The Effective-One-Body Approach to Compact Binaries and its Synergy with Other Approaches

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In memoriam Vishu, aka « LOUis Quasimodo»

33rd International IAP Colloquium
The Era of Gravitational Wave Astronomy

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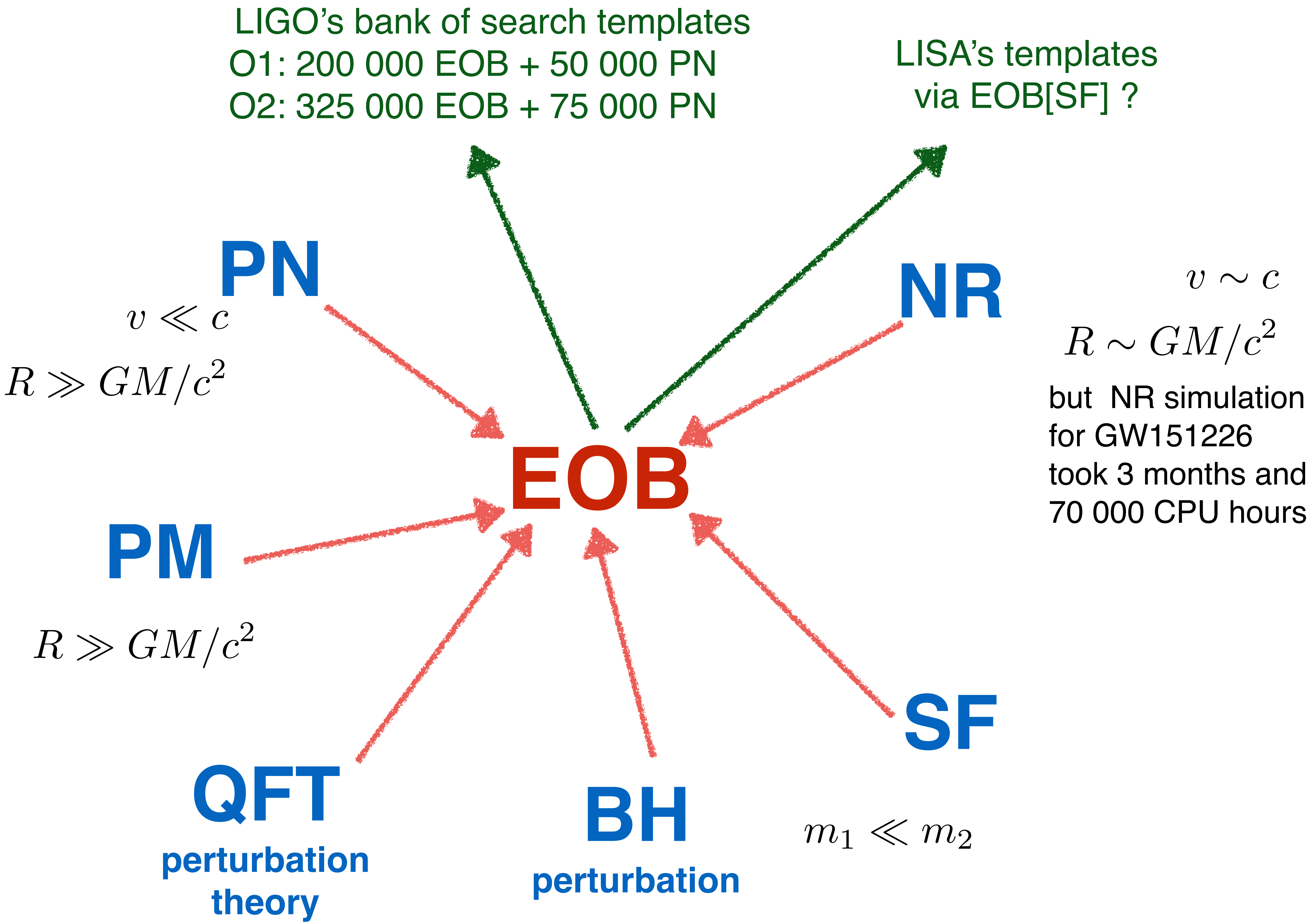

LIGO's bank of search templates
O1: 200 000 EOB + 50 000 PN
O2: 325 000 EOB + 75 000 PN
LISA's templates
via EOB[SF] ?
PN
$v \ll c$
$R \gg GM/c^2$
NR
$v \sim c$
$R \sim GM/c^2$
but NR simulation
for GW151226
took 3 months and
70 000 CPU hours
EOB
PM
$R \gg GM/c^2$
SF
$m_1 \ll m_2$
QFT
perturbation
theory
BH
perturbation

Analytical GW Templates for BBH Coalescences ?

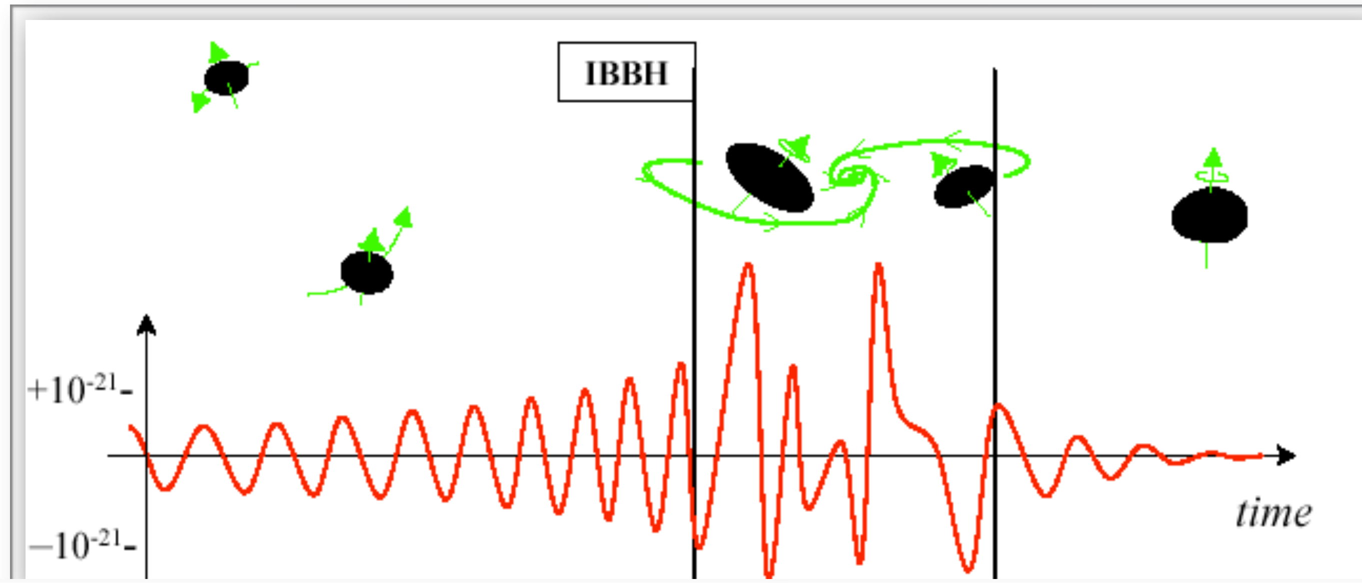
PN corrections to Einstein's quadrupole frequency « chirping »
from PN-improved balance equation dE(f)/dt = - F(f)

$$\frac{d\phi}{d\ln f} = \frac{\omega^2}{d\omega/dt} = Q_\omega^N \widehat{Q}_\omega \qquad \frac{v}{c} = \left(\frac{\pi G(m_1+m_2)f}{c^3}\right)^{\frac{1}{3}}$$

$$Q_\omega^N = \frac{5\,c^5}{48\,\nu\,v^5}\,;\ \widehat{Q}_\omega = 1 + c_2\left(\frac{v}{c}\right)^2 + c_3\left(\frac{v}{c}\right)^3 + \cdots \qquad \nu = \frac{m_1 m_2}{(m_1+m_2)^2}$$

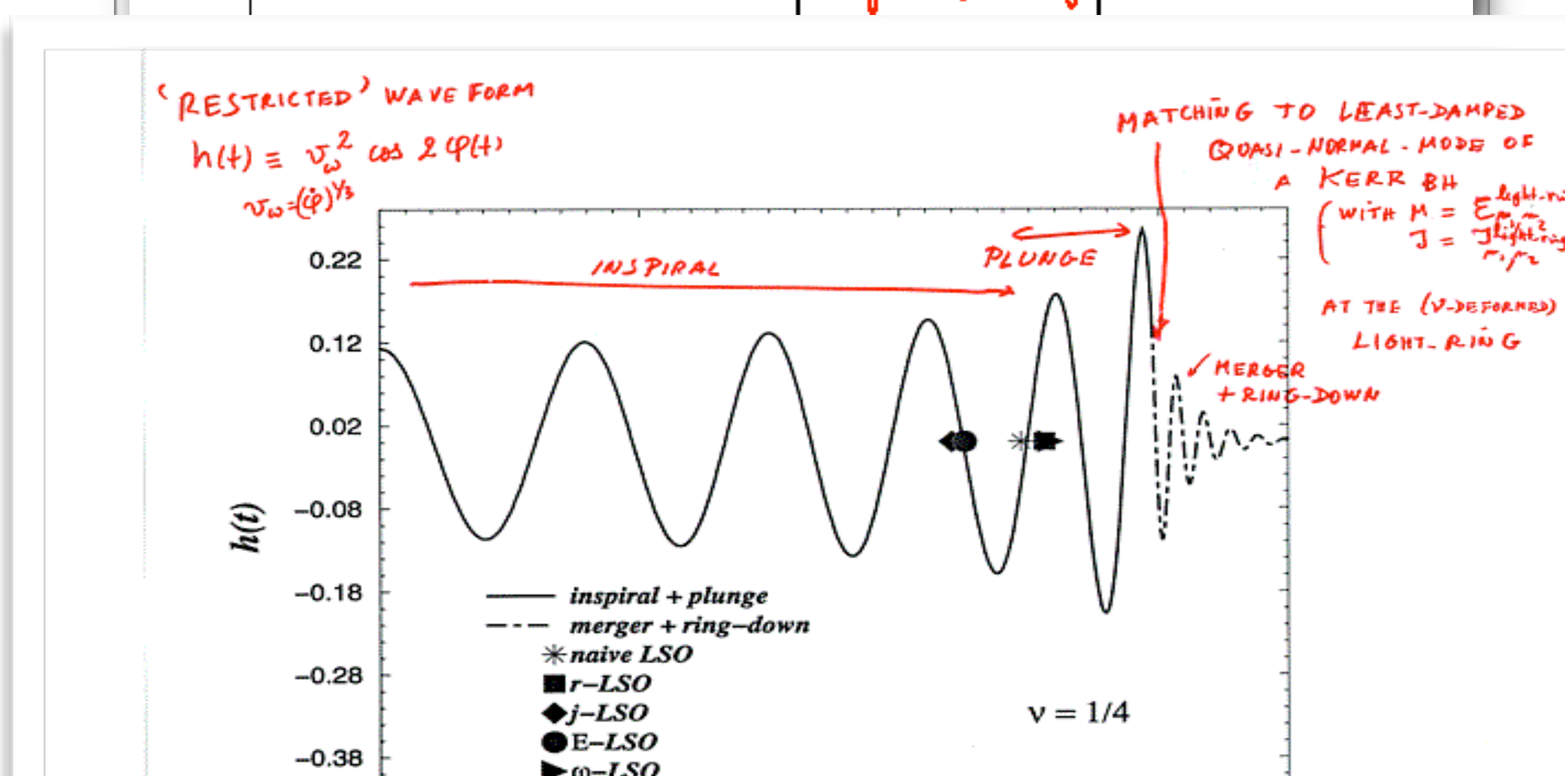
Cutler et al. '93:
« slow convergence of PN »

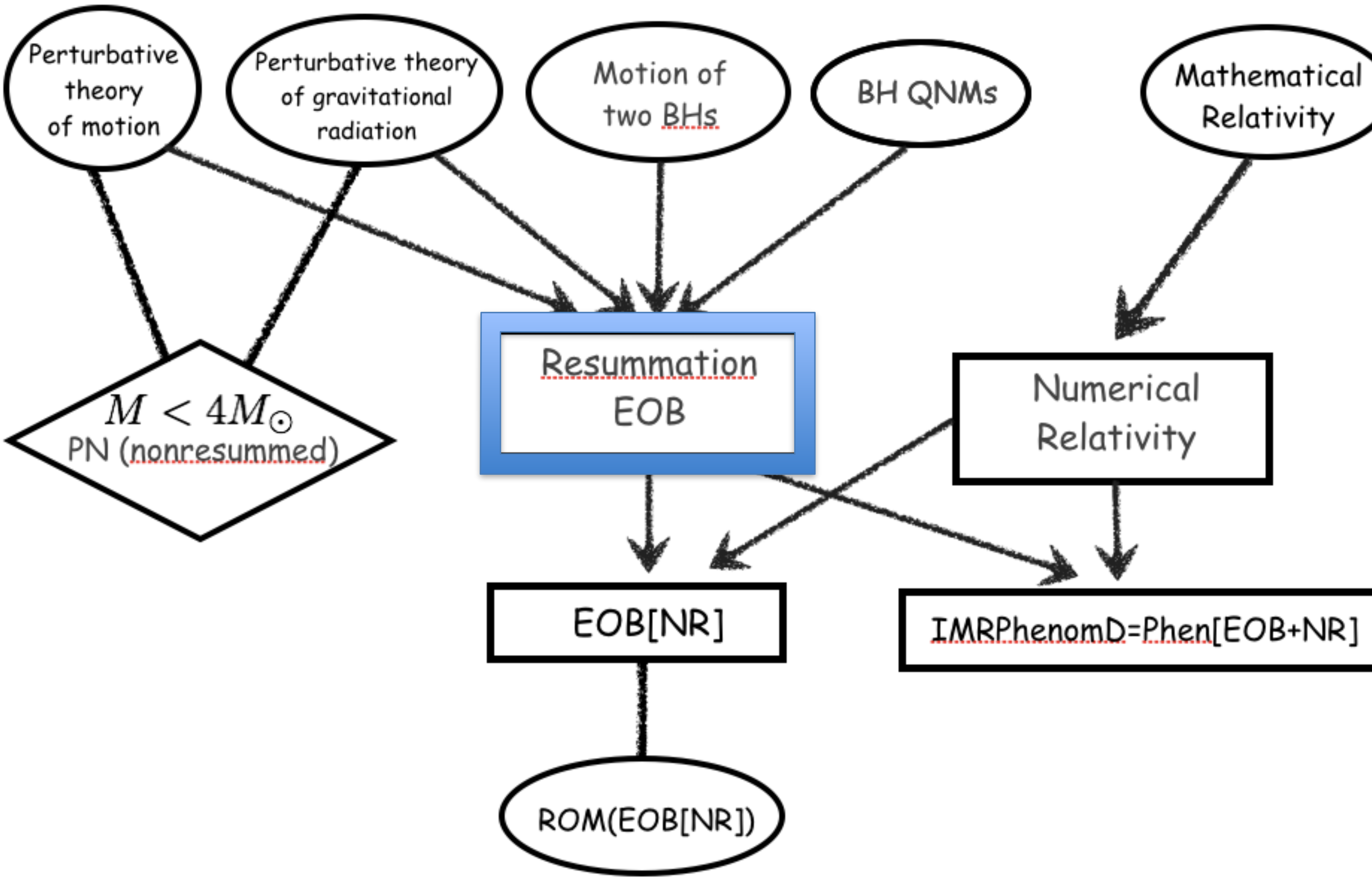
Brady-Creighton-Thorne'98:
« inability of current computational techniques to evolve a BBH through its last ~10 orbits of inspiral » and to compute the merger



Damour-Iyer-Sathyaprakash'98:
use resummation methods for E and F

Buonanno-Damour '99-00: novel, resummed approach: Effective-One-Body analytical formalism



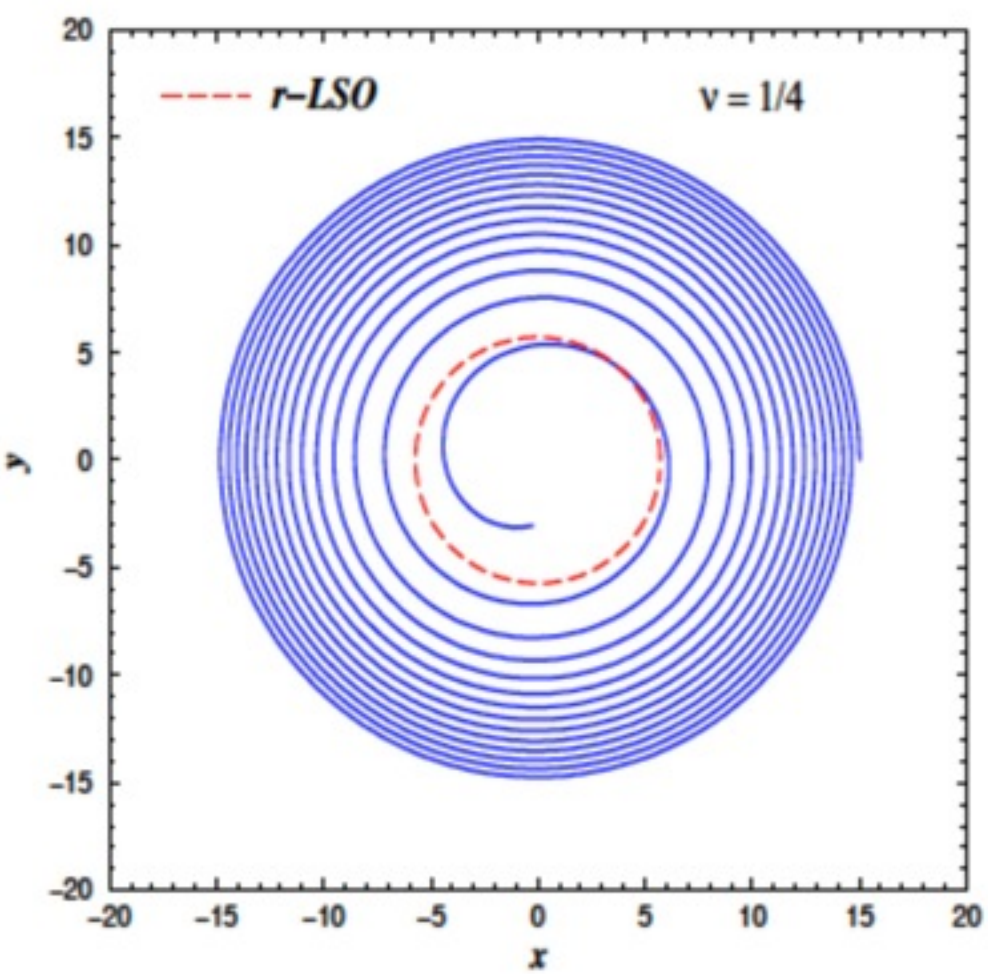

Perturbative theory of motion
Perturbative theory of gravitational radiation
Motion of two BHs
BH QNMs
Mathematical Relativity
$M < 4M_\odot$
PN (nonresummed)
Resummation EOB
Numerical Relativity
EOB[NR]
IMRPhenomD=Phen[EOB+NR]
ROM(EOB[NR])

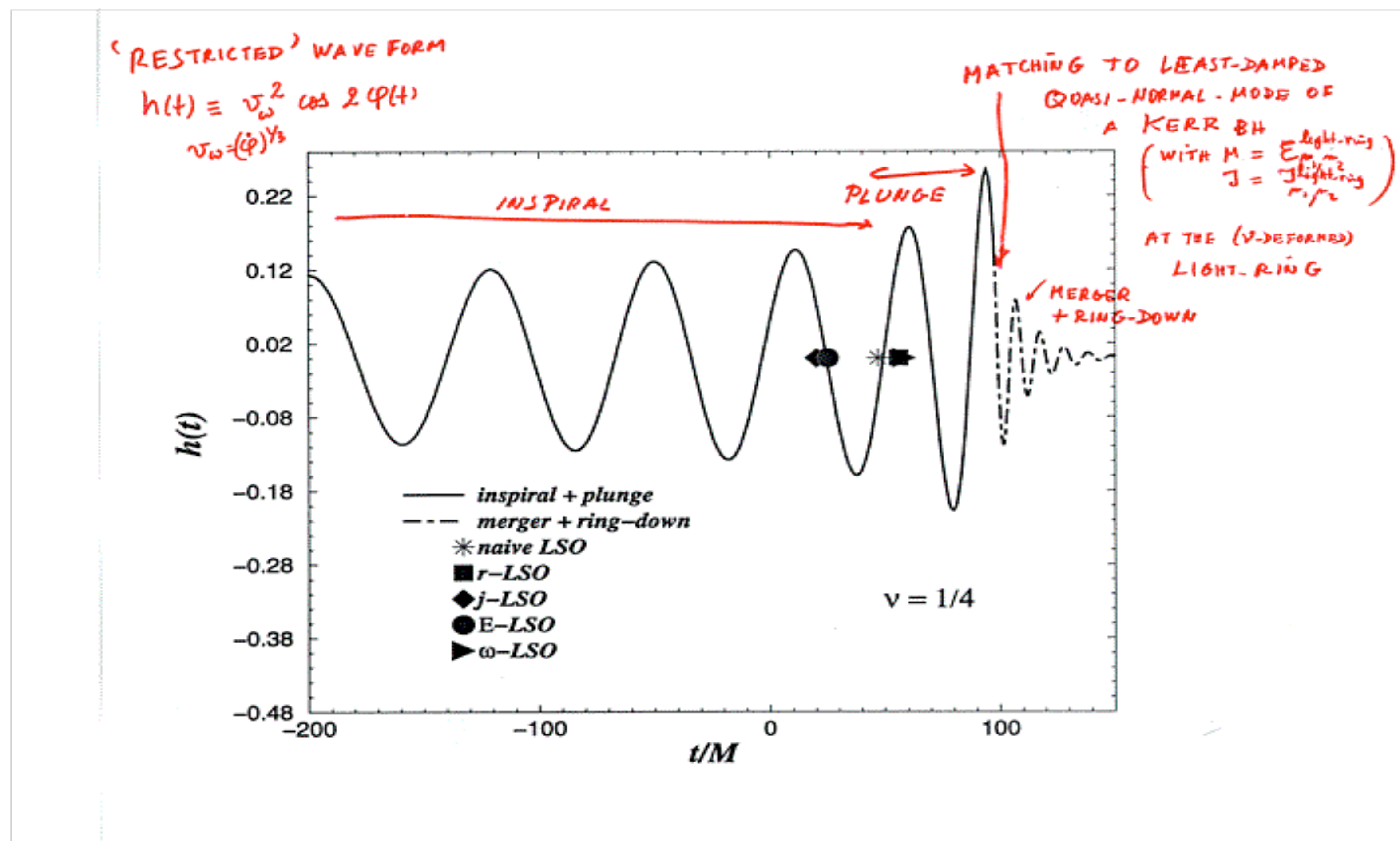
Effective One Body (EOB) Method)

Buonanno-Damour 1999, 2000; Damour-Jaranowski-Schaefer 2000; Damour 2001 (SEOB)

Resummation of perturbative PN results $\longrightarrow$ description of the coalescence
+ addition of ringdown (Vishveshwara 70, Davis-Ruffini-Tiomno 1972)

Buonanno-Damour 2000

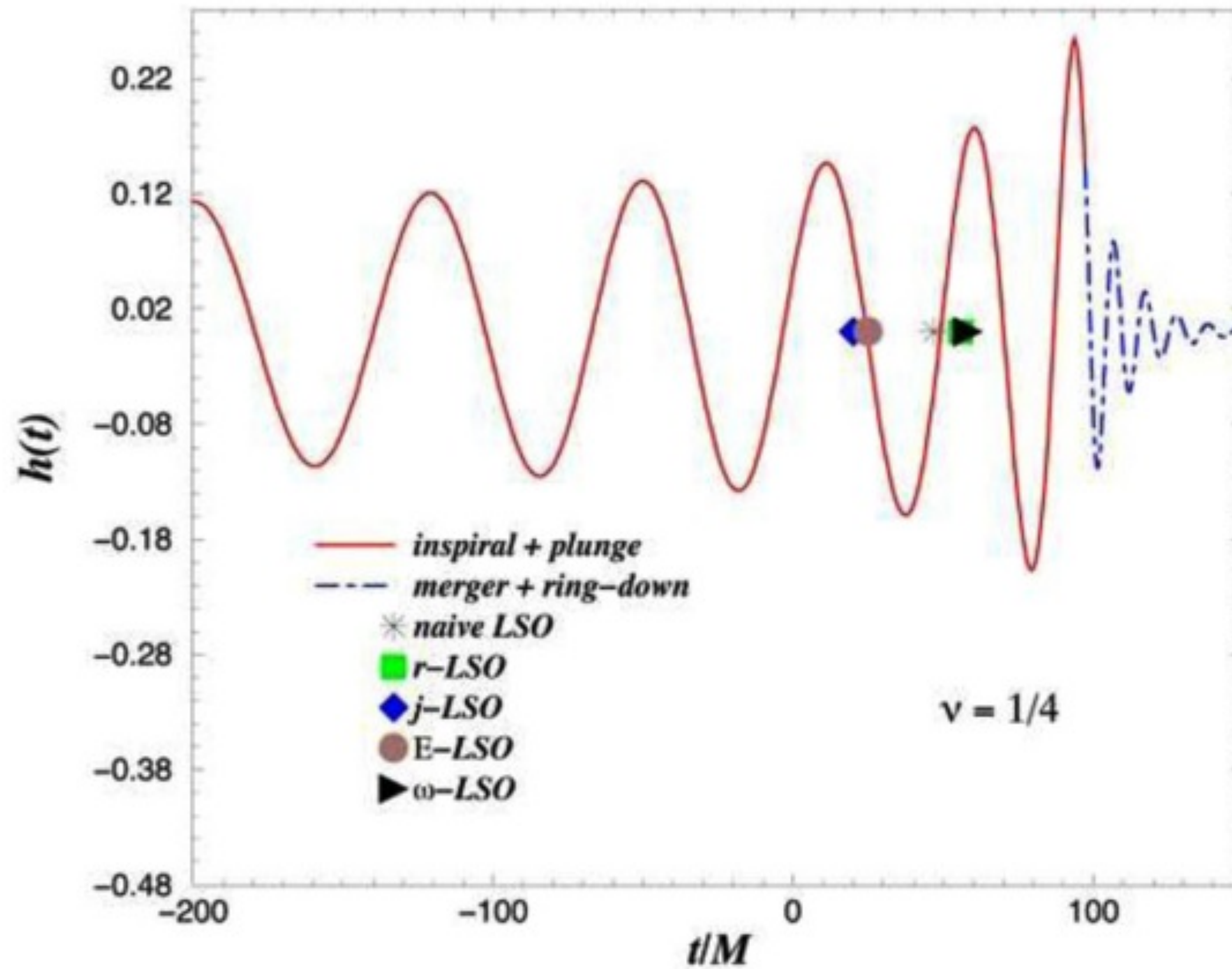




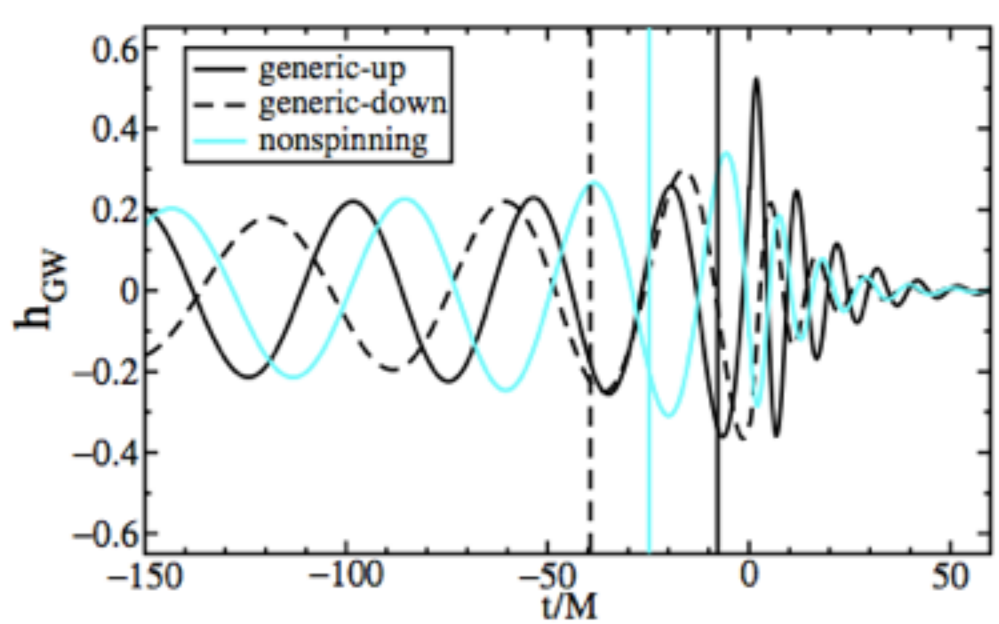
Predictions as early as 2000 :
continued transition, non adiabaticity, first complete waveform, final spin (OK within 10%), final mass

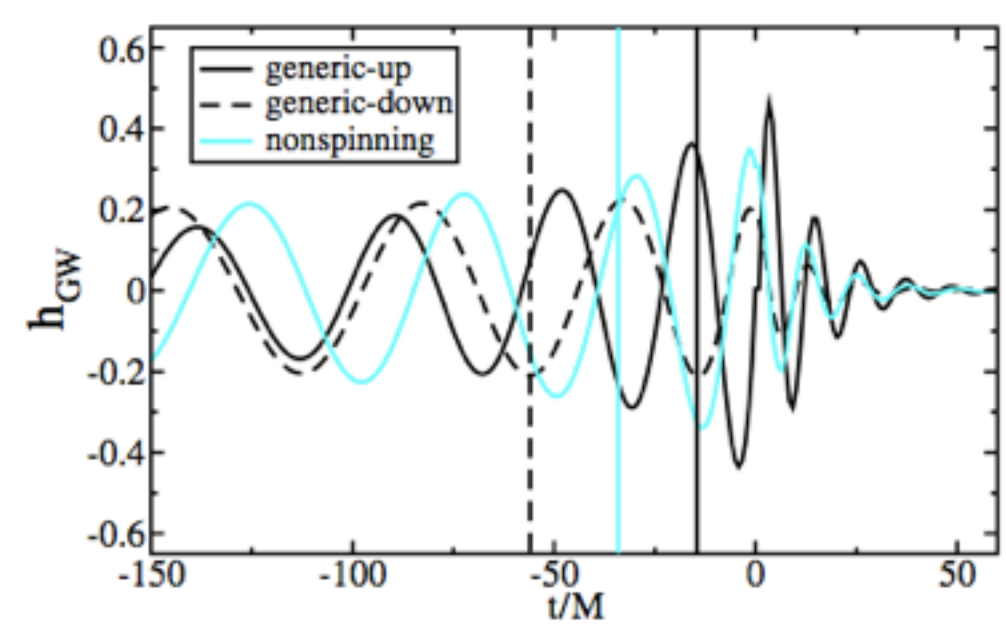
First complete waveforms for BBH coalescences: analytical EOB

Non-spinning BHs
Buonanno-Damour 2000



Spinning BHs
Buonanno-Chen-Damour
Nov 2005:
« to show the promise
of a purely analytical
EOB-based
approach »





EOB THEORY + EOB[NR] + EOB[SF] DEVELOPMENTS

Buonanno,Damour 99 (2 PN Hamiltonian)
Buonanno,Damour 00 (Rad.Reac. full waveform)
Damour, Jaranowski,Schäfer 00 (3 PN Hamiltonian)
Damour 01, (spinning bodies)
Buonanno, Chen, Damour 05,
Damour-Jaranowski,Schäfer 08, Barausse, Buonanno, 10, Nagar 11,
Balmelli-Jetzer 12, Taracchini et al 12,14, Damour,Nagar 14
Damour, Nagar 07, (factorized waveform)
Damour, Iyer, Nagar 08,
Pan et al. 11
Damour, Nagar 10 (BNS tidal effects)
Bini-Damour-Faye 12
Bini, Damour 13, Damour, Jaranowski, Schäfer 15 (4 PN Hamiltonian)

EOB vs NR and EOB[NR]
Buonanno, Cook, Pretorius 07,
Buonanno, Pan, Taracchini 08-
Damour-Nagar 08-

Reduced Order Model version (Pürrer 2014, 2016) of EOB[NR] (Taracchini et al 2014)

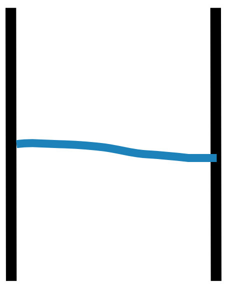
Phenomenological model (Ajith et al 2007, Hannam et al 2014, Husa et al 2016, Kahn et al 2016) of FFT of hybrids EOB + NR

EOB vs SF and EOB[SF]
Damour 09
Barack-Sago-Damour 10
Barausse-Buonanno-LeTiec 12
Akcay-Barack-Damour-Sago 12
Bini-Damour 13-16
LeTiec 15
Bini-Damour-Geralico 16
Hopper-Kavanagh-Ottewill 16
Akcay-vandeMeent 16

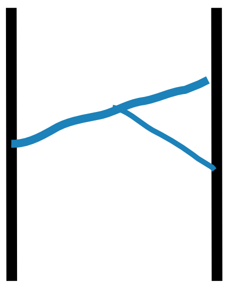
EOB vs PM
Damour 16

Real dynamics versus Effective dynamics

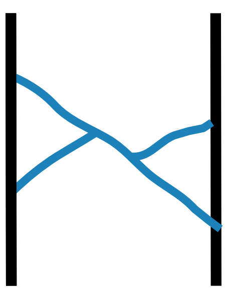
Real dynamics



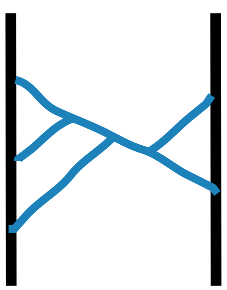
G



G^2
1 loop

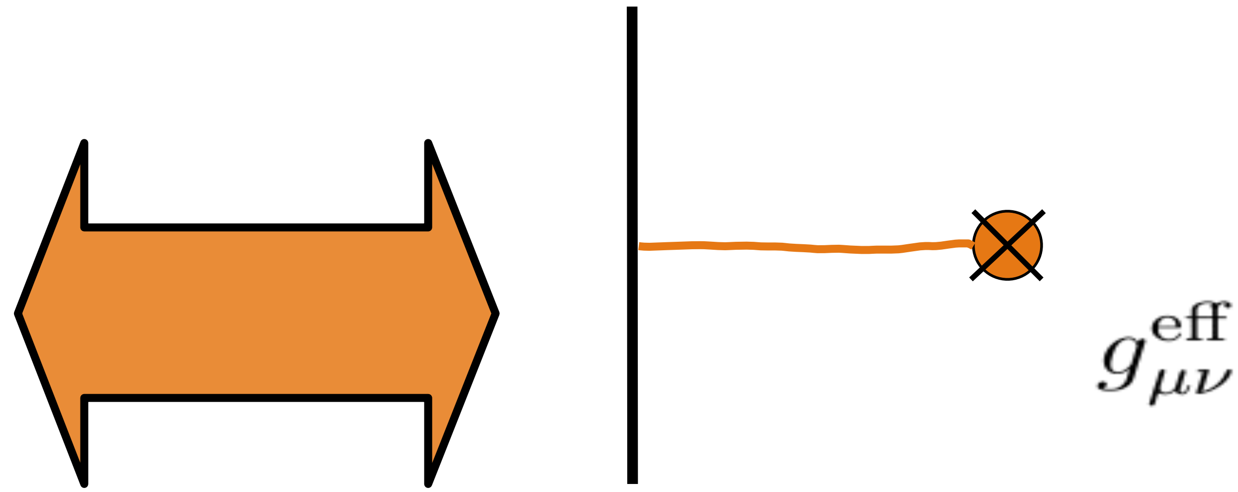


G^3
2 loops



G^4
3 loops

Effective dynamics



$$S = - \int \mu ds + \ldots$$

$$H = H_0 + \left(G H_1 + \frac{G^2}{c^2} H_2 + \frac{G^3}{c^4} H_3 + \frac{G^4}{c^6} H_4 \right) \left(1 + \frac{1}{c^2} + \ldots \right)$$

Effective metric for non-spinning bodies: a nu-deformation of Schwarzschild

$$\nu = \frac{m_1 m_2}{(m_1 + m_2)^2}$$

$$ds^2_{\mathrm{eff}} = -A(r;\nu)\, dt^2 + B(r;\nu)\, dr^2 + r^2 \left(d\theta^2 + \sin^2 \theta \, d\varphi^2 \right)$$

TWO-BODY/EOB "CORRESPONDENCE":

THINK QUANTUM-MECHANICALLY (J.A. WHEELER)

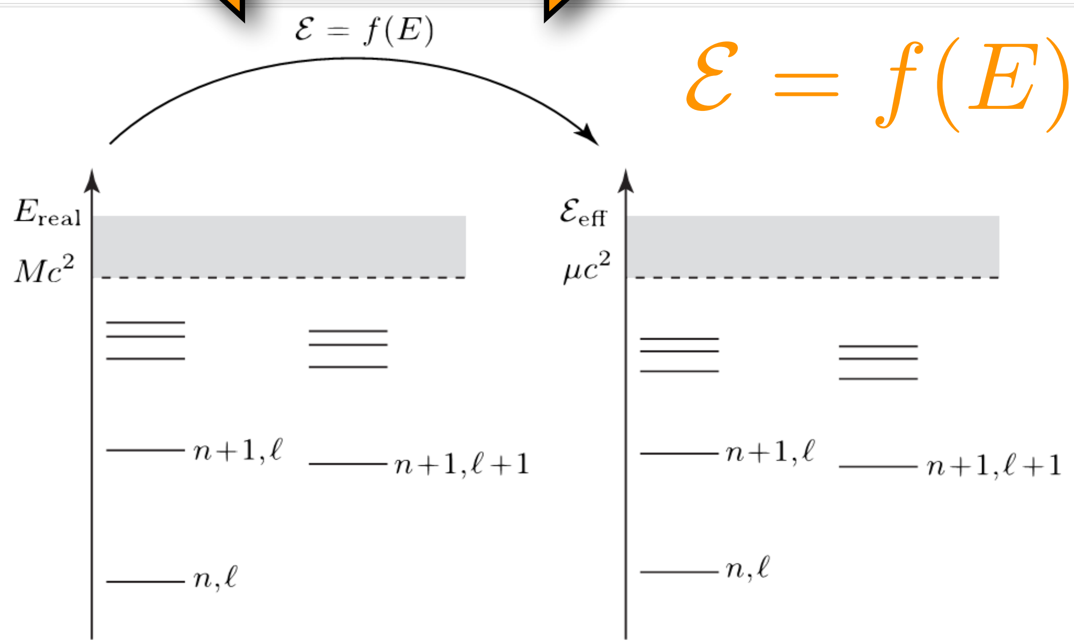
Real 2-body system
(in the c.o.m. frame)
(m_1, m_2)

1:1 map

An effective particle
in some effective metric

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$g_{\mu\nu}^{\mathrm{eff}}$$



$$\mathcal{E} = f(E)$$

Figure 1: Sketch of the correspondence between the quantized energy levels of the real and effective conservative dynamics. n denotes the 'principal quantum

$$\mu^2 + g_{\mathrm{eff}}^{\mu\nu} \frac{\partial S_{\mathrm{eff}}}{\partial x^\mu} \frac{\partial S_{\mathrm{eff}}}{\partial x^\nu} + \mathcal{O}(p^4) = 0$$

Bohr-Sommerfeld's
Quantization Conditions
(action-angle variables &
Delaunay Hamiltonian)

$$\begin{aligned} J &= \ell\hbar = \frac{1}{2\pi} \oint p_\varphi d\varphi \\ N &= n\hbar = I_r + J \\ I_r &= \frac{1}{2\pi} \oint p_r dr \end{aligned}$$

$$H^{\mathrm{classical}}(q,p) \longrightarrow H^{\mathrm{classical}}(I_a) \longrightarrow \boxed{E^{\mathrm{quantum}}(I_a = n_a h) = f^{-1}[\mathcal{E}_{\mathrm{eff}}^{\mathrm{quantum}}(I_a^{\mathrm{eff}} = n_a h)]}$$

2-body Taylor-expanded N + 1PN + 2PN Hamiltonian

$$H_{\mathrm{N}}(\mathbf{x}_a, \mathbf{p}_a) = \frac{\mathbf{p}_1^2}{2m_1} - \frac{1}{2}\frac{Gm_1m_2}{r_{12}} + (1 \leftrightarrow 2)$$

$$c^2 H_{1\mathrm{PN}}(\mathbf{x}_a, \mathbf{p}_a) = -\frac{1}{8}\frac{(\mathbf{p}_1^2)^2}{m_1^3} + \frac{1}{8}\frac{Gm_1m_2}{r_{12}}\left(-12\frac{\mathbf{p}_1^2}{m_1^2} + 14\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1m_2} + 2\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1m_2}\right)$$
$$+\frac{1}{4}\frac{Gm_1m_2}{r_{12}}\frac{G(m_1+m_2)}{r_{12}} + (1 \leftrightarrow 2),$$

$$c^4 H_{2\mathrm{PN}}(\mathbf{x}_a, \mathbf{p}_a) = \frac{1}{16}\frac{(\mathbf{p}_1^2)^3}{m_1^5} + \frac{1}{8}\frac{Gm_1m_2}{r_{12}}\left(5\frac{(\mathbf{p}_1^2)^2}{m_1^4} - \frac{11}{2}\frac{\mathbf{p}_1^2\mathbf{p}_2^2}{m_1^2m_2^2} - \frac{(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{m_1^2m_2^2} + 5\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^2m_2^2}\right.$$
$$\left.-6\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^2m_2^2} - \frac{3}{2}\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^2m_2^2}\right)$$
$$+\frac{1}{4}\frac{G^2m_1m_2}{r_{12}^2}\left(m_2\left(10\frac{\mathbf{p}_1^2}{m_1^2} + 19\frac{\mathbf{p}_2^2}{m_2^2}\right) - \frac{1}{2}(m_1+m_2)\frac{27(\mathbf{p}_1\cdot\mathbf{p}_2) + 6(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1m_2}\right)$$
$$-\frac{1}{8}\frac{Gm_1m_2}{r_{12}}\frac{G^2(m_1^2 + 5m_1m_2 + m_2^2)}{r_{12}^2} + (1 \leftrightarrow 2),$$

2-body Taylor-expanded 3PN Hamiltonian [JS 98, DJS 01]

$$
\begin{aligned}
c^6 H_{3\mathrm{PN}}(\mathbf{x}_a, \mathbf{p}_a) = & -\frac{5}{128}\frac{(\mathbf{p}_1^2)^4}{m_1^7} + \frac{1}{32}\frac{G m_1 m_2}{r_{12}}\left(-14\frac{(\mathbf{p}_1^2)^3}{m_1^6} + 4\frac{((\mathbf{p}_1\cdot\mathbf{p}_2)^2 + 4\mathbf{p}_1^2\mathbf{p}_2^2)\mathbf{p}_1^2}{m_1^4 m_2^2} + 6\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^4 m_2^2}\right.\\
& - 10\frac{(\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2 + \mathbf{p}_2^2(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2)\mathbf{p}_1^2}{m_1^4 m_2^2} + 24\frac{\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^4 m_2^2}\\
& + 2\frac{\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^3 m_2^3} + \frac{(7\mathbf{p}_1^2\mathbf{p}_2^2 - 10(\mathbf{p}_1\cdot\mathbf{p}_2)^2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^3 m_2^3}\\
& + \frac{(\mathbf{p}_1^2\mathbf{p}_2^2 - 2(\mathbf{p}_1\cdot\mathbf{p}_2)^2)(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1^3 m_2^3} + 15\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^3 m_2^3}\\
& \left. - 18\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3}{m_1^3 m_2^3} + 5\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3}{m_1^3 m_2^3}\right) + \frac{G^2 m_1 m_2}{r_{12}^2}\left(\frac{1}{16}(m_1 - 27m_2)\frac{(\mathbf{p}_1^2)^2}{m_1^4}\right.\\
& - \frac{115}{16}m_1\frac{\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1^3 m_2} + \frac{1}{48}m_2\frac{25(\mathbf{p}_1\cdot\mathbf{p}_2)^2 + 371\mathbf{p}_1^2\mathbf{p}_2^2}{m_1^2 m_2^2} + \frac{17}{16}\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2}{m_1^3} + \frac{5}{12}\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4}{m_1^3}\\
& - \frac{1}{8}m_1\frac{(15\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_2) + 11(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1))(\mathbf{n}_{12}\cdot\mathbf{p}_1)}{m_1^3 m_2} - \frac{3}{2}m_1\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^3 m_2}\\
& + \frac{125}{12}m_2\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^2 m_2^2} + \frac{10}{3}m_2\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^2 m_2^2}\\
& \left. - \frac{1}{48}(220m_1 + 193m_2)\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^2 m_2^2}\right) + \frac{G^3 m_1 m_2}{r_{12}^3}\left(-\frac{1}{48}\left(425m_1^2 + \left(473 - \frac{3}{4}\pi^2\right)m_1 m_2 + 150m_2^2\right)\frac{\mathbf{p}_1^2}{m_1^2}\right.\\
& + \frac{1}{16}\left(77(m_1^2 + m_2^2) + \left(143 - \frac{1}{4}\pi^2\right)m_1 m_2\right)\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1 m_2} + \frac{1}{16}\left(20m_1^2 - \left(43 + \frac{3}{4}\pi^2\right)m_1 m_2\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2}{m_1^2}\\
& \left. + \frac{1}{16}\left(21(m_1^2 + m_2^2) + \left(119 + \frac{3}{4}\pi^2\right)m_1 m_2\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1 m_2}\right)\\
& + \frac{1}{8}\frac{G^4 m_1 m_2^3}{r_{12}^4}\left(\left(\frac{227}{3} - \frac{21}{4}\pi^2\right)m_1 + m_2\right) + (1 \leftrightarrow 2).
\end{aligned}
$$

2-body Taylor-expanded 4PN Hamiltonian [DJS, 2014]

$$
\begin{aligned}
c^8 H^{\mathrm{local}}_{4\mathrm{PN}}(\mathbf{x}_a,\mathbf{p}_a) &= \frac{7(\mathbf{p}_1^2)^5}{256 m_1^9} + \frac{G m_1 m_2}{r_{12}} H_{48}(\mathbf{x}_a,\mathbf{p}_a) + \frac{G^2 m_1 m_2}{r_{12}^2} m_1 H_{46}(\mathbf{x}_a,\mathbf{p}_a) \\
&+ \frac{G^3 m_1 m_2}{r_{12}^3}\left(m_1^2 H_{441}(\mathbf{x}_a,\mathbf{p}_a) + m_1 m_2 H_{442}(\mathbf{x}_a,\mathbf{p}_a)\right) \\
&+ \frac{G^4 m_1 m_2}{r_{12}^4}\left(m_1^3 H_{421}(\mathbf{x}_a,\mathbf{p}_a) + m_1^2 m_2 H_{422}(\mathbf{x}_a,\mathbf{p}_a)\right) \\
&+ \frac{G^5 m_1 m_2}{r_{12}^5} H_{40}(\mathbf{x}_a,\mathbf{p}_a) + (1 \leftrightarrow 2).
\end{aligned}
\tag{A3}
$$

$$
\begin{aligned}
H_{48}(\mathbf{x}_a,\mathbf{p}_a) =& \frac{45(\mathbf{p}_1^2)^4}{128 m_1^8} - \frac{9(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1^2)^2}{64 m_1^6 m_2^2} + \frac{15(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1^2)^3}{64 m_1^6 m_2^2} - \frac{9(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1^2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{16 m_1^6 m_2^2} \\
& - \frac{3(\mathbf{p}_1^2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{32 m_1^6 m_2^2} + \frac{15(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1^2)^2\mathbf{p}_2^2}{64 m_1^6 m_2^2} - \frac{21(\mathbf{p}_1^2)^3\mathbf{p}_2^2}{64 m_1^6 m_2^2} - \frac{35(\mathbf{n}_{12}\cdot\mathbf{p}_1)^5(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3}{256 m_1^5 m_2^3} \\
& + \frac{25(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3\mathbf{p}_1^2}{128 m_1^5 m_2^3} + \frac{33(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3(\mathbf{p}_1^2)^2}{256 m_1^5 m_2^3} - \frac{85(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{256 m_1^5 m_2^3} \\
& - \frac{45(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{128 m_1^5 m_2^3} - \frac{(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1^2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{256 m_1^5 m_2^3} + \frac{25(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{64 m_1^5 m_2^3} \\
& + \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{64 m_1^5 m_2^3} - \frac{3(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1\cdot\mathbf{p}_2)^3}{64 m_1^5 m_2^3} + \frac{3\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)^3}{64 m_1^5 m_2^3} + \frac{55(\mathbf{n}_{12}\cdot\mathbf{p}_1)^5(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_2^2}{256 m_1^5 m_2^3} \\
& - \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2\mathbf{p}_2^2}{128 m_1^5 m_2^3} - \frac{25(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1^2)^2\mathbf{p}_2^2}{256 m_1^5 m_2^3} - \frac{23(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{256 m_1^5 m_2^3} \\
& + \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{128 m_1^5 m_2^3} - \frac{7(\mathbf{p}_1^2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{256 m_1^5 m_2^3} - \frac{5(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^4\mathbf{p}_1^2}{64 m_1^4 m_2^4} + \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_2)^4(\mathbf{p}_1^2)^2}{64 m_1^4 m_2^4} \\
& - \frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{4 m_1^4 m_2^4} + \frac{(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{16 m_1^4 m_2^4} - \frac{5(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_2^2}{64 m_1^4 m_2^4} + \frac{21(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2\mathbf{p}_2^2}{64 m_1^4 m_2^4} \\
& - \frac{3(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1^2)^2\mathbf{p}_2^2}{32 m_1^4 m_2^4} - \frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{4 m_1^4 m_2^4} + \frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{16 m_1^4 m_2^4} + \frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2\mathbf{p}_2^2}{16 m_1^4 m_2^4} \\
& - \frac{\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2\mathbf{p}_2^2}{32 m_1^4 m_2^4} + \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4(\mathbf{p}_2^2)^2}{64 m_1^4 m_2^4} - \frac{3(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2\mathbf{p}_1^2(\mathbf{p}_2^2)^2}{32 m_1^4 m_2^4} - \frac{7(\mathbf{p}_1^2)^2(\mathbf{p}_2^2)^2}{128 m_1^4 m_2^4}.
\end{aligned}
\tag{A4a}
$$

$$
\begin{aligned}
H_{46}(\mathbf{x}_a,\mathbf{p}_a) =& \frac{369(\mathbf{n}_{12}\cdot\mathbf{p}_1)^6}{160 m_1^6} - \frac{889(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4\mathbf{p}_1^2}{192 m_1^6} + \frac{49(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1^2)^2}{16 m_1^6} - \frac{63(\mathbf{p}_1^2)^3}{64 m_1^6} - \frac{549(\mathbf{n}_{12}\cdot\mathbf{p}_1)^5(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{128 m_1^5 m_2} \\
& + \frac{67(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2}{16 m_1^5 m_2} - \frac{167(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1^2)^2}{128 m_1^5 m_2} + \frac{1547(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4(\mathbf{p}_1\cdot\mathbf{p}_2)}{256 m_1^5 m_2} - \frac{851(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{128 m_1^5 m_2} \\
& + \frac{1099(\mathbf{p}_1^2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{256 m_1^5 m_2} + \frac{3263(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{1280 m_1^4 m_2^2} + \frac{1067(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2}{480 m_1^4 m_2^2} - \frac{4567(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1^2)^2}{3840 m_1^4 m_2^2} \\
& - \frac{3571(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1\cdot\mathbf{p}_2)}{320 m_1^4 m_2^2} + \frac{3073(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{480 m_1^4 m_2^2} + \frac{4349(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{1280 m_1^4 m_2^2} \\
& - \frac{3461\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{3840 m_1^4 m_2^2} + \frac{1673(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4\mathbf{p}_2^2}{1920 m_1^4 m_2^2} - \frac{1999(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2\mathbf{p}_1^2\mathbf{p}_2^2}{3840 m_1^4 m_2^2} + \frac{2081(\mathbf{p}_1^2)^2\mathbf{p}_2^2}{3840 m_1^4 m_2^2} - \frac{13(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3}{8 m_1^3 m_2^3} \\
& + \frac{191(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3\mathbf{p}_1^2}{192 m_1^3 m_2^3} - \frac{19(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{384 m_1^3 m_2^3} - \frac{5(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{384 m_1^3 m_2^3} \\
& + \frac{11(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{192 m_1^3 m_2^3} + \frac{77(\mathbf{p}_1\cdot\mathbf{p}_2)^3}{96 m_1^3 m_2^3} + \frac{233(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_2^2}{96 m_1^3 m_2^3} - \frac{47(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2\mathbf{p}_2^2}{32 m_1^3 m_2^3} \\
& + \frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{384 m_1^3 m_2^3} - \frac{185\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{384 m_1^3 m_2^3} - \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^4}{4 m_1^2 m_2^4} + \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_2)^4\mathbf{p}_1^2}{4 m_1^2 m_2^4} \\
& - \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3(\mathbf{p}_1\cdot\mathbf{p}_2)}{2 m_1^2 m_2^4} + \frac{21(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{16 m_1^2 m_2^4} + \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_2^2}{6 m_1^2 m_2^4} + \frac{49(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2\mathbf{p}_2^2}{48 m_1^2 m_2^4} \\
& - \frac{133(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{24 m_1^2 m_2^4} - \frac{77(\mathbf{p}_1\cdot\mathbf{p}_2)^2\mathbf{p}_2^2}{96 m_1^2 m_2^4} + \frac{197(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_2^2)^2}{96 m_1^2 m_2^4} - \frac{173\mathbf{p}_1^2(\mathbf{p}_2^2)^2}{48 m_1^2 m_2^4} + \frac{13(\mathbf{p}_2^2)^3}{8 m_2^6}.
\end{aligned}
\tag{A4b}
$$

$$
\begin{aligned}
H_{441}(\mathbf{x}_a,\mathbf{p}_a) =& \frac{5027(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4}{384 m_1^4} - \frac{22993(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2\mathbf{p}_1^2}{960 m_1^4} - \frac{6695(\mathbf{p}_1^2)^2}{1152 m_1^4} - \frac{3191(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{640 m_1^3 m_2} \\
& + \frac{28561(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2}{1920 m_1^3 m_2} + \frac{8777(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{384 m_1^3 m_2} + \frac{752969\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{28800 m_1^3 m_2} \\
& - \frac{16481(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{960 m_1^2 m_2^2} + \frac{94433(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2}{4800 m_1^2 m_2^2} - \frac{103957(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1\cdot\mathbf{p}_2)}{2400 m_1^2 m_2^2} \\
& + \frac{791(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{400 m_1^2 m_2^2} + \frac{26627(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2\mathbf{p}_2^2}{1600 m_1^2 m_2^2} - \frac{118261\mathbf{p}_1^2\mathbf{p}_2^2}{4800 m_1^2 m_2^2} + \frac{105(\mathbf{p}_2^2)^2}{32 m_2^4},
\end{aligned}
\tag{A4c}
$$

$$
\begin{aligned}
H_{442}(\mathbf{x}_a,\mathbf{p}_a) =& \left(\frac{2749\pi^2}{8192} - \frac{211189}{19200}\right)\frac{(\mathbf{p}_1^2)^2}{m_1^4} + \left(\frac{63347}{1600} - \frac{1059\pi^2}{1024}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2\mathbf{p}_1^2}{m_1^4} + \left(\frac{375\pi^2}{8192} - \frac{23533}{1280}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4}{m_1^4} \\
& + \left(\frac{10631\pi^2}{8192} - \frac{1918349}{57600}\right)\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{m_1^2 m_2^2} + \left(\frac{13723\pi^2}{16384} - \frac{2492417}{57600}\right)\frac{\mathbf{p}_1^2\mathbf{p}_2^2}{m_1^2 m_2^2} \\
& + \left(\frac{1411429}{19200} - \frac{1059\pi^2}{512}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2}{m_1^2 m_2^2} + \left(\frac{248991}{6400} - \frac{6153\pi^2}{2048}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1^2 m_2^2} \\
& - \left(\frac{30383}{960} + \frac{36405\pi^2}{16384}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^2 m_2^2} + \left(\frac{1243717}{14400} - \frac{40483\pi^2}{16384}\right)\frac{\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1^3 m_2} \\
& + \left(\frac{2369}{60} + \frac{35655\pi^2}{16384}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^3 m_2} + \left(\frac{43101\pi^2}{16384} - \frac{391711}{6400}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2}{m_1^3 m_2} \\
& + \left(\frac{56955\pi^2}{16384} - \frac{1646983}{19200}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1^3 m_2}.
\end{aligned}
\tag{A4d}
$$

$$
H_{421}(\mathbf{x}_a,\mathbf{p}_a) = \frac{64861\mathbf{p}_1^2}{4800 m_1^2} - \frac{91(\mathbf{p}_1\cdot\mathbf{p}_2)}{8 m_1 m_2} + \frac{105\mathbf{p}_2^2}{32 m_2^2} - \frac{9841(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2}{1600 m_1^2} - \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{2 m_1 m_2}.
\tag{A4e}
$$

$$
\begin{aligned}
H_{422}(\mathbf{x}_a,\mathbf{p}_a) =& \left(\frac{1937033}{57600} - \frac{199177\pi^2}{49152}\right)\frac{\mathbf{p}_1^2}{m_1^2} + \left(\frac{176033\pi^2}{24576} - \frac{2864917}{57600}\right)\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1 m_2} + \left(\frac{282361}{19200} - \frac{21837\pi^2}{8192}\right)\frac{\mathbf{p}_2^2}{m_2^2} \\
& + \left(\frac{698723}{19200} + \frac{21745\pi^2}{16384}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2}{m_1^2} + \left(\frac{63641\pi^2}{24576} - \frac{2712013}{19200}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1 m_2} \\
& + \left(\frac{3200179}{57600} - \frac{28691\pi^2}{24576}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_2^2}.
\end{aligned}
\tag{A4f}
$$

$$
H_{40}(\mathbf{x}_a,\mathbf{p}_a) = -\frac{m_1^4}{16} + \left(\frac{6237\pi^2}{1024} - \frac{169799}{2400}\right)m_1^3 m_2 + \left(\frac{44825\pi^2}{6144} - \frac{609427}{7200}\right)m_1^2 m_2^2.
\tag{A4g}
$$

$$
\begin{aligned}
H^{\mathrm{nonloc}}_{4\mathrm{PN}}(t) &= -\frac{1}{5}\frac{G^2 M}{c^8} I^{(3)}_{ij}(t) \\
&\quad \times \mathrm{Pf}_{2r_{12}/c}\int_{-\infty}^{+\infty}\frac{\mathrm{d}v}{|v|} I^{(3)}_{ij}(t+v),
\end{aligned}
$$

Resummed (non-spinning) 4PN EOB interaction potentials

$$M = m_1 + m_2 \,, \qquad \mu = \frac{m_1 m_2}{m_1 + m_2} \,, \qquad \nu = \frac{m_1 m_2}{(m_1 + m_2)^2} = \frac{\mu}{M} \qquad u \equiv \frac{GM}{R\,c^2}$$

$$ds^2_{\rm eff} = -A(r;\nu)\,dt^2 + B(r;\nu)\,dr^2 + r^2\left(d\theta^2 + \sin^2\theta\,d\varphi^2\right) \qquad \bar{D} \equiv (A\,B)^{-1}$$

$$A(u) = 1 - 2u + 2\nu u^3 + \left(\frac{94}{3} - \frac{41\pi^2}{32}\right)\nu u^4 + \left(\left(\frac{2275\pi^2}{512} - \frac{4237}{60} + \frac{128}{5}\gamma_{\rm E} + \frac{256}{5}\ln 2\right)\nu + \left(\frac{41\pi^2}{32} - \frac{221}{6}\right)\nu^2 + \frac{64}{5}\nu \ln u\right)u^5,$$

$$A^{\rm EOB}(u) = {\rm Pade}^1_4[A^{PN}(u)]$$

$$\bar{D}(u) = 1 + 6\nu u^2 + (52\nu - 6\nu^2)u^3 + \left(\left(-\frac{533}{45} - \frac{23761\pi^2}{1536} + \frac{1184}{15}\gamma_{\rm E} - \frac{6496}{15}\ln 2 + \frac{2916}{5}\ln 3\right)\nu \right.$$
$$\left. + \left(\frac{123\pi^2}{16} - 260\right)\nu^2 + \frac{592}{15}\nu \ln u\right)u^4,$$

$$\hat{Q}(\mathbf{r}', \mathbf{p}') = \left(2(4 - 3\nu)\nu u^2 + \left(\left(-\frac{5308}{15} + \frac{496256}{45}\ln 2 - \frac{33048}{5}\ln 3\right)\nu - 83\nu^2 + 10\nu^3\right)u^3\right)(\mathbf{n}' \cdot \mathbf{p}')^4$$
$$+ \left(\left(-\frac{827}{3} - \frac{2358912}{25}\ln 2 + \frac{1399437}{50}\ln 3 + \frac{390625}{18}\ln 5\right)\nu - \frac{27}{5}\nu^2 + 6\nu^3\right)u^2(\mathbf{n}' \cdot \mathbf{p}')^6 + \mathcal{O}[\nu u(\mathbf{n}' \cdot \mathbf{p}')^8].$$

Spinning EOB effective Hamiltonian

$$H_{\rm eff} = H_{\rm orb} + H_{\rm so} \quad \rightarrow H_{\rm EOB} = Mc^2\sqrt{1 + 2\nu\left(\frac{H_{\rm eff}}{\mu c^2} - 1\right)}$$

$$\hat{H}^{\rm eff}_{\rm orb} = \sqrt{A\left(1 + B_p \boldsymbol{p}^2 + B_{np}(\boldsymbol{n}\cdot\boldsymbol{p})^2 - \frac{1}{1 + \frac{(\boldsymbol{n}\cdot\boldsymbol{\chi}_0)^2}{r^2}}\frac{(r^2 + 2r + (\boldsymbol{n}\cdot\boldsymbol{\chi}_0)^2)}{\mathcal{R}^4 + \Delta(\boldsymbol{n}\cdot\boldsymbol{\chi}_0)^2}((\boldsymbol{n}\times\boldsymbol{p})\cdot\boldsymbol{\chi}_0)^2 + Q_4\right)}.$$

$$H_{\rm so} = G_S \boldsymbol{L}\cdot\boldsymbol{S} + G_{S^*}\boldsymbol{L}\cdot\boldsymbol{S}^*,$$

$$\mathbf{S} = \mathbf{S_1} + \mathbf{S_2}\,;\ \mathbf{S}_* = \frac{m_2}{m_1}\mathbf{S_1} + \frac{m_1}{m_2}\mathbf{S_2}\,,$$

Gyrogravitomagnetic ratios (when neglecting spin^2 effects)

$$r^3 G_S^{\rm PN} = 2 - \frac{5}{8}\nu u - \frac{27}{8}\nu p_r^2 + \nu\left(-\frac{51}{4}u^2 - \frac{21}{2}u p_r^2 + \frac{5}{8}p_r^4\right) + \nu^2\left(-\frac{1}{8}u^2 + \frac{23}{8}u p_r^2 + \frac{35}{8}p_r^4\right)$$

$$r^3 G_{S_*}^{\rm PN} = \frac{3}{2} - \frac{9}{8}u - \frac{15}{8}p_r^2 + \nu\left(-\frac{3}{4}u - \frac{9}{4}p_r^2\right) - \frac{27}{16}u^2 + \frac{69}{16}u p_r^2 + \frac{35}{16}p_r^4 + \nu\left(-\frac{39}{4}u^2 - \frac{9}{4}u p_r^2 + \frac{5}{2}p_r^4\right) + \nu^2\left(-\frac{3}{16}u^2 + \frac{57}{16}u p_r^2 + \frac{45}{16}p_r^4\right)$$

Resummed EOB waveform

(Damour-Iyer-Sathyaprakash 1998) Damour-Nagar 2007, Damour-Iyer -Nagar 2008

$$h_{\ell m} \equiv h_{\ell m}^{(N,\epsilon)} \hat{h}_{\ell m}^{(\epsilon)} \hat{h}_{\ell m}^{\rm NQC}$$

$$\hat{h}_{\ell m}^{(\epsilon)} = \hat{S}_{\rm eff}^{(\epsilon)} T_{\ell m} e^{i\delta_{\ell m}} \rho_{\ell m}^{\ell}$$

$$T_{\ell m} = \frac{\Gamma(\ell+1-2i\hat{\hat{k}})}{\Gamma(\ell+1)} e^{\pi\hat{\hat{k}}} e^{2i\hat{\hat{k}}\ln(2kr_0)}$$

NB: T_lm resums an infinite number of terms and already contains, eg, 4.5PN tail^3 terms (Messina-Nagar17)

$$\begin{aligned}
\rho_{22}(x;\nu) = 1 &+ \left(\frac{55\nu}{84} - \frac{43}{42}\right)x + \left(\frac{19583\nu^2}{42336} - \frac{33025\nu}{21168} - \frac{20555}{10584}\right)x^2 \\
&+ \left(\frac{10620745\nu^3}{39118464} - \frac{6292061\nu^2}{3259872} + \frac{41\pi^2\nu}{192} - \frac{48993925\nu}{9779616} - \frac{428}{105}\mathrm{eulerlog}_2(x) + \frac{1556919113}{122245200}\right)x^3 \\
&+ \left(\frac{9202}{2205}\mathrm{eulerlog}_2(x) - \frac{387216563023}{160190110080}\right)x^4 + \left(\frac{439877}{55566}\mathrm{eulerlog}_2(x) - \frac{16094530514677}{533967033600}\right)x^5 + \mathcal{O}(x^6),
\end{aligned}$$

$$\boxed{\mathcal{F}_\varphi \equiv -\frac{1}{8\pi\Omega}\sum_{\ell=2}^{\ell_{\max}}\sum_{m=1}^{\ell}(m\Omega)^2|Rh_{\ell m}^{(\epsilon)}|^2}$$

STRUCTURE OF THE EOB FORMALISM

PN dynamics
(DD81,D82,DJS01,IF03,BDIF04)

PN rad losses
WW76, BDIWW95, BDEFI 05

PN waveform
BD89, B95&05,ABIQ04,

BH perturbations
RW57, Z70, Z72

Resummed (BD99)

Resummed (DIS98)

Resummed (DN07,DIN08)

QNMs spectrum
$\sigma_N = \alpha_N + i\omega_N$

EOB Hamiltonian
$H_{\rm EOB}$

EOB Rad. Reac. force
$\hat{\mathcal{F}}_\varphi$

Factorized waveform

Matching at merger time

$$
\begin{aligned}
h_{\ell m} &= h^{(N,\epsilon)}_{\ell m}\hat{h}^{(\epsilon)}_{\ell m}\\
\hat{h}^{(\epsilon)}_{\ell m} &= \hat{S}^{(\epsilon)}_{eff}T_{\ell m}e^{i\delta_{\ell m}}\rho^{\ell}_{\ell m}
\end{aligned}
$$

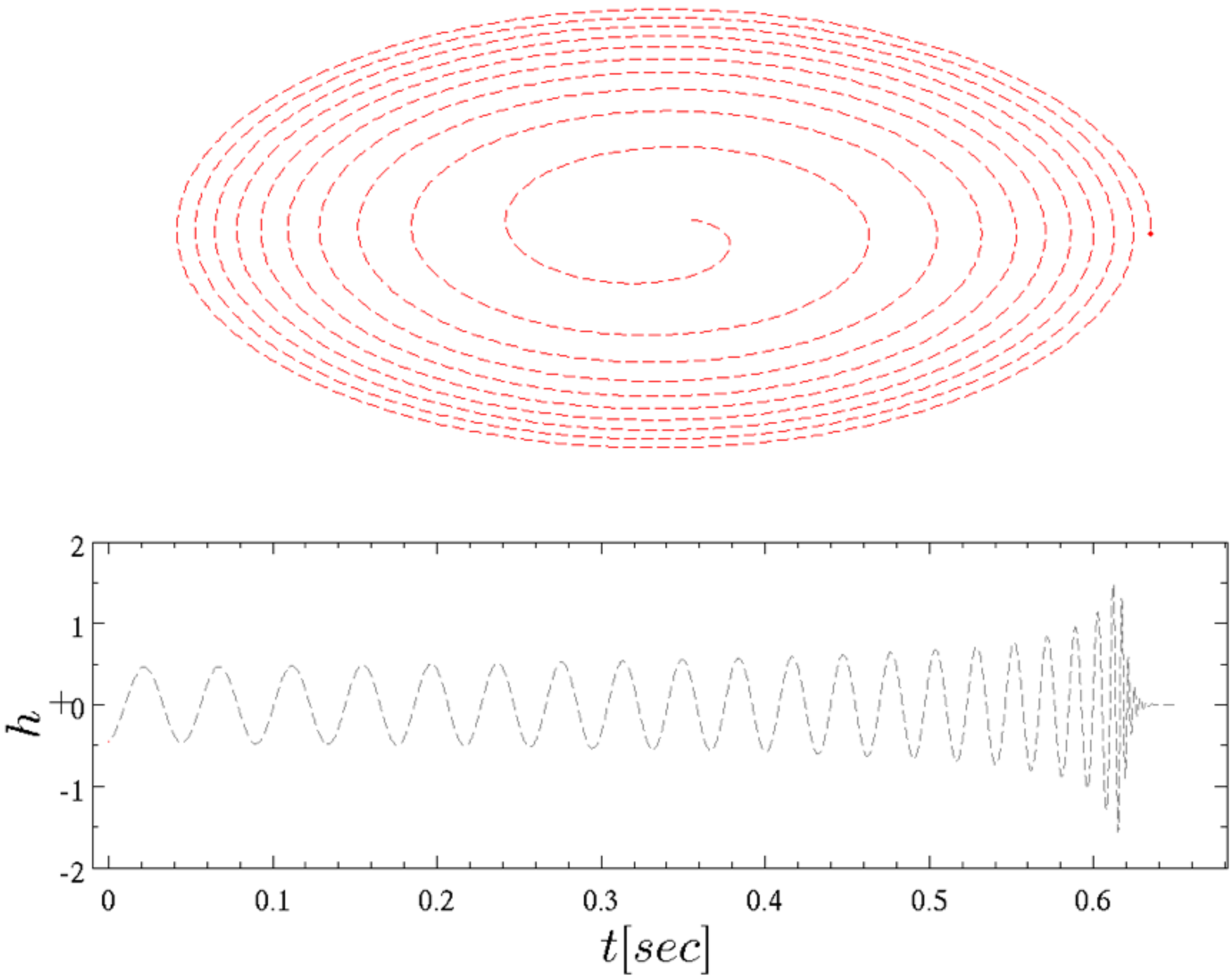
$$
\begin{aligned}
\frac{dr}{dt} &= \left(\frac{A}{B}\right)^{1/2}\frac{\partial \hat{H}_{\rm EOB}}{\partial p_{r_*}},\\
\frac{dp_{r_*}}{dt} &= -\left(\frac{A}{B}\right)^{1/2}\frac{\partial \hat{H}_{\rm EOB}}{\partial r},\\
\Omega \equiv \frac{d\varphi}{dt} &= \frac{\partial \hat{H}_{\rm EOB}}{\partial p_\varphi},\\
\frac{dp_\varphi}{dt} &= \hat{\mathcal{F}}_\varphi .
\end{aligned}
$$

BNS: tides
(Love numbers)

$$h^{\rm ringdown}_{\ell m}(t) = \sum_N C^+_N e^{-\sigma^+_N(t-t_m)}$$

EOB waveform

$$h^{\rm EOB}_{\ell m} = \theta(t_m - t)h^{\rm insplunge}_{\ell m}(t) + \theta(t-t_m)h^{\rm ringdown}_{\ell m}(t)$$


2
1
0
-1
-2
h_+
0
0.1
0.2
0.3
0.4
0.5
0.6
$t[sec]$

NR-completed resummed 5PN EOB radial A potential

« We think, however, that a suitable "numerically fitted" and, if possible, "analytically extended" EOB Hamiltonian should be able to fit the needs of upcoming GW detectors. » (TD 2001)

here Damour-Nagar-Bernuzzi '13, Nagar-etal '16; alternative: Taracchini et al '14, Bohe et al '17

4PN analytically complete + 5 PN logarithmic term in the A(u, nu) function,
With u = GM/R and nu = m1 m2 / (m1 + m2)^2
[Damour 09, Blanchet et al 10, Barack-Damour-Sago 10, Le Tiec et al 11, Barausse et al 11, Akcay et al 12, Bini-Damour 13, Damour-Jaranowski-Schäfer 14, Nagar-Damour-Reisswig-Pollney 15]

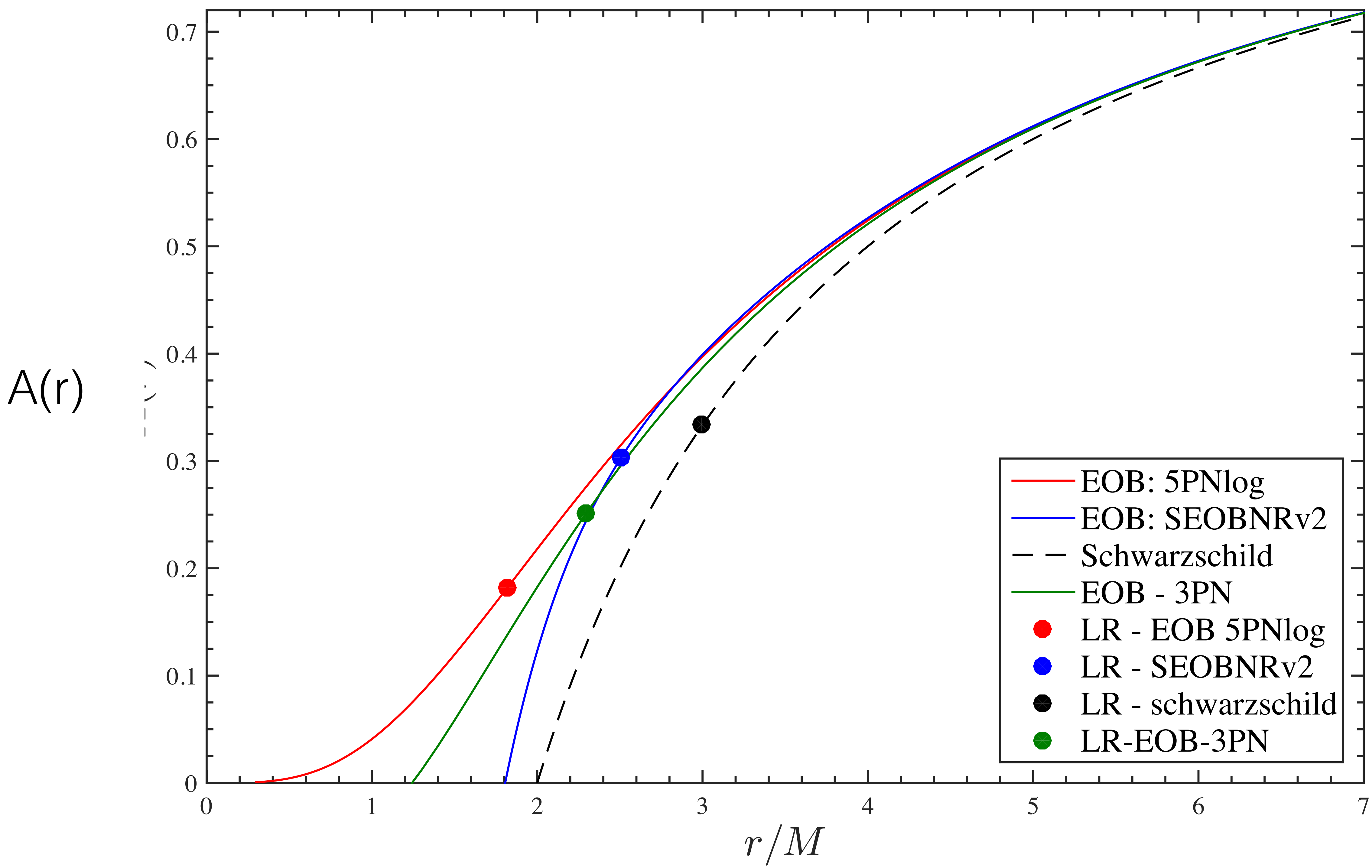
$$u = \frac{GM}{c^2\, R} \qquad \nu = \frac{m_1 m_2}{(m_1+m_2)^2}$$

$$\begin{aligned} A(u;\nu,a_6^c) &= \boxed{P_5^1}\Big[1-2u+2\nu\,u^3+\nu\left(\frac{94}{3}-\frac{41}{32}\pi^2\right)u^4 \\ &+ \nu\left[-\frac{4237}{60}+\frac{2275}{512}\pi^2+\left(-\frac{221}{6}+\frac{41}{32}\pi^2\right)\nu+\frac{64}{5}\ln(16e^{2\gamma}u)\right]u^5 \\ &+ \nu\left[\boxed{a_6^c(\nu)}-\left(\frac{7004}{105}+\frac{144}{5}\nu\right)\ln u\right]u^6\Big] \end{aligned}$$

$$\boxed{a_6^{c\,\text{NR-tuned}}(\nu)} = 81.38-1330.6\,\nu+3097.3\,\nu^2$$

MAIN RADIAL RADIAL EOB POTENTIAL A(R)

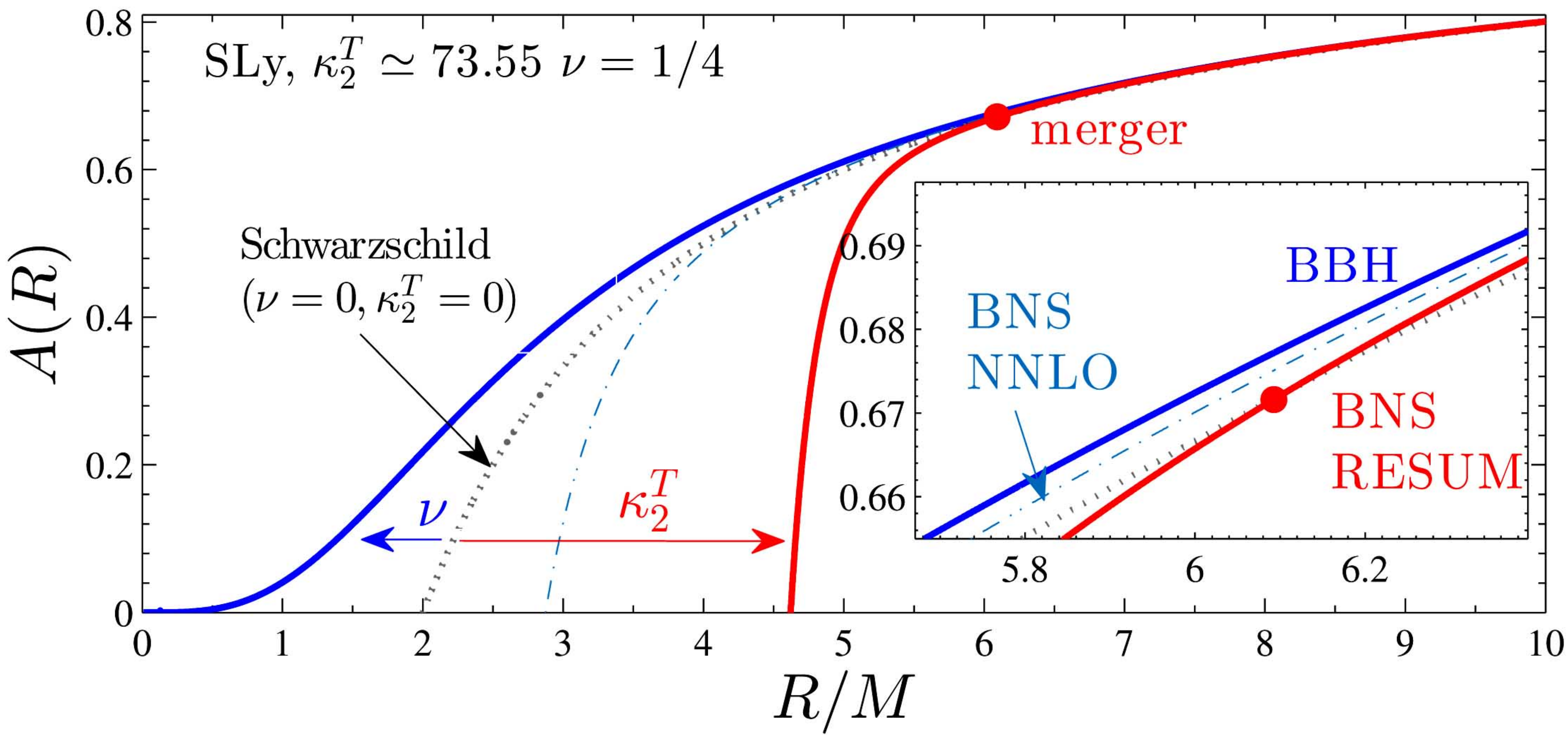
m1=m2 case

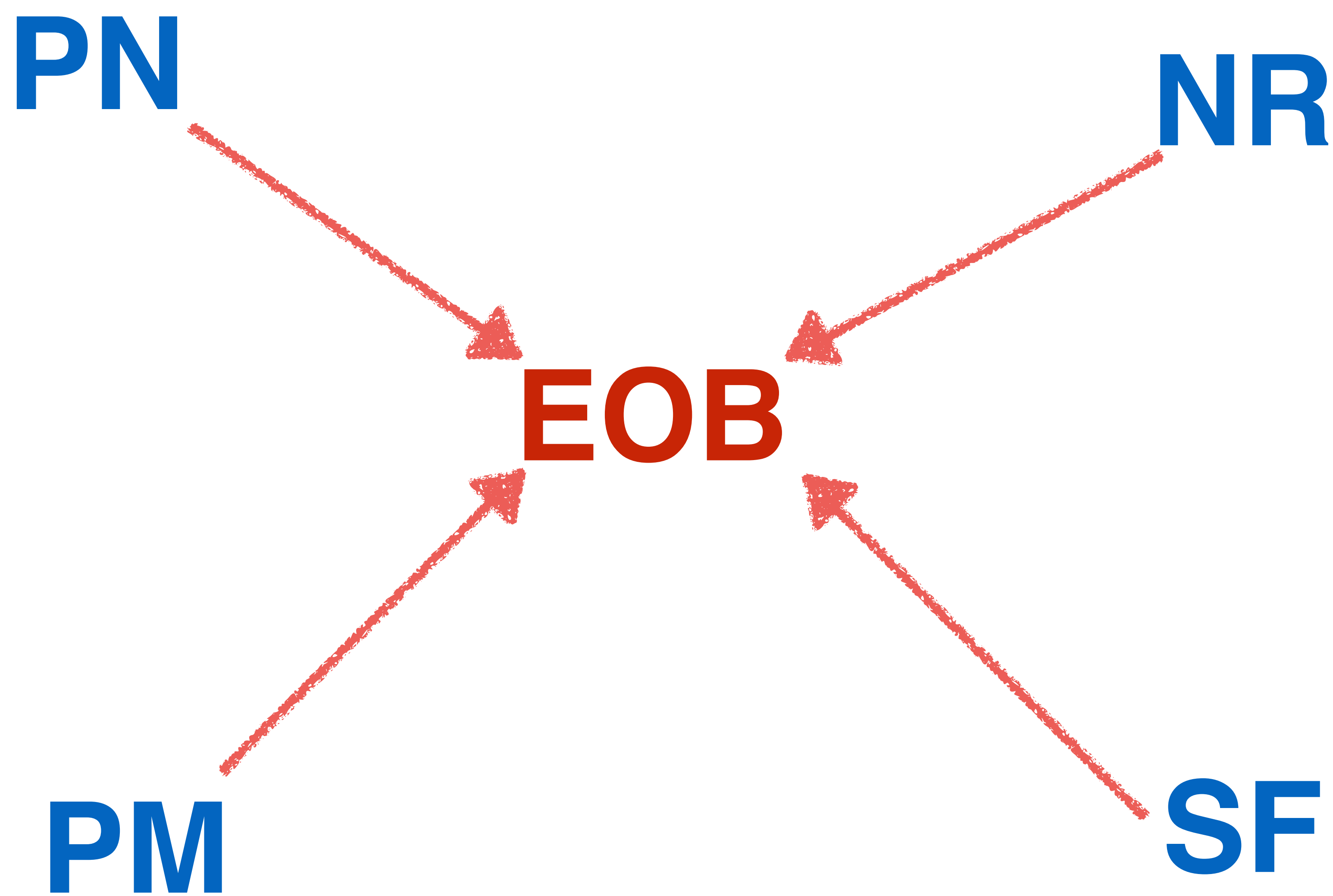


Tidal extension of EOB (TEOB) [Damour-Nagar 09]

$$\begin{aligned} A(r) &= A_r^0 + A^{\rm tidal}(r) \\ A^{\rm tidal}(r) &= -\kappa_2^T u^6 \left(1 + \bar\alpha_1 u + \bar\alpha_2 u^2 + \dots\right) + \dots \end{aligned}$$

TEOB[NR] A(R) potential (Bernuzzi et al. 2015)





See talks by: C. Kavanagh, L. Kidder, W. Han, F-L Julié, …

4 PN periastron precession and scattering and EOB (DJS15, Bini-Damour17)

Technically convenient to use EOB:

Hamilton-Jacobi, use of EOB E_eff,inclusion of spin terms, time-localization of tail action

First analytical computation of 4PN periastron precession (DJS15)

$$\begin{aligned}\chi(E,L,S_1,S_2) &= \chi_{\rm orb}(E,L)\\ &\quad+\chi_{S_1}(E,L)S_1+\chi_{S_2}(E,L)S_2\\ &\quad+O(\mathrm{spin}^2)\,,\end{aligned}$$

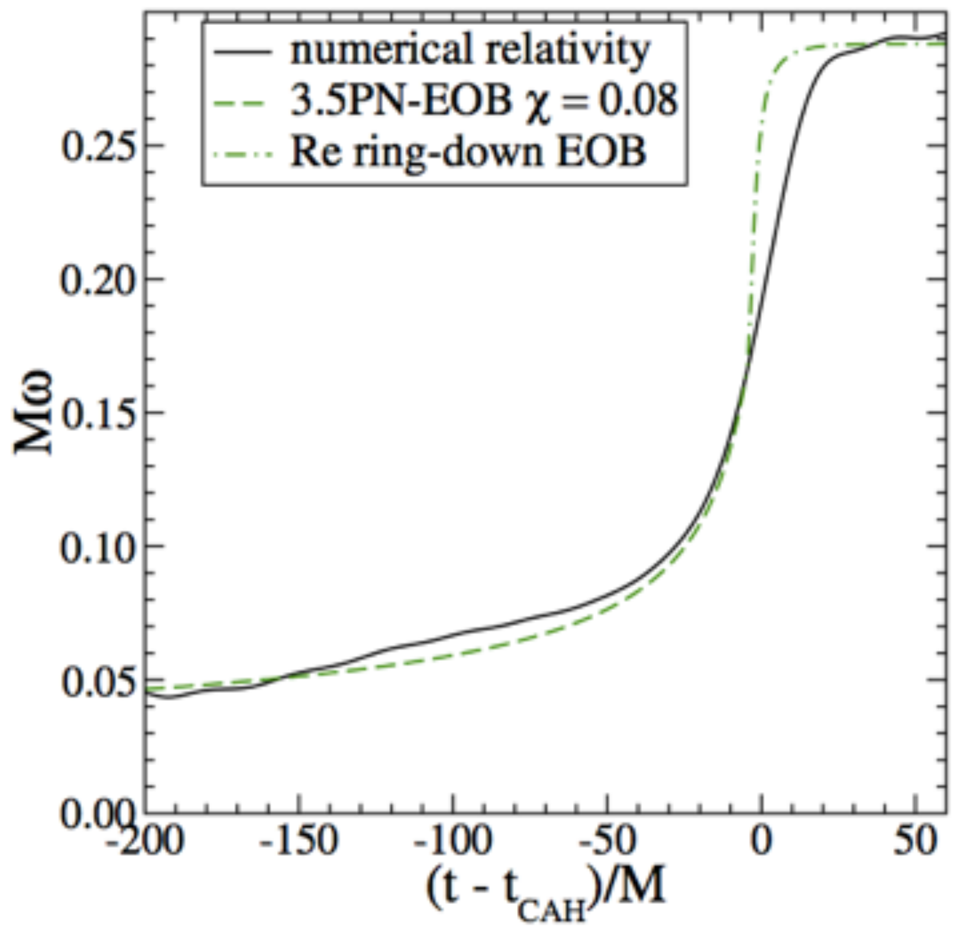
$$\chi(E,L)=\chi_{\rm loc}(E,L)+\chi_{\rm tail}(E,L)\,,$$

$$\begin{aligned}\chi_{\rm loc}(E,L) &= \chi^{(\rm N)}(\bar{E},L)+\frac{1}{c^2}\chi^{(1\rm PN)}(\bar{E},L)\\ &+\frac{1}{c^4}\chi^{(2\rm PN)}(\bar{E},L)+\frac{1}{c^6}\chi^{(3\rm PN)}(\bar{E},L)\\ &+\frac{1}{c^8}\chi^{(4\rm PN)}_{\rm loc}(\bar{E},L)+O\left(\frac{1}{c^{10}}\right)\,.\end{aligned}$$

$$\chi_{\rm tail}(E,L)=\frac{1}{c^8}\chi^{(4\rm PN)}_{\rm tail}(\bar{E},L)\,,$$

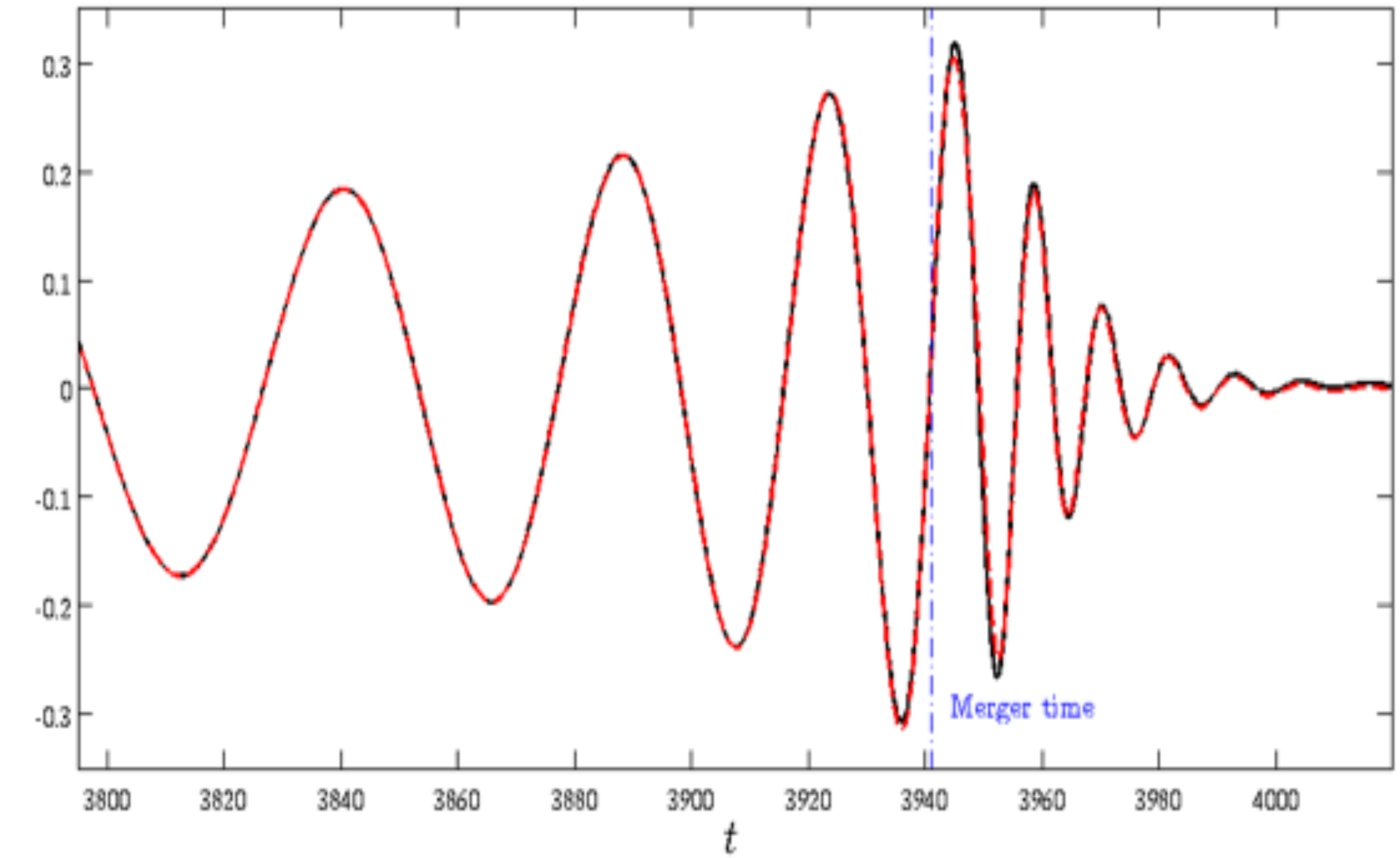
EOB-NR waveform comparison

Buonanno-Cook-Pretorius07

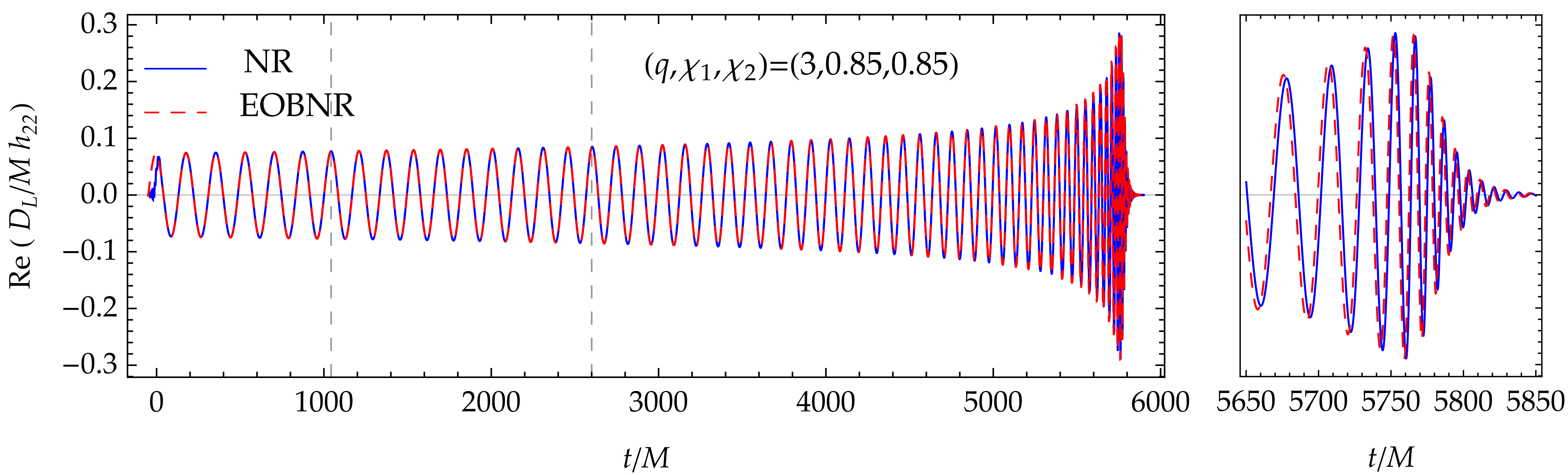


q=1,no spins

Damour-Nagar09

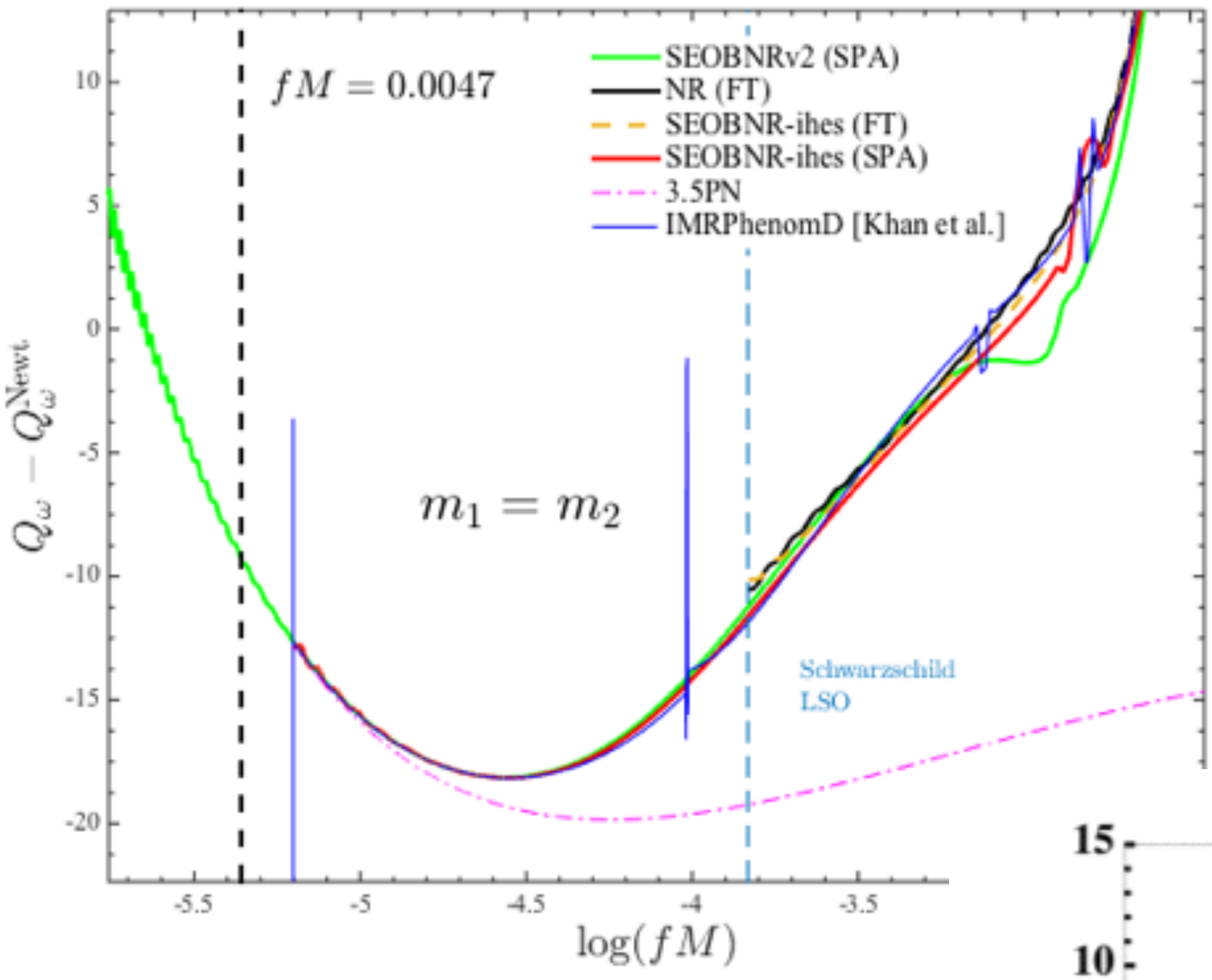


q=3, chi1=chi2=0.85, Bohe etal17



$Q_\omega - Q_\omega^N$

EOB VS PN

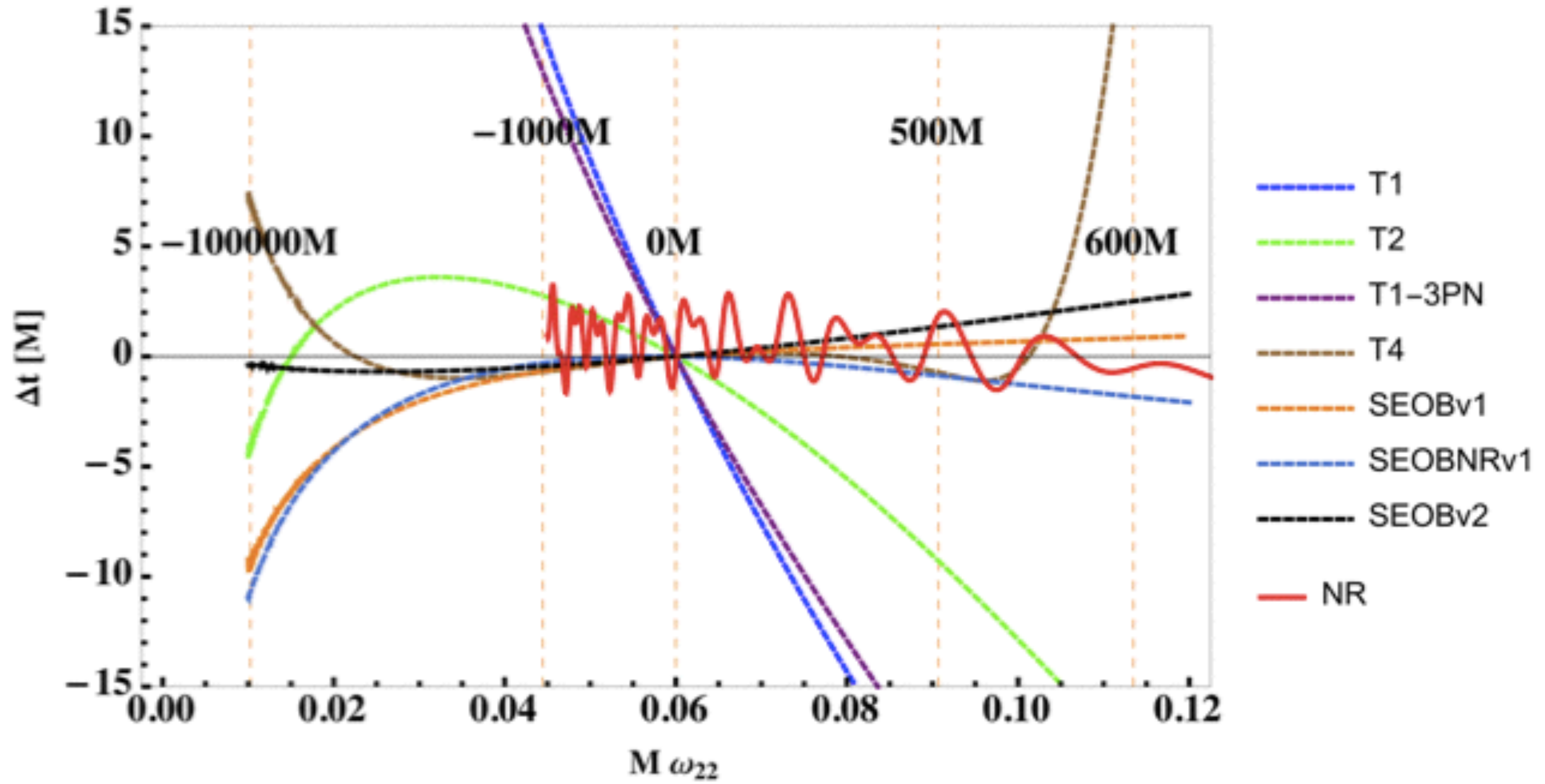


« quality factor » of GW phase

$$Q_\omega = f^2 \frac{d^2\psi(f)}{df^2} \approx \frac{\omega^2}{\dot{\omega}}$$

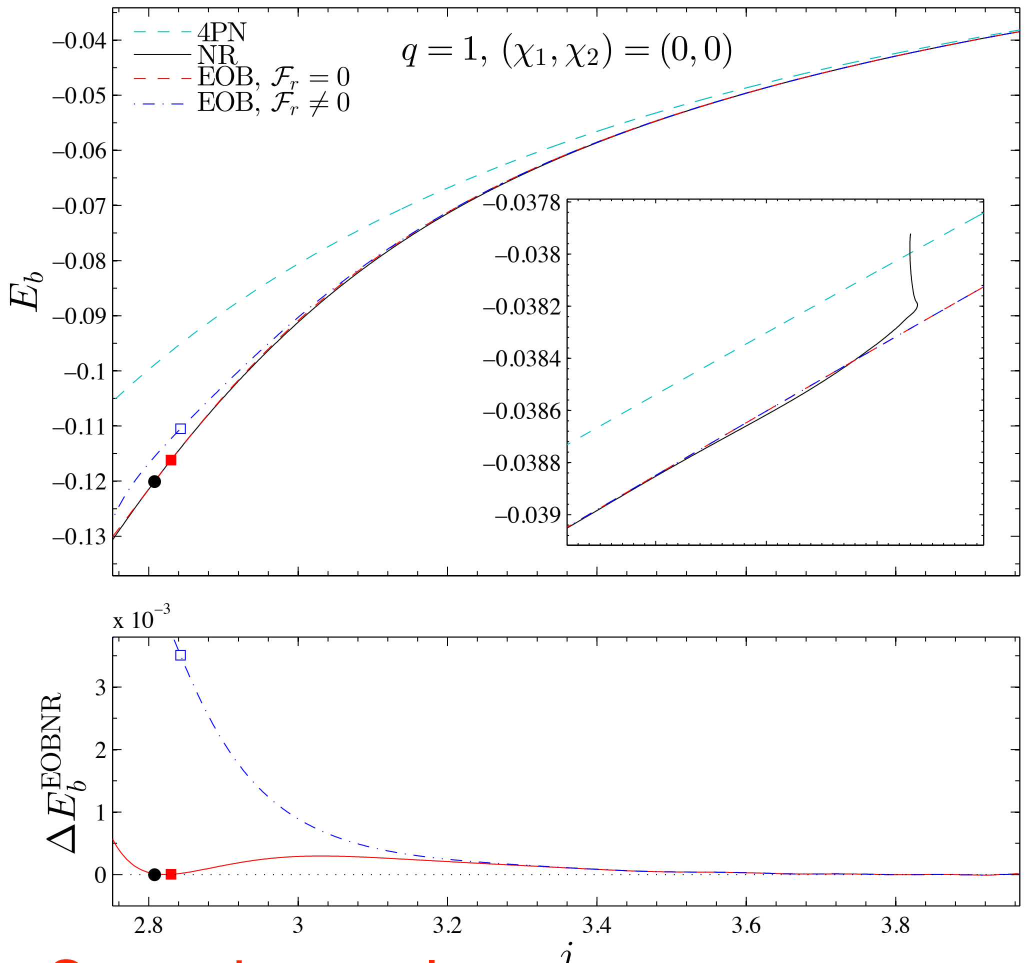
Husa et al 16

PN accuracy loss
during inspiral



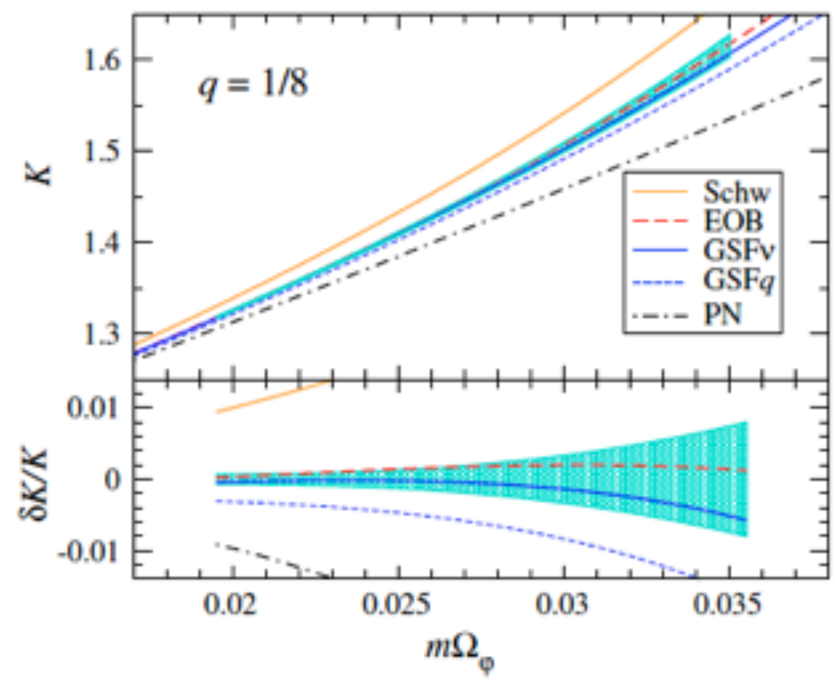
OTHER EOB-NR COMPARISONS

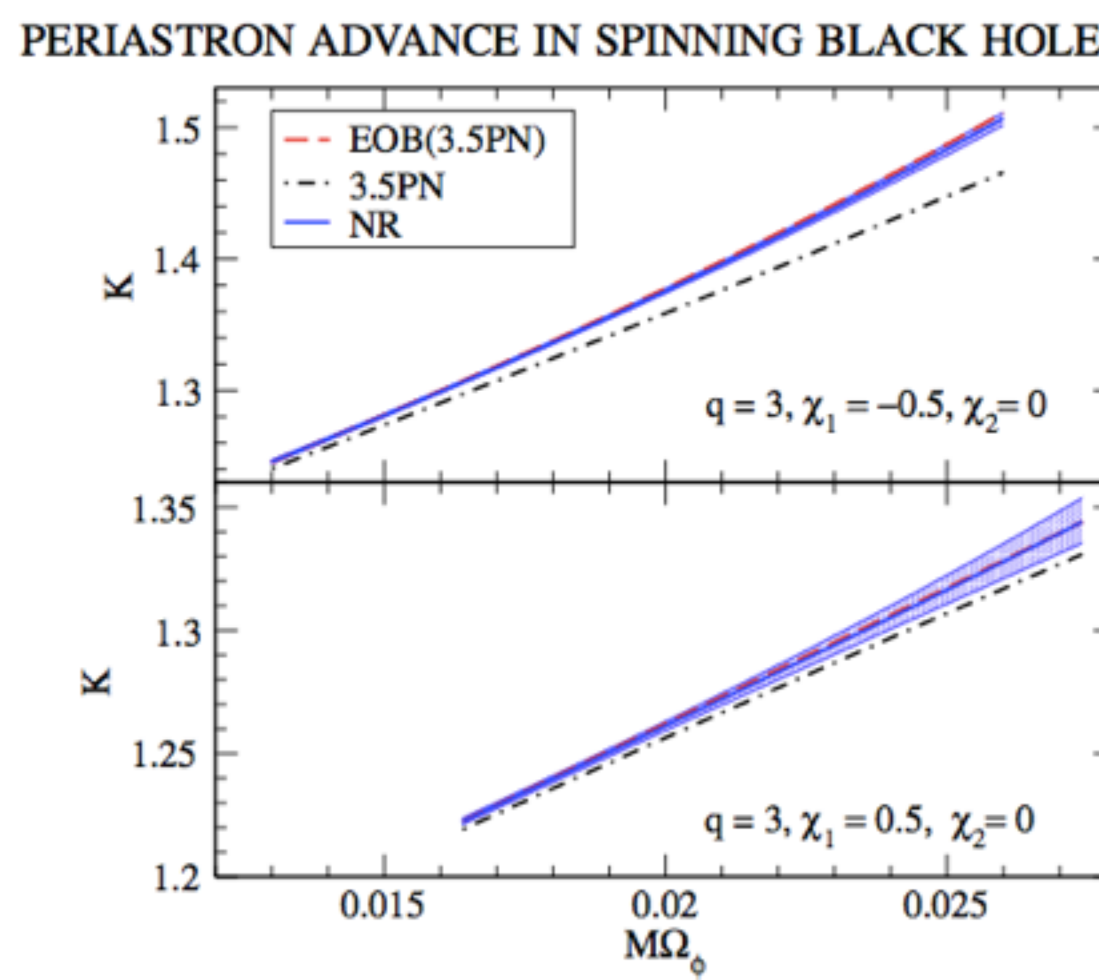
Energetics (Nagar-Damour-Reisswig-Pollney 16



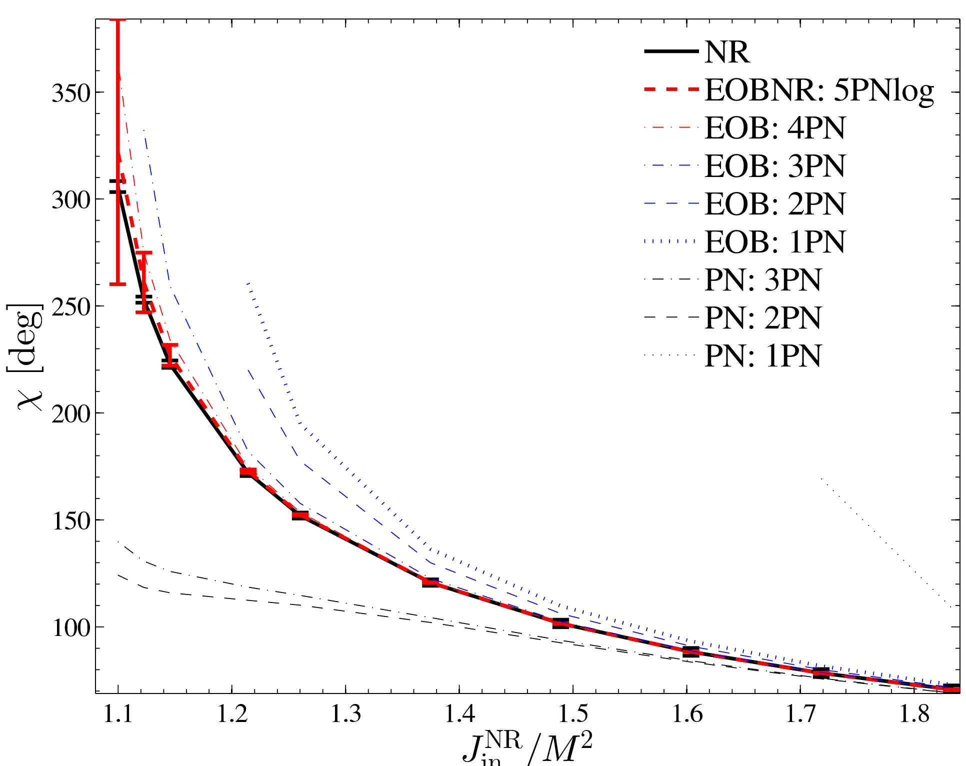
Periastron precession

(LeTiec-Mroue-Barack-Buonanno-Pfeiffer-Sago-Tarachini 11, Hinderer et al 13)



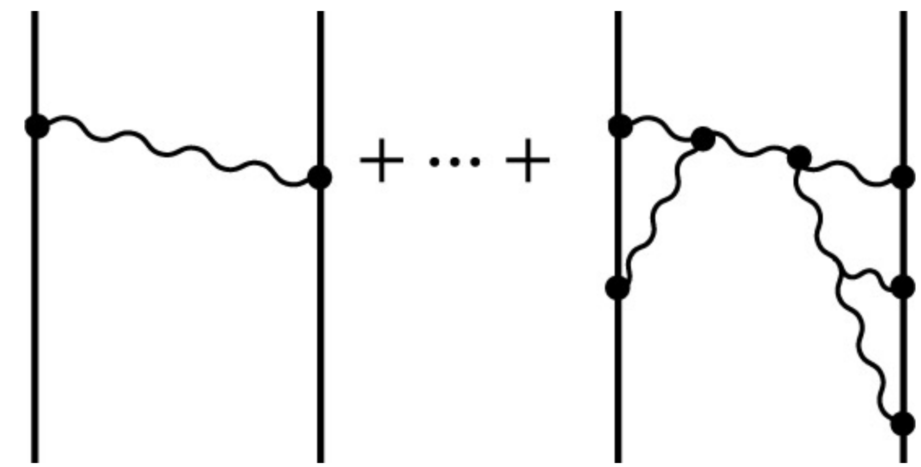


Scattering angle (Damour-Guercilena-Hinder-Hopper-Nagar-Rezzolla 14)

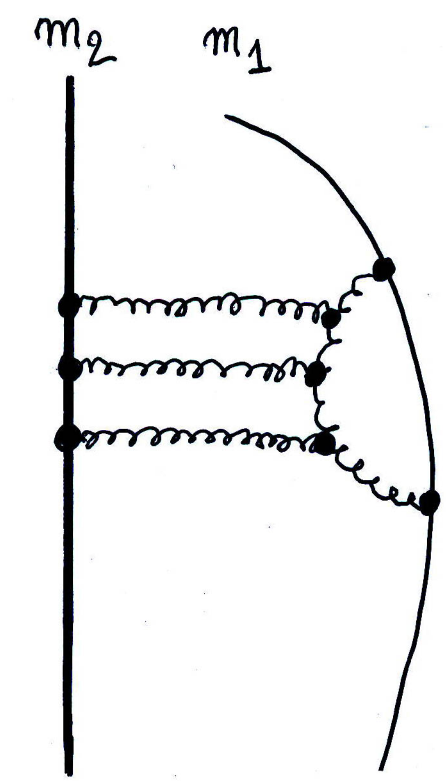


EOB AND GSF

Comparable-mass case: $m_1 \sim m_2$



Gravitational Self-Force Theory : $m_1 << m_2$



• Analytical high-PN results : Blanchet-Detweiler-LeTiec-Whiting '10, Damour '10, Blanchet et al '10, LeTiec et al '12, Bini-Damour '13-15, Kavanagh-Ottewill-Wardell '15 Bini-Damour-Geralico'16, Hopper-Kavanagh-Ottewill'16

• (gauge-invariant) Numerical results : Detweiler '08, Barack-Sago '09, Blanchet-Detweiler-LeTiec-Whiting '10, Barack-Damour-Sago '10, Shah-Friedman-Keidl '12, Dolan et al '14, Nolan et al '15, Akcay-van de Meent '16

• Analytical PN results from high-precision (hundreds to thousands of digits !) numerical results : Shah-Friedman-Whiting '14, Johnson-McDaniel-Shah-Whiting '15

EOB, SF, EOB[SF], LISA ETC

Remarkable EOB fact about expansions in nu=m1m2/(m1+m2)^2: while

$$E_{\leq 4\mathrm{PN}}(x;\nu) = -\frac{\mu c^2 x}{2}\left(1 - \left(\frac{3}{4} + \frac{\nu}{12}\right)x + \left(-\frac{27}{8} + \frac{19\nu}{8} - \frac{\nu^2}{24}\right)x^2 + \left(-\frac{675}{64} + \left(\frac{34445}{576} - \frac{205\pi^2}{96}\right)\nu - \frac{155\nu^2}{96} - \frac{35\nu^3}{5184}\right)x^3 \right.$$
$$+ \left(-\frac{3969}{128} + \left(\frac{9037\pi^2}{1536} - \frac{123671}{5760} + \frac{448}{15}(2\gamma_E + \ln(16x))\right)\nu \right.$$
$$\left.\left. + \left(\frac{3157\pi^2}{576} - \frac{498449}{3456}\right)\nu^2 + \frac{301\nu^3}{1728} + \frac{77\nu^4}{31104}\right)x^4\right). \quad (5.5$$

$$A(u) = 1 - 2u + 2\nu u^3 + \left(\frac{94}{3} - \frac{41\pi^2}{32}\right)\nu u^4 + \left(\left(\frac{2275\pi^2}{512} - \frac{4237}{60} + \frac{128}{5}\gamma_E + \frac{256}{5}\ln 2\right)\nu + \left(\frac{41\pi^2}{32} - \frac{221}{6}\right)\nu^2 + \frac{64}{5}\nu \ln u\right)u^5,$$

Computation of 4PN O(nu) term in A from numerical (Barausse-Buonanno-LeTiec’12) and analytical (Bini-Damour’13) SF computation;
Confirmation of all 4PN O(nu) terms of Damour-Jaranowski-Schaefer’14’15 from SF computations (Barack-Damour-Sago, Bini-Damour-Geralico, van de Meent)

EOB[SF] program: improve the few EOB gauge-invariant potentials by SF-computing (analytically or numerically) the contributions linear in nu. Recently implemented for A, B, Q, gS, gS* (Bini,Damour,Geralico,Kavanagh,Akcay, van de Meent, Hopper,Wardell,Ottewill,…

Aim: define template banks for LISA

GSF : ANALYTICAL HIGH-PN RESULTS

Bini-Damour 15

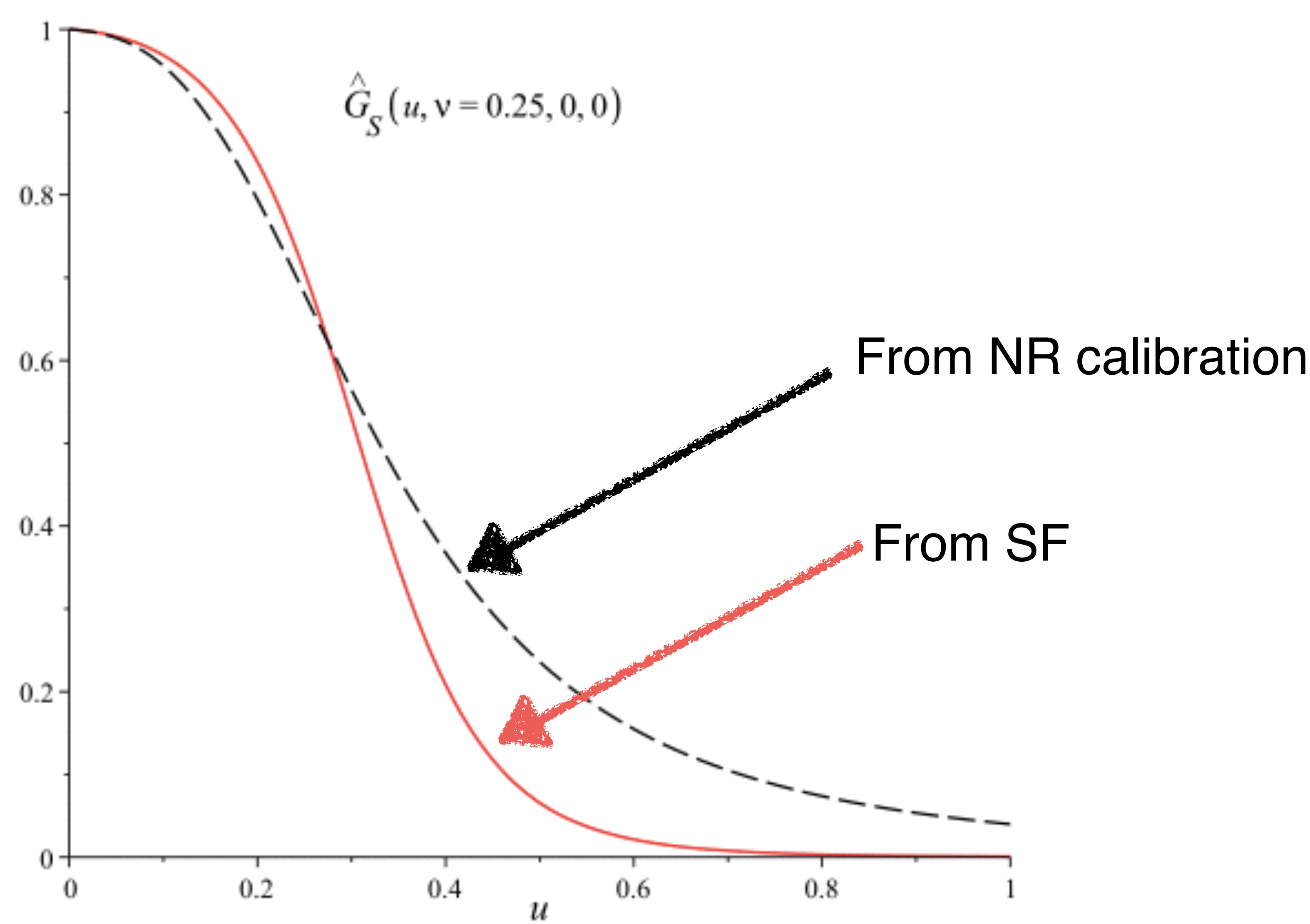
Kavanagh et al 15

$$
\begin{aligned}
a_{10}^{c} &= \frac{18605478842060273}{7079830758000}\ln(2) - \frac{1619008}{405}\zeta(3) - \frac{21339873214728097}{1011404394000}\gamma \\
&\quad + \frac{27101981341}{100663296}\pi^6 - \frac{6236861670873}{125565440}\ln(3) + \frac{360126}{49}\ln(2)\ln(3) + \frac{180063}{49}\ln(3)^2 \\
&\quad - \frac{121494974752}{9823275}\ln(2)^2 - \frac{24229836023352153}{549755813888}\pi^4 + \frac{1115369140625}{124540416}\ln(5) + \frac{96889}{2779} \\
&\quad + \frac{75437014370623318623299}{18690753201120000} - \frac{60648244288}{9823275}\ln(2)\gamma + \frac{200706848}{280665}\gamma^2 \\
&\quad + \frac{11980569677139}{2306867200}\pi^2 + \frac{360126}{49}\gamma\ln(3), \\
a_{10}^{\ln} &= -\frac{21275143333512097}{2022808788000} + \frac{200706848}{280665}\gamma - \frac{30324122144}{9823275}\ln(2) + \frac{180063}{49}\ln(3), \\
a_{10}^{\ln^2} &= \frac{50176712}{280665}, \\
a_{10.5}^{c} &= -\frac{185665618769828101}{24473489040000}\pi + \frac{377443508}{77175}\ln(2)\pi + \frac{2414166668}{1157625}\pi\gamma - \frac{5846788}{11025}\pi^3 - \frac{24}{} \\
a_{10.5}^{\ln} &= \frac{1207083334}{1157625}\pi.
\end{aligned}
$$

$$
\begin{aligned}
c_{15} = &- \frac{2069543450583769619340376724}{325477442086506084375}\zeta(3) + \frac{65195026298245007936}{22370298575625}\gamma\zeta(3) - \frac{5049442304}{25725}\gamma^2\zeta(3) + \frac{1262360576}{15435}\pi^2\zeta(3) \\
&+ \frac{171722752}{441}\zeta(3)^2 + \frac{1613866959570176}{496621125}\zeta(5) - \frac{343445504}{441}\gamma\zeta(5) - \frac{146997248}{105}\zeta(7) + \frac{56314978304}{385875}\zeta(3)\log^2(2) \\
&- \frac{106445664}{343}\zeta(3)\log^2(3) + \frac{151670998244849797696}{22370298575625}\zeta(3)\log(2) - \frac{190336581632}{1157625}\gamma\zeta(3)\log(2) \\
&+ \frac{28863591064624341}{4909804900}\zeta(3)\log(3) - \frac{212891328}{343}\gamma\zeta(3)\log(3) - \frac{212891328}{343}\zeta(3)\log(2)\log(3) - \frac{77186767578125}{19876428}\zeta(3)\log \\
&- \frac{2039263232}{3675}\zeta(5)\log(2) - \frac{49128768}{49}\zeta(5)\log(3) + \frac{298267427515018397019736592175289419501391539444290849}{6587612222544653226142468405031917319531250} \\
&- \frac{6807661768453637768313286948060329087501419}{704310948124803722562607729544062500}\gamma + \frac{1598346944412603247831006289829388}{526171715038677033591890625}\gamma^2 - \frac{1007647146215971027644}{335890033113009375} \\
&+ \frac{461219496448}{72930375}\gamma^4 - \frac{28338275082077591587855063450276303790065762907243197}{999703155845143418115744045792755712000000}\pi^2 + \frac{25191178655399275691104}{67178006622601875}\gamma\pi^2 \\
&- \frac{230609748224}{14586075}\gamma^2\pi^2 + \frac{105480323357757226894713787760391180776248036241}{304245354831316028025099055320268800000}\pi^4 + \frac{1262360576}{385875}\gamma\pi^4 \\
&- \frac{6208472839612966972691457131143}{266930151354100246118400}\pi^6 + \frac{3573178781920929118281329}{151996487423754240}\pi^8 - \frac{10136323685888}{72930375}\log^4(2) + \frac{38438712}{2401}\log^4(3) \\
&- \frac{177896086126482679647872}{54963823600310625}\log^3(2) - \frac{89686013106176}{364651875}\gamma\log^3(2) + \frac{153754848}{2401}\log^3(2)\log(3) \\
&- \frac{131463845322790269123}{245735735245000}\log^3(3) + \frac{153754848}{2401}\gamma\log^3(3) + \frac{153754848}{2401}\log(2)\log^3(3) + \frac{11933074267578125}{51161925672}\log^3(5) \\
&+ \frac{3878258674166628974595420635200204}{189421817413923732093080625}\log^2(2) - \frac{3440856379914601692151168}{1007670099339028125}\gamma\log^2(2) - \frac{16582891400192}{121550625}\gamma^2\log^2(2) \\
&+ \frac{4145722850048}{72930375}\pi^2\log^2(2) - \frac{523697163373483905609}{245735735245000}\log^2(2)\log(3) + \frac{461264544}{2401}\gamma\log^2(2)\log(3) \\
&+ \frac{45454535766189065888302299261759}{6569728226789883034880000}\log^2(3) - \frac{394391535968370807369}{245735735245000}\gamma\log^2(3) + \frac{230632272}{2401}\gamma^2\log^2(3) \\
&- \frac{96096780}{2401}\pi^2\log^2(3) - \frac{437493411770075173449}{245735735245000}\log(2)\log^2(3) + \frac{461264544}{2401}\gamma\log(2)\log^2(3) \\
&+ \frac{230632272}{2401}\log^2(2)\log^2(3) + \frac{11933074267578125}{17053975224}\log^2(2)\log(5) - \frac{2505842696993145943705498046875}{402136320895332222431232}\log^2(5) \\
&+ \frac{11933074267578125}{17053975224}\gamma\log^2(5) + \frac{11933074267578125}{17053975224}\log(2)\log^2(5) + \frac{47929508316470415142010251}{56464635170211840000}\log^2(7) \\
&- \frac{181636067216895220421537747685253699734494659}{6338798533123233503063469565896562500}\log(2) + \frac{74203662155219108543799531653010136}{4735545435348093302327015625}\gamma\log(2) \\
&- \frac{1482169326522492515499392}{1007670099339028125}\gamma^2\log(2) - \frac{4905667647488}{364651875}\gamma^3\log(2) + \frac{371228115490667668451168}{604602059603416875}\pi^2\log(2) \\
&+ \frac{1226416911872}{72930375}\gamma\pi^2\log(2) + \frac{23792072704}{17364375}\pi^4\log(2) - \frac{4141158375397180302387095124935855747727}{1082666315962744888801986560000000}\log(3) \\
&+ \frac{9459358001131575454332055276239}{691550339662092951040000}\gamma\log(3) - \frac{394391535968370807369}{245735735245000}\gamma^2\log(3) + \frac{153754848}{2401}\gamma^3\log(3) \\
&+ \frac{131463845322790269123}{196588588196000}\pi^2\log(3) - \frac{192193560}{2401}\gamma\pi^2\log(3) + \frac{8870472}{1715}\pi^4\log(3) \\
&+ \frac{214411501060211389845962927148381}{13139456453579766069760000}\log(2)\log(3) - \frac{437493411770075173449}{122867867622500}\gamma\log(2)\log(3) \\
&+ \frac{461264544}{2401}\gamma^2\log(2)\log(3) - \frac{192193560}{2401}\pi^2\log(2)\log(3) + \frac{978612948501709853277095576118865234375}{17942749191956127021132384903168}\log(5) \\
&- \frac{2505842696993145943705498046875}{201068160447666111215616}\gamma\log(5) + \frac{11933074267578125}{17053975224}\gamma^2\log(5) - \frac{59665371337890625}{204647702688}\pi^2\log(5) \\
&- \frac{2505842696993145943705498046875}{201068160447666111215616}\log(2)\log(5) + \frac{11933074267578125}{8526987612}\gamma\log(2)\log(5) \\
&- \frac{5858006173792308915665113013914648081}{3239191932075128029777920000000}\log(7) + \frac{47929508316470415142010251}{28232317585105920000}\gamma\log(7) \\
&+ \frac{47929508316470415142010251}{28232317585105920000}\log(2)\log(7) + \frac{7400249944258160101211}{65676344832000000}\log(11),
\end{aligned}
$$

FIRST EOB GYROGRAVITOMAGNETIC RATIO FROM SF

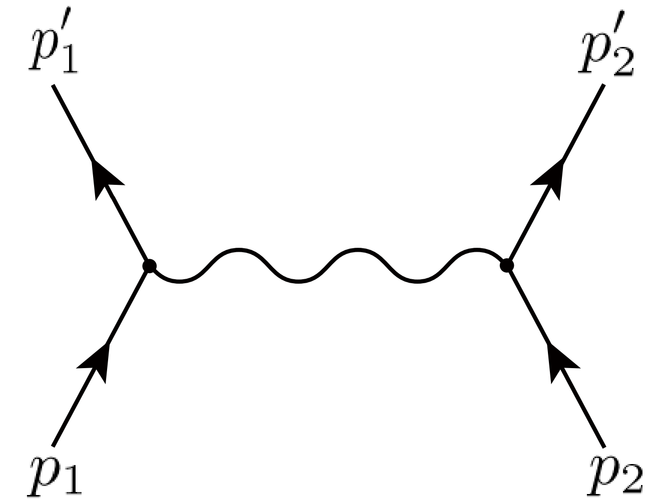
Bini-Damour-Geralico'15



EOB, PM, EFT, QFT and all that

Original EOB dictionary based on bound states.
New (equivalent) dictionary for scattering states:
applicable to the PM approximation (no restriction on v/c).
[Damour2016]

$$\chi_{\mathrm{eff}}(\mathcal{E}_{\mathrm{eff}}, J) = \chi_{\mathrm{real}}(\mathcal{E}_{\mathrm{real}}, J)$$



Direct link between Feynman-like scattering diagrams and EOB Hamiltonian.

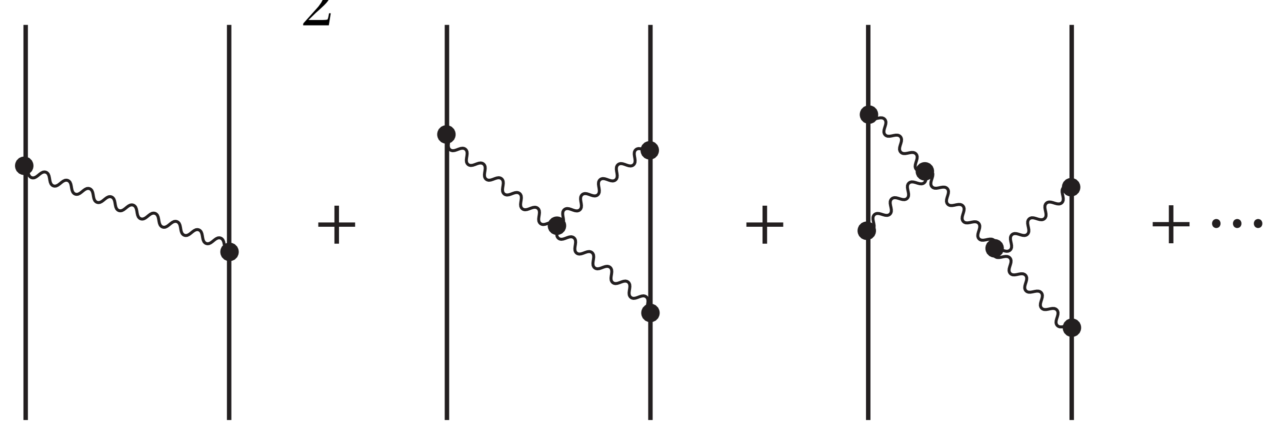
Different from the Feynman-like diagrams giving the Fokker-type action in gravity [Damour-Esposito-Farese ’96]

$$g = \eta + h$$

$$S(h,T) = \int \left(\frac{1}{2} h \Box h + \partial\partial hhh + ... + (h + hh + ...)T \right)$$

$$\Box h = -T + ... \rightarrow h = G\,T + ...$$

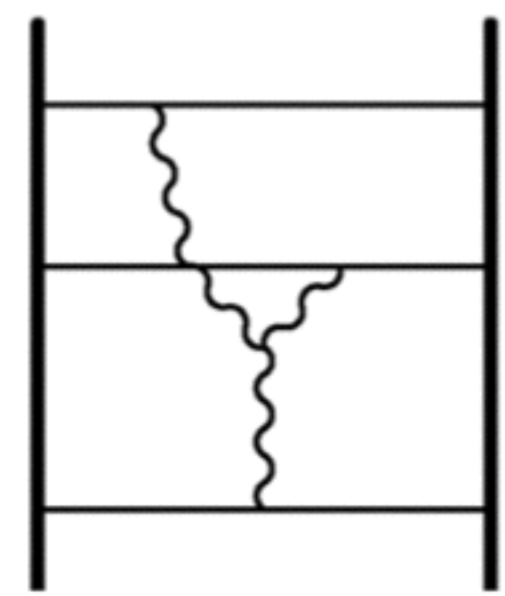
$$S_{\mathrm{red}}(T) = \frac{1}{2} T\,G\,T + V_3(G\,T, G\,T, G\,T) + ...$$



O(G)= Newtonian + (v/c)^n corrections

O(G^2)=1PN =1 loop

O(G^3)=2PN =2 loop



O(G^5)=4PN =4 loop

Recently (Damour-Jaranowski ’17) corrected an error in the EFT computation (by Foffa-Mastrolia-Sturani-Sturm’16) of the above static 4-loop diagram.

New results already at the 1PM order (linear in G)

Derivation of EOB energy map to all orders in v/c:

$$\mathcal{E}_{\text{eff}} = \frac{(\mathcal{E}_{\text{real}})^2 - m_1^2 - m_2^2}{2(m_1 + m_2)}.$$

to order G^1, the relativistic dynamics of a two-body system (of masses m_1, m_2) is equivalent to the relativistic dynamics of an effective test particle of mass $\mu = m_1 m_2/(m_1 + m_2)$ moving in a Schwarzschild metric of mass $M = m_1 + m_2$, i.e. the rather complicated 1PM Hamiltonian of Ledvinka-Schaefer-Bicak2010: with

$$\begin{aligned}
H_{\text{lin}} = {} & \sum_a \overline{m}_a + \frac{1}{4} G \sum_{a, b \neq a} \frac{1}{r_{ab}} \left(7\, \mathbf{p}_a \cdot \mathbf{p}_b + (\mathbf{p}_a \cdot \mathbf{n}_{ab})(\mathbf{p}_b \cdot \mathbf{n}_{ab})\right) - \frac{1}{2} G \sum_{a, b \neq a} \frac{\overline{m}_a \overline{m}_b}{r_{ab}} \\
& \times \left(1 + \frac{p_a^2}{\overline{m}_a^2} + \frac{p_b^2}{\overline{m}_b^2}\right) - \frac{1}{4} G \sum_{a, b \neq a} \frac{1}{r_{ab}} \frac{(\overline{m}_a \overline{m}_b)^{-1}}{(y_{ba} + 1)^2 y_{ba}} \Bigg[2\Big(2 (\mathbf{p}_a \cdot \mathbf{p}_b)^2 (\mathbf{p}_b \cdot \mathbf{n}_{ba})^2 \qquad (6) \\
& - 2 (\mathbf{p}_a \cdot \mathbf{n}_{ba})(\mathbf{p}_b \cdot \mathbf{n}_{ba})(\mathbf{p}_a \cdot \mathbf{p}_b)\mathbf{p}_b^2 + (\mathbf{p}_a \cdot \mathbf{n}_{ba})^2 \mathbf{p}_b^4 - (\mathbf{p}_a \cdot \mathbf{p}_b)^2 \mathbf{p}_b^2 \Big) \frac{1}{\overline{m}_b^2} + 2 \Big[-\mathbf{p}_a^2 (\mathbf{p}_b \cdot \mathbf{n}_{ba})^2 \\
& + (\mathbf{p}_a \cdot \mathbf{n}_{ba})^2 (\mathbf{p}_b \cdot \mathbf{n}_{ba})^2 + 2 (\mathbf{p}_a \cdot \mathbf{n}_{ba})(\mathbf{p}_b \cdot \mathbf{n}_{ba})(\mathbf{p}_a \cdot \mathbf{p}_b) + (\mathbf{p}_a \cdot \mathbf{p}_b)^2 - (\mathbf{p}_a \cdot \mathbf{n}_{ba})^2 \mathbf{p}_b^2 \Big] \\
& + \Big[-3 \mathbf{p}_a^2 (\mathbf{p}_b \cdot \mathbf{n}_{ba})^2 + (\mathbf{p}_a \cdot \mathbf{n}_{ba})^2 (\mathbf{p}_b \cdot \mathbf{n}_{ba})^2 + 8 (\mathbf{p}_a \cdot \mathbf{n}_{ba})(\mathbf{p}_b \cdot \mathbf{n}_{ba})(\mathbf{p}_a \cdot \mathbf{p}_b) \\
& + \mathbf{p}_a^2 \mathbf{p}_b^2 - 3 (\mathbf{p}_a \cdot \mathbf{n}_{ba})^2 \mathbf{p}_b^2 \Big] y_{ba} \Bigg],
\end{aligned}$$

$$y_{ba} = \frac{1}{\overline{m}_b} \sqrt{m_b^2 + (\mathbf{n}_{ba} \cdot \mathbf{p}_b)^2}. \qquad \overline{m}_a = (m_a^2 + \mathbf{p}_a^2)^{\frac{1}{2}}$$

is fully described by the EOB energy map applied to

$$ds_{\text{lin}}^2 = -(1 - 2\frac{GM}{r})dt^2 + (1 + 2\frac{GM}{r})dr^2 + r^2 d\Omega^2$$

TESTING THEORIES

Phenomenological (theory-independent) approach

e.g. Mercury's periastron advance: $\dot{\omega}$

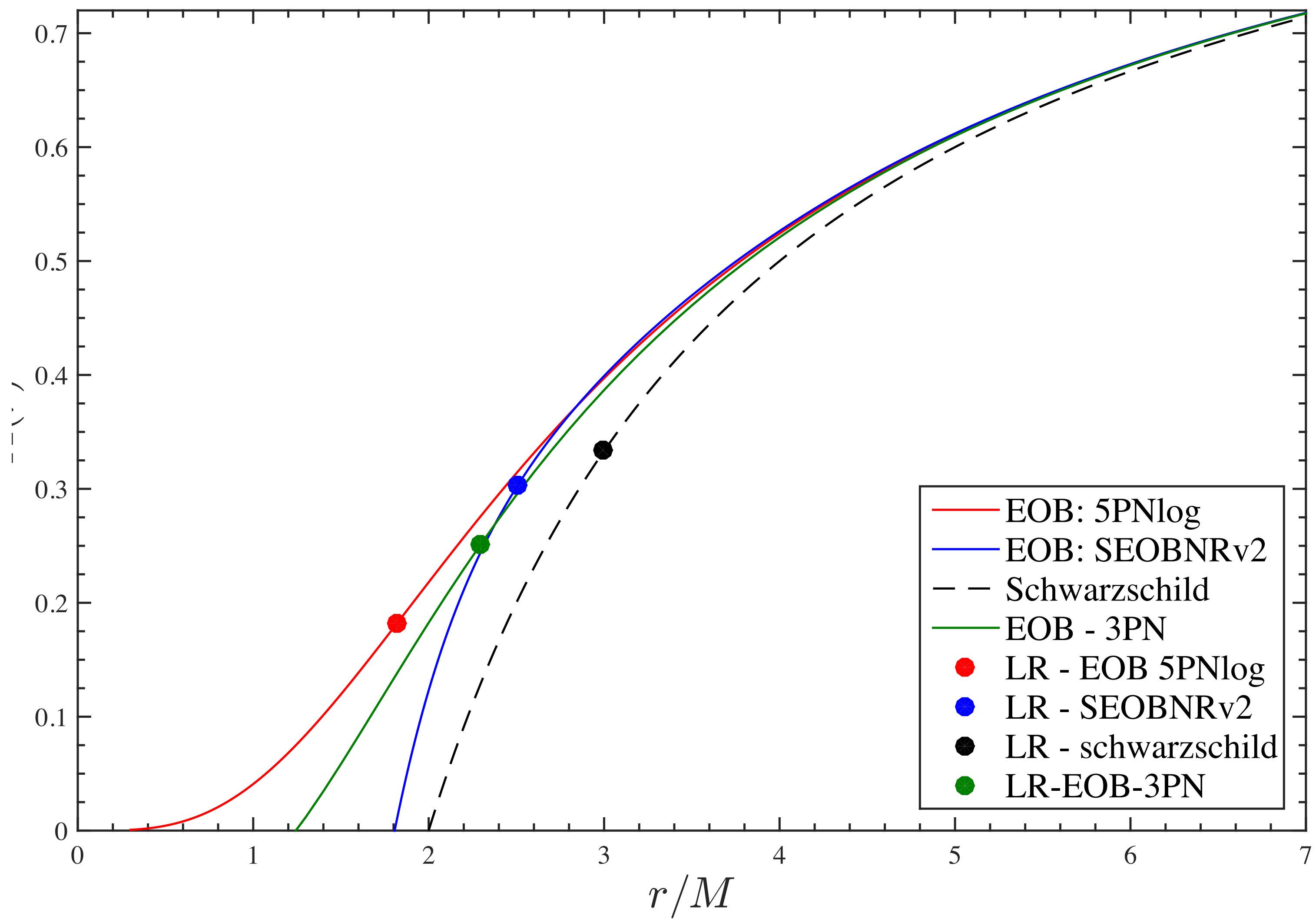
Theory-space approach:
consider a multi-dimensional space of theories:
e.g. tensor-scalar gravity with free parameters
and/or free functions.
Problem: scarcity of sound, well-motivated alternatives to GR,
predicting non-GR effects for BBH systems.
Standard tensor-scalar theories need NS to predict non-GR effects
Lack of proof that currently considered alternative theories are
theoretically, and phenomenologically, sound
(Vainshtein mechanism ??; higher-derivative ghosts ??).
Use of models containing unmotivated scales.
In addition, lack of complete theoretical derivations in most
alternative theories.

NEW PHENOMENOLOGICAL APPROACH TO TESTING STRONG-FIELD GRAVITY IN BBH

Parametrized EOB (PEOB) approach:

Use the analytical flexibility of EOB: flex some of the crucial EOB functions determining the complete EOB waveform (including ring down) by modifying them in the strong-field (u =O(1)) or relativistic (x=O(1)) domain

A(r)



Calculable theory-space alternative for BBH: String-inspired gravity

Consider a 4-parameter, two-derivative deformation of BBH in GR

$$L[g_{\mu\nu},\varphi,A_\mu) = \frac{1}{16\pi G}(R - 2(\partial\varphi)^2 - \frac{1}{4}e^{g\varphi}F_{\mu\nu}F^{\mu\nu})$$

$$g_{\mu\nu}^{\text{obs}} = e^{g'\varphi}g_{\mu\nu}$$

Here, A_mu is a « graviphoton » (Scherk), that could be coupled to dark matter, or to some shadow matter. Dimensionless parameters for « electric-type charges » (assuming some type of charge separation during gravitational collapse; differently from the NS case: Q_NS=0):

$$g; g'; q_1 = \frac{Q_1}{16\pi G m_1}; q_2 = \frac{Q_2}{16\pi G m_2}$$

The scalar hair of each (isolated) BH is a function of g, and q <~ 1.
The 4 parameters will coherently and smoothly deform the dynamics, the radiation damping, the merger, the ringdown, and the observed waveform (adding a spin-0 polarization). By restricting the parameters to special sub-spaces one can explore the sensitivity of GW150914 to various consistent strong-field effects (e.g. q1 + q2=0 or not =0)

CONCLUSIONS

The EOB formalism led (in 2000) to the first quantitative predictions for the waveform, and physical characteristics (notably final spin) of merging BBHs.

NR-completed EOB waveforms (in Reduced Order form) are being employed in LIGO/Virgo data analyses [O1: 200 000 EOB templates, O2: 325 000 EOB templates] and, have played a central role in the search, significance-assessment, parameter-estimation analyses, and GR tests of the GW observations announced so far.

EOB waveform models have also been employed to build frequency-domain, phenomenological models for the inspiral, merger, and ringdown stages of the BBH coalescence. [The latter models have also been used to infer the properties and carry out tests of GR with GW observations.]

The EOB formalism has been extended to tidally interacting systems (BNS, NSBH)

The EOB formalism might also (after SF-completion) be an efficient way of defining accurate templates for LISA.

Beyond its role in defining accurate waveform templates for GW detectors, the EOB formalism is also a new way of describing both the dynamics and the GW radiation of compact binaries. It notably led to accurate descriptions of: periastron precession, energetics [E(J) curve], and scattering of BBH.

EOB theory has also revealed several remarkably simple features (hidden in other formulations) of BBH dynamics such as: linearity in nu=m1m2/(m1+m2)^2 of A(u;nu) up to the 4PN level; validity of a simple energy map to all orders in v/c

$$\mathcal{E}_{\text{eff}} = \frac{(\mathcal{E}_{\text{real}})^2 - m_1^2 - m_2^2}{2(m_1 + m_2)}.$$