (2008)*

*see also Gold & Soter (1969), Correia, Laskar, Néron de Surgy (2001), Correia & Laskar (2003)*

**Need for a realistic physical modeling of atmospheric tides!**

# Tidal waves properties



# Atmospheric tides dynamics

Inertia frequency

$$\frac{\partial V_\theta}{\partial t} - 2\Omega V_\varphi \cos\theta = -\frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{\delta p}{\rho_0} + U\right),$$

Gravitational forcing

$$\frac{\partial V_\varphi}{\partial t} + 2\Omega \cos\theta V_\theta = -\frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}\left(\frac{\delta p}{\rho_0} + U\right),$$

$$\rho_0 \frac{\partial V_r}{\partial t} = -\frac{\partial \delta p}{\partial r} - g\delta\rho - \rho_0 \frac{\partial U}{\partial r}.$$

$$\frac{\partial \delta\rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \rho_0 V_r\right) + \frac{\rho_0}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta V_\theta) + \frac{\partial V_\varphi}{\partial\varphi}\right] = 0$$

Thermal forcing

$$\frac{1}{\Gamma_1 p_0}\left(\frac{\partial \delta p}{\partial t} + \Gamma_1 \sigma_0 \delta p\right) + \frac{N^2}{g}\frac{\partial \xi_r}{\partial t} = \frac{\kappa \rho_0}{p_0} J + \frac{1}{\rho_0}\left(\frac{\partial \delta\rho}{\partial t} + \sigma_0 \delta\rho\right)$$

Brunt-Väisälä frequency

Thermal frequency

Reference model:
Chapman & Lindzen (1970)

Navier Stokes

Conservation of mass

Heat transport

Added terms

# Horizontal structure

$$\delta p = \sum_{\sigma,s} \delta p^{\sigma,s}\left(\theta,x\right) e^{i(\sigma t + s\varphi)}$$

**Expansion in Fourier series**

$$\delta p^{\sigma,s} = \sum_{n} \delta p_n\left(x\right) \Theta_n\left(\theta\right)$$

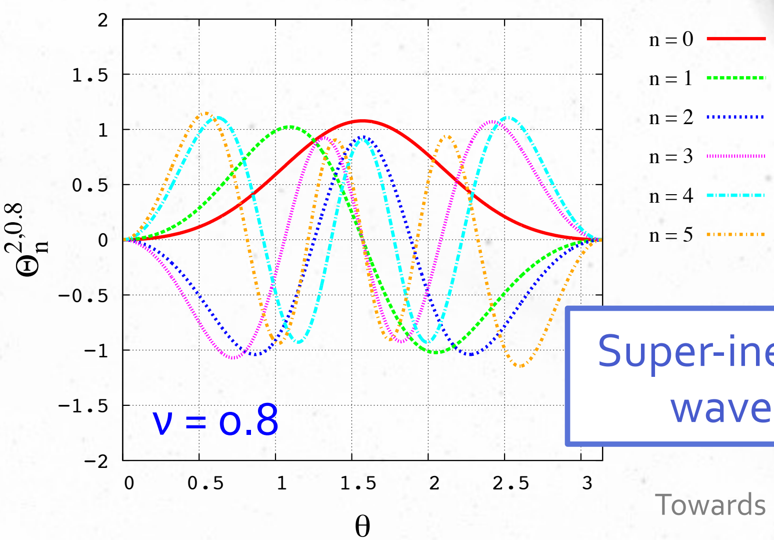
Radial profiles — Hough functions

**Expansion in Hough functions**

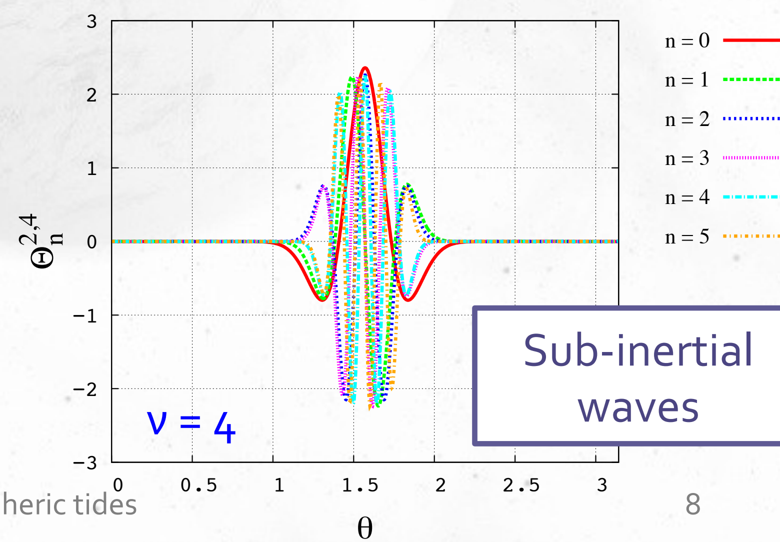
**Laplace's tidal equation**

$$\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\frac{\nu^2\sin\theta}{1-\nu^2\cos^2\theta}\frac{\partial}{\partial\theta}\right) - \frac{\nu^2}{1-\nu^2\cos^2\theta}\left(s\nu\frac{1+\nu^2\cos^2\theta}{1-\nu^2\cos^2\theta}+\frac{s^2}{\sin^2\theta}\right)\right]\Theta_n = -\Lambda_n\Theta_n$$

$$\nu = 2\Omega/\sigma$$




Super-inertial waves




Sub-inertial waves

# Vertical structure

$$\delta p = \sum_{\sigma,s} \delta p^{\sigma,s}(\theta, x)\, e^{i(\sigma t + s\varphi)}$$

**Expansion in Fourier series**

$$\delta p^{\sigma,s} = \sum_{n} \delta p_n(x)\, \Theta_n(\theta)$$

Radial profiles ($\delta p_n(x)$) — Hough functions ($\Theta_n(\theta)$)

**Expansion in Hough functions**

**Schrödinger-like equation**

$$\frac{d^2\Psi_n}{dx^2} + \lambda_n^2 \Psi_n = H^2 e^{-x/2} C(x)$$

$$\lambda_n^2 = f(\sigma_0, \sigma)$$

Thermal frequency ($\sigma_0$)

VELOCITY FIELD

DISPLACEMENT

PRESSURE

DENSITY

TEMPERATURE

# Frequency regimes: comparison with Chapman & Lindzen

ATMOSPHERE THICKNESS

**Relative difference between vertical wave numbers:**

**$k_v^{Chapman-Lindzen}$**

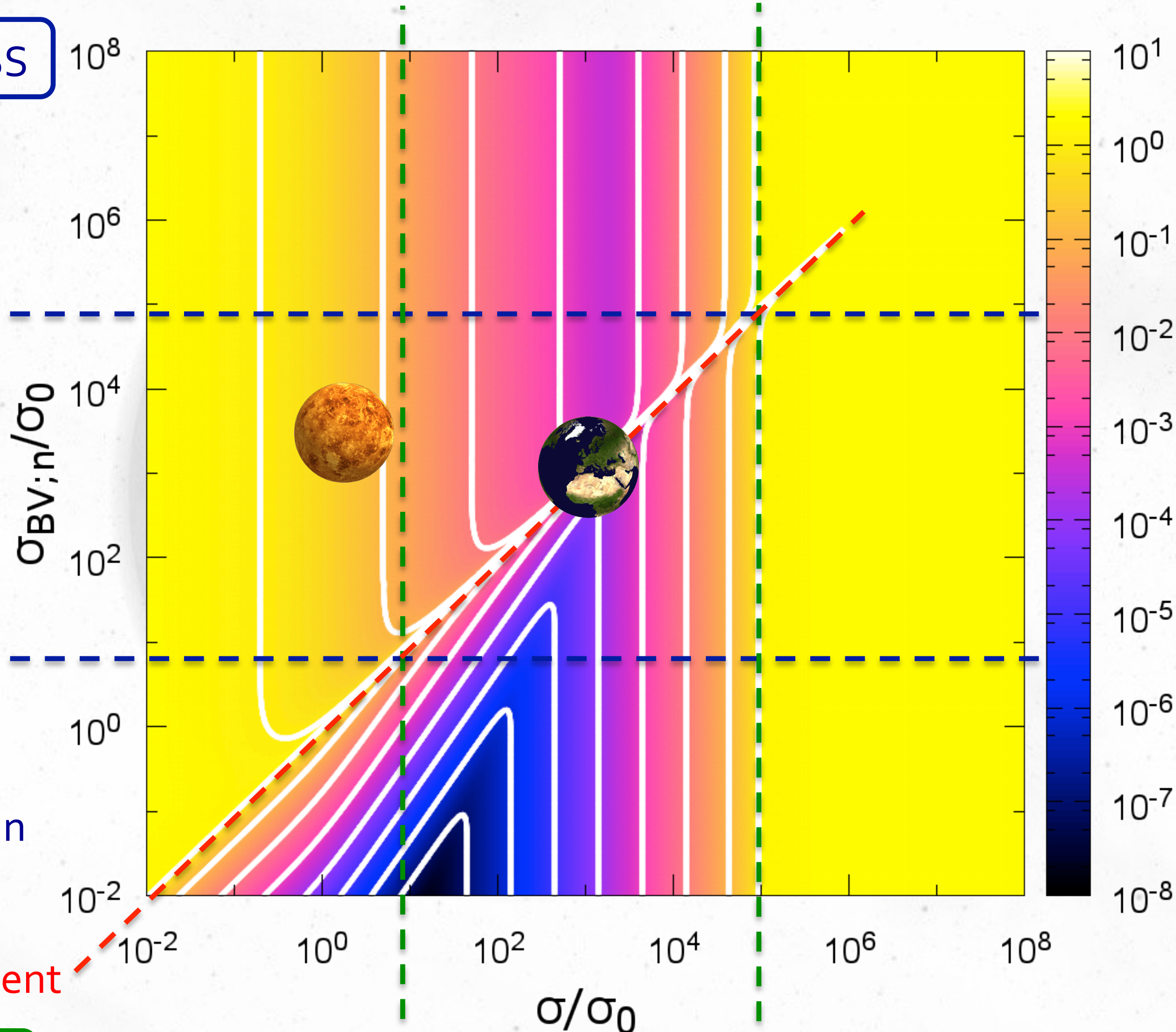
**VS**

**$k_v^{this\_work}$**

Thick

Thin

Very thin



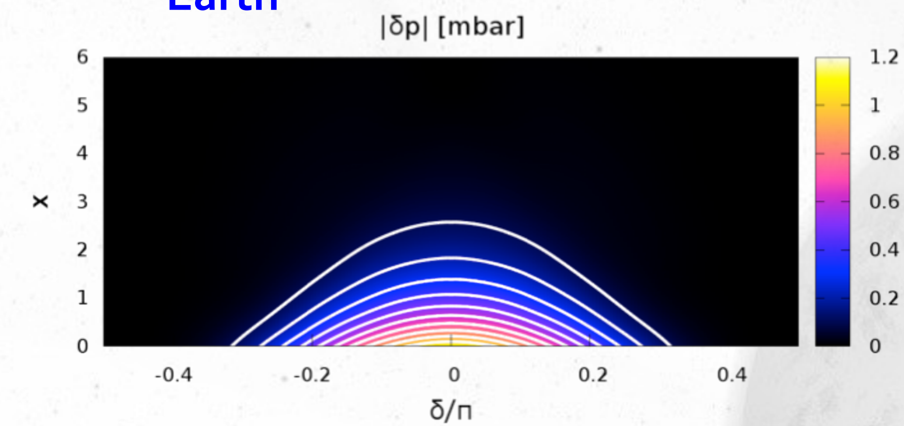

Transition propagative/evanescent

REGIMES

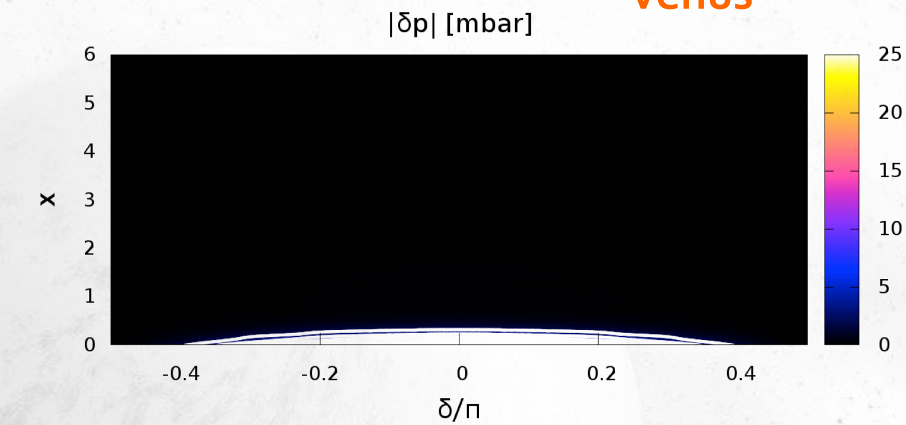
Thermal regime | Dynamic regime | Acoustic regime

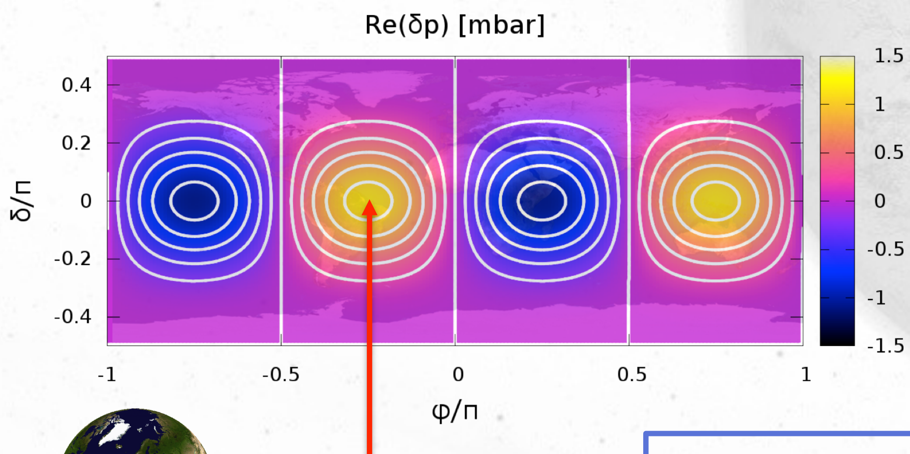
# Spatial distribution of perturbed quantities (preliminary results)

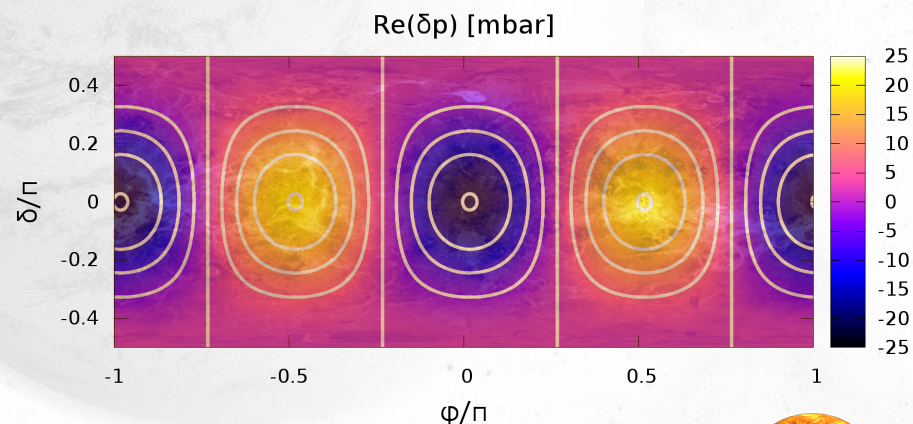
**Earth**




**Venus**



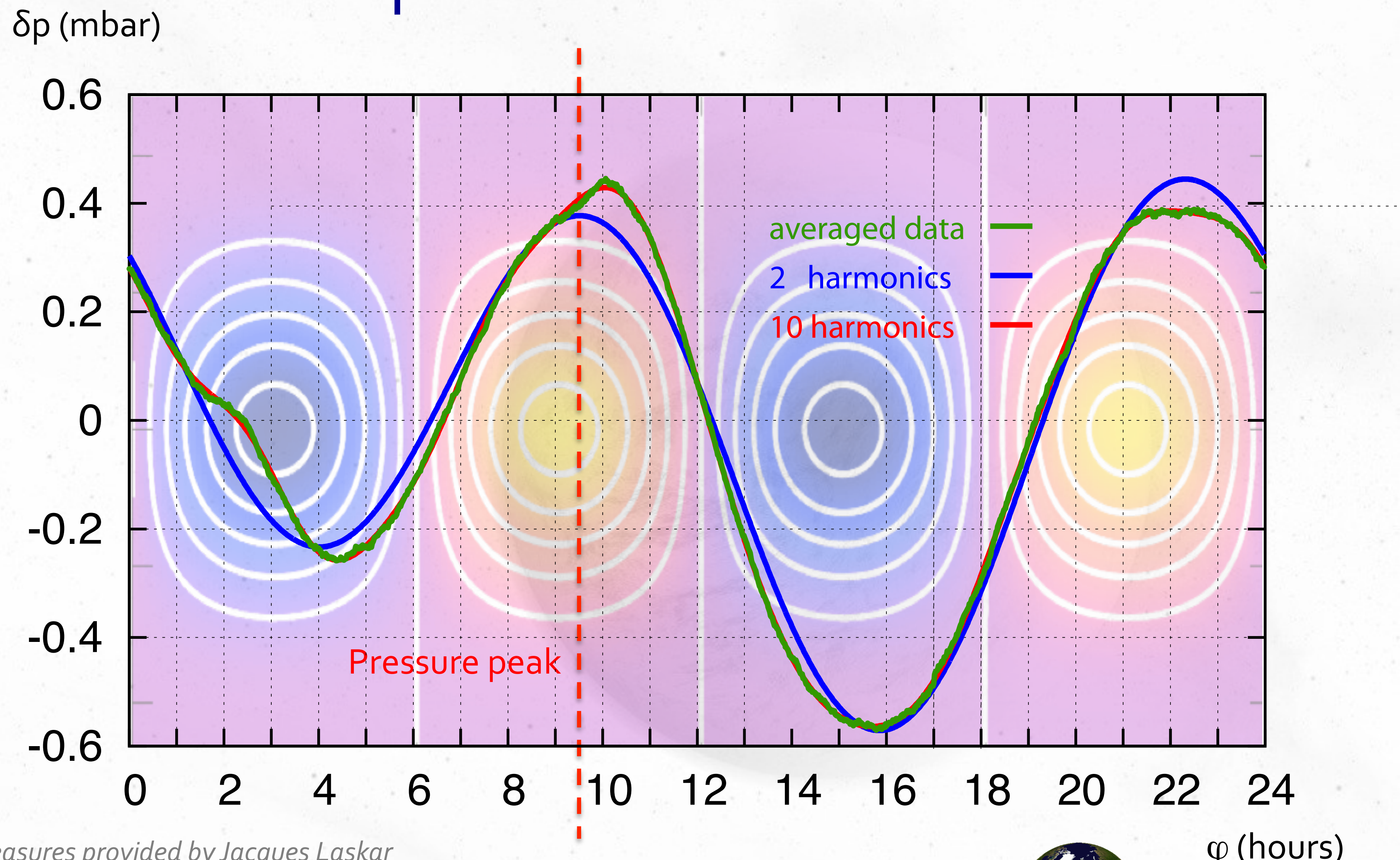




Pressure peak

**Semi-diurnal tide**

In good agreement with the GCM simulations of Leconte, Wu, Menou, Murray (2015)

# Comparison with measures




*Measures provided by Jacques Laskar*

**Earth's semi-diurnal tide**

# Conclusions and prospects

- Earth's semi-diurnal tide explained by the analytical model
- Identification of tidal regimes
- Dependence of the tidal torque on the tidal frequency
- Exploration of the domain of parameters
- Application to Venus and typical super-Earths
- Coupling with solid tides models (cf. Remus & al. 2012)

  ➢ Publication A&A in preparation