Field theoretical formulations of MOND-like gravity

Modified gravity at astrophysical scales

Jean-Philippe Bruneton

Gilles Esposito-Farèse

bruneton@iap.fr
gef@iap.fr

Field theoretical formulations of MOND-like gravity

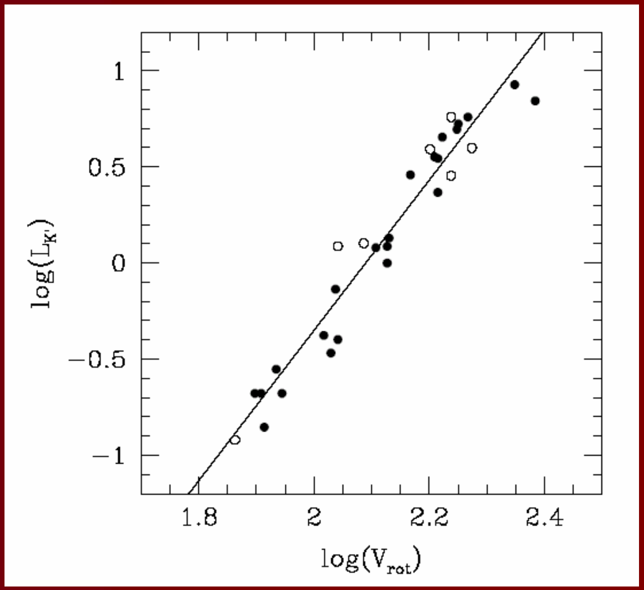
- Introduction - MOND's phenomenology
- Model building
- MOND as K-essence models
- The problem of light deflection
- Nonminimal metric couplings

Introduction

MOND from the Tully-Fisher's law

- Rotation curves

$V^4 \sim L_b$ [Tully-Fisher's law]



Kinematically: $a^2 \sim L_b / r^2$

Thus: $a^2 = G\, M_b\, a_0 / r^2$

[Degeneracy with the ratio M/L]

• Possible modification of the Newton's law of gravity beyond an universal scale of acceleration a_0

• $V^4_{rot} = G\, M\, a_0$

Non relativistic MOND

[Milgrom 1983]

- Modification of inertia:

$$m\, a\, \mu(|a|/a_0) = \Sigma F$$

With $\mu(x>>1) = 1$ and $\mu(x<<1) = x$

[Does not respect the usual conservation law of energy and momentum; Felten 1984]

- Modification of the gravitational force:

$$g = a_0\, f\, [GM_b/a_0 r^2]$$

With $f(x>>1) = x$ and $f(x<<1) = \mathrm{Sqrt}[x]$

[Inequivalent theories]

MOND vs CDM

- In a sense, MOND can be interpreted as providing an universal profile of dark halos:

$$a_0 \, f[GM_b/a_0r^2] = G \, M_{DM}(r) \, / \, r^2$$

- But standard CDM does not involve this universal scale a_0
- a_0 is of order of H_0 => cosmological origin of a_0 ?
- Prediction of MOND:
  - LSB galaxies are DM dominated, no DM in HSB galaxies
  - No DM in the center of galaxies (cusp problem in CDM)
  - Existence of a correlation between baryonic and dark matter [MacGaugh 2005]

  - More details : Sanders&McGaugh 2002

MOND's phenomenology

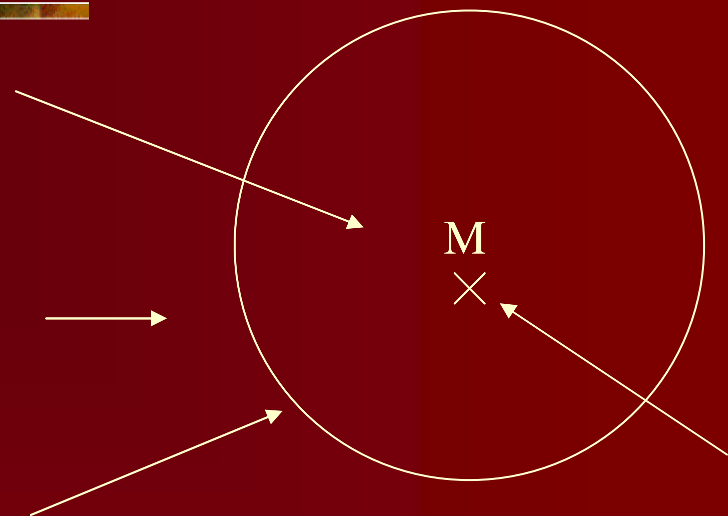
- Newtonian regime:

  $a \gg a_0$, $a = GM/r^2$

- MOND regime:

  $g \ll a_0$, $a = (GMa_0)^{1/2} / r$

- Transition : $r_M \sim (GM/a_0)^{1/2}$



GR (strong fields)

Fits: $a_0 \sim 1.2\ 10^{-10}\ m.s^{-2} \sim c\ H_0$

Model building

General Relativity

- GR's action: $S = S_{\text{gravity}} + S_{\text{matter}}$ $S_{\text{gravity}} = \frac{c^4}{16\pi G}\int \frac{d^4x}{c}\sqrt{-g_*}R^*$ $S_{\text{matter}}[\psi; \tilde{g}_{\mu\nu}]$

  And: $\tilde{g}_{\mu\nu} = g^*_{\mu\nu}$
- Options:
  - Modify the kinetic term R* and/or add new gravitational fields that couple to matter and/or spin-2 field g*: « modified gravity »
  - Consider that $\tilde{g}_{\mu\nu} \neq g^*_{\mu\nu}$ « modified inertia »

- [Very imprecise terminology. E.g. scalar-tensor theories modify inertia in the Einstein frame but modify gravity in the Jordan frame…]
- This difficulty is deeply rooted in the fact that both inertia an gravity are described by the same entity, namely the metric. Thus, this is a consequence of the weak equivalence principle: locally, inertia and gravity cannot be distinguished.

« Modified Inertia »
[Milgrom]

- In a non-metric context, and notably non-relativistic context, modifying inertia has an intrinsic meaning.
- Milgrom considered point particles actions that may depend on higher derivatives of the position.
- He showed that Galilean covariance + MOND requires the action to be non local. Stability and causality may be ok.

- But the relativistic generalization seems not straightforward. This may lead to a non metric theory.

Higher order gravity

- One loop divergences of quantized GR generate terms proportional to Riemann squared. [t'Hooft & Veltman 1974]
- Such terms may thus be naturally added to the classical action:

$$S_{\rm gravity} = \frac{c^4}{16\pi G}\int \frac{d^4x}{c}\sqrt{-g}\;\left[R + \alpha\, C^2_{\mu\nu\rho\sigma} + \beta\, R^2 + \gamma\, {\rm GB}\right]$$

- These theories are however unstable. The schematic propagator may indeed be written as:

$$\frac{1}{p^2+\alpha p^4} = \frac{1}{p^2} - \frac{1}{p^2+1/\alpha}$$

- Negative energy (or ghost) d.o.f. ! Generically, this happens for all Lagrangians of the form $f(R, R_{\mu\nu}, R_{\mu\nu\rho\sigma})$ except GR and f(R) theories.

Avoiding the ghost?

- The GB term does not provide any d.o.f around the Minkowski metric.
- Theories of the form $R + f(\mathrm{GB})$ may thus avoid the ghost around flat spacetimes [Elizalde&Odintsov,Van Acoleyen&Navarro]
- But flat spacetime is generically *not* the vacuum solution!
- Moreover, even if the ghost d.o.f. does not appear around any background, the theory is still unstable, because of Ostrogradski's theorem

Ostrogradski's theorem

- Consider for instance $\mathcal{L}(q,\dot{q},\ddot{q})$
- Then define the canonical variables and momentas:

  $q_1 \equiv q$ $\quad p_1 \equiv \frac{\partial \mathcal{L}}{\partial \dot{q}} - \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \ddot{q}}\right)$ and $q_2 \equiv \dot{q}$ $\quad p_2 \equiv \frac{\partial \mathcal{L}}{\partial \ddot{q}}$

- The Hamiltonian reads $\mathcal{H} \equiv p_1 \dot{q}_1 + p_2 \dot{q}_2 - \mathcal{L}(q,\dot{q},\ddot{q})$

  Inverting, we find $\mathcal{H} = p_1 q_2 + p_2 f(q_1,q_2,p_2) - \mathcal{L}(q_1,q_2,f(q_1,q_2,p_2))$
- This is linear in p_1, and thus not bounded by below

(Mono)scalar-tensor theories

- General action: $S = \frac{c^4}{4\pi G} \int \frac{d^4x}{c} \sqrt{-g_*} \left\{ \frac{R^*}{4} - \frac{1}{2} g_*^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) \right\} + S_{\text{matter}}[\psi; \tilde{g}_{\mu\nu} = A^2(\varphi) g^*_{\mu\nu}]$

- L=F(R*) theories are particular cases of ST theories.
- This may not lead to MOND's phenomenology in general. Indeed
  - If V has a negligible influence, then $\varphi \propto GM/rc^2$
  - If V has a minimum => Yukawa $\varphi \propto GMe^{-mr}/rc^2$
  - If $V(\varphi) = -2a^2 e^{-b\varphi}$, $\varphi = (2/b)\ln(abr)$ is a solution. But b is independent of M.

MOND as K-essence models

Non relativistic field theory of MOND

[Bekenstein-Milgrom 1984]

- Newtonian action with an unusual « kinetic » term (Aquadratic Lagrangian, or AQUAL)

$$S = -\int \frac{a_0^2}{8\pi G}\mathcal{F}\left(\frac{(\nabla\varphi)^2}{a_0^2}\right) + \rho\varphi$$

$$\nabla\left(\nabla\varphi\mu\left(\frac{\nabla\varphi}{a_0}\right)\right) = 4\pi G\rho$$

- Modified Poisson equation
- Galilean covariance + least action principle

  => Noether's theorem holds

  => Conservation of energy-momentum

Relativistic generalization
[Bekenstein-Milgrom 1984]

- « Detection » of DM by weak-lensing + MONDian cosmology? => the need for a relativistic theory of MOND
- Relativistic AQUAL (RAQUAL)
  - Einstein Hilbert action for the metric g*
  - K-essence scalar field (aquadratic kinetic term)
  - Matter couples to a second metric, conformally related to g*.
  - [Finally, this is a scalar-tensor theory with a non standard kinetic term]

$$S = \frac{c^4}{4\pi G}\int \frac{d^4x}{c}\sqrt{-g_*}\left\{\frac{R^*}{4} - \frac{1}{2}f(s,\varphi) - V(\varphi)\right\} + S_{\rm matter}[\psi; \tilde{g}_{\mu\nu} = A^2(\varphi) g^*_{\mu\nu}]$$

$$s \equiv g_*^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$$

MOND as a RAQUAL model

- With: $f(s,\varphi) = f(s)$ $V(\varphi) = 0$ $A(\varphi) = \exp(\alpha\varphi)$

$$\nabla^*_\mu \left[f'(s)\nabla^\mu_*\varphi\right] = -\frac{4\pi G}{c^4}\,\alpha\, T^*,$$

- Hence the μ function reads: $\mu\left(\frac{\nabla\varphi}{a_0}\right) = \frac{df}{ds} \equiv f'(s)$

- MOND is thus recovered with any smooth function f such that

$f'(s) \sim \sqrt{s}$ for $s \ll 1$, and $f'(s) \sim 1$ for $s \gg 1$

Theoretical considerations (1)

- Stability:
  - The condition $f'(s) > 0$ is necessary for the Hamiltonian to be bounded by below
  - But not sufficient
- Hyperbolicity of the field equation:
  - The scalar field propagates along the effective metric $G^{\mu\nu} \equiv f' g_*^{\mu\nu} + 2 f'' \partial^\mu \varphi \partial^\nu \varphi$ ie $G^{\mu\nu} \nabla_\mu^* \nabla_\nu^* \varphi = \text{sources}$
  - Lorentzian signature of G => $f'(s) + 2s f''(s) > 0$

- These two conditions are sufficient for the Hamiltonian to be bounded by below

Theoretical considerations (2)

- The scalar field propagates superluminally if $f''(s) > 0$

  And MOND requires $f'(s) \sim \sqrt{s}$ for $s \ll 1$

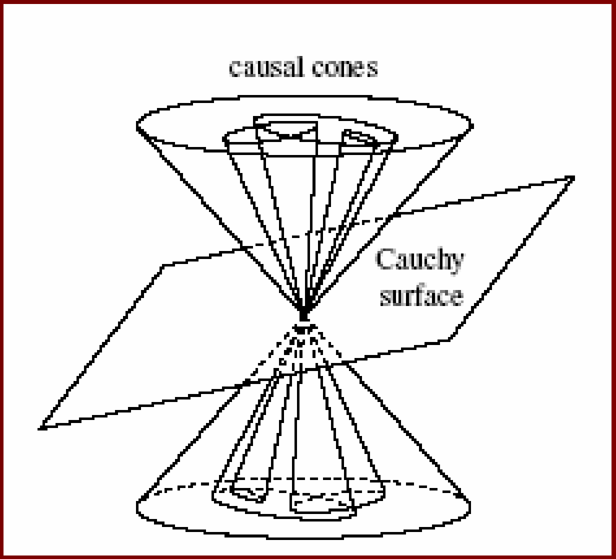
  => Superluminal propagations are unavoidable (in that framework)

- Does it threaten causality? Should we impose $f''(s) \leq 0$?

  [See Bekenstein 1984, Aharonov et al 1969, Adams et al: hep-th 0602178]

Causality in a « multi-metric » scenario

- Let's consider various fields that propagate along non conformally related metrics
- Cauchy surfaces always exist if the union of causal cones has a non vanishing exterior.
- Then general theorems (depending on the precise form of the field equation) ensure that the Cauchy problem is well-posed, ie that the theory is causal.
- Notably, k-essence theories are well-posed if $f'(s) > 0$ $f'(s) + 2sf''(s) > 0$



[gr-qc/0607055]

The origin of the controversy

[hep-th/0612113]

- Basic idea: causally connected events must be time-ordered.
- In the relativistic picture of spacetime, any Lorentzian metric defines a local time-ordering [or chronology]
- There are as many notions of causality as there are non-conformally related metrics.
- … and thus, there is no reasons to favor the chronology induced by the propagation of the gravitational or EM field, etc.
- Which field propagates faster or slower than the others (if any), is thus only an experimental question, not a theoretical one.

Phenomenological considerations (1)

- Superluminal behaviors do not ruin RAQUAL models
- But MOND requires that $f'(s) \sim \sqrt{s} \text{ for } s \ll 1$

  and thus f '(0) = 0
- The Cauchy problem is not well-posed at s=0, i.e. at the transition between local and cosmological scales.
- The theory needs to be cured by introducing a new parameter:

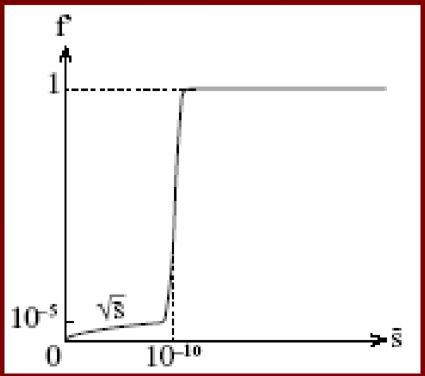
$$f'(s) \sim \varepsilon + \sqrt{s} \text{ for } s \ll 1$$

- Thus at large distance the potential is Newtonian again, with a renormalized gravitational constant G/ε.
- Rotation curves then decline at radius $r = \sqrt{GM/a_0\varepsilon^2}$

Phenomenological considerations (2)

- The conformal coupling implies $|\gamma^{\mathrm{PPN}} - 1| = 2\alpha^2/(1+\alpha^2)$
- Solar system experiments: $\alpha^2 < 10^{-5}$
- But the extra MOND force starts manifesting at $r_{\mathrm{trans}} = \alpha^2 \sqrt{\frac{GM}{a_0}}$

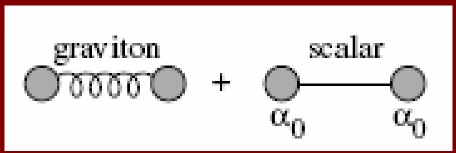
  whose value is 0.1 AU. Excluded by test of Kepler's law.
- Unless one tunes the free function f '



The problem of light deflection

The position of the problem

- The EM sector is conformally invariant and thus unsensitive to the scalar field strength in (ST or) RAQUAL models.
- Thus the light bending reads $\Delta\theta = \frac{4GM}{bc^2}$ like in GR.
- But the effective gravitational constant (measured by Cavendish experiments) reads $G_{\rm eff} = G(1+\alpha_0^2)$
- Thus the above RAQUAL models actually predict less light bending than GR.



Disformal coupling

[Bekenstein-Sanders 94, Sanders 97]

- Simple solution: consider $\tilde{g}_{\mu\nu} = A^2(s,\varphi) g^*_{\mu\nu} + B(s,\varphi)\partial_\mu\varphi\partial_\nu\varphi$
- But increasing light deflection needs B>0 => gravitons are superluminal

- Or, by introducing a unit timelike vector field: $\tilde{g}_{\mu\nu} = A^2(\varphi) g^*_{\mu\nu} + B(\varphi) u^*_\mu u^*_\nu$
- One gets the right amount of light deflection if $-\tilde{g}_{00} \sim \tilde{g}_{rr}^{-1}$ in Schwarzschild coordinates.

- Using $u^\mu_* = (1,0,0,0)$ $g^*_{\mu\nu} + u^*_\mu u^*_\nu \sim g^*_{ij}$ $-u^*_\mu u^*_\nu \sim g^*_{00}$

define $\tilde{g}_{\mu\nu} = -e^{2\alpha\varphi} u^*_\mu u^*_\nu + e^{-2\alpha\varphi}(g^*_{\mu\nu} + u^*_\mu u^*_\nu)$ TeVeS [Bekenstein 2004]

PS : LISA ?

Theoretical considerations

- Coupling a vector field to matter may lead to difficulties:

$$\nabla^*_\mu(\partial^{[\mu}u^{\nu]}) \propto \frac{\sqrt{-\tilde{g}}}{\sqrt{-g}}\tilde{T}^{\rho\nu}u^*_\rho \quad \Rightarrow \quad \nabla^*_\nu(\sqrt{-\tilde{g}}/\sqrt{-g}\tilde{T}^{\rho\nu}u^*_\rho) = 0$$

- But enforcing the vector field to have a fixed norm with a Lagrange multiplier cures the problem
- Full action: $S = \int \mathcal{L} d^4x = -\frac{Kc^3}{32\pi G}\int \sqrt{-g^*}d^4x\,(g_*^{\mu\rho}g_*^{\nu\sigma}F_{\rho\sigma}F_{\mu\nu} - 2\lambda(g_*^{\mu\nu}u_\mu u_\nu + 1))$
- Constraint: $u^*_\mu u^\mu = -1$
- Problem: the Hamiltonian is not bounded by below!

$$H = \int d^3x \left[\frac{4\pi G}{Kc^3}\pi^2 + \frac{Kc^3}{4\pi G}(\nabla \times \mathbf{u})^2 + \sqrt{1+\mathbf{u}^2}\nabla\pi\right]$$

[Clayton&Moffat]

Phenomenological considerations (1)

- Gravitons propagate slower than light if $\alpha\varphi \geq 0$
- In fact the only known experimental constraint is in favor of superluminal propagation of gravitons.
- Indeed matter that propagates faster than gravitons may emit gravitational waves by a « gravi-Cerenkov » process [Elliot et al. hep-ph/0106220, hep-ph/0505211]

$$\frac{dE}{dt} \approx \frac{Gp^4(n-1)^2}{\hbar^2 c_{\text{light}}} \qquad L \sim \frac{\hbar^2 c_{\text{light}}^3}{Gp_f^3(n-1)^2} \qquad p_0 = n|\mathbf{p}|$$

- Thus $n - 1 \lesssim 10^{-16}$, ie $C_{light} < C_{grav}$ ou $C_{light} / C_{grav} < 1 + 10^{-15}$ using UHECR

  Here: $n - 1 = 2\alpha\varphi$

Phenomenological considerations (2)

- Schwarzschild-like metric => $\gamma^{\mathrm{PPN}} = 1$
- No solar-system constraints on α
- But this disformal theory behaves as a scalar-tensor one in strong fields. Thus the way the scalar waves extract energy of binary-pulsar is known even if dynamics of the scalar is subtler in MOND than in ST at large distances.
- Binary-pulsar observations thus impose $\alpha^2 < 4 \times 10^{-4}$
- The fine-tuning problem of the function f ’ is recovered

Nonminimal metric couplings

First idea (1)

- Consider the action $$S = \frac{c^4}{16\pi G}\int \frac{d^4x}{c}\sqrt{-g_*}\,R^* + S_{\text{matter}}[\psi;\tilde{g}_{\mu\nu}]$$
- The metric and its derivatives can be combined to get a local access to the mass and the radius.
- Then define for instance

$$\tilde{g}_{\mu\nu} \equiv g^*_{\mu\nu} + \frac{\sqrt{a_0/3}}{4\,c}\,\frac{(\partial_\lambda \text{GB})^2\,h(X)\,g^*_{\mu\nu} + 2\,\partial_\mu\text{GB}\,\partial_\nu\text{GB}}{(\Box^*\text{GB}/10)^{7/4}}$$

$$X \equiv \frac{1}{\ell_0}\sqrt{\frac{30\,\text{GB}}{\Box^*\text{GB}}}$$

- Hence MOND is entirely coded in the (nonminimal) coupling to matter.
- Great advantage : the theory is purely GR in vacuum.

First idea (2)

- Thus a massive spherical body generates the Schwarzschild solution for g*.
- In that case the matter metric reproduces the MOND's phenomenology (with $h(X) = (1+X)^{-1} + \ln(1+X)$)

- The deadly problem, however, is the (un)stability. This is in fact an higher order gravity theory => Ostrogradski's theorem

Nonminimal scalar-tensor model (1)

- The above instability may be avoided with the help of a scalar field
- Very similar idea :

$$
\begin{aligned}
S &= \frac{c^4}{4\pi G}\int \frac{d^4x}{c}\sqrt{-g_*}\left\{\frac{R^*}{4} - \frac{1}{2}s - \frac{1}{2}\left(\frac{mc}{\hbar}\right)^2\varphi^2\right\} + S_{\text{matter}}[\psi; \tilde{g}_{\mu\nu}],\\
s &\equiv g_*^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi,\\
\tilde{g}_{\mu\nu} &\equiv \left[e^{\alpha\varphi} - \frac{\varphi X}{\alpha}h(X)\right]^2 g^*_{\mu\nu} - 4\,\frac{\varphi X}{\alpha}\,\frac{\partial_\mu\varphi\partial_\nu\varphi}{s},\\
X &\equiv \frac{\sqrt{\alpha a_0}}{c}\,s^{-1/4},\\
h(X) &= (1+X)^{-1} + \ln(1+X).
\end{aligned}
$$

- I.e. a disformal-like theory, but with a quadratic kinetic term
- Purely Brans-Dicke theory in vacuum

Nonminimal scalar-tensor model (2)

- We may thus expect that $\varphi \approx -\alpha GM/rc^2$ outside matter
- In that case the matter metric reads:

$$\tilde{g}_{\mu\nu} = \left[1 - \frac{2\alpha^2 GM}{rc^2} + \frac{2\sqrt{GMa_0}}{c^2} h\left(\frac{r}{\sqrt{GM/a_0}}\right)\right] g^*_{\mu\nu} + \frac{4\sqrt{GMa_0}}{c^2}\delta^r_\mu \delta^r_\nu + \mathcal{O}\left(\frac{1}{c^4}\right)$$

- Ostrogradski's theorem ?
- The Christoffel symbols of the matter metric involve secon derivative of the scalar field.
- But: gauge bosons are described by one-forms, and thus their action does not involve the Christoffel symbols (e.g. EM)
- And the action of fermions depends on the derivative of the metric (or more precisely of the tetrad field), but only linearly

$$\bar{\psi}\tilde{g}^m \partial\!\!\!/ \psi + \bar{\psi}\tilde{g}^n (\partial\!\!\!/ \tilde{g})\psi$$

Consistency of the scalar field equation within matter (1)

- The matter metric is of the type $\tilde{g}_{\mu\nu} = A^2(s,\varphi) g^*_{\mu\nu} + B(s,\varphi)\partial_\mu\varphi\partial_\nu\varphi$
- The scalar field equation is then

$$\sqrt{-g^*}\nabla^*_\mu \nabla^\mu_* \varphi - \frac{4\pi G}{c^4}\partial_\mu \left[\sqrt{-\tilde{g}}\left(B\tilde{T}^{\mu\nu} + \frac{2}{A}\frac{\partial A}{\partial s}\tilde{g}_{\rho\sigma}\tilde{T}^{\rho\sigma} g^{\mu\nu}_* + \left(\frac{\partial B}{\partial s} - \frac{2B}{A}\frac{\partial A}{\partial s}\right)\tilde{T}^{\rho\sigma}\partial_\rho\varphi\partial_\sigma\varphi g^{\mu\nu}_*\right)\partial_\nu\varphi\right]$$
$$= -\frac{4\pi G}{c^4}\sqrt{-\tilde{g}}\left[\frac{1}{A}\frac{\partial A}{\partial \varphi}\tilde{g}_{\rho\sigma}\tilde{T}^{\rho\sigma} + \left(\frac{1}{2}\frac{\partial B}{\partial\varphi} - \frac{B}{A}\frac{\partial A}{\partial\varphi}\right)\tilde{T}^{\rho\sigma}\partial_\rho\varphi\partial_\sigma\varphi\right]$$

- Its hyperbolcity is thus an quite involved question.
- Simple approach: the case of an perfect pressureless fluid. Then the effective kinetic term of the scalar field is simpler and reads $L \propto s + \frac{8\pi G\rho_*}{c^4} A\sqrt{1 - \frac{(U^\mu\partial_\mu\varphi)^2}{A^2}}$

Consistency of the scalar field equation within matter (2)

- The negative sign of B is such that the time-time component of the effective metric G_00 is even more negative.
- Specializing to a point-like mass surounded by tenuous gas, hyperbolicity requires
$$1 + \frac{8\pi G\rho_*}{c^4} A'(s) > 0$$
$$1 + \frac{8\pi G\rho_*}{c^4} (A'(s) + 2sA''(s)) > 0$$
- Unfortunately MOND requires $A(s) \sim s^{-1/4}$
- Hence A' and A'' are of opposite signs. Moreover the order of magnitude are such that the scalar field equation may become non hyperbolic in the tenuous gas in the outskirts of galaxies!

Solution: fine-tuning?

- Add a large and positive term in A(s), that does not spoil the MOND phenomenology. E.g : $(\phi X/\alpha)^{-2}$
- Problem: this term actually dominates the source term, and the scalar field may not be such that $\varphi \approx -\alpha GM/rc^2$
- Solution: keep this term in tenuous gas surrounding a galaxy, but kill it inside dense matter. $(\phi X/\alpha)^{-2} g(X)$
- The best model (but very fine tuned) we can obtain is however such that the scalar field is only generated by the center of extended bodies.

Conclusions

- The success of MOND's phenomenology may signal a breakdown of Newtonian gravity at small accelerations
- Any competing model of DM should therefore explains the success of MOND, and notably the existence of an universal acceleration scale
- K-essence models nicely embed the MOND paradigm in a relativistic field formulation. But the actual difficulty is to reproduce the light deflection.
- TeVeS-like models suffer from unstabilities of the vector field. Moreover the free function must be fine-tuned.
- Scalar disformal models - and nonminimal metric couplings – that reproduce MOND's phenomenology generically lead to nonhyperbolic equation for the scalar field within matter. Fine-tuning is thus also required.
- To date thus, no consistent relativistic theories of MOND exists.
- These difficulties may signal that MOND needs a more general framework than (pseudo-)Riemannian geometry. Finsler geometry? Nonlinear realization of local symetries? Etc.