Testing modified gravity with non-Gaussianity

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Based on:

- XG, Phys.Rev.D82:103004,2010. [1005.1219];

- ***XG, [1008.2123];***

- ***work in progress***

Outline

- A short introduction to non-Gaussianity:
  - Concept, characterization, physical origin, theoretical results and observational constraints;
- Higher-order temperature anisotropy from gravitational perturbations;
- Nonlinear CMB anisotropy in f(R) gravity:
  - Nonlinear mapping from primordial perturbation ζ to today's observable $\Delta T/T$.

Inflation and cosmic perturbations

Cosmology: a Golden Era
A "6-parameter model" can now explain (almost) all observations, ranging from the intergalactic neutral hydrogen to the Cosmic Microwave Background (CMB);
Cosmological parameters are now measured with exquisite precision.

Inflation:
solve the problem of Big-Bang, provides the primordial seeds for CMB and LSS;
Cosmic theory based on inflation predicts a **nearly scale-invariant, adiabatic, nearly Gaussian primordial density perturbation**.

$$\langle \zeta(\boldsymbol{k}_1) \zeta(\boldsymbol{k}_2) \rangle = (2\pi)^3 \delta^3(\boldsymbol{k}_1 + \boldsymbol{k}_2) \frac{2\pi^2}{k^3} \mathcal{P}_\zeta(k_1) \qquad \mathcal{P}_\zeta(k) \simeq \left(\frac{H_*}{2\pi} \right)^2 \frac{1}{2 c_s \epsilon}$$

Gaussian v.s. non-Gaussian

Power spectrum (2 point function): $\left\langle \frac{\Delta T}{T}(\hat{n}_1)\frac{\Delta T}{T}(\hat{n}_2) \right\rangle$

Higher-order correlation function:

$$\left\langle \frac{\Delta T}{T}(\hat{n}_1)\cdots\frac{\Delta T}{T}(\hat{n}_p) \right\rangle \neq 0\,, \qquad p = 3, 4, \cdots$$

$$p(x) \propto e^{-sx^2+bx^3-tx^4+\cdots}$$

$$p\left[\frac{\Delta T}{T}\right] \propto \exp\left\{\int -S*\left(\frac{\Delta T}{T}\right)^2 + B*\left(\frac{\Delta T}{T}\right)^3 - T*\left(\frac{\Delta T}{T}\right)^4 + \cdots\right\}$$

Free = linear = Gaussian, **Interaction = nonlinear = non-Gaussian**

Why non-Gaussianity?

Distinguishing various (non)inflation models/mechanism (multifield, noncanonical kinetic term, fast-roll, initial vacuum, curvaton, end-of-inflation, …);

More information concerning the evolution of the universe;

Interactions in the early universe (inflaton, gravitation, …); …

Gaussian v.s. non-Gaussian

Power spectrum (2 point function): $\left\langle \frac{\Delta T}{T}(\hat{n}_1) \frac{\Delta T}{T}(\hat{n}_2) \right\rangle$

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Free = linear = Gaussian, **Interaction = nonlinear = non-Gaussian**

Why non-Gaussianity?

Most important:

There are observations.

Characterizing the non-Gaussianity

Bispectrum (3 point function):

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = (2\pi)^3 B_\zeta(k_1, k_2, k_3) \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)$$

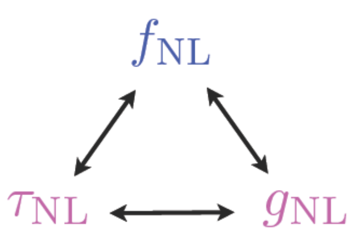
$$B_\zeta(k_1, k_2, k_3) = \frac{6}{5} f_{\rm NL} \left(P_\zeta(k_1) P_\zeta(k_2) + P_\zeta(k_2) P_\zeta(k_3) + P_\zeta(k_3) P_\zeta(k_1) \right)$$

Trispectrum (4 point function):

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \zeta_{\vec{k}_4} \rangle = (2\pi)^3 T_\zeta(k_1, k_2, k_3, k_4) \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4)$$

$k_{13} = k_1 + k_3$

$$T_\zeta(k_1, k_2, k_3, k_4) = \tau_{\rm NL} \left(P_\zeta(k_{13}) P_\zeta(k_3) P_\zeta(k_4) + 11 \text{ perms.} \right)$$
$$+ \frac{54}{25} g_{\rm NL} \left(P_\zeta(k_2) P_\zeta(k_3) P_\zeta(k_4) + 3 \text{ perms.} \right)$$

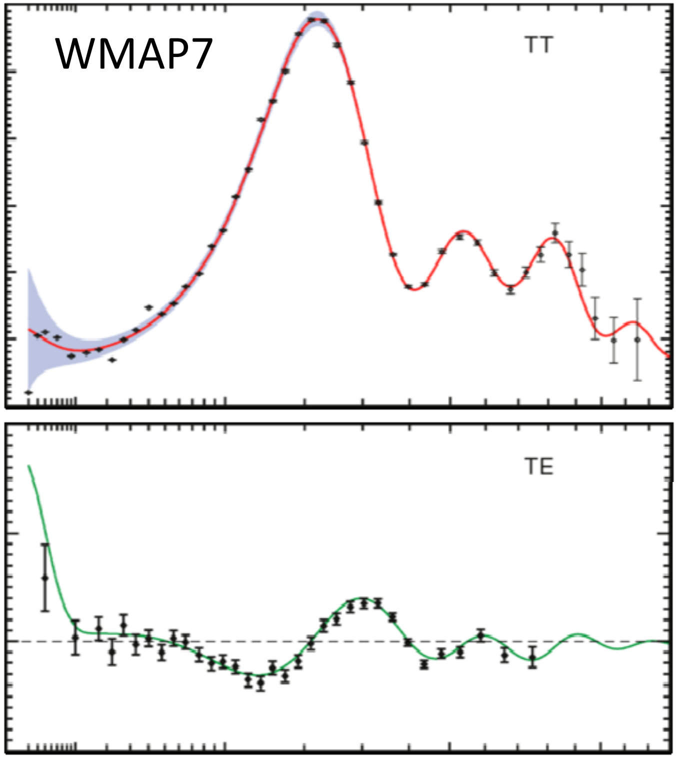


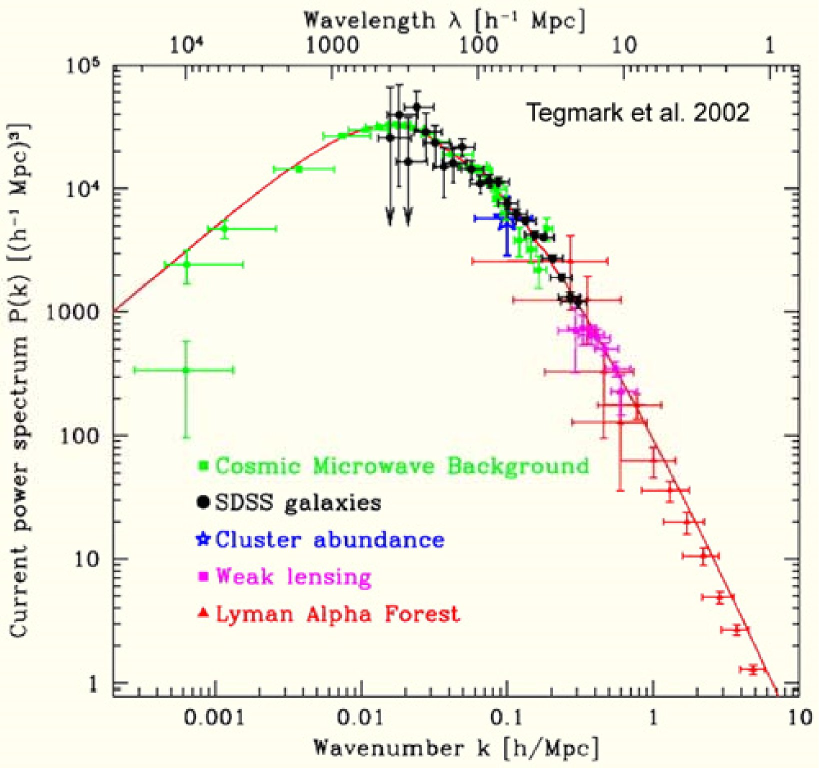
:Bispectrum

:Tripectrum

Momentum shapes: (1)

- Power spectrum has simple *k*-dependence --- the **shape**.
- Most of the information is encoded in the **shape** of the spectrum.

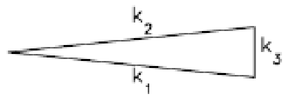




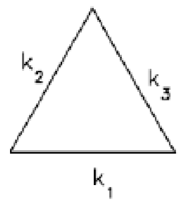
Momentum shapes: (2)

However, $B(k_1, k_2, k_3)$ and $T(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4)$ have complicated momentum-dependence.

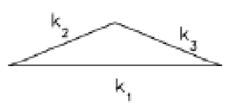
Squeezed triangle $k_1=k_2 \gg k_3$



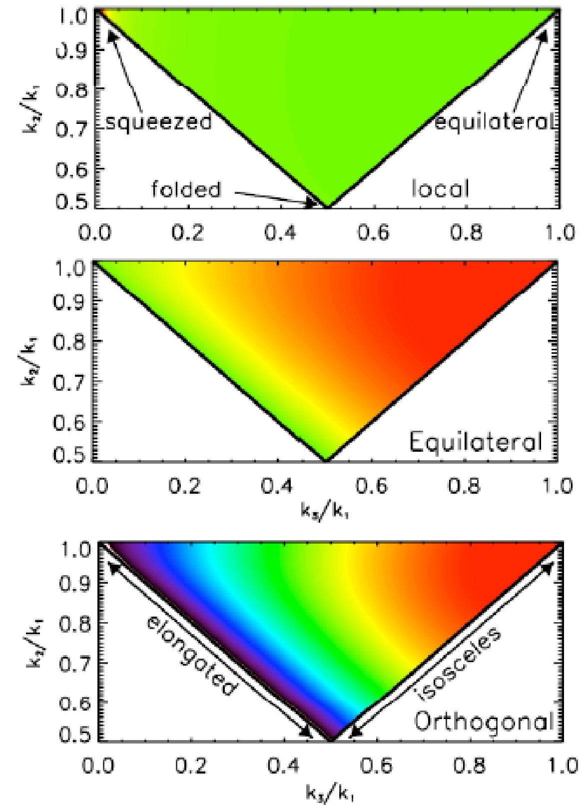
Equilateral triangle $k_1=k_2= k_3$



Folded triangle $k_2=k_3=1/2\ k_1$



[Komatsu (2010)]



Typicial "templates": local, equilateral, orthogonal

[Komatsu et al (2001); Creminelli et al (2006); Senatore et al. (2009)]

$$B_{\rm local}(k_1,k_2,k_3) \propto f_{\rm NL}^{\rm local}\left[\frac{1}{(k_1k_2)^{4-n_s}}+\frac{1}{(k_2k_3)^{4-n_s}}+\frac{1}{(k_3k_1)^{4-n_s}}\right]$$

$$B_{\rm equilateral}(k_1,k_2,k_3) \propto f_{\rm NL}^{\rm equil}\left[-\frac{1}{(k_1k_2)^{4-n_s}}-\frac{1}{(k_2k_3)^{4-n_s}}-\frac{1}{(k_3k_1)^{4-n_s}}-\frac{2}{(k_1k_2k_3)^{\frac{2(4-n_s)}{3}}}+\left(\frac{1}{(k_1k_2^2k_3^3)^{\frac{4-n_s}{3}}}+5perms\right)\right]$$

$$B_{\rm orthogonal}(k_1,k_2,k_3) \propto f_{\rm NL}^{\rm equil}\left[-\frac{3}{(k_1k_2)^{4-n_s}}-\frac{3}{(k_2k_3)^{4-n_s}}-\frac{3}{(k_3k_1)^{4-n_s}}-\frac{8}{(k_1k_2k_3)^{\frac{2(4-n_s)}{3}}}+\left(\frac{1}{(k_1k_2^2k_3^3)^{\frac{4-n_s}{3}}}+5perms\right)\right]$$

Current observational limit

No definitive proof of the existence of NG.
Slow-roll single-field inflation is consistent with observation.

$$f_{\rm NL}^{\rm local} \approx r \approx 1 - n_s, \qquad f_{\rm NL}^{\rm equilateral} \approx f_{\rm NL}^{\rm orthogonal} \approx 0.$$

Current limits:

$-9 < f_{\rm NL}^{\rm local} < +111$ (95% C.L.)	WMAP5
$-10 < f_{\rm NL}^{\rm local} < +74$ (95% C.L.)	WMAP7
$f_{\rm NL} = -12 \pm 62$ (68% C.L.)	Calabrese et al. (2009), WMAP5
$f_{\rm NL} = +84 \pm 40$ (68% C.L.)	Rudjord et al. (2009), WMAP5
$-29 < f_{\rm NL} < +70$ (95% C.L.)	Slosar et al. (2008), SDSS
$+25 < f_{\rm NL} < +117$ (95% C.L.)	Xia et al. (2010), WMAP7+2dFGRS+SN+VLA

$-214 < f_{\rm NL}^{\rm equilateral} < +266$ (95% C.L.)	WMAP7
$-410 < f_{\rm NL}^{\rm orthogonal} < +6$ (95% C.L.)	WMAP7

$-3.2 \times 10^5 < \tau_{\rm NL} < 3.3 \times 10^5$ (95% C.L.)	Smidt et al. (2010), WMAP5
$-3.8 \times 10^6 < g_{\rm NL} < 3.9 \times 10^6$ (95% C.L.)	Smidt et al. (2010), WMAP5

Planck will reduce the error bar of $f_{\rm NL}$ with a factor 4~5;

Planck: $\Delta\tau_{\rm NL} = 560$ (95%C.L.); $\Delta g_{\rm NL} \sim 10^4$

We have known…

- "Local-type" non-Gaussianity:
  - If $f_{NL}^{(local)}$>>1, all single field inflation models will be rull out, [*Creminelli & Zaldarriaga* (2004)];
  - $\tau_{NL} \geq (6/5\, f_{NL})^2$ for single-field local-type non-Gaussianity [Komatsu (2010)];

- **Relation between momentum shapes and fundamental physics** [*Creminelli* (2003); *Babich, Creminelli & Zaldarriaga* (2004); *Chen et al*. (2007)];

- **Are higher-order effects (transfer) important?** $$\frac{\Delta T}{T} = -\frac{1}{3}\Phi = -\frac{1}{5}\zeta$$

- **Secondary effects**: The most important one is "ISW + WL" [*Serra & Cooray* (2008)], especially for the local-type non-Gaussianity.

Bispectrum v.s. Trispectrum

- **Observational side:** upcoming observations will give possibly positive and even precise evidence of f_{NL}; while for τ_{NL} & g_{NL}, there are only very weak limits, even with Planck.
- **Theoretical side**:
  - **NG$_3$** is larger than **NG$_4$** in a typical model [*Creminelli et al* (2010)]:

$$\mathrm{NG}_3 \equiv \frac{\langle \zeta^3 \rangle}{\langle \zeta^2 \rangle^{3/2}} \simeq \left. \frac{\mathcal{L}_3}{\mathcal{L}_2} \right|_{E \sim H} \qquad \mathrm{NG}_4 \equiv \frac{\langle \zeta^4 \rangle}{\langle \zeta^2 \rangle^2} \simeq \left. \frac{\mathcal{L}_4}{\mathcal{L}_2} \right|_{E \sim H}$$

$$\mathrm{NG}_3 \simeq f_{\mathrm{NL}} \Delta_\zeta^{1/2} \qquad \mathrm{NG}_4 \simeq \tau_{\mathrm{NL}} \Delta_\zeta$$

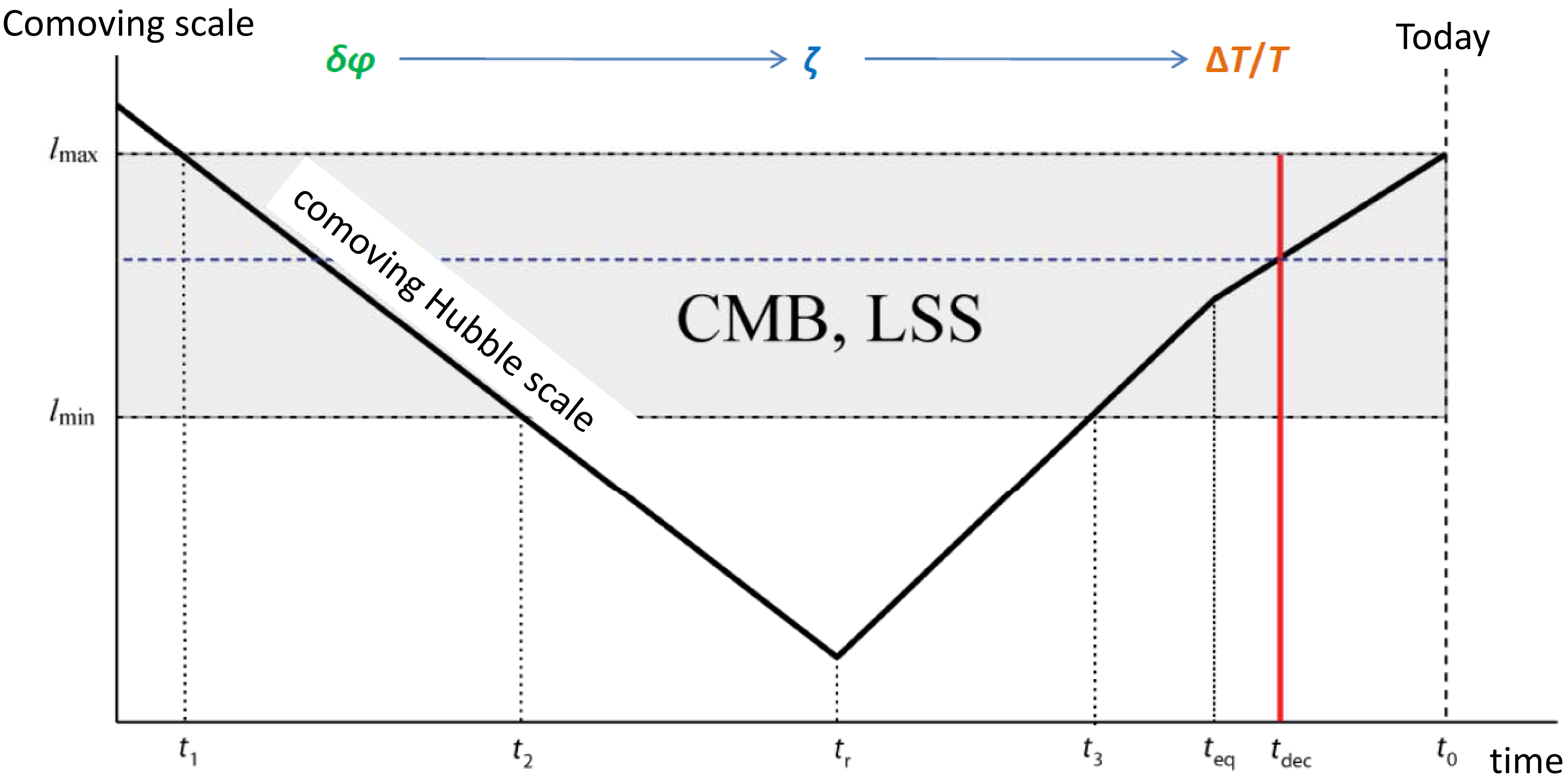
$$\mathrm{NG}_4 \sim \mathrm{NG}_3^2$$

f_{NL}~100 ⟹ τ_{NL}~10^4

  - Models with large trispectrum but negligible bispectrum? [*Senatore* (2010)]

- Higher-order correlation functions (5, 6 points functions) are almost impossible to be detected.

Evolution of Cosmic Perturbations



Initial NG:
interaction of inflaton(s)

Primordial NG:
nonlinearity of gravitation

Observed NG:
nonlinearies in gravitational & acoustic & plasma physics

Non-Gaussianity of initial quantum fluctuations: (1)

--- higher-order correlators due to interactions of scalar field quantum fluctuations during inflation.

Cosmological Perturbation Theory + Quantum Field Theory [*Maldacena* (2002)]

$$S[g_{\mu\nu},\phi] \xrightarrow[g_{\mu\nu}=\bar{g}_{\mu\nu}+\delta g_{\mu\nu}, \quad \phi^I=\bar{\phi}^I+\delta\phi^I]{\text{perturbative expansion}} \bar{S}[\bar{g}_{\mu\nu},\bar{\phi}] + S_2[\delta g_{\mu\nu},\delta\phi] + S_3[\delta g_{\mu\nu},\delta\phi] + S_4[\delta g_{\mu\nu},\delta\phi] + \cdots$$

Distributional functional: $p[\delta g_{\mu\nu},\delta\phi] \propto e^{i(S_2+S_3+S_4+\cdots)}$

Typical interaction terms:

"**Non-local**":
(non-canonical kinetic terms, DBI, k-inflation)

"**Local**":
suppressed by slow-roll!

$$\mathcal{L}_3 \propto \left(\delta\dot{\phi}\right)^3, \qquad \delta\dot{\phi}\left(\partial_i\delta\phi\right)^2, \qquad \left(\delta\phi\right)^2\partial^2\delta\phi, \qquad \left(\delta\phi\right)^3, \quad \cdots$$

$$\mathcal{L}_4 \propto \left(\delta\dot{\phi}\right)^4, \qquad \left(\delta\dot{\phi}\right)^2\left(\partial_i\delta\phi\right)^2, \qquad \left(\delta\phi\right)^2\left(\delta\dot{\phi}\right)^4, \qquad \left(\delta\phi\right)^4, \quad \cdots$$

Non-local NG (equilateral/orthogonal-type…)

Local-type NG

Non-Gaussianity of initial quantum fluctuations: (2)

Currently, concerning the initial non-Gaussianity around the time of Hubble-exiting, we have known:

• **All** single-field inflation models give negligible local non-Gaussianity (local interactions are suppressed by slow-rolling):

$$B\left(k_1, k_1, k_3 \rightarrow 0\right) = \frac{5}{3}\left(1 - n_s\right) P\left(k_1\right) P\left(k_3\right)$$

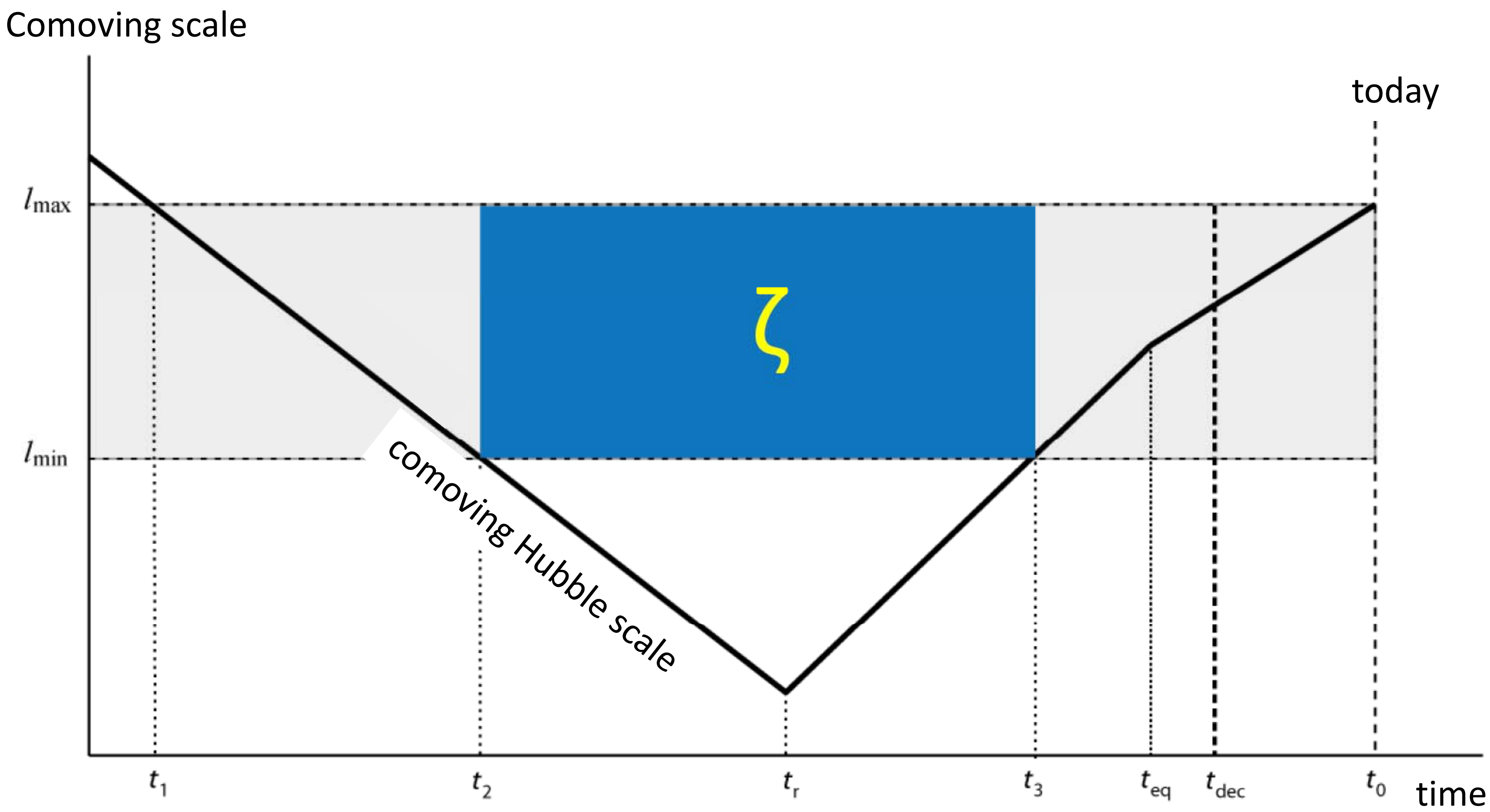
Due to the conservation on super-Hubble scales, **single-field models give negligible local-type primordial non-Gaussianity.**

• Non-local type non-Gaussianities can be generated from: non-canonical kinetic terms, non-Bunch-Davis initial vacuum, etc;

• Slow-roll **multi-field** models typically generate non-local non-Gaussianity;

• **Local-type** non-Gaussianity arises mainly from the **super-Hubble evolution** in mult-filed/curvaton/end-of-inflation models.

Super-Hubble primordial curvature perturbation



Conserved ζ

Due to the energy-momentum conservation, there must exist a non-perturbative and gauge-invariant conserved variable ζ:

$$\zeta = -\psi + \int_{\bar{\rho}}^{\rho} \frac{d\tilde{\rho}}{3\left(\tilde{\rho} + \tilde{p}\right)}$$

The conservation of ζ makes it possible to relate the perturbations around Hubble-exiting and re-entering, no matter what happens during the intermediate period.

ζ is the perturbation of e-folding numbers in uniform density slices: **δ*N*-formula** [*Sasaki* et al (1994)]

$$\zeta = \delta N = \delta \int dt \frac{N}{3} \nabla_\mu n^\mu = \delta \int dt \left(H - \dot{\psi}\right) = -\psi$$

Primordial non-Gaussianity of ζ

- Two equivalent approaches:

  1) comoving/uniform density gauge: $S_2[\zeta]$, $S_3[\zeta]$, $S_4[\zeta]$, …

  2) (most popular) calculating **NG of inflaton** in uniform curvature gauge around horizon-crossing, then using δN-formula on super-Hubble scales:

$$\begin{aligned}\zeta &= -\psi|_{\text{uniform density}} \equiv \delta N\left(\phi, \dot{\phi}, \partial_i\phi\right) \approx \delta N\left(\delta\phi\right) \\ &= N_{,\phi}\delta\phi + \frac{1}{2}N_{,\phi\phi}\delta\phi^2 + \cdots\end{aligned}$$

- Essentially, δ*N*-formula is just the nonlinear gauge transformation from $\delta\phi$ to ζ on large-scales,

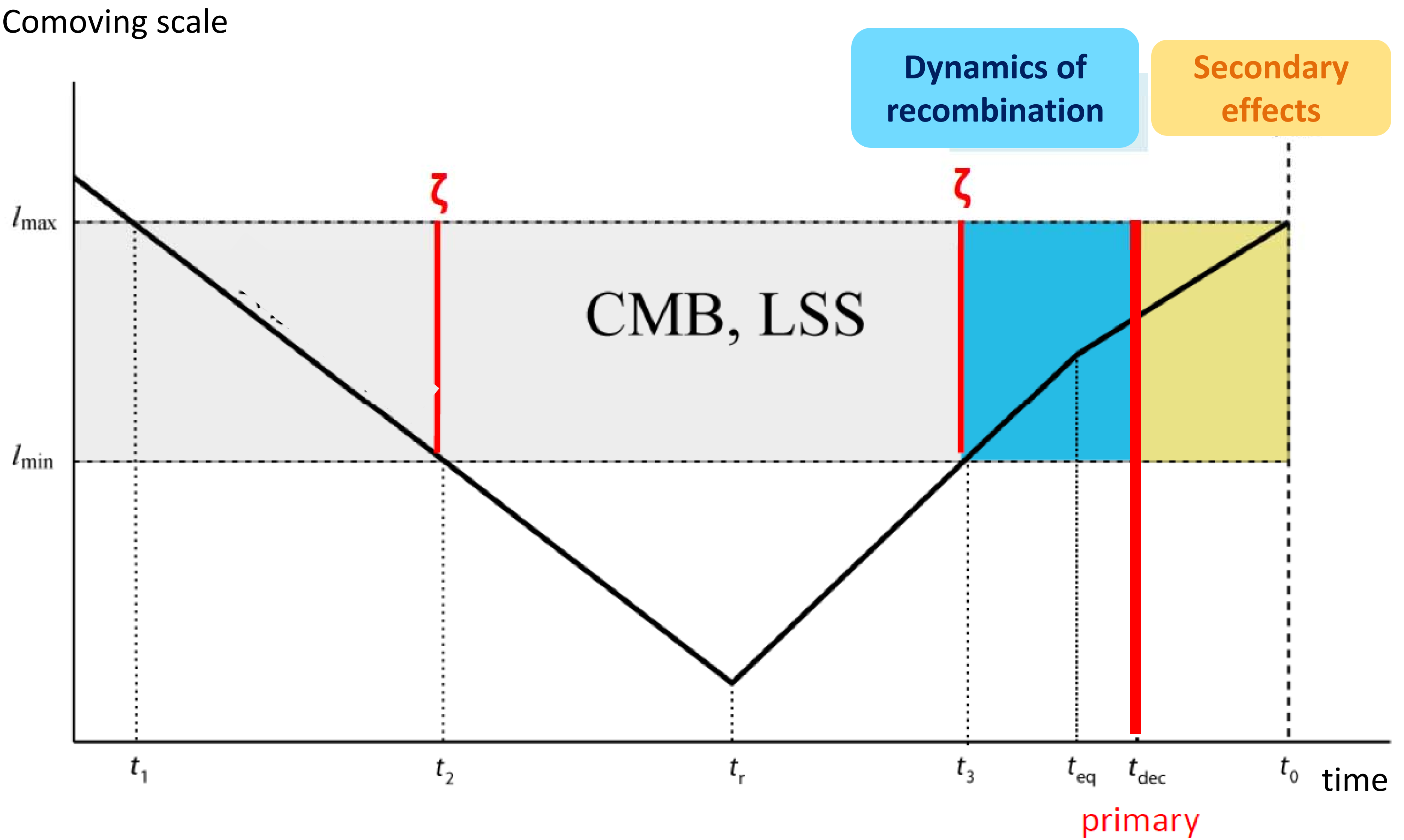
Nonlinear mapping

$\delta\varphi \longrightarrow \zeta$

which provide a **nonlinear mapping between δ*φ* and ζ**.

- δ*N*-formula can gives large **local-type** non-Gaussianity of ζ on large-scales.

Observed non-Gaussianity



Nonlinear mapping/evolution and Secondary Effects

- **Nonlinear (higher-order) mapping and evolution:**
  - Nonlinear generalization of linear relation $\Delta T/T = 1/3\ \Phi = -1/5\ \zeta$;
  - Higher-order Einstein-Boltzmann equation;

- **Secondary effects (after decoupling):**
  - Scattering secondary effects (small-scale)
    - Thermal/kinetic SZ effect; Ostriker-Vishinac effect; Reionization;
  - Graviational secondary effects:
    - ISW effect (Rees-Sciama effect); Gravitational lensing.

Non-Gaussianity: a brief summary

time ↓

Process	Origins of NG	Types of NG
Initial vacuum fluctuation	Excited state	
Sub-Hubble evolution	Potential/derivative interactions	Equilateral + orthogonal
Hubble-exiting	Potential	Local
Super-Hubble evolution	Self-interactions + gravity	Local
End-of-inflation	Conditions of end-of-inflation	Local
(p)Reheating	Modulated reheating	Local
Post-inflation	Curvaton	Local
Primordial non-Gaussianity		
Radiation + matter + last-scattering	Primordial anisotropy (nonlinear mapping/evolution)	Local + equilateral
ISW + lensing	Secondary anisotropies	Local + equilateral
Observed non-Gaussianity		

Using non-Gaussianity to test gravity?

We will use the large-scale nonlinear "mapping" from ζ to ΔT/T to constraint modifield gravity.

Why we focus on these non-primordial effects?

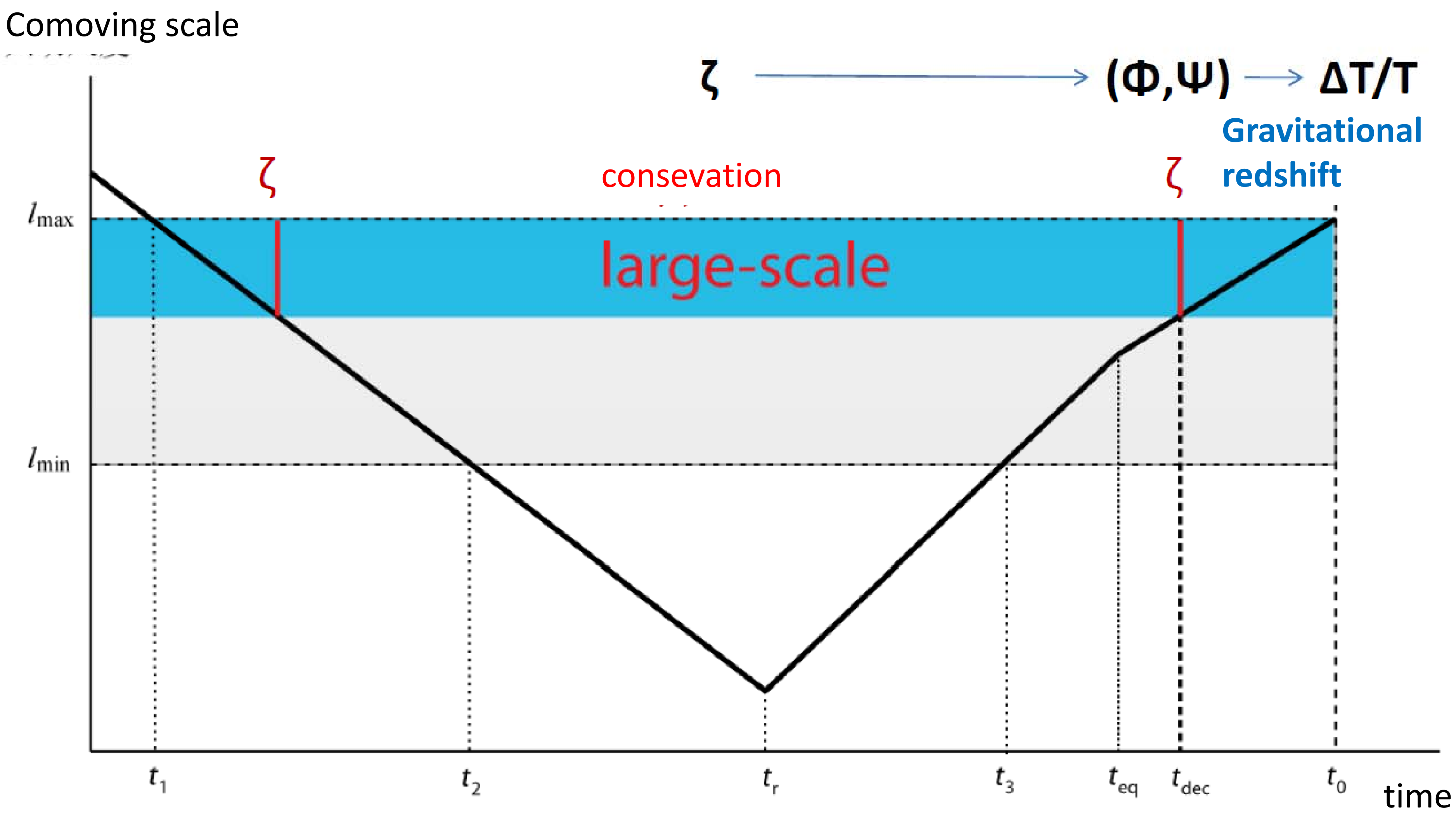
1) Most of previous studies on non-Gaussianity focus on the "**primordial**" non-Gaussianity of ζ during inflation;

2) We will explore the ability this **post-inflationary contribution** to the observed non-Gaussainity in probing new physics.

3) To determine their contributions (contaminations) to the final observed non-Gaussianity and to construct appropriate "template" **in order to abstract the real primordial NG**.

(Investigations regarding the primordial NG in modified gravity are in progress…)

Large-scale anisotropy



Nonlinear SW in GR: an example

On large-scales: $ds^2 = -e^{2\Phi}dt^2 + a^2 e^{-2\Psi} d\boldsymbol{x}^2 = -d\tilde{t}^2 + \tilde{a}^2(t,\boldsymbol{x})\, d\boldsymbol{x}^2$

Blackbody radiation: $\frac{\omega}{T} = \text{const.} \longrightarrow T_o = T_e \frac{\omega_o}{\omega_e}$

1) **Graviational redshift** from surface of last-scattering (SLS) to observer:

$$\frac{\omega_o}{\omega_e} = \frac{\bar{\omega}_o}{\bar{\omega}_e} e^{\Phi_e - \Phi_o}$$

2) **Intrinsic temperature fluctuation** on SLS:

$$\frac{1}{3}\ln\rho_m = \frac{1}{4}\ln\rho_\gamma \propto \ln T$$

$$\frac{1}{3}\rho_m = \tilde{H}^2 \equiv \left(\frac{d\ln\tilde{a}}{d\tilde{t}}\right)^2 \approx e^{-2\Phi} H^2$$

$$\longrightarrow \frac{T}{\bar{T}} = e^{-\frac{2}{3}\Phi}$$

Fully nonlinear (non-perturbative) Sachs-Wolfe effect

[*Bartolo et al. (2005)*]:

$$T_o \propto e^{-\frac{2}{3}\Phi_e} e^{\Phi_e} = e^{\frac{1}{3}\Phi_e}$$

Initial conditions on SLS

According to the conserved ζ, to determine the metric perturbations on SLS:

$$\zeta = -\Psi + \int_{\bar{\rho}}^{\rho} \frac{d\tilde{\rho}}{3\left(\tilde{\rho}+\tilde{p}\right)} = -\Psi + \frac{1}{3\left(1+w\right)} \ln \frac{\rho}{\bar{\rho}} = -\Psi - \frac{2}{3}\Phi$$

Constraint from Einstein equation: $\Psi - \Phi = \mathcal{K}\left[\Phi, \Psi\right]$

Initial conditions for metric perturbations on the SLS:

$$\Phi = \Phi_e\left[\zeta\right], \qquad \Psi = \Psi_e\left[\zeta\right]$$

$$T_o \propto e^{-\frac{1}{5}\zeta + \text{higher-order nonlocal terms}}$$

[*Bartolo et al. (2005)*]:

The final temperature fluctuation is always non-Gaussian, even the primordial curvature perturbation is exact Gaussian!

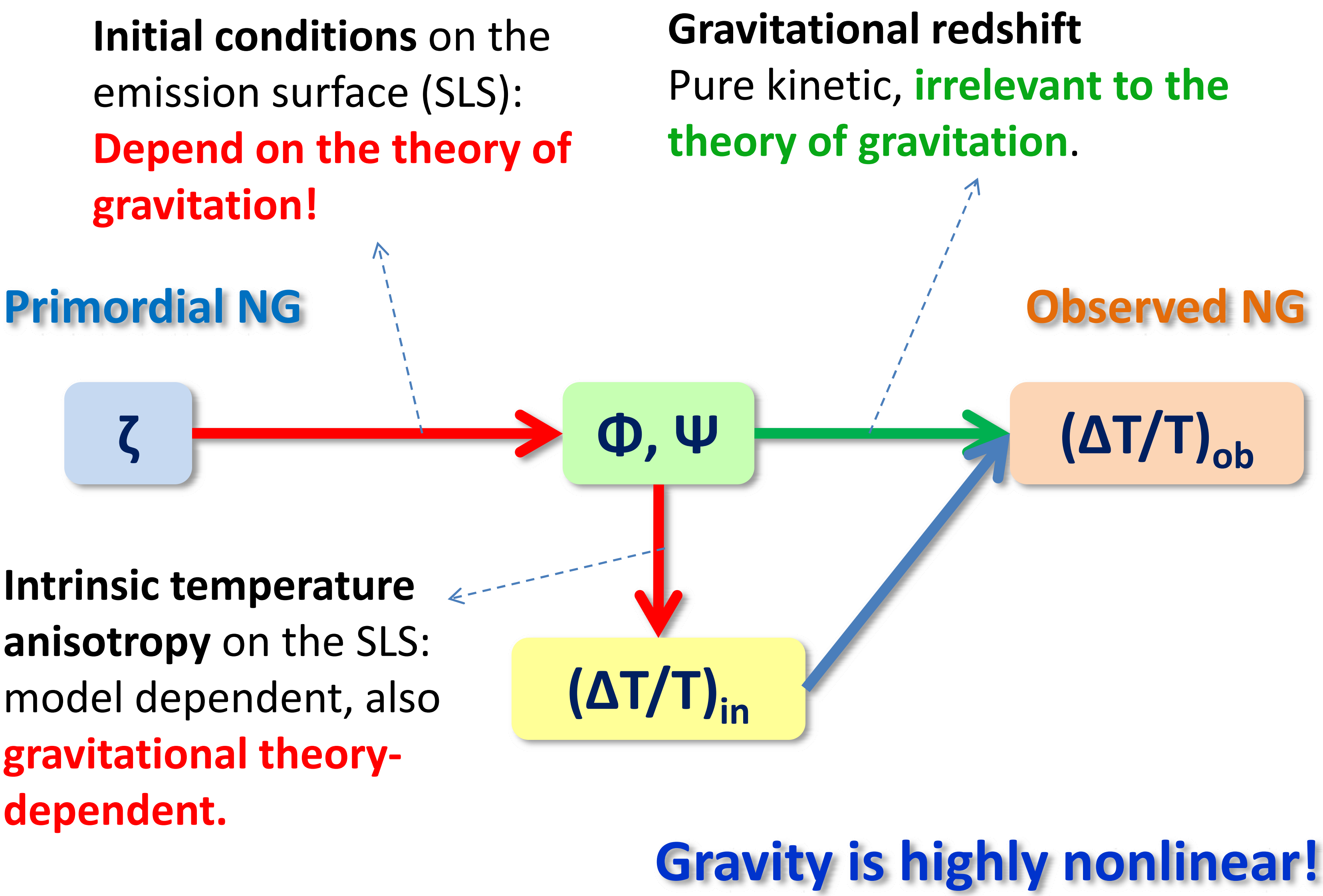
Summary of the results in GR

• The nonlinear SW effect contributes to the final non-Gaussianity <O(1);
• Cross-correlation between ISW and Lensing would contribute to ~O(5). [*Pitrou, Uzan, Bernadeau* (2010)];

$$T_o \propto e^{-\frac{1}{5}\zeta + \text{higher-order nonlocal terms}}$$

The factor "-1/5" is too small!
Can we enhance it, in order to enhance the nonlinear mapping from ζ to ΔT/T?

Where does gravity enter



Sachs-Wolfe in f(R) gravity

"f(R) + minimally-coupled matter" system (Jordan frame):

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} f(R) + \mathcal{L}_m \right)$$

Large-scale metric perturbation: $ds^2 = a^2 \left(-e^{2\Phi} d\eta^2 + e^{-2\Psi} dx^i dx^i \right)$

1) **Gravitational redshift**: $a\omega = -\Phi + \frac{1}{2}\Phi^2$

2) **Intrinsic anisotropy**: $T_e/\bar{T}_e = (\rho_\gamma/\bar{\rho}_\gamma)^{\frac{1}{4}} = (\rho_m/\bar{\rho}_m)^{\frac{1}{3}}$

Being modified due to the deviation from GR!

$$\ln \frac{\rho_m}{\bar{\rho}_m} = -(2-\sigma_1)\Phi + \frac{1}{2}\left(\sigma_2 - \sigma_1^2\right)\Phi^2$$

Nonlinear generalization of (large-scale) SW effect in f(R) gravity :

$$\frac{\Delta T}{T} = \frac{1}{3}(1+\sigma_1)\Phi + \frac{1}{18}\left(1 + 2\sigma_1 - 2\sigma_1^2 + 3\sigma_2\right)\Phi^2$$

[*Gao* [1008.2123]]

GR: ($\sigma_1 = \sigma_2 = 0$) ⟹ $\rho_m/\bar{\rho}_m = e^{-2\Phi}$ ⟹ $\frac{\Delta T}{T} = e^{\Phi/3}$

σ_1 and σ_2

σ_1 and σ_2 are complicated combinations of parameters that depend on the structure of f(R):

$$\sigma_1 = 2 - 6\frac{1+\beta(\epsilon-1)+(2\beta+\gamma)\epsilon_R}{2\epsilon-\gamma\epsilon_R^2-\beta\epsilon_R\left(\epsilon_{R'}-2\right)},$$
$$\sigma_2 = 4 + 12\frac{\beta-1+\gamma(\epsilon-1)+(3\gamma+\delta)\epsilon_R}{2\epsilon-\gamma\epsilon_R^2-\beta\epsilon_R\left(\epsilon_{R'}-2\right)},$$

Expansion history parameters:

$$\epsilon = 1 - \frac{d\ln\mathcal{H}}{d\ln a}, \qquad \epsilon_R = \frac{d\ln R}{d\ln a}, \qquad \epsilon_{R'} = \frac{d\ln R'}{d\ln a}.$$

Parameters which charactrize the structure of f(R):

$$\beta = \frac{Rf_{,RR}}{f_{,R}}, \quad \gamma = \frac{R^2 f_{,RRR}}{f_{,R}}, \quad \delta = \frac{R^3 f_{,RRRR}}{f_{,R}}.$$

"Compton parameter" [*Hu et al* (2006)]: $B = -\beta\epsilon_R/\epsilon$

Initial conditions in f(R) in matter era

Traceless part of the generalized Einstein equation gives the "constraints":

$$\Psi_1 = (1-2\beta)\,\Phi$$

$$\Psi_2 = K_2[\Phi] \;\equiv\; \partial^{-4}\Big[\left(3\lambda - 2\beta + 8\beta^2 + 4\gamma\right)\left(\partial^2\Phi\right)^2 + \left(\lambda + 6\beta - 4\beta^2 + 8\gamma\right)\left(\partial_i\partial_j\Phi\right)^2 + 4\left(\lambda + 2\beta + 4\gamma\right)\partial_i\Phi\partial_i\partial^2\Phi + 4\left(\beta - \beta^2 + \gamma\right)\Phi\partial^4\Phi\Big].$$

Conserved primordial curvature perturbation: $\zeta = -\Psi + \frac{1}{3}\ln\frac{\rho_m}{\bar{\rho}_m}$

Initial conditions:

$$\Phi_1 = -\frac{3\zeta}{5 - 6\beta - \sigma_1},$$

$$\Phi_2 = \frac{9\left[\left(\sigma_2 - \sigma_1^2\right)\zeta^2 - 6K_2[\zeta]\right]}{2\left(5 - 6\beta - \sigma_1\right)^3}$$

Second-order anisotropy in f(R)

The final temperature fluctuation: $\frac{\Delta T}{T} = \left(\frac{\Delta T}{T}\right)_{(1)} + \left(\frac{\Delta T}{T}\right)_{(2)} + \cdots$

$$\left(\frac{\Delta T}{T}\right)_{(1)} = -\frac{1+\sigma_1}{5-6\beta-\sigma_1}\zeta$$

$$\left(\frac{\Delta T}{T}\right)_{(2)}(\boldsymbol{k}) = \frac{1}{2}\int \frac{d^3p_1 d^3p_2}{(2\pi)^3} b\left(\boldsymbol{p}_1, \boldsymbol{k}-\boldsymbol{p}_1\right)\zeta_{\boldsymbol{p}_1}\zeta_{\boldsymbol{k}-\boldsymbol{p}_1},$$

$$b\left(\boldsymbol{p}_1, \boldsymbol{p}_2\right) = b_0 - b_1 g\left(\boldsymbol{p}_1, \boldsymbol{p}_2\right)$$

$$g\left(\boldsymbol{p}_1, \boldsymbol{p}_2\right) = 1 + 2\frac{p_1^2+p_2^2}{\left(\boldsymbol{p}_1+\boldsymbol{p}_2\right)^2} - 3\frac{\left(p_1^2-p_2^2\right)^2}{\left(\boldsymbol{p}_1+\boldsymbol{p}_2\right)^4}.$$

$$g\left(\boldsymbol{p}_1, \boldsymbol{p}_2\right) \rightarrow 0$$

$$b(\boldsymbol{p}_1, \boldsymbol{k}-\boldsymbol{p}_1) \rightarrow b_0$$

in the limit when p_1 or p_2 vanishes, which corresponds to the "**squeezed**" configuration and thus the **local-type** NG.

A large b_0 implies a large contribution to the local NG!

GR: $b(\boldsymbol{p}_1, \boldsymbol{p}_2) \xrightarrow{\mathrm{GR}} \frac{1}{25} - \frac{3}{50} g(\boldsymbol{p}_1, \boldsymbol{p}_2)$ cannot contribute large local NG.

Nonlinear parameter

$$\Phi = \Phi_{\mathrm{L}} + f_{\mathrm{NL}} * \Phi_{\mathrm{L}}^2$$

[*Komatsu et al* (2001)]

Ansatz for the primordial NG: $\zeta = \zeta_{\mathrm{L}} + \frac{3}{5-6\beta-\sigma_1} f_{\mathrm{NL}}^{\zeta} * \zeta_{\mathrm{L}}^2$

$$\frac{\Delta T}{T} \equiv -\frac{1}{3}\left(1+\sigma_1\right)\Phi \text{ and } \Phi_{\mathrm{L}} = \frac{3}{5-6\beta-\sigma_1}\zeta_{\mathrm{L}}$$

$$f_{\mathrm{NL}} = f_{\mathrm{NL}}^{\zeta} - \frac{\left(5-6\beta-\sigma_1\right)^2}{6\left(1+\sigma_1\right)} b\left(\boldsymbol{p}_1, \boldsymbol{k}-\boldsymbol{p}_1\right)$$

Primordial NG

Contribution from nonlinear mapping from ζ to ΔT/T, which in principle can be enhanced when gravity is modified.

Parameters

$$b_0 \equiv \frac{1}{(5-6\beta-\sigma_1)^3}\Big[5-6\beta\left(7+2\left(4-\sigma_1\right)\sigma_1+3\sigma_2\right) +36\left(\beta^2-\gamma\right)\left(1+\sigma_1\right) +9\sigma_1-\sigma_1^2\left(15+\sigma_1\right)+18\sigma_2\Big]$$

$$b_1 \equiv \frac{3\left(1+\sigma_1\right)\left(3\lambda+6\beta(2\beta-1)\right)}{2\left(5-6\beta-\sigma_1\right)^3},$$

Sadly, there seems no simple dependences of b_0 and b_1 on the functional structure of f(R), which makes the constraint cumbersome and thus weak.

Numerical results

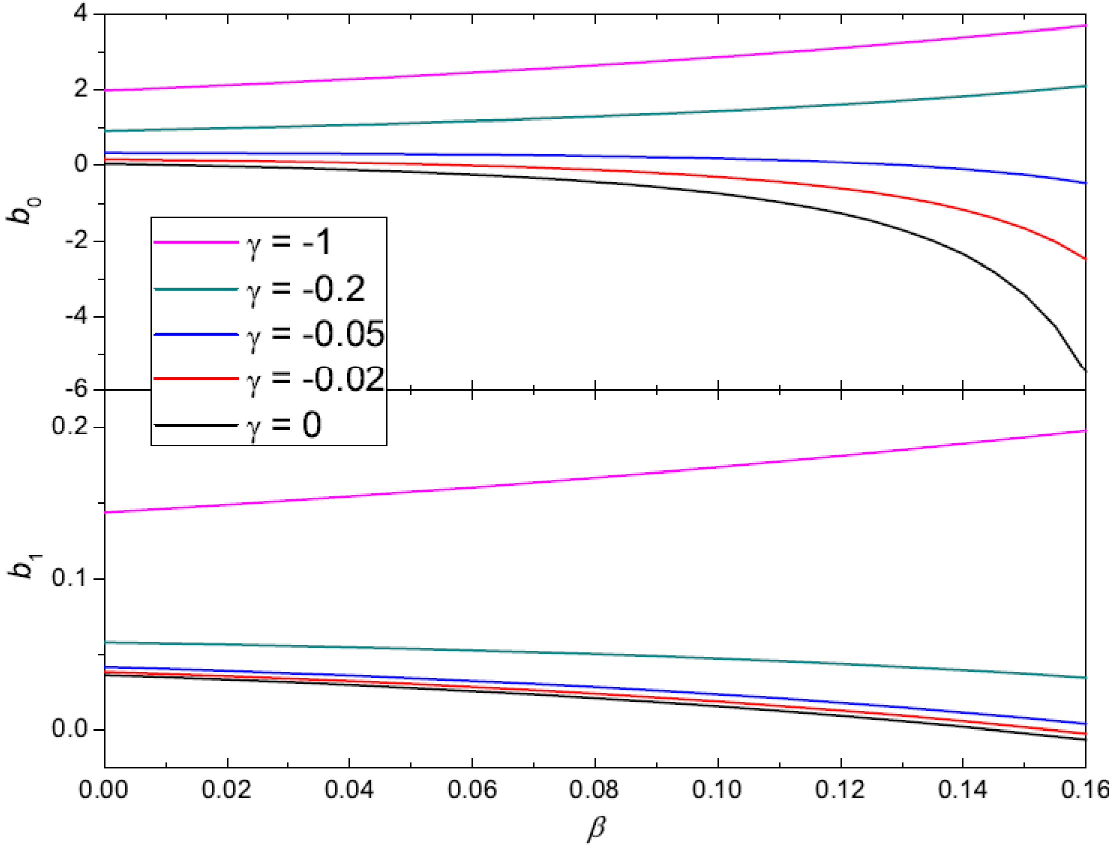
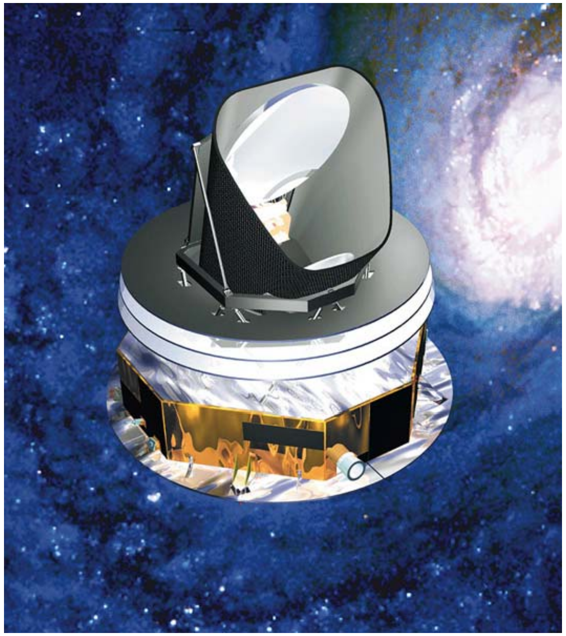


FIG. 1: (color online). b_0 and b_1 as functions of β with diverse values of γ and vanishing δ for the ΛCDM expansion history with $\Omega_\Lambda = 0.74$. In the matter-dominated era, the parameters introduced in (6)-(7) are $\epsilon = 1.5$, $\epsilon_R = -3$ and $\epsilon_{R'} = -3.5$ respectively. We assume the range of values of β, γ and δ ensures such an expansion history.

Conclusion

•Non-Gaussianity will open a new window and bring us more information to the early Universe, which cannot be got from the study of power spectrum.
•Planck will improve WMAP f_{NL} local error bars by a factor 4.
•Non-Gaussianity cannot be ignored in analysis of upcoming CMB observations, even if the primordial perturbations are Gaussian.
•The post-inflationary non-Gaussianity can also be used to probe fundamental physics;
• We have shown that non-Gaussianity due to the nonlinear mappring from ζ to $\Delta T/T$ can be enhanced in principle in f(R) gravity;
• This result provides a new observational window to test modified gravity, which is independent from previous tests;
•More detailed studies are needed and are in progress…

Thanks a lot for your attention!



General form for large-scale anisotropy

Why can we use the late-time evolution to test gravitation?

$$\frac{\Delta T}{T} = \frac{\Delta T}{T}\left[\Phi_e, \Psi_e, \cdots\right] = \frac{\Delta T}{T}\left[\Phi_e\left[\zeta\right], \Psi_e\left[\zeta\right], \cdots\right] = \frac{\Delta T}{T}\left[\zeta\right]$$

Gravitational redshift
Pure kinetic, irrelevant to the theory of gravitation.

Initial conditions on the emission surface (SLS)
Depends on the theory of gravitation!

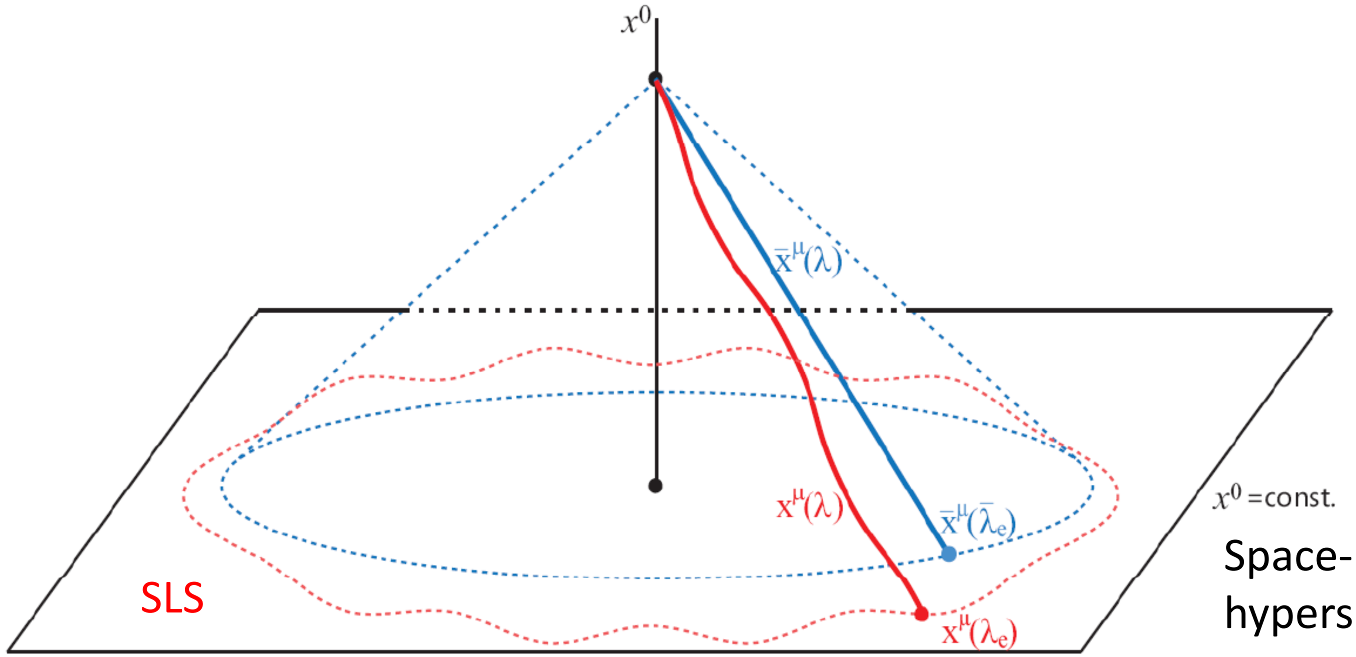
Intrinsic temperature anisotropy on the emission surface (model dependent, also gravitational theory-dependent!):

$$T\left(\eta_e, x_e^i; n_e^i\right) = \bar{T}\left(\eta_e, \bar{x}_e^i; n^i\right) \underline{e^{\tau\left(\eta_e, x_e^i; n_e^i\right)}}$$

Observed anisotropy:

$$\frac{\Delta T}{T}\left(x_o^i, n^i\right) \equiv \frac{T\left(x_o^i, n^i\right) - \bar{T}\left(x_o^i, n^i\right)}{\bar{T}\left(x_o^i, n^i\right)} = \frac{a_o}{a_e}\frac{\omega_o\left(\eta_o, x_o^i; p_o^\mu\right)}{\omega_e\left(\eta_e, x_e^i; p_e^\mu\right)} e^{\tau\left(\eta_e, x_e^i; n_e^i\right)} - 1$$

A closer look at the anisotropy



Space-like hypersurface

$$\frac{\Delta T}{T}\left(x_o^i, n^i\right) \equiv \frac{T\left(x_o^i, n^i\right) - \bar{T}\left(x_o^i, n^i\right)}{\bar{T}\left(x_o^i, n^i\right)} = \frac{a_o}{a_e}\frac{\omega_o\left(\eta_o, x_o^i; p_o^\mu\right)}{\omega_e\left(\eta_e, x_e^i; p_e^\mu\right)} e^{\tau\left(\eta_e, x_e^i; n_e^i\right)} - 1$$

$$\omega = -g_{\mu\nu} u^\mu P^\nu$$

A closer looking at the anisotropy (1)

$$\frac{\Delta T}{T}\left(x_o^i, n^i\right) \equiv \frac{T\left(x_o^i, n^i\right) - \bar{T}\left(x_o^i, n^i\right)}{\bar{T}\left(x_o^i, n^i\right)} = \frac{a_o}{a_e}\frac{\omega_o\left(\eta_o, x_o^i; p_o^\mu\right)}{\omega_e\left(\eta_e, x_e^i; p_e^\mu\right)} e^{\tau\left(\eta_e, x_e^i; n_e^i\right)} - 1$$

Large-scale metric perturbation: $ds^2 = -e^{2\Phi}dt^2 + a^2 e^{-2\Psi} d\boldsymbol{x}^2$

Redshift: $\omega = -g_{\mu\nu} u^\mu P^\nu$

Intrinsic anisotropy (adiabatic): $T_e = \bar{T}_e e^{-\frac{2}{3}\Phi}$

A closer looking at the anisotropy (2)

Nonlinear anisotropy in terms of metric perturbations up to the 3rd-order:
[Bartolo (????); Pitrou (????); Gao (2010)]

$$\left(\frac{\Delta T}{T}\right)_{(1)} = \frac{\Phi}{3} - I_1,$$

$$\left(\frac{\Delta T}{T}\right)_{(2)} = \frac{\Phi^2}{18} + \frac{1}{3}\partial_i\Phi\left(n^i x^0_{(1)} + x^i_{(1)}\right) - \frac{\Phi I_1}{3} - I_2 + x^0_{(1)}A',$$

$$\begin{aligned}\left(\frac{\Delta T}{T}\right)_{(3)} =& \frac{\Phi^3}{162} - \frac{\Phi^2 I_1}{18} + x^0_{(2)}A' + \frac{1}{3}\Phi\left(x^0_{(1)}A' - I_2\right) - I_3 + \frac{1}{2}x^0_{(1)}\left[\partial_i A'\left(n^i x^0_{(1)} + 2x^i_{(1)}\right) - 2I_1 A' + x^0_{(1)}A''\right] \\ &+ \partial_i\Phi\left[\frac{1}{3}\left(x^i_{(2)} + x^0_{(1)}I^i_1\right) + \frac{1}{9}n^i\left(3x^0_{(2)} + x^0_{(1)}\left(\Phi + 6A - 18x^0_{(1)} - 6I_1\right)\right) + x^i_{(1)}\left(\Phi - 3I_1\right)\right] \\ &+ \frac{1}{6}\partial_i\partial_j\Phi\left(x^i_{(1)} + n^i x^0_{(1)}\right)\left(x^j_{(1)} + n^j x^0_{(1)}\right) + 2n^i\partial_i\Phi'\left(x^0_{(1)}\right)^2.\end{aligned}$$

$$\left(\frac{\Delta T}{T}\right) = \frac{\Phi}{3} + \frac{\Phi^2}{18} + \frac{\Phi^3}{162} + \mathrm{ISW} + \mathrm{Lensing} \simeq e^{\frac{\Phi}{3}} + \cdots$$

[Bartolo (????)]

A closer looking at the anisotropy (3)

- (i, j)-component of Einstein equation gives the constraint between Ψ and Φ (up to 3rd order);
- Conservated ζ gives the initial values of Ψ and Φ: $\zeta = -\Psi - \frac{2}{3}\Phi.$

Initial conditions during matter era:

$$\Phi = \Phi_{(1)} + \Phi_{(2)} + \Phi_{(3)} + \cdots$$

$$\Phi_{(1)} = -\frac{3}{5}\zeta.$$

$$\Phi_{(2)} = -\frac{9}{25}\partial^{-4}\left[3\left(\partial^2\zeta\right)^2 + \left(\partial_i\partial_j\zeta\right)^2 + 4\partial_i\zeta\partial_i\partial^2\zeta\right]$$

$$\Phi_{(3)} = -\frac{54}{125}\partial^{-4}\Big[\left(3\partial^2\zeta\partial^{-2} + \partial_i\partial_j\zeta\partial_i\partial_j\partial^{-4} + 2\partial_i\zeta\partial_i\partial^{-2} + 2\partial_i\partial^2\zeta\partial_i\partial^{-4}\right) \times \left(3\left(\partial^2\zeta\right)^2 + \left(\partial_i\partial_j\zeta\right)^2 + 4\partial_i\zeta\partial_i\partial^2\zeta\right)\Big]$$

Non-local

Nonlinear mapping from ζ to ΔT/T on large-scales

$$\zeta \to \frac{\Delta T}{T}[\zeta] \equiv \left(\frac{\Delta T}{T}\right)_{(1)}[\zeta] + \left(\frac{\Delta T}{T}\right)_{(2)}[\zeta] + \left(\frac{\Delta T}{T}\right)_{(3)}[\zeta] + \cdots$$

Linear order: $\left(\frac{\Delta T}{T}\right)_{(1)} = -\frac{1}{5}\zeta$

Nonlinear order [Gao PRD (2010)]:

$$\left(\frac{\Delta T}{T}\right)_{(2)}(\boldsymbol{k}) = \frac{1}{2}\int \frac{d^3p_1 d^3p_2}{(2\pi)^3}\delta^3(\boldsymbol{k}-\boldsymbol{p}_1-\boldsymbol{p}_2)\,\beta(k;p_1,p_2)\,\zeta_{\boldsymbol{p}_1}\zeta_{\boldsymbol{p}_2},$$

$$\left(\frac{\Delta T}{T}\right)_{(3)}(\boldsymbol{k}) = \frac{1}{3!}\int \frac{d^3p_1 d^3p_2 d^3p_3}{(2\pi)^6}\delta^3(\boldsymbol{k}-\boldsymbol{p}_1-\boldsymbol{p}_2-\boldsymbol{p}_3)\,\gamma(\boldsymbol{k};\boldsymbol{p}_1,\boldsymbol{p}_2,\boldsymbol{p}_3)\,\zeta_{\boldsymbol{p}_1}\zeta_{\boldsymbol{p}_2}\zeta_{\boldsymbol{p}_3},$$

$$\beta(k;p_1,p_2) = -\frac{1}{50} + \frac{9\left(p_1^2-p_2^2\right)^2}{50k^4} - \frac{3\left(p_1^2+p_2^2\right)}{25k^2},$$

$$\gamma(\boldsymbol{k};\boldsymbol{p}_1,\boldsymbol{p}_2,\boldsymbol{p}_3) = -\frac{1}{125} + \left[\left(1-g(\boldsymbol{p}_1,\boldsymbol{p}_{23})\right)g(\boldsymbol{p}_2,\boldsymbol{p}_3) + 2\text{ cyclic}\right],$$

$$g(\boldsymbol{p},\boldsymbol{q}) = \frac{3}{250}\left[1 + 2\frac{p^2+q^2}{(\boldsymbol{p}+\boldsymbol{q})^2} - 3\frac{\left(p^2-q^2\right)^2}{(\boldsymbol{p}+\boldsymbol{q})^4}\right]$$