What galaxy surveys really measure: relativisitic corrections to the measured galaxy power spectrum

Ruth Durrer

Geneva and Saclay

UNIVERSITÉ DE GENÈVE
FACULTÉ DES SCIENCES
Département de physique théorique

avril 4, 2011

What do very large scale galaxy catalogs really measure?

- Until now, galaxy surveys have been assumed to measure the the matter density fluctuations apart from aspects like biasing and maybe redshift space distortions.
- We have not taken into account that all observations are actually made on our past lightcone, for example, we see density fluctuations which are further away from us, further in the past.
- the measured redshift is not simply the background redshift $\bar{z}$,
- not only the number of galaxies but also the volume is distorted
- the angles we are looking into are not the ones into which the photons from a given galaxy arriving at our position have been emitted.
- For small galaxy catalogs, such effects are not very relevant, but when we go out to redshifts of order $z \sim 1$ or more, they become very important. Already for SDSS which goes out to $z \simeq 0.2$ (main catalog) or even $z = 0.4$ (LRG's) they become relevant.

Introduction

- Until now, galaxy surveys have been assumed to measure the the matter density fluctuations apart from aspects like biasing and maybe redshift space distortions.
- We have not taken into account that all observations are actually made on our past lightcone, for example, we see density fluctuations which are further away from us, further in the past.
- the measured redshift is not simply the background redshift $\bar{z}$,
- not only the number of galaxies but also the volume is distorted
- the angles we are looking into are not the ones into which the photons from a given galaxy arriving at our position have been emitted.
- For small galaxy catalogs, such effects are not very relevant, but when we go out to redshifts of order $z \sim 1$ or more, they become very important. Already for SDSS which goes out to $z \simeq 0.2$ (main catalog) or even $z = 0.4$ (LRG's) they become relevant.

Introduction

- Until now, galaxy surveys have been assumed to measure the the matter density fluctuations apart from aspects like biasing and maybe redshift space distortions.
- We have not taken into account that all observations are actually made on our past lightcone, for example, we see density fluctuations which are further away from us, further in the past.
- the measured redshift is not simply the background redshift $\bar{z}$,
- not only the number of galaxies but also the volume is distorted
- the angles we are looking into are not the ones into which the photons from a given galaxy arriving at our position have been emitted.
- For small galaxy catalogs, such effects are not very relevant, but when we go out to redshifts of order $z \sim 1$ or more, they become very important. Already for SDSS which goes out to $z \simeq 0.2$ (main catalog) or even $z = 0.4$ (LRG's) they become relevant.

Introduction

- Until now, galaxy surveys have been assumed to measure the the matter density fluctuations apart from aspects like biasing and maybe redshift space distortions.
- We have not taken into account that all observations are actually made on our past lightcone, for example, we see density fluctuations which are further away from us, further in the past.
- the measured redshift is not simply the background redshift $\bar{z}$,
- not only the number of galaxies but also the volume is distorted
- the angles we are looking into are not the ones into which the photons from a given galaxy arriving at our position have been emitted.
- For small galaxy catalogs, such effects are not very relevant, but when we go out to redshifts of order $z \sim 1$ or more, they become very important. Already for SDSS which goes out to $z \simeq 0.2$ (main catalog) or even $z = 0.4$ (LRG's) they become relevant.

Introduction

- Until now, galaxy surveys have been assumed to measure the the matter density fluctuations apart from aspects like biasing and maybe redshift space distortions.
- We have not taken into account that all observations are actually made on our past lightcone, for example, we see density fluctuations which are further away from us, further in the past.
- the measured redshift is not simply the background redshift $\bar{z}$,
- not only the number of galaxies but also the volume is distorted
- the angles we are looking into are not the ones into which the photons from a given galaxy arriving at our position have been emitted.
- For small galaxy catalogs, such effects are not very relevant, but when we go out to redshifts of order $z \sim 1$ or more, they become very important. Already for SDSS which goes out to $z \simeq 0.2$ (main catalog) or even $z = 0.4$ (LRG's) they become relevant.

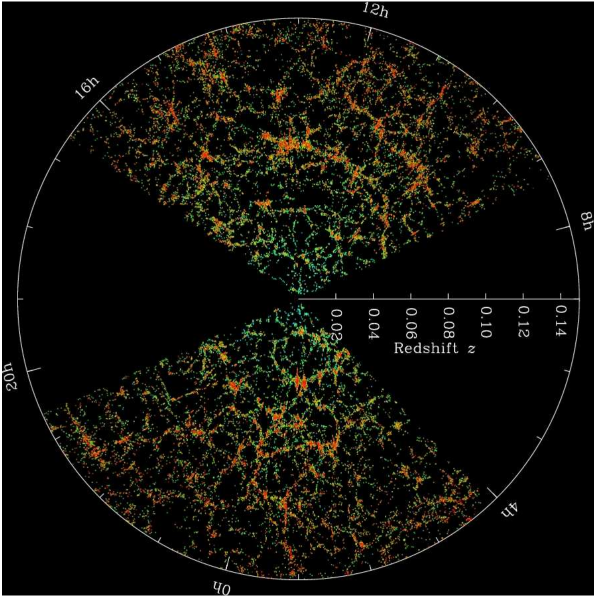
Introduction

- Until now, galaxy surveys have been assumed to measure the the matter density fluctuations apart from aspects like biasing and maybe redshift space distortions.
- We have not taken into account that all observations are actually made on our past lightcone, for example, we see density fluctuations which are further away from us, further in the past.
- the measured redshift is not simply the background redshift $\bar{z}$,
- not only the number of galaxies but also the volume is distorted
- the angles we are looking into are not the ones into which the photons from a given galaxy arriving at our position have been emitted.
- For small galaxy catalogs, such effects are not very relevant, but when we go out to redshifts of order $z \sim 1$ or more, they become very important. Already for SDSS which goes out to $z \simeq 0.2$ (main catalog) or even $z = 0.4$ (LRG's) they become relevant.

Introduction

- Until now, galaxy surveys have been assumed to measure the the matter density fluctuations apart from aspects like biasing and maybe redshift space distortions.
- We have not taken into account that all observations are actually made on our past lightcone, for example, we see density fluctuations which are further away from us, further in the past.
- the measured redshift is not simply the background redshift $\bar{z}$,
- not only the number of galaxies but also the volume is distorted
- the angles we are looking into are not the ones into which the photons from a given galaxy arriving at our position have been emitted.
- For small galaxy catalogs, such effects are not very relevant, but when we go out to redshifts of order $z \sim 1$ or more, they become very important. Already for SDSS which goes out to $z \simeq 0.2$ (main catalog) or even $z = 0.4$ (LRG's) they become relevant.

Introduction



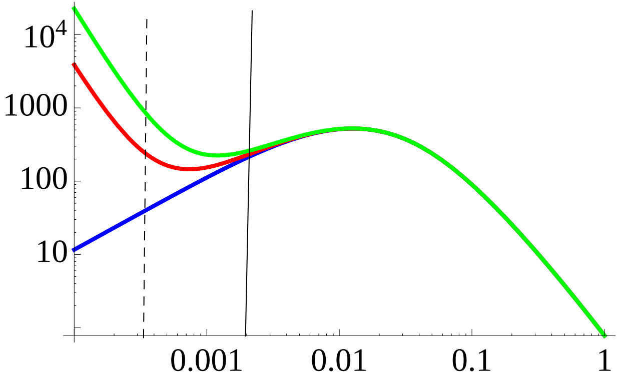
M. Blanton and the Sloan Digital Sky Survey.

- Until now, galaxy surveys have been assumed to measure the the matter density fluctuations apart from aspects like biasing and maybe redshift space distortions.
- We have not taken into account that all observations are actually made on our past lightcone, for example, we see density fluctuations which are further away from us, further in the past.
- the measured redshift is not simply the background redshift $\bar{z}$,
- not only the number of galaxies but also the volume is distorted
- the angles we are looking into are not the ones into which the photons from a given galaxy arriving at our position have been emitted.
- For small galaxy catalogs, such effects are not very relevant, but when we go out to redshifts of order $z \sim 1$ or more, they become very important. Already for SDSS which goes out to $z \simeq 0.2$ (main catalog) or even $z = 0.4$ (LRG's) they become relevant.
- But of course much more for future surveys like DES, BOSS and Euclid.
- We shall see that these relativistic effects do not only complicate the interpretation of galaxy surveys, but they also represent a new opportunity!

Introduction

- Until now, galaxy surveys have been assumed to measure the the matter density fluctuations apart from aspects like biasing and maybe redshift space distortions.
- We have not taken into account that all observations are actually made on our past lightcone, for example, we see density fluctuations which are further away from us, further in the past.
- the measured redshift is not simply the background redshift $\bar{z}$,
- not only the number of galaxies but also the volume is distorted
- the angles we are looking into are not the ones into which the photons from a given galaxy arriving at our position have been emitted.
- For small galaxy catalogs, such effects are not very relevant, but when we go out to redshifts of order $z \sim 1$ or more, they become very important. Already for SDSS which goes out to $z \simeq 0.2$ (main catalog) or even $z = 0.4$ (LRG's) they become relevant.
- But of course much more for future surveys like DES, BOSS and Euclid.
- We shall see that these relativistic effects do not only complicate the interpretation of galaxy surveys, but they also represent a new opportunity!

To illustrate our point let us first just note that the density fluctuation is a gauge dependent quantity and we obtain different results on large scales whether we calculate it in comoving gauge or longitudinal gauge



The matter power spectrum in comoving gauge (blue), longitudinal gauge (red) and spatially flat gauge (green) as function of $k\,[h\mathrm{Mpc}^{-1}]$.
$H_0[h\mathrm{Mpc}^{-1}] = 0.33 \times 10^{-3}$ (dashed grey), $k_{sloan}[h\mathrm{Mpc}^{-1}] \simeq 2 \times 10^{-3}$ (soldi grey).

What are very large scale galaxy catalogs really measuring?

Following C. Bonvin & RD, in preparation 2011.
Relativisitc corrections to galaxy surveys are also discussed int: J. Yoo el al. 2009, J. Yoo 2010

For each galaxy in a catalog we measure

$$(z, \theta, \phi) = (z, \mathbf{n}) \quad + \text{info about mass, spectral type...}$$

We can count the galaxies inside a redshift bin and small solid angle, $N(z, \mathbf{n})$ and measure the fluctuation of this count:

$$\Delta(z, \mathbf{n}) = \frac{N(z, \mathbf{n}) - \bar{N}(z)}{\bar{N}(z)}.$$

This quantity is directly measurable.
It must therefore be possible to express it in terms of gauge invariant perturbation variables to first order in perturbation theory.

On small scales, gauge artefacts and fluctuations in the spacetime geometry can be neglected, $\Psi \propto (H_0/k)^2\delta$. Then the number density fluctuation it is simply related to the density contrast $\delta = (\rho(\mathbf{x}, t) - \bar{\rho}(t))/\bar{\rho}(t)$ (up to the problem of biasing).

What are very large scale galaxy catalogs really measuring?

Following C. Bonvin & RD, in preparation 2011.
Relativisitc corrections to galaxy surveys are also discussed int: J. Yoo el al. 2009, J. Yoo 2010

For each galaxy in a catalog we measure

$$(z, \theta, \phi) = (z, \mathbf{n}) \quad + \text{info about mass, spectral type...}$$

We can count the galaxies inside a redshift bin and small solid angle, $N(z, \mathbf{n})$ and measure the fluctuation of this count:

$$\Delta(z, \mathbf{n}) = \frac{N(z, \mathbf{n}) - \bar{N}(z)}{\bar{N}(z)}.$$

This quantity is directly measurable.
It must therefore be possible to express it in terms of gauge invariant perturbation variables to first order in perturbation theory.

On small scales, gauge artefacts and fluctuations in the spacetime geometry can be neglected, $\Psi \propto (H_0/k)^2\delta$. Then the number density fluctuation it is simply related to the density contrast $\delta = (\rho(\mathbf{x}, t) - \bar{\rho}(t))/\bar{\rho}(t)$ (up to the problem of biasing).

What are very large scale galaxy catalogs really measuring?

Following C. Bonvin & RD, in preparation 2011.
Relativisitc corrections to galaxy surveys are also discussed int: J. Yoo el al. 2009, J. Yoo 2010

For each galaxy in a catalog we measure

$$(z, \theta, \phi) = (z, \mathbf{n}) \quad + \text{info about mass, spectral type...}$$

We can count the galaxies inside a redshift bin and small solid angle, $N(z, \mathbf{n})$ and measure the fluctuation of this count:

$$\Delta(z, \mathbf{n}) = \frac{N(z, \mathbf{n}) - \bar{N}(z)}{\bar{N}(z)}.$$

This quantity is directly measurable.
It must therefore be possible to express it in terms of gauge invariant perturbation variables to first order in perturbation theory.

On small scales, gauge artefacts and fluctuations in the spacetime geometry can be neglected, $\Psi \propto (H_0/k)^2\delta$. Then the number density fluctuation it is simply related to the density contrast $\delta = (\rho(\mathbf{x}, t) - \bar{\rho}(t))/\bar{\rho}(t)$ (up to the problem of biasing).

What are very large scale galaxy catalogs really measuring?

Following C. Bonvin & RD, in preparation 2011.
Relativisitc corrections to galaxy surveys are also discussed int: J. Yoo el al. 2009, J. Yoo 2010

For each galaxy in a catalog we measure

$$(z, \theta, \phi) = (z, \mathbf{n}) \quad + \text{info about mass, spectral type...}$$

We can count the galaxies inside a redshift bin and small solid angle, $N(z, \mathbf{n})$ and measure the fluctuation of this count:

$$\Delta(z, \mathbf{n}) = \frac{N(z, \mathbf{n}) - \bar{N}(z)}{\bar{N}(z)}.$$

This quantity is directly measurable.
It must therefore be possible to express it in terms of gauge invariant perturbation variables to first order in perturbation theory.

On small scales, gauge artefacts and fluctuations in the spacetime geometry can be neglected, $\Psi \propto (H_0/k)^2\delta$. Then the number density fluctuation it is simply related to the density contrast $\delta = (\rho(\mathbf{x}, t) - \bar{\rho}(t))/\bar{\rho}(t)$ (up to the problem of biasing).

What are very large scale galaxy catalogs really measuring?

Following C. Bonvin & RD, in preparation 2011.
Relativisitc corrections to galaxy surveys are also discussed int: J. Yoo el al. 2009, J. Yoo 2010

For each galaxy in a catalog we measure

$$(z, \theta, \phi) = (z, \mathbf{n}) \quad + \text{info about mass, spectral type...}$$

We can count the galaxies inside a redshift bin and small solid angle, $N(z, \mathbf{n})$ and measure the fluctuation of this count:

$$\Delta(z, \mathbf{n}) = \frac{N(z, \mathbf{n}) - \bar{N}(z)}{\bar{N}(z)}.$$

This quantity is directly measurable.
It must therefore be possible to express it in terms of gauge invariant perturbation variables to first order in perturbation theory.

On small scales, gauge artefacts and fluctuations in the spacetime geometry can be neglected, $\Psi \propto (H_0/k)^2\delta$. Then the number density fluctuation it is simply related to the density contrast $\delta = (\rho(\mathbf{x}, t) - \bar{\rho}(t))/\bar{\rho}(t)$ (up to the problem of biasing).

What are very large scale galaxy catalogs really measuring?

We first define the density fluctuation per redshift bin dz as $\delta_z(\mathbf{n}, z)$.

$$\delta_z(\mathbf{n}, z) = \frac{\rho(\mathbf{n}, z) - \bar{\rho}(z)}{\bar{\rho}(z)} = \frac{\frac{N(\mathbf{n},z)}{V(\mathbf{n},z)} - \frac{\bar{N}(z)}{V(z)}}{\frac{\bar{N}(z)}{V(z)}}$$

This together with the volume fluctuations, results in the directly observed number fluctuations

$$\Delta(\mathbf{n}, z) = \delta_z(\mathbf{n}, z) + \frac{\delta V(\mathbf{n}, z)}{V(z)}$$

Both these terms are in principle measurable and therefore gauge invariant. We want to express them in terms of standard gauge invariant perturbation variables.

What are very large scale galaxy catalogs really measuring?

We first define the density fluctuation per redshift bin dz as $\delta_z(\mathbf{n},z)$.

$$\delta_z(\mathbf{n},z) = \frac{\rho(\mathbf{n},z)-\bar\rho(z)}{\bar\rho(z)} = \frac{\frac{N(\mathbf{n},z)}{V(\mathbf{n},z)} - \frac{\bar N(z)}{V(z)}}{\frac{\bar N(z)}{V(z)}}$$

This together with the volume fluctuations, results in the directly observed number fluctuations

$$\Delta(\mathbf{n},z) = \delta_z(\mathbf{n},z) + \frac{\delta V(\mathbf{n},z)}{V(z)}$$

Both these terms are in principle measurable and therefore gauge invariant. We want to express them in terms of standard gauge invariant perturbation variables.

We first define the density fluctuation per redshift bin dz as $\delta_z(\mathbf{n}, z)$.

$$\delta_z(\mathbf{n}, z) = \frac{\rho(\mathbf{n}, z) - \bar{\rho}(z)}{\bar{\rho}(z)} = \frac{\frac{N(\mathbf{n},z)}{V(\mathbf{n},z)} - \frac{\bar{N}(z)}{V(z)}}{\frac{\bar{N}(z)}{V(z)}}$$

This together with the volume fluctuations, results in the directly observed number fluctuations

$$\Delta(\mathbf{n}, z) = \delta_z(\mathbf{n}, z) + \frac{\delta V(\mathbf{n}, z)}{V(z)}$$

Both these terms are in principle measurable and therefore gauge invariant. We want to express them in terms of standard gauge invariant perturbation variables.

We consider a photon emitted from a galaxy (S), moving in direction $\mathbf{n}$ into our telescope. The observer (O) receives the photon redshifted by a factor

$$1 + z = \frac{(n \cdot u)_S}{(n \cdot u)_O} .$$

To first order in perturbation theory one finds (in longitudinal gauge)

$$\frac{\delta z}{(1+z)} = -\left[(\mathbf{n} \cdot \mathbf{V} + \Psi)(\mathbf{n}, z) + \int_0^{r_S} (\dot\Phi + \dot\Psi) d\lambda\right]$$

With this, the density fluctuation in redshift space becomes

$$\delta_z(\mathbf{n}, z) = D_g(\mathbf{n}, z) + 3(\mathbf{V} \cdot \mathbf{n})(\mathbf{n}, z) + 3(\Psi + \Phi)(\mathbf{n}, z) + 3\int_0^{r_S} (\dot\Psi + \dot\Phi)(\mathbf{n}, z(\lambda)) d\lambda$$

This quantity is gauge invariant and therefore, in principle, measurable. E.g. if we could compare fluctuations of two different classes of objects, we could eliminate the common volume distortion and directly measure the density fluctuation in redshift space.

We consider a photon emitted from a galaxy (S), moving in direction **n** into our telescope. The observer (O) receives the photon redshifted by a factor

$$1 + z = \frac{(n \cdot u)_S}{(n \cdot u)_O} \ .$$

To first order in perturbation theory one finds (in longitudinal gauge)

$$\frac{\delta z}{(1+z)} = -\left[\left(\mathbf{n} \cdot \mathbf{V} + \Psi \right)(\mathbf{n}, z) + \int_0^{r_S} (\dot{\Phi} + \dot{\Psi}) d\lambda \right]$$

With this, the density fluctuation in redshift space becomes

$$\delta_z(\mathbf{n}, z) = D_g(\mathbf{n}, z) + 3(\mathbf{V} \cdot \mathbf{n})(\mathbf{n}, z) + 3(\Psi + \Phi)(\mathbf{n}, z) + 3\int_0^{r_S} (\dot{\Psi} + \dot{\Phi})(\mathbf{n}, z(\lambda)) d\lambda$$

This quantity is gauge invariant and therefore, in principle, measurable. E.g. if we could compare fluctuations of two different classes of objects, we could eliminate the common volume distortion and directly measure the density fluctuation in redshift space.

Matter fluctuations per redshift bin

We consider a photon emitted from a galaxy (S), moving in direction $\mathbf{n}$ into our telescope. The observer (O) receives the photon redshifted by a factor

$$1 + z = \frac{(n \cdot u)_S}{(n \cdot u)_O} \ .$$

To first order in perturbation theory one finds (in longitudinal gauge)

$$\frac{\delta z}{(1+z)} = -\left[\left(\mathbf{n} \cdot \mathbf{V} + \Psi \right)(\mathbf{n}, z) + \int_0^{r_S} (\dot{\Phi} + \dot{\Psi}) d\lambda \right]$$

With this, the density fluctuation in redshift space becomes

$$\delta_z(\mathbf{n}, z) = D_g(\mathbf{n}, z) + 3(\mathbf{V} \cdot \mathbf{n})(\mathbf{n}, z) + 3(\Psi + \Phi)(\mathbf{n}, z) + 3 \int_0^{r_S} (\dot{\Psi} + \dot{\Phi})(\mathbf{n}, z(\lambda)) d\lambda$$

This quantity is gauge invariant and therefore, in principle, measurable. E.g. if we could compare fluctuations of two different classes of objects, we could eliminate the common volume distortion and directly measure the density fluctuation in redshift space.

Volume perturbations

We consider a small volume element at the source position. By this we mean the spatial volume of an source with 4-velocity u^μ.

$$
\begin{aligned}
dV &= \sqrt{-g}\;\epsilon_{abcd}\; u^a \mathrm{d}x^b \mathrm{d}x^c \mathrm{d}x^d \\
&= \sqrt{-g}\;\epsilon_{abcd} u^a \frac{\partial x^b}{\partial z}\frac{\partial x^c}{\partial \theta_S}\frac{\partial x^d}{\partial \varphi_S}\left|\frac{\partial(\theta_S,\varphi_S)}{\partial(\theta_O,\varphi_O)}\right| \mathrm{d}z\,\mathrm{d}\theta_O\,\mathrm{d}\varphi_O \\
&\equiv v(z,\theta_O,\varphi_O)\mathrm{d}z\,\mathrm{d}\theta_O\,\mathrm{d}\varphi_O\,.
\end{aligned}
$$

In a perturbed universe the angles at the source are perturbed with respect to the angle at the observer and we have $\theta_S = \theta_O + \delta\theta$ and $\varphi_S = \varphi_O + \delta\varphi$. To first order the Jacobian determinant becomes

$$
\left|\frac{\partial(\theta_S,\varphi_S)}{\partial(\theta_O,\varphi_O)}\right| = 1 + \frac{\partial\delta\theta}{\partial\theta} + \frac{\partial\delta\varphi}{\partial\varphi}.
$$

The metric determinant to first order is (in longitudinal gauge), $\sqrt{-g} = a^4(1+\Psi-3\Phi)$. Expanding all the terms to first order one obtains for the volume perturbation at observed redshift z

$$
\begin{aligned}
\frac{\delta v}{\bar v}(\mathbf{n},z) &= \frac{v(z)-\bar v(z)}{\bar v(z)} = -3\Phi + \left(\cot\theta_O + \frac{1}{\partial\theta}\right)\delta\theta + \frac{\partial\delta\varphi}{\partial\varphi} - \mathbf{V}\cdot\mathbf{n} + \frac{2\delta r}{r} \\
&\quad - \frac{d\delta r}{d\lambda} + \frac{1}{\mathcal{H}(1+z)}\frac{d\delta z}{d\lambda} - \left(-4 + \frac{2}{r\mathcal{H}} + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2}\right)\frac{\delta z}{1+z}.
\end{aligned}
$$

Volume perturbations

We consider a small volume element at the source position. By this we mean the spatial volume of an source with 4-velocity u^μ.

$$
\begin{aligned}
dV &= \sqrt{-g}\;\epsilon_{abcd}\;u^a \mathrm{d}x^b \mathrm{d}x^c \mathrm{d}x^d \\
&= \sqrt{-g}\;\epsilon_{abcd} u^a \frac{\partial x^b}{\partial z}\frac{\partial x^c}{\partial \theta_S}\frac{\partial x^d}{\partial \varphi_S}\left|\frac{\partial(\theta_S,\varphi_S)}{\partial(\theta_O,\varphi_O)}\right| \mathrm{d}z\,\mathrm{d}\theta_O\,\mathrm{d}\varphi_O \\
&\equiv v(z,\theta_O,\varphi_O)\mathrm{d}z\,\mathrm{d}\theta_O\,\mathrm{d}\varphi_O\,.
\end{aligned}
$$

In a perturbed universe the angles at the source are perturbed with respect to the angle at the observer and we have $\theta_S = \theta_O + \delta\theta$ and $\varphi_S = \varphi_O + \delta\varphi$. To first order the Jacobian determinant becomes

$$
\left|\frac{\partial(\theta_S,\varphi_S)}{\partial(\theta_O,\varphi_O)}\right| = 1 + \frac{\partial\delta\theta}{\partial\theta} + \frac{\partial\delta\varphi}{\partial\varphi}.
$$

The metric determinant to first order is (in longitudinal gauge), $\sqrt{-g} = a^4(1+\Psi-3\Phi)$. Expanding all the terms to first order one obtains for the volume perturbation at observed redshift z

$$
\begin{aligned}
\frac{\delta v}{\bar v}(\mathbf{n},z) &= \frac{v(z)-\bar v(z)}{\bar v(z)} = -3\Phi + \left(\cot\theta_O + \frac{1}{\partial\theta}\right)\delta\theta + \frac{\partial\delta\varphi}{\partial\varphi} - \mathbf{V}\cdot\mathbf{n} + \frac{2\delta r}{r} \\
&\quad - \frac{d\delta r}{d\lambda} + \frac{1}{\mathcal{H}(1+z)}\frac{d\delta z}{d\lambda} - \left(-4 + \frac{2}{r\mathcal{H}} + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2}\right)\frac{\delta z}{1+z}.
\end{aligned}
$$

Volume perturbations

We consider a small volume element at the source position. By this we mean the spatial volume of an source with 4-velocity u^μ.

$$\begin{aligned} dV &= \sqrt{-g}\ \epsilon_{abcd}\ u^a \mathrm{d}x^b \mathrm{d}x^c \mathrm{d}x^d \\ &= \sqrt{-g}\ \epsilon_{abcd} u^a \frac{\partial x^b}{\partial z}\frac{\partial x^c}{\partial \theta_S}\frac{\partial x^d}{\partial \varphi_S}\left|\frac{\partial(\theta_S,\varphi_S)}{\partial(\theta_O,\varphi_O)}\right| \mathrm{d}z\,\mathrm{d}\theta_O\,\mathrm{d}\varphi_O \\ &\equiv v(z,\theta_O,\varphi_O)\mathrm{d}z\,\mathrm{d}\theta_O\,\mathrm{d}\varphi_O\,. \end{aligned}$$

In a perturbed universe the angles at the source are perturbed with respect to the angle at the observer and we have $\theta_S = \theta_O + \delta\theta$ and $\varphi_S = \varphi_O + \delta\varphi$. To first order the Jacobian determinant becomes

$$\left|\frac{\partial(\theta_S,\varphi_S)}{\partial(\theta_O,\varphi_O)}\right| = 1 + \frac{\partial\delta\theta}{\partial\theta} + \frac{\partial\delta\varphi}{\partial\varphi}.$$

The metric determinant to first order is (in longitudinal gauge), $\sqrt{-g} = a^4(1+\Psi-3\Phi)$. Expanding all the terms to first order one obtains for the volume perturbation at observed redshift z

$$\begin{aligned} \frac{\delta v}{\bar v}(\mathbf{n},z) &= \frac{v(z)-\bar v(z)}{\bar v(z)} = -3\Phi + \left(\cot\theta_O + \frac{1}{\partial\theta}\right)\delta\theta + \frac{\partial\delta\varphi}{\partial\varphi} - \mathbf{V}\cdot\mathbf{n} + \frac{2\delta r}{r} \\ &\quad - \frac{d\delta r}{d\lambda} + \frac{1}{\mathcal{H}(1+z)}\frac{d\delta z}{d\lambda} - \left(-4 + \frac{2}{r\mathcal{H}} + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2}\right)\frac{\delta z}{1+z}. \end{aligned}$$

Volume perturbations

We consider a small volume element at the source position. By this we mean the spatial volume of an source with 4-velocity u^μ.

$$\begin{aligned} dV &= \sqrt{-g}\ \epsilon_{abcd}\ u^a \mathrm{d}x^b \mathrm{d}x^c \mathrm{d}x^d \\ &= \sqrt{-g}\ \epsilon_{abcd} u^a \frac{\partial x^b}{\partial z}\frac{\partial x^c}{\partial \theta_S}\frac{\partial x^d}{\partial \varphi_S}\left|\frac{\partial(\theta_S,\varphi_S)}{\partial(\theta_O,\varphi_O)}\right| \mathrm{d}z\mathrm{d}\theta_O\mathrm{d}\varphi_O \\ &\equiv v(z,\theta_O,\varphi_O)\mathrm{d}z\mathrm{d}\theta_O\mathrm{d}\varphi_O\,. \end{aligned}$$

In a perturbed universe the angles at the source are perturbed with respect to the angle at the observer and we have $\theta_S = \theta_O + \delta\theta$ and $\varphi_S = \varphi_O + \delta\varphi$. To first order the Jacobian determinant becomes

$$\left|\frac{\partial(\theta_S,\varphi_S)}{\partial(\theta_O,\varphi_O)}\right| = 1 + \frac{\partial\delta\theta}{\partial\theta} + \frac{\partial\delta\varphi}{\partial\varphi}.$$

The metric determinant to first order is (in longitudinal gauge), $\sqrt{-g} = a^4(1+\Psi-3\Phi)$.

Expanding all the terms to first order one obtains for the volume perturbation at observed redshift z

$$\begin{aligned} \frac{\delta v}{\bar{v}}(\mathbf{n},z) &= \frac{v(z)-\bar{v}(z)}{\bar{v}(z)} = -3\Phi + \left(\cot\theta_O + \frac{1}{\partial\theta}\right)\delta\theta + \frac{\partial\delta\varphi}{\partial\varphi} - \mathbf{V}\cdot\mathbf{n} + \frac{2\delta r}{r} \\ & -\frac{d\delta r}{d\lambda} + \frac{1}{\mathcal{H}(1+z)}\frac{d\delta z}{d\lambda} - \left(-4 + \frac{2}{r\mathcal{H}} + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2}\right)\frac{\delta z}{1+z}. \end{aligned}$$

Volume perturbations

We consider a small volume element at the source position. By this we mean the spatial volume of an source with 4-velocity u^μ.

$$
\begin{aligned}
dV &= \sqrt{-g}\;\epsilon_{abcd}\;u^a \mathrm{d}x^b \mathrm{d}x^c \mathrm{d}x^d \\
&= \sqrt{-g}\;\epsilon_{abcd} u^a \frac{\partial x^b}{\partial z}\frac{\partial x^c}{\partial \theta_S}\frac{\partial x^d}{\partial \varphi_S}\left|\frac{\partial(\theta_S,\varphi_S)}{\partial(\theta_O,\varphi_O)}\right| \mathrm{d}z\,\mathrm{d}\theta_O\,\mathrm{d}\varphi_O \\
&\equiv v(z,\theta_O,\varphi_O)\mathrm{d}z\,\mathrm{d}\theta_O\,\mathrm{d}\varphi_O\,.
\end{aligned}
$$

In a perturbed universe the angles at the source are perturbed with respect to the angle at the observer and we have $\theta_S = \theta_O + \delta\theta$ and $\varphi_S = \varphi_O + \delta\varphi$. To first order the Jacobian determinant becomes

$$
\left|\frac{\partial(\theta_S,\varphi_S)}{\partial(\theta_O,\varphi_O)}\right| = 1 + \frac{\partial\delta\theta}{\partial\theta} + \frac{\partial\delta\varphi}{\partial\varphi}.
$$

The metric determinant to first order is (in longitudinal gauge), $\sqrt{-g} = a^4(1+\Psi-3\Phi)$. Expanding all the terms to first order one obtains for the volume perturbation at observed redshift z

$$
\begin{aligned}
\frac{\delta v}{\bar v}(\mathbf{n},z) &= \frac{v(z)-\bar v(z)}{\bar v(z)} = -3\Phi + \left(\cot\theta_O + \frac{1}{\partial\theta}\right)\delta\theta + \frac{\partial\delta\varphi}{\partial\varphi} - \mathbf{V}\cdot\mathbf{n} + \frac{2\delta r}{r} \\
&\quad - \frac{d\delta r}{d\lambda} + \frac{1}{\mathcal{H}(1+z)}\frac{d\delta z}{d\lambda} - \left(-4 + \frac{2}{r\mathcal{H}} + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2}\right)\frac{\delta z}{1+z}.
\end{aligned}
$$

A somewhat lengthy but straight forward calculation of the perturbed photon geodesic allows to calculate the angle perturbations $\delta\theta$ and $\delta\varphi$ and the radial perturbation δr. Putting it all together one obtains after several integrations by part the volume perturbation

$$\begin{aligned}\frac{\delta v}{v} &= -2(\Psi+\Phi) - 3\mathbf{V}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\left[\dot\Phi + \partial_r(\mathbf{V}\cdot\mathbf{n})\right] \\ &+ \left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2}{r_S\mathcal{H}}\right)\left(\Psi + \mathbf{V}\cdot\mathbf{n} + \int_0^{r_S} d\lambda(\dot\Phi+\dot\Psi)\right) \\ &-3\int_0^{r_S} d\lambda(\dot\Phi+\dot\Psi) + \frac{2}{r_S}\int_0^{r_S} d\lambda(\Phi+\Psi) - \frac{1}{r_S}\int_0^{r_S} d\lambda\frac{r_S - r}{r}\Delta_\Omega(\Phi+\Psi)\,.\end{aligned}$$

The total galaxy density fluctuation per redshift bin, per sold angle

Putting the density and volume fluctuations together one obtains the galaxy number density fluctuations

$$\begin{aligned}\Delta(\mathbf{n},z) \quad &= \quad D_g + \Phi + \Psi + \frac{1}{\mathcal{H}}\left[\dot{\Phi} - \partial_r(\mathbf{V}\cdot\mathbf{n})\right] \\ &+ \left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2}{r_S\mathcal{H}}\right)\left(\Psi + \mathbf{V}\cdot\mathbf{n} + \int_0^{r_S} d\lambda(\dot{\Phi}+\dot{\Psi})\right) \\ &+ \frac{1}{r_S}\int_0^{r_S} d\lambda\left[2 - \frac{r_S - r}{r}\Delta_\Omega\right](\Phi+\Psi).\end{aligned}$$

Putting the density and volume fluctuations together one obtains the galaxy number density fluctuations

$$\begin{aligned}\Delta(\mathbf{n},z) &= \boxed{D_g} + \Phi + \Psi + \frac{1}{\mathcal{H}}\left[\dot{\Phi} - \boxed{\partial_r(\mathbf{V}\cdot\mathbf{n})}\right] \\ &+ \left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2}{r_S\mathcal{H}}\right)\left(\Psi + \boxed{\mathbf{V}\cdot\mathbf{n}} + \int_0^{r_S} d\lambda(\dot{\Phi} + \dot{\Psi})\right) \\ &+ \frac{1}{r_S}\int_0^{r_S} d\lambda\left[2(\Phi+\Psi) - \boxed{\frac{r_S - r}{r}\Delta_\Omega(\Phi+\Psi)}\right].\end{aligned}$$

The angular power spectrum of galaxy density fluctuations

For fixed z, we can expand $\Delta(\mathbf{n}, z)$ quantity in spherical harmonics,

$$\Delta(\mathbf{n}, z) = \sum_{\ell m} a_{\ell m}(z) Y_{\ell m}(\mathbf{n}), \qquad C_\ell(z) = \langle |a_{\ell m}|^2 \rangle .$$

The transversal power spectrum

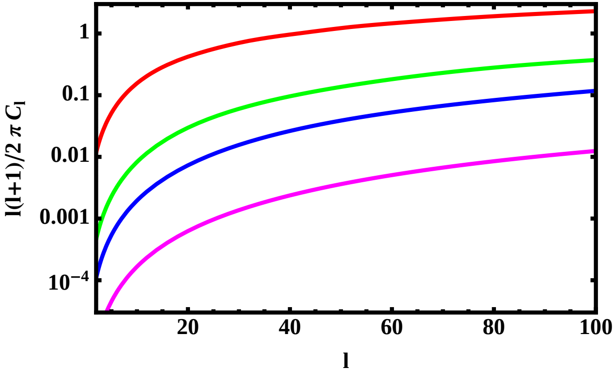
The $C_\ell(z)$'s at fixed redshift z are the transversal power spectrum. They are dominated by the density contribution and the redshift space distortion which correspond to integrals of the form

$$\propto \int \frac{dk}{k} j_\ell^2(r_s k) \left(\frac{k}{H_0}\right)^4 T_\Psi^2(k)$$

They measure the perturbation spectrum not at the scale $k \sim \ell H_0$ but at the maximum of the transfer function since $\int \frac{dk}{k} j_\ell^2(r_s k) \left(\frac{k}{H_0}\right)^4$ is UV divergent.

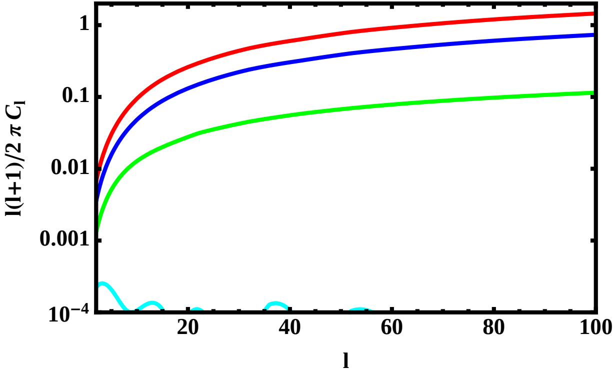
$$\int \frac{dk}{k} j_\ell^2(r_s k) \left(\frac{k}{H_0}\right)^4 \sim (r_S H_0)^{-2} \left(\frac{k_{\max}}{H_0}\right)^2$$

The transverse power spectrum (from Bonvin & RD '11)



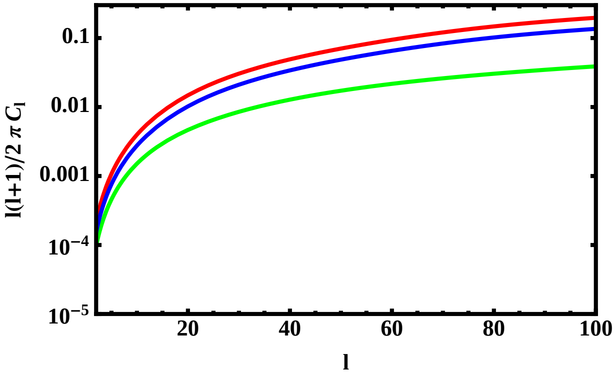
$z = 0.1$ (red), $z = 0.5$ (green), $z = 1$ (blue), $z = 3$ (magenta).

Contributions to the transverse power spectrum at redshift $z = 0.1$ (from Bonvin & RD '11)



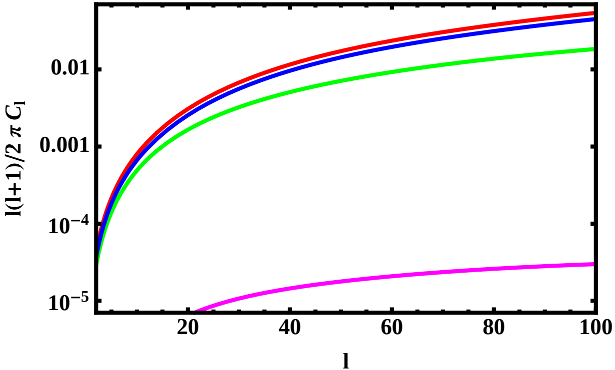
C_ℓ^{DD} (red), C_ℓ^{zz} (green), C_ℓ^{Dz} (blue), C_ℓ^{VV} (cyan).

Contributions to the transverse power spectrum at redshift $z = 0.5$ (from Bonvin & RD '11)



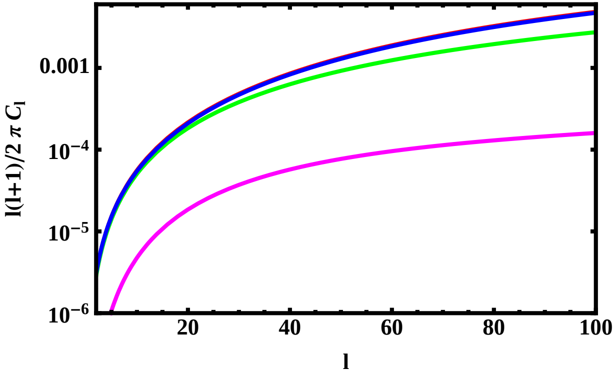
C_ℓ^{DD} (red), C_ℓ^{zz} (green), C_ℓ^{Dz} (blue).

Contributions to the transverse power spectrum at redshift $z = 1$ (from Bonvin & RD '11)



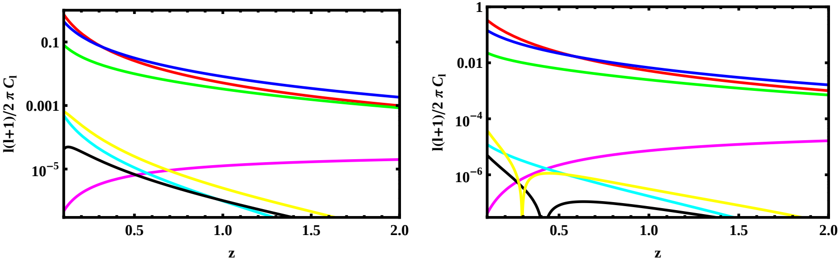
C_ℓ^{DD} (red), C_ℓ^{zz} (green), C_ℓ^{Dz} (blue), $C_\ell^{lensing}$ (magenta).

Contributions to the transverse power spectrum at redshift $z = 3$
(from Bonvin & RD '11)



C_ℓ^{DD} (red), C_ℓ^{zz} (green), C_ℓ^{Dz} (blue), $C_\ell^{lensing}$ (magenta).

Contributions to the transversal power spectrum as function of the redshift, $\ell = 10$ (left) and $\ell = 50$ (right) (from Bonvin & RD '11)



C_ℓ^{DD} (red), C_ℓ^{zz} (green), C_ℓ^{Dz} (blue), C_ℓ^{LL} (magenta), C_ℓ^{VV} (cyan), C_ℓ^{zV} (black), C_ℓ^{DV} (yellow).

The radial power spectrum can be obtained from $C_\ell(z, z')$. This can be approximated by computing the $C_\ell(z)$'s with a window function of width $\Delta z = z' - z$. Perturbations on scales smaller than $\sim \Delta z/\mathcal{H}(z)$ are then 'smeared out' and this yields to a reduction especially of the density and redshift space contributions.

$$C_\ell(z, z') = \frac{2A}{\pi} \int \frac{dk}{k} (kt_O)^{n-1} F_\ell(k, z) F_\ell^*(k, z')$$

$$C_\ell(z, \Delta z) = \int d\Delta z C_\ell(z, z + \Delta z) W(\Delta z)$$

The lensing contribution, which is in principle of the same order but measures the power at scale $k \simeq \ell/r_S$ is not reduced by windowing.

The radial power spectrum

The radial power spectrum can be obtained from $C_\ell(z,z')$. This can be approximated by computing the $C_\ell(z)$'s with a window function of width $\Delta z = z' - z$. Perturbations on scales smaller than $\sim \Delta z/\mathcal{H}(z)$ are then 'smeared out' and this yields to a reduction especially of the density and redshift space contributions.

$$C_\ell(z,z') = \frac{2A}{\pi}\int \frac{dk}{k}(kt_O)^{n-1}F_\ell(k,z)F_\ell^*(k,z')$$

$$C_\ell(z,\Delta z) = \int d\Delta z C_\ell(z,z+\Delta z)W(\Delta z)$$

The lensing contribution, which is in principle of the same order but measures the power at scale $k \simeq \ell/r_S$ is not reduced by windowing.

The radial power spectrum

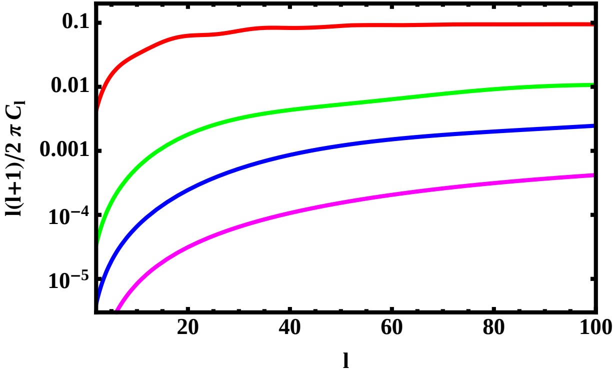
The radial power spectrum can be obtained from $C_\ell(z,z')$. This can be approximated by computing the $C_\ell(z)$'s with a window function of width $\Delta z = z' - z$. Perturbations on scales smaller than $\sim \Delta z/\mathcal{H}(z)$ are then 'smeared out' and this yields to a reduction especially of the density and redshift space contributions.

$$C_\ell(z,z') = \frac{2A}{\pi}\int \frac{dk}{k}(kt_O)^{n-1}F_\ell(k,z)F_\ell^*(k,z')$$

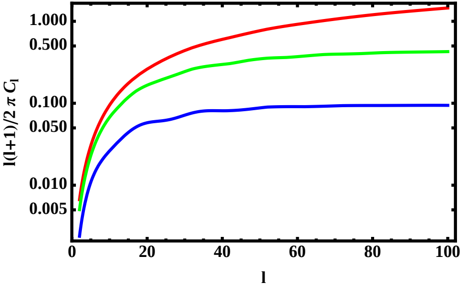
$$C_\ell(z,\Delta z) = \int d\Delta z C_\ell(z,z+\Delta z)W(\Delta z)$$

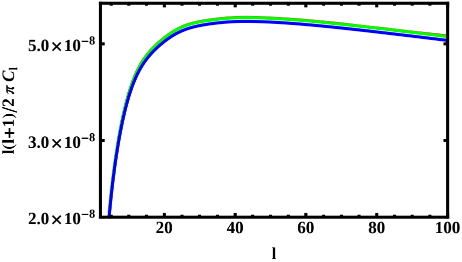
The lensing contribution, which is in principle of the same order but measures the power at scale $k \simeq \ell/r_S$ is not reduced by windowing.

The total power spectrum with window function $\sigma_z = 0.1 \times z$
(from Bonvin & RD '11)

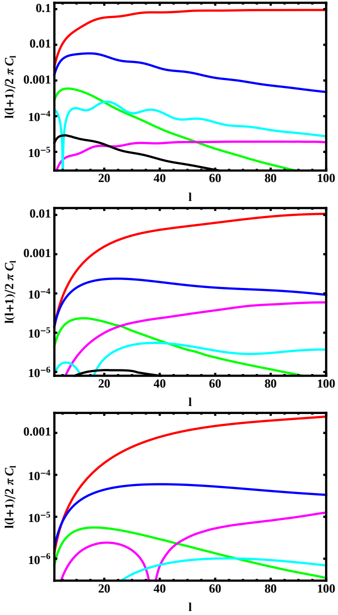


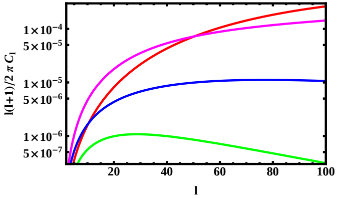
$z = 0.1$ (red), $z = 0.5$ (green), $z = 1$ (blue), $z = 3$ (magenta).





C_ℓ^{DD} (left), $C_\ell^{lensing}$ (right) without window (red), $\sigma_z = 0.002$ (green), $\sigma_z = 0.01$ (blue), for z=0.1.





Top to bottom: $z = 0.1,\ 0.5,\ 1,$
top right: $z = 3,$
$\Delta z = 0.01 \times z$

C_ℓ^{DD} (red), C_ℓ^{zz} (green), C_ℓ^{Dz} (blue),
C_ℓ^{lens} (magenta), C_ℓ^{vel} (cyan), $C_\ell^{potential}$ (black),

Conclusions

- So far cosmological precision data mainly comes from the CMB, but in the future we expect very large & precise 3d galaxy catalogs.
- On scales larger than about 10% of the Hubble scale, $z \gtrsim 0.1$ relativistic corrections have to be taken into account.
- It will be interesting to split the observed galaxy power spectrum into its radial and transversal components and determine directly $C_\ell(z)$ and $C_\ell(z, z')$ from the data.
- These spectra are not only sensitive to the matter distribution but also to the velocity via redshift space distortions and to the perturbations of spacetime geometry.
- The spectra will depend sensitively and in several different ways on dark energy (growth factor, distance redshift relation), on the matter and baryon densities etc. Their measurements therefore provide a new route to estimate cosmological parameters.
- Evolutionary effects and biasing not discussed here will also have to be included...

Conclusions

- So far cosmological precision data mainly comes from the CMB, but in the future we expect very large & precise 3d galaxy catalogs.
- On scales larger than about 10% of the Hubble scale, $z \gtrsim 0.1$ relativistic corrections have to be taken into account.
- It will be interesting to split the observed galaxy power spectrum into its radial and transversal components and determine directly $C_\ell(z)$ and $C_\ell(z, z')$ from the data.
- These spectra are not only sensitive to the matter distribution but also to the velocity via redshift space distortions and to the perturbations of spacetime geometry.
- The spectra will depend sensitively and in several different ways on dark energy (growth factor, distance redshift relation), on the matter and baryon densities etc. Their measurements therefore provide a new route to estimate cosmological parameters.
- Evolutionary effects and biasing not discussed here will also have to be included...

Conclusions

- So far cosmological precision data mainly comes from the CMB, but in the future we expect very large & precise 3d galaxy catalogs.
- On scales larger than about 10% of the Hubble scale, $z \gtrsim 0.1$ relativistic corrections have to be taken into account.
- It will be interesting to split the observed galaxy power spectrum into its radial and transversal components and determine directly $C_\ell(z)$ and $C_\ell(z, z')$ from the data.
- These spectra are not only sensitive to the matter distribution but also to the velocity via redshift space distortions and to the perturbations of spacetime geometry.
- The spectra will depend sensitively and in several different ways on dark energy (growth factor, distance redshift relation), on the matter and baryon densities etc. Their measurements therefore provide a new route to estimate cosmological parameters.
- Evolutionary effects and biasing not discussed here will also have to be included...

Conclusions

- So far cosmological precision data mainly comes from the CMB, but in the future we expect very large & precise 3d galaxy catalogs.
- On scales larger than about 10% of the Hubble scale, $z \gtrsim 0.1$ relativistic corrections have to be taken into account.
- It will be interesting to split the observed galaxy power spectrum into its radial and transversal components and determine directly $C_\ell(z)$ and $C_\ell(z, z')$ from the data.
- These spectra are not only sensitive to the matter distribution but also to the velocity via redshift space distortions and to the perturbations of spacetime geometry.
- The spectra will depend sensitively and in several different ways on dark energy (growth factor, distance redshift relation), on the matter and baryon densities etc. Their measurements therefore provide a new route to estimate cosmological parameters.
- Evolutionary effects and biasing not discussed here will also have to be included...

Conclusions

- So far cosmological precision data mainly comes from the CMB, but in the future we expect very large & precise 3d galaxy catalogs.
- On scales larger than about 10% of the Hubble scale, $z \gtrsim 0.1$ relativistic corrections have to be taken into account.
- It will be interesting to split the observed galaxy power spectrum into its radial and transversal components and determine directly $C_\ell(z)$ and $C_\ell(z, z')$ from the data.
- These spectra are not only sensitive to the matter distribution but also to the velocity via redshift space distortions and to the perturbations of spacetime geometry.
- The spectra will depend sensitively and in several different ways on dark energy (growth factor, distance redshift relation), on the matter and baryon densities etc. Their measurements therefore provide a new route to estimate cosmological parameters.
- Evolutionary effects and biasing not discussed here will also have to be included...

Conclusions

- So far cosmological precision data mainly comes from the CMB, but in the future we expect very large & precise 3d galaxy catalogs.
- On scales larger than about 10% of the Hubble scale, $z \gtrsim 0.1$ relativistic corrections have to be taken into account.
- It will be interesting to split the observed galaxy power spectrum into its radial and transversal components and determine directly $C_\ell(z)$ and $C_\ell(z, z')$ from the data.
- These spectra are not only sensitive to the matter distribution but also to the velocity via redshift space distortions and to the perturbations of spacetime geometry.
- The spectra will depend sensitively and in several different ways on dark energy (growth factor, distance redshift relation), on the matter and baryon densities etc. Their measurements therefore provide a new route to estimate cosmological parameters.
- Evolutionary effects and biasing not discussed here will also have to be included...