

# Gravitational waves from neutron-star binaries

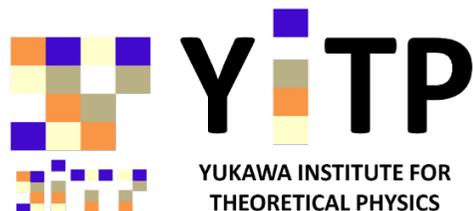
- Constraining Neutron Star EOS -

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# Outline

- 0. Brief introduction**
- 1. Typical scenarios of NS-mergers**
- 2. Gravitational waves & equations of state**
- 3. Viscous hydrodynamics for post-merger of NS-NS**
- 4. High-accuracy simulations for NS-NS inspiral**
- 5. Summary**

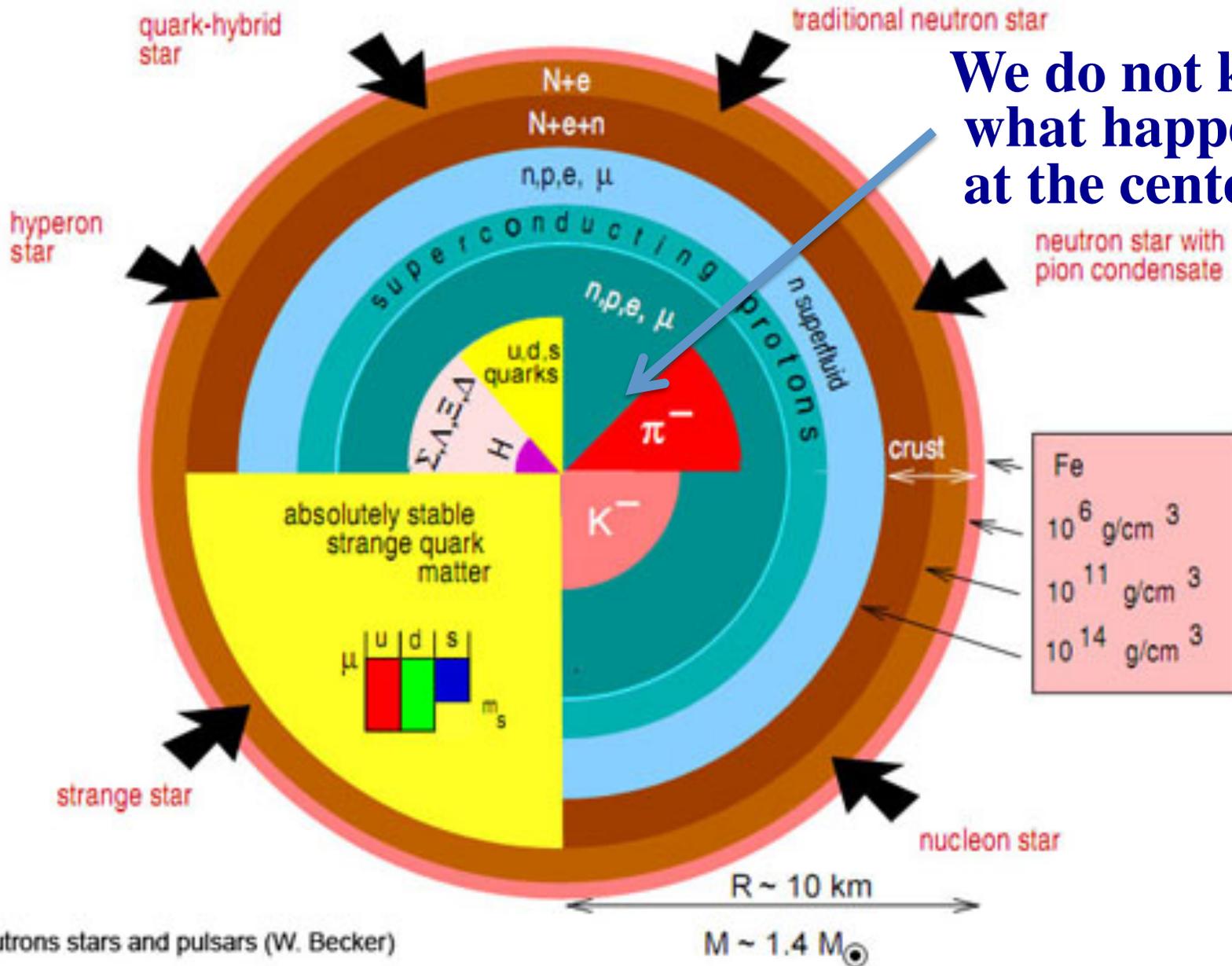
# Many (young) people are exploring NS binaries in numerical relativity

- Shibata & Uryu (1999), Taniguchi
- Sekiguchi, Kiuchi, Kyutoku, Hotokezaka, Kawaguchi
- Rezzolla, Baiotti, Giacomazzo, Kastaun, Ciolfi, Radice, Takami..
- Shapiro, Liu, Etienne, Pachalidis, ..
- Bernuzzi, Dietrich, Bruegmann, Gold, ..
- Lehner, Palenzuele, Liebling, Nielsen, Anderson, ..
- Foucart, Duez, O'Connor, Ott, Haas, Scheel, Kidder, Pfeiffer,...
- Loeffler and his colleagues & many others



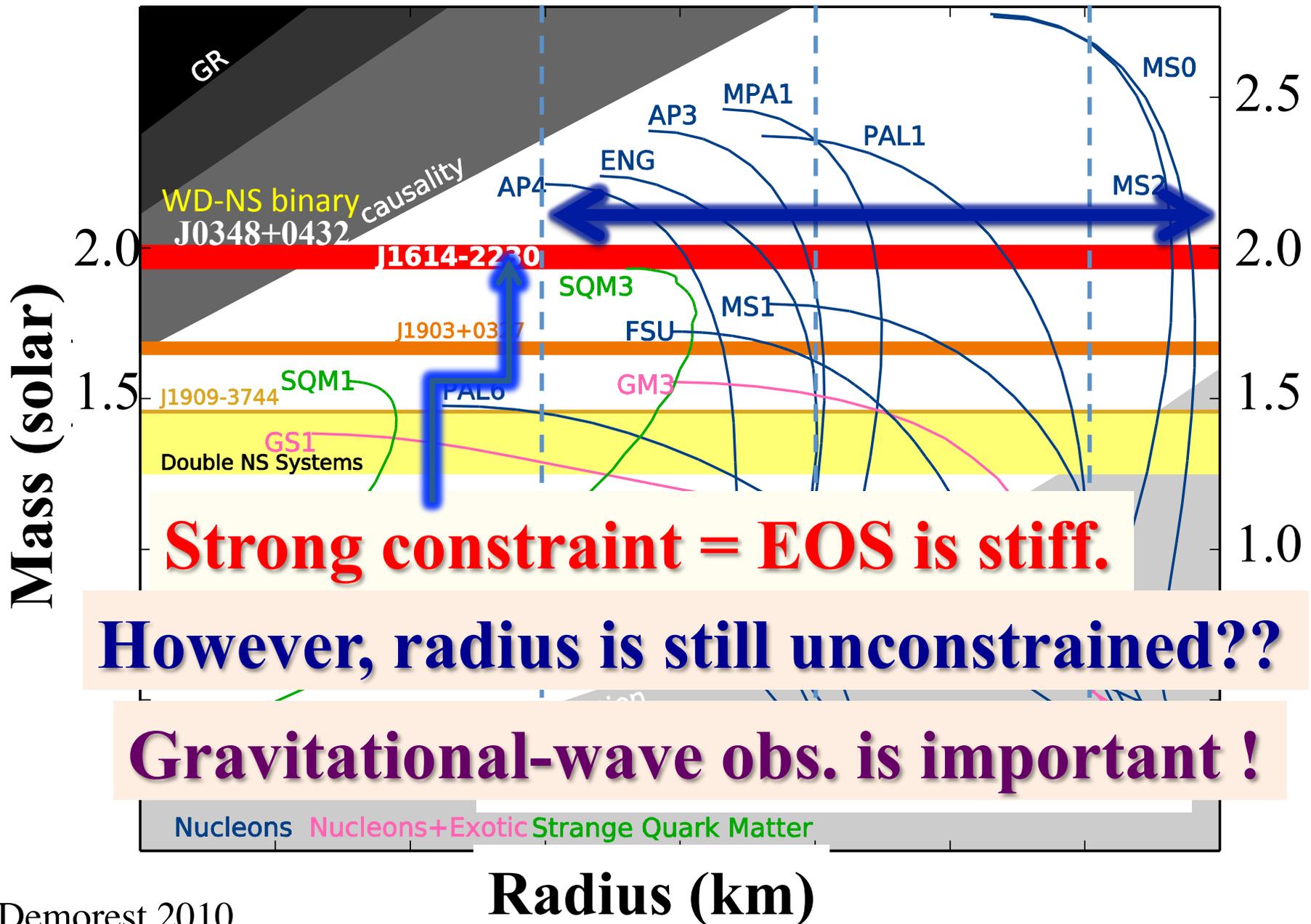
- **Solid progress on understanding NS-NS/NS-BH binary by numerical relativity**

# Introduction: Neutron star structure is still unsolved



source : Neutrons stars and pulsars (W. Becker)

# Mass-radius relation for various EOS



# **1 Typical scenarios of NS-NS/BH-NS merger**

## **1-A Binary neutron stars**

# Boundary conditions from radio pulsar observation

➤ **Total Mass of NS in compact NS-NS** is likely to be in a narrow range,  $m \approx 2.73 \pm 0.15 M_{\text{sun}}$

	PSR	Orbital period	Eccentricity	Each mass		lifetime
		$P(\text{day})$	$e$	$M(M_{\text{sun}})$	$M_1$ $M_2$	$T_{\text{GW}}$
1.	B1913+16	0.323	0.617	2.828	1.441 1.387	3.0
2.	B1534+12	0.421	0.274	2.678	1.333 1.345	27
3.	B2127+11C	0.335	0.681	2.71	1.35 1.36	2.2
4.	J0737-3039	0.102	0.088	2.58	1.34 1.25	0.86
5.	J1756-2251	0.32	0.18	2.57	1.34 1.23	17
6.	J1906+746	0.166	0.085	2.61	1.29 1.32	3.1
7.	J1913+1102	0.206	0.090	2.875	1.65 1.24	~5
8.	A24	0.184	0.606	2.74	1.35 1.39	~0.75

$\times 10^8 \text{ yrs}$

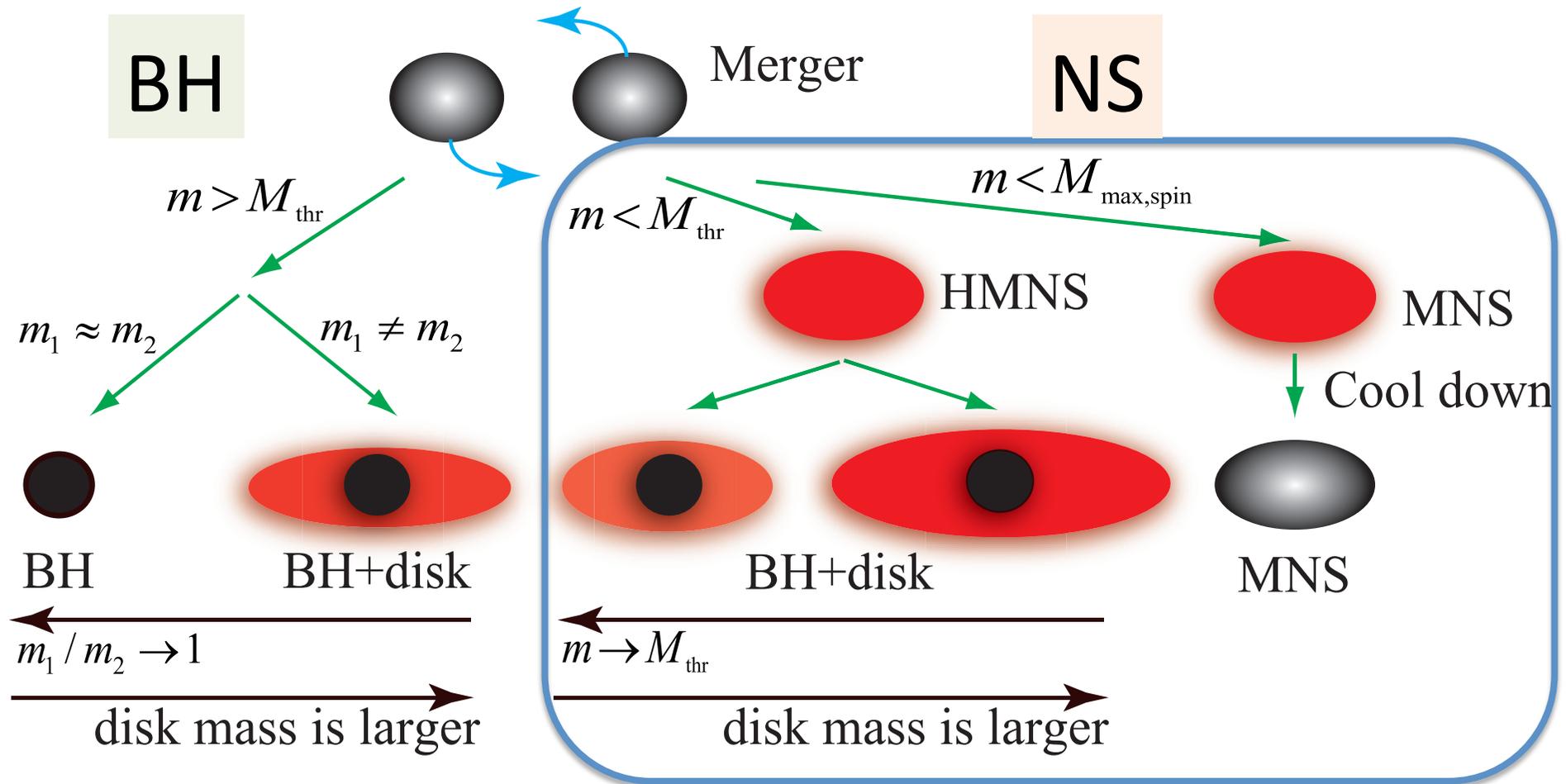
# Boundary conditions from radio pulsar observation

- **Total Mass of NS in compact NS-NS** is likely to be in a narrow range,  $m \approx 2.73 \pm 0.15 M_{\text{sun}}$
- **Spin of NS** is likely to be not very high,  $P_{\text{rot}} > \sim 10 \text{ ms}$  or  $\chi < \sim 0.04$  (1<sup>st</sup> NS=weakly recycled)
- **NS radius (EOS)** is still uncertain, but **maximum mass of NS would be  $\geq 2 M_{\text{sun}}$**   
(Demorest 2010; Antoniadis 2013)  
→ **EOS of NS has to be sufficiently stiff**



- **Numerical relativity simulations have shown that massive neutron stars are formed after the merger**

# Possible outcomes of NS-NS mergers



Likely for  $M_{\text{tot}} < \sim 2.8M_{\text{sun}}$

I.e., irrespective of EOS, threshold mass  $> \sim 2.8M_{\text{sun}}$

# 1-B Black hole-neutron star binaries

# Two possibilities: **Tidal disruption or not**

For tidal disruption, (Self gravity of NS) < (BH tidal force)

$$\frac{M_{\text{NS}}}{(\alpha R_{\text{NS}})^2} < \frac{M_{\text{BH}} (\alpha R_{\text{NS}})}{r^3} \quad (\alpha > 1) \Rightarrow 1 \leq \left( \frac{M_{\text{BH}}}{r_{\text{ISCO}}} \right)^3 \left( \frac{M_{\text{NS}}}{M_{\text{BH}}} \right)^2 \left( \frac{\alpha R_{\text{NS}}}{M_{\text{NS}}} \right)^3$$

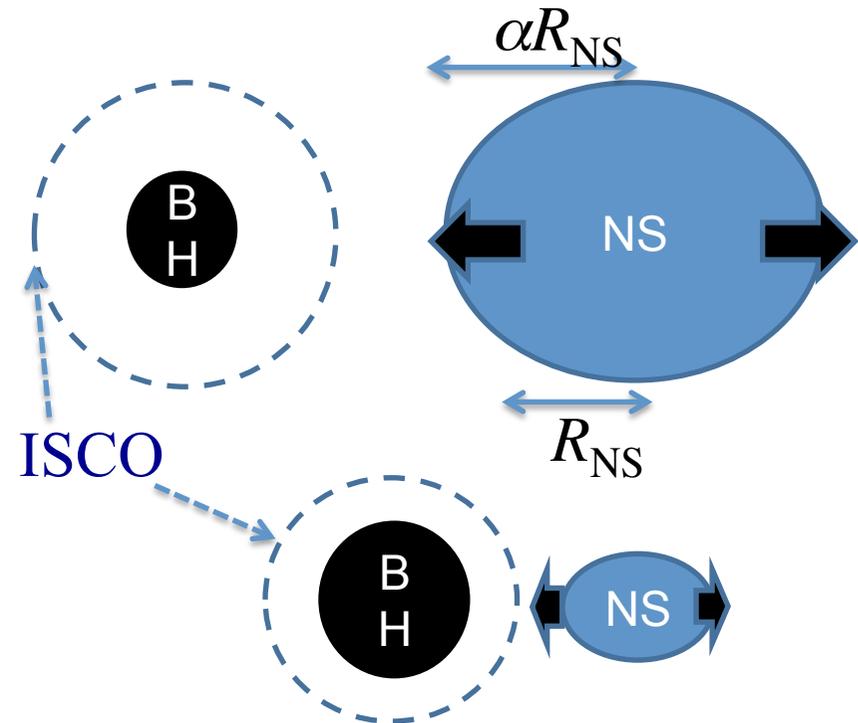
- **For tidal disruption**

- ❖ Large NS Radius or

- ❖ Small BH mass or

- ❖ High corotation spin

**is necessary**



## BH-NS with aligned BH spin

$$M_{\text{BH}}=6.75M_{\text{sun}}$$

$$a=0.75$$

$$M_{\text{NS}}=1.35M_{\text{sun}}$$

$$R=11.1 \text{ km}$$



$$M_{\text{BH}}=4.05M_{\text{sun}}$$

$$a=0$$

$$M_{\text{NS}}=1.35M_{\text{sun}}$$

$$R=11.0 \text{ km}$$

# BH-NS with aligned BH spin

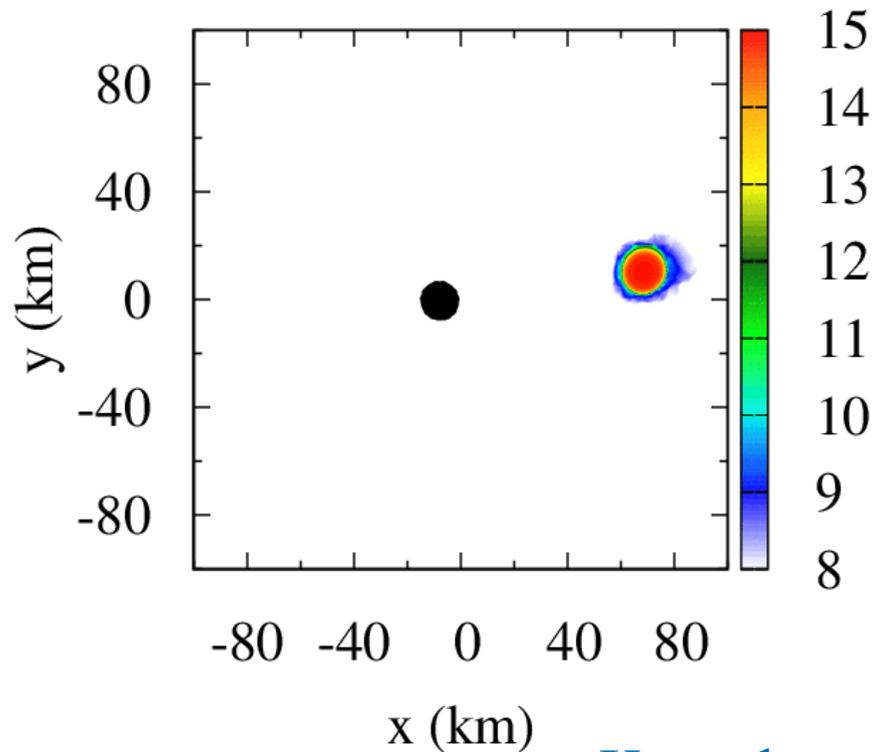
$$M_{\text{BH}} = 6.75 M_{\text{sun}}$$

$$a = 0.75$$

$$M_{\text{NS}} = 1.35 M_{\text{sun}}$$

$$R = 11.1 \text{ km}$$

t = 11.56 ms      log  $\rho$  (g/cm<sup>3</sup>)



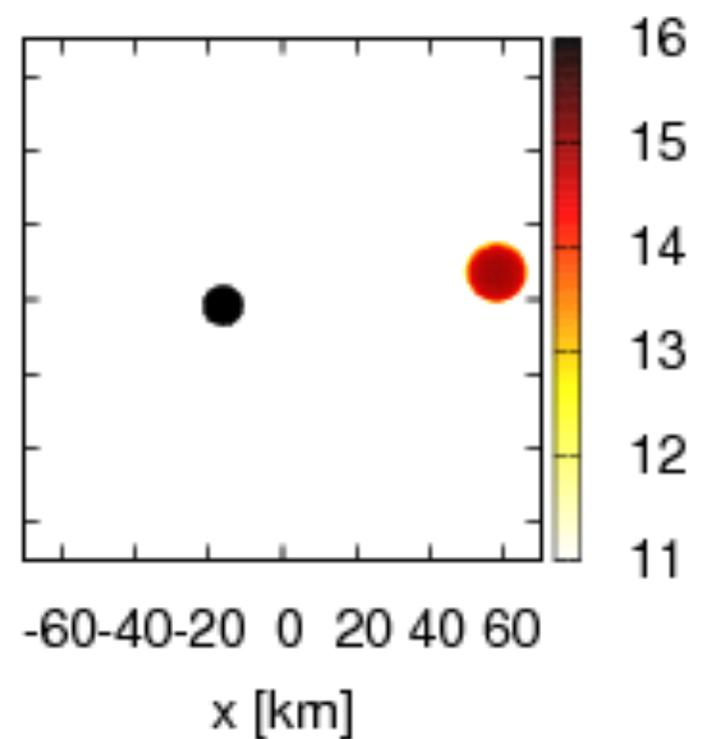
$$M_{\text{BH}} = 4.05 M_{\text{sun}}$$

$$a = 0$$

$$M_{\text{NS}} = 1.35 M_{\text{sun}}$$

$$R = 11.0 \text{ km}$$

t = 156.4008  $\mu$ s



Kyutoku et al. 2011, 2015

**For tidal disruption of plausible BH-NS with**  
 $M_{\text{NS}}=1.35M_{\text{sun}}, R_{\text{NS}} \sim 12 \text{ km}, \text{ \& } M_{\text{BH}} > 6 M_{\text{sun}}$



**High BH spin is necessary  $> \sim 0.75$**

Foucart et al. (2013, 2014); Kyutoku et al. (2015)

$$1 \leq \left( \frac{M_{\text{BH}}}{r_{\text{ISCO}}} \right)^3 \left( \frac{M_{\text{NS}}}{M_{\text{BH}}} \right)^2 \left( \frac{\alpha R_{\text{NS}}}{M_{\text{NS}}} \right)^3$$

- Note 1: BH mass should be smaller than  $\sim 20$  solar mass for BH spin  $< \sim 0.9$
- Note 2 : If high-mass BH,  $\sim 30$  solar mass, is standard, *ultra high spin* is needed for tidal disruption

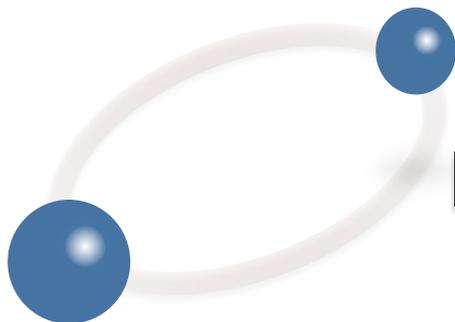
# **2 Gravitational waves & Equations of state**

# 2-A NS-NS case

$M \sim 2.5 - 2.8 M_{\text{sun}}$

Early Inspiral

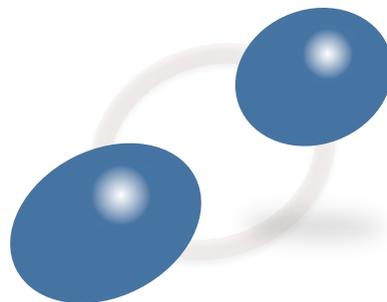
$(r_{\text{orb}} \gg R_{\text{NS}})$



Point mass +  
adiabatic phase

Late inspiral

$(r_{\text{orb}} \leq 5R_{\text{NS}})$



Tidally deformed phase

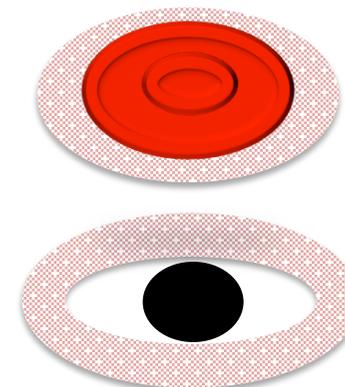
Merger =>

Massive NS



Dynamical & GR phase

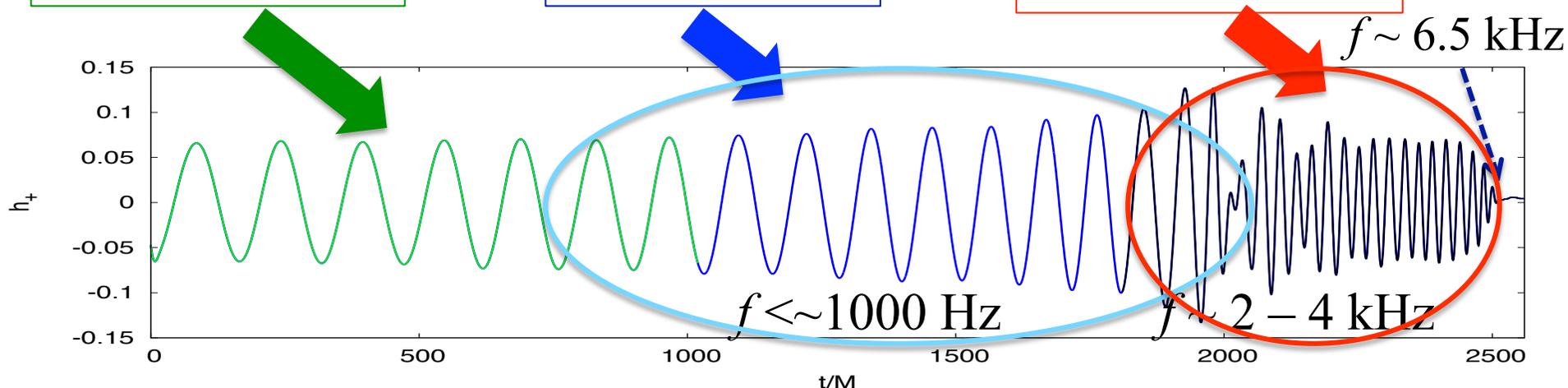
Black hole/MNS  
+ torus → GRB?



Post-Newtonian

Late inspiral

Post merger  
Massive NS/BH



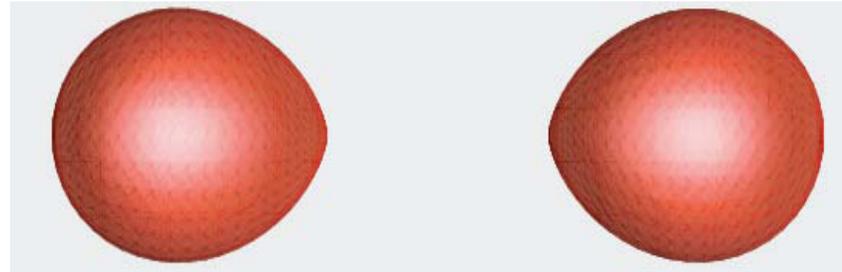
# Imprint of EOS on late inspiral waveform

In a binary system, the tides raised on each NS depend on the **deformability** of that NS:

**Stiff EOS = larger radius = large deformability**



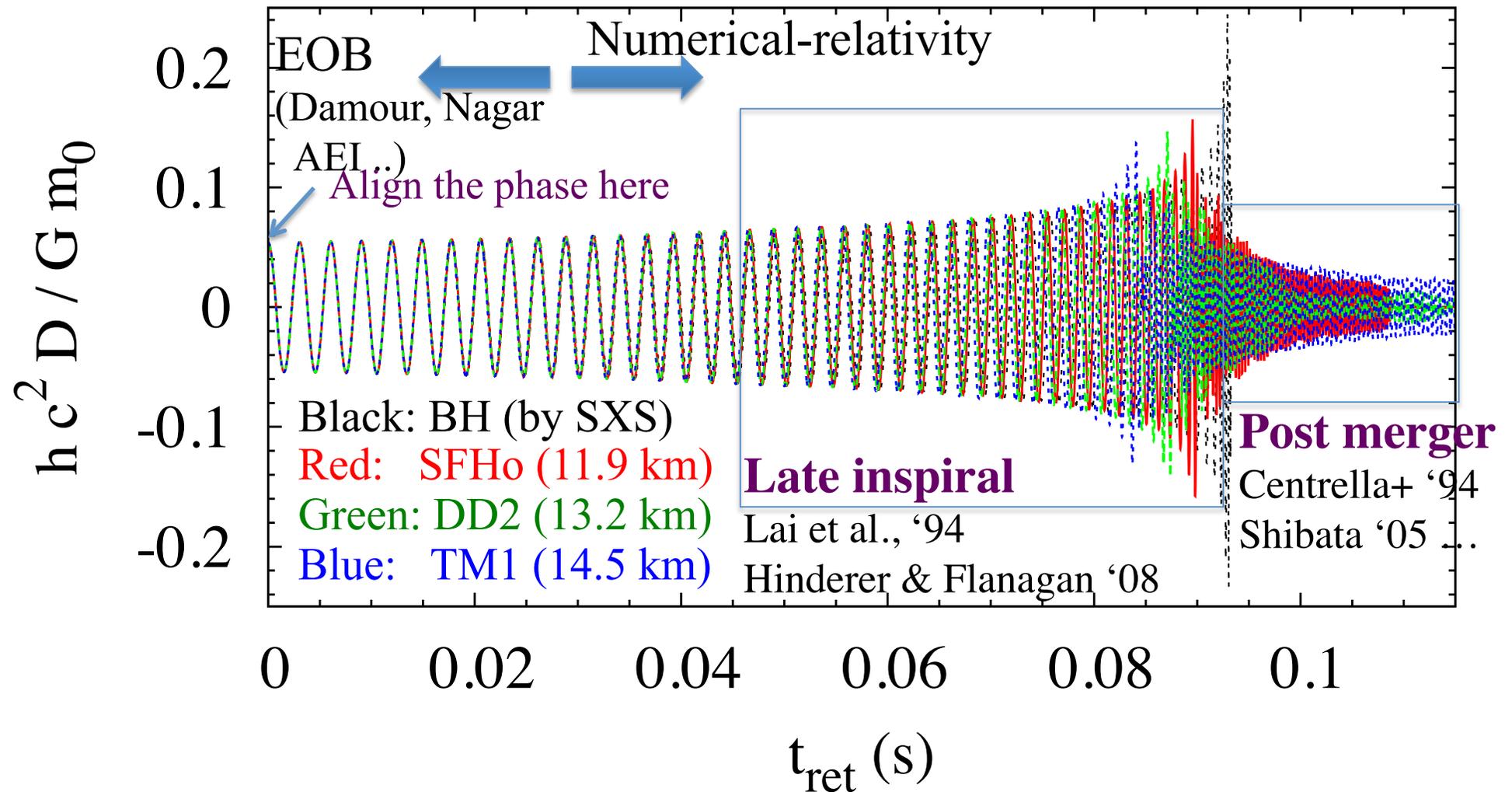
**Soft EOS = small radius = small deformability**



Courtesy J. Friedman

$$\phi \sim -\frac{GM}{r} - \frac{3I_{ij}^{TF} n^i n^j}{2r^3} : I_{ij}^{TF} = O(r^{-3}) \quad \text{Lai et al. (1994)}$$

# Gravitational waveform from NS-NS: hybrid waveforms (1.35-1.35 solar mass)



Hotokezaka et al. 2016 (see also efforts by Bernuzzi, ...2011-)

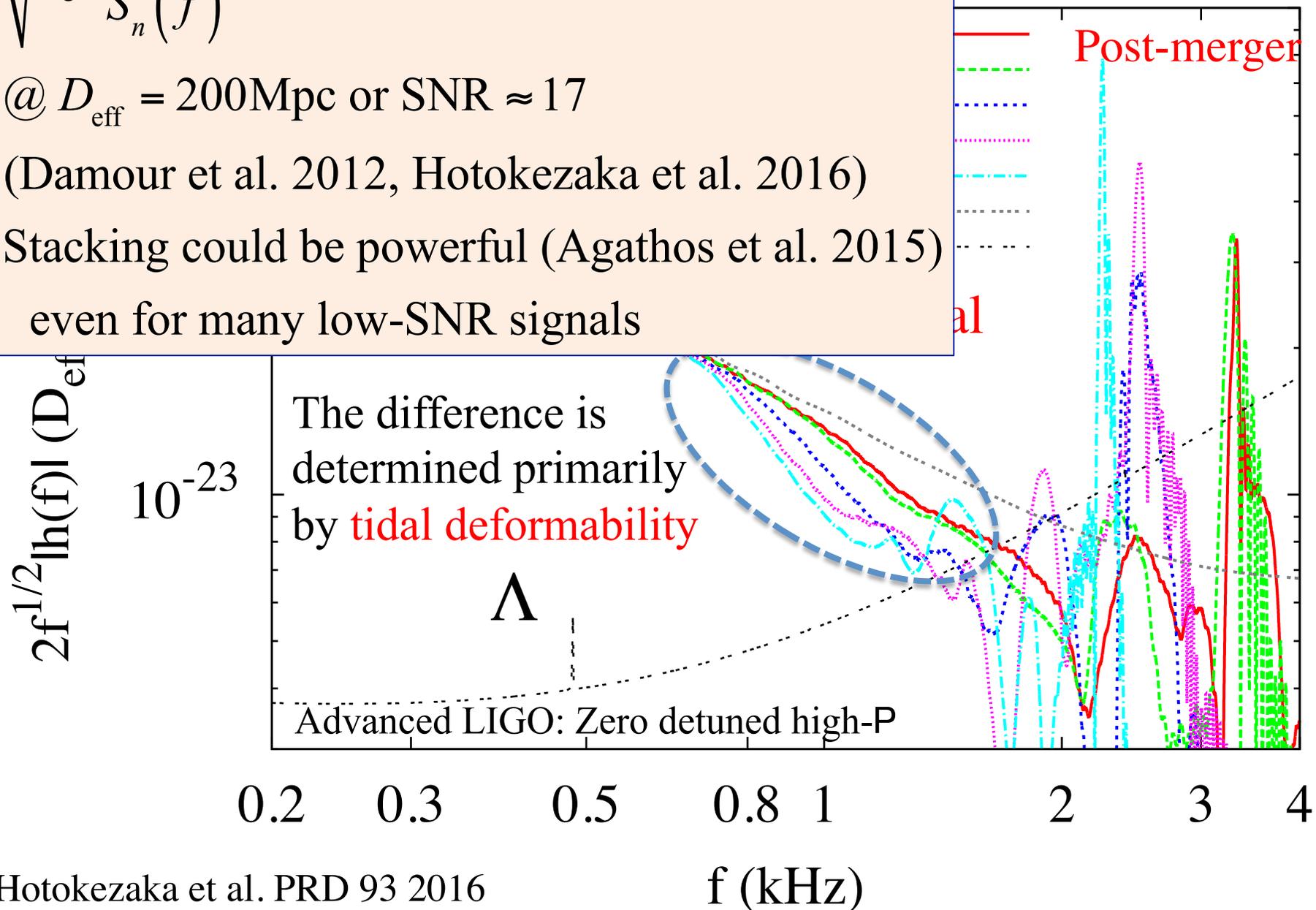
$$\sqrt{4 \int \frac{\delta h^2}{S_n(f)} df} \approx 2 \text{ for } \delta\Lambda=400 \Rightarrow \delta R \approx 1 \text{ km}$$

@  $D_{\text{eff}} = 200\text{Mpc}$  or  $\text{SNR} \approx 17$

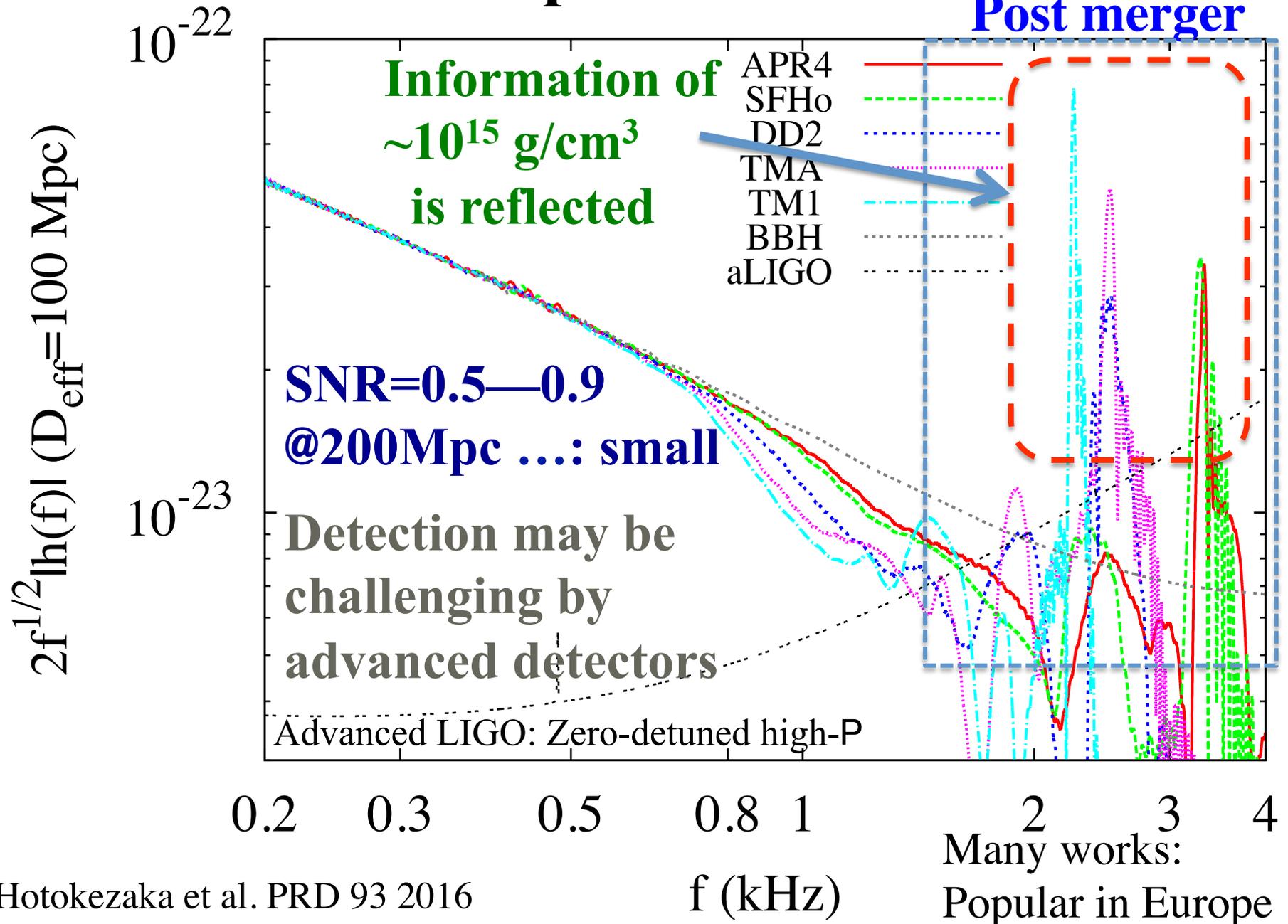
(Damour et al. 2012, Hotokezaka et al. 2016)

Stacking could be powerful (Agathos et al. 2015)

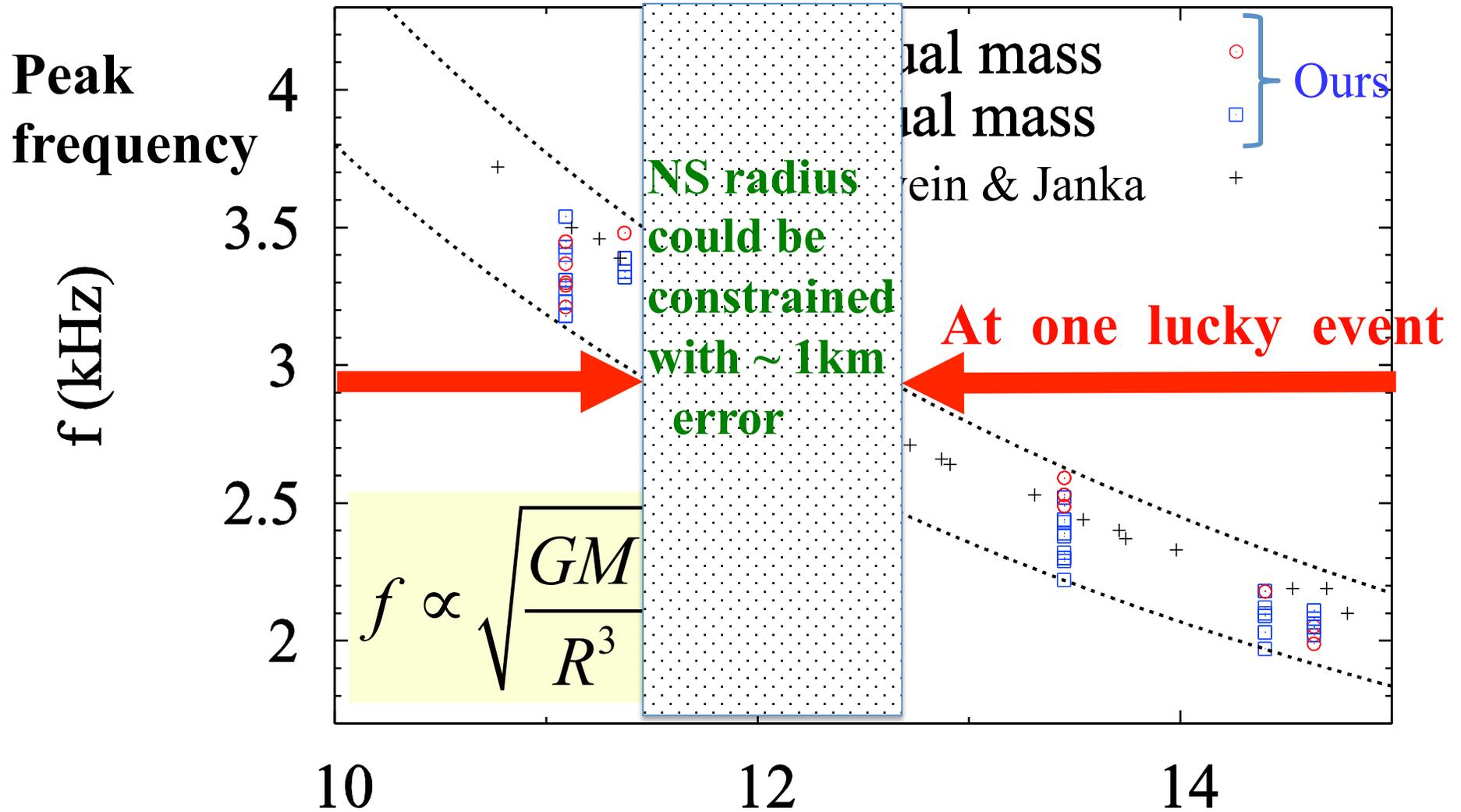
even for many low-SNR signals



# Spectrum



# Clear correlation between peak and radius



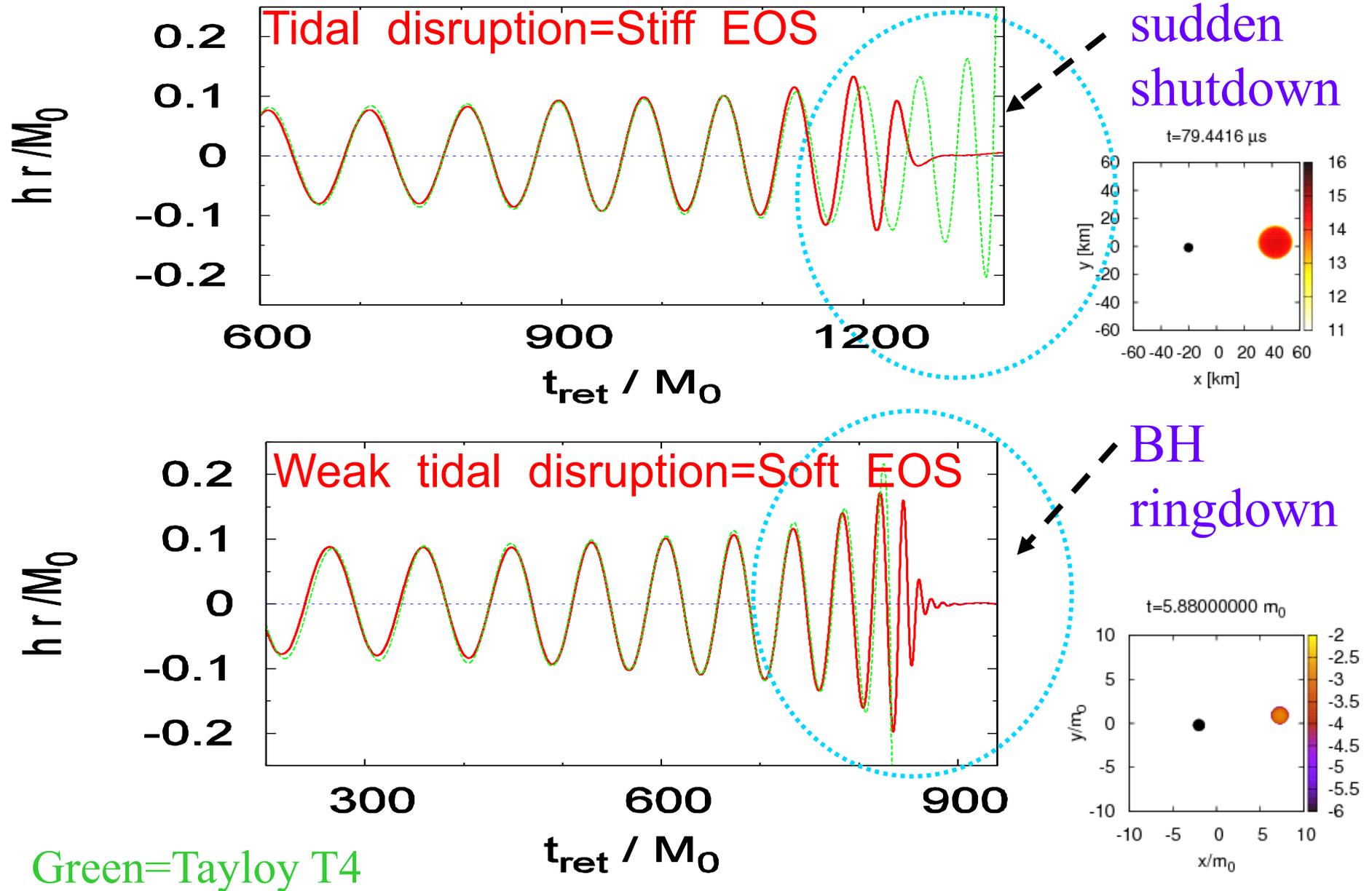
See also Rezzola et al,...

## Radius of 1.6 solar-mass NS

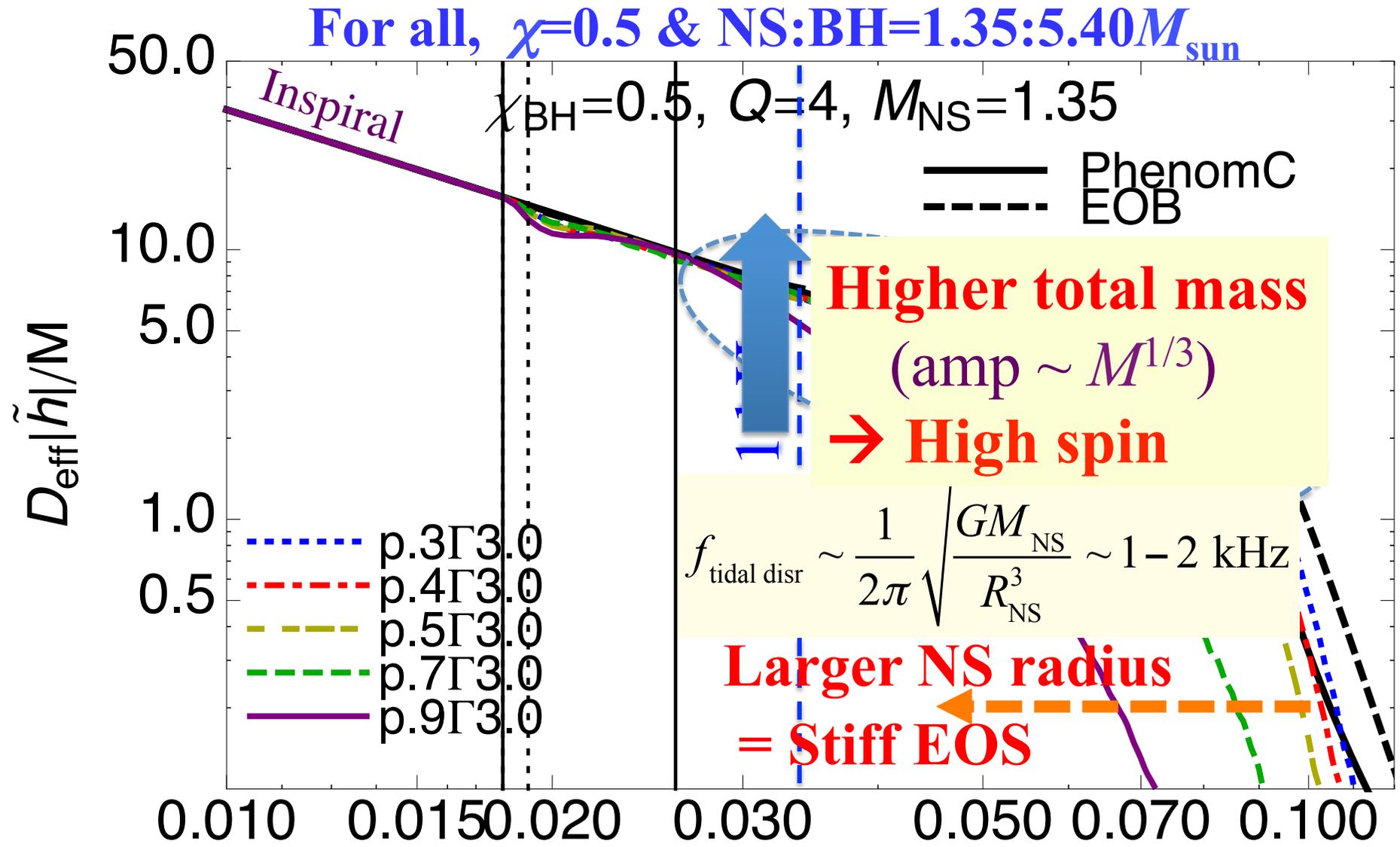
# Current issues for NS-NS

- For late inspiral (*clean system*):  
Need to construct accurate measurement templates  
→ **high-resolution** numerical relativity simulations  
+ **sophisticated modeling** (e.g., TEOB) are necessary  
(section 4 for our latest efforts)
- For post-merger phase (*many physics play roles*):  
**Careful physical modeling** is necessary:  
Most of previous studies have neglected **systematics**  
(section 3)

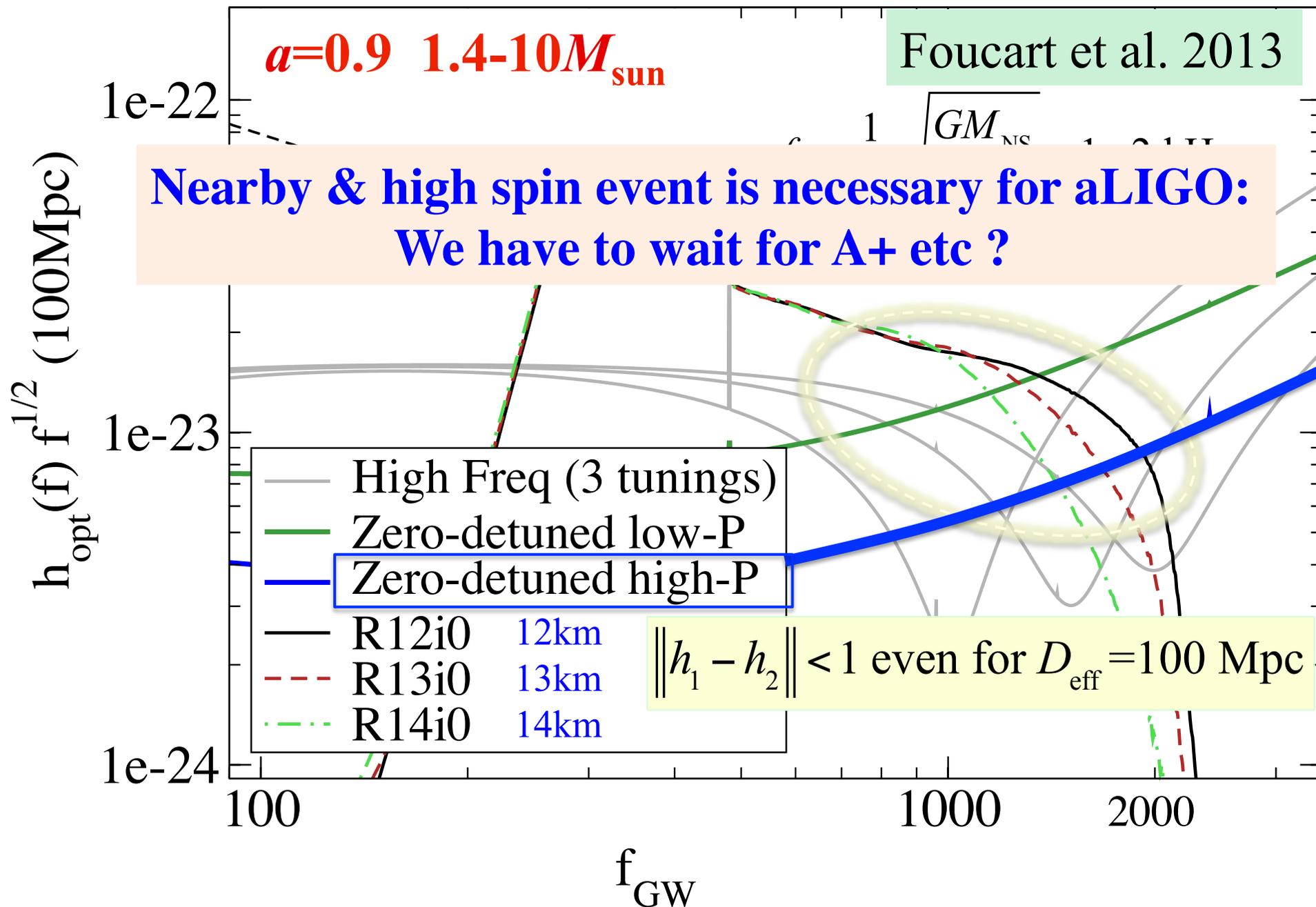
# 2-B BH-NS: Signal of tidal disruption



# BH-NS Fourier spectrum



# Cutoff frequency is high, & SNR is still small



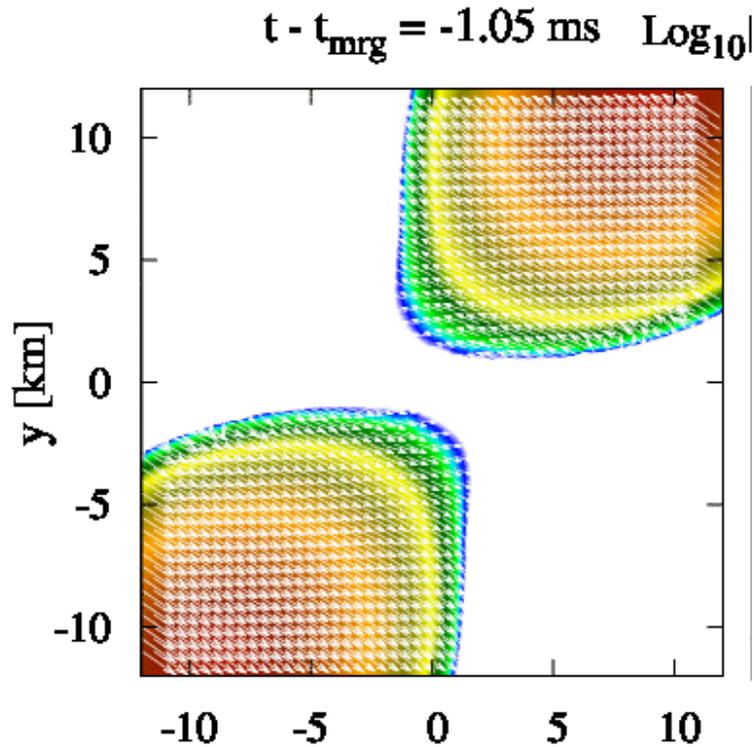
### 3 Viscous hydrodynamics of post-merger of NS-NS

#### ❖ Physical state for the merger remnants

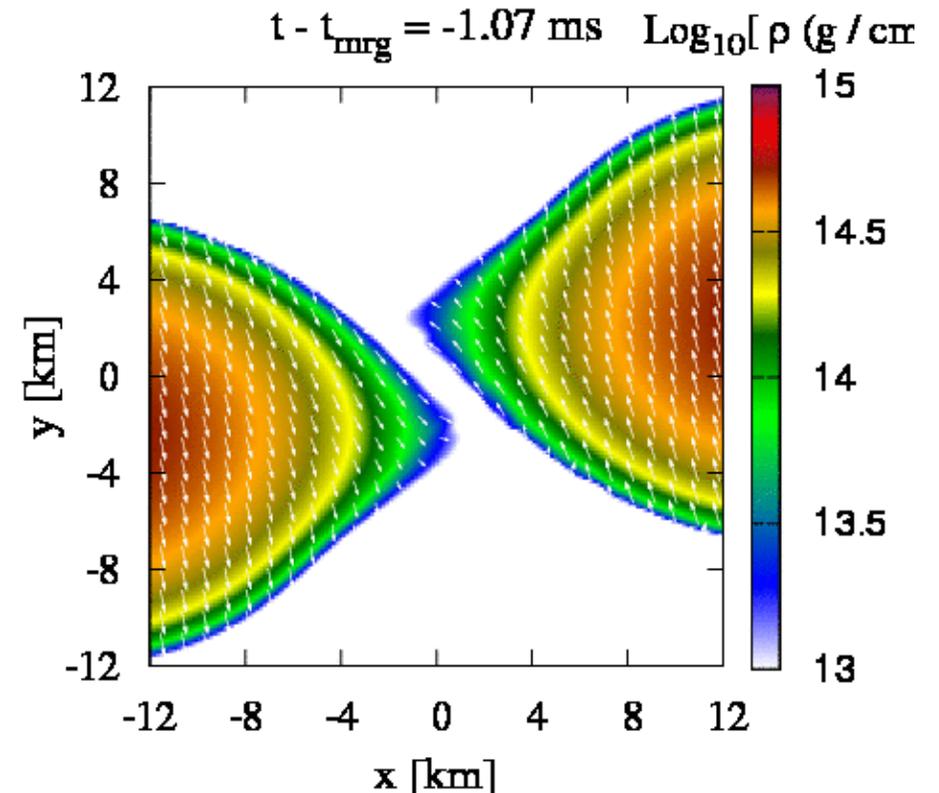
- Massive neutron stars (MNS) are typical remnants
- MNS are *magnetized & differentially rotating*  
→ **subject to MHD instabilities**
- MHD simulations (e.g., Price & Rosswog, '07, Kiuchi et al. '14, '15) suggest that **magnetic fields would be significantly amplified by Kelvin-Helmholtz instability and subsequent quick winding**  
→ **turbulence could be induced**

# High-resolution GRMHD for NS-NS

Kiuchi et al. 2015



$\Delta x = 17.5 \text{ m}$



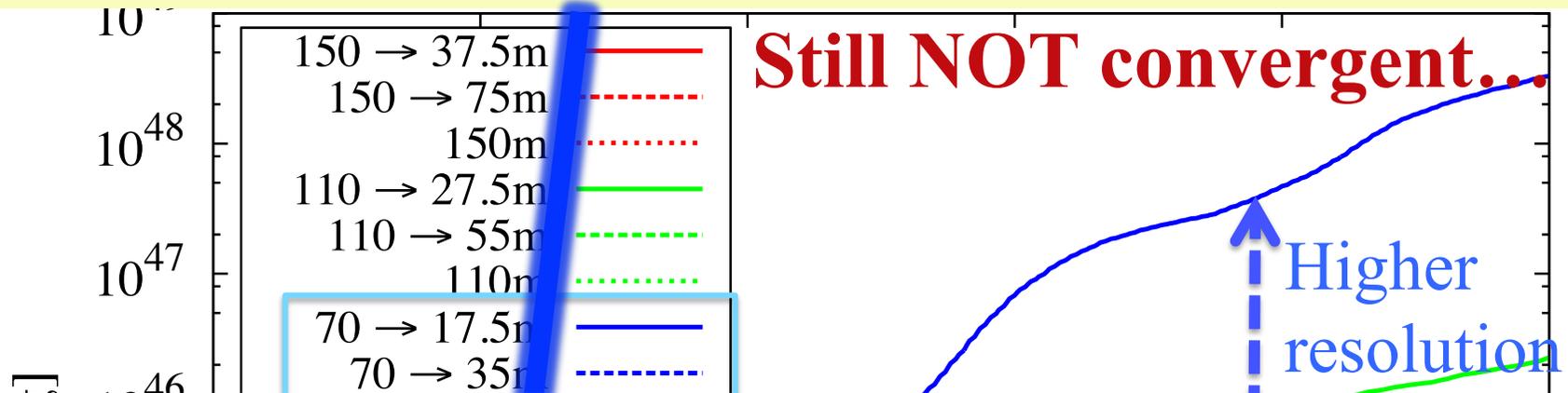
$\Delta x = 70 \text{ m}$

**Kelvin-Helmholtz instability:**

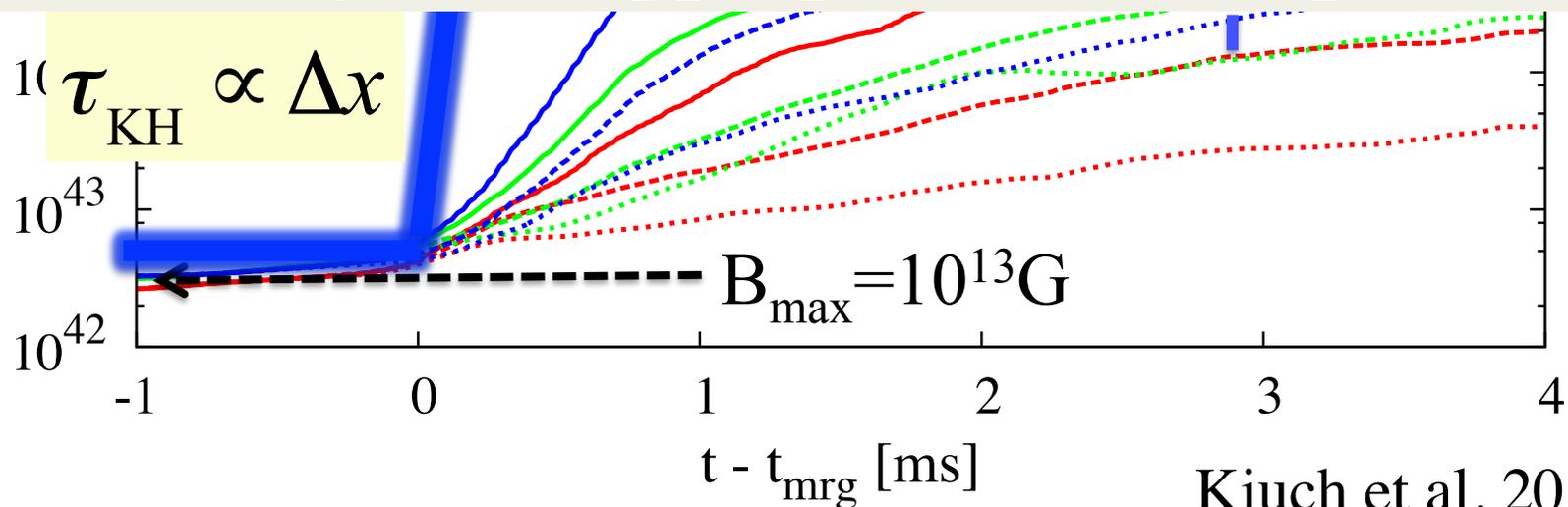
- **Magnetic field should be amplified by winding**
- Quick angular momentum transport ? (not yet seen)

# Magnetic energy: Resolution dependence

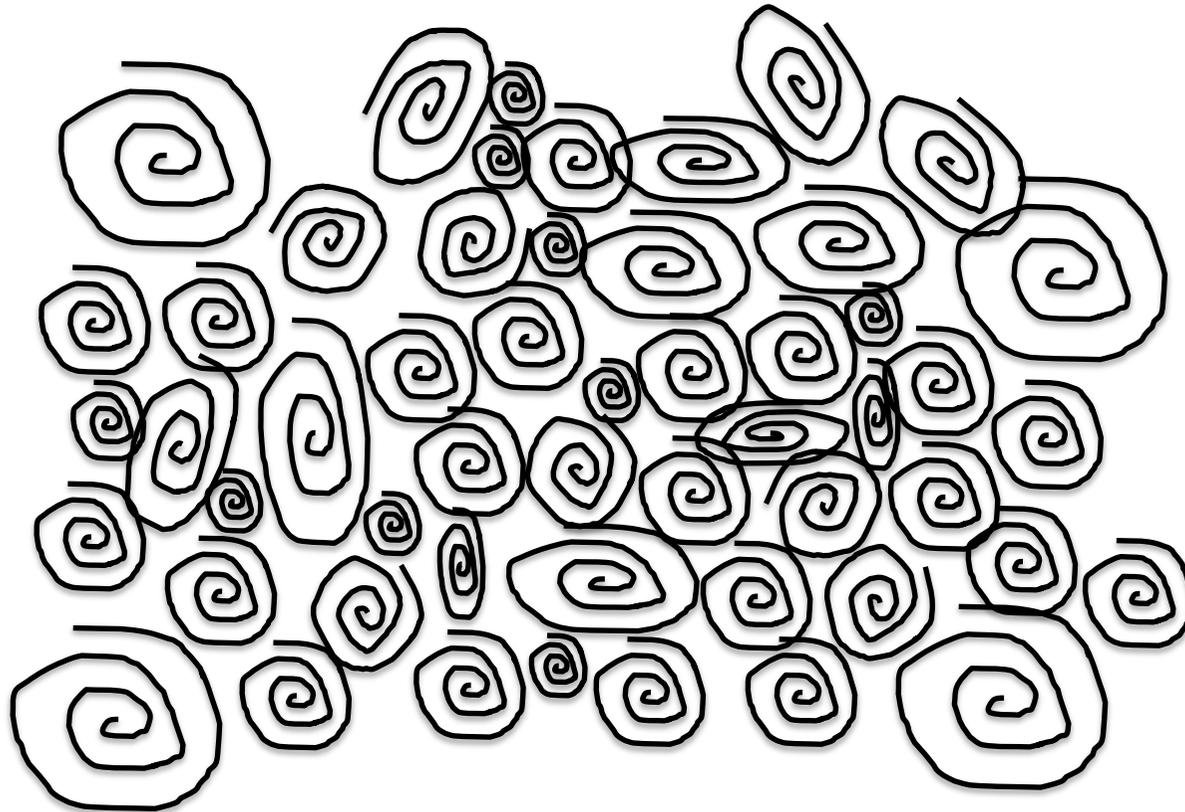
B field would be amplified in  $\Delta t \ll 1$  ms  $\rightarrow$  turbulence ?



Purely hydrodynamics or radiation hydrodynamics is not likely to be appropriate for this problem



Shear motion at the merger →  
huge number of vortexes are formed and  
magnetic field is quickly amplified  
→ further shear motion → turbulence  
→ turbulent (effectively global) viscosity



## Current status in this issue

- High-resolution MHD simulation indicates that **obviously more resolved simulation is needed**  
→ But it is not feasible due to the restriction of the computational resources (in future we have to do)
- **One alternative for exploring the possibilities is viscous hydrodynamics** (Radice '17, Shibata et al. '17)
- ✓ Note that we do not know whether viscous hydrodynamics can appropriately describe the state resulting from turbulence fluid: But, viscous hydro would be able to explore one possible limiting case.

# A GR viscous hydrodynamics

Well-known viscous hydrodynamics formulation  
(e.g., “Classical theory of fields” by Landau-Lifshitz)

$$\nabla_b T^b_a = 0 : T_{ab} = \rho h u_a u_b + P g_{ab} - \underline{\underline{\rho v \sigma_{ab}}}$$

$$\text{where } \sigma_{ab} := h^c_a h^d_b \left( \nabla_c u_d + \nabla_d u_c - \frac{2}{3} g_{cd} \nabla_e u^e \right)$$

$$\text{and } h_{ab} := g_{ab} + u_a u_b. \quad \underline{\underline{v: viscous coefficient}}$$

- ❖ In this case, **parabolic equations** are derived  
→ causality is violated and hence not physical

# Israel-Stewart formalism

To guarantee causality, Israel and Stewart (1979) set

$$\nabla_b T^b_a = 0 : T_{ab} = \rho h u_a u_b + P g_{ab} - \rho \nu \sigma_{ab}$$

$$\mathcal{L}_u \sigma_{ab} = -\xi \left[ \sigma_{ab} - h^c_a h^d_b \left( \nabla_c u_d + \nabla_d u_c - \frac{2}{3} g_{cd} \nabla_e u^e \right) \right]$$

$\xi$  : [Time]<sup>-1</sup> const parameter, short timescale

Lie derivative

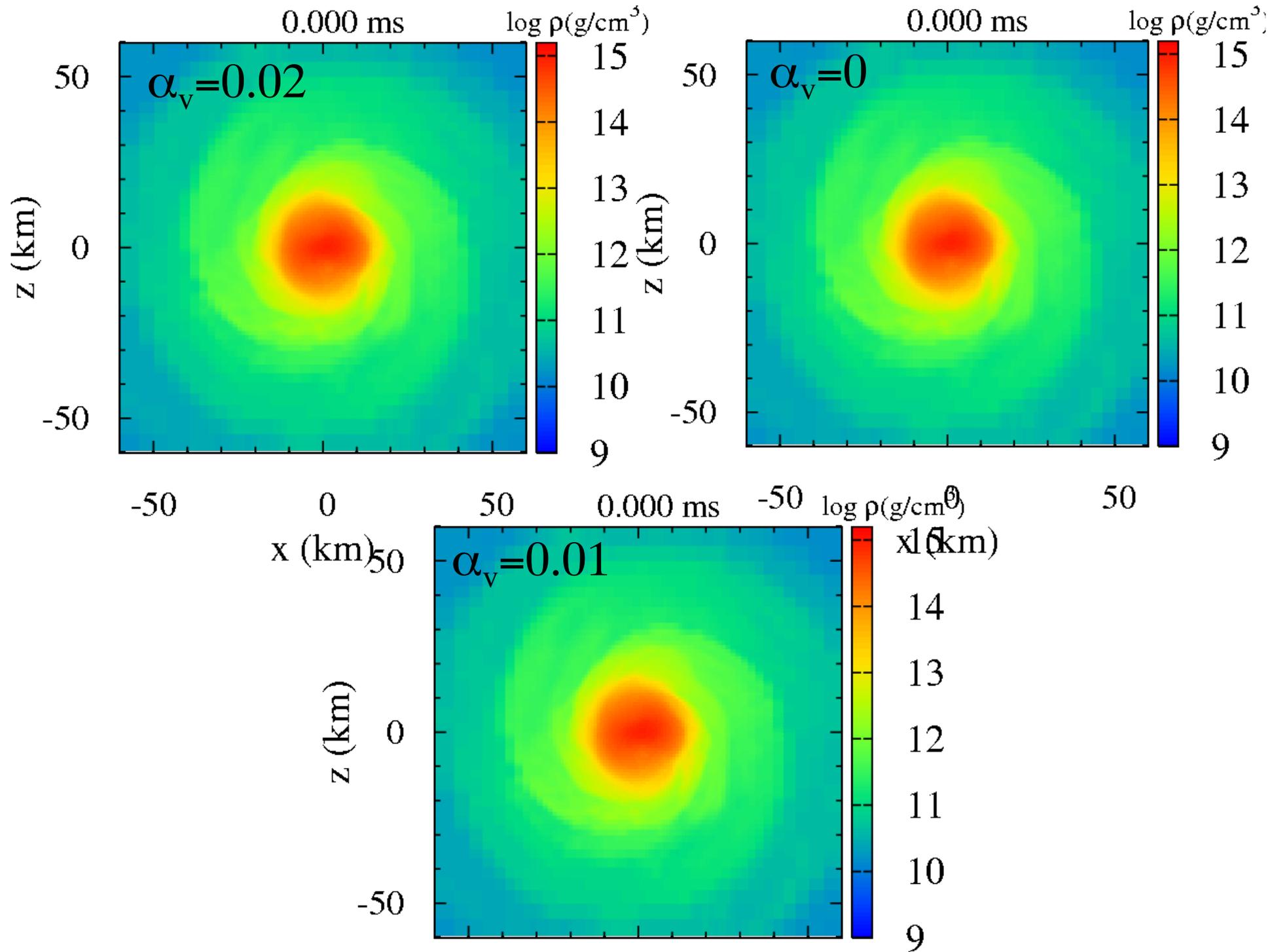
- **Telegraph-type equation is derived:** causality preserving
- If we neglect the last term, the equations are simplified  
→ Pay attention to shear viscous hydrodynamics  
(Shibata et al. '2017)

# 3D viscous hydrodynamics simulation for remnant of binary neutron star merger

(Shibata & Kiuchi PRD June 2017)

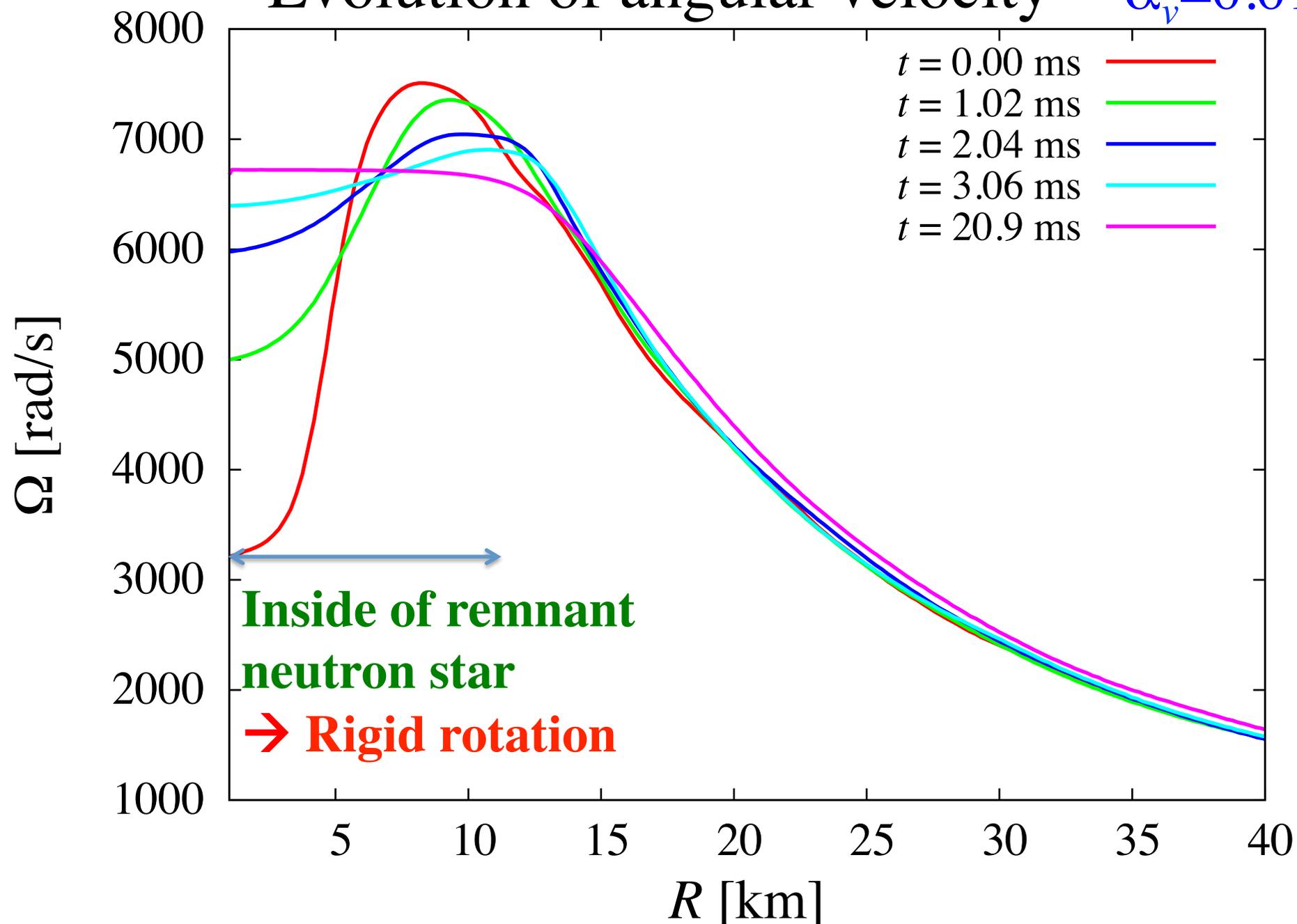
- Merger remnant is used as initial condition
- ✓ H4 EOS (stiff EOS)
- ✓ Mass = 1.35-1.35 solar mass
- Simulation is started at  $\sim 5$ ms after the onset of merger
- $\nu$  is set to be  $\alpha_\nu c_s^2 \Omega^{-1} \sim \alpha_\nu c_s X$  ( $X \sim 10$  km):  $\alpha$  model
- $\alpha$  parameter = 0.01—0.02 taking into account the latest MHD simulation results for accretion disks (such as Jim Stone and his colleagues have been doing)

See also recent work by Radice (2017)

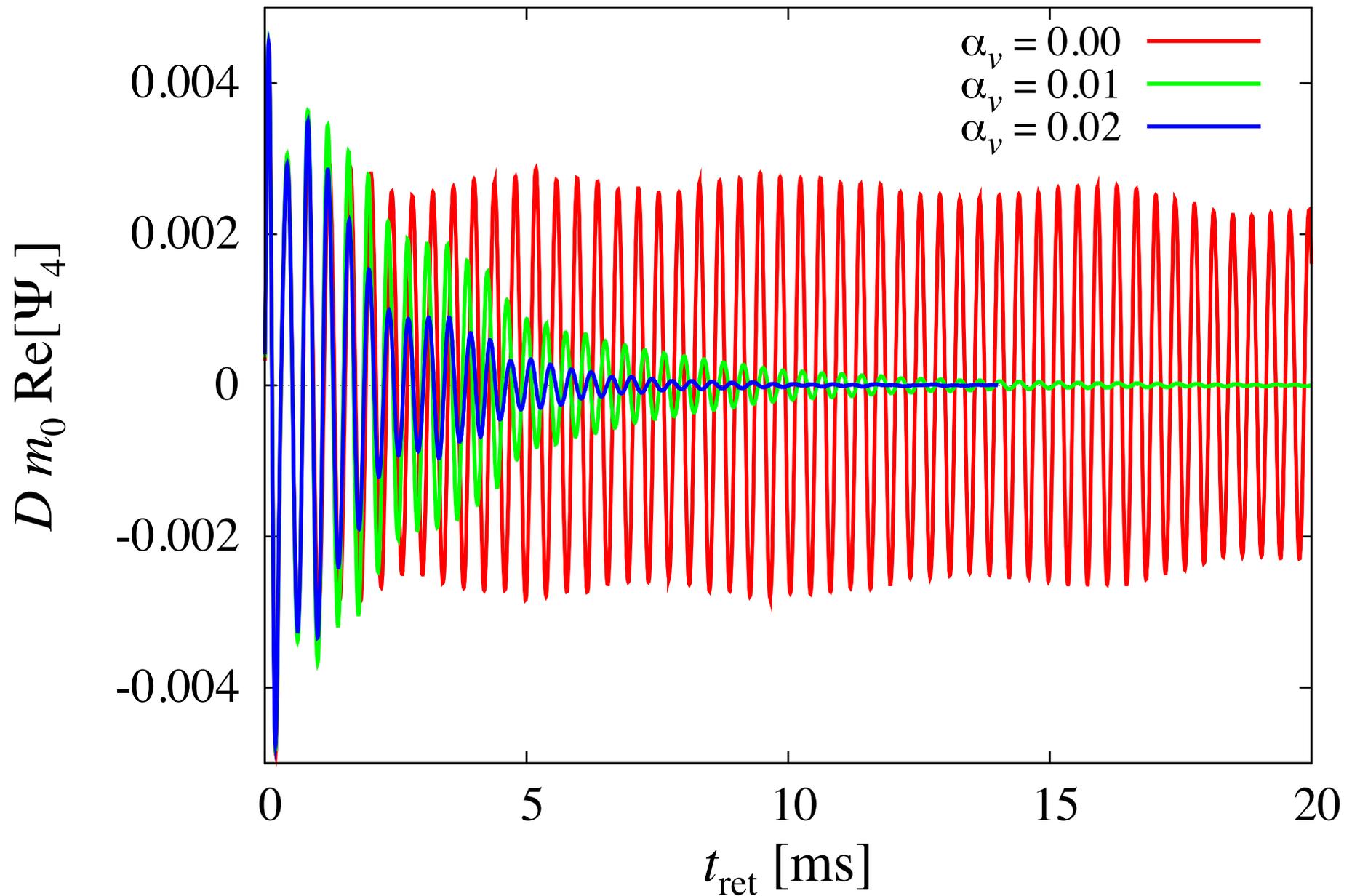


# Evolution of angular velocity

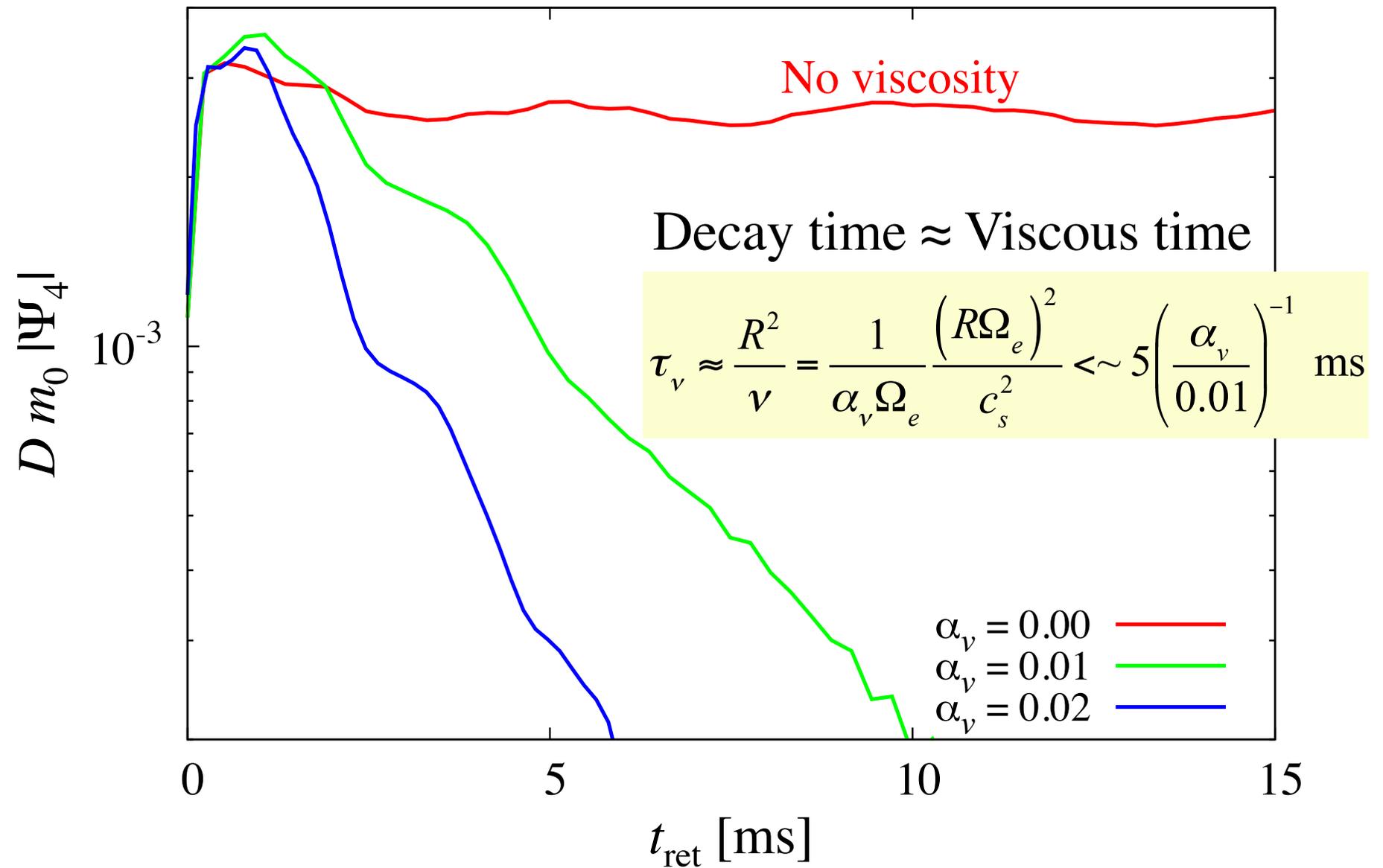
$\alpha_v=0.01$



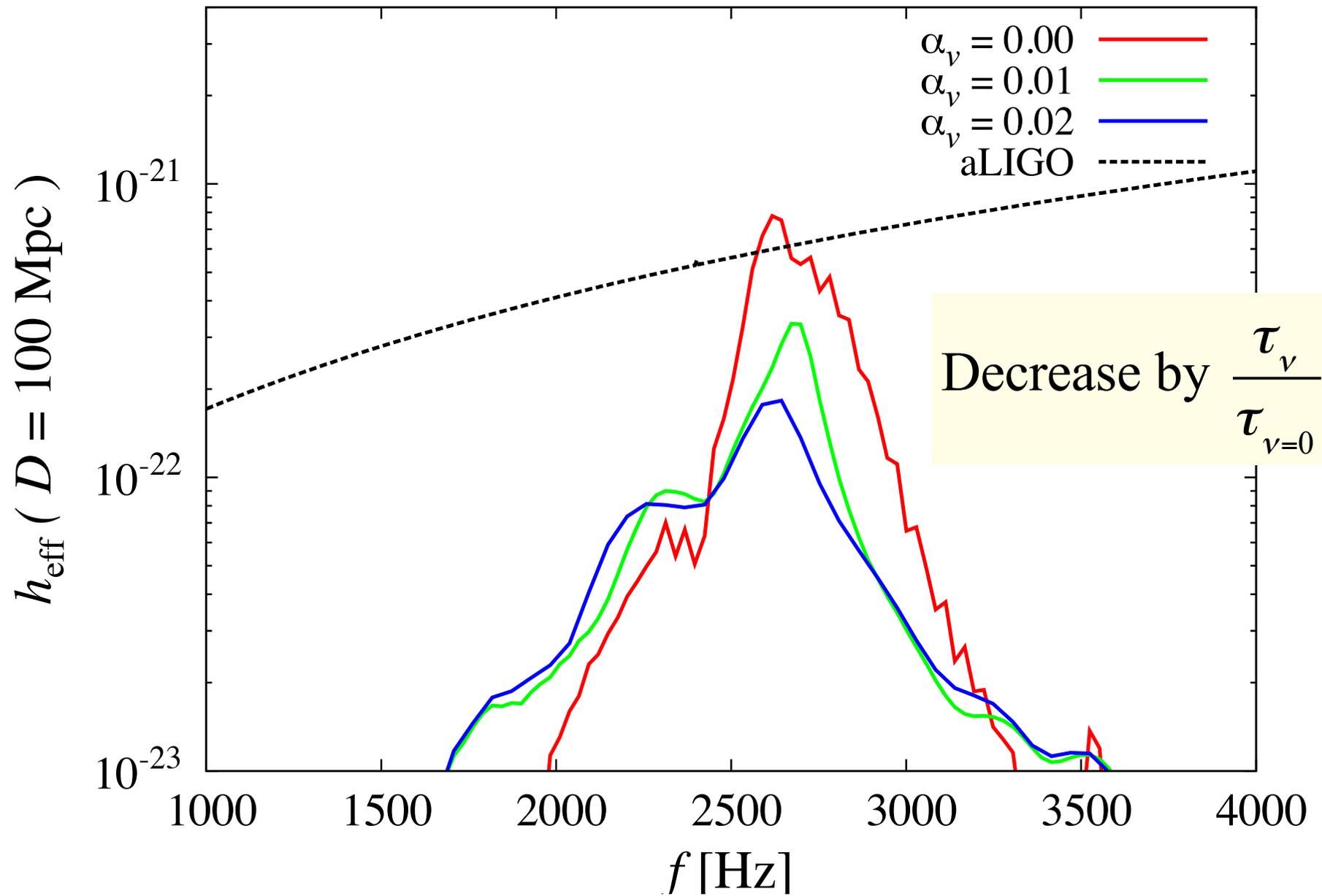
# Gravitational waveforms



# Amplitude of gravitational waves



# Spectrum



# Short summary

- If MHD turbulence  $\approx$  viscous hydrodynamics with  $\alpha_v \geq 0.01$ , evolution of merger remnant of NS-NS would be **highly different from that by ideal fluid dynamics**
- Viscous hydrodynamics suggests that **post-merger gravitational waves could be quite weak**
- Caution is needed to GW community  
→ No detection of post-merger gravitational waves does not always imply BH formation (MNS may exist)
- How large is  $\alpha_v$  ?  
→ **High-resolution MHD is necessary in the future**

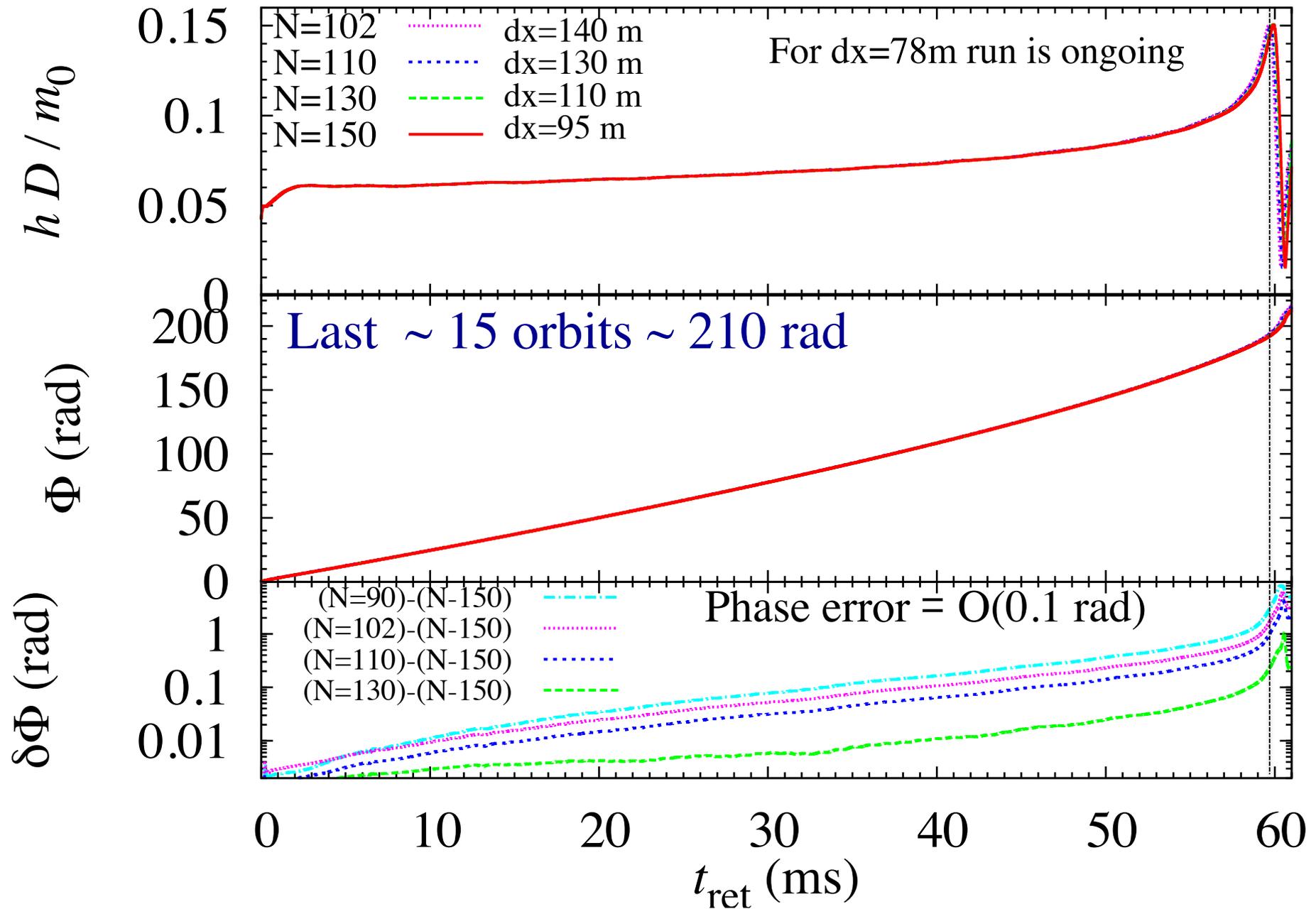
# 4 High-resolution simulation of inspiraling NS-NS

K. Kiuchi, K. Kawaguchi, K. Kyutoku, Y. Sekiguchi, M. Shibata

- For this issue, high-resolution run is needed anyhow
- Our previous simulation for this project :  $dx \sim 150$  m
- Thorough resolution study is ongoing with  $dx$  up to 63—86 m (for EOS of radius 10.9—13.7 km)
- Piecewise polytropic EOS is employed:  
# only tidal deformability is important in this problem
- Initial eccentricity  $\sim 0.001$ : low eccentricity is the key for carefully comparing numerical data with EOB data

See also the efforts by Dietrich, Bernuzzi et al

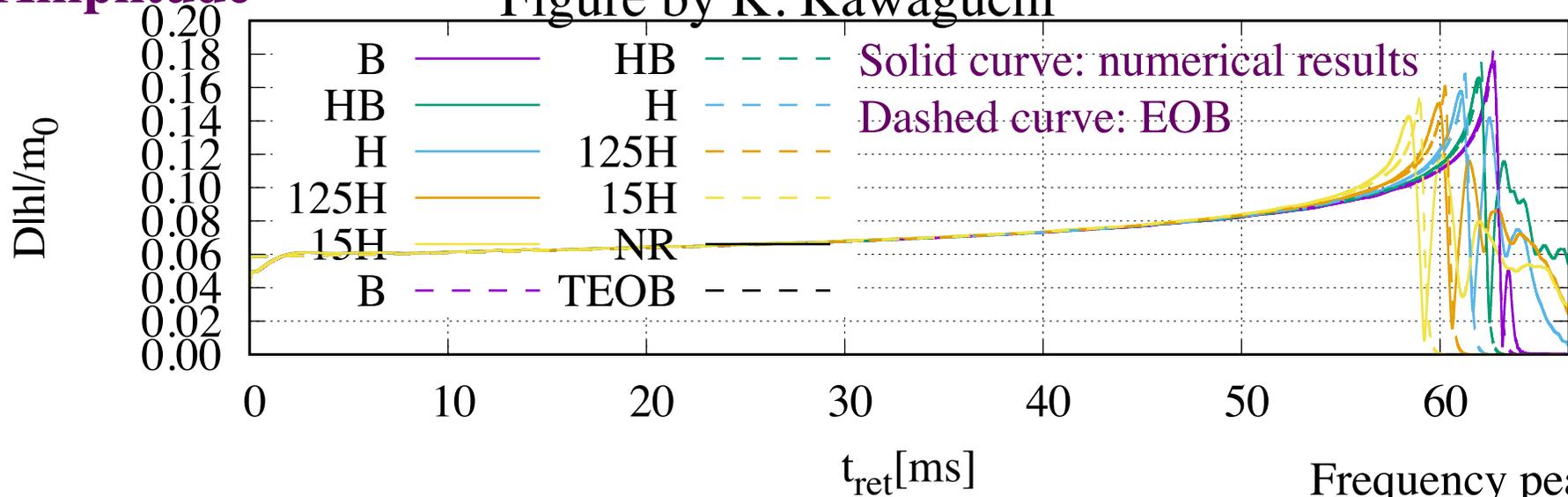
For EOS with  $1.35-1.35M_{\text{sun}}$  &  $R=13.0$  km



# Comparison with a TEOB (Hinderer et al. 2016)

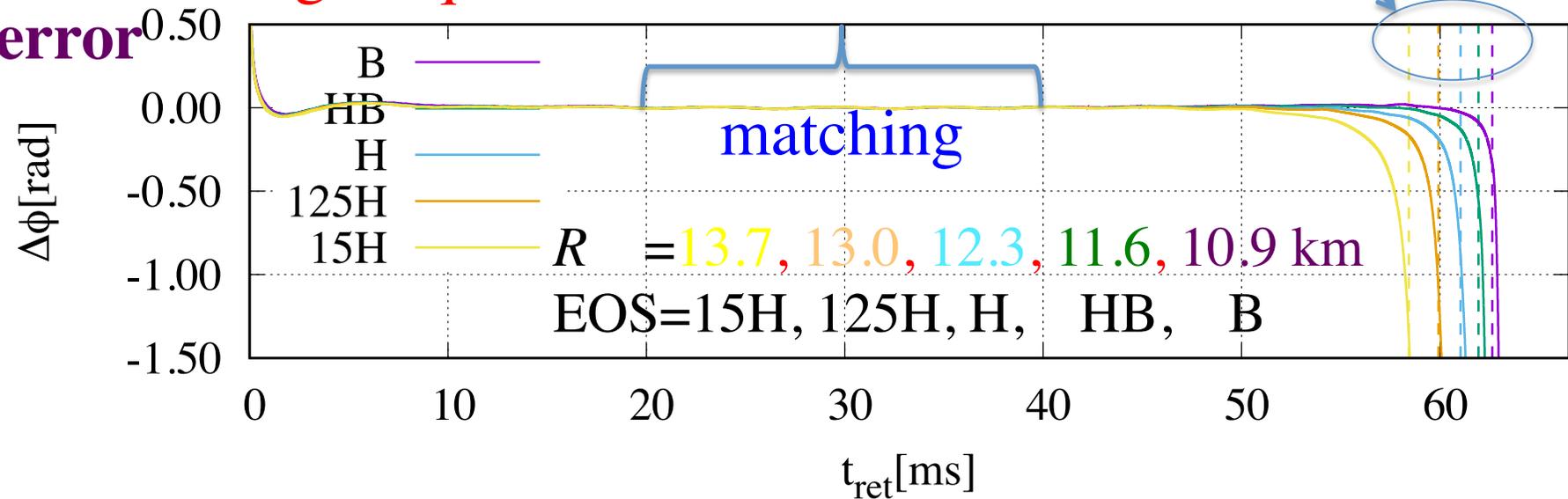
Amplitude

Figure by K. Kawaguchi

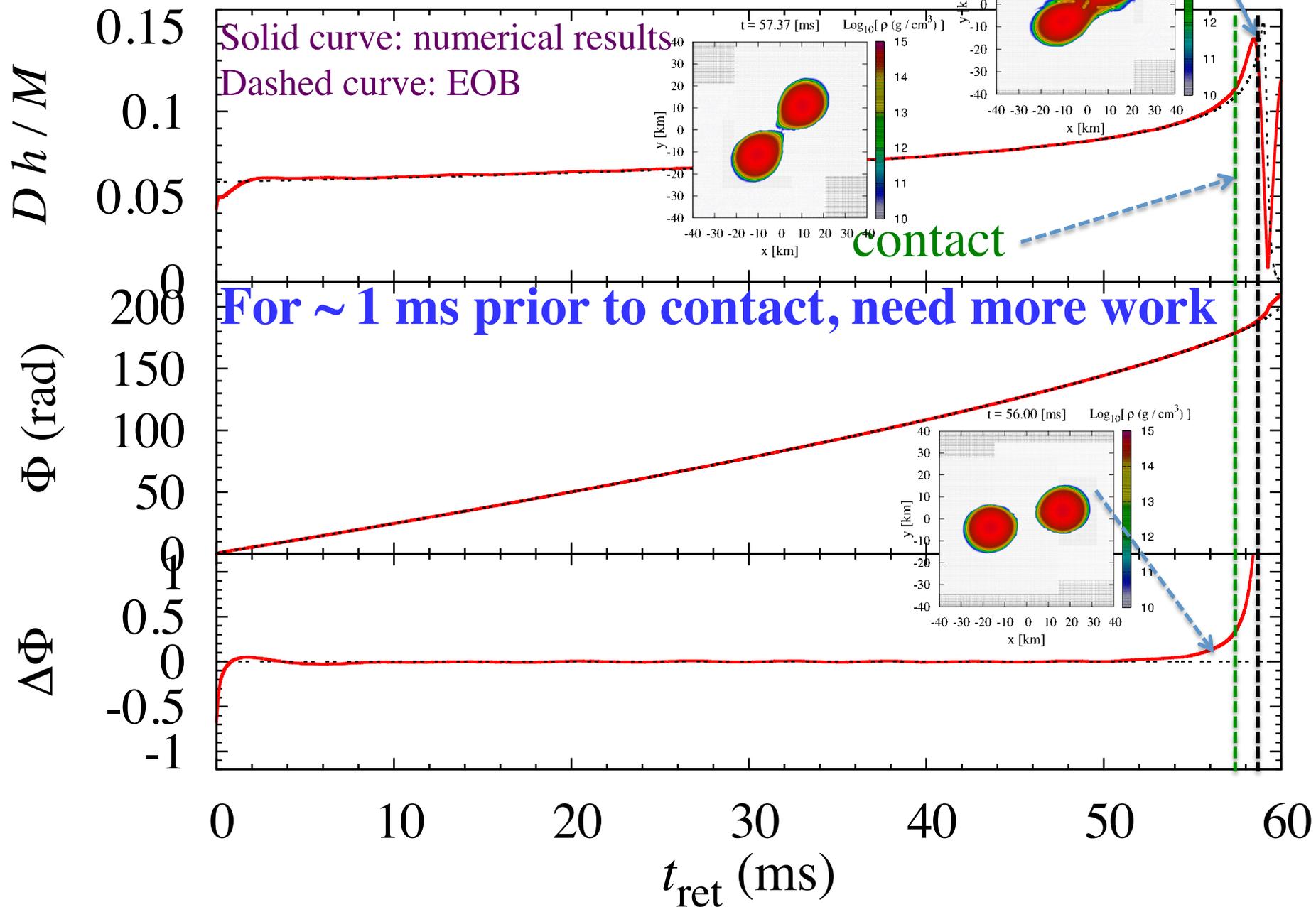


Phase error

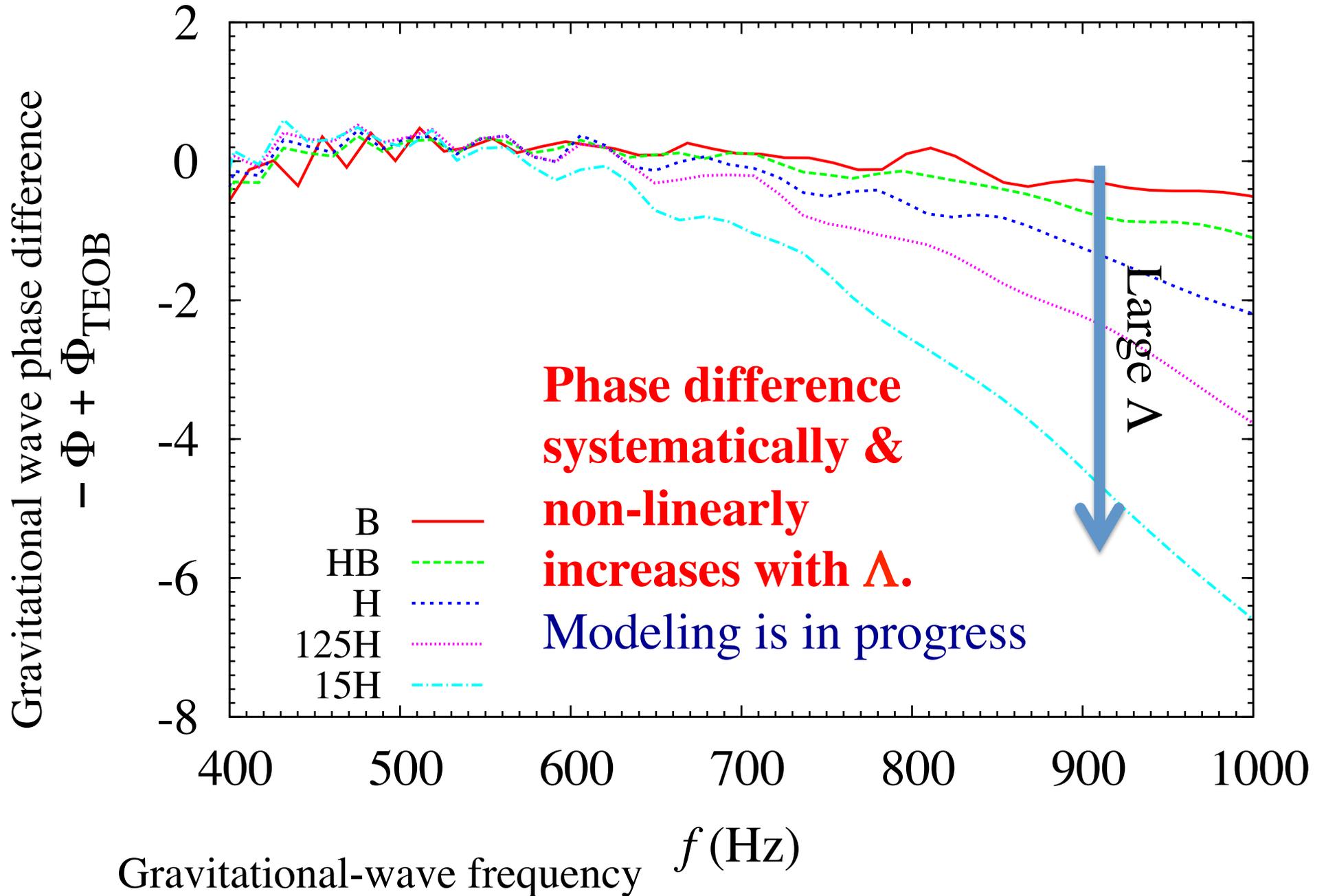
Agree quite well for small-radius neutron stars



# Comparison for $R=13.7$ km case



# Phase difference between NR and TEOB



# EOB is promising but need improvement

- For soft EOS, it works well up to amplitude peak
  - For stiff EOS, it works well approximately  
up to  $\sim 1\text{m}$  prior to contact of two NSs
  - After the contact, inspiral-like waveform continues for a few ms, in particular for stiff EOS (large-radius NSs)
  - ✓ Modeling for such final phase would be the final piece
- ✧ Note: In the final phase, the dependence on the tidal deformability becomes most remarkable.

# 5 Summary

- **Detecting late-inspiral gravitational waves** from NS-NS will constrain EOS for  $D_{\text{eff}} < \sim 200\text{Mpc}$  or by stacking:  
The GW frequency is  $\sim$  a few 100—1 kHz.
- **Post-Merger waveforms** for NS-NS may reflect the EOS of NS: **But** SNR would be too low for advanced detectors.
- ✧ **Physical consideration suggests that post-merger GWs could be even weaker: We should consider systematics**
- **Gravitational waves at tidal disruption of NS** in BH-NS will reflect the EOS of NS:  $f \sim 1\text{—}2$  kHz:  
Rapidly spinning BH will be needed for the detection
- **Further high-resolution run is the key** for getting reliable prediction of late inspiraling gravitational waves