Why is the Planck satellite necessary for inflationary cosmology?

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Based on C. Pahud, A. Liddle, P. Mukherjee, and D. Parkinson, astro-ph/0605004 & astro-ph/0701481;

J. Barrow, A. Liddle, and C. Pahud, astro-ph/0610807;

and ongoing work C. Pahud, M. Kamionkowski, and A. Liddle.

Spectral index and Running

One of the key goal of cosmology is to probe the nature of the primordial perturbations, for instance to seek support for the inflationary cosmology.

- Parameters of interest: n_S, $\alpha \equiv dn_s/d\ln k$, r, …

$$P_S(k) = A_S \cdot (k/k_0)^{n_S(k) - 1}$$

$$n_S(k) = n_S(k_0) + \alpha \ln (k/k_0)$$

General point of view

In these papers, we carry out model selection forecasting for the Planck satellite, focussing on its ability to measure the scalar spectral index n_s and its running α. For this purpose, we are considering the three following models:

- M_0) A flat, Harrison-Zel'dovich model with a cosmological constant.
  Parameters: Ω_B, Ω_{cdm}, τ, h, A_S
- M_1) The same as M_0, except allowing n_S to vary in the range of 0.8 - 1.2.
- M_2) The same as M_1, except allowing α to vary in the range of -0.1 - 0.1.

Outlook

- Introduction
  - Bayesian model selection technique
  - WMAP3 vs the Planck satellite
  - Tensor perturbations
- Model selection forecasts for n_S and α
  - Simulating Planck data
  - Results
- Conclusions

Bayesian model selection technique

- To evaluate the models under assumption, we determine their *evidence*:

$$E(M) \equiv P(D|M) = \int d\theta\ \underbrace{P(D|\theta,M)}_{\text{Likelihood}}\ \underbrace{P(\theta|M)}_{\text{Prior}},$$

- The integral is calculated using a nested sampling algorithm developed by P. Mukherjee, D. Parkinson and A.R. Liddle.

  Website: www.cosmonest.org

Which model is preferred ?

- Finally, in order to compare the models in pairs, we consider the *Bayes factor*:

$$B_{ij} \equiv E(M_i) \,/\, E(M_j) \text{ , for } i,j = 0,1,2 \ (i \neq j)$$

- We use the *Jeffreys' scale* to determine the significance of any difference in evidence between the two models:

$\ln B_{ij} < 1$	=>	not worth more than a bare mention
$1 < \ln B_{ij} < 2.5$	=>	substantial
$2.5 < \ln B_{ij} < 5$	=>	strong to very strong
$5 < \ln B_{ij}$	=>	decisive

WMAP3 vs the Planck satellite

- WMAP3 has measured $n_S = 0.958 \pm 0.016$, which excludes HZ at the 95% level of confidence. However, a statistical approach is still necessary to decisively exclude it.

- In a companion paper (astro-ph/0605003) D. Parkinson, P. Mukherjee and A.R. Liddle arrive to the conclusion, for WMAP3 data:

$$\ln B_{01} = -0.34 \pm 0.26$$

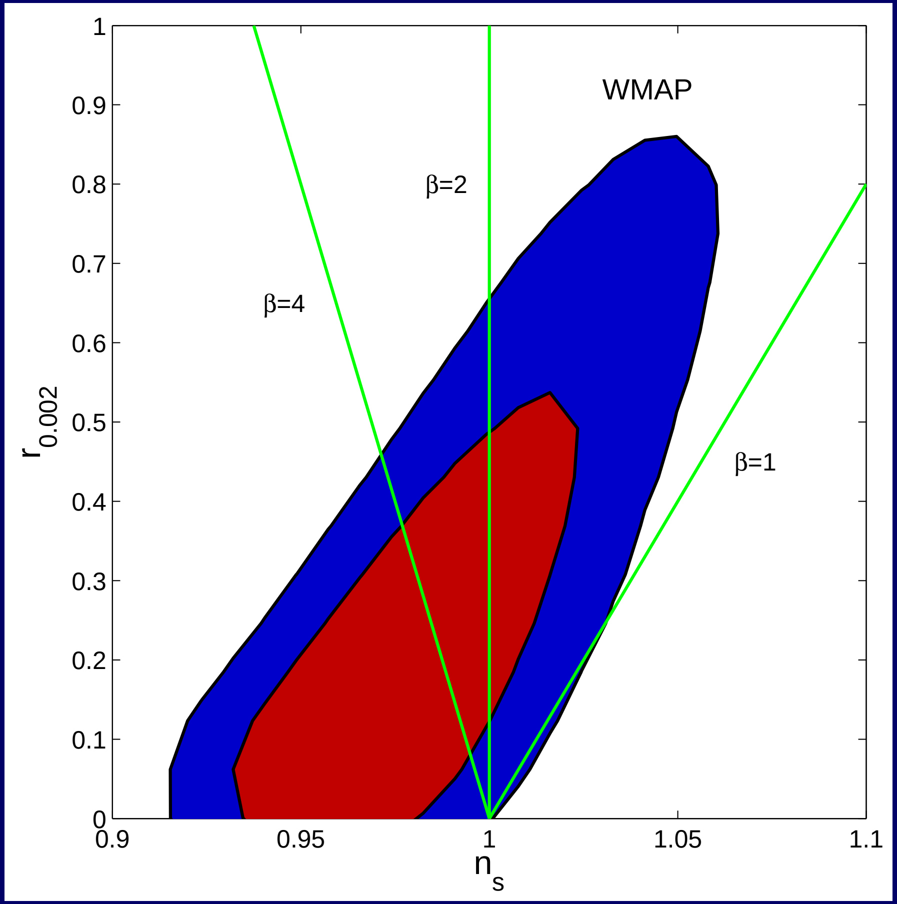
- Planck, on an other hand, will measure n_s with the greatest precision ever:

$$\pm 0.005 \text{ !!}$$

Tensor perturbations

- In presence of tensor perturbations WMAP3 has measured, for a power-law model, $n_S = 0.984^{+0.029}_{-0.028}$
- … and if the running is included in the model, $n_S = 1.16 \pm 0.10$ and $\alpha = -0.085 \pm 0.043$.
- The "Intermediate Inflation" model, $V \propto \phi^{-\beta}$, gives $n_S = 1$ and $r > 0$ providing $\beta = 2$, to first order in slow-roll (J. Barrow & A. Liddle).
- To first order, $n_s = 1 - \dfrac{\beta(\beta-2)}{\phi^2}$ and $r = \dfrac{8\beta^2}{\phi^2}$.

Tensor perturbations



Trajectories for different values of the parameter β in the n_S-r plane, to first-order in slow-roll.

(J. Barrow, A. Liddle, and C. Pahud)

Simulating Planck data

- Our aim is to plot the *Bayes factor* using datasets generated as a function of the parameters of interest, n_S and α.

  Datasets generated with the best-fit WMAP data with n_S in the range of 0.8-1.2 and α between -0.1 and 0.1.

- In doing so, we uncover the regions of this parameters space in which Planck would be able to decisively select between the models, and also those regions where the comparison would be inconclusive.

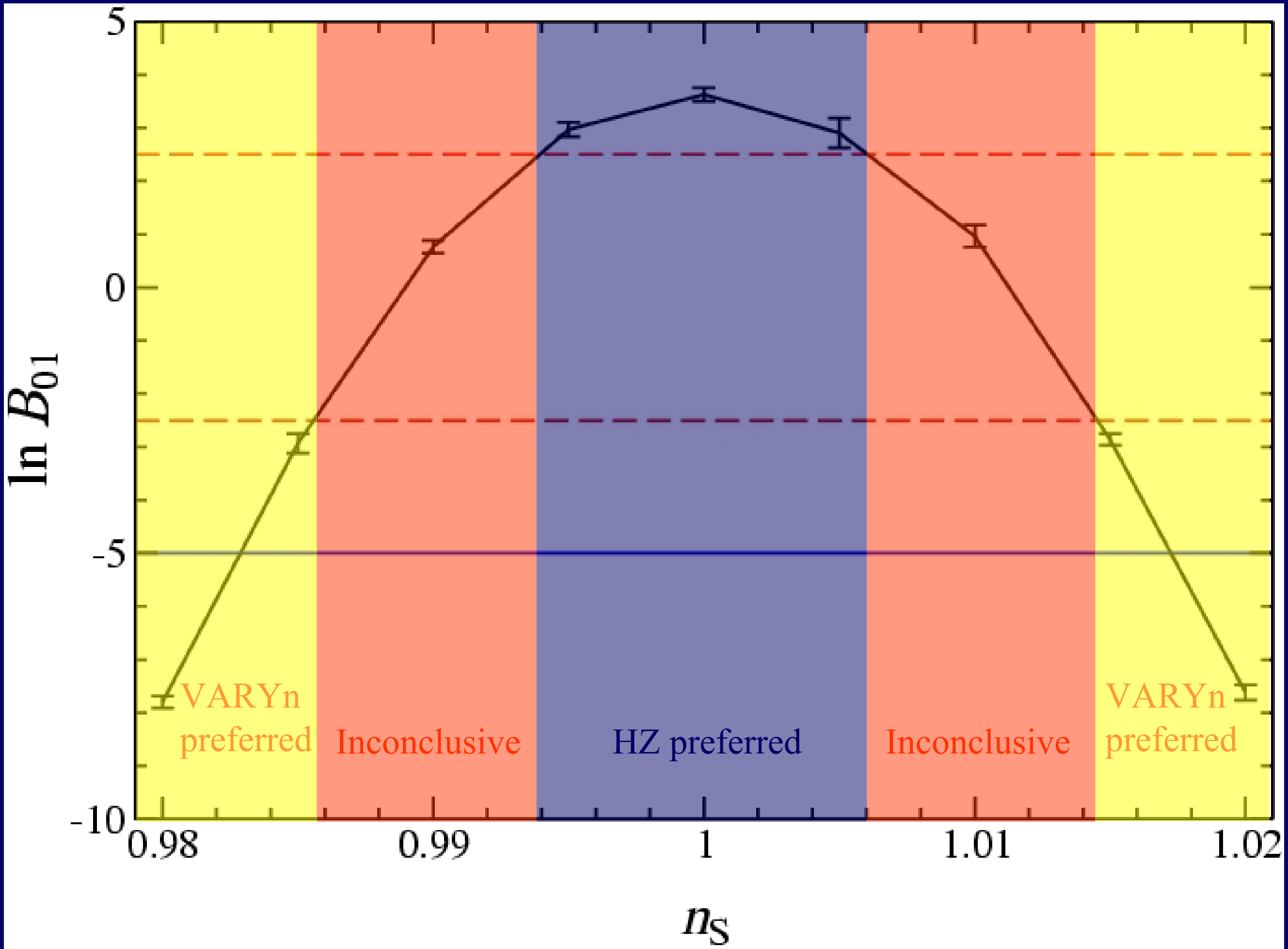
The three models

- M_0 = 5 basic parameters = HZ
- M_1 = 5 basic parameters + n_S = VARYn
- M_2 = 5 basic parameters + n_S + α = VARYn α

$$n_S(k) = n_S(k_0) + \alpha \ln (k/k_0)$$

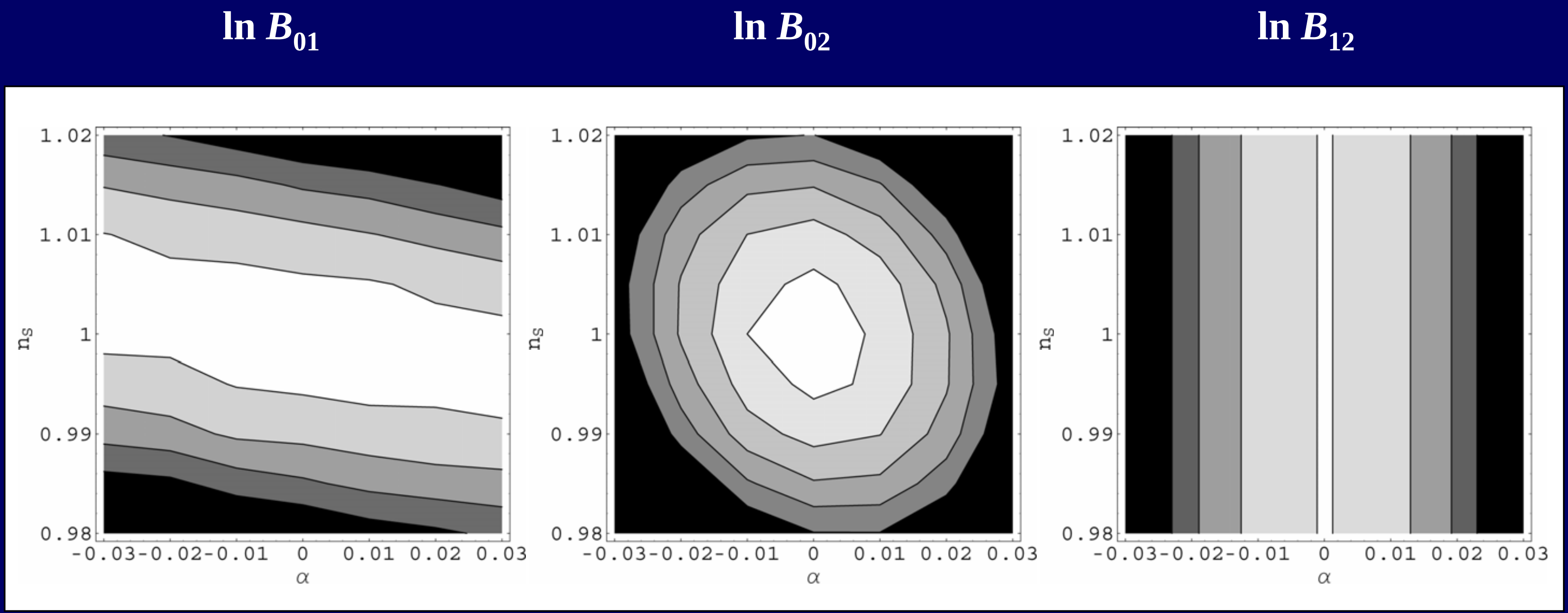
with the pivot scale $k_0 = 0.05\ \mathrm{Mpc}^{-1}$

Results: Spectral index



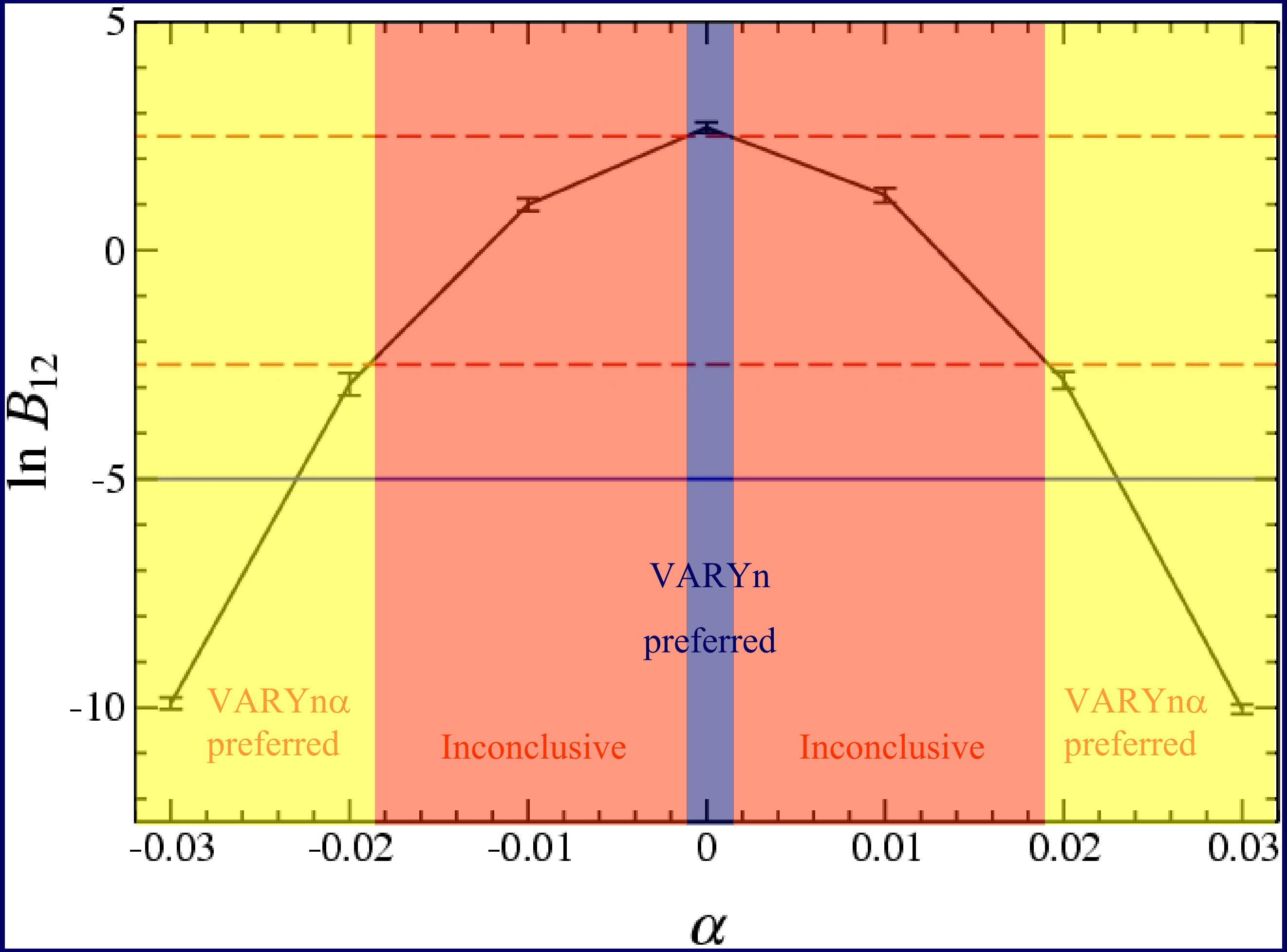
The horizontal lines indicate where the comparison becomes 'strong' (dashed) and 'decisive' (solid) on the Jeffreys' scale.

Spectral index and its running



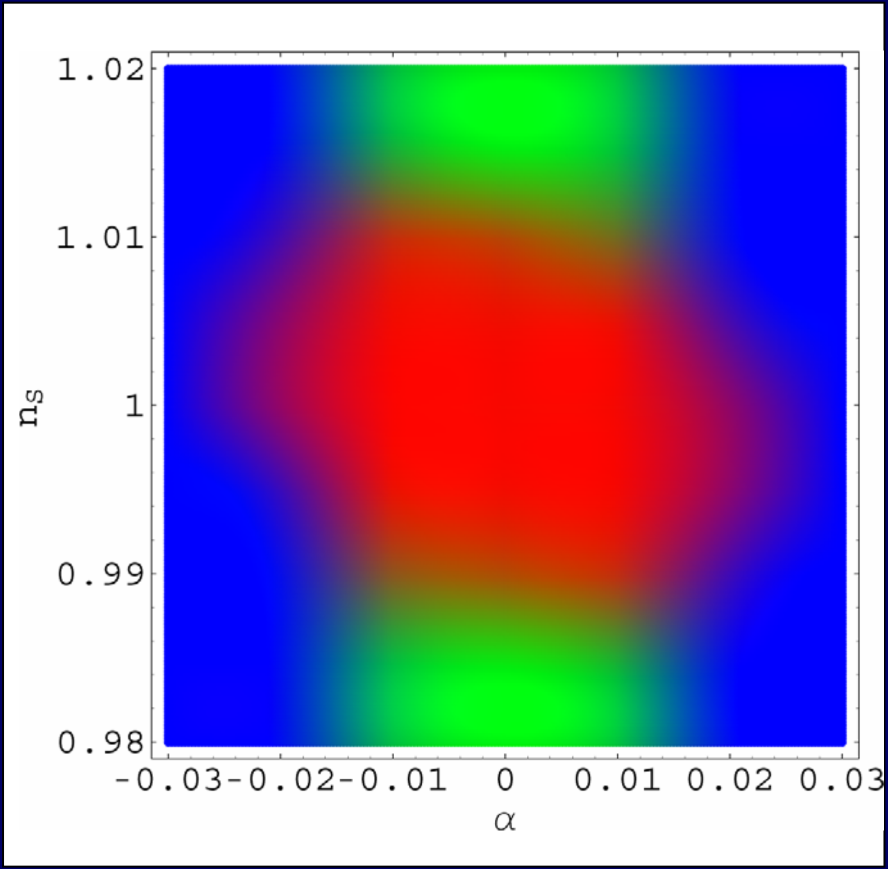
The contour lines represent different steps in the Jeffreys' scale. From the plot centres, the levels are 2.5, 0, -2.5, -5 in the left and right panels, with the centre panel contours starting at 5.

Running of spectral index

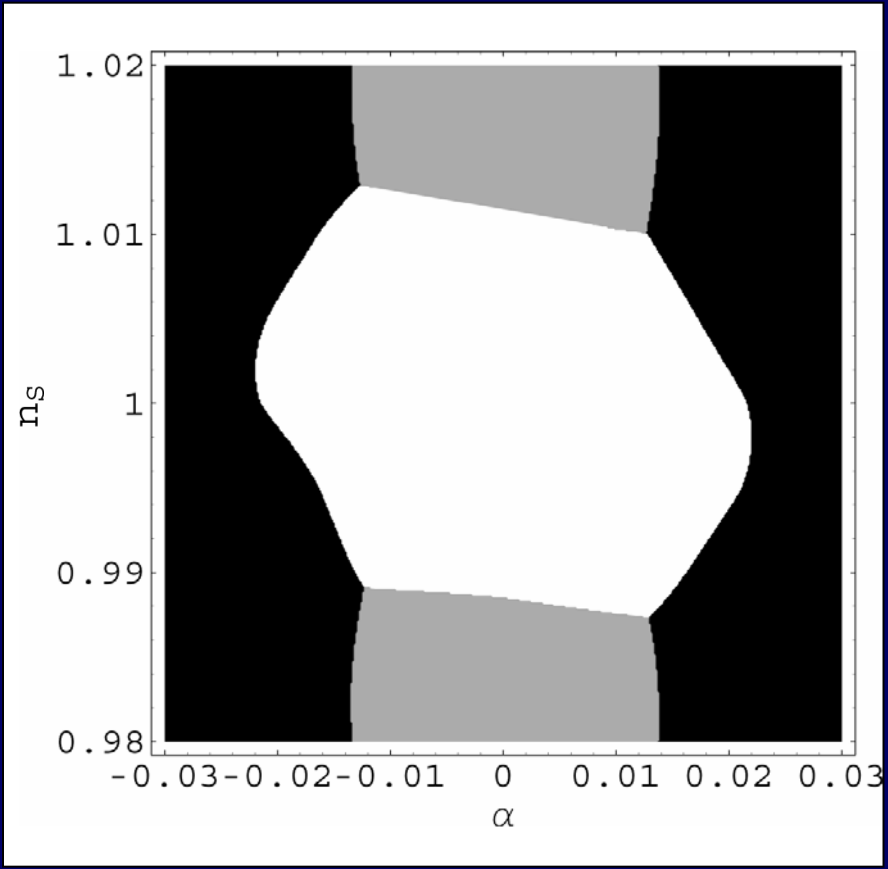


The horizontal lines indicate where the comparison becomes 'strong' (dashed) and 'decisive' (solid) on the Jeffreys' scale.

Three-way model comparison



False-colour RGB plot, with Red = HZ, Green = VARYn and Blue = VARYnα



Sector plot, with White = HZ, Grey = VARYn and Black = VARYnα

Conclusions

- Model selection analyses complement the usual parameter error forecasts, and can robustly identify the need for new fit parameters.

- It is not as easy to rule out $n_S=1$ as suggested by parameter error forecasts. If HZ is the true model, VARYn will be strongly disfavoured, but not decisively.

- If VARYn is the true model, it will be strongly, but not decisively, preferred over VARYnα. However, n_S away from [0.986, 1.014] is needed to strongly favour VARYn over HZ.

- Finally, suppose VARYnα is the true model, the alternatives will be only ruled out if the true value satisfies $|\alpha|>0.02$. WMAP3 gives at the 95% level of confidence $-0.17<\alpha<+0.01$.

Osci-llations in the inflaton potential?

General point of view

- We consider a class of inflationary models with small oscillations imprinted on an otherwise smooth inflaton potential.
- These oscillations are manifest as oscillations in the power spectrum for primordial perturbations, which then give rise to oscillating departures from the standard CMB power spectrum.
- We quantify the smallest detectable oscillations in the CMB power spectrum, and thus the smallest detectable amplitude of oscillations in the inflaton potential.

Outlook

- Introduction

- Features analysis
  - Motivation
  - The oscillating model
  - Power spectra
  - Detectability

- Conclusions & Future work

Introduction

- CMB experiments continue to be consistent with the simplest predictions of inflationary models, even if the data become increasingly precise.

- Many parameters like

$$A_S, n_S, A_T, \ldots$$

  can now be used to constrain the parameters space of the inflaton potential. The simplest model being

$$V(\phi) = \frac{1}{2} m^2 \phi^2, \text{ with } m \simeq \sqrt{8\pi} \times 10^{-6} M_{\mathrm{PL}}$$

- However it is worth asking whether the CMB data, present or future, can be used to constrain more complicated forms of the potential.

Motivation

- Oscillations in the primordial power spectrum arising from a step in the inflaton potential:
  - L. Covi *et al.* (Phys. Rev. D **74**, 2006)
  - J. Hamann *et al.* (arXiv:astro-ph/0701380)
- Oscillations in the primordial power spectrum coming from a rapid phase transition in multiple-fields models:
  - P. Hunt and S. Sarkar (Phys. Rev. D **70**, 2004 & arXiv:0706.2443)
- Oscillations in the inflaton potential in natural-inflation models, constrained from existing data:
  - X. Wang *et al.* (Int. J. Mod. Phys. D **14**, 2005)

The oscillating model

- The inflationary background dynamics is governed by the Friedmann and the Klein-Gordon equations ($8\pi G = c = \hbar = 1$)

$$3H^2 \equiv 3\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad \text{and} \quad \ddot{\phi} + 3H\dot{\phi} = -\frac{dV}{d\phi}$$

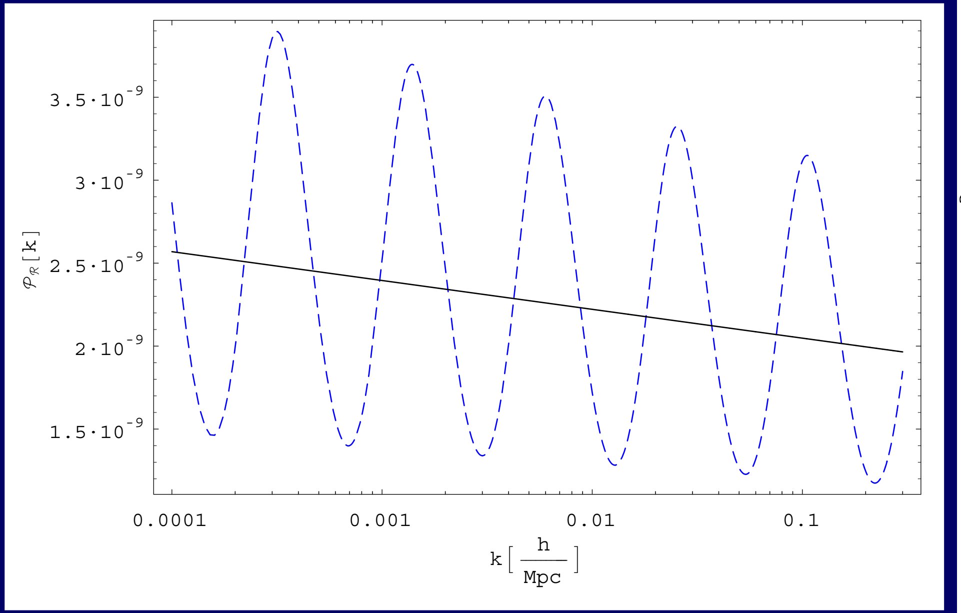
- We consider then sinusoidal fluctuations on the quadratic smooth potential

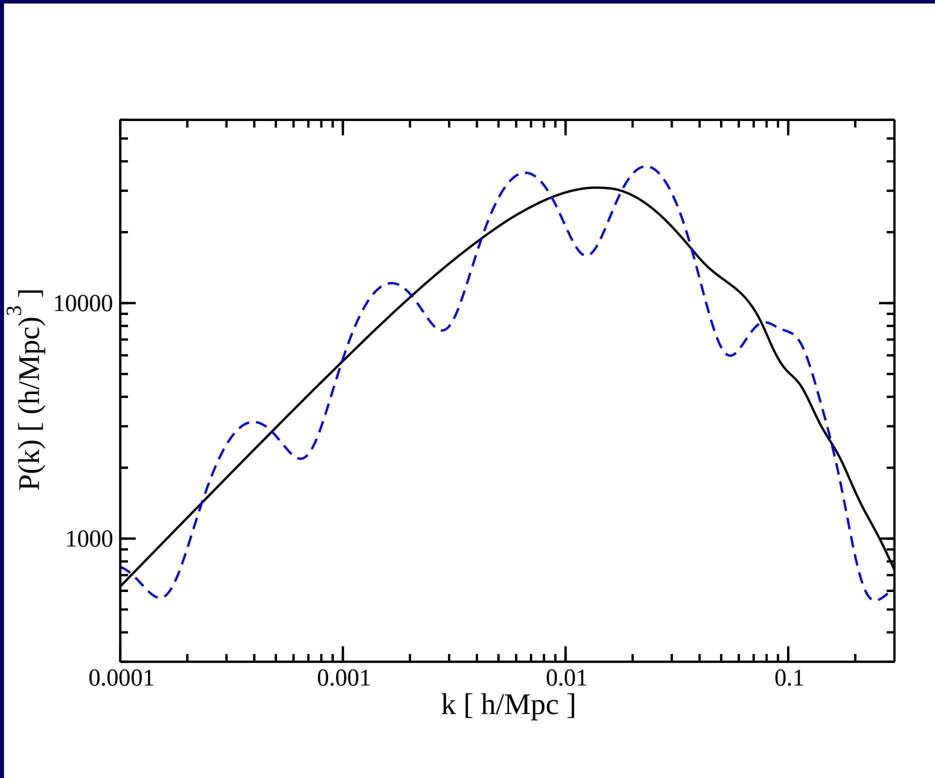
$$V(\phi) = \frac{1}{2}m^2\phi^2\left[1 + \alpha \sin\left(\frac{\phi}{\beta}\right)\right] \text{ with } \alpha << 1$$

- The power spectrum P_R of the primordial curvature perturbation is expressed with the horizon-crossing approximation

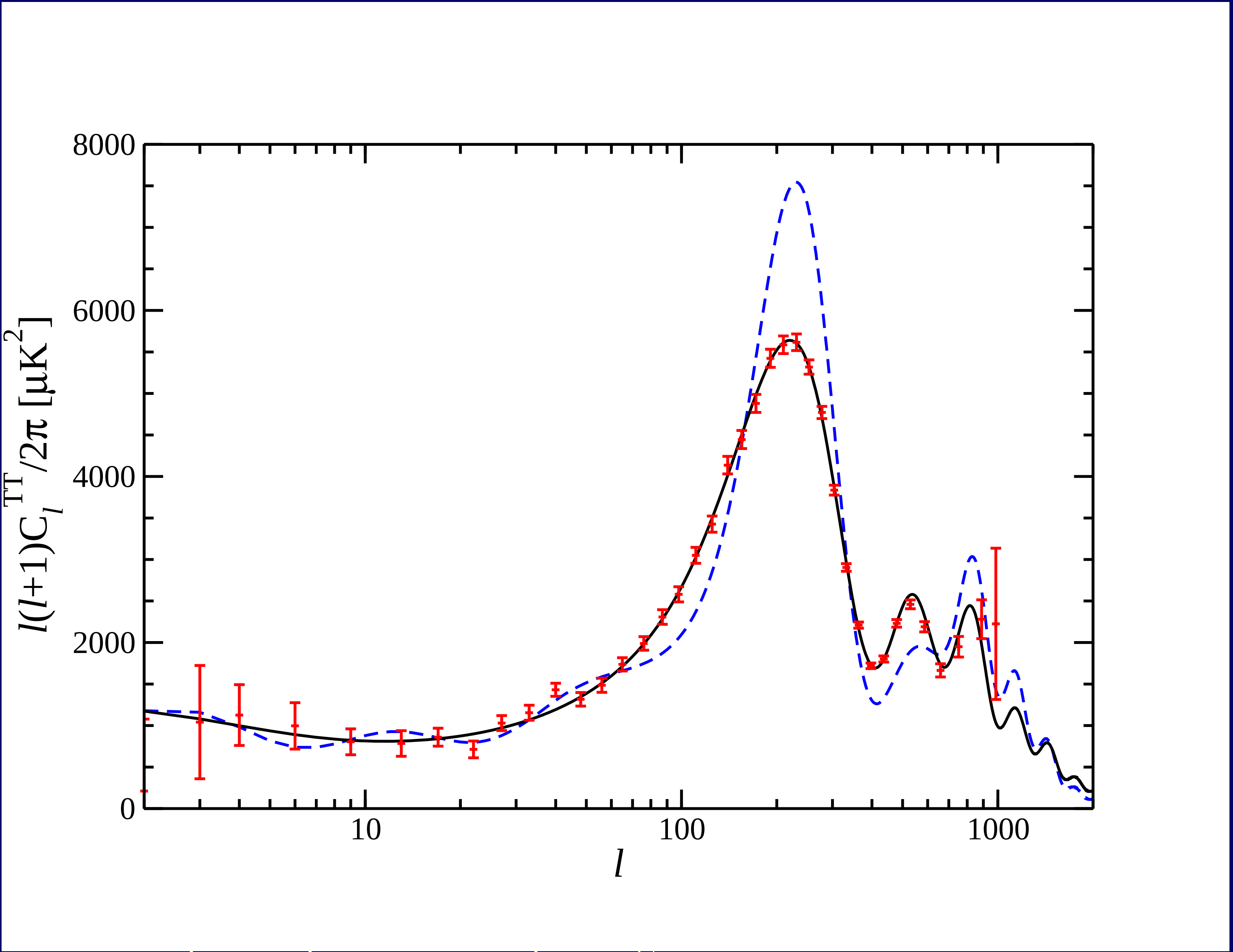
$$P_R^{1/2}(k) = \left[1 - (2C+1)\varepsilon_H + C\eta_H\right]\frac{H^2}{2\pi|\dot{\phi}|}\bigg|_{k=aH} \quad \text{with } \varepsilon_H \equiv \frac{1}{2}\frac{\dot{\phi}^2}{H^2} \text{ and } \eta_H \equiv -\frac{\ddot{\phi}}{H\dot{\phi}}$$

Primordial and matter spectra





Primordial power spectrum (left) and matter power spectrum (right) of the smooth inflaton potential (solid) and oscillating potential (dashed). The latter's parameters are $[\alpha, \beta] = [5 \times 10^{-4}, 3 \times 10^{-2}]$ in order to clearly show their effect.



previous plots. The WMAP3 data are superimposed. The error bars include both the cosmic variance and instrumental noise.

Detectability

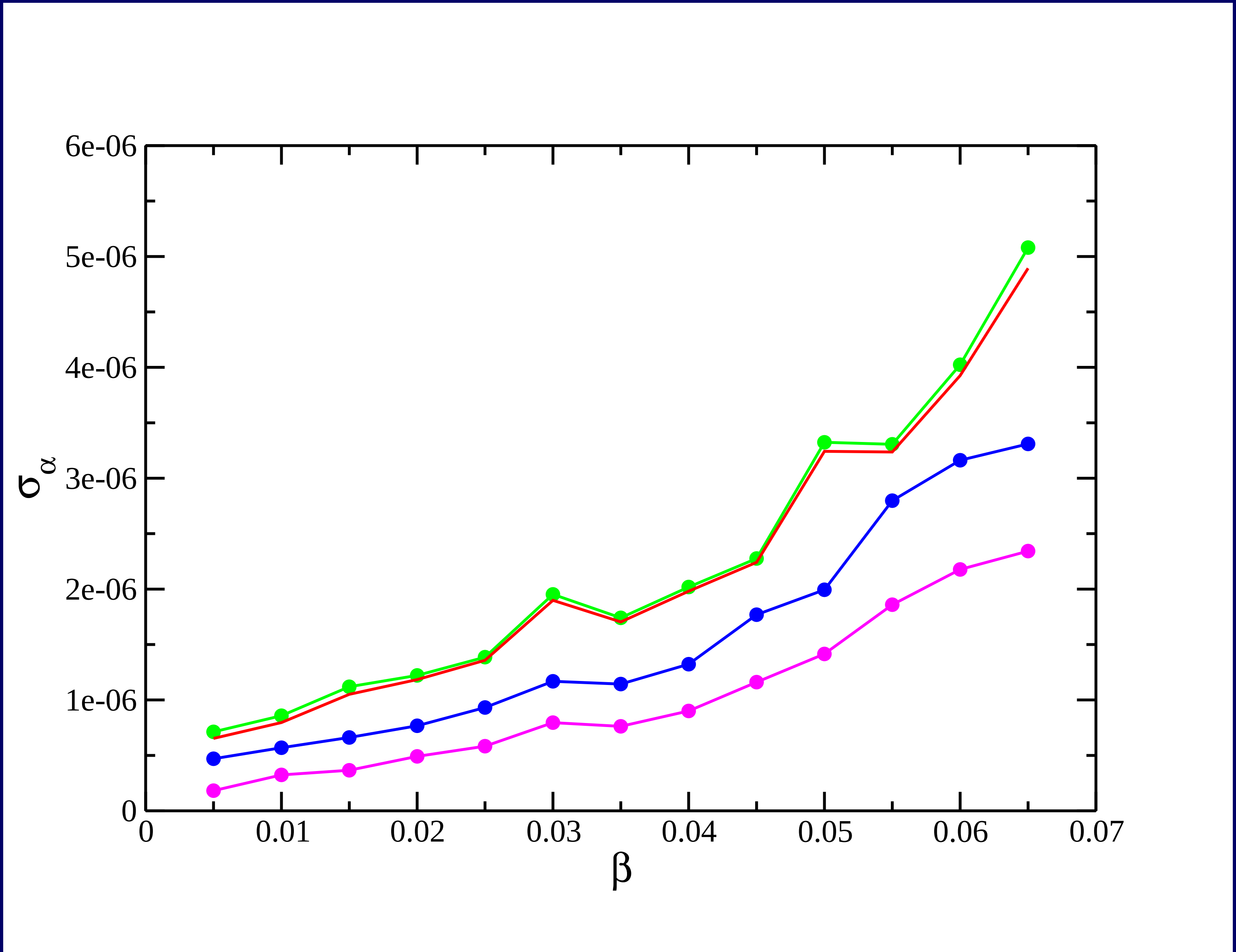
- We perform an estimate of the smallest oscillation amplitude α that will be detectable with Planck. To do so, we suppose that each multipole moment *l* can be measured with a standard error

$$\sigma_l = \sqrt{\frac{2}{(2l+1)\,f_{sky}}}(C_l + N_l)$$

where N_l is the contribution from the detector noise.

- We then estimate the error to α by

$$\left(\frac{1}{\sigma_\alpha}\right)^2 = \sum_l \left(\frac{\partial C_l}{\partial \alpha}\right)^2 \frac{1}{\sigma_l^2(\alpha)}$$



The smallest detectable amplitude σ_α as a function of β, for Planck.
An amplitude $\alpha = 5 \times 10^{-5}$ has been chosen for the analysis.

Conclusions & Future work

- Despite the constraint due to the cosmic variance, very small oscillations $O(10^{-6})$ may still be detectable by future experiments, like Planck, if they do exist.
- Our results are only slightly improved when the polarization and temperature-polarization power spectra are included.
- A marginalization over all the parameters in our model should bring some extra information.
- A comparison with WMAP3 through a MCMC analysis would be interesting to consider as well.