Dynamics of compact binary systems in scalar-tensor theories at the third post-Newtonian order

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grit gravitation in técnico

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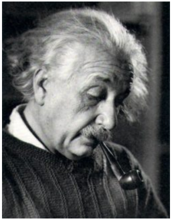
Plan

1. Scalar-tensor theories
2. The 3PN scalar-tensor Fokker Lagrangian
3. First results

Motivations

Tests of Einstein's general relativity using gravitational waves

- We need precise gravitational waveforms for alternative theories of gravity,



Why scalar-tensor theories ?

- It passes weak-field tests, *i.e.* in the Solar System,
- **It predicts large deviation from GR in the strong-field regime**,
- Hawking theorem (1976) : Binary BHs gravitational radiation indistinguishable from GR,
- Deviations from GR are expected for neutron stars.

Scalar-tensor theories

The action

$$S_{\mathrm{ST}} = \frac{c^3}{16\pi G} \int \mathrm{d}^4 x \sqrt{-g} \left[\phi R - \frac{\omega(\phi)}{\phi} g^{\alpha\beta} \partial_\alpha \partial_\beta \phi \right] + S_m \left(\mathfrak{m}, g_{\alpha\beta} \right)$$

- Scalar field ϕ and scalar function $\omega(\phi)$,
- Matter fields $\mathfrak{m}$,
- **Physical metric $g_{\alpha\beta}$** : Scalar field only coupled to the gravitational sector,
- Conformal metric $\tilde{g}_{\alpha\beta}$: Scalar field only coupled to the matter sector.

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The matter part

- **Self-gravitating bodies : the masses depend on the scalar field $M_A(\phi)$ (Eardley, 1975),**

$$S_{\mathrm{m}} = -\sum_A \int \mathrm{d}t\, M_A(\phi)\, c^2 \sqrt{-g_{\alpha\beta}\frac{v_A^\alpha v_A^\beta}{c^2}}$$

Scalar-tensor theories

Far from the system

- Scalar field $\phi = \phi_0$, scalar function $\omega(\phi_0) = \omega_0$,

Scalar-tensor theories

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The set of ST parameters

- Sensitivities : $s_A = \left.\frac{\mathrm{d}\ln M_A(\phi)}{\mathrm{d}\ln\phi}\right|_0$, and all higher order derivatives,
- Derivatives of the scalar function $\omega(\phi)$, *i.e.* $\left.\frac{\mathrm{d}\omega}{\mathrm{d}\phi}\right|_0$,
- ST parameters : $\tilde{G} = \frac{G(4+2\omega_0)}{\phi_0(3+2\omega_0)}$, $\alpha = \frac{2+\omega_0-s_1-s_2+2s_1s_2}{2+\omega_0}$

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Newtonian result

$$a^i_{1,\,N} = -\frac{\tilde{G}\alpha m_2}{r^2_{12}} n^i_{12}$$

▷ **Indistinguishable from GR**, effective gravitational constant $G_{\text{eff}} = \tilde{G}\alpha$.

What has been done so far

Scalar tensor waveforms

- Equations of motion at 2.5PN,
- Tensor gravitational waveform to 2PN,
- Scalar waveform to 1.5PN (starts at -0.5PN),
- Energy flux to 1PN beyond the leading order (starts at -1PN),

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Some remarks

- ▷ Done using the DIRE method (Pati & Will, 2000),
- ▷ To go to 2PN in the flux we need the EoM at 3PN .

This talk

Our goal

Compute the equations of motion at 3PN order.

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The method

We will use the multipolar post-Newtonian formalism, in particular the method based on a Fokker Lagrangian that was developped for the 4PN EoM in GR.

The multipolar post-Newtonian formalism

- **In the near zone :** post-Newtonian expansion

$$\bar{h}^{\mu\nu} = \sum_{m=2}^{\infty} \frac{1}{c^m} \bar{h}^{\mu\nu}_m \,, \quad \text{with} \quad \Box \bar{h}^{\mu\nu}_m = 16\pi G \, \bar{\tau}^{\mu\nu}_m \,,$$

$$\bar{\psi} = \sum_{m=2}^{\infty} \frac{1}{c^m} \bar{\psi}_m \,, \quad \text{with} \quad \Box \bar{\psi}_m = -8\pi G \, \bar{\tau}^{(s)}_m$$

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- **In the wave zone :** multipolar expansion

$$\mathcal{M}(h)^{\alpha\beta} = \sum_{n=1}^{\infty} G^n h^{\alpha\beta}_{(n)} \,, \quad \text{with} \quad \Box h^{\alpha\beta}_{(n)} = \Lambda^{\alpha\beta}_n \left[h_{(1)}, \ldots, h_{(n-1)}; \psi \right] ,$$

$$\mathcal{M}(\psi) = \sum_{n=1}^{\infty} G^n \psi_{(n)} \,, \quad \text{with} \quad \Box \psi_{(n)} = \Lambda^{(s)}_n \left[\psi_{(1)}, \ldots, \psi_{(n-1)}; h \right] ,$$

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- **Buffer zone :** matching between the near zone and far zone solutions :

$$\overline{\mathcal{M}(h)} = \mathcal{M}\left(\bar{h}\right) \qquad \text{everywhere,}$$

$$\overline{\mathcal{M}(\psi)} = \mathcal{M}\left(\bar{\psi}\right) \qquad \text{everywhere.}$$

What is a Fokker Lagrangian ?

Fokker (1929)

- Replace the gravitational degrees of freedom by their solution

$$S_{\text{Fokker}}\left[y_A, v_A, \dots\right] = S\left[g_{\text{sol}}\left(y_B, v_B, \dots\right), \phi_{\text{sol}}\left(y_B, v_B, \dots\right); v_A\right]$$

- **Generalized Lagrangian** : dependent on the accelerations,
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Why a Fokker Lagrangian ?

- Simpler calculation, only for the conservative part,
- The "$\mathbf{n+2}$" method : we need to know the metric at only half the order we would have expected, $\mathcal{O}(n+2)$ instead of $\mathcal{O}(2n+2, 2n+1, 2n; 2n+2)$.

Scalar-tensor theories

The gravitational part

- Rescaled scalar field : $\varphi = \frac{\phi}{\phi_0}$,
- From the conformal metric $\tilde{g}_{\mu\nu} = \varphi g_{\mu\nu}$ to the gothic metric $\tilde{\mathfrak{g}}^{\mu\nu} = \sqrt{\tilde{g}}\tilde{g}^{\mu\nu}$,

$$S_{\mathrm{ST}} = \frac{c^3\phi_0}{32\pi G}\int \mathrm{d}^4x \bigg[-\frac{1}{2}\left(\tilde{\mathfrak{g}}_{\mu\sigma}\tilde{\mathfrak{g}}_{\mu\rho} - \frac{1}{2}\tilde{\mathfrak{g}}_{\mu\nu}\tilde{\mathfrak{g}}_{\rho\sigma}\right)\tilde{\mathfrak{g}}^{\lambda\gamma}\partial_\lambda\tilde{\mathfrak{g}}^{\mu\nu}\partial_\gamma\tilde{\mathfrak{g}}^{\rho\sigma} + \tilde{\mathfrak{g}}_{\mu\nu}\left(\partial_\sigma\tilde{\mathfrak{g}}^{\rho\mu}\partial_\rho\tilde{\mathfrak{g}}^{\sigma\nu} - \partial_\rho\tilde{\mathfrak{g}}^{\rho\mu}\partial_\sigma\tilde{\mathfrak{g}}^{\sigma\nu}\right) - \frac{3+2\omega}{\varphi^2}\tilde{\mathfrak{g}}^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi\bigg]$$

The matter part

$$S_{\mathrm{m}} = -\sum_A \int \mathrm{d}t\, M_A(\phi)\, c^2 \sqrt{-g_{\alpha\beta}\frac{v_A^\alpha v_A^\beta}{c^2}}$$

▷ depends on the scalar field through the masses and the physical metric $g_{\alpha\beta}$.

Post-Newtonian formalism from GR to scalar-tensor theories

- Perturbed metric $h^{\mu\nu} = \tilde{\mathfrak{g}}^{\mu\nu} - \eta^{\mu\nu}$ and scalar field $\psi = \varphi - 1$,
- At leading order $(h, \psi) = (h^{00ii} = h^{00} + h^{ii}, h^{0i}, h^{ij}; \psi) = \mathcal{O}(2, 3, 4; 2)$

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The "$n+2$" method in ST : $\mathcal{O}(4, 5, 4; 4)$

$$h^{00ii} = -\frac{4V}{c^2} - \frac{8V^2}{c^4} + \mathcal{O}\left(\frac{1}{c^6}\right) ,$$

$$h^{0i} = -\frac{4V^i}{c^3} - \frac{8}{c^5}\left(R_i + VV^i\right) + \mathcal{O}\left(\frac{1}{c^7}\right) ,$$

$$h^{ij} = -\frac{4}{c^4}\left(W_{ij} - \frac{1}{2}\delta_{ij}W\right) + \mathcal{O}\left(\frac{1}{c^6}\right)$$

$$\psi = -\frac{4\psi_{(0)}}{c^2} + \frac{2}{c^4}\left(1 - \frac{\phi_0\omega_0'}{3 + 2\omega_0}\right)\psi_{(0)}^2 + \mathcal{O}\left(\frac{1}{c^6}\right) ,$$

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▷ We need V, V^i and $\psi_{(0)}$ at 1PN and R^i, W_{ij} at N,

$$\Delta W_{ij} = -\frac{4\pi G}{\phi_0}\left(\sigma_{ij} - \delta_{ij}\sigma_{kk}\right) - \partial_i V\,\partial_j V - (3 + 2\omega_0)\partial_i\psi_{(0)}\,\partial_j\psi_{(0)}$$

How does it work in practice...

1. Compute the (local) Fokker Lagrangian using Hadamard-type regularisation for both :
   - the divergences at the position of the particles (ultraviolet),
   - the divergences of the PN solution at infinity (infrared),

$$L = \mathrm{FP}_{B=0} \int \mathrm{d}^3x \left(\frac{r}{r_0}\right)^B \overline{\mathcal{L}}$$

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3. Add the tail term, if present (in GR it starts at 4PN).

1- Compute the 3PN ST Lagrangian using Hadamard regularisation

UV divergences

- Compact bodies $\longrightarrow$ modelised by point particles, *i.e.* $\delta^{(3)}\left(\mathbf{x}-\mathbf{y}_A(t)\right)$
- **Two constants of regularisation** l_1 **and** l_2

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IR divergence

- post-Newtonian solution valid only in the near zone $\Longrightarrow$ divergences at infinty,

$$L = \mathrm{FP}_{B=0} \int \mathrm{d}^3x \left(\frac{r}{r_0}\right)^B \overline{\mathcal{L}}$$

- in GR at 3PN no contribution,
- in ST contributions at 3PN !
  - **constant of regularisation r_0** : does not vanish through a shift,
  - vanishes in the GR limit ($\omega_0 \to 0$) and when $s_1 = s_2$ or $s_{1\,\mathrm{or}\,2} = \frac{1}{2}$ (BHs).

2- Use dimensional regularisation to treat the UV divergences

Principle

- Go to d spatial dimensions, with $d = 3 + \varepsilon$.
  - $G \to G\, l_0^{d-3}$,
  - Expand all functions when $r_1 \to 0$.
- Compute the difference between HR and DR through the formula

$$\mathcal{D}I = \frac{1}{\varepsilon} \sum_{q=q_0}^{q_1} \left[\frac{1}{q+1} + \varepsilon \ln l_1 \right] \langle f_{-3,q}^{(\varepsilon)} \rangle + (1 \leftrightarrow 2)$$

- Expand the Lagrangian in the limit $\varepsilon \to 0$.

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- Expand the Lagrangian in the limit $\varepsilon \to 0$.

Result

- No more constant l_1 and l_2 : ok,
- Presence of a **pole** $\frac{1}{\varepsilon}$: should vanish through a redefinition of the trajectory of the particles.

3- Tail effects and IR divergences

A scalar tail effect ?

- Constant r_0 still present in the end result of the local part $\Longrightarrow$ **Presence of tail terms in the conservative Lagrangian at 3PN, originating from the scalar field ?**
- New effect in ST theories, due to the fact that the scalar field flux starts at -1PN,
- At the end the constant r_0 should disappear.

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Test of the IR divergences

- Use dimensional regularisation also for the IR divergences to test how strong the regularisation procedure is :
  - If no difference with Hadamard : OK,
  - If different results : switch to the more powerful DimReg (for the tails also).
- Constant $r_0 \longrightarrow$ pole $1/\varepsilon$.

What has been done

Equations of motion at 2PN

- Easy and "quick" calculation : $\mathcal{O}(4,3,4;\,4)$, only Hadamard regularisation,
- Confirmation of the previous result by Mirshekari & Will (2013).

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At 3PN

- **Fokker Lagrangian using Hadamard regularisation**,
- Some consistency checks :
  - GR limit : $\omega_0 \to \infty \Longrightarrow$ GR result,
  - Two black hole limit : $s_1 = s_2 = \frac{1}{2} \Longrightarrow$ indistinguishable from GR.

Where we are

On-going calculations

- From Hadamard to dimensional regularisation : very long calculation,
- Eventual tail effects at 3PN coming from the scalar field.

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What's next ?

- ▷ Conserved quantities (energy, angular momentum . . .),
- ▷ Ready to use eom to be incorporated in the scalar waveform and the scalar flux at 2PN.

Conclusion

Equations of motion at 3PN in scalar-tensor theories

- First part computed using Hadamard regularisation ,
- Dimentional regularisation and investigation of the tail effect : work in progress,
- Conserved quantities : to be done.

Prospects

- Waveform for ST theories at 2PN (inspiral phase),
- Can be EOB waveform to have the full IMR waveform,
- Do the same for other modified theories of gravity.