inflation

@landscape

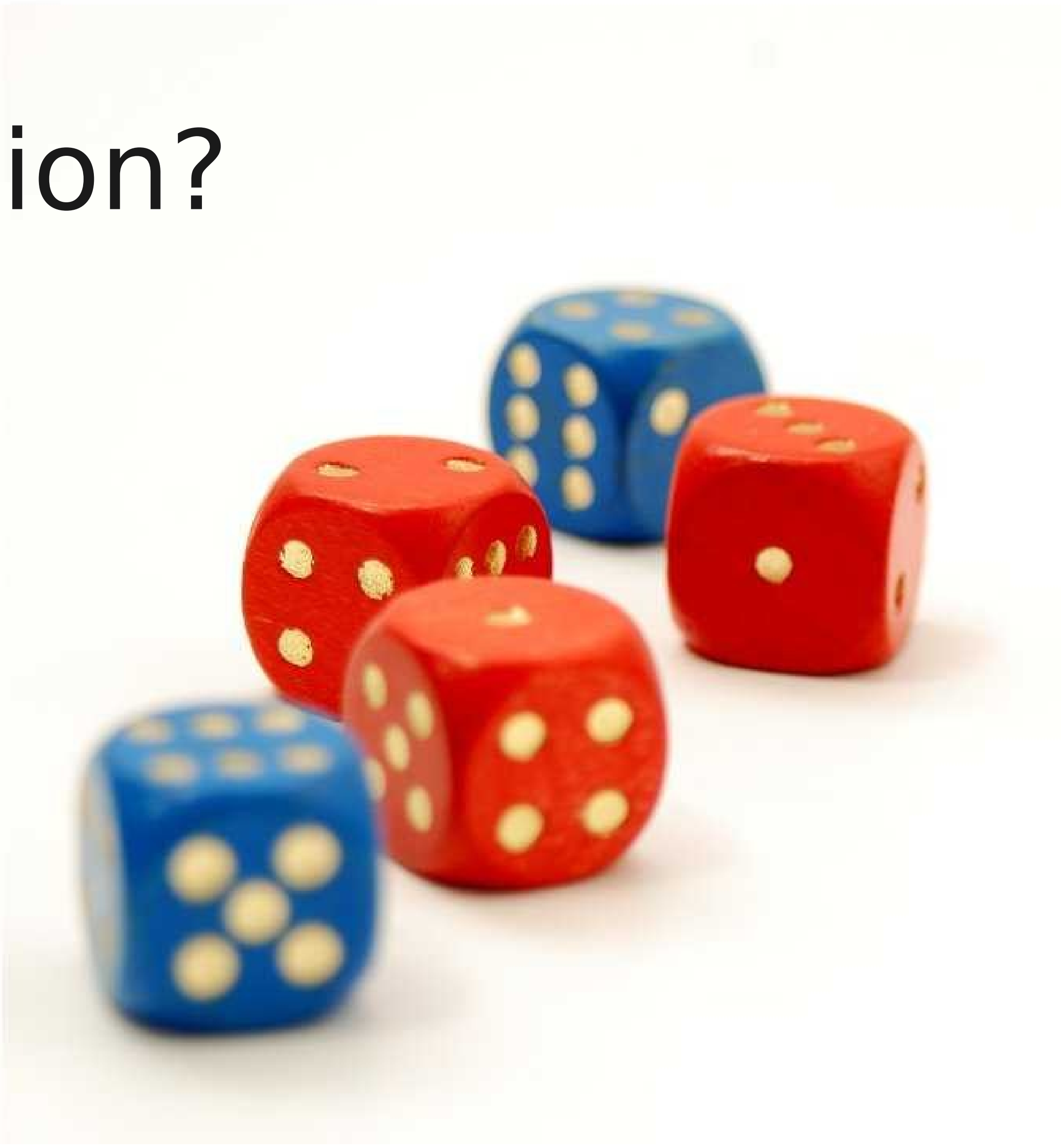
Inflation in a Landscape

Yi Wang, McGill University, June 2011

Based on:

- X. Chen, YW, 0909.0496, 0911.3380
- M. Li, YW 0903.2123
- N. Afshordi, A. Slosar, YW, 1006.5021
- Y. Cai, S. Pi, YW, 110?.*
- R. Brandenberger, F. Duplessis, YW, 110?.*
- J. Cline, G. Moore, YW, 1106.2188

Bet on inflation?



multiple fields

or

single field

The potential is

flat or bumpy

Dynamics: Simple,

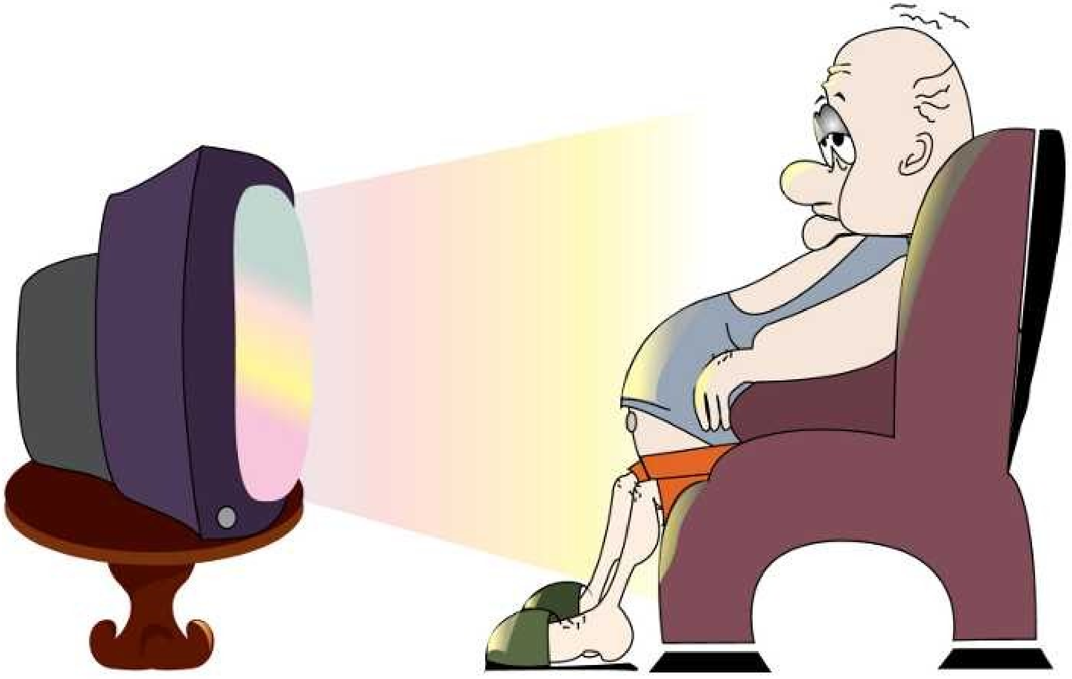
or

Complicated?

Alternatives ...

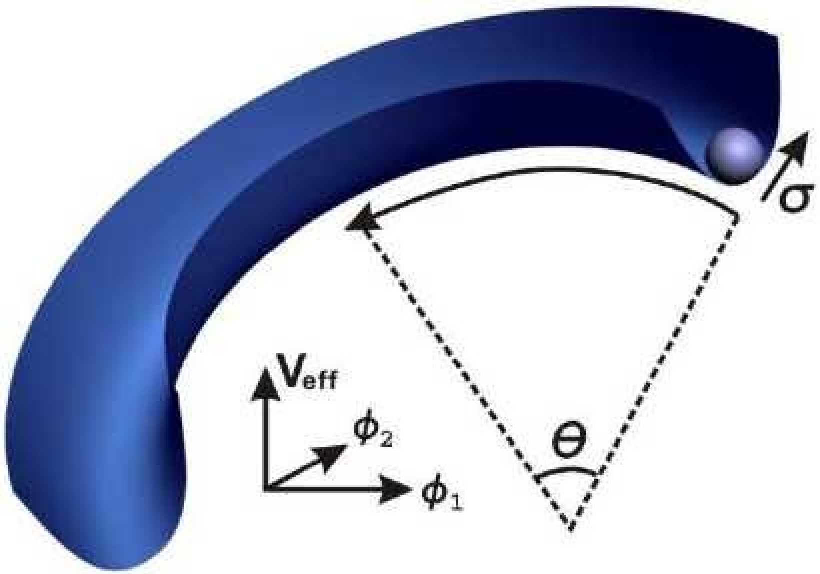
work hard or

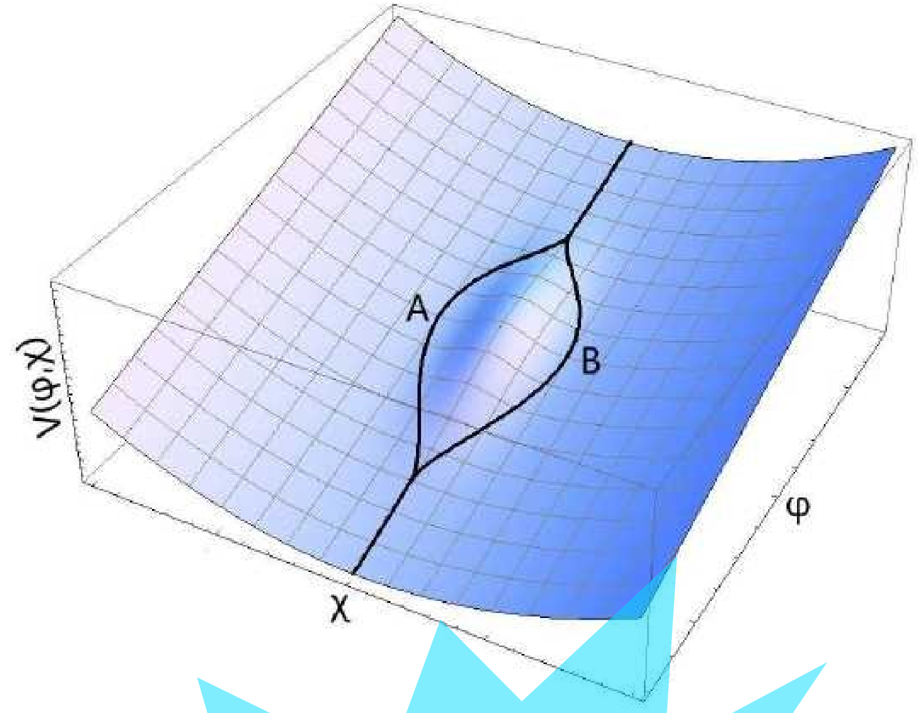
go home & watch TV

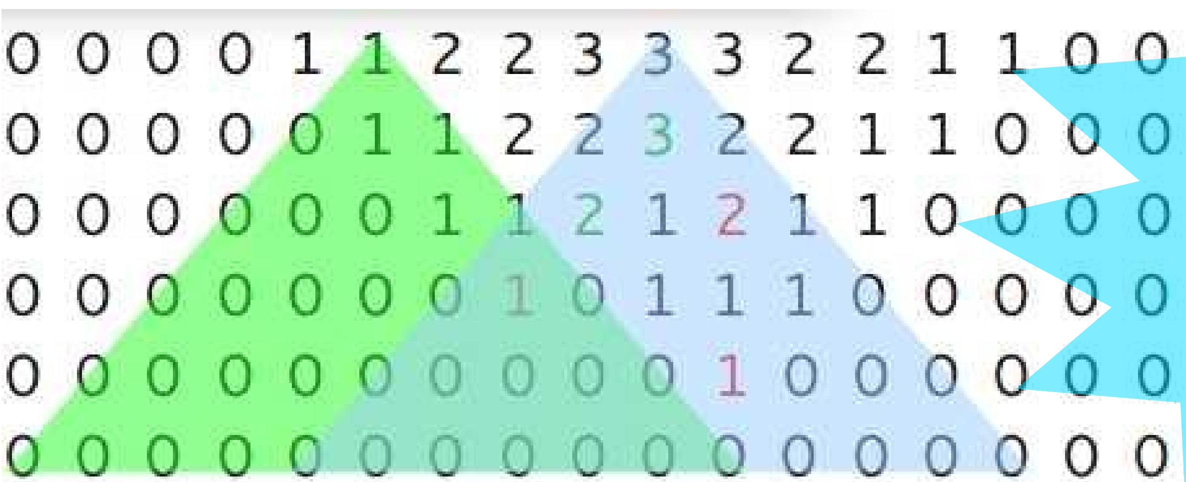


work hard

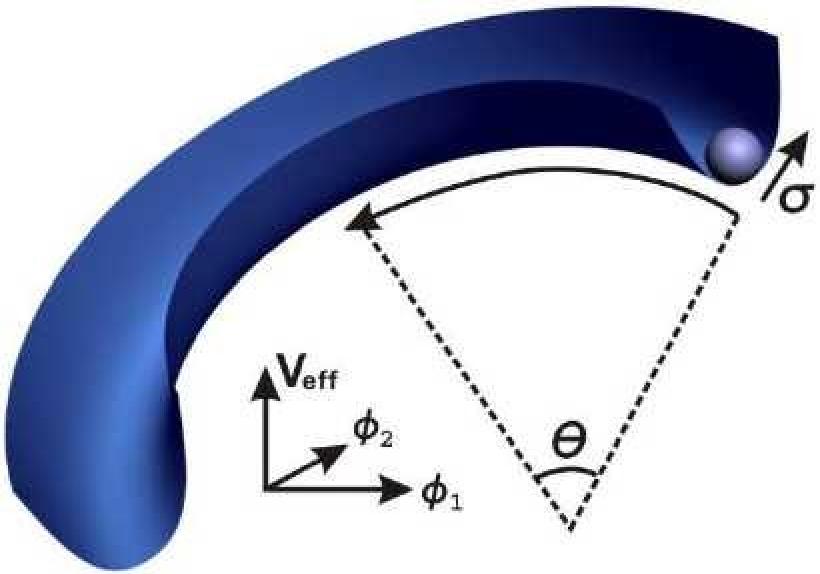
BET


σ
V_eff
ϕ2
ϕ1
Θ


V(φ,χ)
A
B
χ
φ


0 0 0 0 1 1 2 2 3 3 3 2 2 1 1 0 0
0 0 0 0 0 1 1 2 2 3 2 2 1 1 0 0 0
0 0 0 0 0 0 1 1 2 1 2 1 1 0 0 0 0
0 0 0 0 0 0 0 1 0 1 1 1 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

BET



Part I: quasi-single field inflation

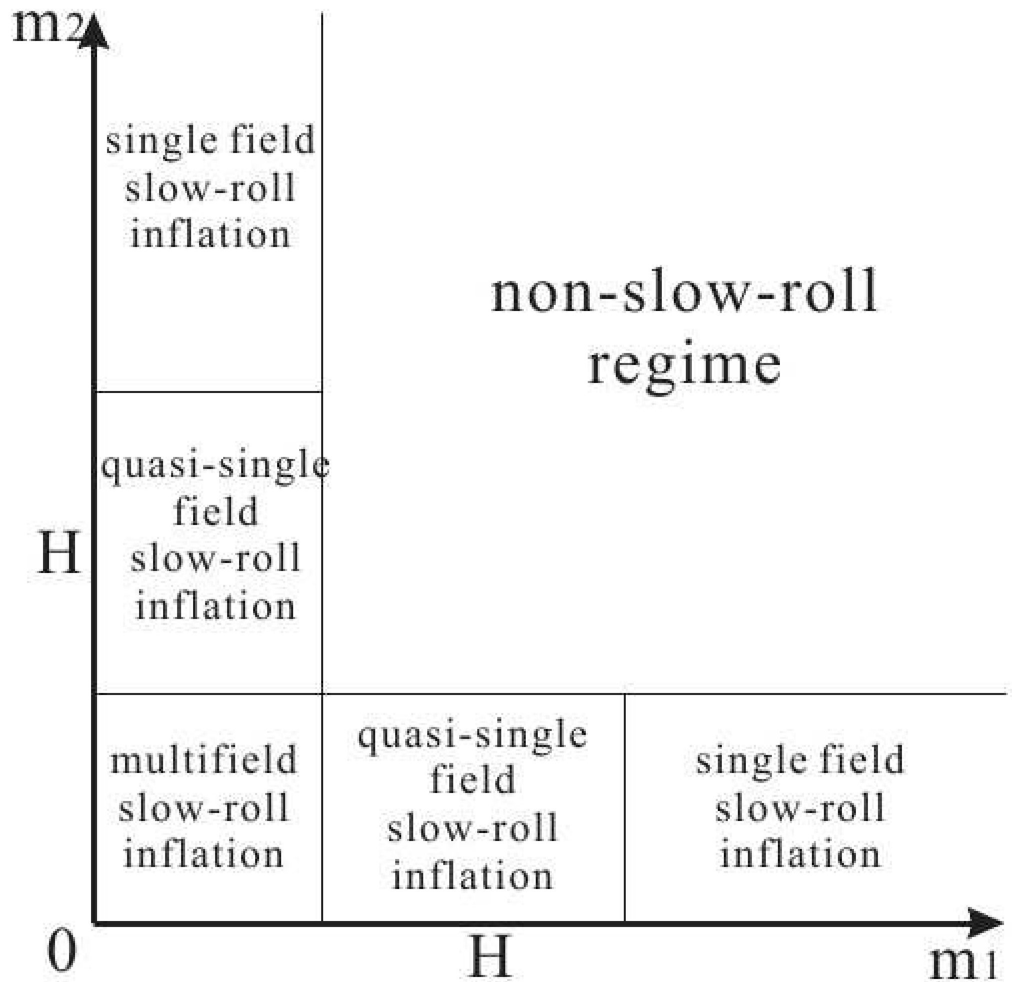
Definition of single field?

Background:
$m^2 < \eta H^2$

Perturbations:
$m < H$

Quasi-single field:

$\eta H^2 < m^2 < H^2$


m2
single field slow-roll inflation
non-slow-roll regime
quasi-single field slow-roll inflation
H
multifield slow-roll inflation
quasi-single field slow-roll inflation
single field slow-roll inflation
0
H
m1

Why to care $\eta H^2 < m^2 < H^2$?

a priori large parameter space!

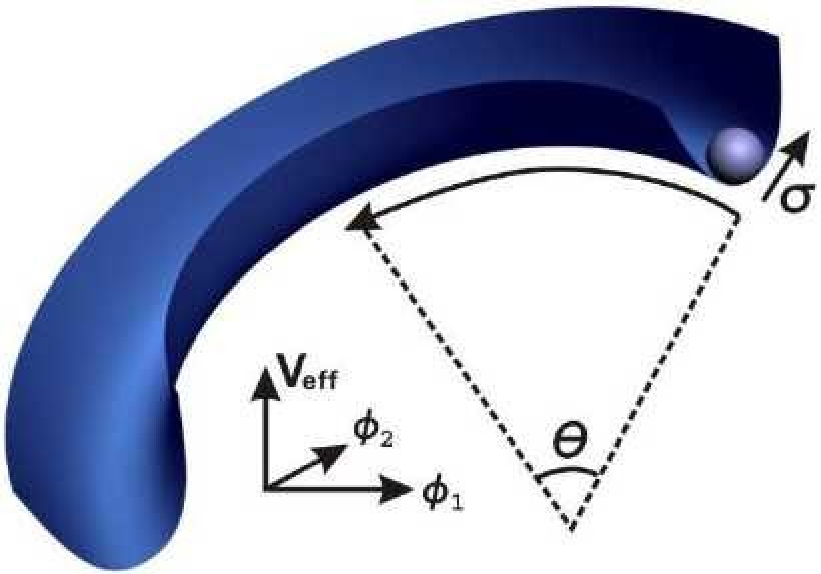
Why to care $\eta H^2 < m^2 < H^2$?

a priori large parameter space!

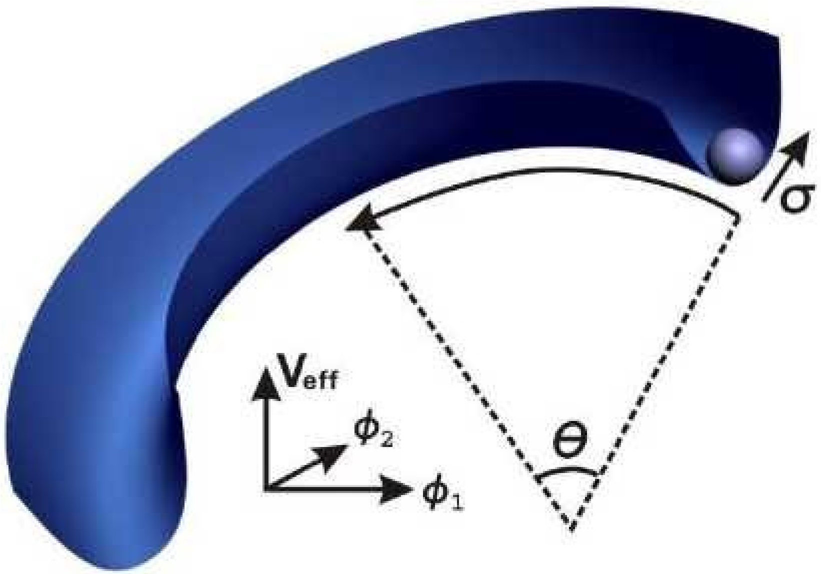
Fine tuning problem of inflation:

To tune once is already a lot

A simple model:



A simple model:



$$S_m = \int d^4x\sqrt{-g}\left[-\frac{1}{2}(\tilde{R}+\sigma)^2 g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V_{\rm sr}(\theta) - V(\sigma)\right]$$

Method of calculation:

in-in formalism

$$\langle Q(t)\rangle \equiv \langle 0|\left[\bar{T}\exp\left(i\int_{t_0}^{t} dt' H_I(t')\right)\right] Q_I(t)\left[T\exp\left(-i\int_{t_0}^{t} dt' H_I(t')\right)\right]|0\rangle$$

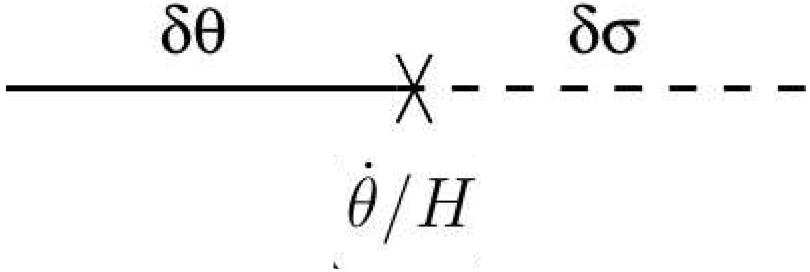
$$i^n \int_{t_0}^{t} dt_1 \int_{t_1}^{t} dt_2 \cdots \int_{t_{n-1}}^{t} dt_n \,\langle [H_I(t_1),[H_I(t_2),\cdots,[H_I(t_n),Q_I(t)]\cdots]]\rangle$$

Transfer vertex

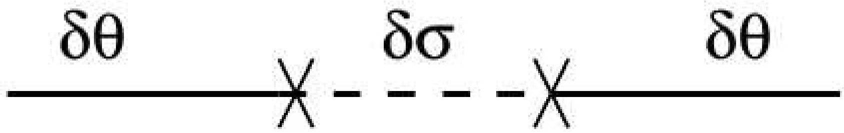
$$S_m = \int d^4x\sqrt{-g}\left[-\frac{1}{2}(\tilde{R}+\sigma)^2 g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V_{\rm sr}(\theta) - V(\sigma)\right]$$

2pt coupling from turning trajectory

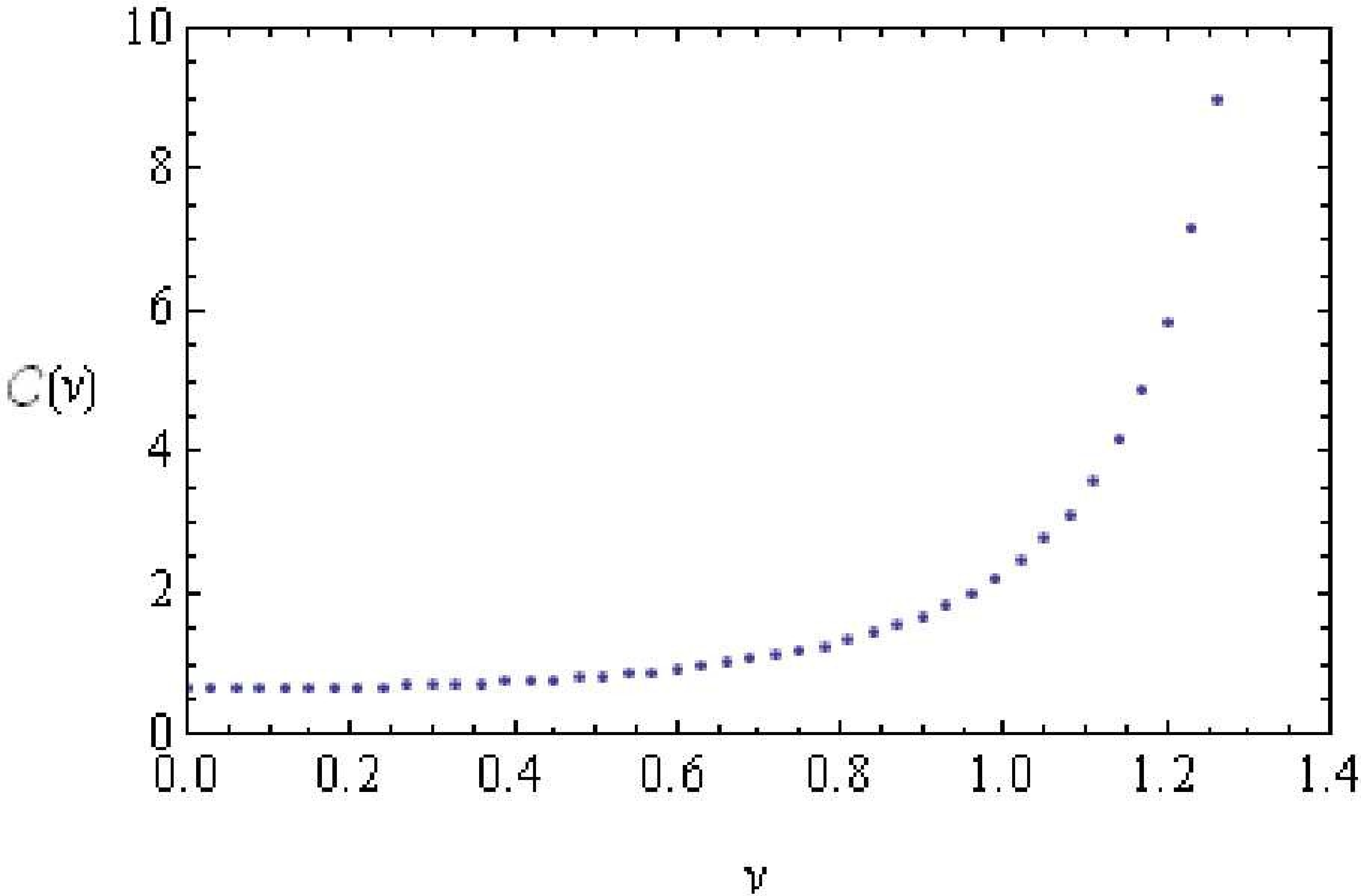
$$\delta\mathcal{L}_2 \;=\; 2a^3R\dot{\theta}_0\;\delta\sigma\delta\dot{\theta}$$



Power spectrum

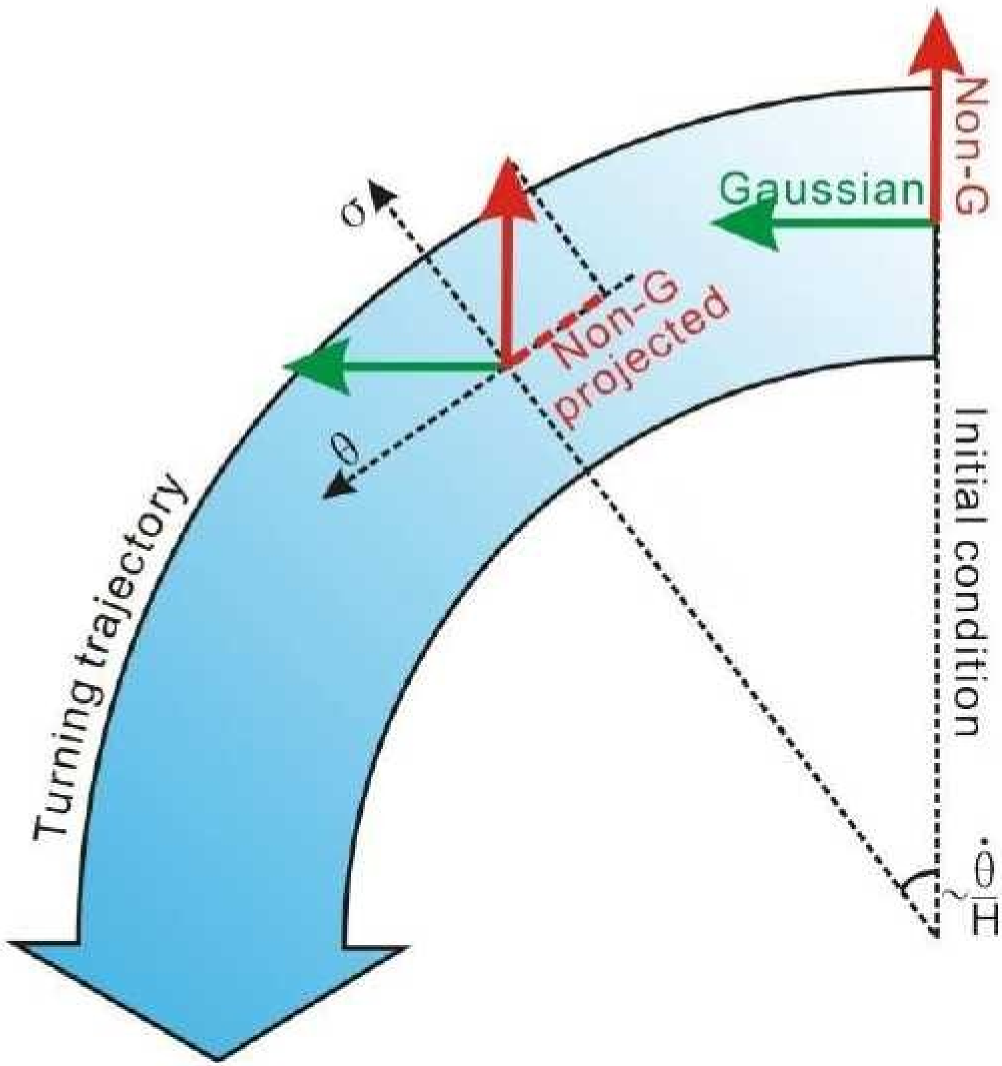


$$\delta P_\zeta \sim \left(\dot{\theta}/H\right)^2 P_\zeta$$

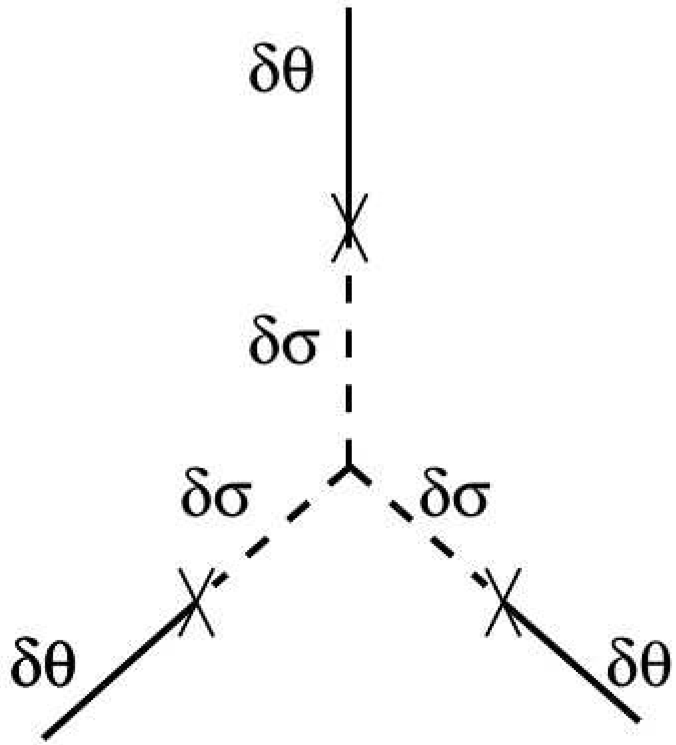


$$\delta P_\zeta = \mathcal{C}(\nu)\left(\dot{\theta}/H\right)^2 P_\zeta \qquad \nu = \sqrt{9/4 - m^2/H^2}$$

Non-Gaussianity



Bispectrum: amplitude



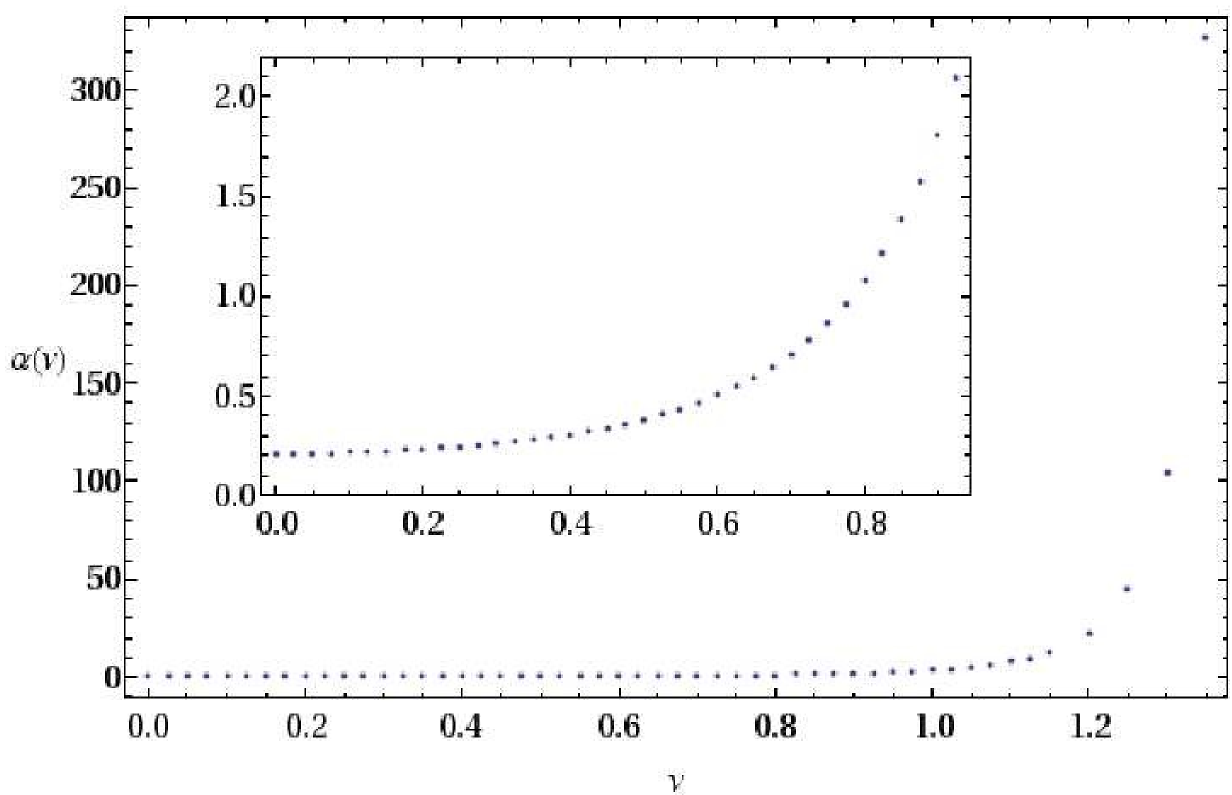
$$f_{NL} \sim P_\zeta^{-1/2} \left(\dot{\theta}/H\right)^3 (V'''/H)$$

$\langle \zeta^3 \rangle \sim \text{coupling} \times P_\zeta^{3/2}$

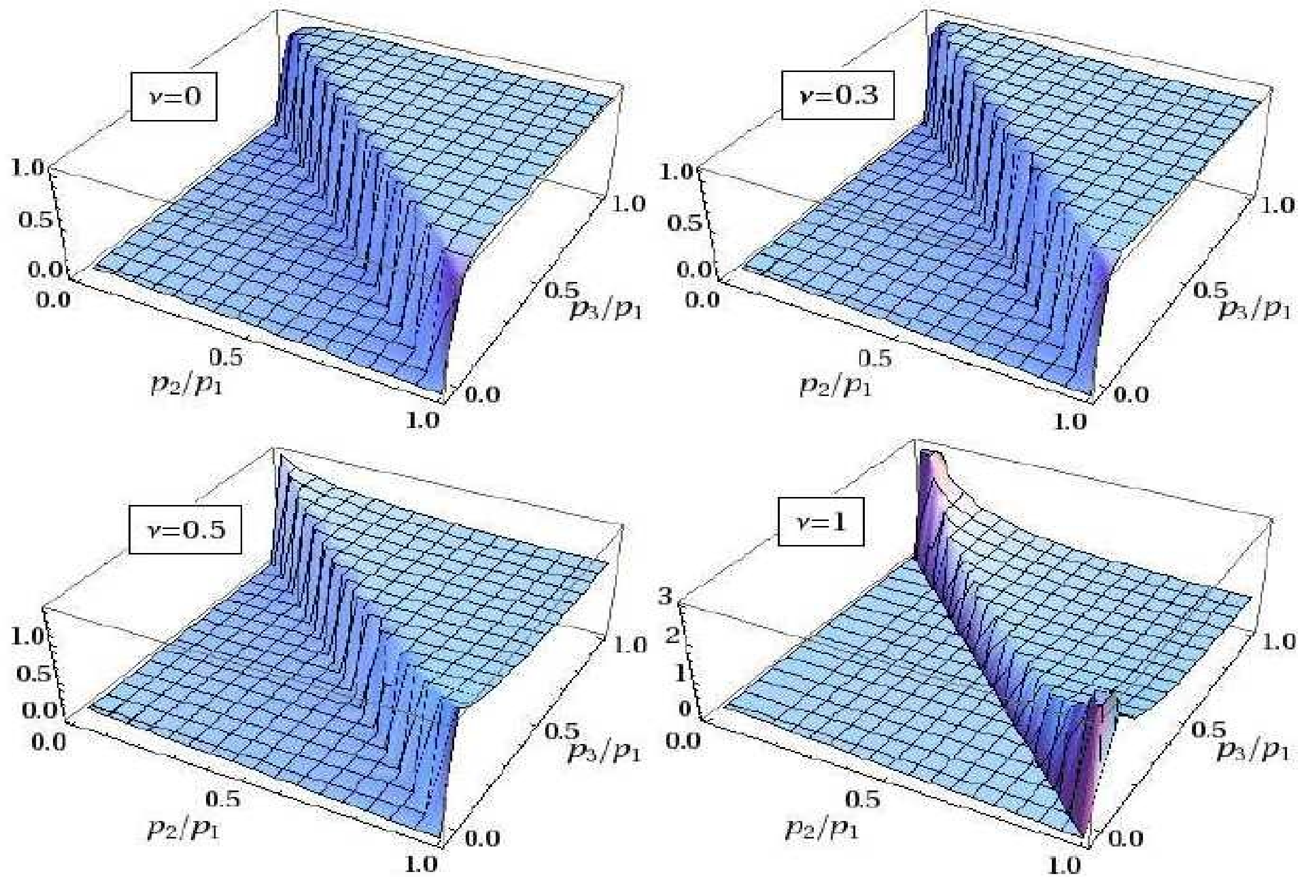
$\sim f_{NL} \times P_\zeta^{2}$

Bispectrum: amplitude

$$f_{NL} = \alpha(\nu) P_\zeta^{-1/2} \left(\dot{\theta}/H\right)^3 \left(V'''/H\right)$$



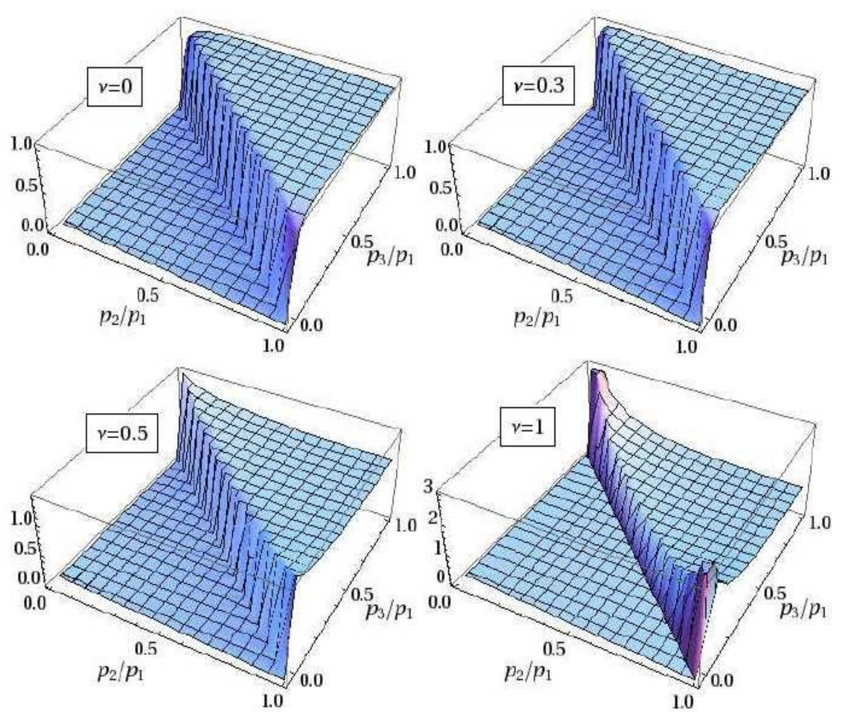
Bispectrum: shape



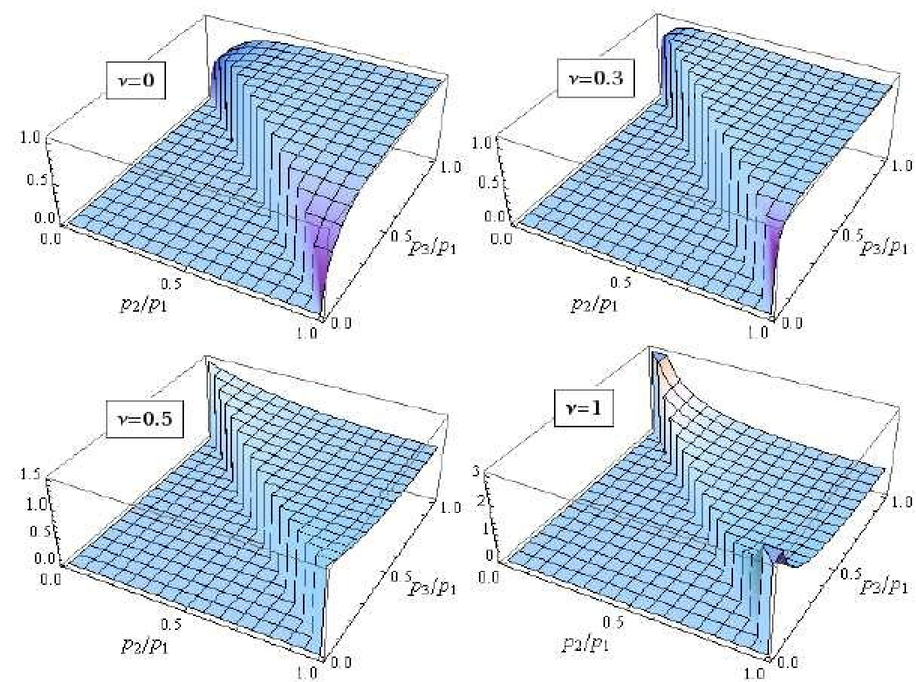
$$\nu = \sqrt{9/4 - m^2/H^2}$$

Bispectrum: shape ansatz

$$F = \frac{3^{\frac{9}{2}-3\nu}}{10}\frac{f_{NL}^{\text{int}}(p_1^2+p_2^2+p_3^2)}{(p_1p_2p_3)^{\frac{3}{2}+\nu}(p_1+p_2+p_3)^{\frac{7}{2}-3\nu}}$$

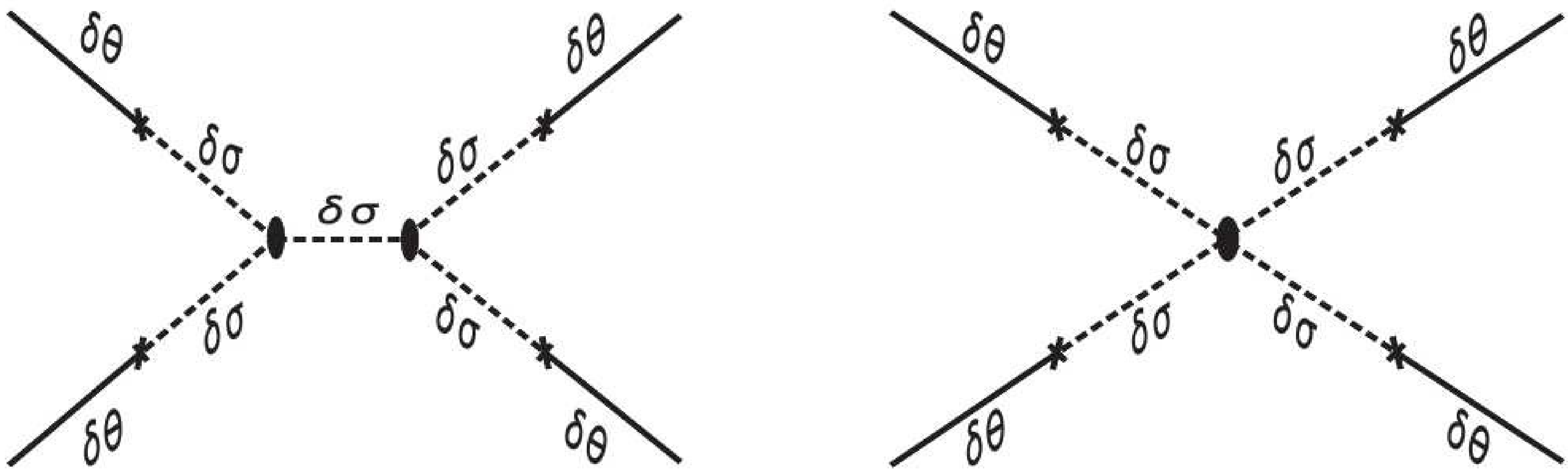


True numeric



Shape ansatz

Trispectrum



$$t_{NL} \sim \max \left\{ P_\zeta^{-1} \left(\dot{\theta}/H \right)^4 (V'''/H)^2 \ , \ P_\zeta^{-1} \left(\dot{\theta}/H \right)^4 V'''' \right\}$$

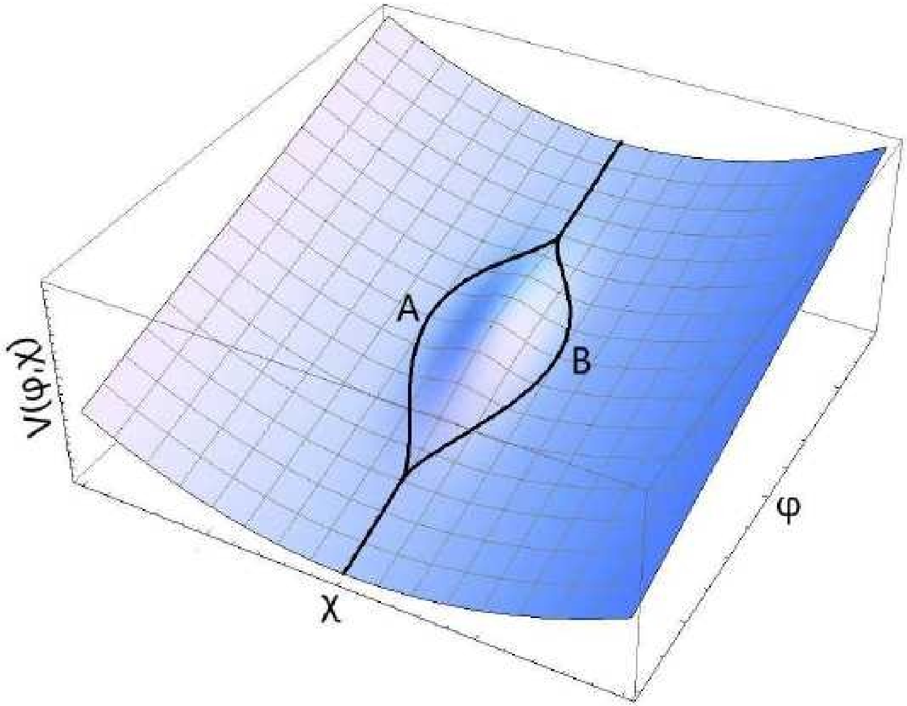
$$t_{NL} \gg f_{NL}^2 \text{ for } \dot{\theta}/H \ll 1$$

Higher point correlations

$$h_{NL} \sim P_\zeta^{-3/2} \left(\dot{\theta}/H\right)^5 (V'''/H)^3 \sim (\dot{\theta}/H)^{-4} f_{NL}^3 \,,$$
$$i_{NL} \sim P_\zeta^{-2} \left(\dot{\theta}/H\right)^6 (V'''/H)^4 \sim (\dot{\theta}/H)^{-6} f_{NL}^4 \,.$$

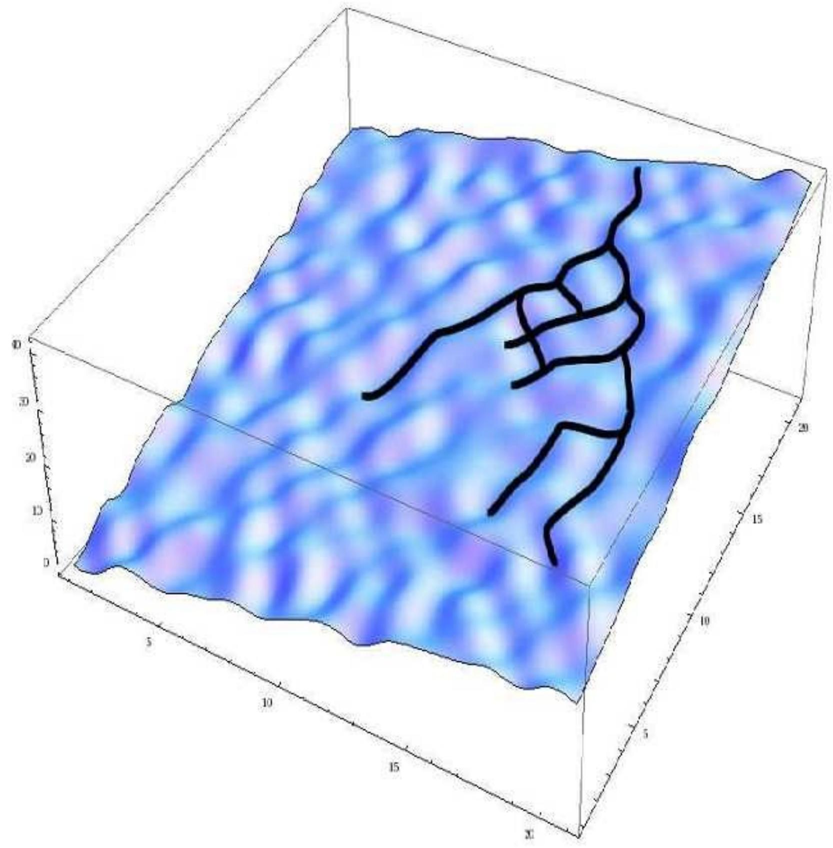
Might be difficult to probe.

But ... who knows?



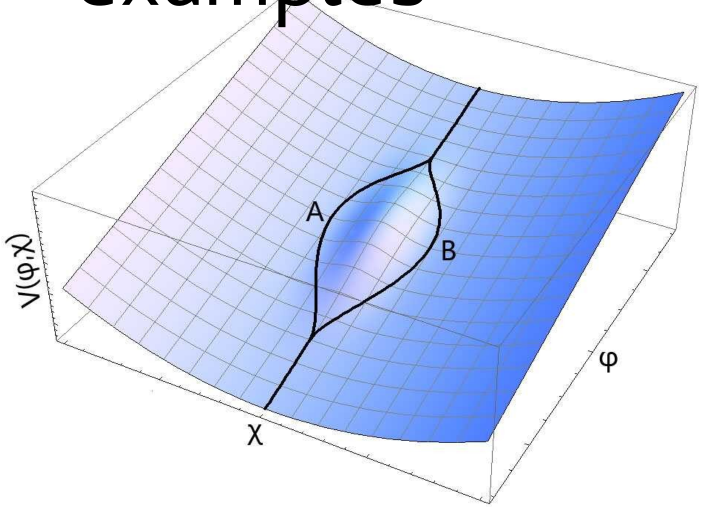
Part II: multi-stream inflation

Multi-stream trajectories for inflation

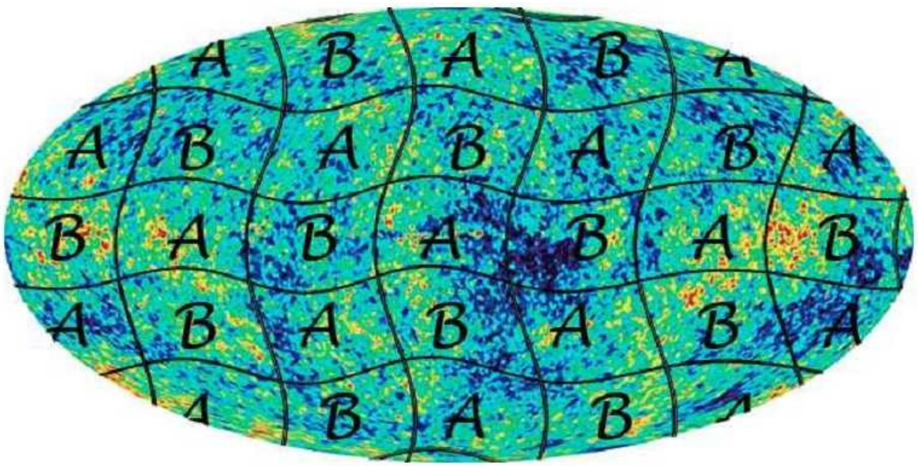


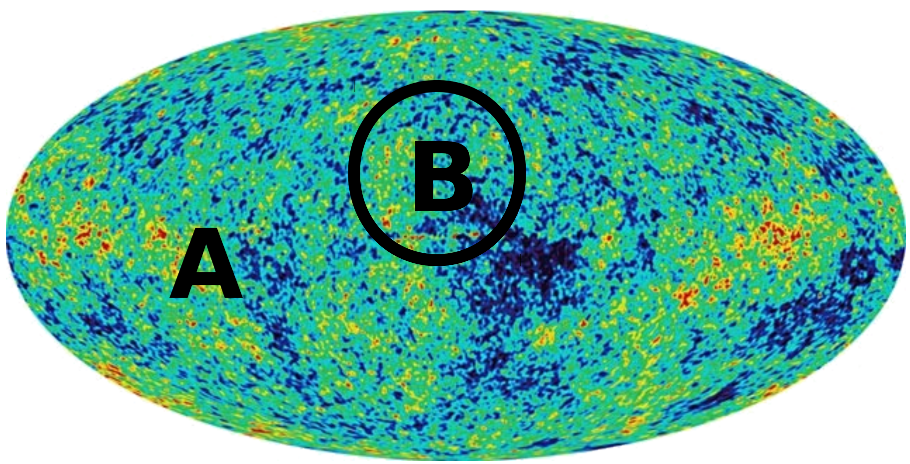
statistics

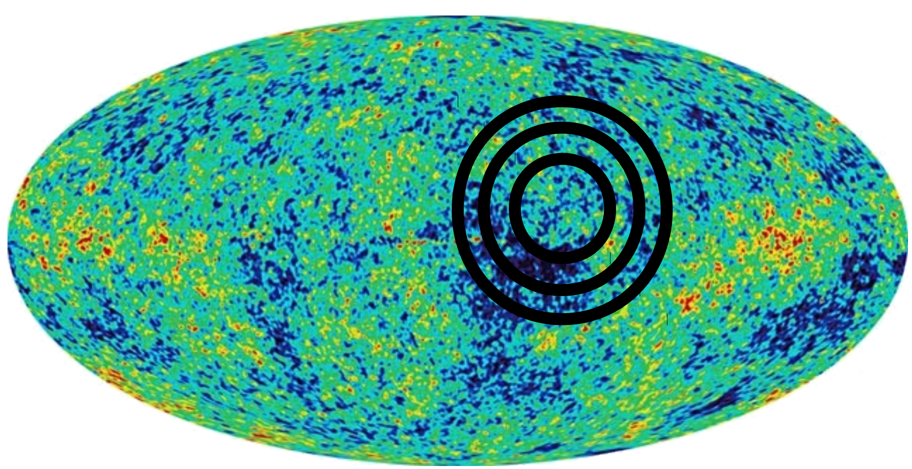
examples



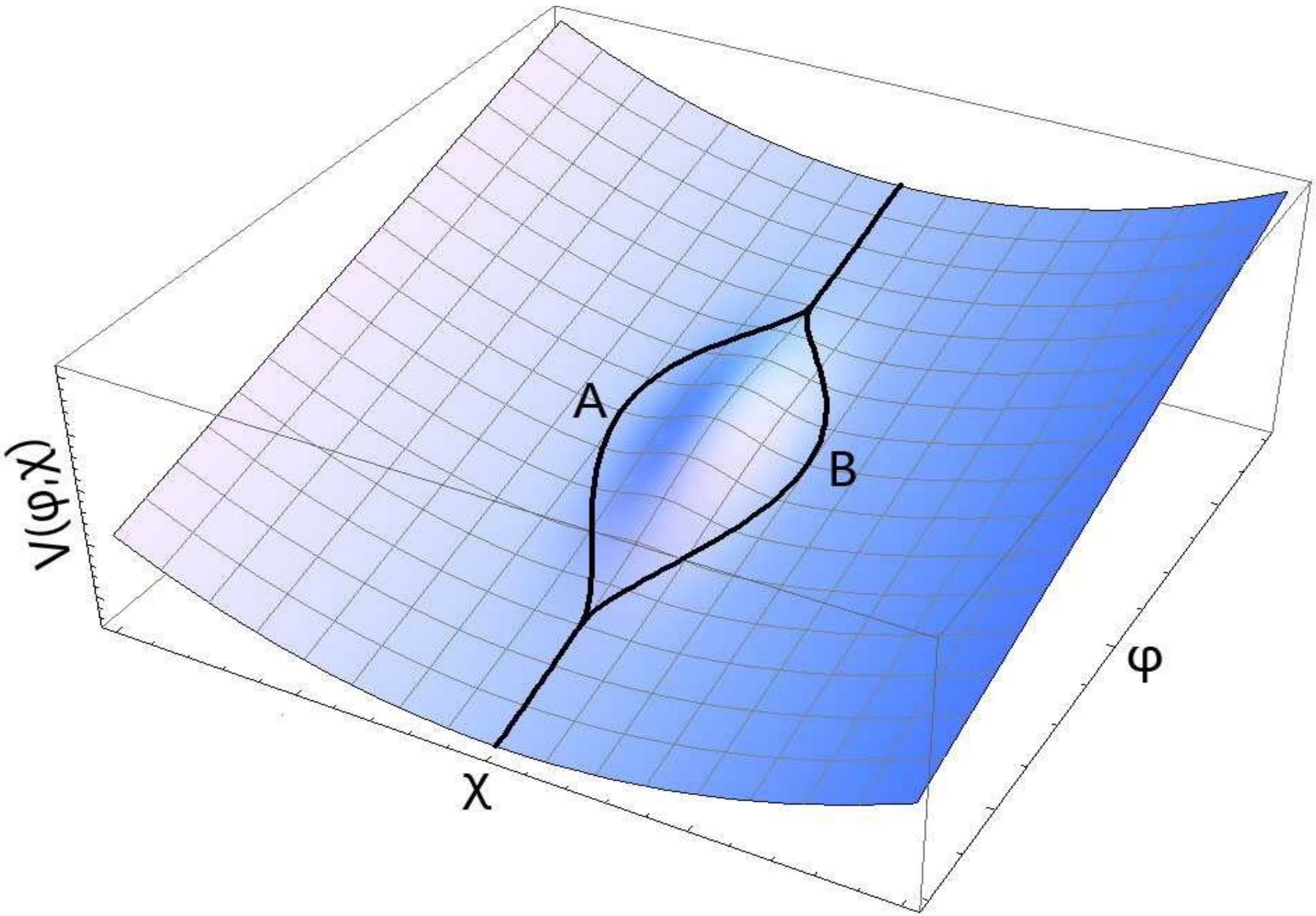
examples (case study):







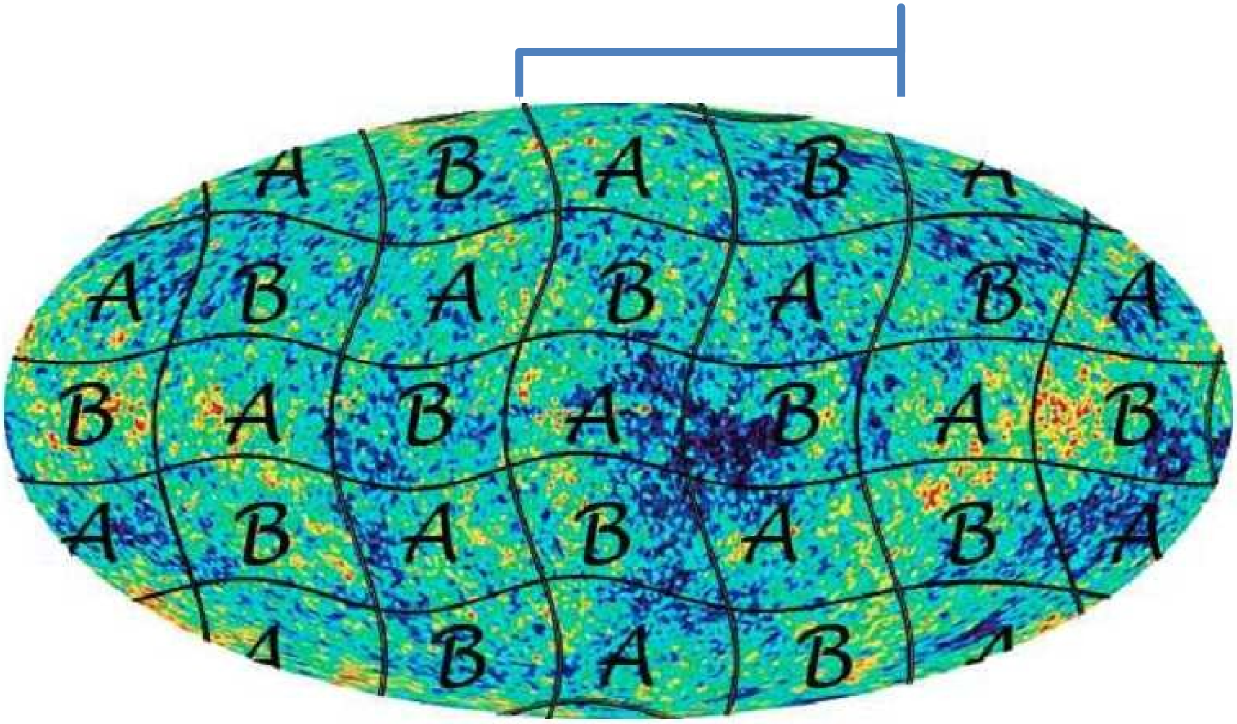
Nearly symmetric bifurcations



Nearly symmetric bifurcations

– – – Pert. feature

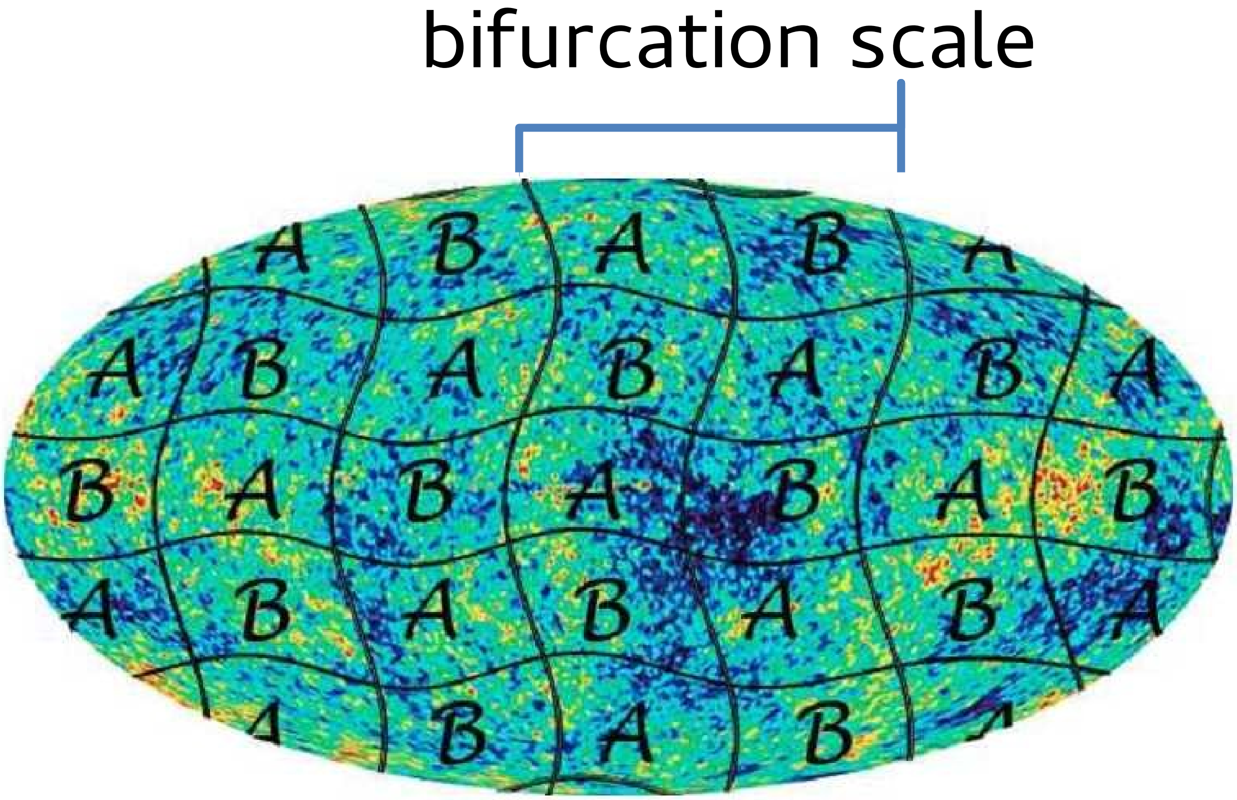
bifurcation scale



$\zeta = \delta N$

Nearly symmetric bifurcations

---- Pert. asym.



$P_\zeta(A) \neq P_\zeta(B)$

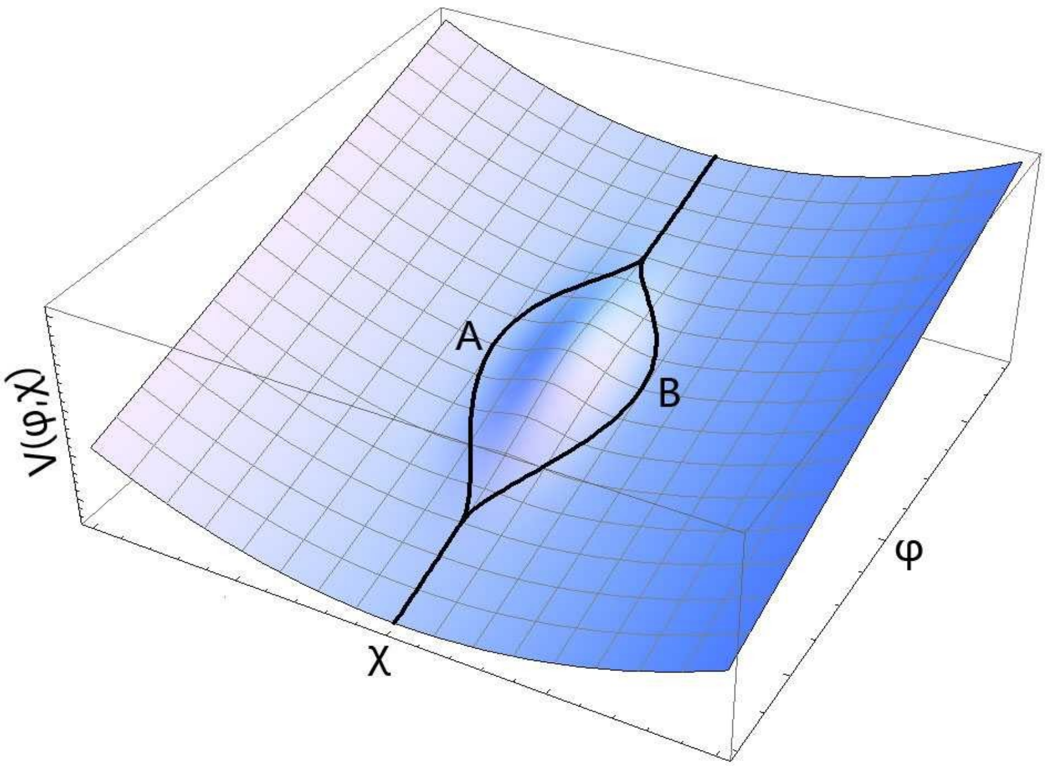
Nearly symmetric bifurcations

---- Non-G

Correlation between

feature

asymmetry



Nearly symmetric bifurcations

---- Non-G

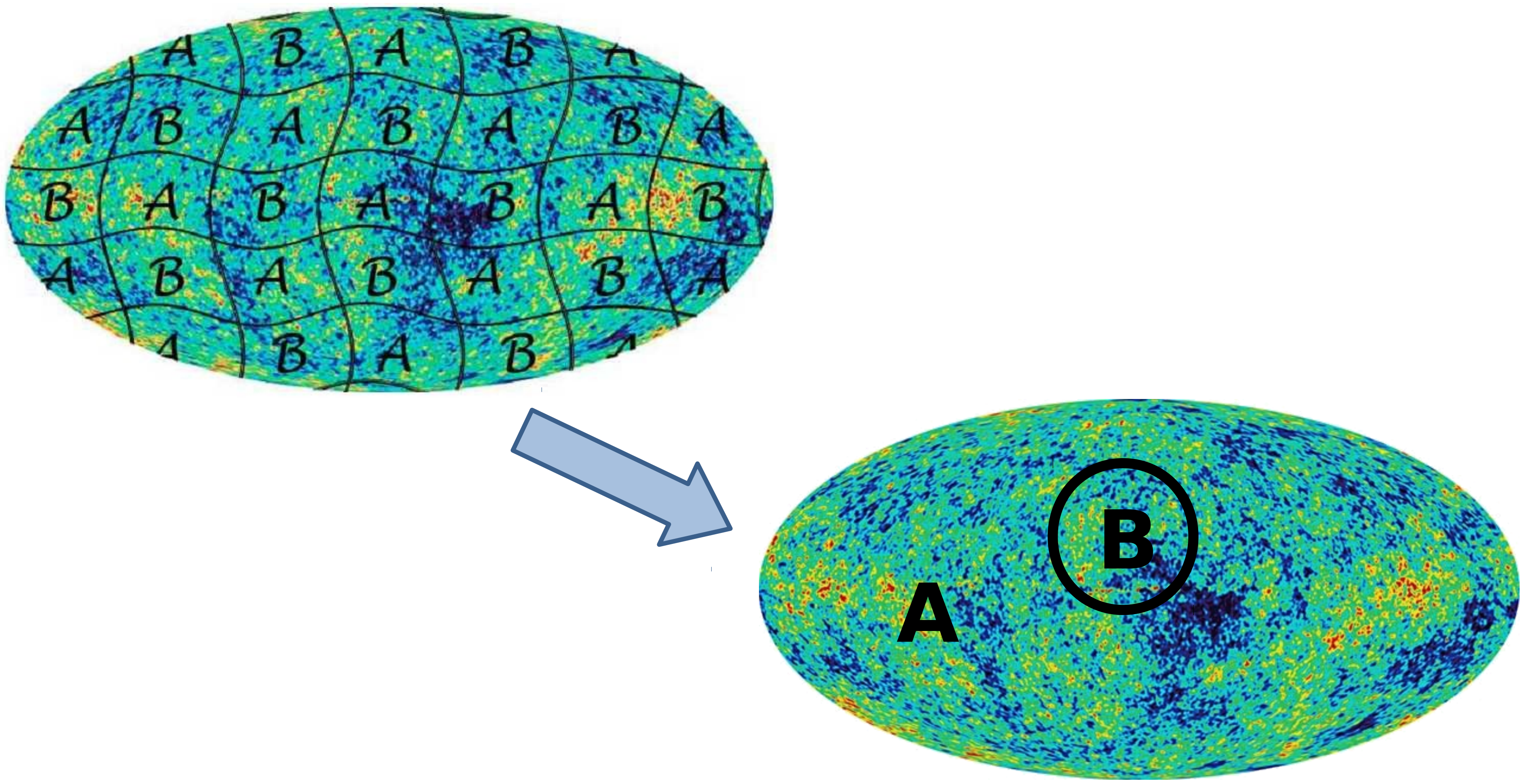
$$P(\delta\zeta_{k_1}^{S,N},\zeta_k) = P(\delta\zeta_{k_1}^{S,N})\left[\frac{e^{-\frac{\zeta_k^2}{2\sigma_A^2}}}{\sqrt{2\pi}\sigma_A}\theta(\delta\zeta_{k_1}^S) + \frac{e^{-\frac{\zeta_k^2}{2\sigma_B^2}}}{\sqrt{2\pi}\sigma_B}\theta(-\delta\zeta_{k_1}^S)\right]$$

$$f_{NL} \simeq x P_\zeta^{-1/2}\left(\frac{P_\zeta^A - P_\zeta^B}{P_\zeta}\right)$$

$x \equiv \delta\zeta_{k_1}/\zeta_{k_1}$ denotes the fraction of extra fluctuation from the multi-stream effect.

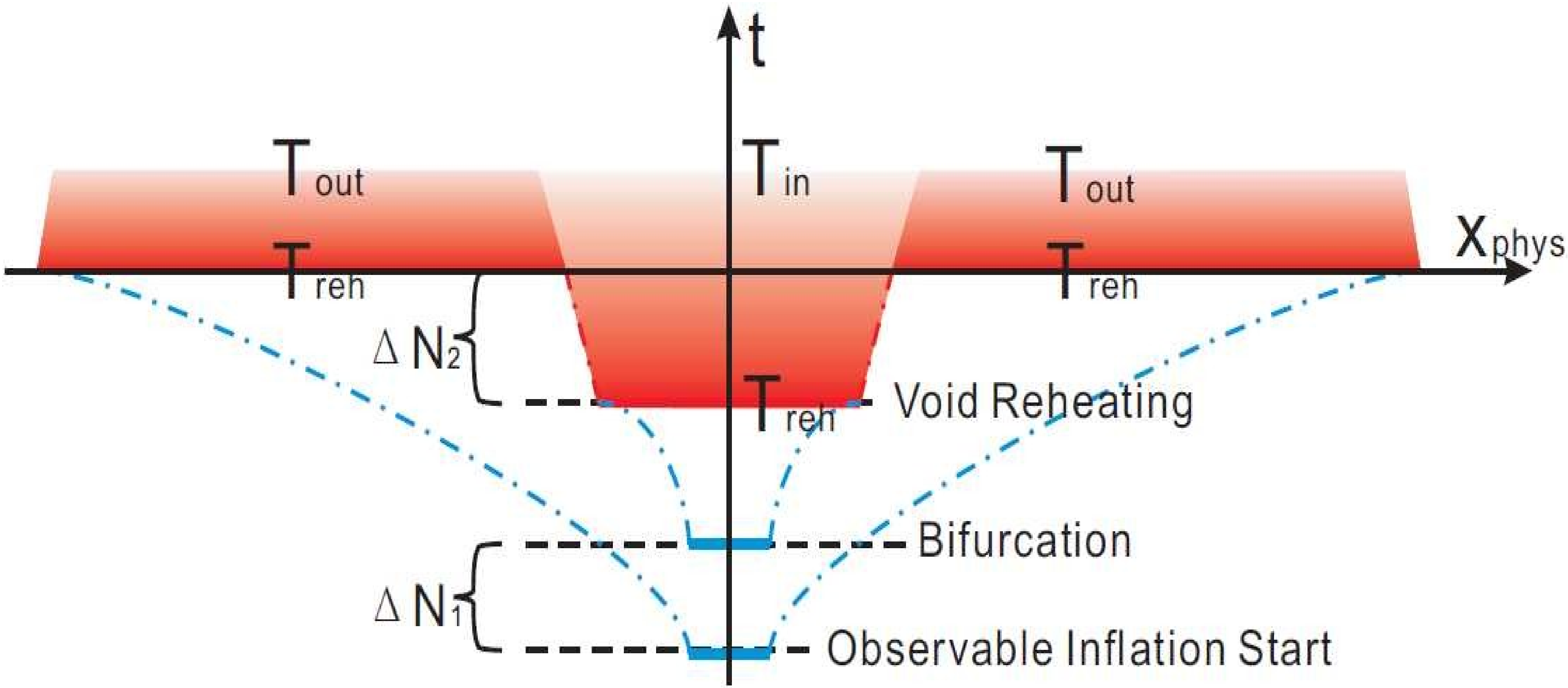
Very asymmetric bifurcations

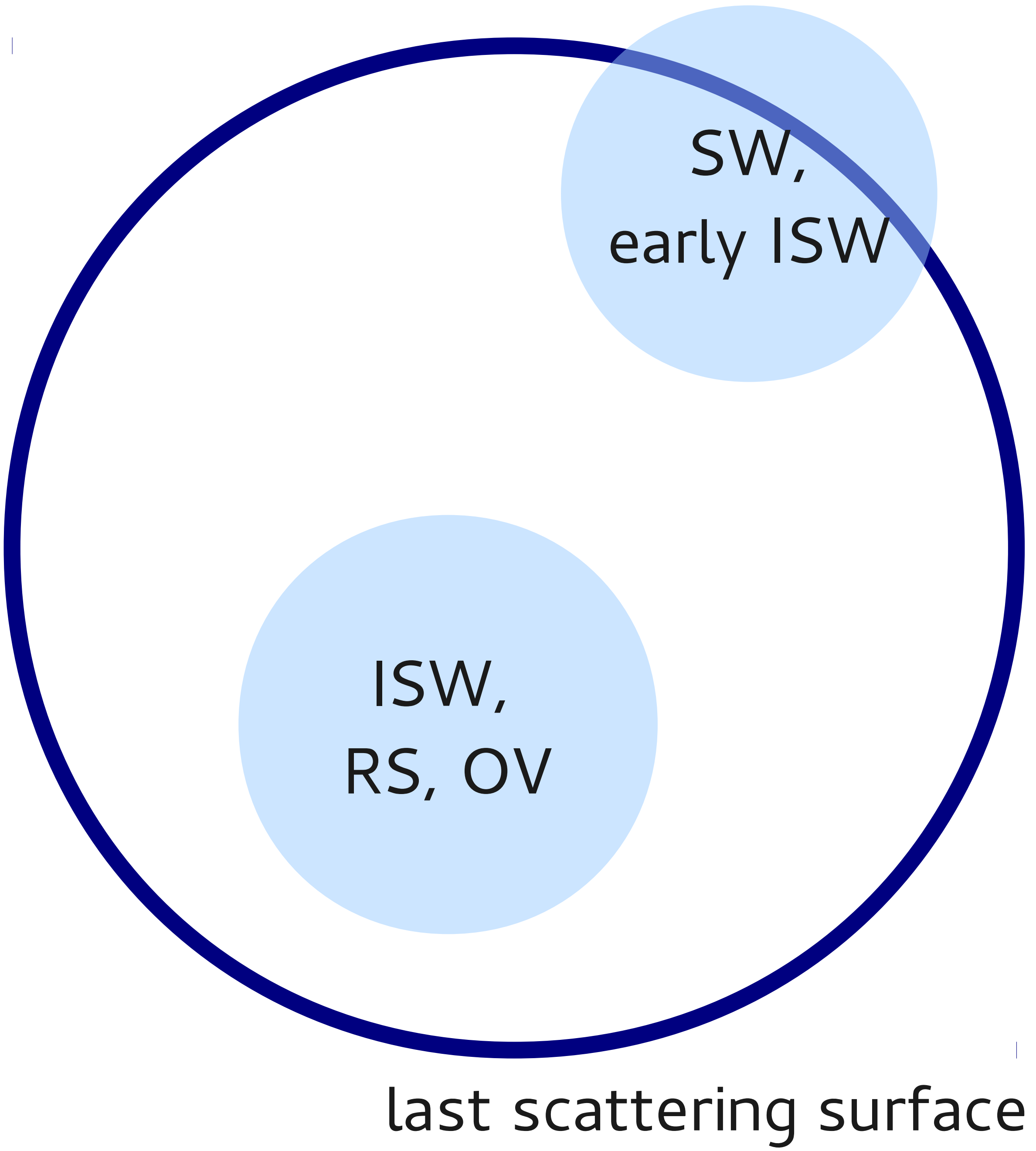
—--— Overview



Very asymmetric bifurcations

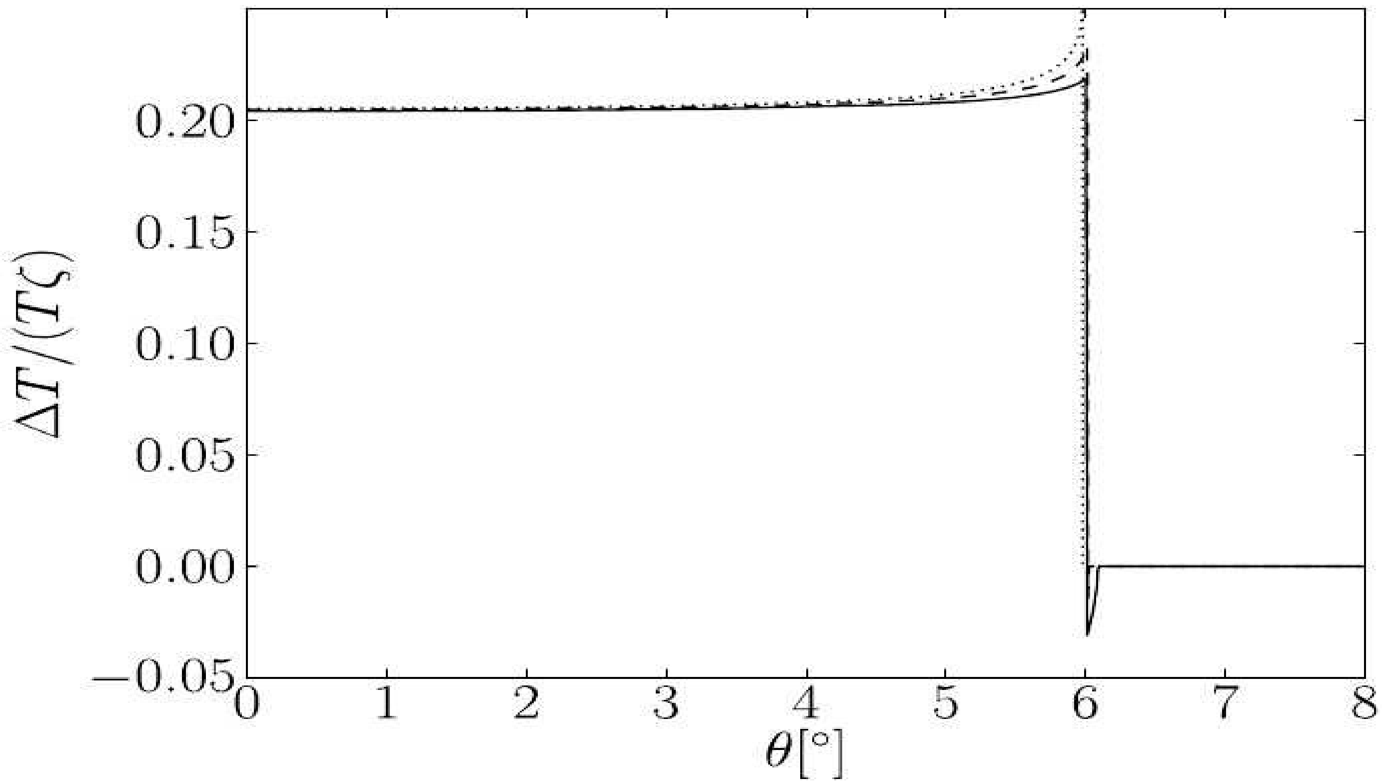
---- Primordial




SW,
early ISW
ISW,
RS, OV
last scattering surface

Very asymmetric bifurcations

—--— SW, early ISW

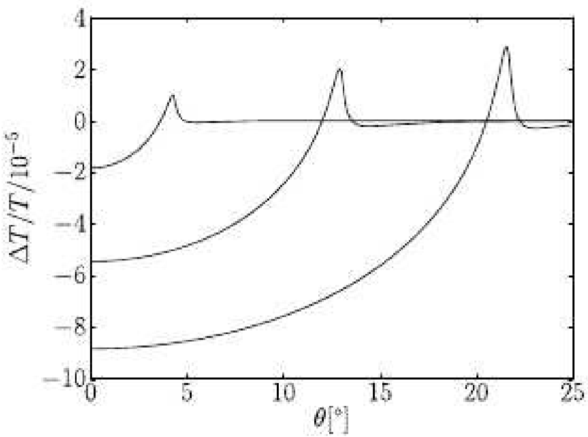


ζ>0: under-density

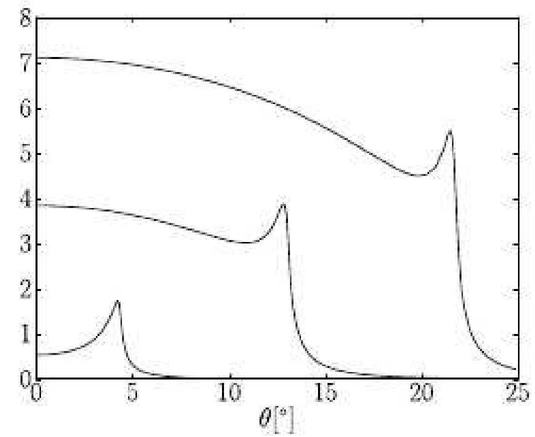
Very asymmetric bifurcations

---- ISW,RS,OV

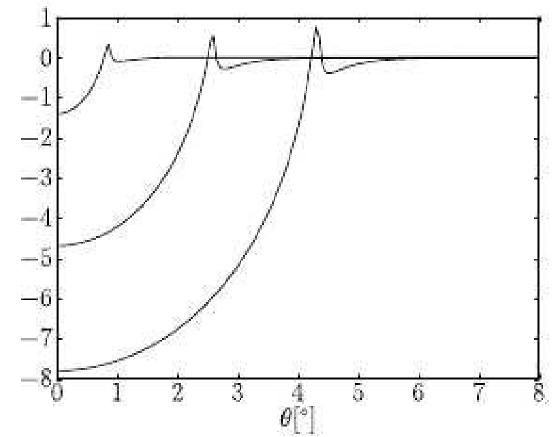
$z = 0.5$ with $\zeta_* = 10^{-3}$



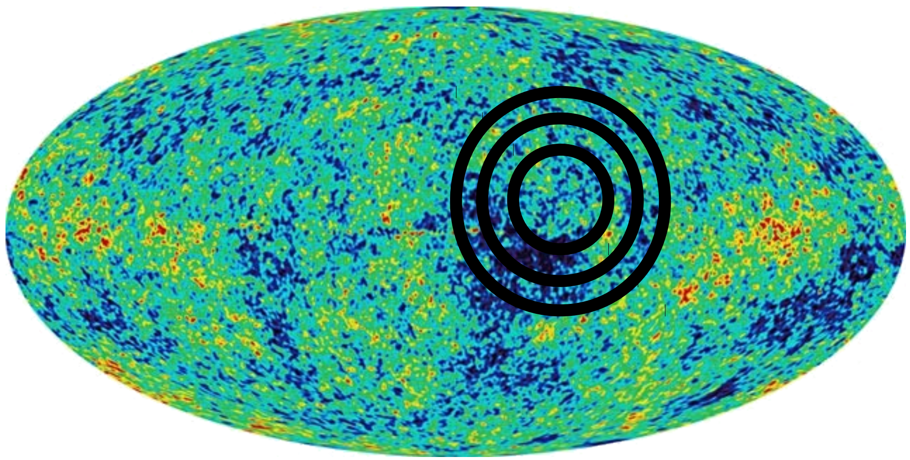
$\zeta_* = -10^{-3}$



$z = 10$ with $\zeta_* = 10^{-3}$

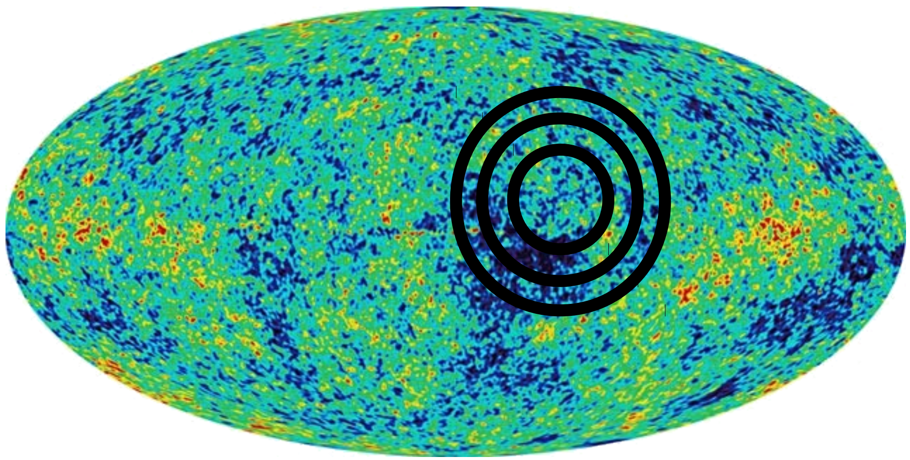


$\zeta_*>0$: under-density



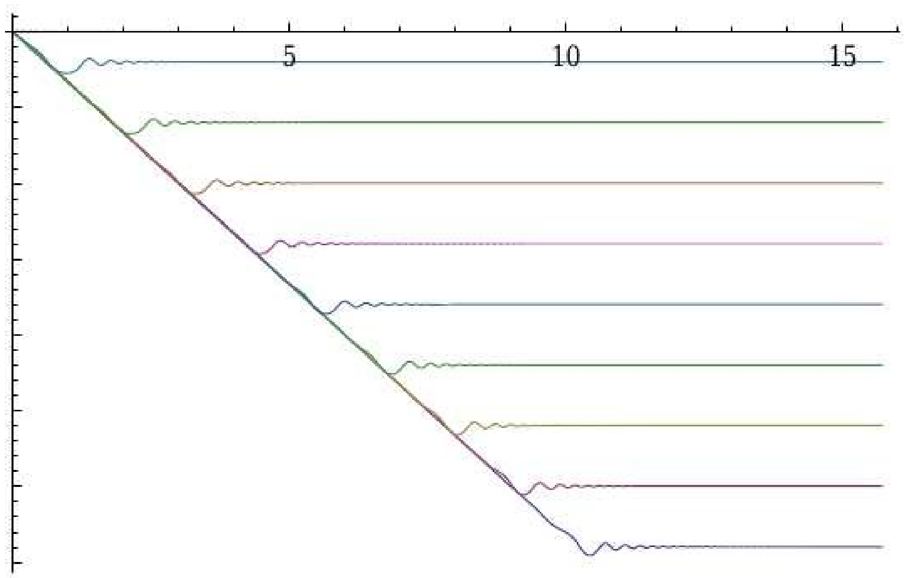
topological defect
induced MSI

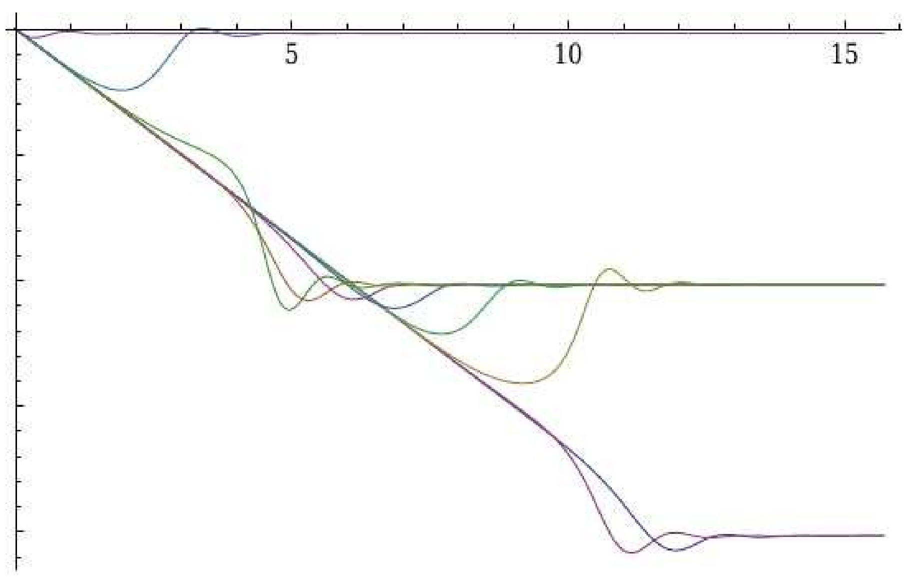
$$V = V_{sr}(\varphi) + s\chi + m^2\,\Psi^a\Psi^a \cos(\chi/\chi_0)$$



topological defect induced MSI

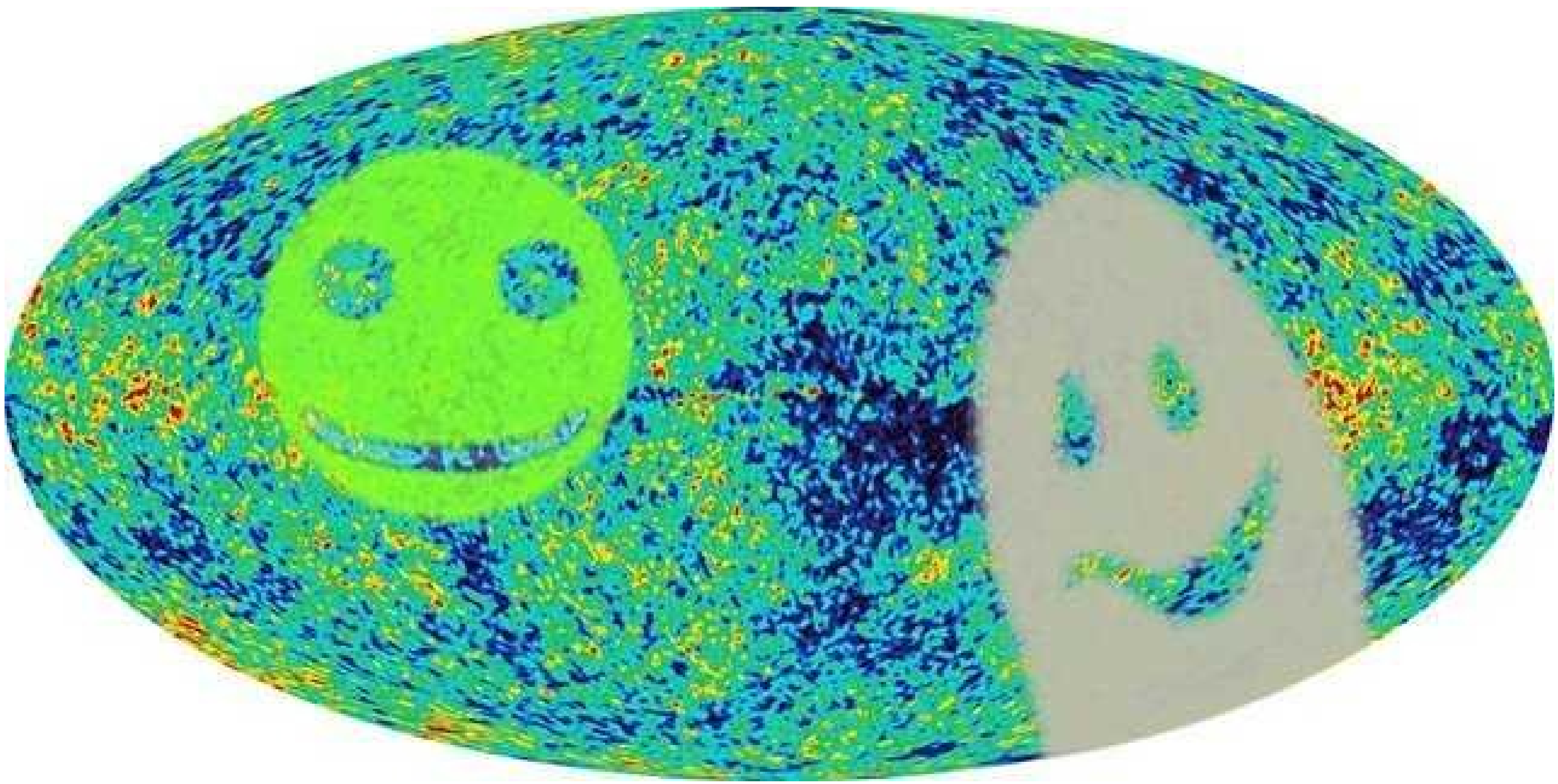
$$V = V_{sr}(\varphi) + s\chi + m^2\,\Psi^a\Psi^a \cos(\chi/\chi_0)$$





A feature in position space?

Reminder: multi-stream inflation



Inflation in a random potential

statistics

Inflation in a random potential

{ amplification

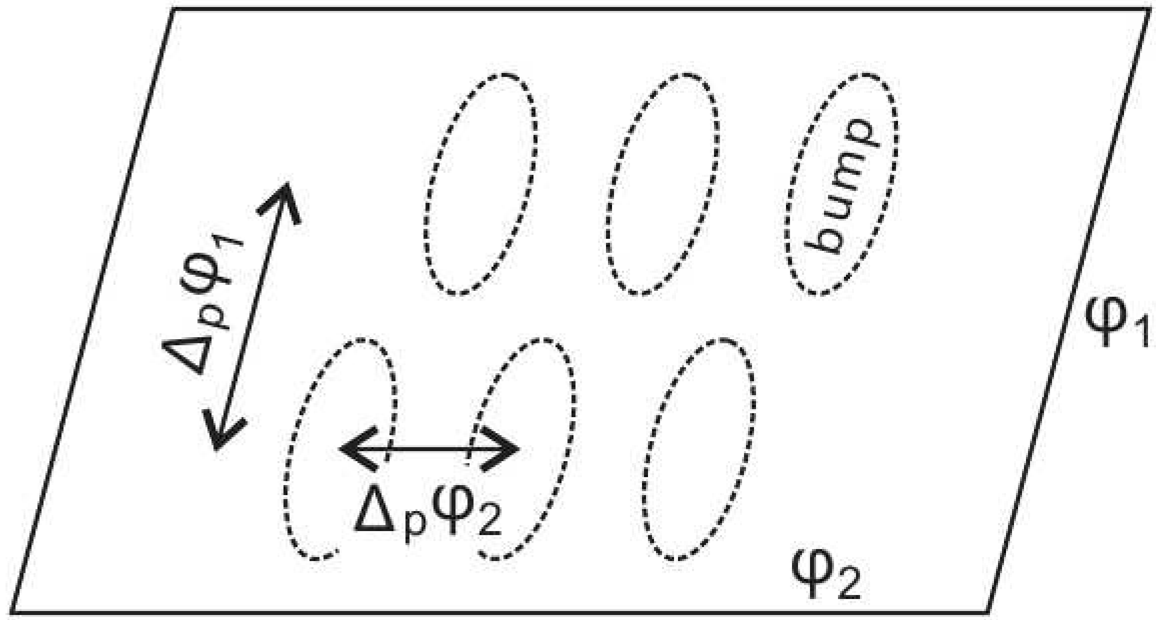
random walk

isocurvature direction

Inflation in a random potential

- amplification
- random walk

isocurvature direction



$$\xi = \Delta_p\varphi_1 / \Delta_p\varphi_2$$

$$\ddot{\varphi}_1 + 3H\dot{\varphi}_1 + \partial_1 V(\varphi_1) + \partial_1 U(\varphi_1, \varphi_2) = 0 \ ,$$

$$\ddot{\varphi}_2 + 3H\dot{\varphi}_2 + \partial_2 U(\varphi_1, \varphi_2) = 0 \ ,$$

$$\lambda \equiv \sqrt{\langle(\partial_1 U)^2\rangle}/|\partial_1 V|.$$

$$\delta \equiv \varphi_2^{(A)} - \varphi_2^{(B)}$$

$$\ddot{\delta} + 3H\dot{\delta} + (\partial_2^2 U)\delta = 0$$

$$\partial_2^2 U \simeq \frac{\lambda\xi\partial_1 V}{\Delta_p\varphi_2} \sin\left(\frac{2\pi\dot{\varphi}_1 t}{\Delta_p\varphi_1}\right)$$

oscillate?

or grow?

Mathieu equation,
Analog: broad resonance preheating

$$\delta \equiv \varphi_2^{(A)} - \varphi_2^{(B)}$$

$$\ddot{\delta} + 3H\dot{\delta} + (\partial_2^2 U)\delta = 0$$

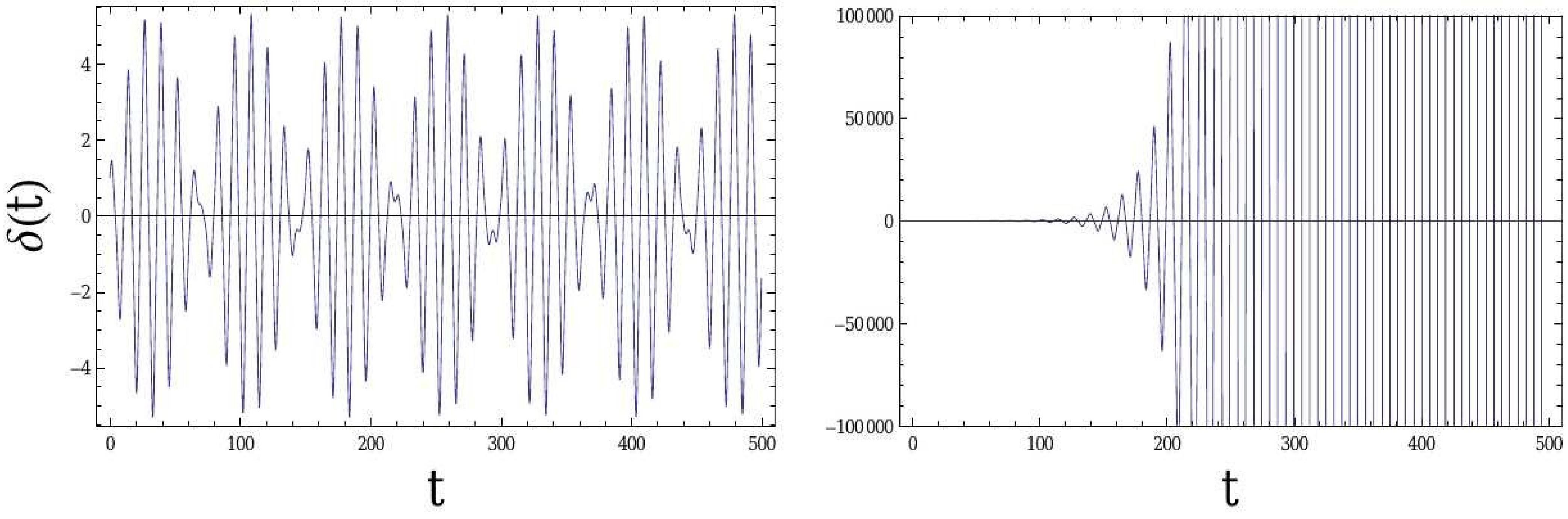
$$\partial_2^2 U \simeq \frac{\lambda \xi \partial_1 V}{\Delta_p \varphi_2} \sin\left(\frac{2\pi \dot{\varphi}_1 t}{\Delta_p \varphi_1}\right)$$

Rough estimate: neglect friction

$$\ddot{\delta} + 3H\dot{\delta} + (\partial_2^2 U)\delta = 0$$

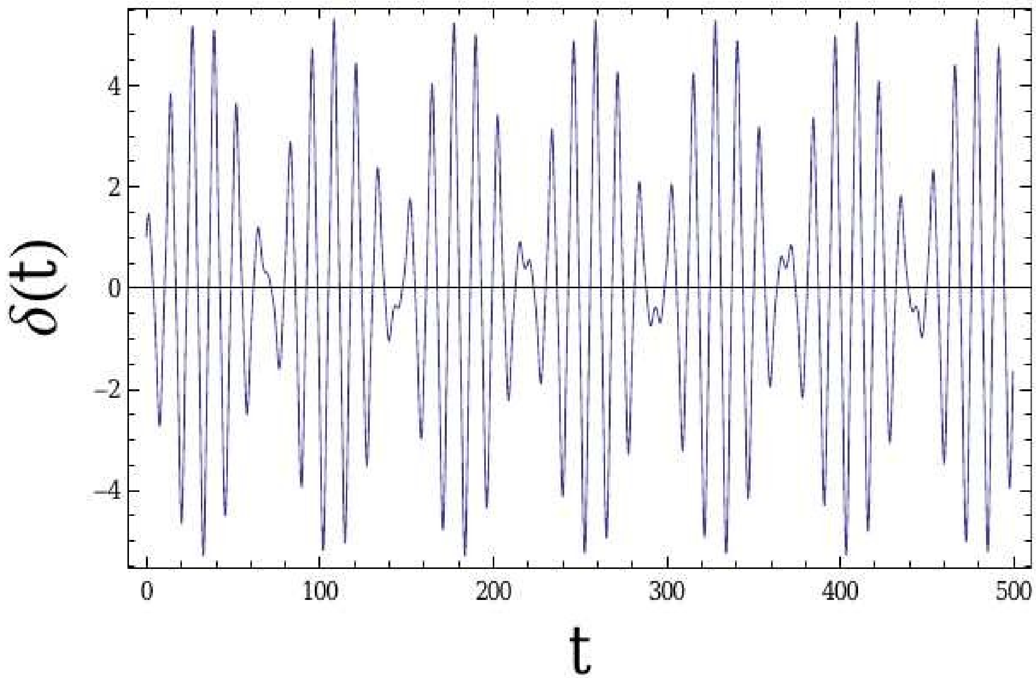
$$\ddot{\delta} + \alpha \sin(\beta t)\delta = 0$$

$\alpha/\beta^2 = 0.45$ $\qquad$ $\alpha/\beta^2 = 0.46$

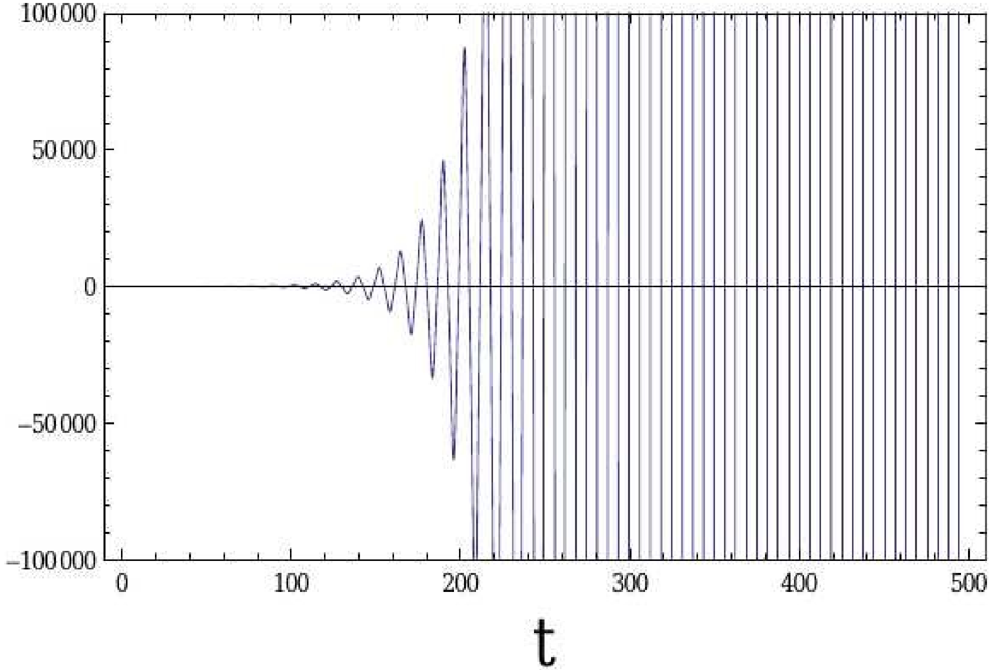


Transition between oscillation and growing

$\alpha/\beta^2 = 0.45$



$\alpha/\beta^2 = 0.46$

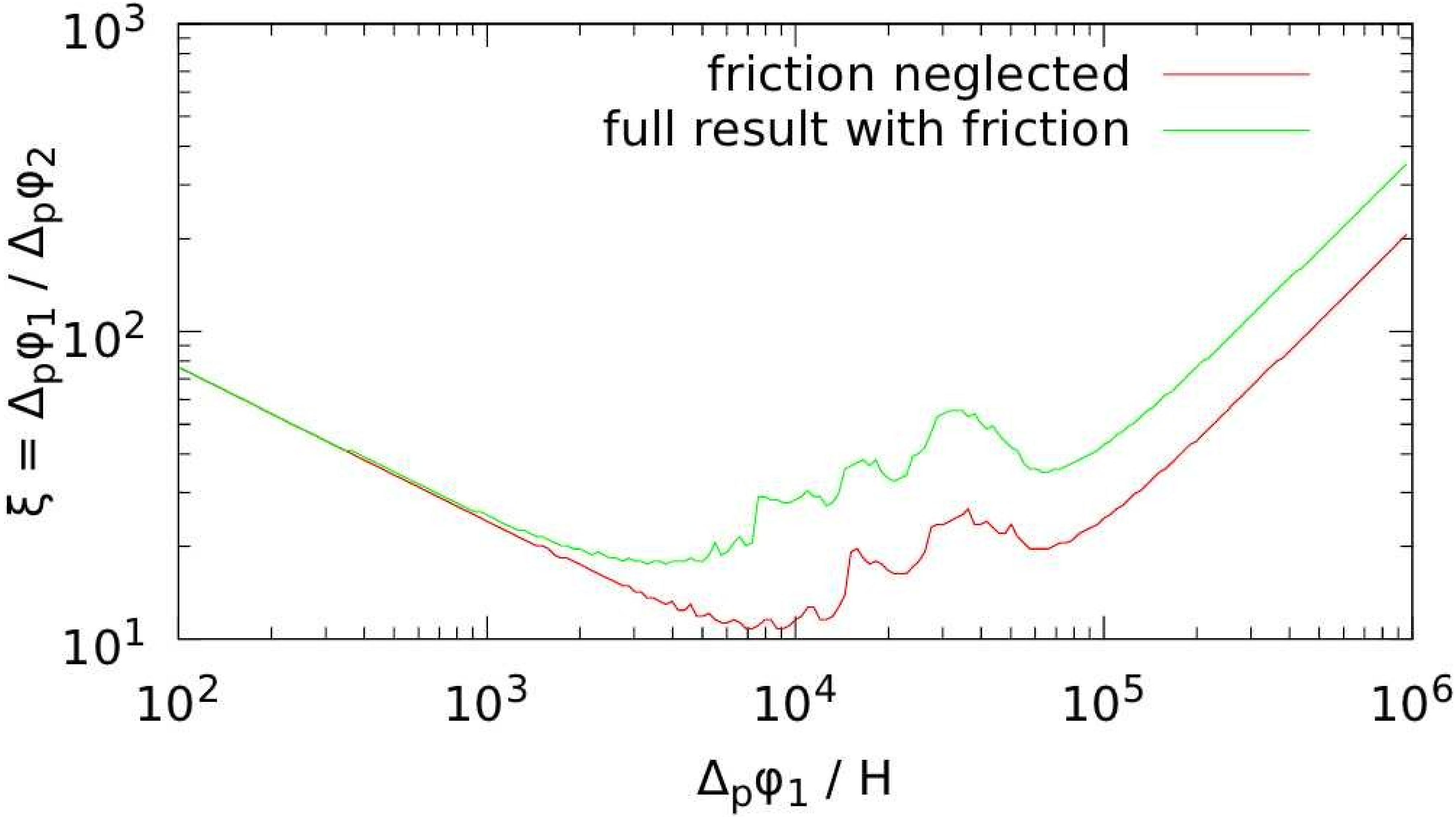


growing solution

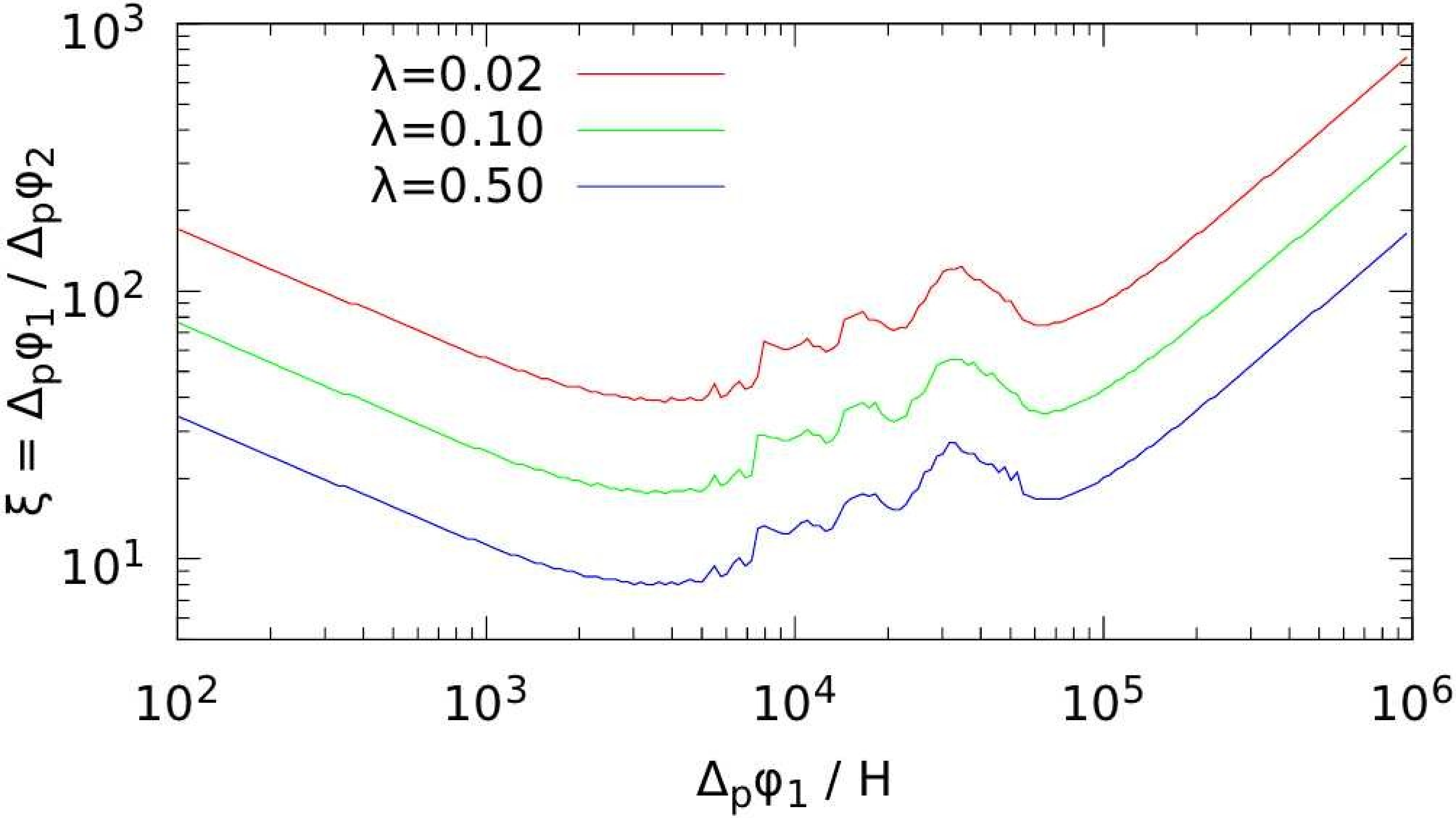
$$\frac{1}{\lambda\xi^2 P_\zeta^{1/2}} < \frac{\Delta_p \varphi_1}{H} < \frac{3\lambda\xi^2}{2\pi P_\zeta^{1/2}}$$

large exponent

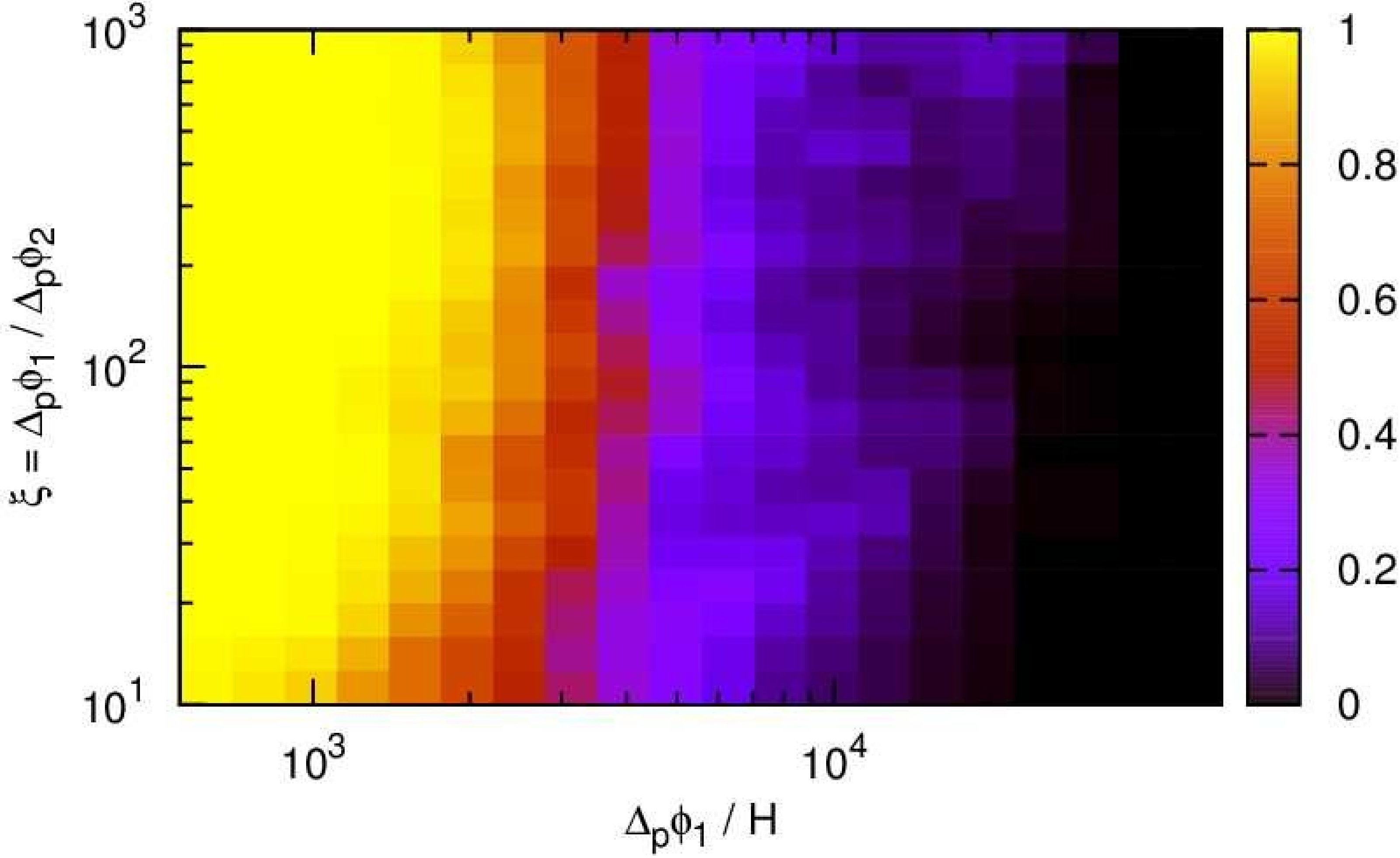
with friction



with friction



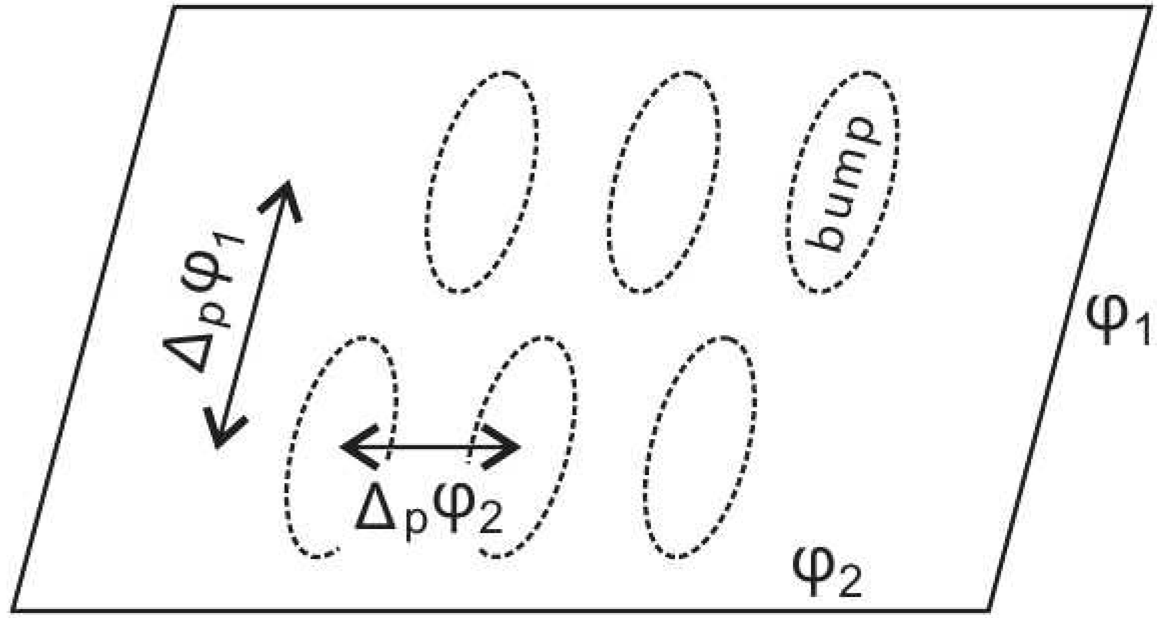
Completely random potential



Inflation in a random potential

{ amplification
  random walk

isocurvature direction



$$\Delta t = \Delta_p \varphi_1 / \dot{\varphi}_1$$

$$\Delta_q \varphi_2 = P_\zeta^{1/4} \sqrt{\frac{H \Delta_p \varphi_1}{2\pi}} = \Delta_q^H \varphi \sqrt{\frac{\Delta_p \varphi_1}{\Delta_c^H \varphi_1}}$$

$$\mathrm{P_{bifur}}(\Delta t) = \Delta_q \varphi_2 / \Delta_p \varphi_2$$

$$\Delta_p \varphi_1 = \Delta_p \varphi_2$$

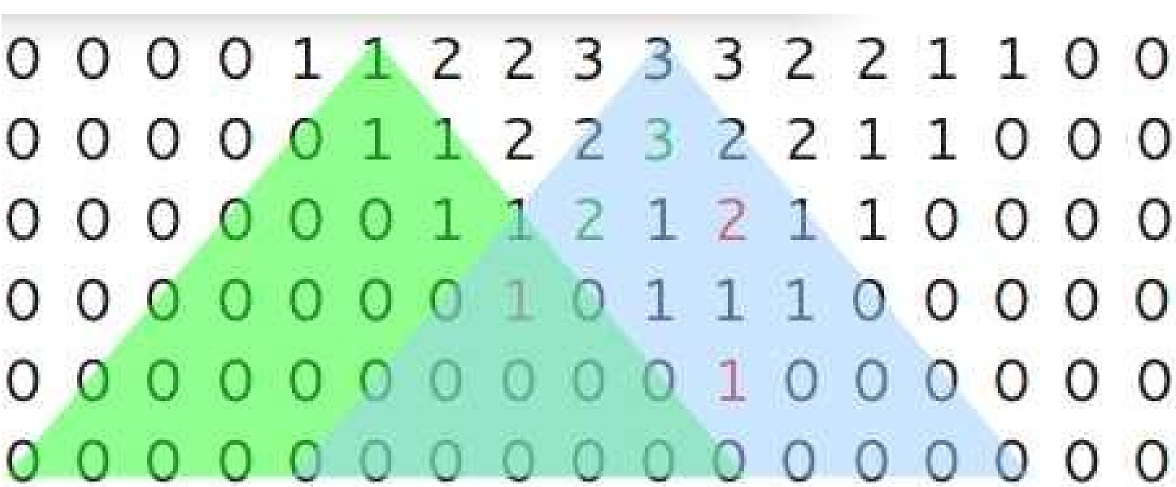
$$\mathrm{P_{bifur}}(N) \begin{cases} \simeq \tilde{N} \mathrm{P_{bifur}} \\ \simeq \frac{2}{3} \tilde{N}^{3/2} \mathrm{P_{bifur}} \end{cases} \qquad \begin{cases} \leq 20H \\ \leq 100H \end{cases}$$

×(6N)½ for the whole observable universe: about 160H or 800H

bifurcation along a world line

$$\Delta_p \varphi_1 = \Delta_p \varphi_2 \quad \begin{cases} \leq 20H \\ \leq 100H \end{cases}$$

Part III: single field chain inflation



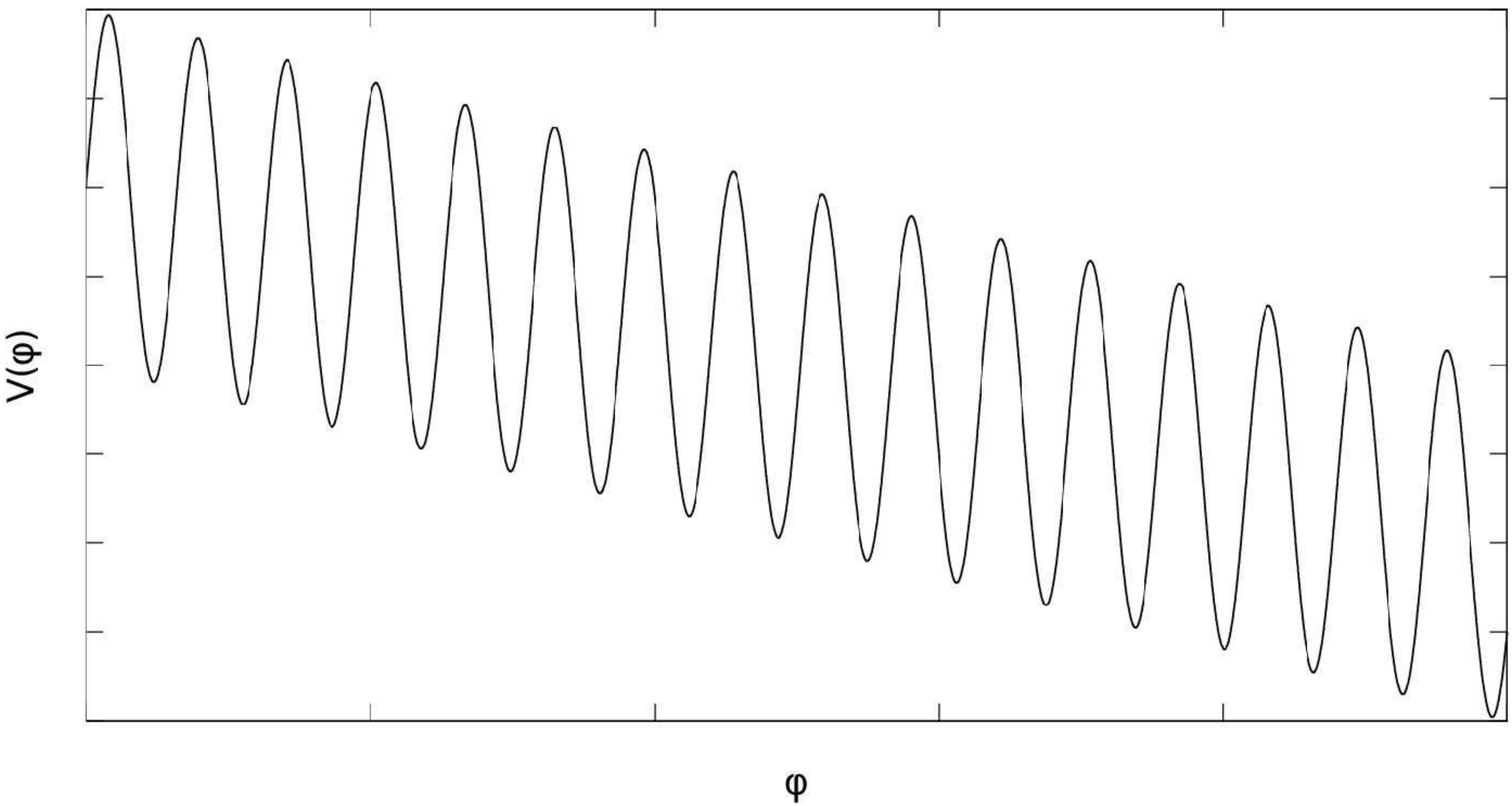
Chain inflation:

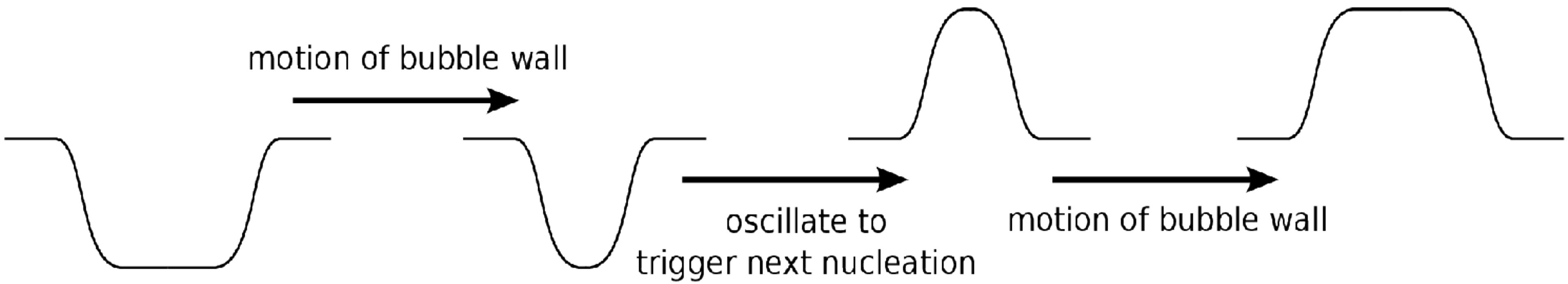
Freese, Spolyar (2004)

See also: Liu, Feldstein, Tweedie, Chialva, Danielsson, Huang, ...

Perturbations: first full, analytical result

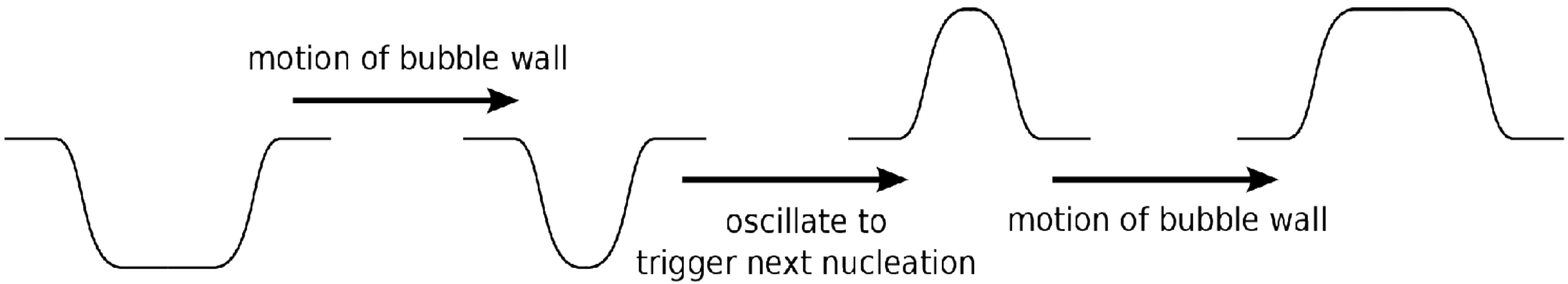
Simplest first
single field chain inflation

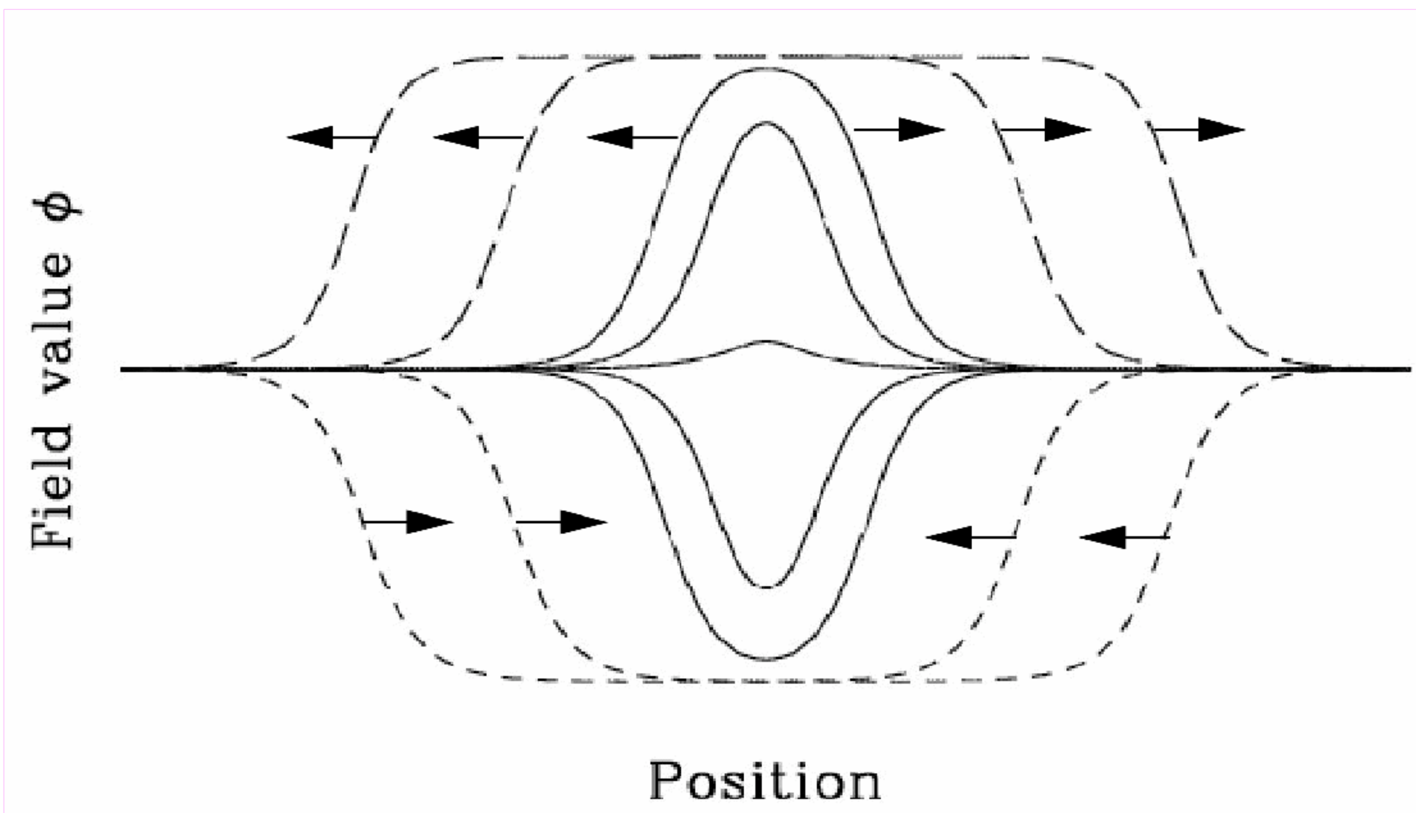




New effect:

walls collide into new bubble

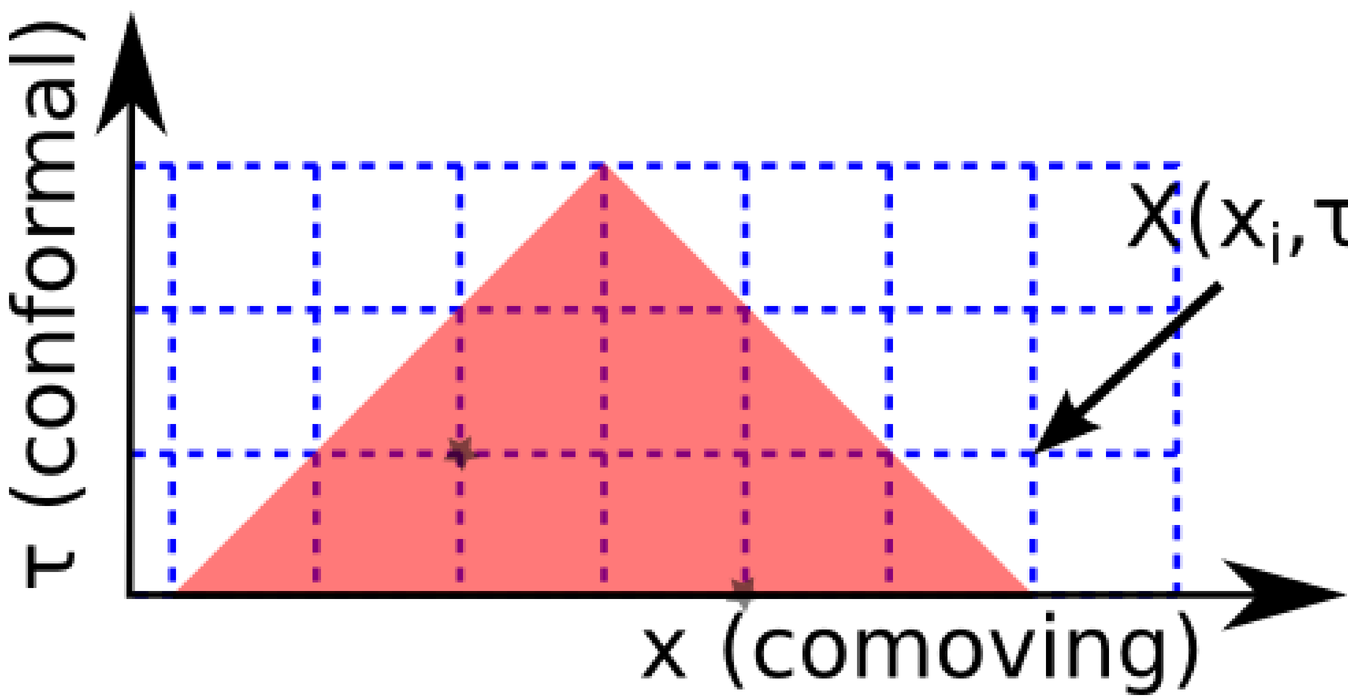

motion of bubble wall
oscillate to
trigger next nucleation
motion of bubble wall

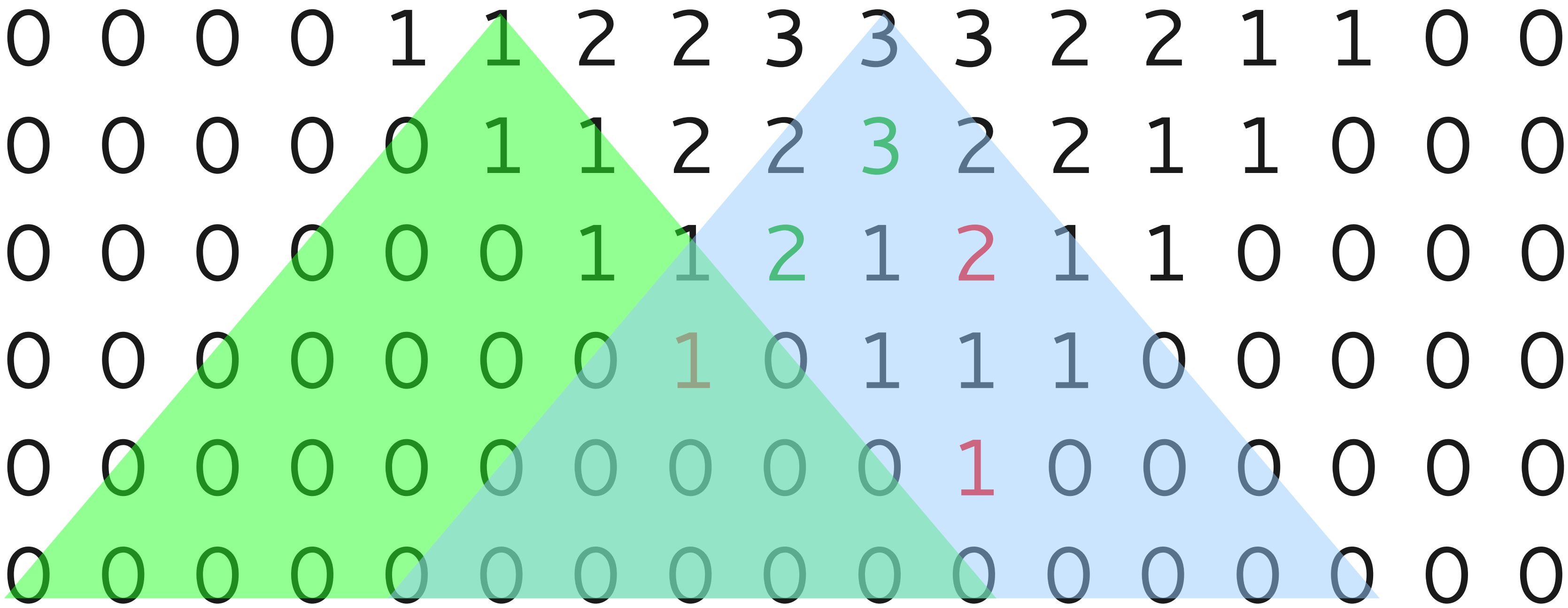

Field value ϕ
Position

Background dynamics

$$\varphi(x,\tau) = \Sigma\, X(x_i, \tau_j)$$

$$X \sim \mathrm{Pois}(\Gamma\, H^{-4} \tau^{-4}\, d^3x d\tau)$$





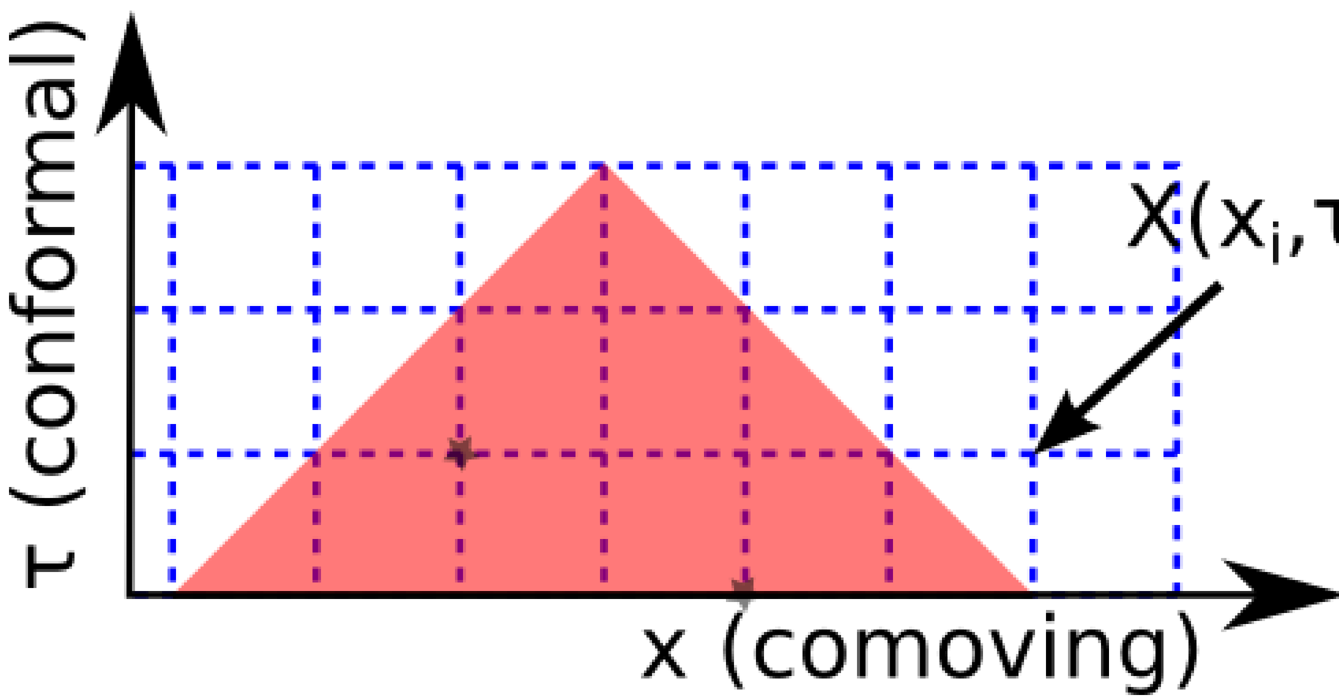
spontaneous

from bubble wall

Background dynamics

$$\varphi(x,\tau) = \Sigma\, X(x_i, \tau_j)$$

$$X \sim \mathrm{Pois}(\Gamma\, H^{-4} \tau^{-4}\, d^3x d\tau)$$



Background dynamics

$$\varphi(x,\tau) = \Sigma\, X(x_i\,,\tau_j)$$

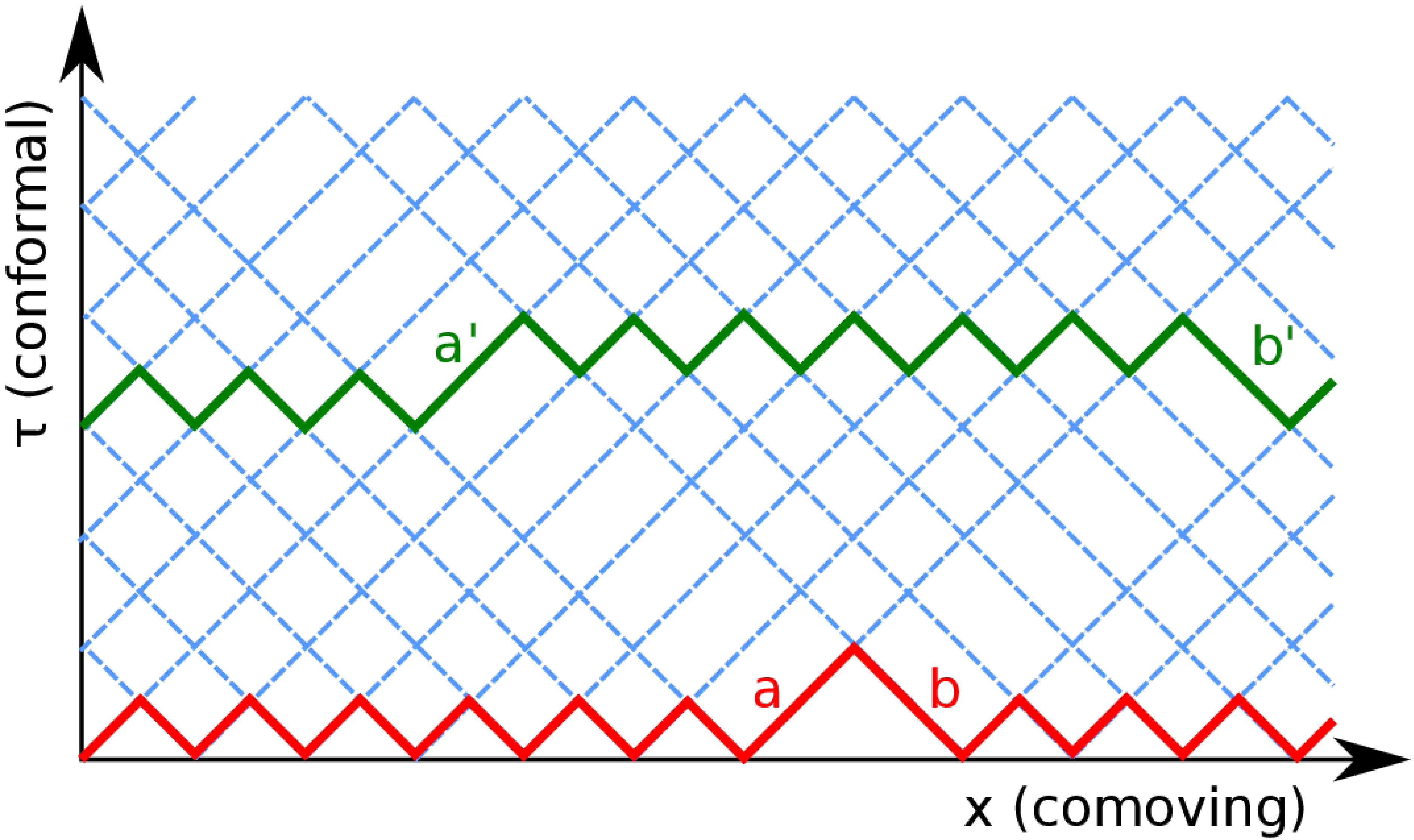
$$X \sim \mathrm{Pois}(\Gamma\, H^{-4}\tau^{-4}\, d^3x d\tau)$$

$$X+Y \sim \mathrm{Pois}(\lambda_X+\lambda_Y)$$

$$\varphi(x,\tau) \sim \mathrm{Pois}(\varphi_i + 4\pi\Gamma H^{-3}(t-t_i)/3)$$

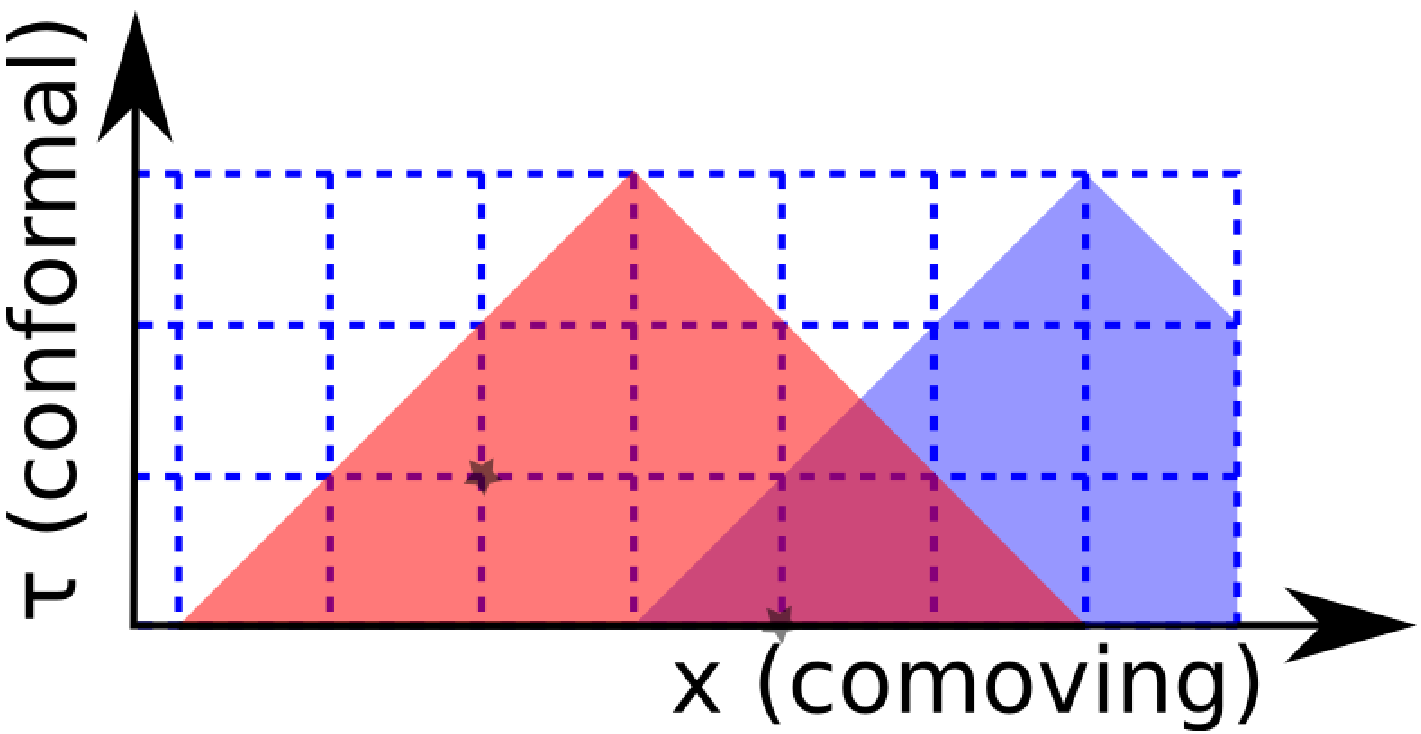
$$<\varphi(x,\tau)> = \varphi_i + 4\pi\Gamma H^{-3}(t-t_i)/3$$

Peturbation theory



$$\langle \delta\varphi(x,\tau)\delta\varphi(x+r,\tau)\rangle_c = \int_{-\infty}^{\tau} \mathrm{d}\eta \; V_2(\eta)\frac{\Gamma}{H^4\eta^4}$$

$$V_2(\eta) \equiv \begin{cases} \frac{4}{3}\pi(\tau-\eta-r/2)^2(\tau-\eta+r/4) & \text{when } r < 2(\tau-\eta) \\ 0 & \text{when } r \geq 2(\tau-\eta) \end{cases}$$



$$\langle \delta\varphi(x,\tau)\delta\varphi(x+r,\tau)\rangle_c = \int_{-\infty}^{\tau} \mathrm{d}\eta\ V_2(\eta)\frac{\Gamma}{H^4\eta^4}$$

$$V_2(\eta) \equiv \begin{cases} \frac{4}{3}\pi(\tau-\eta-r/2)^2(\tau-\eta+r/4) & \text{when } r < 2(\tau-\eta) \\ 0 & \text{when } r \geq 2(\tau-\eta) \end{cases}$$

$$\langle \delta\varphi_{\mathbf{k}_1}(\tau)\delta\varphi_{\mathbf{k}_1}(\tau)\rangle = (2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2)\int_0^{\infty} \mathrm{d}r\frac{4\pi r\sin(kr)}{k}\langle \delta\varphi(0,\tau)\delta\varphi(r,\tau)\rangle$$

$$\langle \delta\varphi_{\mathbf{k}_1}(\tau)\delta\varphi_{\mathbf{k}_1}(\tau)\rangle = (2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2)\frac{8\pi^3\Gamma}{3H^4k^3}$$

$$\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\rangle = (2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2)\frac{3\pi H^4}{2\Gamma k^3} \qquad P_\zeta = \frac{3H^4}{4\pi\Gamma}$$

consistency checks

$V = V_0 - A\varphi + V_1 \sin(2\pi\varphi/\Delta\varphi)$

① $\lambda = V'''' < 2\pi$ everywhere

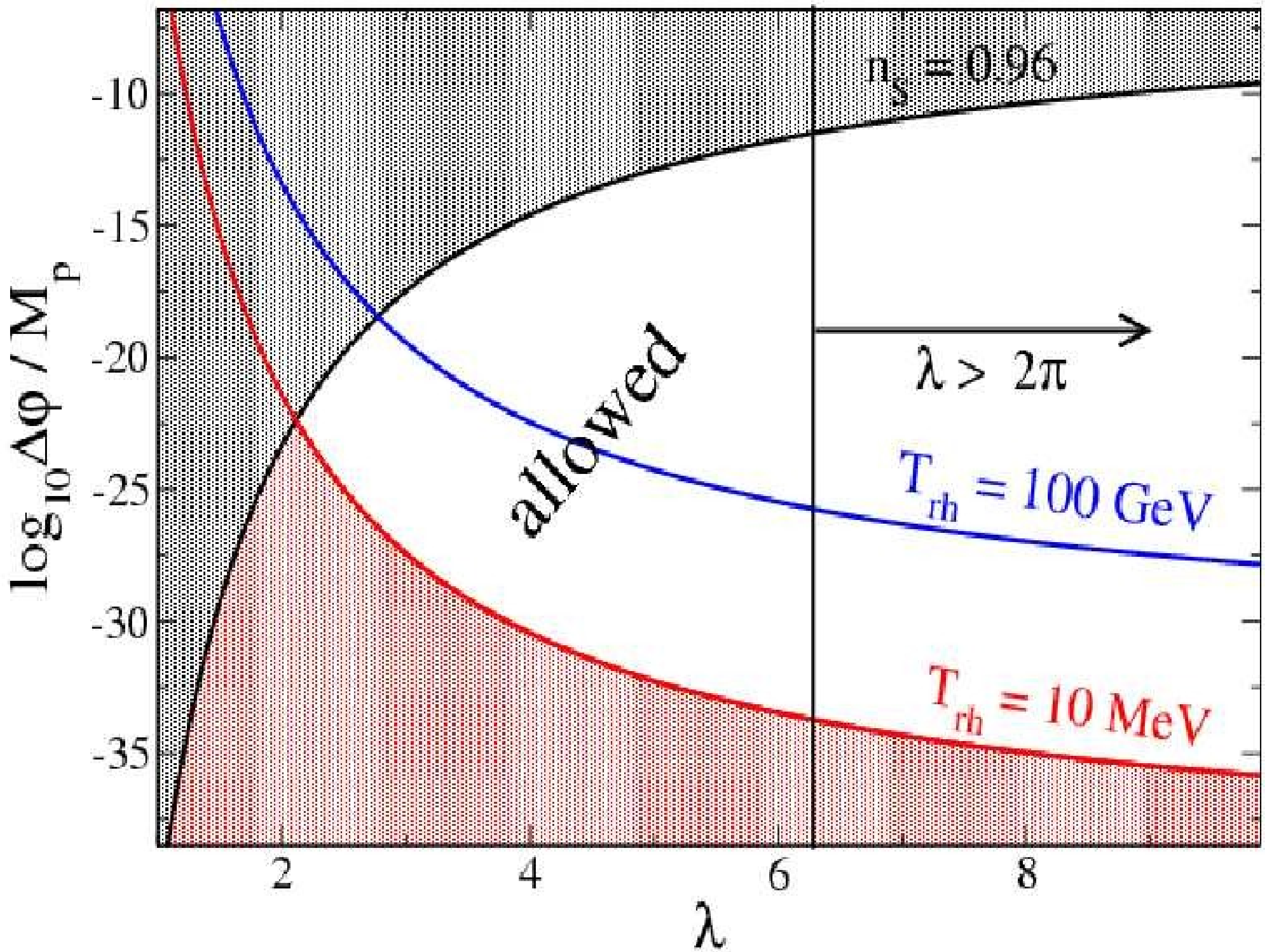
=============

② φ stuck

③ small bubble

④ P_ζ and n_s

⑤ T_{reh}



The potential may get burnt !

(non-) Gaussianity

$$X \sim \mathrm{Pois}(\lambda),\ \langle (X - \langle X \rangle)^3 \rangle = \lambda$$

$$\langle \delta\varphi^3 \rangle = \int_{-\infty}^{\tau} \mathrm{d}\eta\ V_3(\eta) \frac{\Gamma}{H^4 \eta^4}$$

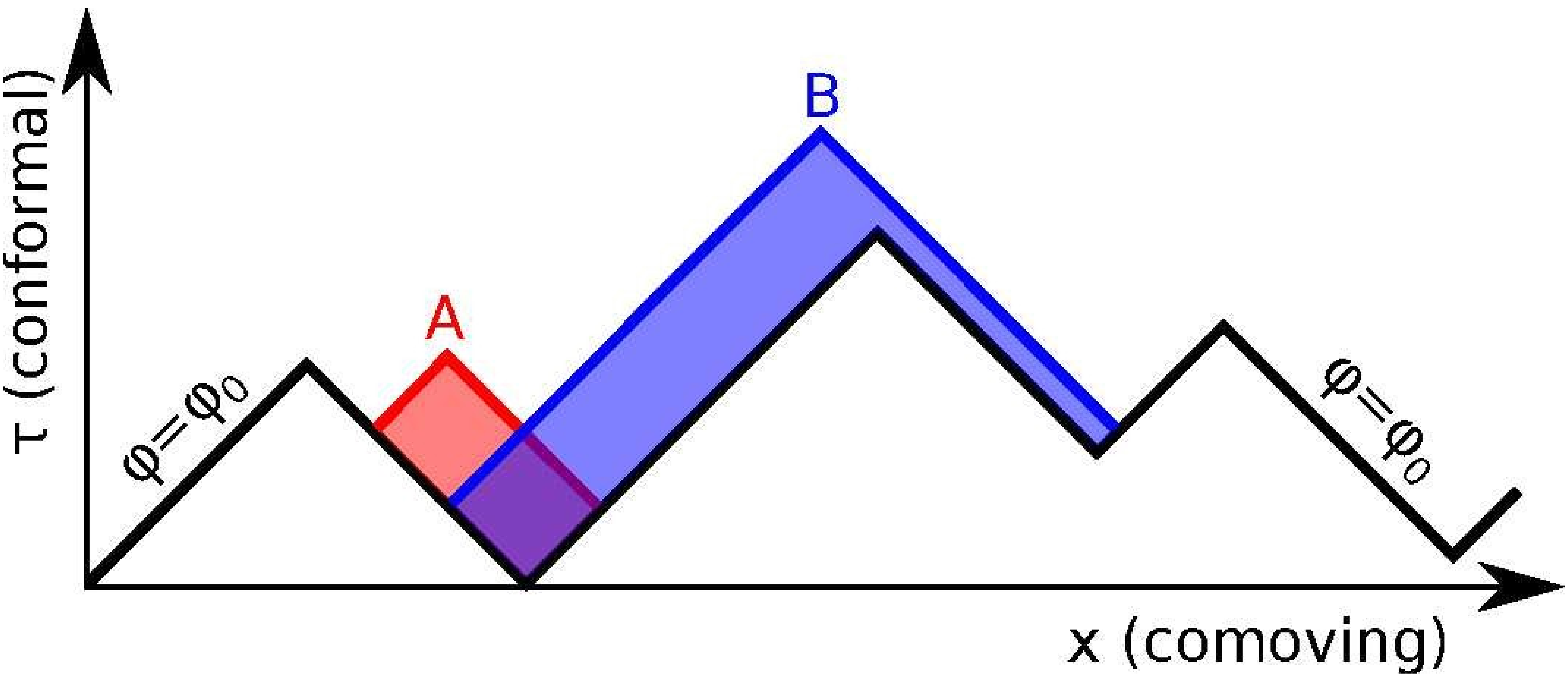
$$\langle \zeta^3 \rangle = (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \frac{3}{5} f_{NL} P_\zeta^2$$

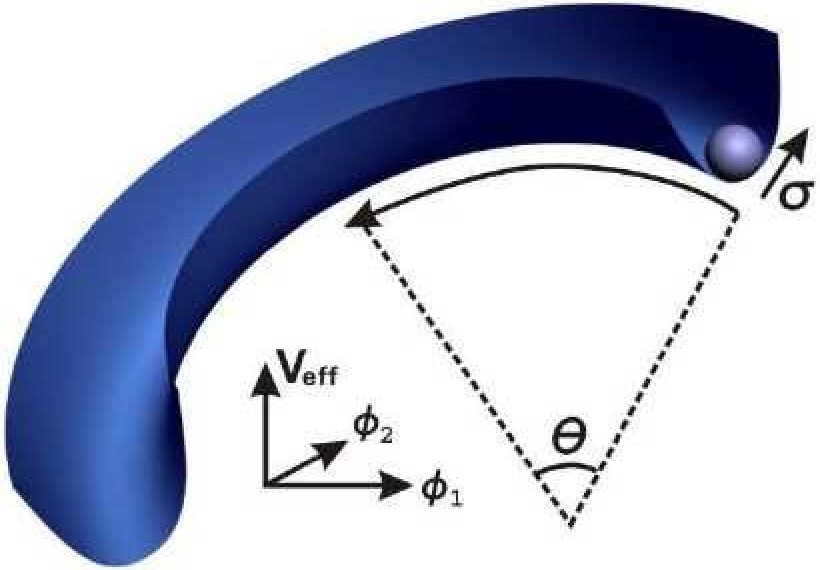
$f_{NL} \sim g_{NL} \sim h_{NL} \sim i_{NL} \sim \ldots \sim O(1)$

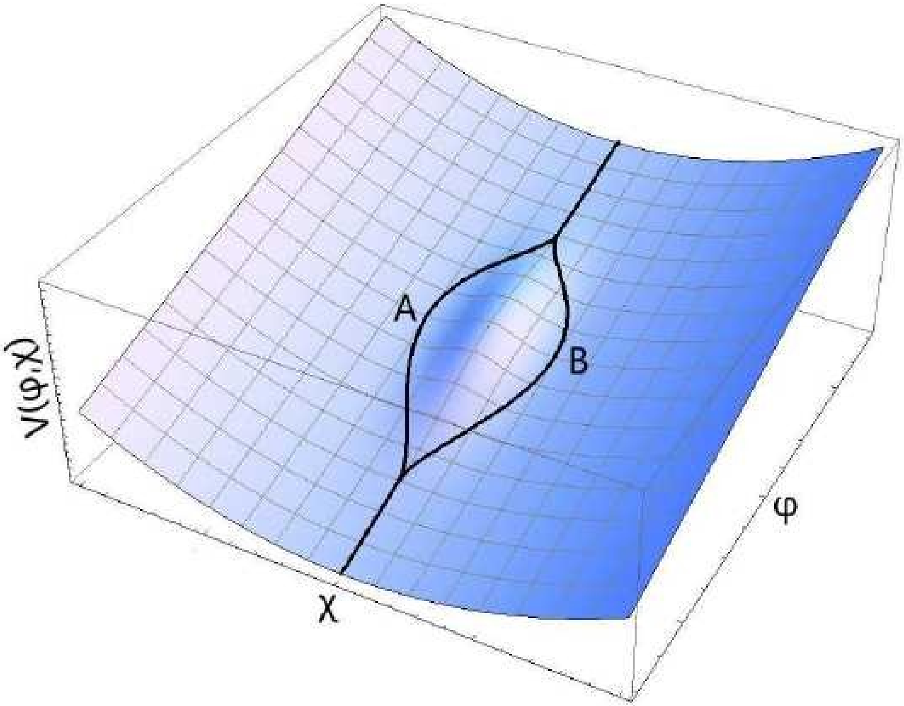
Multiple fields:

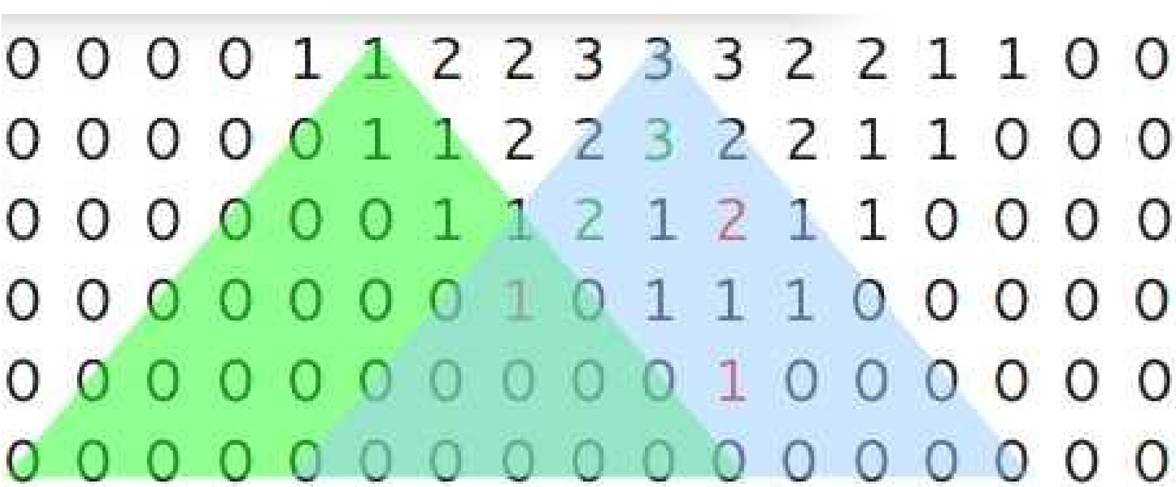
same power spectrum

fewer minimas needed




σ
Θ
V_eff
φ2
φ1


A
B
V(φ,χ)
φ
χ


0 0 0 0 1 1 2 2 3 3 3 2 2 1 1 0 0
0 0 0 0 0 1 1 2 2 3 2 2 1 1 0 0 0
0 0 0 0 0 0 1 1 2 1 2 1 1 0 0 0 0
0 0 0 0 0 0 0 1 0 1 1 1 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

Merci!