Measuring the peculiar acceleration of binary black holes with LISA

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Outline

- The expansion of the universe and the cosmic matter inhomogeneities affect the propagation of GWs
- We identify 3 redshift-dependent effects on the chirp signal:
  - *time variation of the background expansion of the universe*
  - *time variation of the gravitational potential at the GW source*
  - *time variation of the peculiar velocity of the GW source*
- These effects cause a phase drift during the in-spiral:
  - Not relevant for Earth-based detectors
  - Relevant for non-monochromatic LISA sources with many in-spiral cycles in band: *low chirp mass and* $\tau_c \sim \Delta t_{\rm obs}$
- The phase drift due to the peculiar acceleration dominates:
  - Can be used to discriminate between different BBH formation channels

Waveform for an unperturbed universe with constant z

Assuming a FRW metric, the waveform at the observer is

$$h_+(t_O) = \frac{4(G\mathcal{M}_c(z))^{\frac{5}{3}}}{d_L}(\pi f_O)^{\frac{2}{3}}\frac{1+\cos^2\imath}{2}\cos(\Phi_O)$$

time at the observer

luminosity distance

redshifted chirp mass

frequency at the observer

phase at the observer

Where the redshift <u>z is assumed to be constant</u> during the time of observation of the signal:

$$1+z = \frac{a_O}{a_S} \qquad f_O(\tau_O) = \frac{1}{\pi}\left(\frac{5}{256\tau_O}\right)^{3/8}(G\mathcal{M}_c)^{-5/8}$$

$$f_S = (1+z) f_O$$

$$\tau_O = (1+z)\tau_S \qquad \Phi_O(\tau_O) = -2\left(\frac{\tau_O}{5G\mathcal{M}_c}\right)^{5/8} + \Phi_i$$

Considering a varying redshift

Relax the assumption that the redshift is constant during the observational time of the GW signal

$$(1+z)\frac{d}{dt_O}\left[(1+z)f_O\right] = \frac{96}{5}\pi^{8/3}\left(\frac{GM_c}{c^3}\right)^{5/3}\left[(1+z)f_O\right]^{11/3}$$

Two main effects:

- the **background expansion** of the universe varies during the time of observation of the binary

  [Seto *et al* (2001), Takahashi & Nakamura (2005), Nishizawa *et al* (2012)]

- the **redshift perturbations** due to the distribution of matter between the GW source and the observer vary in time during the time of observation of the binary

  [Bonvin, Caprini, Sturani, NT, arXiv:1609.08093]

Considering a varying redshift: homogeneous universe

Background expansion:

[Seto *et al* (2001), Takahashi & Nakamura (2005), Nishizawa *et al* (2012)]

Homogeneous variation of the redshift:

$$1 + z(t) = \frac{a_O(t)}{a_S(t)} \simeq H_O \Delta t_O - H_S \Delta t_S + \mathcal{O}(\Delta t^2)$$

Solving eqs for GW frequency and phase yields:

$$f_O(\tau_O) = \frac{1}{\pi} \left(\frac{5}{256\, \tau_O} \right)^{\frac{3}{8}} (G\mathcal{M}_c(z))^{-\frac{5}{8}} \left(1 + \frac{3}{8} X(z) \tau_O \right)$$

$$\Phi_O(\tau_O) = -2 \left(\frac{\tau_O}{5 G\mathcal{M}_c(z)} \right)^{\frac{5}{8}} \left(1 - \frac{5}{8} X(z) \tau_O \right) + \Phi_c$$

$$X(z) \equiv \frac{1}{2} \left(H_0 - \frac{H_S(z)}{1+z} \right)$$

Considering a varying redshift: perturbed universe

Computing the redshift perturbations:

Consider scalar perturbations on FRW:

$$ds^2 = -(1 + 2\psi)\,dt^2 + a^2 (1 - 2\phi)\,\delta_{ij}\,dx^i dx^j$$

definition of the redshift $$1 + z = \frac{f_S}{f_O} = \frac{E_S}{E_O} = \frac{(k^\mu u_\mu)_S}{(k^\mu u_\mu)_O}$$

$$\frac{dk^\mu}{d\lambda} + \Gamma^\mu_{\alpha\beta} k^\alpha k^\beta = 0 \qquad\qquad u^\mu = \frac{1}{a}(1 - \psi, \mathbf{v})$$

GW wave-vector — four velocity at source and observer

Considering a varying redshift: perturbed universe

Computing the redshift perturbations:

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definition of the redshift $$1+z = \frac{f_S}{f_O} = \frac{E_S}{E_O} = \frac{(k^\mu u_\mu)_S}{(k^\mu u_\mu)_O}$$

$$1+z = \frac{a_O}{a_S}\left[1 + (\mathbf{v}_S - \mathbf{v}_O)\cdot\mathbf{n} + \psi_O - \psi_S - \int_{t_S}^{t_O} dt(\dot\phi + \dot\psi)\right]$$

- $\frac{a_O}{a_S}$ → background expansion
- $(\mathbf{v}_S - \mathbf{v}_O)\cdot\mathbf{n}$ → Doppler term
- $\psi_O - \psi_S$ → gravitational redshift
- $\int_{t_S}^{t_O} dt(\dot\phi + \dot\psi)$ → integrated Sachs Wolfe

Considering a varying redshift: perturbed equations

These effects introduce additional contributions in the frequency and the phase of the chirp signal with new time dependences

$$f(\tau_O) = \frac{1}{\pi}\left(\frac{5}{256\,\tau_O}\right)^{3/8}(G\mathcal{M}_c)^{-5/8}\left(1+\frac{3}{8}Y(z)\tau_O\right)$$

$$\Phi_O(\tau_O) = -2\left(\frac{\tau_O}{5G\mathcal{M}_c}\right)^{5/8}\left(1-\frac{5}{8}Y(z)\tau_O\right)+\Phi_i$$

$$Y(z) = \frac{1}{2}\left(H_0 - \frac{H_S}{1+\bar{z}}\right) + \frac{1}{2}\left[\frac{\dot{\mathbf{v}}_S\cdot\mathbf{n}}{1+\bar{z}} - \dot{\mathbf{v}}_O\cdot\mathbf{n} + \frac{\dot{\phi}_S}{1+\bar{z}} - \dot{\phi}_O\right]$$

variation of the cosmological expansion during observation time

acceleration of the binary and the observer during observation time

time variation of the potentials during observation time

Considering a varying redshift: perturbed equations

$$A(f) = \sqrt{\frac{5}{24\pi^{4/3}}} \frac{(G\mathcal{M}_c)^{5/6}}{d_L(z)} \frac{1}{f^{7/6}} \left[1 - \frac{5\,(G\mathcal{M}_c)^{-5/3}}{384\pi^{8/3}} \frac{Y(z)}{f^{8/3}}\right]$$

$$\Phi(f) = 2\pi f t_c - \frac{\pi}{4} - \Phi_c + \frac{3}{128} (\pi G\mathcal{M}_c)^{-5/3} \frac{1}{f^{5/3}} - \frac{25}{32768\,\pi} (\pi G\mathcal{M}_c)^{-10/3} \frac{Y(z)}{f^{13/3}}$$

Effective –4PN frequency dependence:
(but comparable to max ~2PN once its prefactor is taken into account)

- Frequency dependent shift during the in-spiral phase
- Need observation of many cycles to be relevant
- No application to Earth-based detectors (only few cycles)
- Relevant for slowly evolving LISA sources ($\sim 10^6$ cycles)

Estimate of the amplitude of $Y(z)$

$$Y(z) = \frac{1}{2}\left(H_0 - \frac{H_S}{1+\bar{z}}\right) + \frac{1}{2}\left[\frac{\dot{\mathbf{v}}_S \cdot \mathbf{n}}{1+\bar{z}} - \dot{\mathbf{v}}_O \cdot \mathbf{n} + \frac{\dot{\phi}_S}{1+\bar{z}} - \dot{\phi}_O\right]$$

variation of the cosmological expansion: depends only on the cosmology

peculiar acceleration of the binary: dominates over the effect due to cosmological expansion for realistic situations

subdominant $\dot{\phi} \sim H_0 \phi \sim 10^{-5} H_0$

accounted for by eLISA motion

Estimate of the amplitude of $Y(z)$

$$Y(z) = \frac{1}{2}\left(H_0 - \frac{H_S}{1+\bar{z}}\right) + \frac{1}{2}\left[\frac{\dot{\mathbf{v}}_S \cdot \mathbf{n}}{1+\bar{z}} - \cancel{\dot{\mathbf{v}}_O \cdot \mathbf{n}} + \cancel{\frac{\dot{\phi}_S}{1+\bar{z}}} - \cancel{\dot{\phi}_O}\right]$$

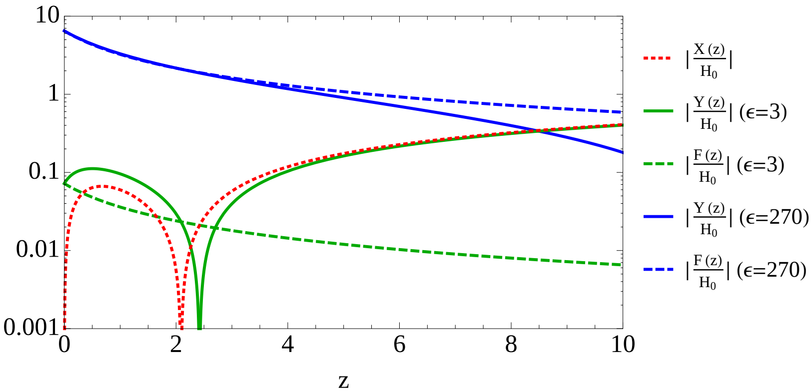
variation of the cosmological expansion: depends only on the cosmology

$$2.4 \times 10^{-2}\,\epsilon\,\frac{H_0}{1+\bar{z}}$$

$$\epsilon \equiv \left(\frac{v_{\rm s}}{100\ \rm kms}\right)^2 \left(\frac{10\ \rm kpc}{r}\right)(\hat{\rm e}\cdot\hat{\rm n})$$

- $v_{\rm s}$ is the CoM velocity of the binary
- r is the distance from the galactic center

Estimate of the amplitude of $Y(z)$



If ϵ is not negligible, then the contribution of peculiar accelerations dominates over the ones due to expansion of the universe, especially at low redshifts

Estimate of the phase shift due to $Y(z)$

$$\Phi_O(\tau_O) = -2\left(\frac{\tau_O}{5G\mathcal{M}_c(z)}\right)^{5/8}\left(1 - \frac{5}{8}\,Y(z)\tau_O\right) + \Phi_c$$

For a typical LISA binary observed up to coalescence:

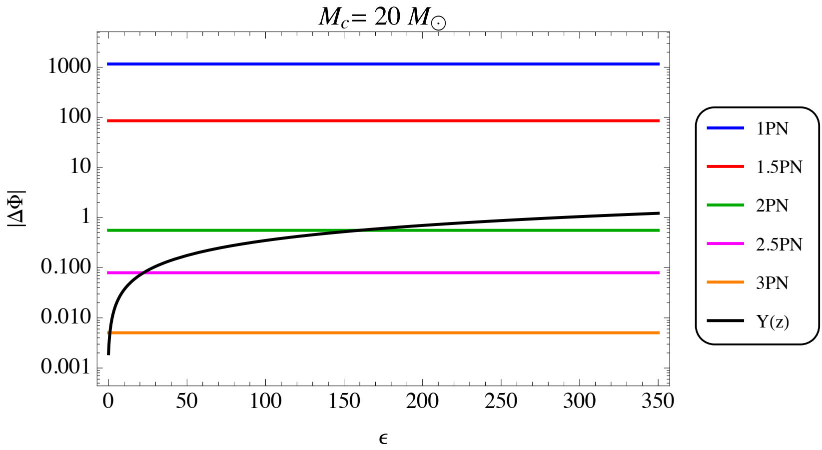
$$\Delta\Phi_{\rm coal} \simeq 3.96\cdot 10^{-5} h \times \frac{Y(z)}{H_0}\left(\frac{5\cdot 10^3 M_\odot}{\mathcal{M}_c(z)}\right)^{\frac{10}{3}}\left(\frac{10^{-3}{\rm Hz}}{f_O}\right)^{\frac{13}{3}}$$

For a typical LISA binary observed for a finite time interval (up to the mission lifetime):

$$\Delta\Phi_{\Delta{\rm t}} \simeq 0.1\, h\,\frac{Y(z)}{H_0}\left(\frac{50 M_\odot}{\mathcal{M}_c(z)}\right)^{\frac{5}{3}}\left(\frac{10^{-3}{\rm Hz}}{f_O}\right)^{\frac{5}{3}}\frac{\Delta t}{\rm year}$$

$\Rightarrow$ need low mass binaries at close redshift (high SNR) [LIGO-like]

Estimate of the phase shift due to $Y(z)$



Implications for GW detection

Question: What kind of peculiar accelerations of BBHs can we detect with LISA? What values of ϵ?

[Inayoshi, NT, Caprini, Haiman, arXiv:1702.06529]

To address this question we performed a Fisher matrix analysis

$$F_{ij} = \left\langle \frac{\partial h}{\partial \theta_i} \middle| \frac{\partial h}{\partial \theta_j} \right\rangle = 2 \int_{f_{\min}}^{f_{\max}} \frac{df}{S_n(f)} \left(\frac{\partial h}{\partial \theta_i} \frac{\partial h^*}{\partial \theta_i} + \mathrm{c.c.} \right)$$

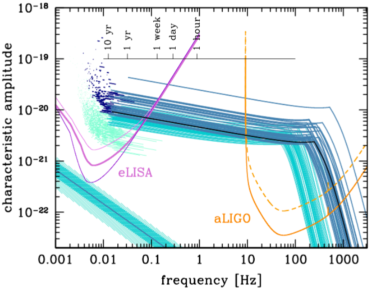
where $h(f)$ is the (sky-averaged and spin-less) 3.5PN waveform in Fourier space including the peculiar acceleration effect, which depends on the 6 parameters (for high accelerations $Y \propto \epsilon$)

$$\theta_i = (\mathcal{M}_c, \Phi_c, t_c, \eta, d_L, Y)$$

Implications for GW detection

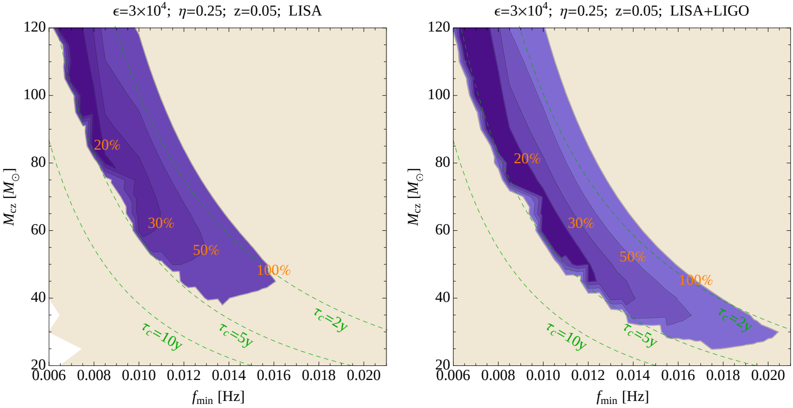
We made two parallel error estimations:

- With LISA alone: ΔY is the 1σ error marginalized over all other waveform parameters
- With LISA + LIGO where the time of coalescence t_c is fixed by an Earth-based detection and ΔY is marginalized only over the remaining parameters



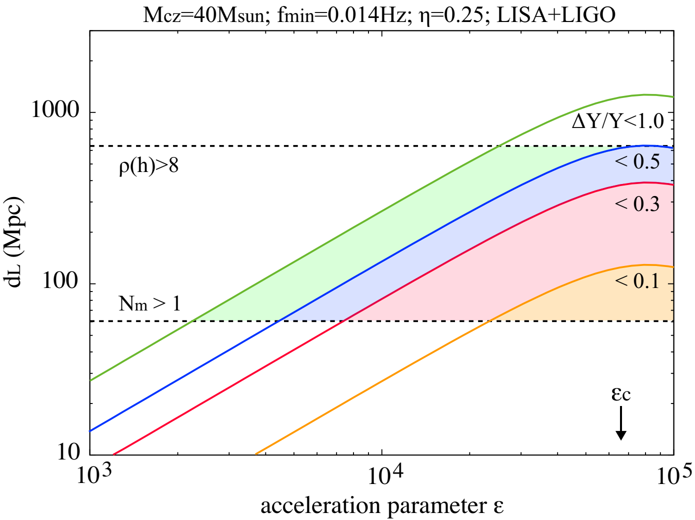
[Sesana, arXiv:1602.06951]

Implications for GW detection



[Inayoshi, NT, Caprini, Haiman, arXiv:1702.06529]

Implications for GW detection



Implications for BBH formation channels

Are we expecting large peculiar accelerations?
Only for specific BBH formation channels:

scenario	v (km/s)	r (pc)	ϵ
FBs at low-z (A)	~ 200	$> 5 \times 10^3$	< 10
FBs at high-z (A)	~ 300	$10^3 - 10^4$	$10 - 100$
GCs (B)	~ 200	$\sim 5 \times 10^4$	
NSCs (B)	$30 - 100$	~ 1	$\mathbf{10^3 - 10^4}$
AGN disks (C)	$200 - 600$	$0.1 - 1$	$\mathbf{10^4 - 10^5}$
Population III (D)	~ 200	$\lesssim 10^3$	$10 - 100$

The phase drift in the GW waveform produced by the peculiar acceleration of BBHs can be used as a discriminator between different BBH formation channels by LISA+LIGO observations

Conclusions

Measuring BBH peculiar accelerations with LISA

- The GW signal is affected by the evolution of the redshift perturbations during the observational time
- This produces a phase drift which is dominated by the peculiar acceleration contribution
- The effect is relevant for low mass LISA sources ($30M_\odot \lesssim \mathcal{M}_c \lesssim 100M_\odot$) with $\tau_c \sim \Delta t_{\rm obs}$
- It can be used to discriminate between different BBH formation channels