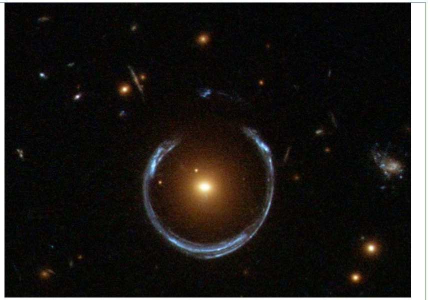
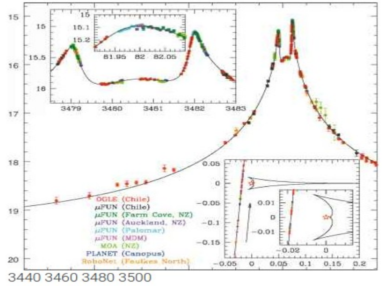
Gravitational lensing
From planets to clusters of galaxies



First lecture

Basic equations

First application: the point mass lens



Gravitational lensing
A short history

Newton realized that masses should deflect light

First Newtonian calculation Johann Soldner (1801)

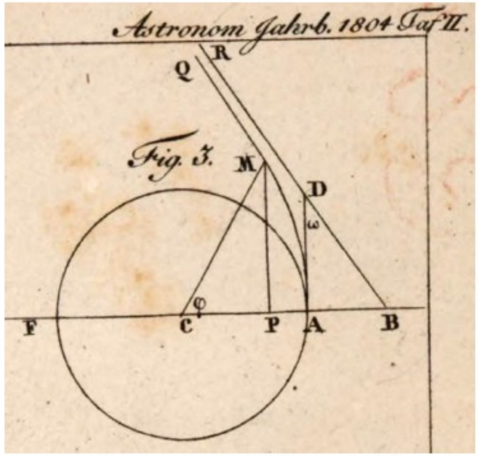
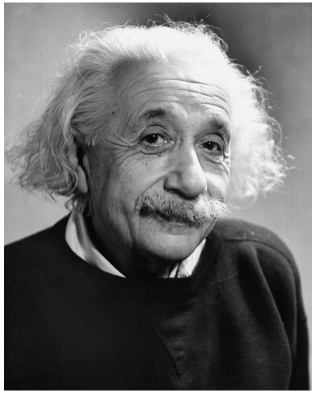
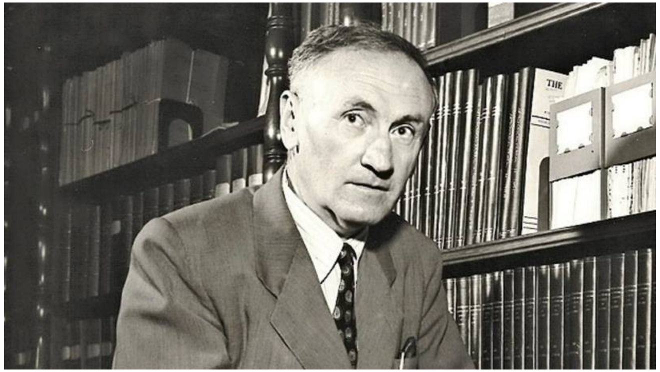


Figure 1. The Figure associated with Soldner's article.[7] Reproduced with permission from copy of the *Bayerische Staatsbibliothek*, Signatur: Eph. astr. 23-1804. urn:nbn:de:bvb:12-bsb105383333-5, p.281

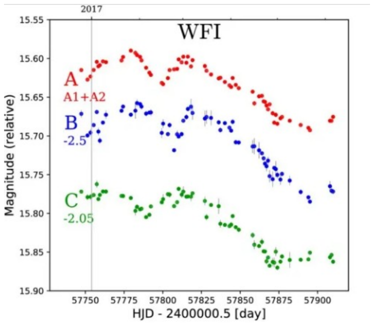
Einstein (1915) the correct deflection angle in general relativity is twice the previous Newtonian value



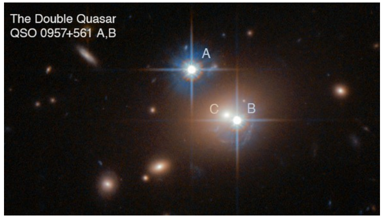
Zwicky (1937) realized that galaxies can split images
With large enough separations to be observable



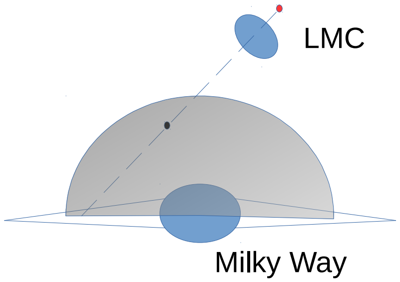
Refsdal (1964) propose to measure the Hubble constant
By using time delays
(trhough the variability of the lensed source)



Walsh, Carswell, & Weymann (1979) discover the
double image of a quasar QSO 0957+561



Paczynski (1986b) propose to monitor millions of star in
LMC and SMC
Now gravitational microlensing can be observed



The era of gravitational lensing is opening

The first case of gravitational arc is there

Lynds & Petrosian (1986)

Galaxy cluster Abell 370

The first gravitational arc

Lynds & Petrosian (1986)

68.01

Giant Luminous Arcs in Galaxy Clusters

R. Lynds (KPNO/NOAO), V. Petrosian (Stanford U.)

We announce the existence of a hitherto unknown type of spatially coherent extragalactic structure having, in the two most compelling known examples, the common properties: location in clusters of galaxies, narrow arc-like shape, enormous length, and situation of center of curvature toward both a cD galaxy and the apparent center of gravity of the cluster. The arcs are in excess of 100 Kpc in length, have luminosities roughly comparable with those of giant E galaxies, and are distinctly bluer than E galaxies - especially so in one case. Interpretations of the nature of the arcs are discussed within the framework of available data.

Soucail et al. (1987)

Galaxy cluster Abell 370

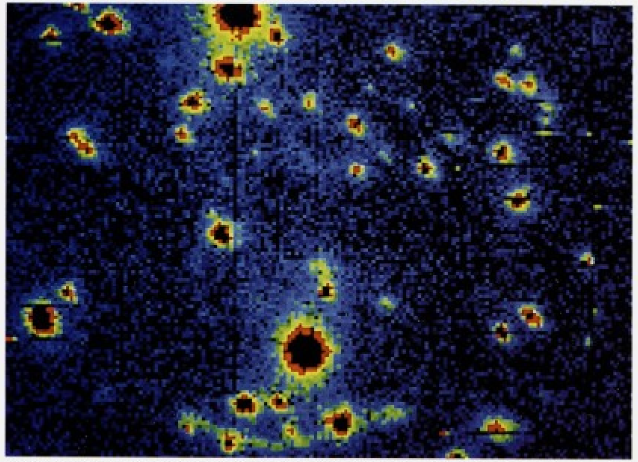


Figure 1: *Image of the core of the cluster of galaxies A 370 (z = 0.374), dominated by two giant galaxies (# 20 and # 35). The arc is located southward galaxy # 35 and has a linear size of ~ 8 kpc wide and 160 kpc long. In the lensing hypothesis it is an image of a galaxy at redshift z = 0.59. Note the galaxies superimposed on the arc, especially the brightest one (# 37) whose influence has been taken into account in the lensing model.*

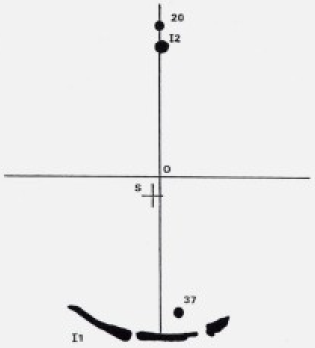
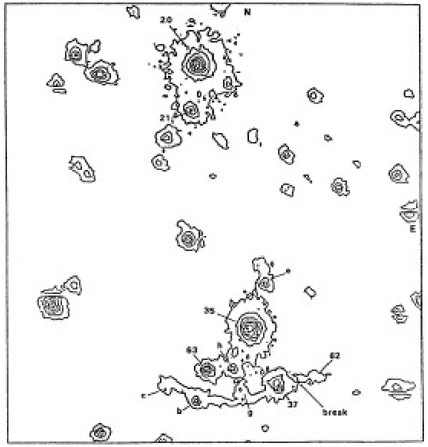
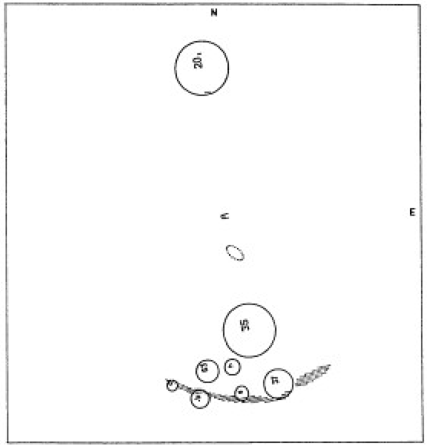


Figure 2: *Schematic diagram of the lensing configuration in a three point mass model: $2.25\ 10^{14}\ M_\odot$ for the cluster core (point 0) $3 \cdot 10^{12}\ M_\odot$ for galaxy # 20 and $0.7\ 10^{12}\ M_\odot$ for galaxy # 37. I1 and I2 are the two images of a circular source which would appear in S without lensing. Note the large break to the right of I1. The details of such a configuration will be given in a paper submitted to Nature.*





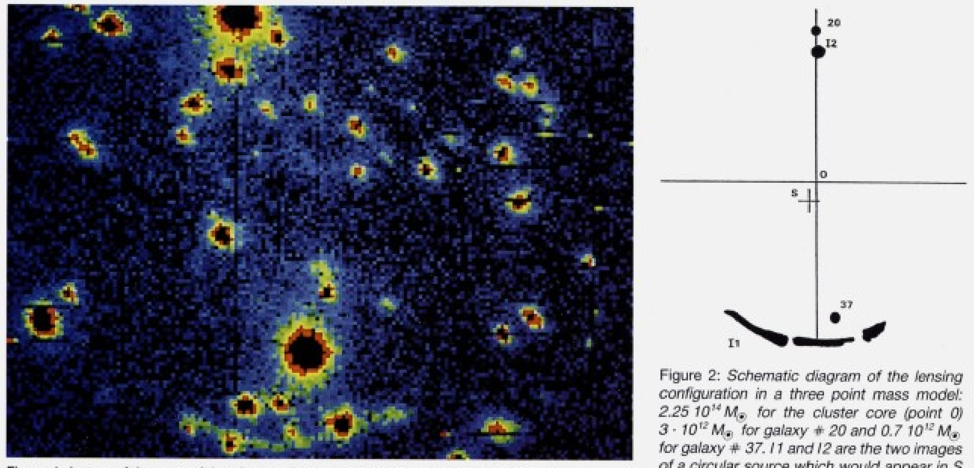
Bogdan Paczynski
Nature (1987)

Paczynski proposed that the arcs are the images of background galaxies which are strongly distorted and elongated by the gravitational lens effect of the foreground cluster.

This model was confirmed when the first arc redshifts were measured and found to be greater than that of the clusters.

HST (2019)

Soucail et al. (1987)



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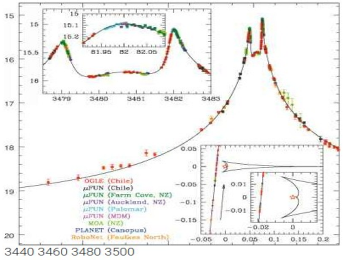
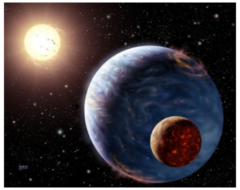
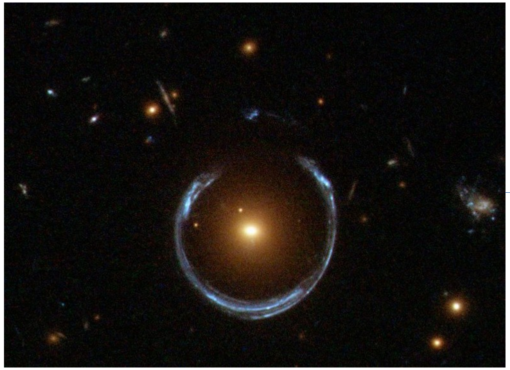
Gravitational lensing probes all astrophysical scales
A journey of increasing scale





From planets
To Galaxies



From Galaxies
To Galaxy groups

From Groups
To clusters of galaxies

What kind of information do we obtain from gravitational lensing ?

Gravitational lensing offers a direct unbiased measure of the mass

Making maps of the mass distribution

Dark matter mapping

Lensing has an ability to resolve very fine structure – un-observable by other means

The structure of the lens

planets

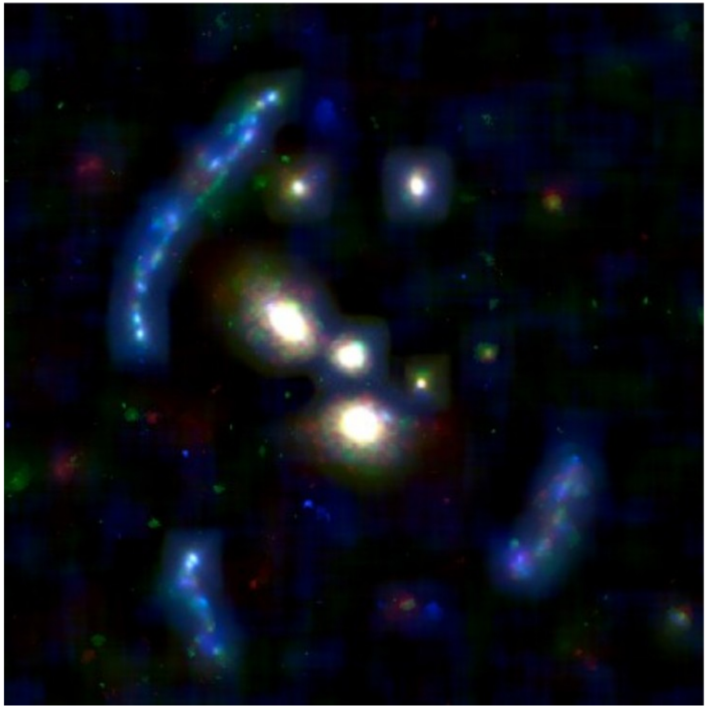
Dark matter substructures

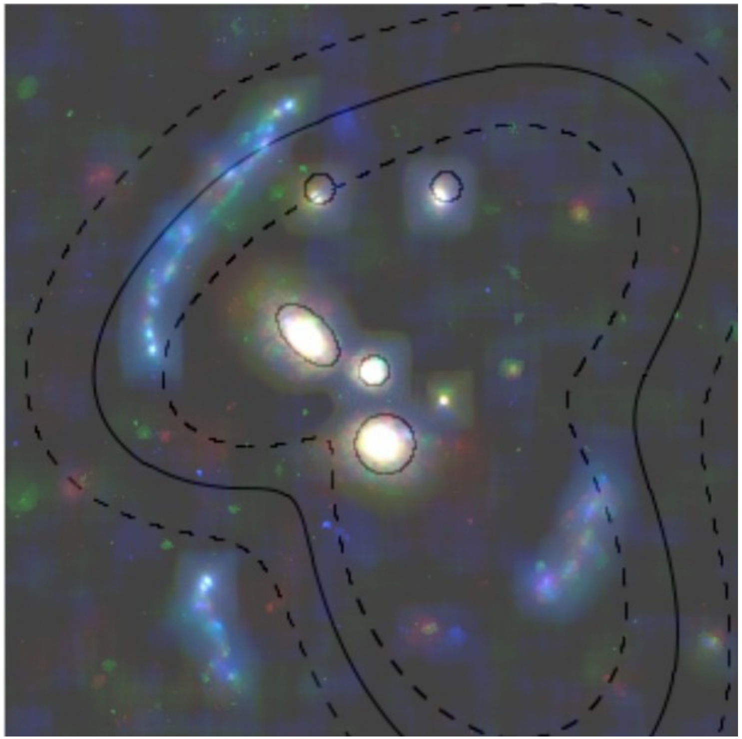
The structure of the source

red giant star

quasar accretion disk

Lensing offers a direct measure of mass visible or not

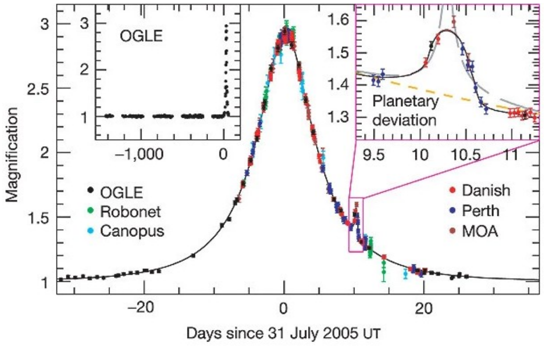




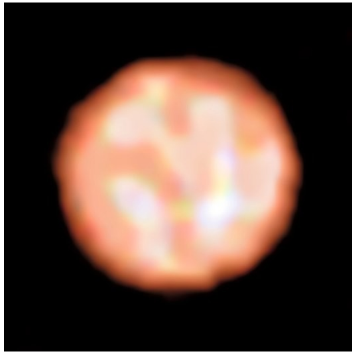
Direct reconstruction of mass

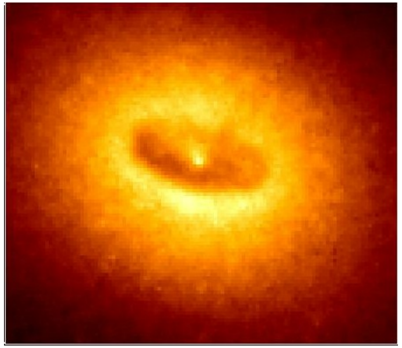
Lensing has an ability to resolve very fine structure

The structure of the lens: planets – Dark matter substructures



The structure of the source: red giant star - quasar accretion disk





What this course does not cover

Cosmological lensing

Cosmic shear

CMB lensing

Galaxy-galaxy lensing

Some reviews

Martin Kilbinger: Cosmology with cosmic shear observations: a review

Lewis & Challinor : Weak gravitational lensing of the CMB

The basics of gravitational lensing

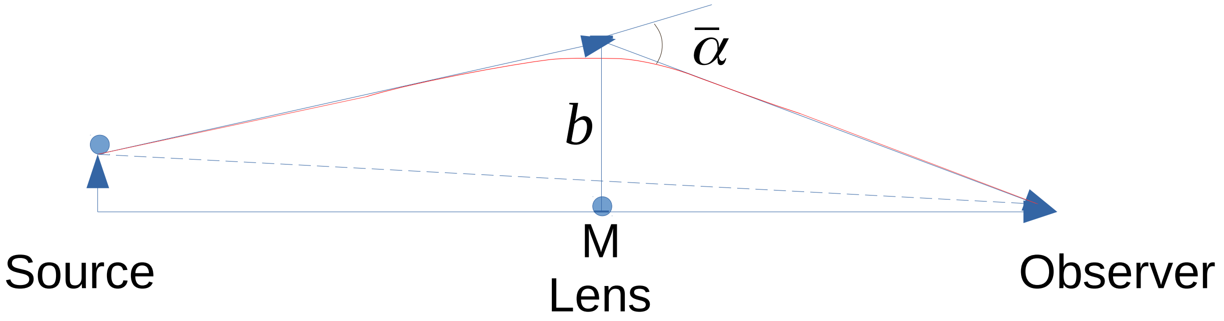
The fundamental scale

Einstein ring

The various lensing regimes

Strong lensing
Weak lensing
Intermediate regime

Lensing: bending of the light trajectory by a massive object



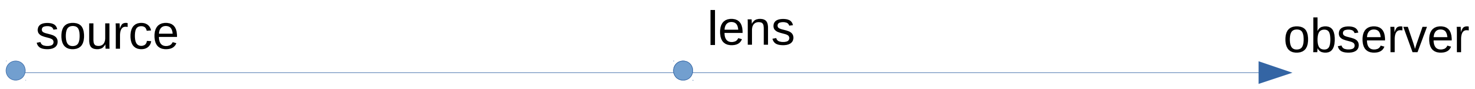
b Is the impact parameter

For small deviations, general relativity gives: $\bar{\alpha} = \frac{4GM}{bc^2}$

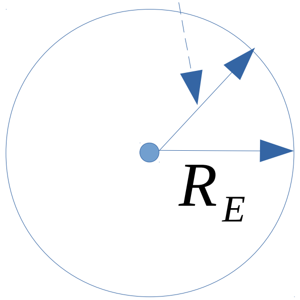
(see for instance Misner, Thorne & Wheeler or Schultz)

Lensing has a fundamental scale

Let's consider a perfectly symmetrical situation



The lens, source and observer are perfectly aligned
In this case due to the symmetry all trajectories are
The same except for a rotation of the plane of the
trajectory

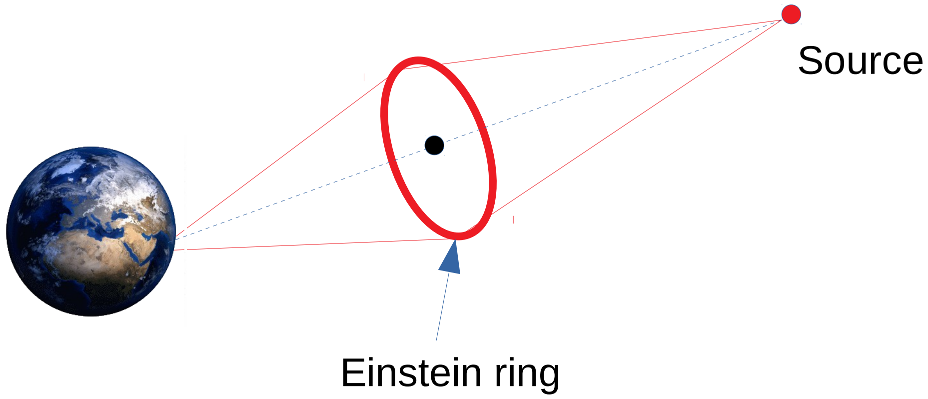


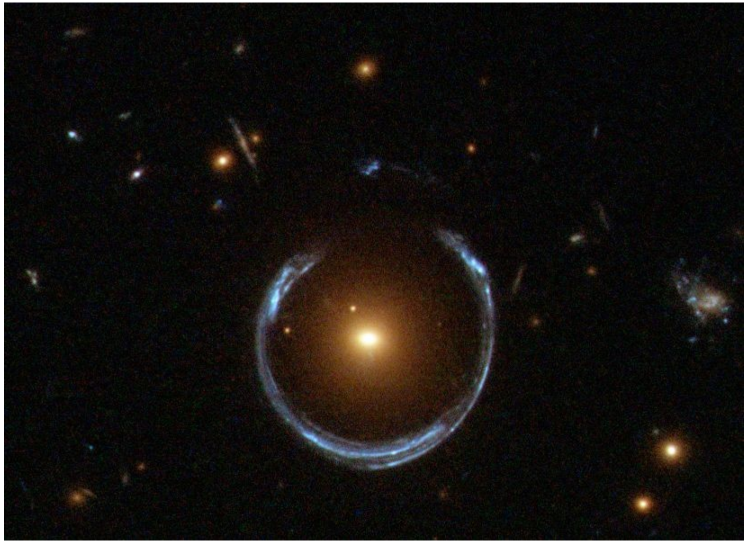
The image of the source is a full Circle

The radius of the circle is the

Einstein radius: R_E

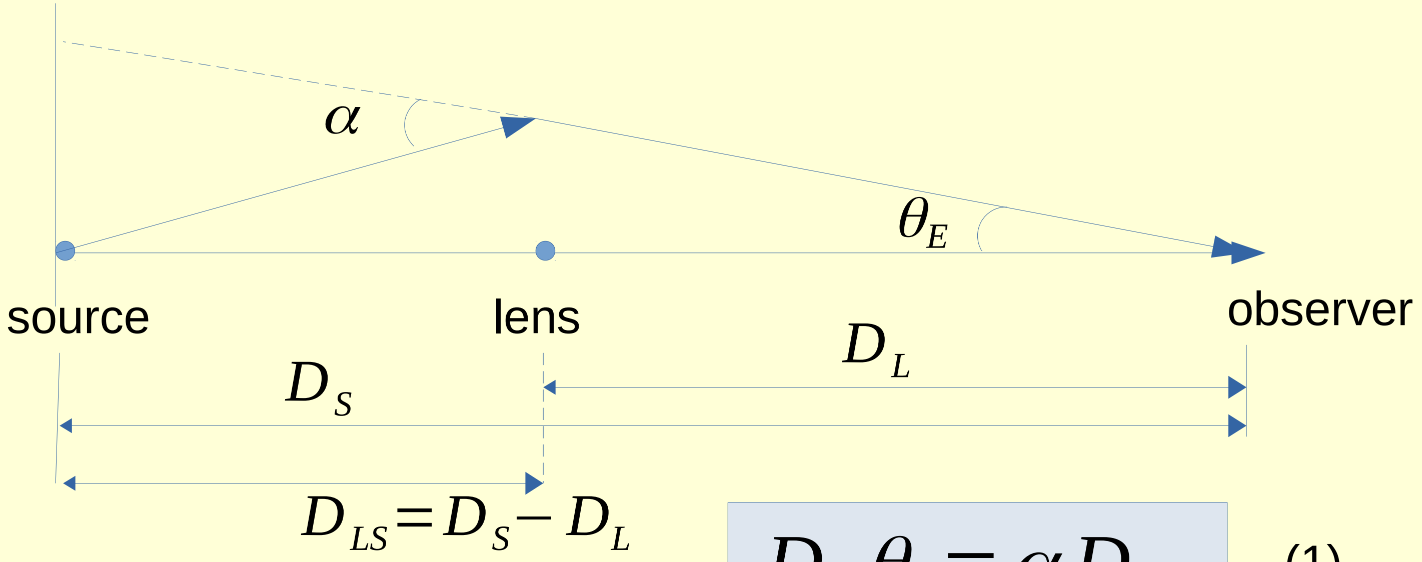
The Einstein ring





The image seen from earth

Estimating the Einstein radius

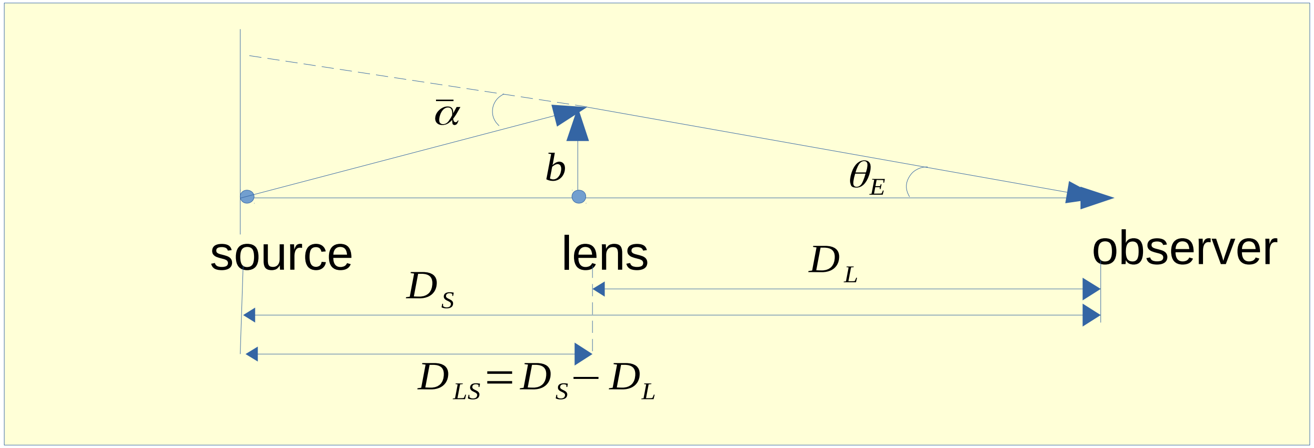


$$D_S \theta_E = \alpha D_{LS} \quad (1)$$

$$\bar{\alpha} = \frac{4GM}{c^2 b} \quad \textit{with} \quad b = \theta_E D_L \quad \textit{combined with} \quad D_S \theta_E = \bar{\alpha} D_{LS}$$

We obtain $$\theta_E = \sqrt{\frac{4GM}{c^2} \frac{D_{LS}}{D_S D_L}}$$

The Einstein radius is: $$R_E = \theta_E D_L = \sqrt{\frac{4GM}{c^2} \frac{D_{LS} D_L}{D_S}}$$

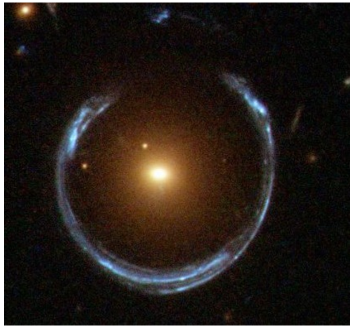


Typical values of the Einstein radius

Star: $\theta_E \simeq 1\,mas$ $R_E \simeq 1\,AU$ Unresolved blend

Galaxies:

$\theta_E \simeq 2\,arcsec$ $R_E \simeq 30\,kpc$



Cluster of galaxies:

$\theta_E \simeq 50\,arcsec$ $R_E \simeq 0.5\,Mpc$

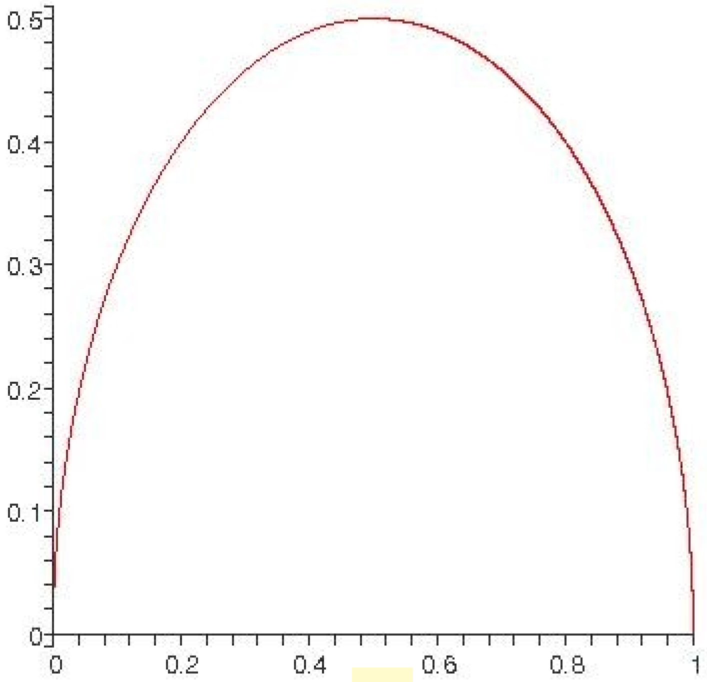
The Einstein radius and the distance between the lens and source

$$R_E \propto \sqrt{\frac{(D_S - D_L) D_L}{D_S}}$$

$$u = \frac{D_L}{D_S}$$

$$R_E \propto \sqrt{u(1-u) D_S} = f(u)\sqrt{D_S}$$

$f(u)$



u

For a source at fixed distance the Einstein radius is maximal
When the lens lens is placed at mid-distance

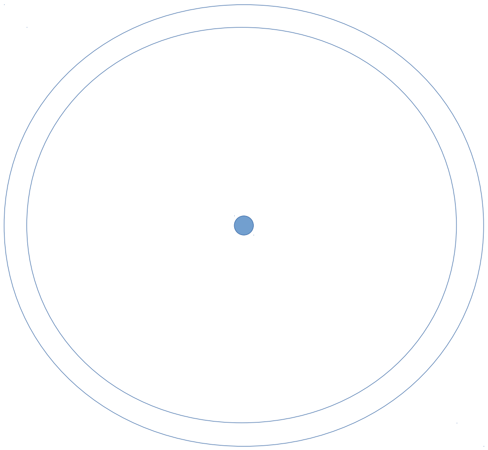
The various lensing regimes

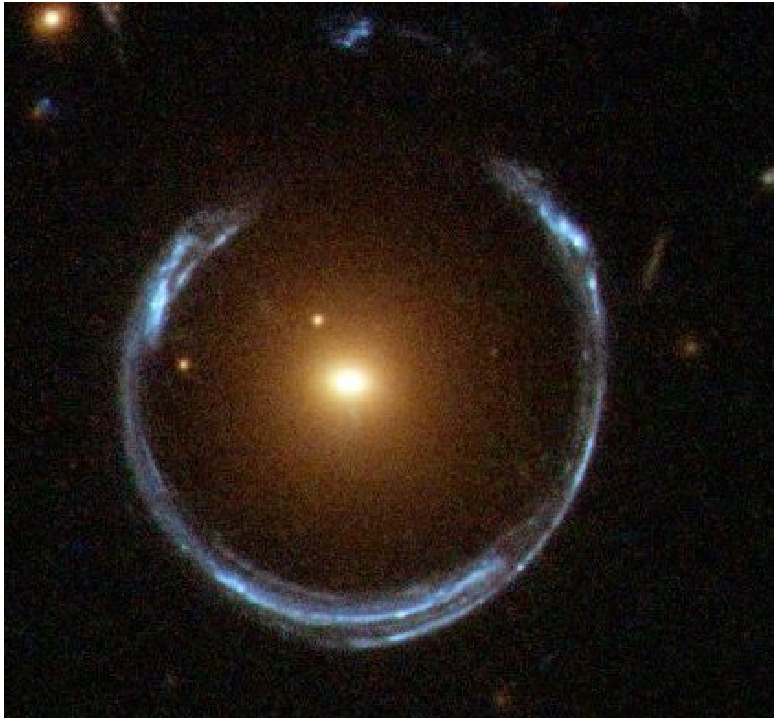
Strong lensing

Weak lensing

Intermediate regime

Consequence of the fundamental scale: the various lensing regimes





Extended source at the center of circularly symmetric lens: thick Einstein ring

Slightly mis-aligned source or not circularly symmetrical potential

Source mis-alignement R_S

$$R_S \leqslant R_E$$

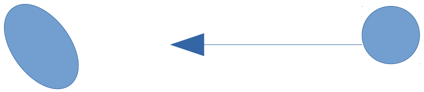
Broken ring: gravitational arcs

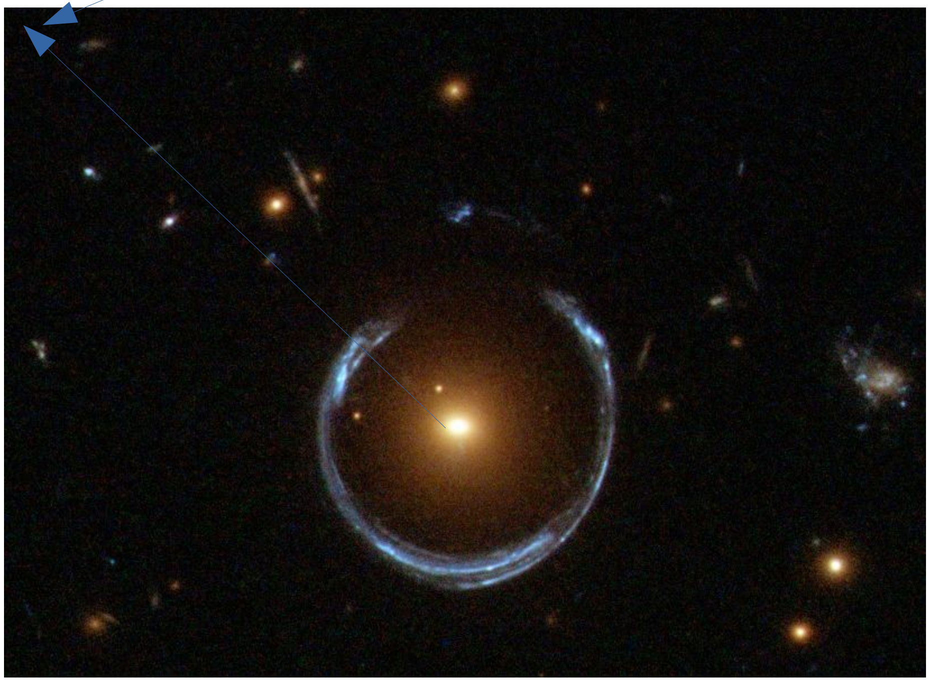
Strong lensing

Source far away from center of lens
(a few times the Einstein radius)

Weak effect

Weak distortion
A round source become
An ellipse

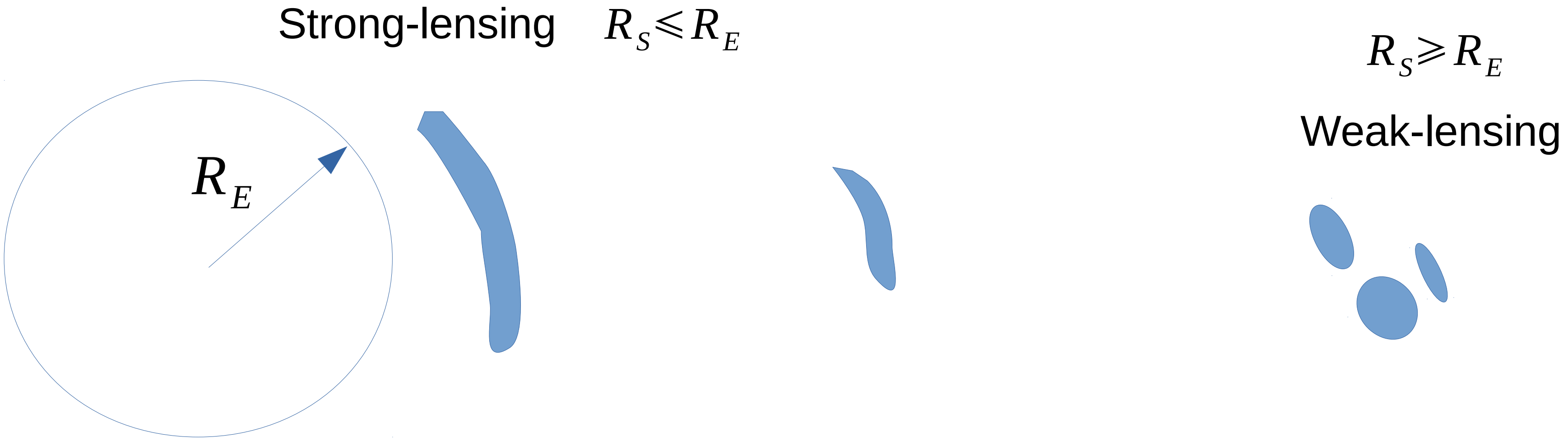




There is a statistical change in the
Ellipticity of background galaxies

Weak-Lensing

Between the weak and strong lensing regime: intermediate regime



Variable elliptical distortion: some curvature

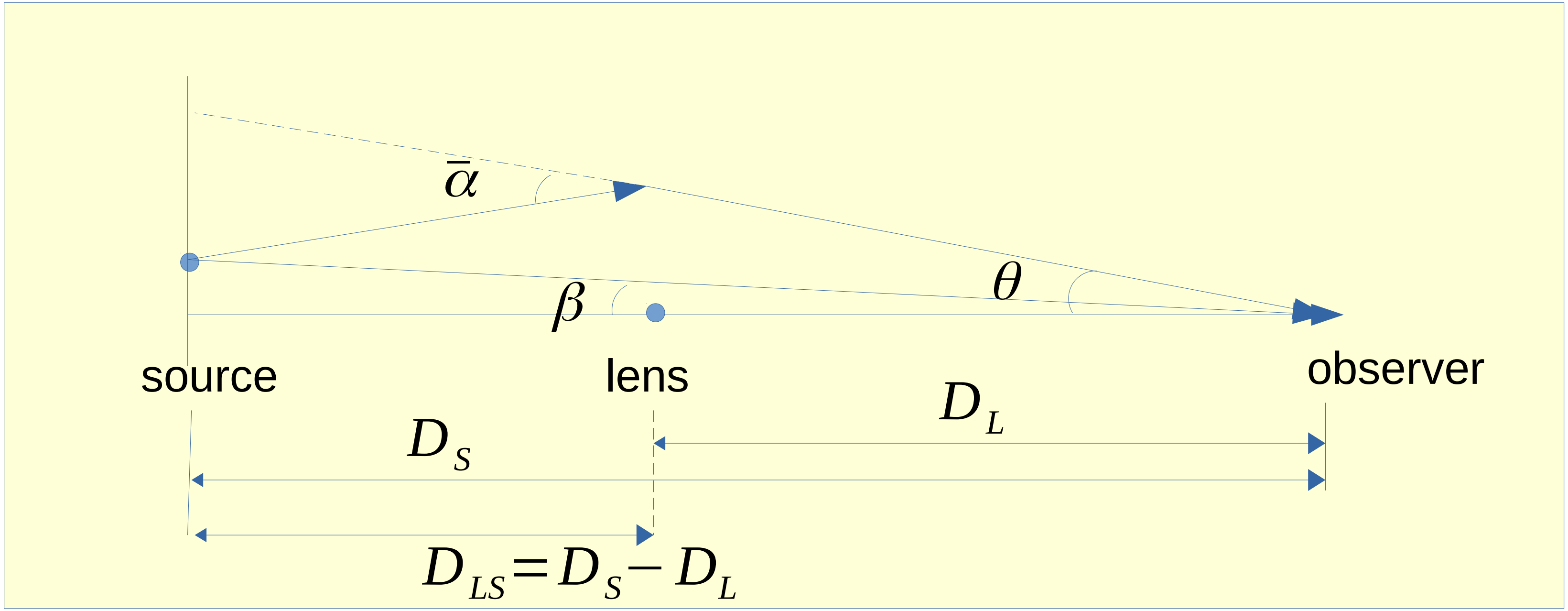
The position of the source defines the regime

General gravitational lensing in astrophysical context

Basic equations

Full mathematical description

The lens equation

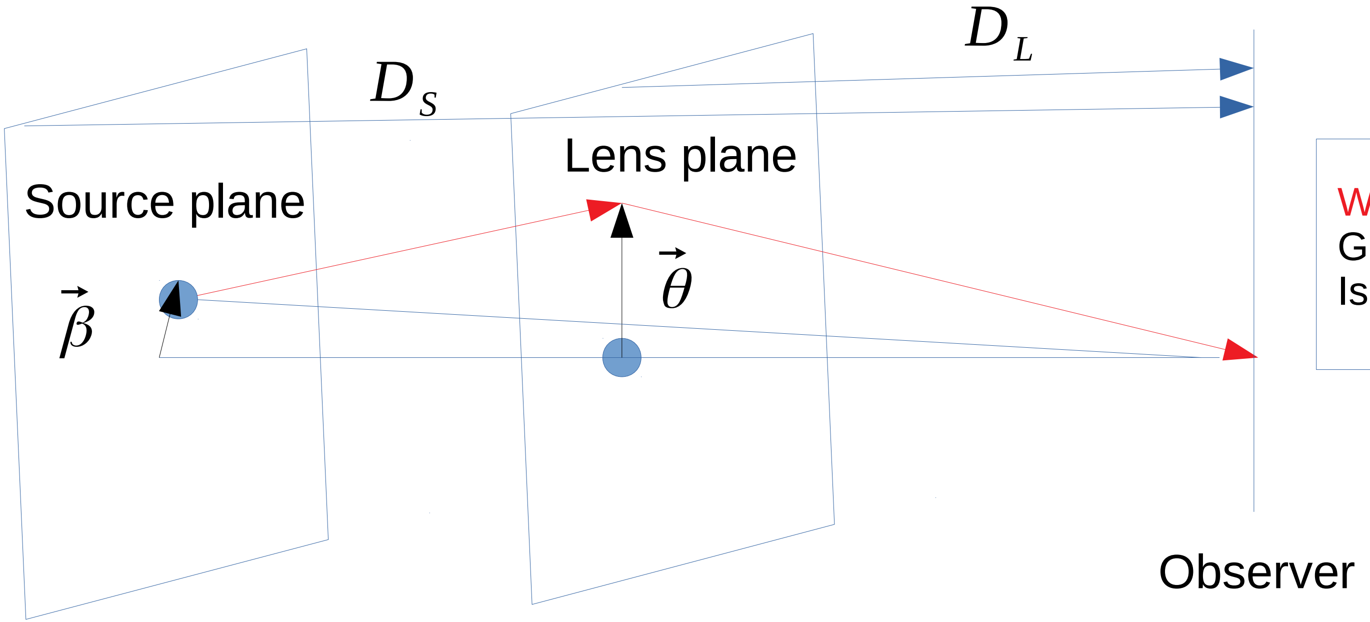


$$\beta D_S+\bar{\alpha} D_{LS}=\theta D_S \longrightarrow \beta=\theta-\bar{\alpha}\frac{D_{LS}}{D_S}=\theta-\alpha$$

$$\beta=\theta-\alpha$$

Reduced deflection angle $\alpha=\bar{\alpha}\frac{D_{LS}}{D_S}$

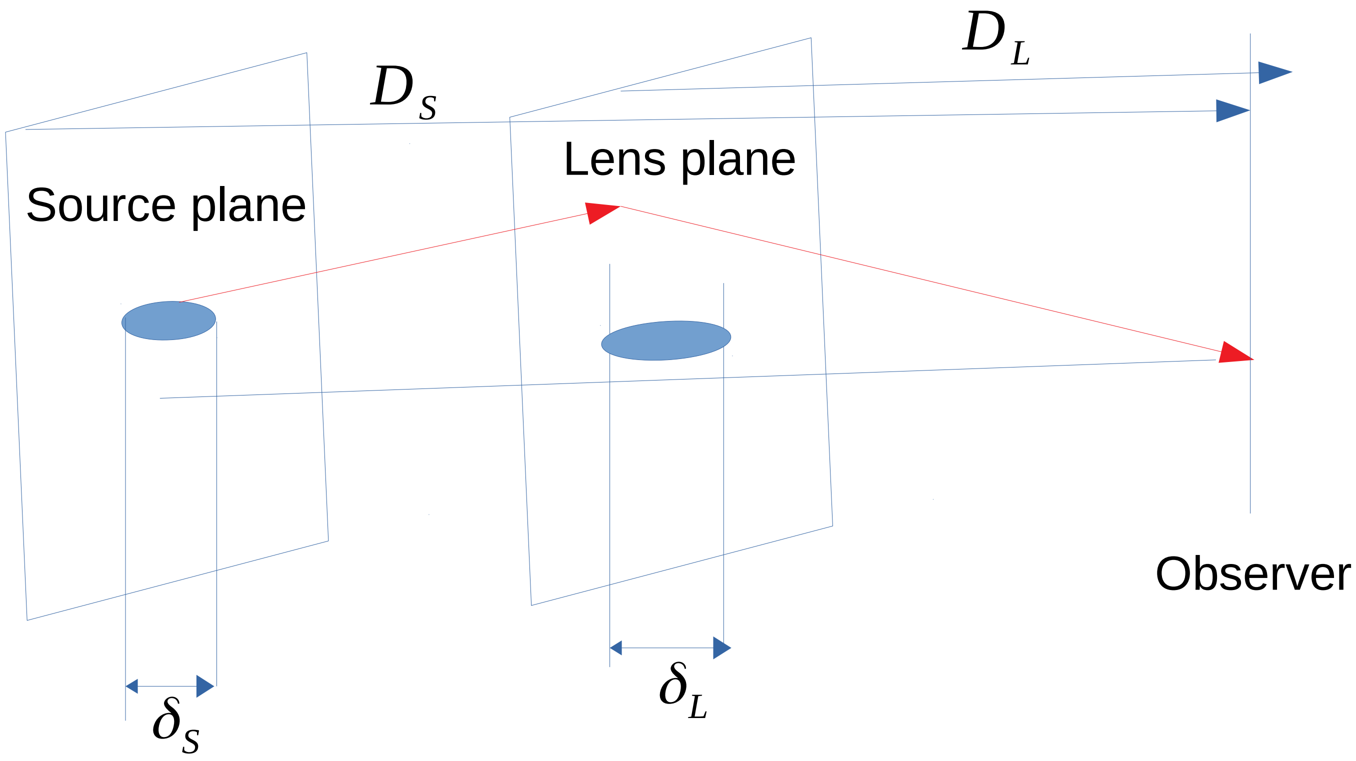
3D representation of the lens equation



Why we work in planes
Galaxies, star,...thickness
Is small with respect to the distances

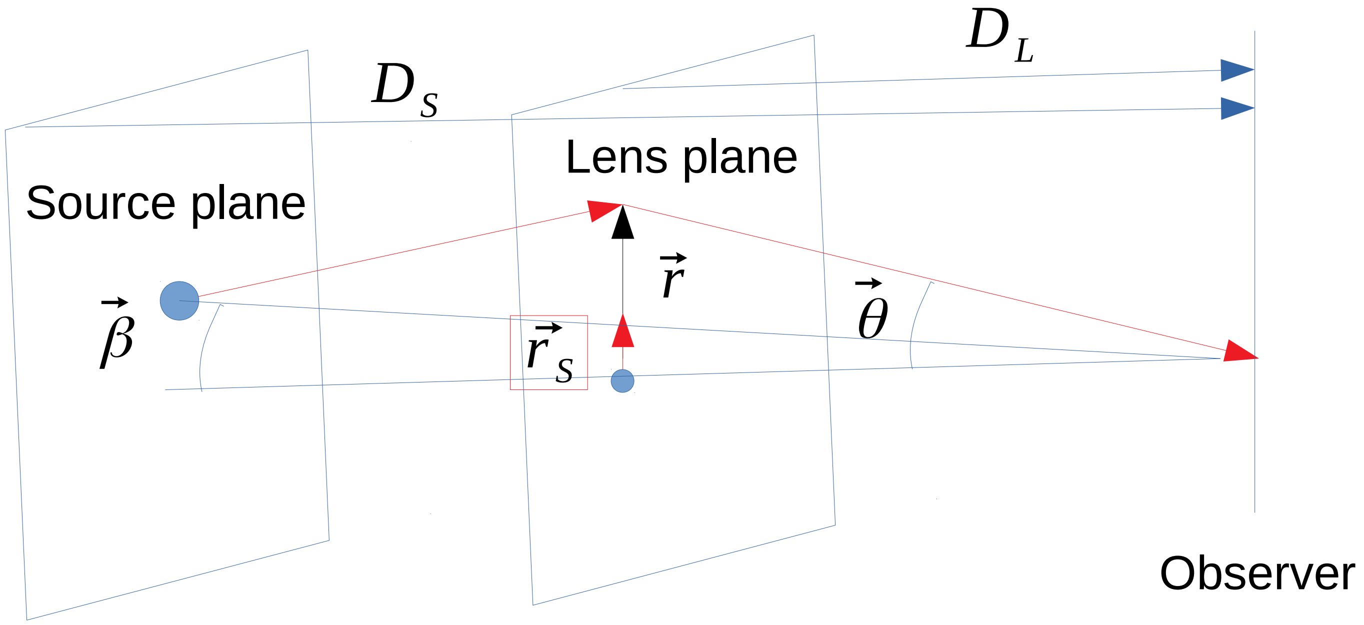
General lens equation in vector form $\vec{\beta} = \vec{\theta} - \vec{\alpha}$

General distribution of lenses: planar approximation
The thin lens model



$$\delta_S \leqslant D \quad ; \quad \delta_L \leqslant D \quad ; \quad D = D_S, D_L$$

The vectors angle are equivalent to vectors in the plane

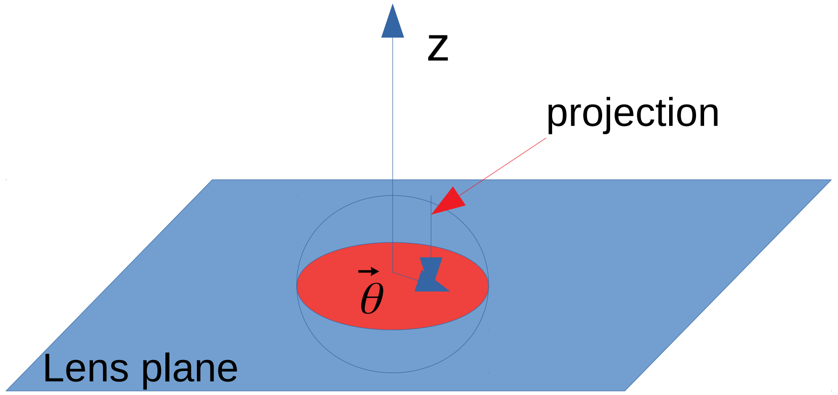


$$\vec{r} = \theta D_L \quad ; \quad \vec{r}_s = \beta D_L$$

$$\vec{\beta} = \vec{\theta} - \vec{\alpha} \longrightarrow \vec{r}_S = \vec{r} - \vec{\hat{\alpha}}$$

The lens equation for a continuous distribution in the lens plane

In the thin lens approximation the density is projected density in the lens plane
Leading to a surface density



$$\Sigma(\vec{\theta}) = \int \rho(\vec{\theta}, z)\, dz$$

$\Sigma(\vec{\theta})$ Is the projected surface density in the lens plane

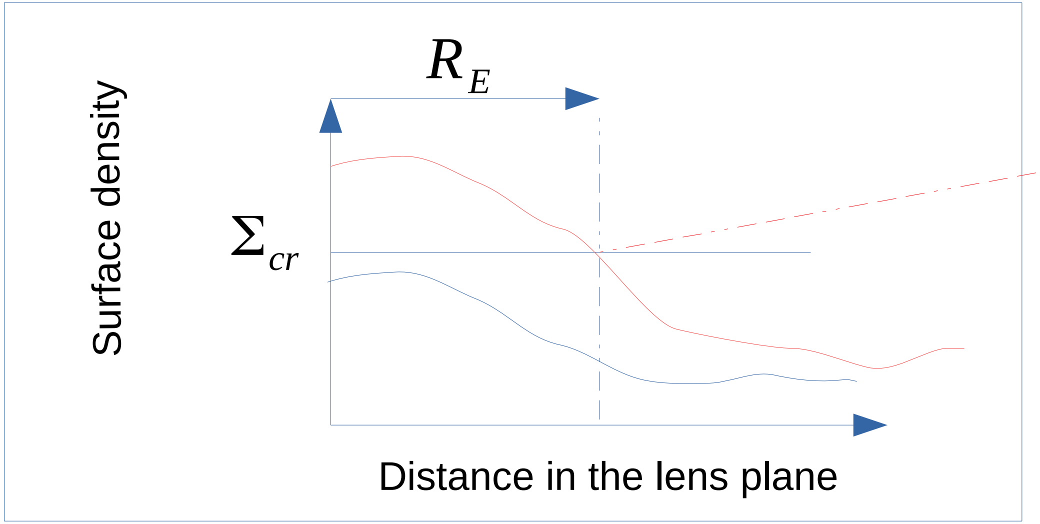
Note: $\vec{\theta}$ is related to the local coordinate in the lens plane: $\vec{r} = \vec{\theta} D_L$

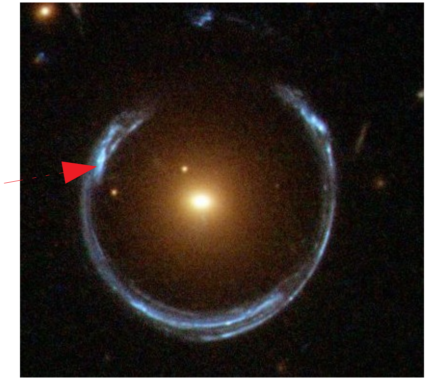
Introducing Σ_{cr} the mean surface density within the Einstein radius

$$M = \Sigma_{cr}\, \pi R_E^2$$

$$R_E^2 = \frac{4GM}{c^2} \frac{D_{LS} D_L}{D_S}$$

$$\Sigma_{cr} = \frac{c^2 D_S}{4 \pi G D_{LS} D_L}$$





Below Σ_{cr} no gravitational arcs

No strong lensing

Deviation due to a point mass lens

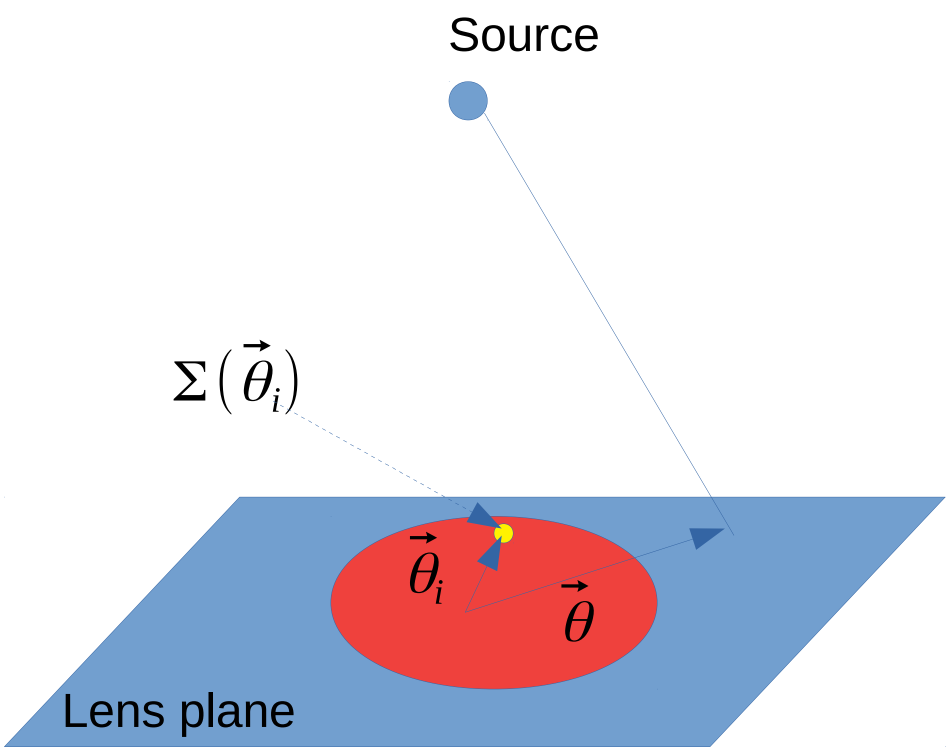
$$\bar{\alpha}=\frac{4GM}{bc^2}$$

$$b=\theta D_L$$

$$\alpha=\bar{\alpha}\frac{D_{LS}}{D_S}$$

$$\alpha=\frac{4GM}{c^2}\frac{D_{LS}}{D_S D_L}\frac{1}{\theta} \longrightarrow \vec{\alpha}=\frac{4GM}{c^2}\frac{D_{LS}}{D_S D_L}\frac{\vec{\theta}}{|\vec{\theta}|^2}$$

The lens equation for a continuous distribution in the lens plane



The deviation produced by a small element of the lens is:

$$\overrightarrow{\delta\alpha} \propto \Sigma(\vec{\theta}_i)\, d^2\theta_i \, \frac{\vec{\theta}-\vec{\theta}_i}{|\vec{\theta}-\vec{\theta}_i|^2}$$

Weight due to local mass

Deviation due to a point mass lens

We introduce the normalized surface density (convergence)

$$\kappa(\vec{\theta}) = \frac{\Sigma(\vec{\theta})}{\Sigma_{cr}}$$

Then: $$\overrightarrow{\delta\alpha} = \frac{1}{\pi}\kappa(\vec{\theta}_i) d^2\theta_i \frac{\vec{\theta} - \vec{\theta}_i}{|\vec{\theta} - \vec{\theta}_i|^2}$$

We co-add the angular deviation for each local element

$$\alpha(\vec{\theta}) = \int \overrightarrow{\delta\alpha} = \frac{1}{\pi}\int_{LP} \kappa(\vec{\theta}_i) \frac{\vec{\theta} - \vec{\theta}_i}{|\vec{\theta} - \vec{\theta}_i|^2} d^2\theta_i$$

$$\alpha(\vec{\theta}) = \frac{1}{\pi} \int_{LP} \kappa(\vec{\theta}_i) \frac{\vec{\theta} - \vec{\theta}_i}{|\vec{\theta} - \vec{\theta}_i|^2} d^2 \theta_i$$

We introduce the potential

$$\phi(\vec{\theta}) = \frac{1}{\pi} \int_{LP} \kappa(\vec{\theta}_i) \log\left(|\vec{\theta} - \vec{\theta}_i|\right) d^2 \theta_i$$

Then: $\vec{\alpha} = \vec{\nabla} \phi$

Then finally the lens equation takes the simple form:

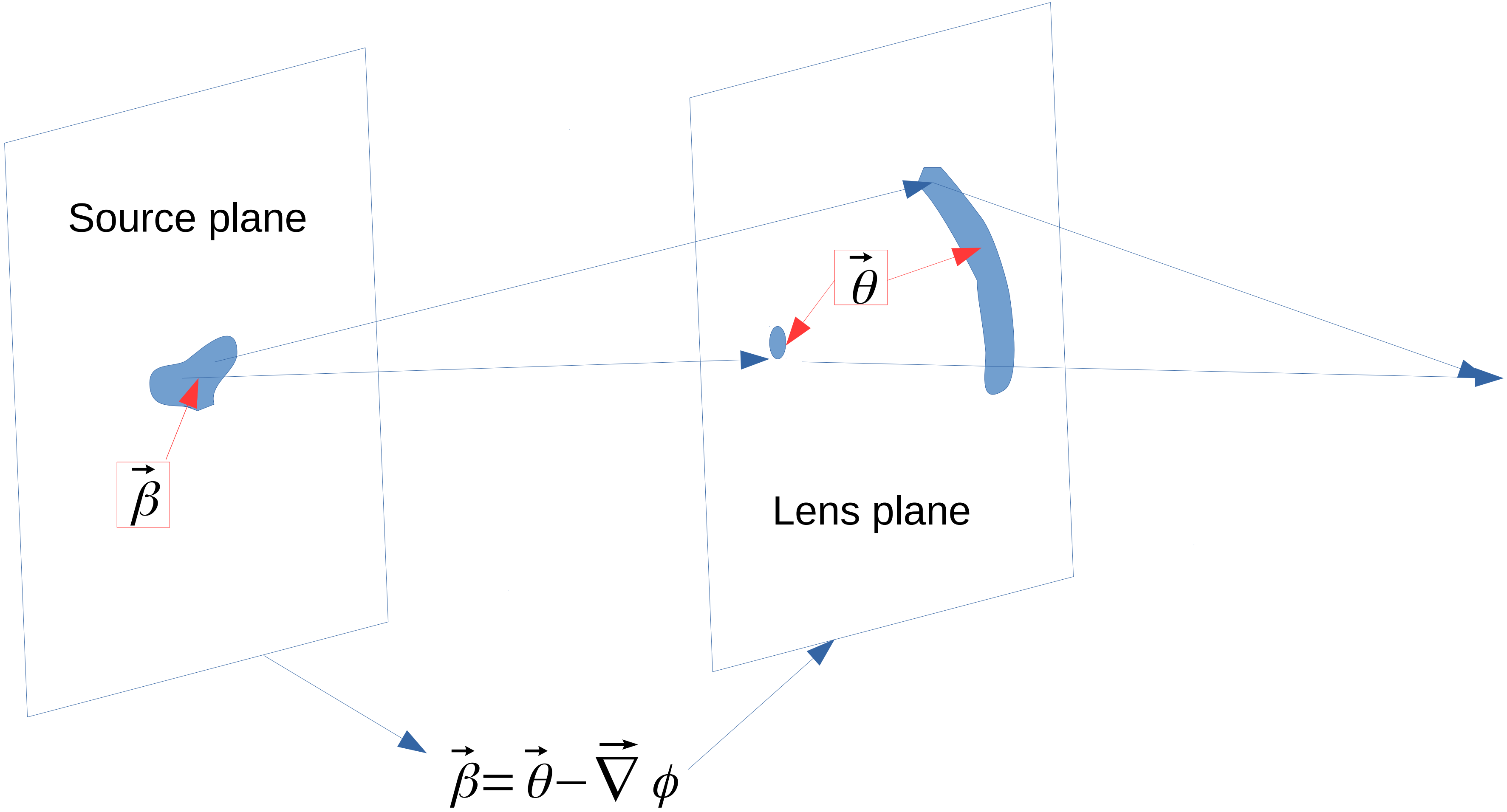
$$\vec{\alpha} = \vec{\nabla}\,\phi$$

$$\vec{\beta} = \vec{\theta} - \vec{\alpha} \quad \longrightarrow \quad \boxed{\vec{\beta} = \vec{\theta} - \vec{\nabla}\,\phi}$$

$$\phi(\vec{\theta}) = \frac{1}{\pi}\int_{LP} \kappa(\vec{\theta}_i)\log\left(\left|\vec{\theta} - \vec{\theta}_i\right|\right) d^2\theta_i \qquad \kappa = \frac{1}{2}\Delta\,\phi$$

The lens equation describe a general change in coordinates from the source coordinates ($\vec{\beta}$) to the lens coordinates ($\vec{\theta}$)

Lensing is a coordinate re-mapping from the source plane to the lens plane



Additionally the coordinates change introduced by lensing
Conserve the surface brightness of the source

(see Misner, Thorne & Wheeler, or Schultz)

The conservation of surface brightness,
plus the coordinates transform provided by the lens equation
is a complete description of gravitational lensing

$$\vec{\beta} = \vec{\theta} - \vec{\nabla}\,\phi \quad \oplus \quad \text{surface brightness conservation}$$

First application: point mass lens

Basic equations

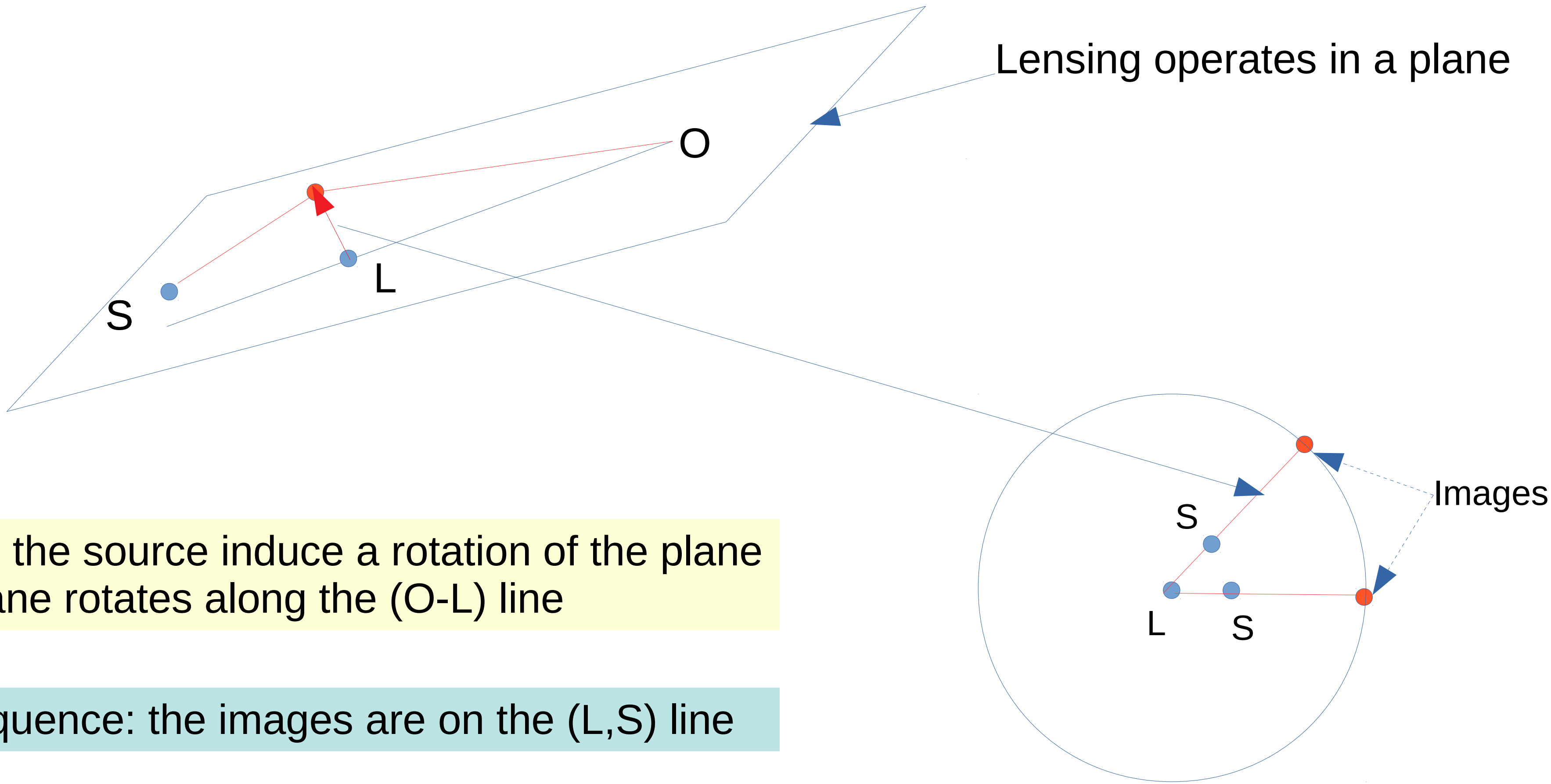
Amplification of the source

Direct calculation
Jacobian
Total amplification
Light curve
Fundamental degeneracies

Astrometric effects

Basic equations
Amplitude of the effect

Lensing by a point mass lens

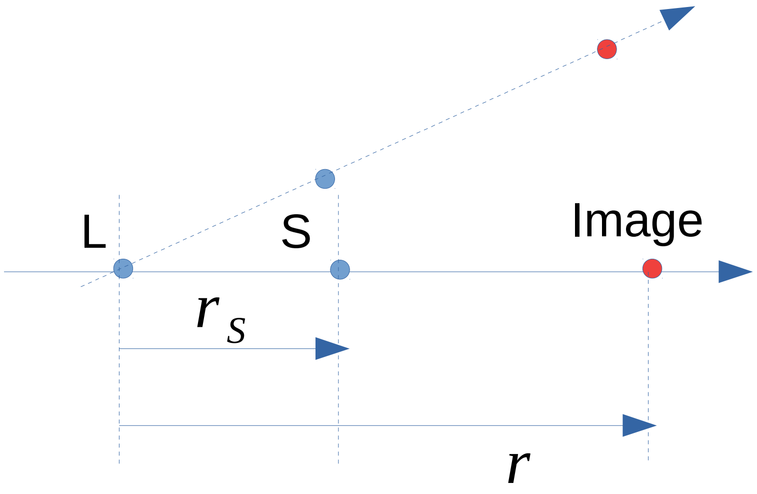


Moving the source induce a rotation of the plane
The plane rotates along the (O-L) line

Consequence: the images are on the (L,S) line

For convenience we use lens plane coordinates: $r_S = \beta D_L \quad r = \theta D_L$

We work along the (L,S) line:



The images are aligned with the (L,S) line
Moving the source rorates the line and images

Lens equation $$r_S = r - \frac{r_E^2}{r}$$

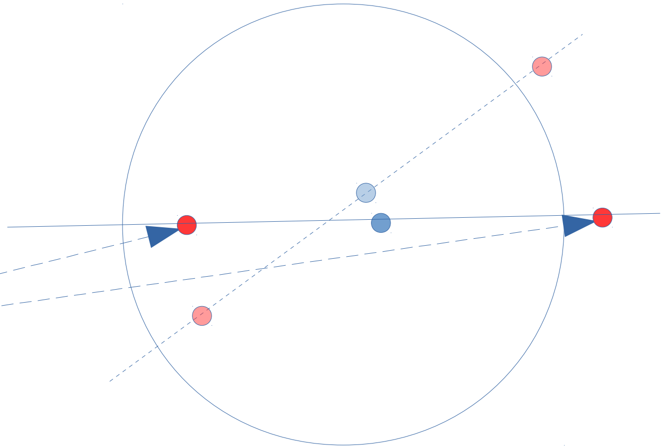
Re-normalization by the Einstein radius, $$r_S = \frac{r_S}{r_E} \quad ; \quad r = \frac{r}{r_E}$$

Lens equation $$r_S = r - \frac{1}{r}$$

Lens equation $r_S = r - \dfrac{1}{r}$

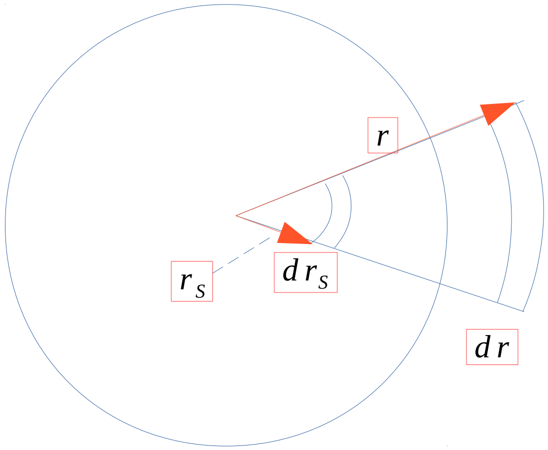
Two solutions: $r = \dfrac{r_s \pm \sqrt{r_s^2 + 4}}{2}$

Two images of the source



Typical separation a few mas

Total amplification: sum of the flux of the two images
(images usually not separable)



Amplification: $A = \frac{r}{r_s} \frac{dr}{dr_s}$

$$A = A_1 + A_2$$

$$r_{1,2} = \frac{r_s \pm \sqrt{r_s^2 + 4}}{2}$$

$$A = \frac{r_S^2 + 2}{r_S \sqrt{r_S^2 + 4}}$$

Other method: lensing is a change in coordinates

The amplification is the change in the volume element in the coordinate transform

This is the determinant of the Jacobian matrix J

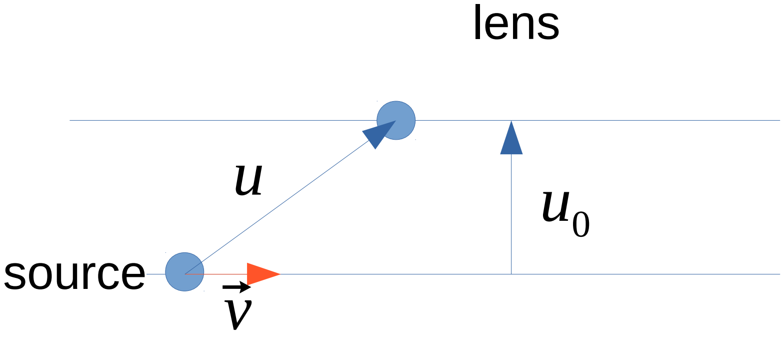
$$J = \begin{vmatrix} \frac{\partial x_S}{\partial x} & \frac{\partial y_S}{\partial x} \\ \frac{\partial x_S}{\partial y} & \frac{\partial y_s}{\partial y} \end{vmatrix} \qquad x_S = x - \frac{x}{r^2} \quad ; \quad y_S = y - \frac{y}{r^2} \longrightarrow J = \frac{r^4 - 1}{r^4}$$

$$\boxed{A = J^{-1}} \quad ; \quad A \rightarrow \infty \longrightarrow r = 1 \longrightarrow \text{Einstein circle=critical line}$$

$$A = A_1 + A_2 = \frac{1}{|J_1|} + \frac{1}{|J_2|} = \frac{r_S^2 + 2}{r_S \sqrt{r_S^2 + 4}}$$

Typical microlensing amplification curve in astrophysical context

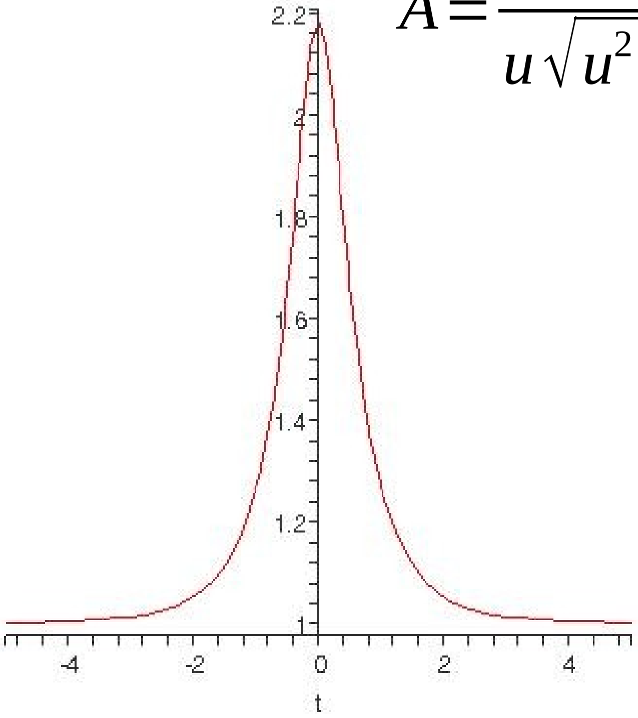
$$u \equiv r_s$$



$$u^2 = u_0^2 + v^2 t^2$$

u_0 Is the impact parameter

$$A = \frac{u^2 + 2}{u\sqrt{u^2 + 4}}$$



$$u^2 = u_0^2 + v^2 t^2 \qquad A = \frac{u^2+2}{u\sqrt{u^2+4}}$$

All length are in units of the Einstein radius

We measure: $u_0 \equiv \frac{u_0}{R_E} \quad ; \quad v \equiv \frac{v}{R_E} = t_E^{-1}$

The crossing time: t_E is directly related to $R_E = \sqrt{\frac{4GM}{c^2} \frac{D_{LS} D_L}{D_S}}$

But the velocity is unknown

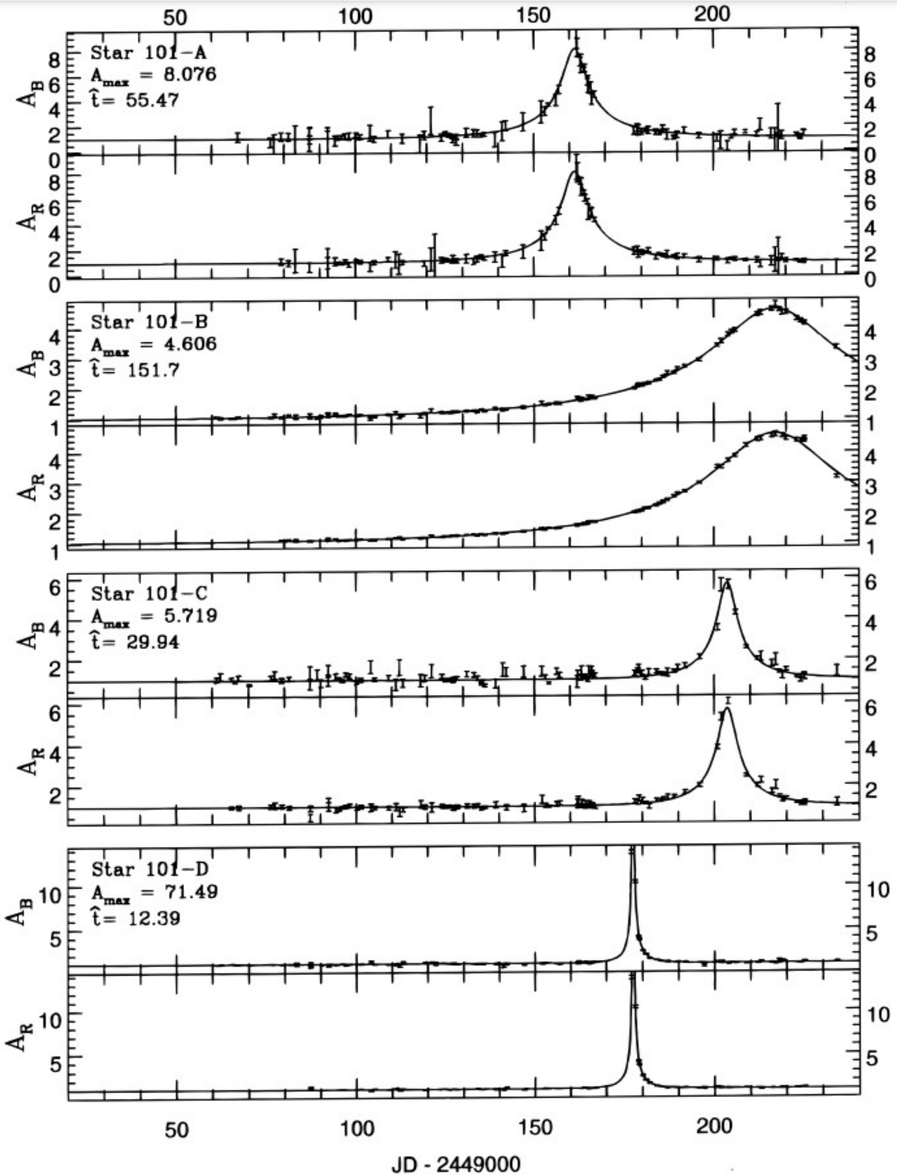
And R_E does not relate directly to the mass since the distances are unknown

Fundamental degeneracies

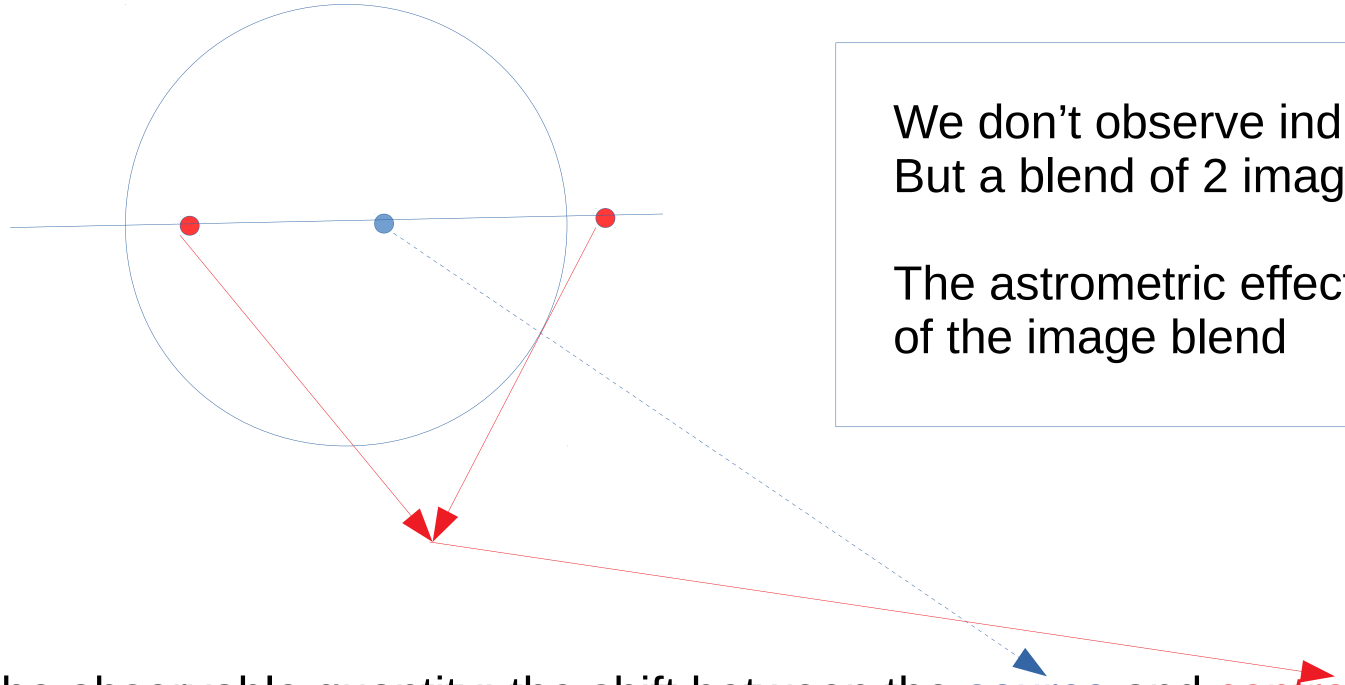
The first light curves
of microlensing events

Alcock etal. (1997)

(Galactic Bulge events)



Astrometric effects



We don’t observe individual images
But a blend of 2 images

The astrometric effect is the shift of the centroid of the image blend

The observable quantity: the shift between the source and centroid position

Calculation of astrometric effects

The position of the images centroid
$$\bar{u} = \frac{u_1 A_1 + u_2 A_2}{A_1 + A_2} = \frac{u(u^2+3)}{u^2+2}$$

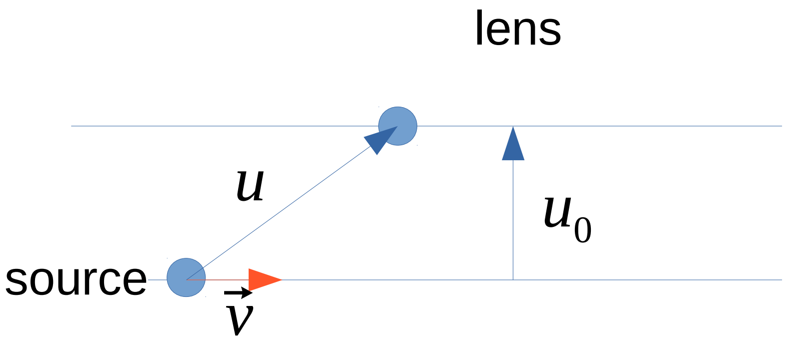
The observable quantity: the shift between the source and centroid position

$$\Delta = u - \bar{u} = \frac{u}{2+u^2}$$

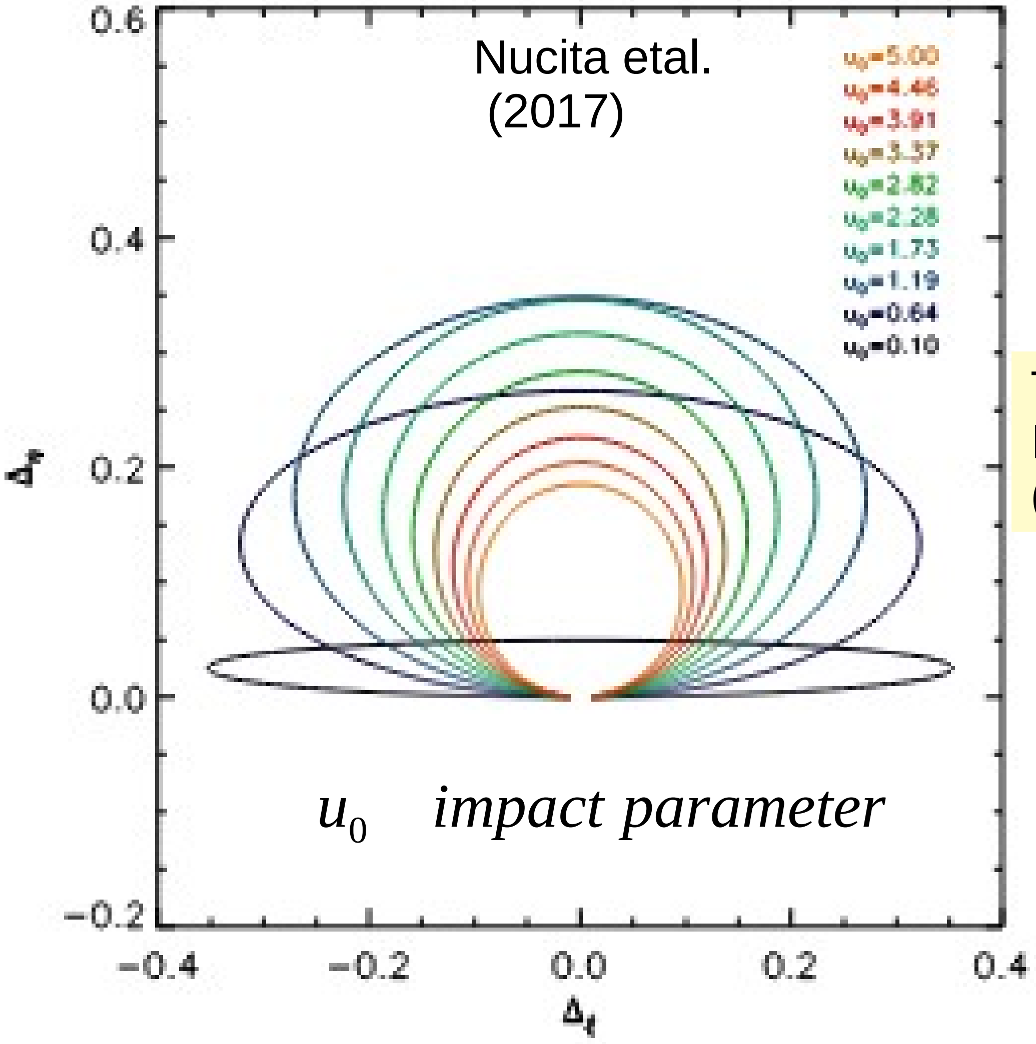
The two projected component of Δ are:

$$\Delta_\xi = \frac{t - t_0}{t_E(2+u^2)}$$

$$\Delta_\eta = \frac{u_0}{(2+u^2)}$$



$$u^2 = u_0^2 + v^2 t^2$$



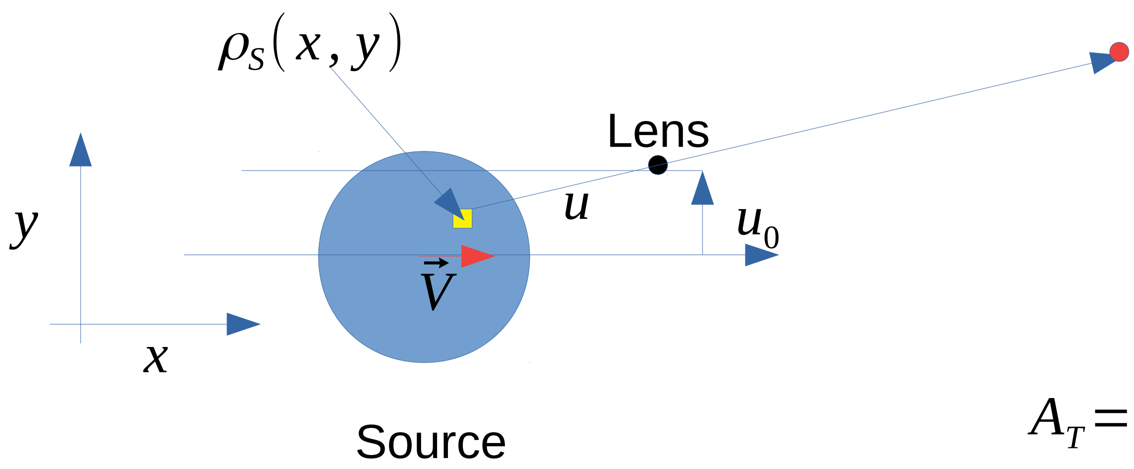
The astrometric effect may increase with
Increasing Impact parameter
(Unlike the amplification)

Lensing by point mass lens:some interesting problems

The extended source problem: the source is not a point
The source has a finite size and surface brightness profile

The moving observer: the effect of the earth orbital motion

The extended source problem



$$\delta A_I = \rho_S(x,y)\, A(u)\, dxdy$$

$$A_T = \int \delta A_I = \int \rho_S(x,y)\, A(u)\, dxdy$$

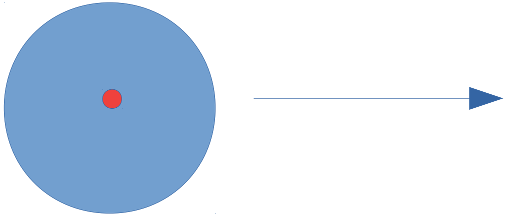
$$u^2 = (x + Vt)^2 + (y - u_0)^2$$

The extended source problem

$$A_T = \int \rho(x, y) A(u)\, dxdy$$

$$A(u) = \frac{u^2 + 2}{u\sqrt{u^2 + 4}}$$

No real singularity: constant circular area at center in polar coordinates



$$A_0 \rightarrow \int_0^R \frac{\sqrt{u^2 + 2}}{u\sqrt{u^2 + 4}}\, u\, du$$

Write a numerical code to integrate over the source

Constant brightness

Limb darkening for stars (color effects ?)

General method to reconstruct the density profile of the source?

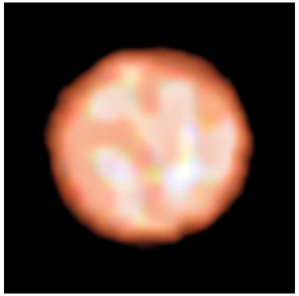
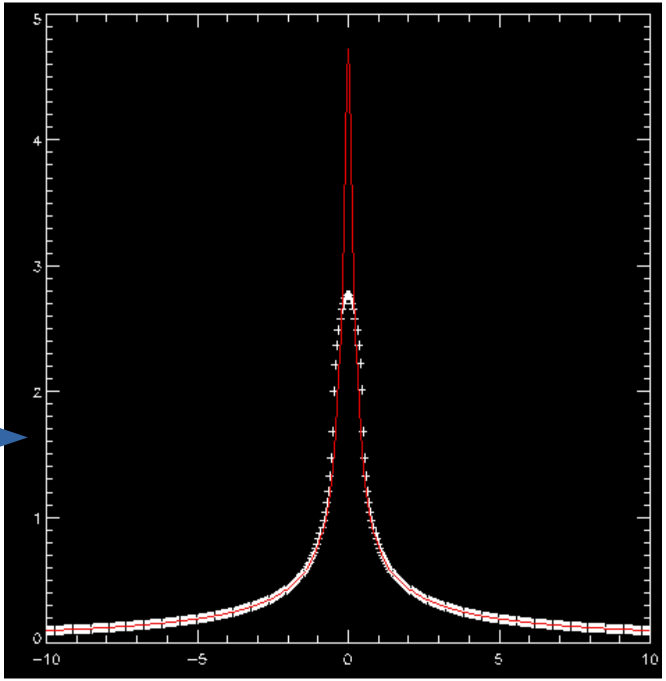
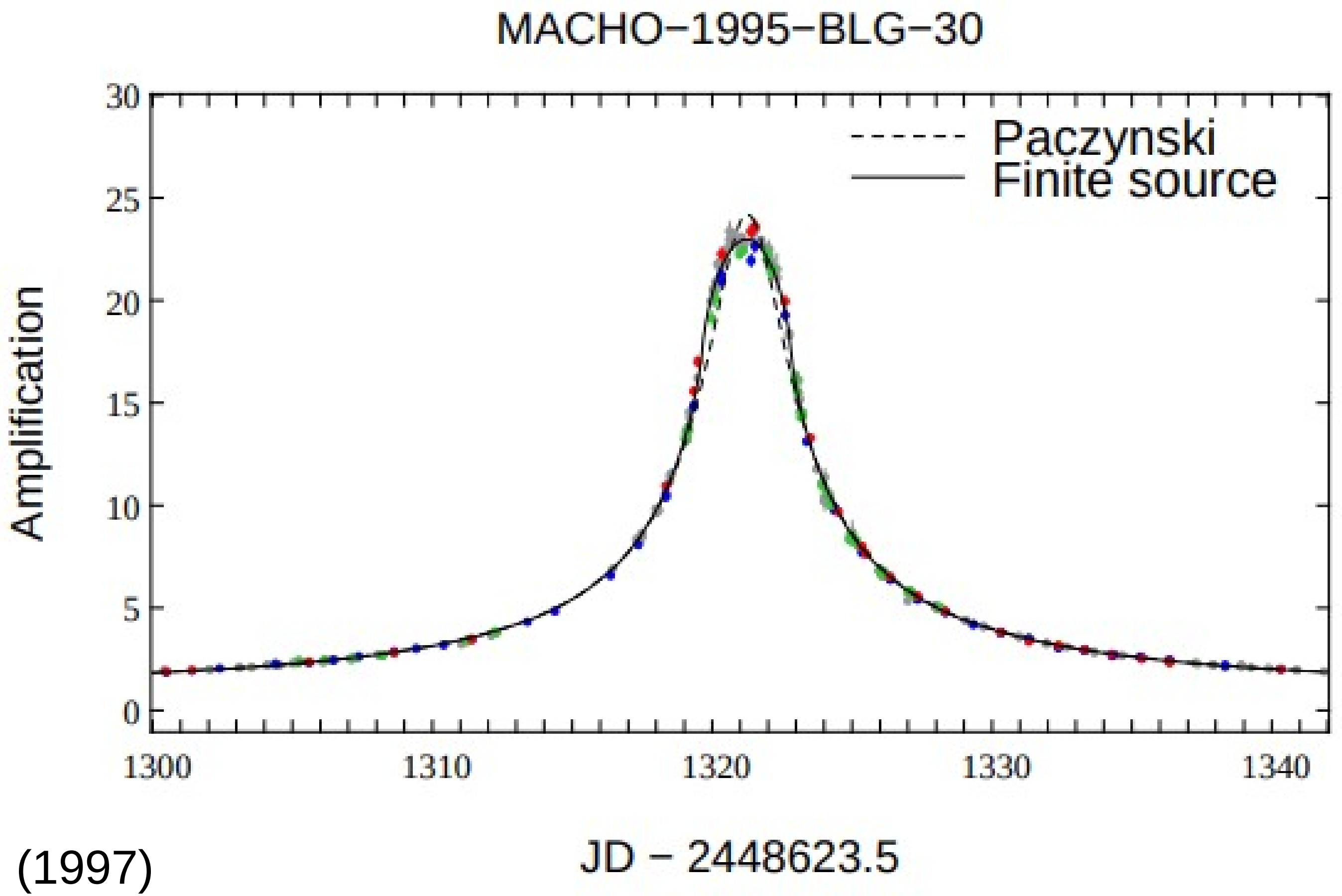


Illustration of the effect



A first case showing finite source size effect

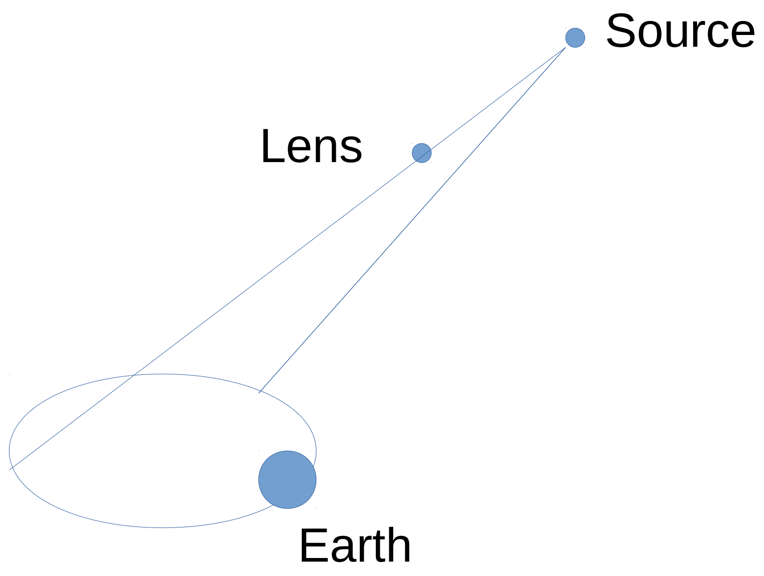


Alcock *etal.* (1997)

Other interesting problem for point mass lenses

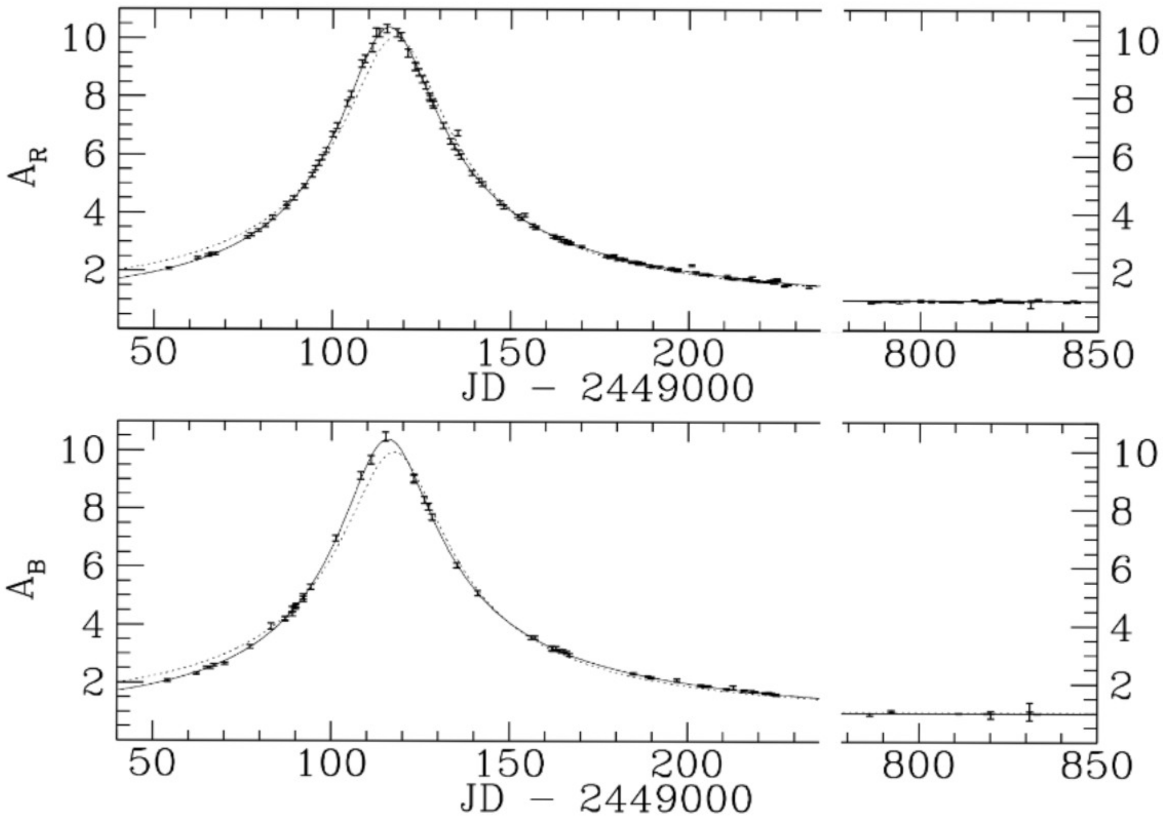
The effect of the earth orbital motion

Parallax effect for the longer microlensing events



The earth motion change the line of sight and the impact parameter: estimate the effect

The first parallax event



Alcock etal. (1995)

The solution with parallax

Velocity: 75 ± 5 km s-1

angle of 28° ± 4°

Dlens = 1.7+1.1−0.7 kpc

M = 1.3+1.3−0.6M☉

continuous line: fit with parallax

dotted line: fit without parallax