Two-body problem in scalar-tensor theories, an effective-one-body approach

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Motivations

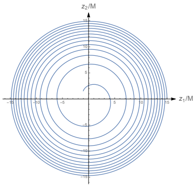
• GW150914 : The very first observation of a BBH coalescence by LIGO-Virgo has opened **a new era in gravitational wave astronomy.**

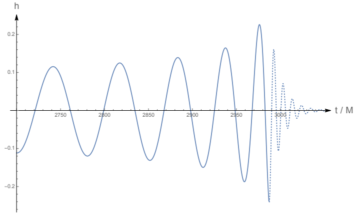
• Opportunity to bring **new tests of modified gravities**, in the strong-field regime near merger.

• EOB is a powerful approach to describe **analytically** the coalescence of 2 compact objects in **General Relativity**, from inspiral **to merger**.

$$H(Q,P)\ , \quad \epsilon = \left(\frac{v}{c}\right)^2 \qquad \longrightarrow \qquad H_e(q,p)\ , \quad ds_e^2 = g^e_{\mu\nu}dx^\mu dx^\nu$$

$$\boxed{H_e = f_{\mathrm{EOB}}(H)}$$





• Instrumental to build libraries of waveform templates for LIGO-VIRGO

[arXiv:1703.05360]

- Can we extend the EOB approach to modified gravities ?
- Consider the simplest and most studied example of **massless scalar-tensor theories**.

- First buiding block : map the conservative part of the two-body dynamics onto the geodesic of an effective metric.

- ST-extension of [Buonanno-Damour 98]

We adopt the conventions of Damour and Esposito-Farèse [DEF 92, 95]

ST action in the Einstein-frame ($G_* \equiv c \equiv 1$)

$$S_{EF} = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\right) + S_m\left[\Psi, \mathcal{A}^2(\varphi)g_{\mu\nu}\right]$$

Skeletonization of compact bodies :

$$S_m = -\sum_A \int d\lambda \sqrt{-g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}}\, m_A(\varphi)$$

$m_A(\varphi)$ depends on the theory $\mathcal{A}(\varphi)$ and on the EOS of body A.

$\rightarrow$ SEP violation [Eardley 75, DEF 92]

Our starting point : what is known today

Two-body Scalar-Tensor Lagrangian [DEF 93][Mirshekari, Will 13]

- Harmonic coordinates $\partial_\mu(\sqrt{-g}g^{\mu\nu}) = 0$
- conservative 2PK dynamics : $\mathcal{O}((\frac{v}{c})^4) \sim \mathcal{O}((\frac{m}{r})^2)$ corrections to Kepler
- Weak field expansion

$$\begin{aligned} g_{\mu\nu} &= \eta_{\mu\nu} + \delta g_{\mu\nu} \\ \varphi &= \varphi_0 + \delta\varphi \end{aligned}$$

- the fundamental functions $m_A(\varphi)$ and $m_B(\varphi)$ are expanded around φ_0 :

$$\begin{aligned} \ln m_A(\varphi) &\equiv \ln m_A^0 + \alpha_A^0(\varphi - \varphi_0) + \beta_A^0(\varphi - \varphi_0)^2 + {\beta'}_A^0(\varphi - \varphi_0)^3 + \cdots \\ \ln m_B(\varphi) &\equiv \ln m_B^0 + \alpha_B^0(\varphi - \varphi_0) + \beta_B^0(\varphi - \varphi_0)^2 + {\beta'}_B^0(\varphi - \varphi_0)^3 + \cdots \end{aligned}$$

i.e. the 2PK Lagrangian depends on 8 fundamental **parameters**.

The two-body Lagrangian

Two-body 2PK Lagrangian

$$L = -m_A^0 - m_B^0 + L_{\rm K} + L_{\rm 1PK} + L_{\rm 2PK} + \cdots$$

$$\vec{N} \equiv \frac{\vec{Z}_A - \vec{Z}_B}{R}, \quad \vec{V}_A \equiv \frac{d\vec{Z}_A}{dt}, \quad R \equiv |\, \vec{Z}_A - \vec{Z}_B \,|, \quad \vec{A}_A \equiv \frac{d\vec{V}_A}{dt}$$

- Keplerian order :

$$\boxed{L_{\rm K} = \frac{1}{2} m_A^0 V_A^2 + \frac{1}{2} m_B^0 V_B^2 + \frac{G_{AB} m_A^0 m_B^0}{R}} \qquad \text{where} \qquad G_{AB} \equiv 1 + \alpha_A^0 \alpha_B^0$$

- post-Keplerian (1PK) :

$$\begin{aligned} L_{\rm 1PK} &= \frac{1}{8} m_A^0 V_A^4 + \frac{1}{8} m_B^0 V_B^4 \\ &+ \frac{G_{AB} m_A^0 m_B^0}{R} \left(\frac{3}{2}(V_A^2 + V_B^2) - \frac{7}{2} \vec{V}_A \cdot \vec{V}_B - \frac{1}{2}(\vec{N} \cdot \vec{V}_A)(\vec{N} \cdot \vec{V}_B) + \bar{\gamma}_{AB} (\vec{V}_A - \vec{V}_B)^2 \right) \\ &- \frac{G_{AB}^2 m_A^0 m_B^0}{2R^2} \left(m_A^0 (1 + 2\bar{\beta}_B) + m_B^0 (1 + 2\bar{\beta}_A) \right) \end{aligned}$$

$$\text{where} \quad \bar{\gamma}_{AB} \equiv -\frac{2\alpha_A^0 \alpha_B^0}{1 + \alpha_A^0 \alpha_B^0} \qquad \bar{\beta}_A \equiv \frac{1}{2} \frac{\beta_A^0 (\alpha_B^0)^2}{(1 + \alpha_A^0 \alpha_B^0)^2} \quad (A \leftrightarrow B)$$

- post-post-Keplerian (2PK) :

$$
\begin{aligned}
L_{\rm 2PK} = & \frac{1}{16} m_A^0 V_A^6 \\
& + \frac{G_{AB} m_A^0 m_B^0}{R} \Bigg[\frac{1}{8}(7+4\bar\gamma_{AB}) \left(V_A^4 - V_A^2 (\vec N \cdot \vec V_B)^2 \right) - (2+\bar\gamma_{AB}) V_A^2 (\vec V_A \cdot \vec V_B) + \frac{1}{8} (\vec V_A \cdot \vec V_B)^2 \\
& \qquad + \frac{1}{16}(15 + 8\bar\gamma_{AB}) V_A^2 V_B^2 + \frac{3}{16} (\vec N \cdot \vec V_A)^2 (\vec N \cdot \vec V_B)^2 + \frac{1}{4}(3+2\bar\gamma_{AB}) \vec V_A \cdot \vec V_B (\vec N \cdot \vec V_A)(\vec N \cdot \vec V_B) \Bigg] \\
& + \frac{G_{AB}^2 m_B^0 (m_A^0)^2}{R^2} \Bigg[\frac{1}{8} \left(2 + 12\bar\gamma_{AB} + 7\bar\gamma_{AB}^2 + 8\bar\beta_B - 4\delta_A \right) V_A^2 + \frac{1}{8} \left(14 + 20\bar\gamma_{AB} + 7\bar\gamma_{AB}^2 + 4\bar\beta_B - 4\delta_A \right) V_B^2 \\
& \qquad - \frac{1}{4} \left(7 + 16\bar\gamma_{AB} + 7\bar\gamma_{AB}^2 + 4\bar\beta_B - 4\delta_A \right) \vec V_A \cdot \vec V_B - \frac{1}{4} \left(14 + 12\bar\gamma_{AB} + \bar\gamma_{AB}^2 - 8\bar\beta_B + 4\delta_A \right) (\vec V_A \cdot \vec N)(\vec V_B \cdot \vec N) \\
& \qquad + \frac{1}{8} \left(28 + 20\bar\gamma_{AB} + \bar\gamma_{AB}^2 - 8\bar\beta_B + 4\delta_A \right) (\vec N \cdot \vec V_A)^2 + \frac{1}{8} \left(4 + 4\bar\gamma_{AB} + \bar\gamma_{AB}^2 + 4\delta_A \right) (\vec N \cdot \vec V_B)^2 \Bigg] \\
& + \frac{G_{AB}^3 (m_A^0)^3 m_B^0}{2R^3} \left[1 + \frac{2}{3}\bar\gamma_{AB} + \frac{1}{6}\bar\gamma_{AB}^2 + 2\bar\beta_B + \frac{2}{3}\delta_A + \frac{1}{3}\epsilon_B \right] + \frac{G_{AB}^3 (m_A^0)^2 (m_B^0)^2}{8R^3} \left[19 + 8\bar\gamma_{AB} + 8(\bar\beta_A + \bar\beta_B) + 4\zeta \right] \\
& - \frac{1}{8} G_{AB} m_A^0 m_B^0 \left(2(7+4\bar\gamma_{AB}) \vec A_A \cdot \vec V_B (\vec N \cdot \vec V_B) + \vec N \cdot \vec A_A (\vec N \cdot \vec V_B)^2 - (7+4\bar\gamma_{AB}) \vec N \cdot \vec A_A V_B^2 \right) \\
& \qquad + (A \leftrightarrow B)
\end{aligned}
$$

where $\delta_A \equiv \frac{(\alpha_A^0)^2}{(1+\alpha_A^0\alpha_B^0)^2}$ $\quad \epsilon_A \equiv \frac{(\beta'_A \alpha_B^3)^0}{(1+\alpha_A^0\alpha_B^0)^3}$ $\quad \zeta \equiv \frac{\beta_A^0 \alpha_A^0 \alpha_B^0 \beta_B^0}{(1+\alpha_A^0\alpha_B^0)^3}$ $(A \leftrightarrow B)$

The centre-of-mass two-body 2PK Hamiltonians

In the centre-of-mass frame : $\boxed{\vec{P}_A + \vec{P}_B \equiv \vec{0}}$

17 coefficients

$$H = M + \left(\frac{P^2}{2\mu} - \mu\frac{G_{AB}M}{R}\right) + H^{\rm 1PK} + H^{\rm 2PK} + \cdots$$

- $\dfrac{H^{\rm 1PK}}{\mu} = \left(h_1^{\rm 1PK}\hat{P}^4 + h_2^{\rm 1PK}\hat{P}^2\hat{P}_R^2 + h_3^{\rm 1PK}\hat{P}_R^4\right) + \dfrac{1}{\hat{R}}\left(h_4^{\rm 1PK}\hat{P}^2 + h_5^{\rm 1PK}\hat{P}_R^2\right) + \dfrac{h_6^{\rm 1PK}}{\hat{R}^2}$
- $\dfrac{H^{\rm 2PK}}{\mu} = \left(h_1^{\rm 2PK}\hat{P}^6 + h_2^{\rm 2PK}\hat{P}^4\hat{P}_R^2 + h_3^{\rm 2PK}\hat{P}^2\hat{P}_R^4 + h_4^{\rm 2PK}\hat{P}_R^6\right)$

$$+ \frac{1}{\hat{R}}\left(h_5^{\rm 2PK}\hat{P}^4 + h_6^{\rm 2PK}\hat{P}_R^2\hat{P}^2 + h_7^{\rm 2PK}\hat{P}_R^4\right) + \frac{1}{\hat{R}^2}\left(h_8^{\rm 2PK}\hat{P}^2 + h_9^{\rm 2PK}\hat{P}_R^2\right) + \frac{h_{10}^{\rm 2PK}}{\hat{R}^3}$$

The 17 $h_i^{\rm NPK}$ coefficients are computed explicitly and depend on :

- coordinate system
- the 8 fundamental parameters built from $m_A(\varphi)$ and $m_B(\varphi)$

Geodesic motion in a static, spherically symmetric metric

In Schwarzschild-Droste coordinates (equatorial plane $\theta = \pi/2$) :

$$\boxed{ds_e^2 = -A(r)dt^2 + B(r)dr^2 + r^2 d\phi^2}$$

$A(r)$ and $B(r)$ are arbitrary.

Effective Hamiltonian $H_e(q, p)$:

$$H_e(q,p) = \sqrt{A\left(\mu^2 + \frac{p_r^2}{B} + \frac{p_\phi^2}{r^2}\right)} \quad \text{with} \quad p_r \equiv \frac{\partial L_e}{\partial \dot{r}} \quad , \quad p_\phi \equiv \frac{\partial L_e}{\partial \dot{\phi}}$$

Can be **expanded** :

$$\boxed{\begin{aligned} A(r) &= 1 + \frac{a_1}{r} + \frac{a_2}{r^2} + \frac{a_3}{r^3} + \cdots \\ B(r) &= 1 + \frac{b_1}{r} + \frac{b_2}{r^2} + \cdots \end{aligned}}$$

i.e. depends on **5 effective parameters** at 2PK order, to be determined.

1) Use of a canonical transformation :

$$H(Q,P) \to H(q,p)$$

Generic ansatz $G(Q,p)$ that depends on 9 parameters at 2PK order :

$$G(Q,p) = R\,p_r\left[\left(\alpha_1\mathcal{P}^2 + \beta_1\hat{p}_r^2 + \frac{\gamma_1}{\hat{R}}\right) + \left(\alpha_2\mathcal{P}^4 + \beta_2\mathcal{P}^2\hat{p}_r^2 + \gamma_2\hat{p}_r^4 + \delta_2\frac{\mathcal{P}^2}{\hat{R}} + \epsilon_2\frac{\hat{p}_r^2}{\hat{R}} + \frac{\eta_2}{\hat{R}^2}\right) + \cdots\right]$$

2) Relate H to H_e through the quadratic relation [Damour 2016]

$$\frac{H_e(q,p)}{\mu} - 1 = \left(\frac{H(q,p)-M}{\mu}\right)\left[1 + \frac{\nu}{2}\left(\frac{H(q,p)-M}{\mu}\right)\right]$$

where $\nu = \frac{m_A^0 m_B^0}{(m_A^0+m_B^0)^2}$, $M = m_A^0 + m_B^0$, $\mu = \frac{m_A^0 m_B^0}{M}$

$$\frac{H_e(q,p)}{\mu} - 1 = \left(\frac{H(q,p)-M}{\mu}\right)\left[1 + \frac{\nu}{2}\left(\frac{H(q,p)-M}{\mu}\right)\right]$$

- H_e depends on 5 parameters

$$A(r) = 1 + \frac{a_1}{r} + \frac{a_2}{r^2} + \frac{a_3}{r^3} + \cdots \ , \quad B(r) = 1 + \frac{b_1}{r} + \frac{b_2}{r^2} + \cdots$$

- H depends on 17 coefficients (h_i^{NPK}) ;
- The canonical transformation depends on 9 parameters (α_i, β_i,...) ;

$$\boxed{17 = 9 + 5 + 3}$$

Hence, 3 constraints on the h_i^{NPK} coefficients of the two-body Hamiltonian.

$\rightarrow$ **The two-body problem can be mapped towards a geodesic only for a subclass of theories** (exclude Lorentz-violating, electrodynamics,...)

$$ds_e^2 = -A(r)dt + B(r)dr^2 + r^2 d\theta^2$$

Yields a **unique** solution in scalar-tensor theories (coordinate-independent)

Scalar-Tensor effective metric

$$A(r) = 1 - 2\left(\frac{G_{AB}M}{r}\right) + 2\left[\langle\bar{\beta}\rangle - \bar{\gamma}_{AB}\right]\left(\frac{G_{AB}M}{r}\right)^2 + \left[2\nu + \delta a_3^{\mathrm{ST}}\right]\left(\frac{G_{AB}M}{r}\right)^3 + \cdots$$

$$B(r) = 1 + 2\left[1 + \bar{\gamma}_{AB}\right]\left(\frac{G_{AB}M}{r}\right) + \left[2(2-3\nu) + \delta b_2^{\mathrm{ST}}\right]\left(\frac{G_{AB}M}{r}\right)^2 + \cdots$$

Reduces to GR when $m_A(\varphi) = cst$

General Relativity 2PN effective metric [Buonanno, Damour 98]

$$A_{\mathrm{GR}}(r) = 1 - 2\left(\frac{G_* M}{r}\right) + 2\nu\left(\frac{G_* M}{r}\right)^3 + \cdots$$

$$B_{\mathrm{GR}}(r) = 1 + 2\left(\frac{G_* M}{r}\right) + 2(2-3\nu)\left(\frac{G_* M}{r}\right)^2 + \cdots$$

(i) The "bare" gravitational constant G_* is replaced by the effective one

$$\boxed{G_* \rightarrow G_{AB} \equiv 1 + \alpha_A^0 \alpha_B^0}$$

at all orders.

(ii) At 1PK level,

$$A(r) = 1 - 2\left(\frac{G_{AB}M}{r}\right) + 2\left[\langle\bar{\beta}\rangle - \bar{\gamma}_{AB}\right]\left(\frac{G_{AB}M}{r}\right)^2 + \cdots$$
$$B(r) = 1 + 2\left[1 + \bar{\gamma}_{AB}\right]\left(\frac{G_{AB}M}{r}\right) + \cdots$$

one recognizes the **PPN Eddington metric** written in Droste coordinates, with :

$$\boxed{\beta^{\mathrm{Edd}} = 1 + \langle\bar{\beta}\rangle \ , \quad \gamma^{\mathrm{Edd}} = 1 + \bar{\gamma}_{AB}}$$

Where

$$\langle\bar{\beta}\rangle \equiv \frac{m_A^0 \bar{\beta}_B^0 + m_B^0 \bar{\beta}_A^0}{m_A^0 + m_B^0} \qquad \bar{\gamma}_{AB} \equiv -\frac{2\alpha_A^0\alpha_B^0}{1 + \alpha_A^0\alpha_B^0} \qquad \bar{\beta}_A \equiv \frac{1}{2}\frac{\beta_A^0(\alpha_B^0)^2}{(1 + \alpha_A^0\alpha_B^0)^2}$$

Scalar-Tensor effective metric

$$A(r) = 1 - 2\left(\frac{G_{AB}M}{r}\right) + 2\left[\langle\bar{\beta}\rangle - \bar{\gamma}_{AB}\right]\left(\frac{G_{AB}M}{r}\right)^2 + \left[2\nu + \delta a_3^{\mathrm{ST}}\right]\left(\frac{G_{AB}M}{r}\right)^3 + \cdots$$

$$B(r) = 1 + 2\left[1 + \bar{\gamma}_{AB}\right]\left(\frac{G_{AB}M}{r}\right) + \left[2(2-3\nu) + \delta b_2^{\mathrm{ST}}\right]\left(\frac{G_{AB}M}{r}\right)^2 + \cdots$$

(iii) 2PK corrections

$$\delta a_3^{\mathrm{ST}} \equiv \frac{1}{12}\Big[-20\bar{\gamma}_{AB} - 35\bar{\gamma}_{AB}^2 - 24\langle\bar{\beta}\rangle(1 - 2\bar{\gamma}_{AB}) + 4\big(\langle\delta\rangle - \langle\epsilon\rangle\big)$$
$$+ \nu\Big(-36(\bar{\beta}_A + \bar{\beta}_B) + 4\bar{\gamma}_{AB}(10 + \bar{\gamma}_{AB}) + 4(\epsilon_A + \epsilon_B) + 8(\delta_A + \delta_B) - 24\zeta\Big)\Big]$$

$$\delta b_2^{\mathrm{ST}} \equiv \left[4\langle\bar{\beta}\rangle - \langle\delta\rangle + \bar{\gamma}_{AB}(9 + \frac{19}{4}\bar{\gamma}_{AB}) + \nu\left(2\langle\bar{\beta}\rangle - 4\bar{\gamma}_{AB}\right)\right]$$

$$\delta_A \equiv \frac{(\alpha_A^0)^2}{(1+\alpha_A^0\alpha_B^0)^2} \quad \epsilon_A \equiv \frac{(\beta'_A\alpha_B^3)^0}{(1+\alpha_A^0\alpha_B^0)^3} \quad \zeta \equiv \frac{\beta_A^0\alpha_A^0\alpha_B^0\beta_B^0}{(1+\alpha_A^0\alpha_B^0)^3}$$

- The inversion of $H_e = f_{\rm EOB}(H)$ defines a "resummed" EOB Hamiltonian :

$$H_{\rm EOB} = M\sqrt{1+2\nu\left(\frac{H_e}{\mu}-1\right)} \quad \text{where} \quad H_e = \sqrt{A\left(\mu^2+\frac{p_r^2}{B}+\frac{p_\phi^2}{r^2}\right)}$$

The dynamics deduced from $H_{\rm EOB}$ and the "real" Hamiltonians H are, by construction, equivalent up to 2PK order.

- $H_{\rm EOB}$ hence defines a resummed dynamics, that may capture some features of the strong field regime.

A typical strong-field feature : ST corrections to orbital frequency at the ISCO (equal-mass case : $\nu = 1/4$)

Last ingredient : the ST-corrected $A(u\,;\nu)$

$$u \equiv \frac{G_{AB} M}{r}\,, \quad \nu = \frac{m_A^0 m_B^0}{(m_A^0 + m_B^0)^2}$$

ST-corrected $A(u\,;\nu)$

$$A(u\,;\nu) = A^{\rm GR}_{\rm 2PN}(u\,;\nu) + 2\epsilon_{\rm 1PK}\, u^2 + (\epsilon^0_{\rm 2PK} + \nu\epsilon^{\nu}_{\rm 2PK})u^3$$

where

$$\epsilon_{\rm 1PK} \equiv \langle\bar{\beta}\rangle - \bar{\gamma}_{AB}$$

$$\epsilon^0_{\rm 2PK} \equiv \frac{1}{12}\Big[-20\bar{\gamma}_{AB} - 35\bar{\gamma}^2_{AB} - 24\langle\bar{\beta}\rangle(1-2\bar{\gamma}_{AB}) + 4\big(\langle\delta\rangle - \langle\epsilon\rangle\big)\Big]$$

$$\epsilon^{\nu}_{\rm 2PK} \equiv -3(\bar{\beta}_A + \bar{\beta}_B) + \frac{1}{3}\bar{\gamma}_{AB}(10+\bar{\gamma}_{AB}) + \frac{1}{3}(\epsilon_A + \epsilon_B) + \frac{2}{3}(\delta_A + \delta_B) - 2\zeta$$

ST-Corrections described by 3 parameters, $(\epsilon_{\rm 1PK}, \epsilon^0_{\rm 2PK}, \epsilon^{\nu}_{\rm 2PK})$

- **BUT** numerically driven by $(\alpha_A^0)^2$ (c.f. DEF, diagrammatic methods)
- When $(\alpha_A^0)^2 << 1$, $\epsilon_{\rm 1PK} \sim \epsilon^0_{\rm 2PK} \sim \epsilon^{\nu}_{\rm 2PK}$ and ST-corrections are perturbative

In this perturbative approach, **best available EOB-NR function** for GR :

$$A^{\mathrm{GR}}_{\mathrm{2PN}}(u\,;\nu) \rightarrow \boxed{A^{\mathrm{GR}}_{\mathrm{EOBNR}}(u\,;\nu) = \mathcal{P}^1_5[A^{Taylor}_{\mathrm{5PN}}]}$$

i.e. the $(1,5)$ Padé approximant of the truncated 5PN expansion :

$$A^{Taylor}_{\mathrm{5PN}} = 1 - 2u + 2\nu u^3 + \nu a_4 u^4 + (a_5^c + a_5^{\ln} \ln u)u^5 + \nu(a_6^c + a_6^{\ln} \ln u)u^6$$

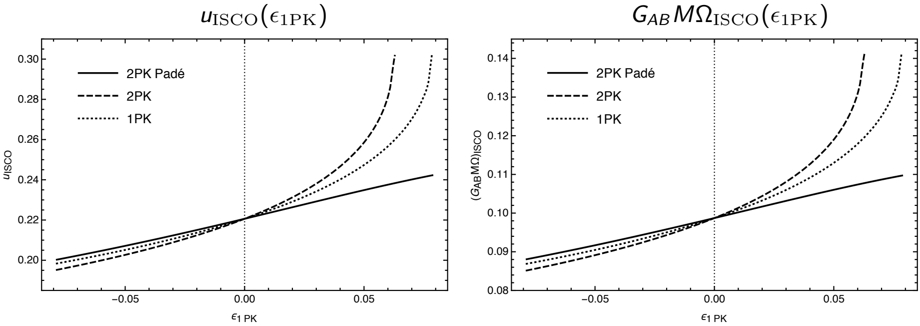
[Damour, Nagar, Reisswig, Pollney 2016]

- smoothly connected to Schwarzschild when $\nu \rightarrow 0$
- $a_6^c(\nu)$ is obtained by calibration with Numerical Relativity

A typical strong-field feature : orbital frequency at the ISCO, equal-mass case ($\nu = 1/4$), setting $\epsilon_{1\mathrm{PK}} \equiv \epsilon^0_{2\mathrm{PK}} \equiv \epsilon^\nu_{2\mathrm{PK}}$

- 2PK Padeed corrections,

$$A = \mathcal{P}^1_5[A^{\mathrm{GR}}_{\mathrm{EOBNR}}(u\,;\nu) + 2\epsilon_{1\mathrm{PK}}u^2 + (\epsilon^0_{2\mathrm{PK}} + \nu\epsilon^\nu_{2\mathrm{PK}})u^3]$$



$$\left.\frac{d(G_{AB}M\Omega)_{ISCO}}{d\epsilon_{1\mathrm{PK}}}\right|_{\nu=1/4} \simeq 0.13$$

relative correction to GR significant ($\sim 10\%$) when $\epsilon_{1\mathrm{PK}} \sim 10^{-2} - 10^{-1}$

One-body problem in scalar-tensor theories ?

SSS metric and scalar field in Just coordinates

$$ds_*^2 = -D_* dt^2 + \frac{d\rho^2}{D_*} + C_* \rho^2 (d\theta^2 + \sin^2\theta \, d\phi^2)$$

$$\text{with} \quad D_*(\rho) = \left(1 - \frac{\mathfrak{a}_*}{\rho}\right)^{\frac{\mathfrak{b}_*}{\mathfrak{a}_*}}, \quad C_*(\rho) = \left(1 - \frac{\mathfrak{a}_*}{\rho}\right)^{1-\frac{\mathfrak{b}_*}{\mathfrak{a}_*}}$$

$$\varphi_*(\rho) = \frac{\mathfrak{q}_*}{\mathfrak{a}_*} \ln(1 - \frac{\mathfrak{a}_*}{\rho}) + \varphi_0$$

and where

$$\mathfrak{a}_*^2 - \mathfrak{b}_*^2 = 4\mathfrak{q}_*^2$$

- **Test particle** $m_*(\varphi)$ orbiting around a central body in scalar-tensor theories :

$$L_* = -m_*(\varphi_*)\sqrt{-g^*_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}}$$

One-body Hamiltonian

$$H_* = m_*^0\sqrt{V_* D_* + D_*^2 \hat{p}_\rho^2 + \frac{D_*}{C_*}\frac{\hat{p}_\phi^2}{\hat{\rho}^2}}$$

$$V_* = \left(\frac{m_*(\varphi)}{m_*^0}\right)^2 \quad , \quad D_*(\rho) = \left(1 - \frac{\mathfrak{a}_*}{\rho}\right)^{\frac{\mathfrak{b}_*}{\mathfrak{a}_*}} , \quad C_*(\rho) = \left(1 - \frac{\mathfrak{a}_*}{\rho}\right)^{1-\frac{\mathfrak{b}_*}{\mathfrak{a}_*}}$$

Effective Hamiltonian

$$\hat{H}_e \equiv \frac{H_e}{\mu} = \sqrt{V_e D_e + D_e^2 \hat{p}_\rho^2 + \frac{D_e}{C_e}\frac{\hat{p}_\phi^2}{\hat{\rho}^2}}$$

$$D_e(\rho) \equiv \left(1 - \frac{a}{\hat{\rho}}\right)^{\frac{b}{a}}, \quad C_e(\rho) \equiv \left(1 - \frac{a}{\hat{\rho}}\right)^{1-\frac{b}{a}}, \quad V_e(\rho) = 1 + \frac{v_1}{\hat{\rho}} + \frac{v_2}{\hat{\rho}^2} + \frac{v_3}{\hat{\rho}^3} + \cdots$$

i.e. **5 effective parameters** at 2PK order (a, b, v_1, v_2, v_3), that are **uniquely fixed** by the EOB mapping.

$$a = 2\mathcal{R}$$

$$b = 2$$

$$v_1 = 2(1 - G_{AB})$$

$$v_2 = 2 - 4G_{AB} + 2G_{AB}^2\left(1 + \langle\bar{\beta}\rangle\right) + (1 - G_{AB})2\mathcal{R}$$

$$\frac{v_3}{4} = 1 - \frac{5}{3}G_{AB} + \left(1 + \langle\bar{\beta}\rangle + \frac{2}{3}\langle\delta\rangle\right)G_{AB}^2 - \frac{1}{3}\left(1 + 3\langle\bar{\beta}\rangle + \frac{1}{4}\langle\epsilon\rangle + 2\langle\delta\rangle\right)G_{AB}^3 + \left(1 - 2G_{AB} + G_{AB}^2\left(1 + \langle\bar{\beta}\rangle\right)\right)\mathcal{R}$$
$$+ \nu\left[\frac{17}{3}G_{AB} - \frac{1}{3}\left(19 + 4\langle\bar{\beta}\rangle + 6\zeta\right)G_{AB}^2 + \left(\frac{2}{3} - \frac{3}{4}(\bar{\beta}_A + \bar{\beta}_B) + \frac{1}{12}(\epsilon_A + \epsilon_B) + \frac{1}{6}(\delta_A + \delta_B) + \frac{3}{2}\langle\bar{\beta}\rangle\right)G_{AB}^3\right]$$

$$\mathcal{R} \equiv \sqrt{1 + G_{AB}^2\langle\delta\rangle + \nu\left[8G_{AB} - 2G_{AB}^2\left(1 + \langle\bar{\beta}\rangle\right)\right]}$$

Test-mass limit ($\nu \to 0$)

The test-mass limit $\nu = 0$ gives back the ST one-body problem

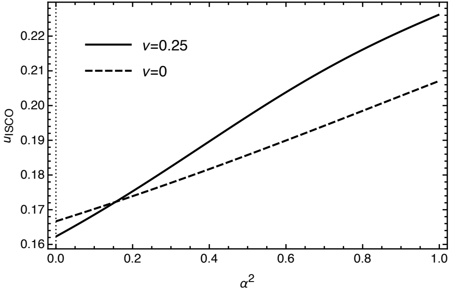
$$\ln M_*(\varphi) = \ln M_*^0 + A_*^0(\varphi - \varphi_0) + \cdots$$
$$\ln m_*(\varphi) = \ln m_*^0 + \alpha_*^0(\varphi - \varphi_0) + \beta_*^0(\varphi - \varphi_0)^2 + {\beta'}_*^0(\varphi - \varphi_0)^3 \cdots$$

$$M_*^0 = m_A^0 + m_B^0 = M$$
$${A_*^0}^2 = \frac{m_A^0(\alpha_A^0)^2 + m_B^0(\alpha_B^0)^2}{m_A^0 + m_B^0}$$
$$m_*^0 = \frac{m_A^0 m_B^0}{m_A^0 + m_B^0} = \mu$$
$$\alpha_*^0 = \frac{\alpha_A^0 \alpha_B^0}{A_*^0}$$
$$\beta_*^0 = \frac{(m_A\alpha_A^2)^0\beta_B^0 + (m_B\alpha_B^2)^0\beta_A^0}{(m_A\alpha_A^2)^0 + (m_B\alpha_B^2)^0}$$
$${\beta'}_*^0 = \frac{(m_A\alpha_A^3)^0{\beta'}_B^0 + (m_B\alpha_B^3)^0{\beta'}_A^0}{(m_A^0 + m_B^0){A_*^0}^3}$$

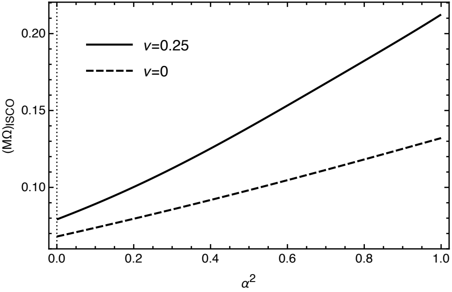
Strong-field feature : ISCO location and frequency

- $\mathcal{A}(\varphi) = e^{\alpha\varphi}, \quad \alpha = cst = 1/\sqrt{3+2\omega}$
- negligible self-gravity limit : $m_A(\varphi) = m_A^0 e^{\alpha(\varphi-\varphi_0)}$

$u_{\rm ISCO}(\alpha^2)$



$M\Omega_{\rm ISCO}(\alpha^2)$



$$\left.\frac{d(M\Omega)_{ISCO}}{d(\alpha^2)}\right|_{\nu=1/4} \simeq 0.13$$

- Consistent with the GR-centered approach, but **valid in regimes that depart strongly from GR** ($\alpha^2 \sim 1$)

Concluding remarks :

• Remarkably, the EOB approach is valid beyond the scope of General Relativity. In **Scalar-Tensor theories :**

$$A^{\rm 2PK}(u) \equiv \mathcal{P}^1_5[A^{Taylor}_{\rm 5PN} + 2\epsilon_{\rm 1PK}\, u^2 + (\epsilon^0_{\rm 2PK} + \nu\, \epsilon^\nu_{\rm 2PK})u^3]$$

• But also applicable for **any theory** whose coefficients $h_i^{\rm NPK}$ satisfy the 3 mapping conditions.

• The Scalar-Tensor example suggests a generic 2PK ansatz

$$\boxed{A^{\rm PEOB}(u) \equiv \mathcal{P}^1_5[A^{Taylor}_{\rm 5PN} + 2(\epsilon^0_{\rm 1PK} + \nu\, \epsilon^\nu_{\rm 1PK})u^2 + (\epsilon^0_{\rm 2PK} + \nu\, \epsilon^\nu_{\rm 2PK})u^3]}$$

where $\epsilon^0_{\rm 1PK}$, $\epsilon^\nu_{\rm 1PK}$, $\epsilon^0_{\rm 2PK}$, and $\epsilon^\nu_{\rm 2PK}$ are theory-agnostic Parametrized EOB (PEOB) coefficients.

Concluding remarks :

• Binary pulsar experiments have put **stringent constraints on ST theories** (no dipolar radiation)

$$\boxed{(\alpha_A^0)^2 < 4 \times 10^{-6}}$$

For **any** body A, regardless of its EOS or self-gravity.

• The ISCO ST-correction (significant for $(\alpha_A^0)^2 \gtrsim 10^{-2}$) seems unlikely to improve binary pulsar constraints.

However :

• The interferometers LIGO-Virgo or even LISA are designed to detect highly redshifted sources. Cosmological history of ST theories ?

• Stars subject to dynamical scalarization can develop non perturbative $(\alpha_A^0)^2$ near merger [Barausse, Palenzuela, Ponce, Lehner 2013].
EOB well-suited to investigate this regime !

Ongoing work : ST corrections to the radiation reaction force

Known at 1.5PN and 2.5PN [Mirshekari, Will 13]

$$\vec{\mathcal{F}} = \frac{8}{5}\frac{(G_{AB}m_A^0 m_B^0)^2}{MR^3}\left[(\vec{N}.\vec{\mathcal{V}})\vec{N}(A_{1.5}+A_{2.5}) - \vec{\mathcal{V}}(B_{1.5}+B_{2.5})\right]$$

$$A_{1.5} = \frac{5}{8}\frac{(\alpha_A^0-\alpha_B^0)^2}{1+\alpha_A^0\alpha_B^0}\ , \quad A_{2.5} = a_1\mathcal{V}^2 + a_2\frac{G_{AB}M}{R} + a_3(\vec{N}.\vec{\mathcal{V}})^2$$

$$B_{1.5} = \frac{5}{24}\frac{(\alpha_A^0-\alpha_B^0)^2}{1+\alpha_A^0\alpha_B^0}\ , \quad B_{2.5} = b_1\mathcal{V}^2 + b_2\frac{G_{AB}M}{R} + b_3(\vec{N}.\vec{\mathcal{V}})^2$$