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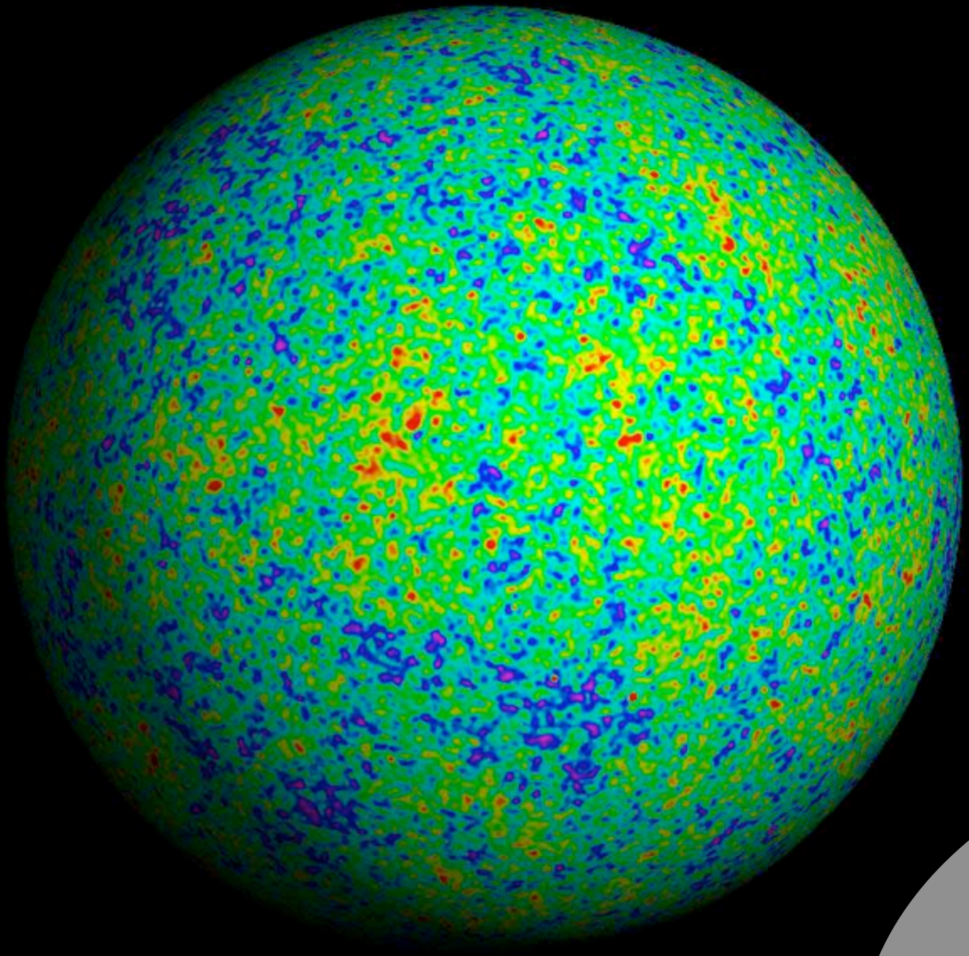
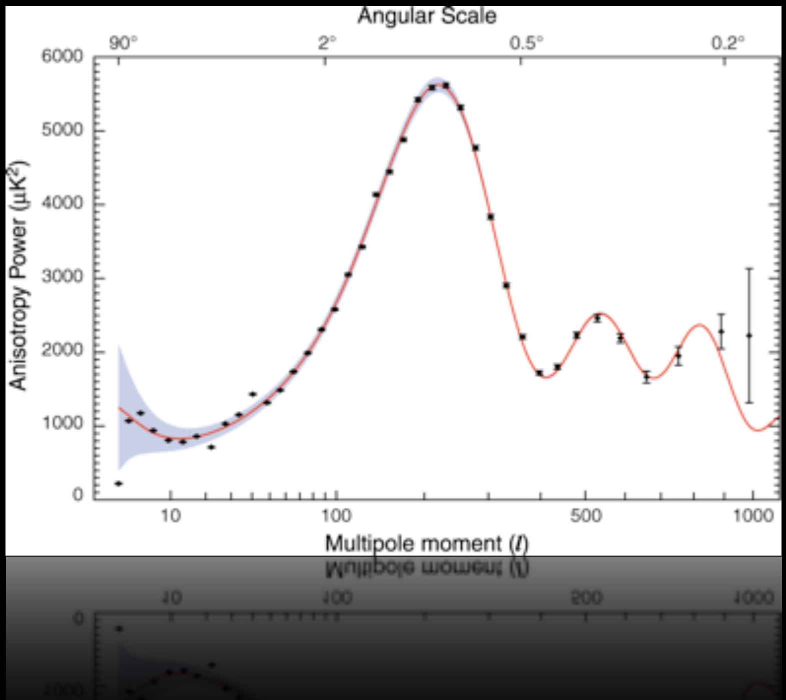
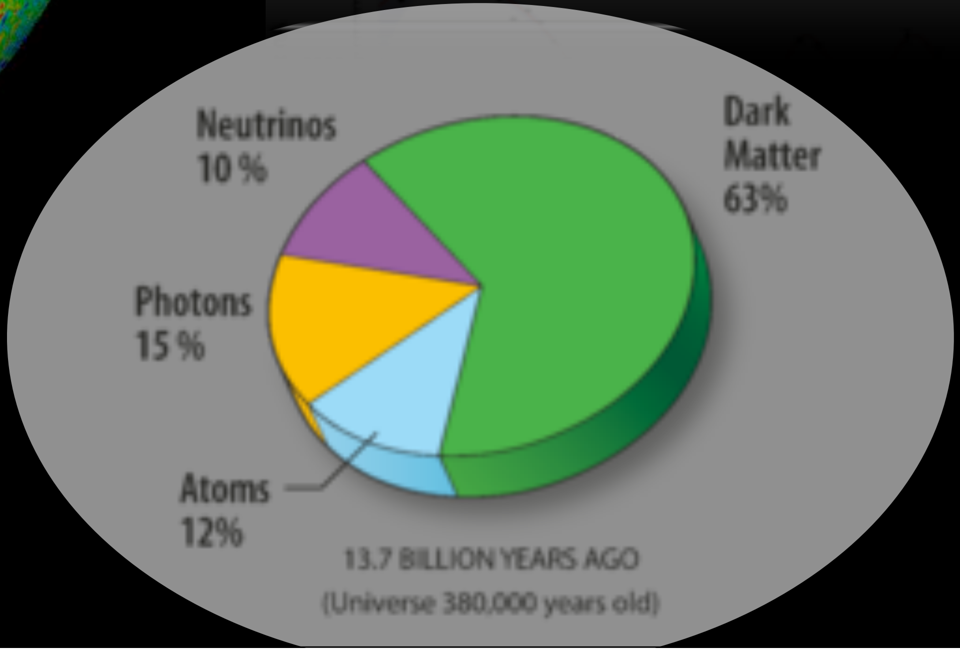
The growth of gravitational instabilities in an expanding universe

IAP november 2012

The current model of cosmology





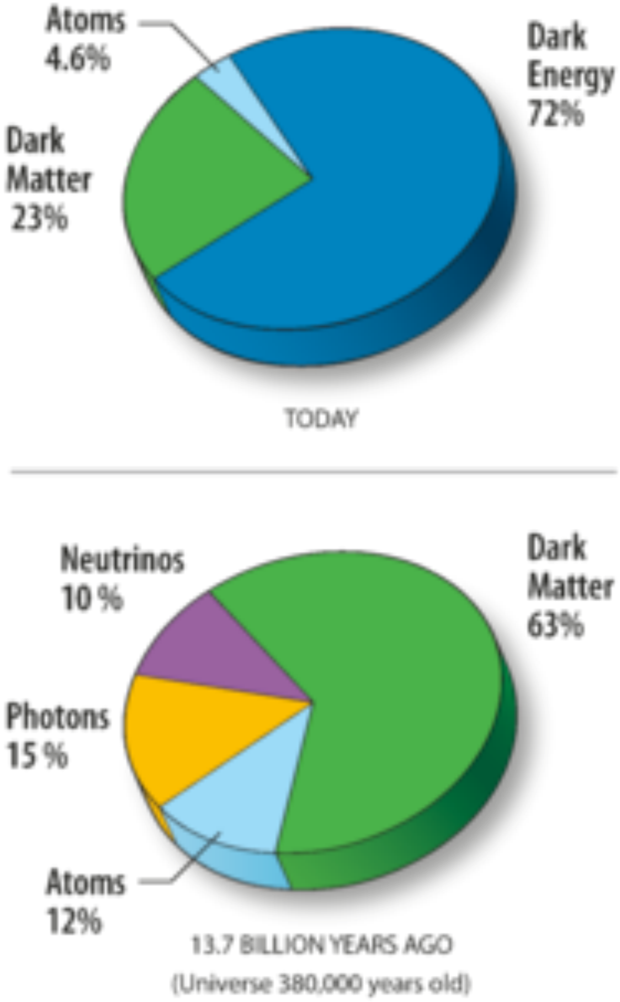


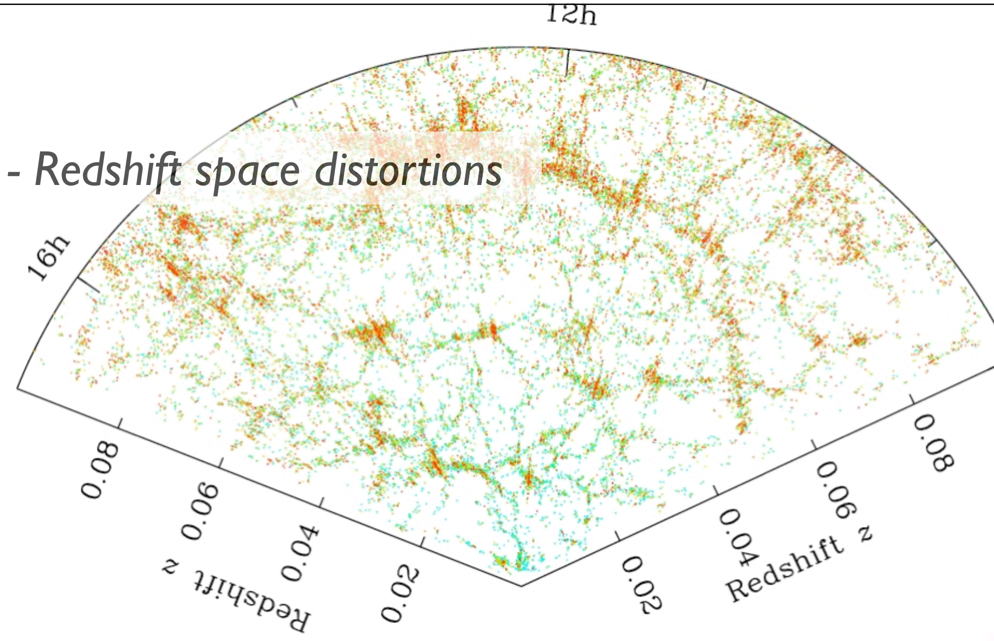
A snapshop of the universe 377,000 years after the Big Bang: CMB temperature fluctuations

A "concordant" model of cosmology but that contains three puzzling ingredients:

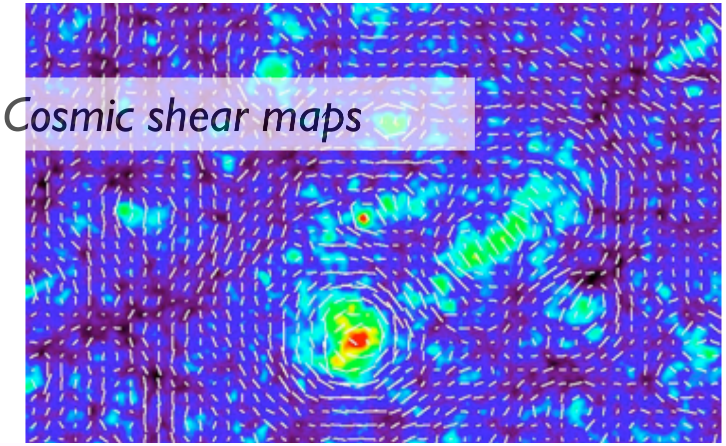
- *An inflationary stage*
- *dark matter*
- *dark energy or a cosmological constant responsible for the (recent) acceleration of the universe*

low redshift manifestations through the way the ***large-scale structure of the universe*** *forms and evolves?*

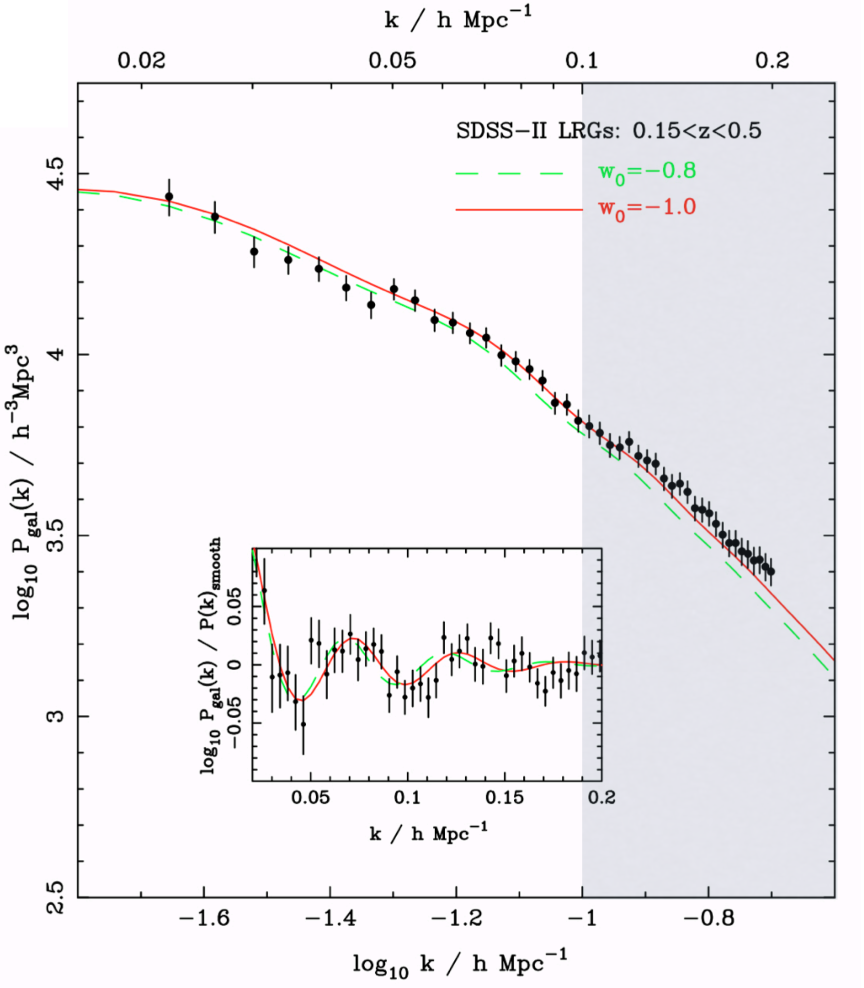




- *Redshift space distortions*



- *Cosmic shear maps*



What is at stake?

- using LSS data to constrain models

What do we want to learn?

- Initial metric perturbations, spectra, primordial non-Gaussianities
- constraints on the dark matter particles - mass of the neutrinos
- dark energy/modification of the gravity in the expansion/growth of structure (fifth force)

Nonlinear effects are ubiquitus!

A self-gravitating expanding dust fluid

A self-gravitating expanding dust fluid

- *Data show that large-scale structure has formed from small density inhomogeneities since time of matter dominated universe with a dominant cold dark matter component*

The Vlasov equation (collision-less Boltzmann equation) - f(x,p) is the phase space density distribution - are fully nonlinear.

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial}{\partial t} f(\mathbf{x},\mathbf{p},t) + \frac{\mathbf{p}}{ma^2}\frac{\partial}{\partial \mathbf{x}} f(\mathbf{x},\mathbf{p},t) - m\frac{\partial}{\partial \mathbf{x}}\Phi(\mathbf{x})\frac{\partial}{\partial \mathbf{p}} f(\mathbf{x},\mathbf{p},t) = 0$$

$$\Delta\Phi(\mathbf{x}) = \frac{4\pi G m}{a}\left(\int f(\mathbf{x},\mathbf{p},t)\mathrm{d}^3\mathbf{p} - \bar{n}\right)$$

This is what N-body codes aim at simulating...

The rules of the game:
single flow equations

Peebles 1980; Fry 1984
FB, Colombi, Gaztañaga, Scoccimarro, Phys. Rep. 2002

$$\begin{aligned}
\frac{\partial}{\partial t}\delta(\mathbf{x},t) + \frac{1}{a}[(1+\delta(\mathbf{x},t))\mathbf{u}_i(\mathbf{x},t)]_{,i} &= 0\\
\frac{\partial}{\partial t}\mathbf{u}_i(\mathbf{x},t) + \frac{\dot{a}}{a}\mathbf{u}_i(\mathbf{x},t) + \frac{1}{a}\mathbf{u}_j(\mathbf{x},t)\mathbf{u}_{i,j}(\mathbf{x},t) &= -\frac{1}{a}\Phi_{,i}(\mathbf{x},t) + \cancel{\ldots}\\
\Phi_{,ii}(\mathbf{x},t) - 4\pi G\overline{\rho}\; a^2\;\delta(\mathbf{x},t) &= 0
\end{aligned}$$

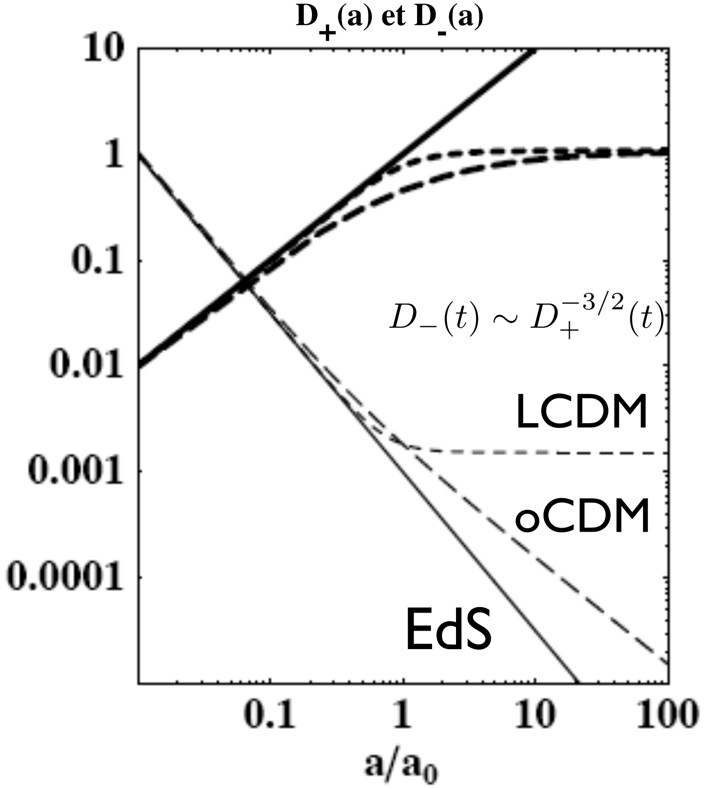
GR correction effects are usually small *Yoo et al., PRD, 2009...*

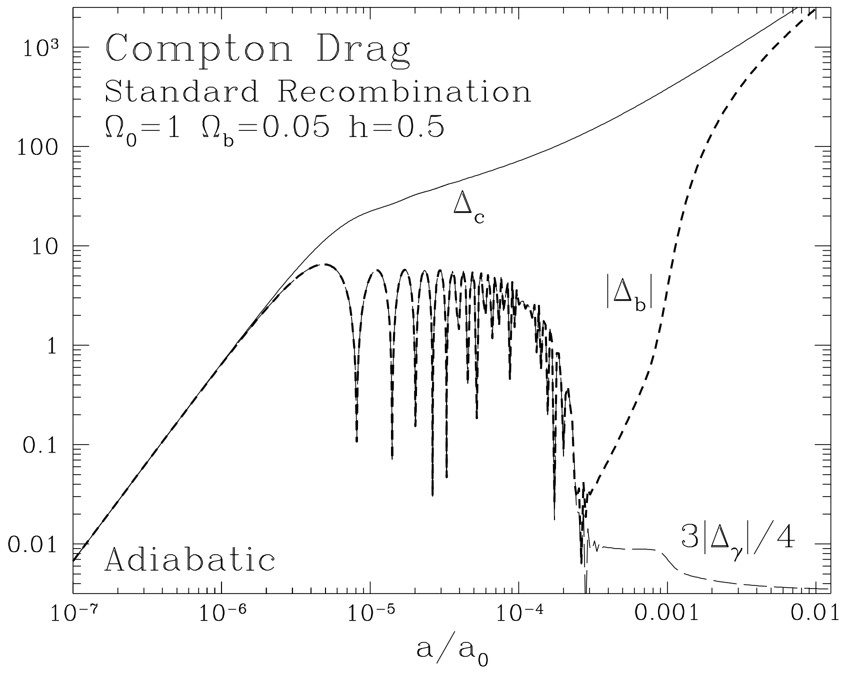
The linear regime

The solution (scalar modes)

$$\delta(\mathbf{x},t) = D_+(t)\delta_+(\mathbf{x}) + D_-(t)\delta_-(\mathbf{x})$$

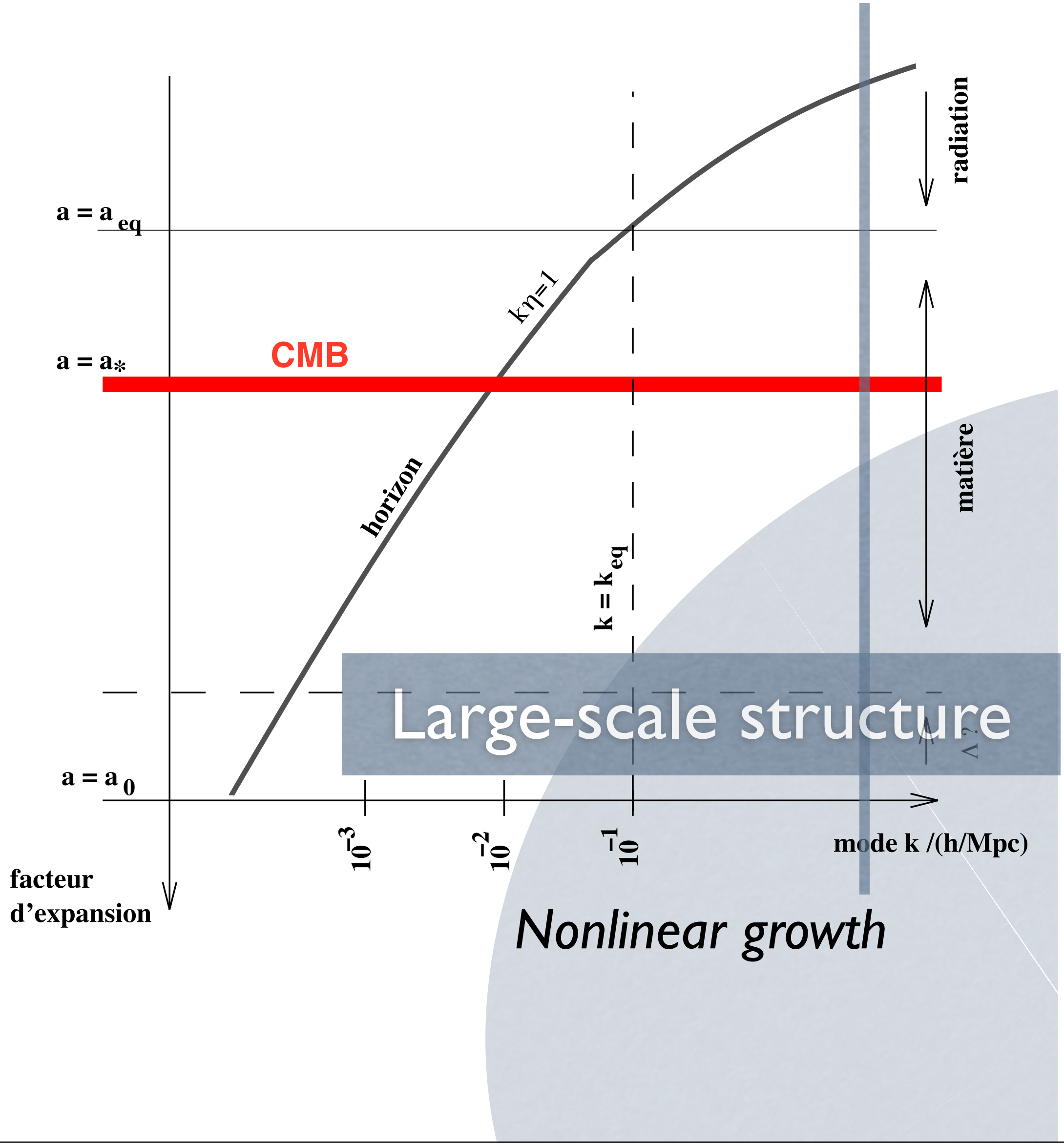
1 growing and 1 decaying mode





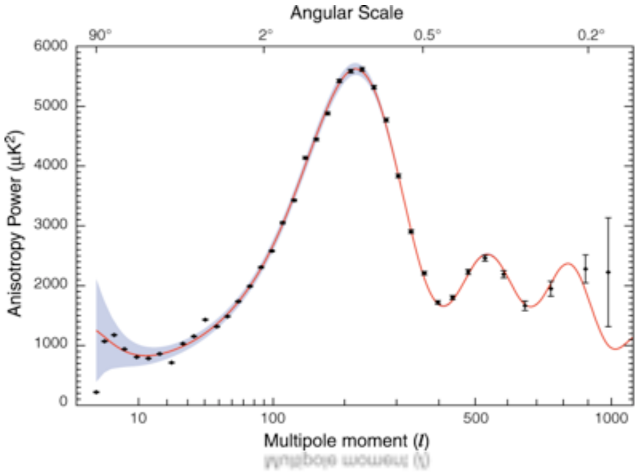
Connexion with the physics of the early universe (Hu, PhD thesis)

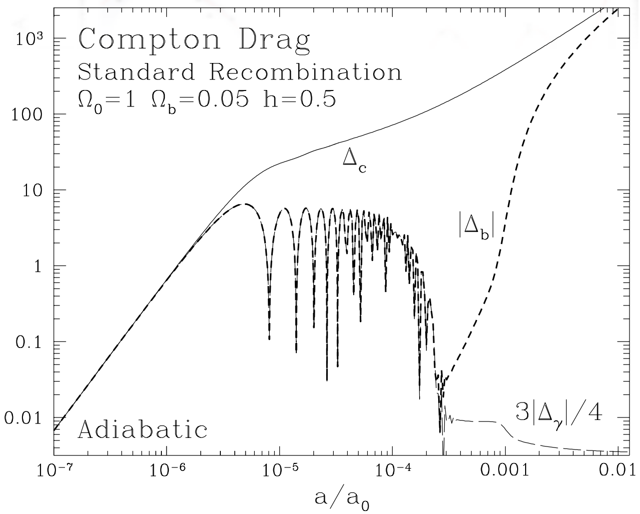
The development of cosmological instabilities across time and scale



Large-scale structure

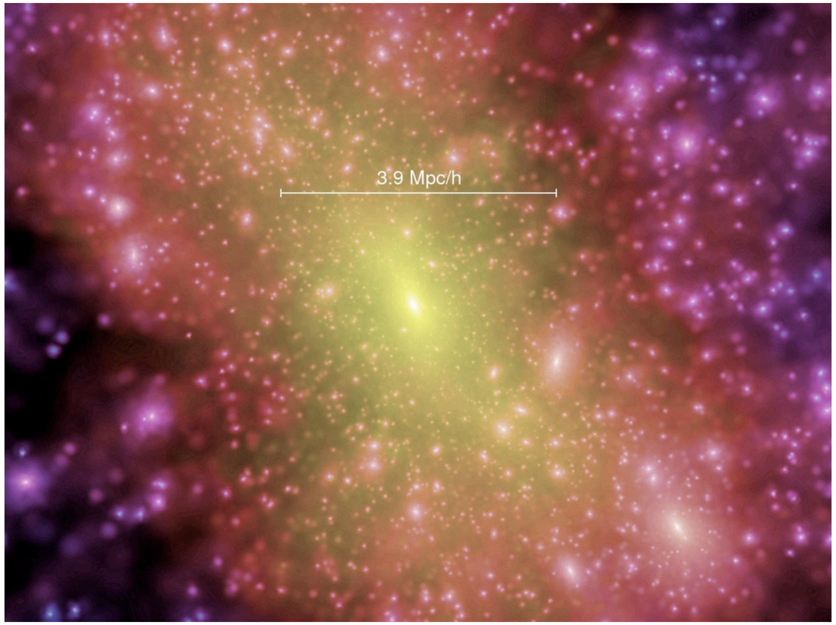
Nonlinear growth





Hu, Sugiyama '95, '96

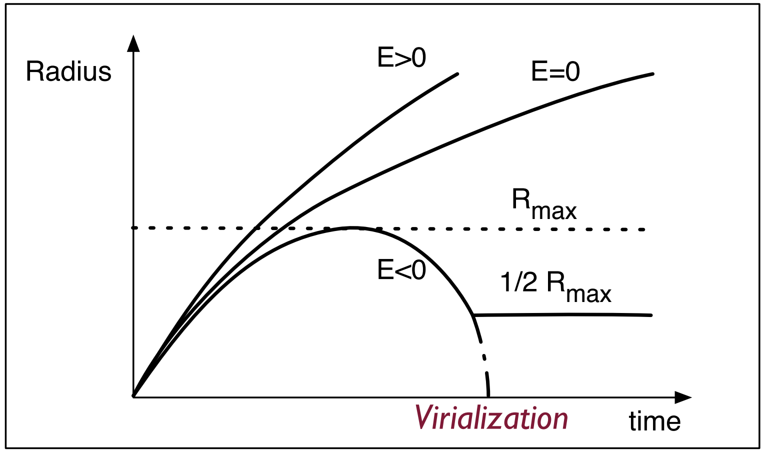
A glimpse into the nonlinear regime



Eventually objects form and their properties decouple from the global expansion

Hierarchical models are based on self-similar growth of correlation functions + stable clustering ansatz. They were popular in the eighties.

Davis, Peebles '77
Balian, Schaeffer '89
Hamilton et al. '95



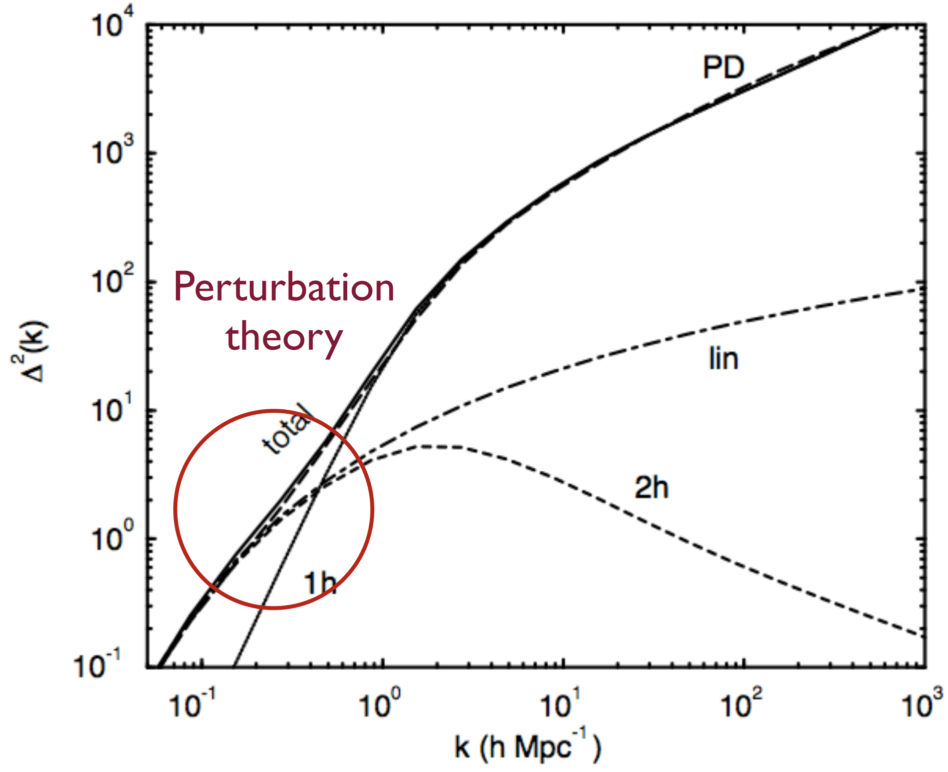
The collapse of a spherical object can be computed exactly.

The virialization processes are complex but should lead to the formation of objects roughly half the size of their maximal extension.

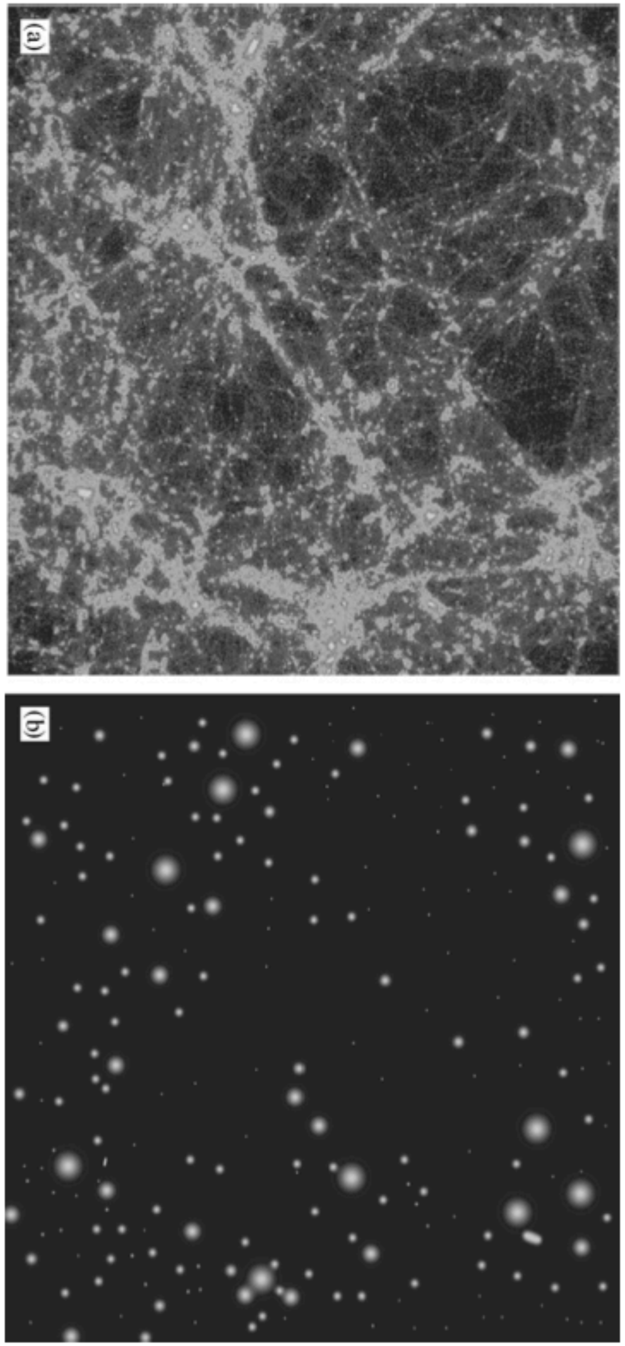
The halo model

The complex matter distribution is replaced by a set of halos characterized by their mass distribution and density profile.

Cooray, Sheth '02



Perturbation theory



Perturbation Theory

- To get insights into the development of gravitational instabilities;
- to test/complement N-body simulations;
- provide predictions from first principles in a large variety of models, and for a large numbers of parameters.

One more rule: it is possible to analytically expand the cosmic fields with respect to initial density fields

$$\delta(\mathbf{x},t)=\delta^{(1)}(\mathbf{x},t)+\delta^{(2)}(\mathbf{x},t)+\dots$$

Vlasov equation of a single flow pressure-less fluid

- *A reformulation of the theory with a FT like approach*

$$\Phi_a(\mathbf{k},\eta) = \left(\begin{array}{c} \delta(\mathbf{k},\eta) \\ \theta(\mathbf{k},\eta)/f_+(\eta) \end{array} \right)$$ *cosmological doublet*

Scoccimarro 1997

- *Dynamical equations (now in Fourier space)*

$$\frac{\partial}{\partial\eta}\boldsymbol{\Phi}_a(\mathbf{k},\eta) + \boldsymbol{\Omega}_a^b(\eta)\boldsymbol{\Phi}_b(\mathbf{k},\eta) = \boldsymbol{\gamma}_a^{bc}(\mathbf{k}_1,\mathbf{k}_2)\boldsymbol{\Phi}_b(\mathbf{k}_1)\boldsymbol{\Phi}_c(\mathbf{k}_2)$$

convolution is implicit

linear structure matrix

$$\Omega_a{}^b(\eta) = \left(\begin{array}{cc} 0 & -1 \\ -\frac{3}{2f^2}\Omega_{\rm m}(\eta) & \frac{3}{2f^2}\Omega_{\rm m}(\eta) - 1 \end{array} \right)$$

$$\gamma_{\rm a}{}^{\rm bc}(\mathbf{k}_1,\mathbf{k}_2) = \begin{cases} \frac{1}{2}\left\{1+\frac{\mathbf{k}_2\cdot\mathbf{k}_1}{|\mathbf{k}_2|^2}\right\} & ; \quad (a,b,c)=(1,1,2) \\ \frac{1}{2}\left\{1+\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{|\mathbf{k}_1|^2}\right\} & ; \quad (a,b,c)=(1,2,1) \\ \frac{(\mathbf{k}_1\cdot\mathbf{k}_2)|\mathbf{k}_1+\mathbf{k}_2|^2}{2|\mathbf{k}_1|^2|\mathbf{k}_2|^2} & ; \quad (a,b,c)=(2,2,2) \\ 0 & ; \quad \text{otherwise} \end{cases}$$

- *Linear solution*

$$\Phi_a(\mathbf{k},\eta) = g_a{}^b(\eta,\eta_0)\,\Phi_b(\mathbf{k},\eta_0)$$

doublet linear propagator $g_a{}^b(\eta,\eta_0) = \frac{e^{(\eta-\eta_0)}}{5}\left[\begin{array}{cc} 3 & 2 \\ 3 & 2 \end{array}\right] - \frac{-e^{3(\eta-\eta_0)/2}}{5}\left[\begin{array}{cc} -2 & 2 \\ 3 & -3 \end{array}\right]$

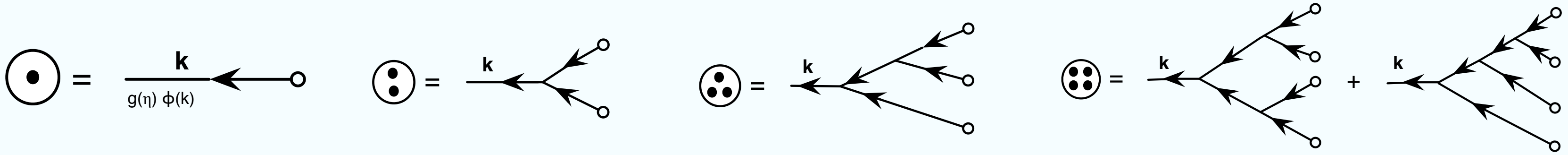
- Integral representation of the motion equations

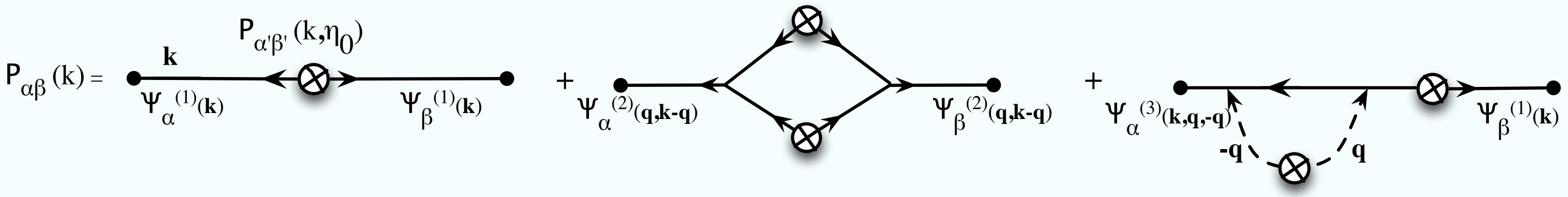
$$\Phi_a(\mathbf{k},\eta) = g_a^{\ b}(\eta)\Phi_b(\mathbf{k},\eta=0) + \int_0^{\eta} d\eta' g_a^{\ b}(\eta-\eta')\gamma_b^{\ cd}(\mathbf{k}_1,\mathbf{k}_2)\Phi_c(\mathbf{k}_1,\eta')\Phi_d(\mathbf{k}_2,\eta')$$

↓ linear evolution

↓ mode coupling terms

- Diagrammatic representation





Note : detailed effects of baryons versus DM can be taken into account (Somogyi & Smith 2010; FB, Van de Rijt, Vernizzi '12) with a 4-component multiplet, for neutrinos it is more complicated...

- Not a quantum field theory problem...

- The system is not invariant over time translation: it is actually an unstable (non-equilibrium) system, where perturbations grow with time (as ~ power-law). The late time behavior of this system is probably non trivial and there is no known solution to it.

- Loop corrections are not due to virtual particle productions but to mode couplings effects, modes being set in the initial conditions.

- Vertices have a non-trivial k-dependence but which is entirely due to the conservation equation and is independent of the energy content of the universe. Only 2 → 1 vertices exist (quadratic couplings). This is not the case generically for modified gravity models (like chameleon, DGP ...)

- Due to the shape of CDM spectrum, there are no UV divergences (nor IR). Loops, e.g. "Renormalizations", are all finite.

- More closely related to hydrodynamic turbulence

Methods of Field Theory

Beyond standard PT : "resumming", redefining the series expansions

Renormalization Perturbation Theory

Crocce & Scoccimarro '05, 06

Inspired by hydro turbulence resummation schemes, see L'vov & Procaccia '95

Time-flow (renormalization) equations

M. Pietroni '08

Anselmi & Pietroni '12

From the field evolution equation to the multi-spectra evolution equation

The closure theory

Taruya & Hiramatsu, ApJ 2008, 2009

Motion equations for correlators are derived using the Direct-Interaction (DI) approximation in which one separates the field expression in a DI part and a Non-DI part. At leading order in Non-DI >> DI, one gets a closed set of equations,

These equations can more rigorously be derived in a large N expansion.

Valageas P., A&A, 2007

The eikonal approximation

FB, Van de Rijt & Vernizzi 2012

Effective Theory approaches

Pietroni et al '12, Carrasco et al. '12

The Multi-Point Propagator expansion (Gamma expansion)

The diagram contributing to the power spectrum up to 2-loop order:

linear power spectrum

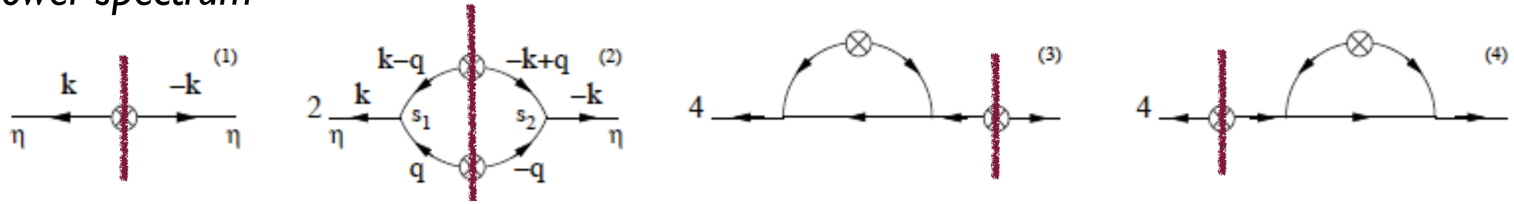


FIG. 5: Diagrams for the correlation function $P_{ab}(\mathbf{k}, \eta)$ up to two-loops (only 7 out of 29 two-loop diagrams are shown here). The dashed lines represent the points at which the two trees representing perturbative solutions to Ψ_a and Ψ_b have been glued together.

The key ingredients : the (multipoint) propagators

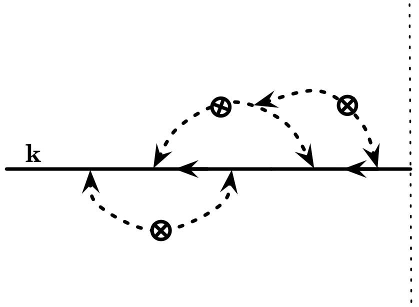
Scoccimarro and Crocce PRD, 2005

Final density / velocity div.

$$G_{ab}(k,\eta)\ \delta_{\rm D}(\mathbf{k}-\mathbf{k}') \equiv \left\langle \frac{\delta\Psi_a(\mathbf{k},\eta)}{\delta\phi_b(\mathbf{k}')} \right\rangle$$

Initial Conditions

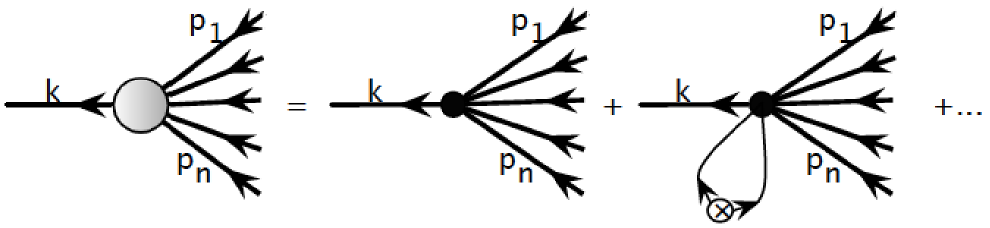
$G_{ab}(k) =$



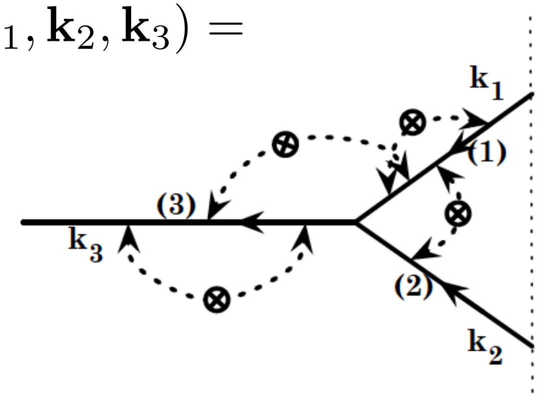
FB, Crocce, Scoccimarro, PRD, 2008

$$\Gamma^{(p)}_{ab_1\ldots b_p}(\mathbf{k}_1,\ldots,\mathbf{k}_p,\eta)\delta_{\rm D}(\mathbf{k}-\mathbf{k_{1\ldots p}}) = \frac{1}{p!}\left\langle \frac{\delta^p\Psi_a(\mathbf{k},\eta)}{\delta\phi_{b_1}(\mathbf{k}_1)\ldots\delta\phi_{b_p}(\mathbf{k}_p)} \right\rangle$$

$\Gamma^{(n)}(k,p_1,\ldots,p_n)=$



$\Gamma^{(2)}_{abc}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) =$



- This suggests another scheme: to use the n-point propagators as the building blocks

FB, Crocce, Scoccimarro, PRD, 2008

  - The reconstruction of the power spectrum :

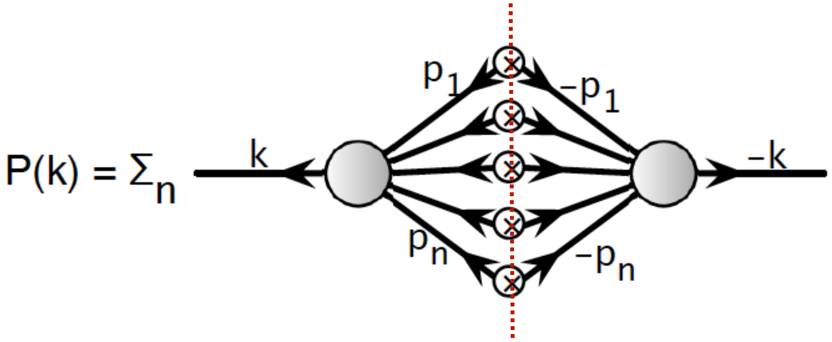
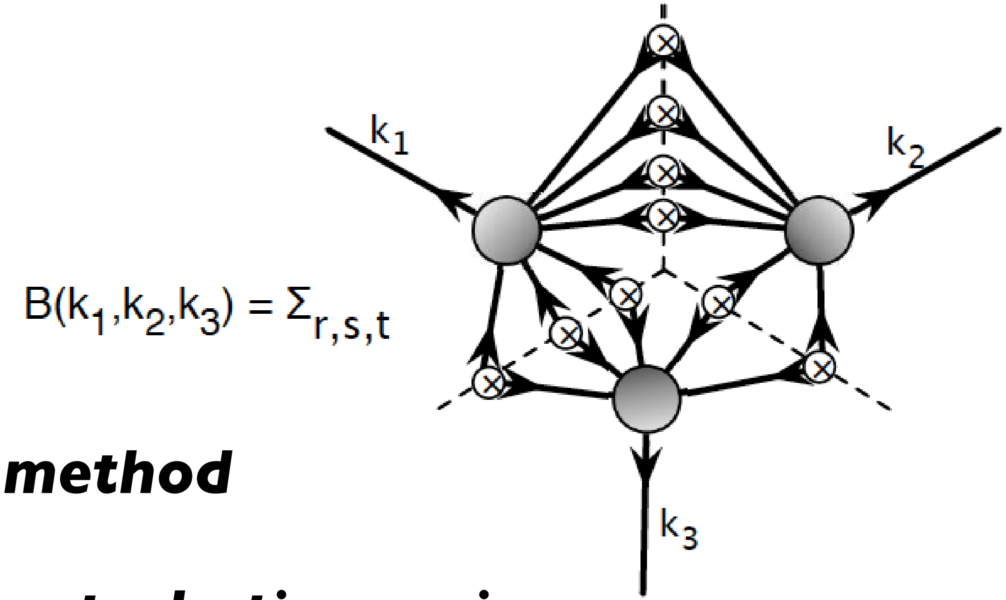


FIG. 3: Reconstruction of the power spectrum out of transfer functions. The crossed circles represent the initial spectrum. The sum runs over the number of internal co ing lines, e.g. the number of such circles. It is to be that each term of this sum is positive.

Sum of positive terms

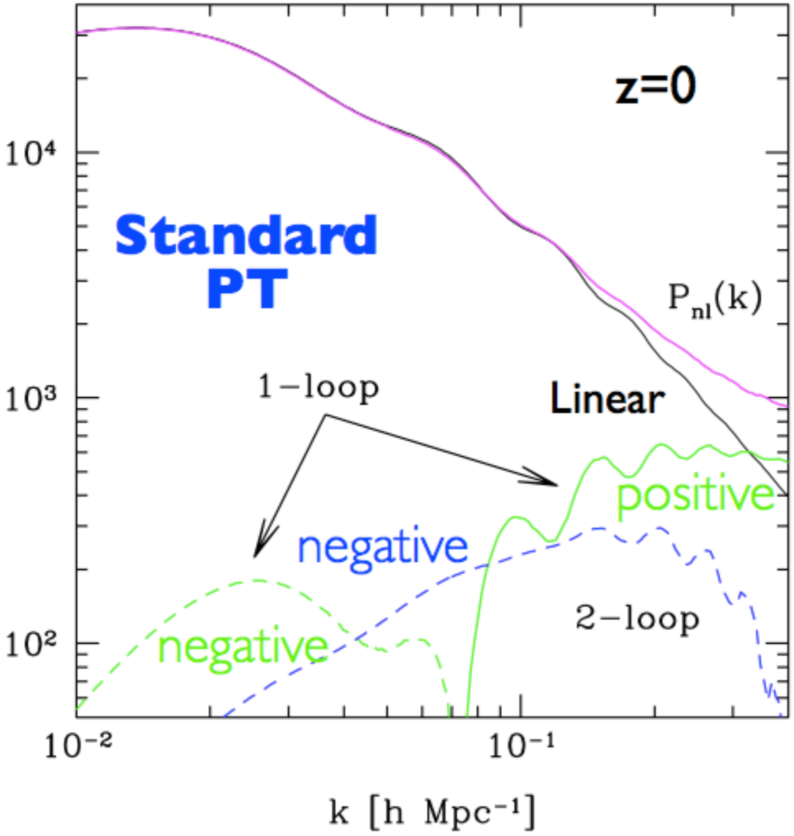
  - Also provide the building blocks for higher order moments...

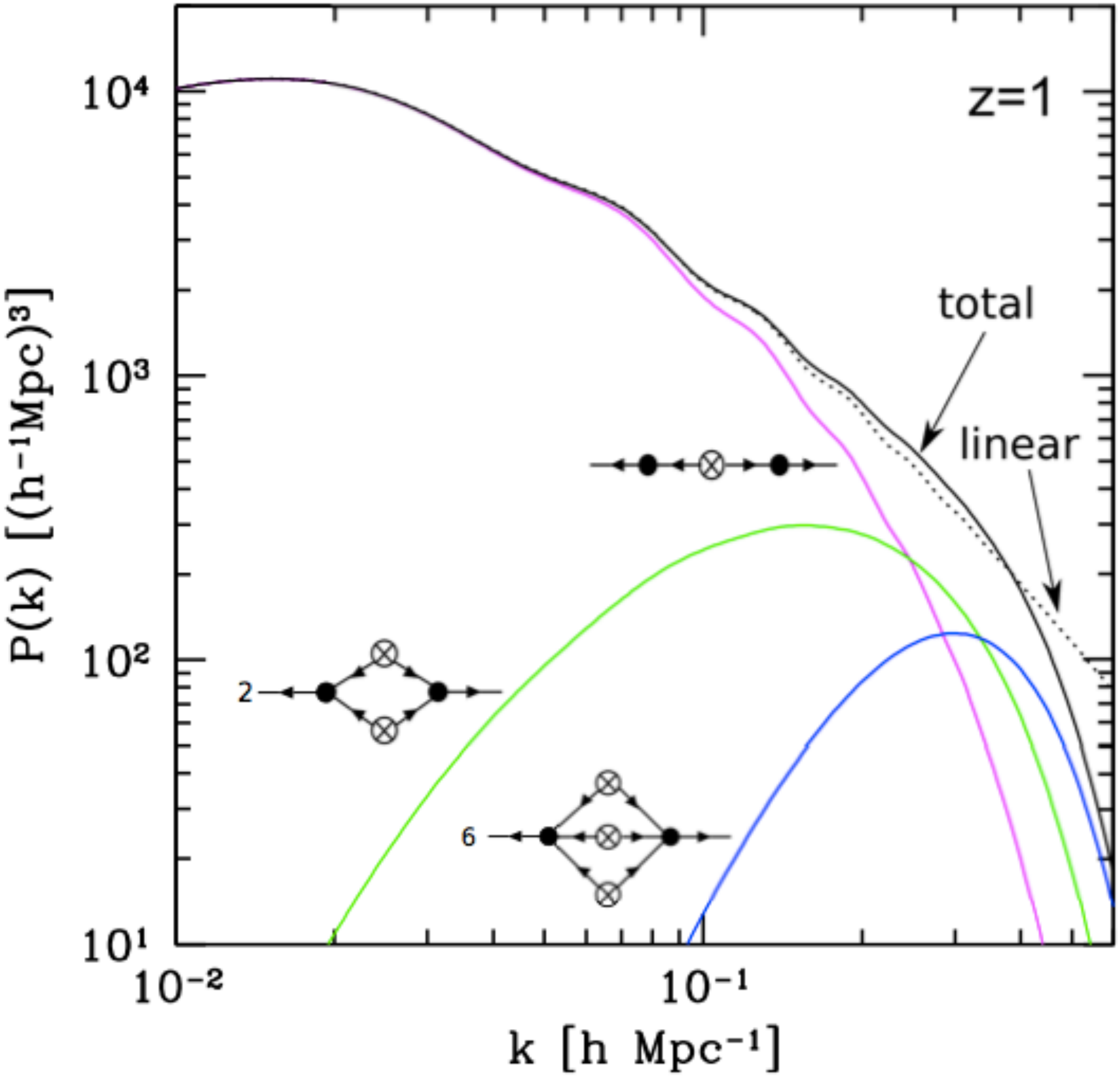


Γ-expansion method

- ***re-organisation(s) of the perturbation series***

Reconstruction of the power spectrum: from sPT to Multi-point propagator reconstruction





The "IR" domain with the eikonal approximation

FB, Van de Rijt, Vernizzi 2011
and 2012

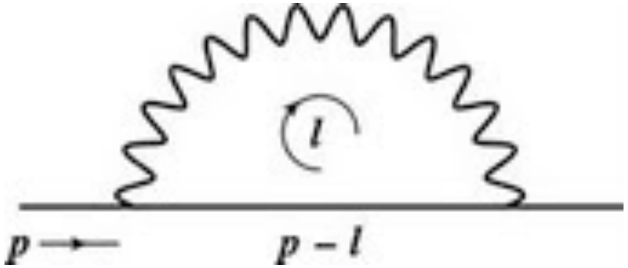
The eikonal approximation :

- In wave propagations: it leads to geometrical optics

  photon wavelength is much shorter than any other lengths

- In quantum field theory such as QED and QCD

$$p \gg l \quad \text{in}$$



"Relativistic eikonal expansion", Abarbanel and Itzykson, 1969

The IR modes in the eikonal approximation :

FB, Van de Rijt, Vernizzi 2011

dynamics :

$$\frac{\partial}{\partial\eta}\Phi_a(\mathbf{k},\eta)+\Omega_a^{\ b}(\eta)\Phi_b(\mathbf{k},\eta)=\boxed{\gamma_a^{bc}(\mathbf{k}_1,\mathbf{k}_2)}\Phi_b(\mathbf{k}_1)\Phi_c(\mathbf{k}_2)$$

Impact of the long-wave modes into the short wave modes (of interest)

Non trivial k dependence!

1. Split the interaction term into 2 parts:

- $k_1 \ll k_2 \quad \text{or} \quad k_2 \ll k_1 \quad \text{(soft domain)}$
- $k_1 \approx k_2 \quad \text{(hard domain)}$

2. Compute the first part using simplified form for the vertices

$$\frac{\partial}{\partial\eta}\Phi_a(\mathbf{k},\eta)+\Omega_a^{\ b}(\eta)\Phi_b(\mathbf{k},\eta)-\Xi_a^{\ b}(\mathbf{k},\eta)\Phi_b(\mathbf{k},\eta)=\gamma_a^{bc}(\mathbf{k}_1,\mathbf{k}_2)\Phi_b(\mathbf{k}_1)\Phi_c(\mathbf{k}_2)|_{\text{hard domain}}$$

$$\Xi_a^{\ b}(\mathbf{k},\eta)=\int \mathrm{d}^3\mathbf{q}\left(\gamma_a^{cb}(\mathbf{q},\mathbf{k})+\gamma_a^{bc}(\mathbf{k},\mathbf{q})\right)\Phi_c(\mathbf{q},\eta)|_{\text{soft domain}}$$

It leads to a "renormalized" theory that takes into account the long wave modes in a nonlinear manner.

3. Taking ensemble average over Ξ leads to the standard results assuming linear growing modes and Gaussian initial conditions.

The "renormalized" theory at linear order

$$\frac{\partial}{\partial\eta}\Phi_a(\mathbf{k},\eta)+\Omega_a^{\ b}(\eta)\Phi_b(\mathbf{k},\eta)-\Xi_a^{\ b}(\mathbf{k},\eta)\Phi_b(\mathbf{k},\eta)=0$$

$$\Xi_a^{\ b}(\mathbf{k},\eta)=\int \mathrm{d}^3\mathbf{q}\left({}^{\rm eik.}\gamma_a^{\ cb}(\mathbf{q},\mathbf{k})+{}^{\rm eik.}\gamma_a^{\ bc}(\mathbf{k},\mathbf{q})\right)\Phi_c(\mathbf{q},\eta)\vert_{\rm soft\ domain}$$

velocity field component only

What is in this new term ?

A **multi-component** fluid analysis with adiabatic modes and iso-curvature/density modes

$$\Xi_a^b(\mathbf{k},\eta)=\Xi^{(\rm ad)}(\mathbf{k},\eta)\delta_a^b+\Xi_a^{b\,(\nabla)}(\mathbf{k},\eta)$$

adiabatic term

$$=\int\frac{\mathbf{k}.\mathbf{q}}{q^2}\,\delta_d(\mathbf{q})\mathrm{d}^3\mathbf{q}$$

non-adiabatic term

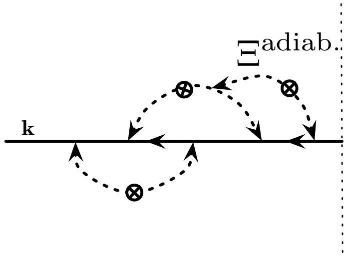
$$\Xi_a^{b\,(\nabla)}=\Xi^{(\nabla)}\,h_a^b\,,\quad h_a^b\equiv\begin{pmatrix} f_2 & 0 & 0 & 0\\ 0 & f_2 & 0 & 0\\ 0 & 0 & -f_1 & 0\\ 0 & 0 & 0 & -f_1\end{pmatrix}$$

Impact of the adiabatic modes :

$$\xi_a{}^b(\mathbf{k},\eta,\eta_0) = g_a{}^b(\eta,\eta_0)\exp\left[\mathrm{i}\mathbf{k}.\mathbf{d}^{\mathrm{adiab.}}(\eta')\right]$$ *(adiabatic) displacement field*

Consequences for propagators (building blocks for PT calculations)

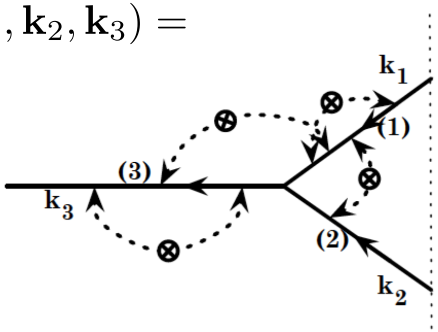
$$G_{ab}(k) =$$



$$= \left\langle \xi_a{}^b(\eta)\right\rangle_{\Xi} = g_a{}^b(\eta)\exp\left(-\frac{k^2\sigma_{\mathrm{d}}^2(\eta-\eta_0)^2}{2}\right)$$

Crocce & Scoccimarro '05, 06

$$\Gamma^{(2)}_{abc}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) =$$



$$= \Gamma^{\mathrm{tree}}_{abc}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)\exp\left(-\frac{k_3^2\sigma_{\mathrm{d}}^2(\eta-\eta_0)^2}{2}\right)$$

FB, Crocce & Scoccimarro '08

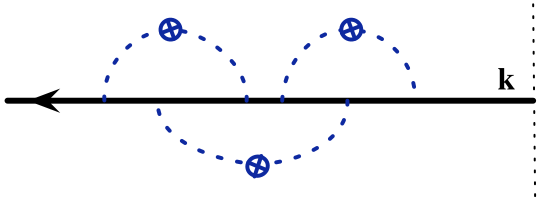
Consequences for equal-time poly-spectra : none

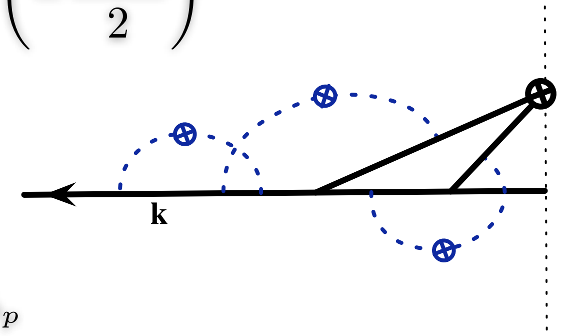
A regularization scheme = how to interpolate between n-loop results and the large-k behavior ?

An ad-hoc solution was provided by Crocce and Scoccimarro (RPT) for the one-point propagator but it cannot be generalized all cases.

- The proposed form is the following

$$
\begin{aligned}
{}^{\mathrm{Reg}}\Gamma^{(p)}{}_{a}^{\;b_1\ldots b_p} = \;& {}^{\mathrm{tree}}\Gamma_{a}^{\;b_1\ldots b_p}\;\exp\left(-\frac{k^2\sigma_d^2}{2}\right) \\
+ \;& \left[{}^{\mathrm{one-loop}}\Gamma_{a}^{\;b_1\ldots b_p} + \frac{1}{2}k^2\sigma_d^2\;{}^{\mathrm{tree}}\Gamma_{a}^{\;b_1\ldots b_p}\right]\exp\left(-\frac{k^2\sigma_d^2}{2}\right) \\
+ \;& \left[{}^{\mathrm{two-loop}}\Gamma_{a}^{\;b_1\ldots b_p} + \mathrm{c.t.}\right]\exp\left(-\frac{k^2\sigma_d^2}{2}\right)
\end{aligned}
$$



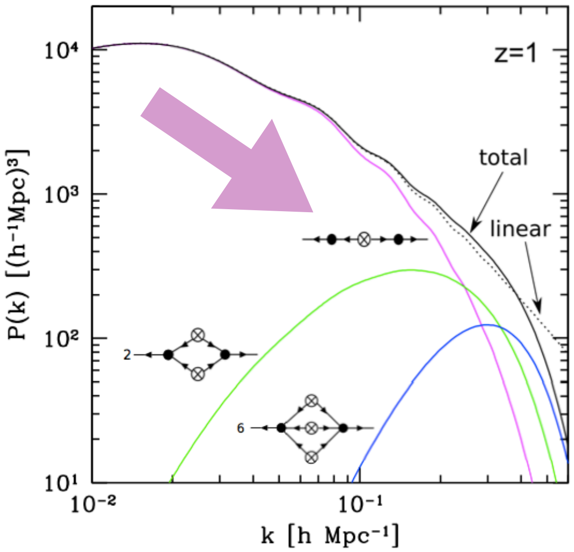


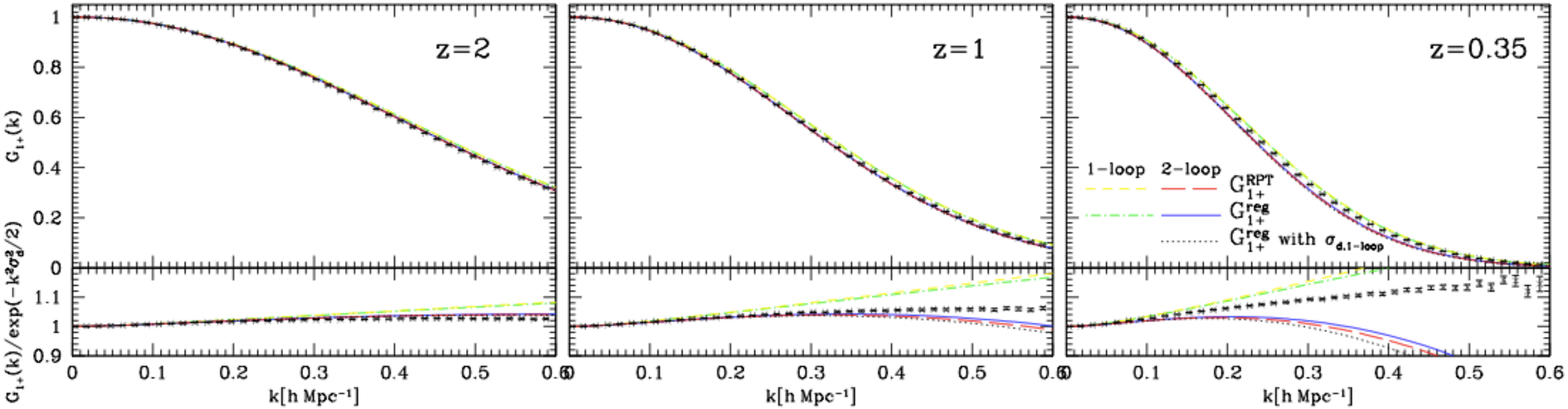
$$
\mathrm{c.t.} = \frac{1}{2}\left(\frac{k^2\sigma_d^2}{2}\right)^2\;{}^{\mathrm{tree}}\Gamma_{a}^{\;b_1\ldots b_p} + \frac{k^2\sigma_d^2}{2}\;{}^{\mathrm{one-loop}}\Gamma_{a}^{\;b_1\ldots b_p}
$$

- This is our proposition for regularized propagators: our best guess!

FB, Crocce, Scoccimarro '12

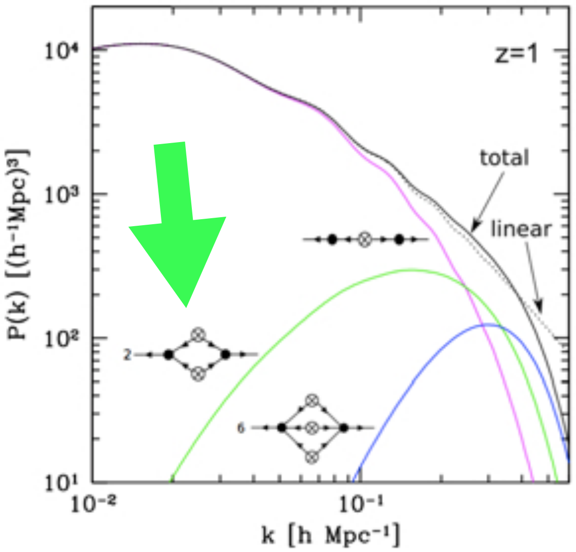
The two-point propagator at 1-loop and 2-loop orders

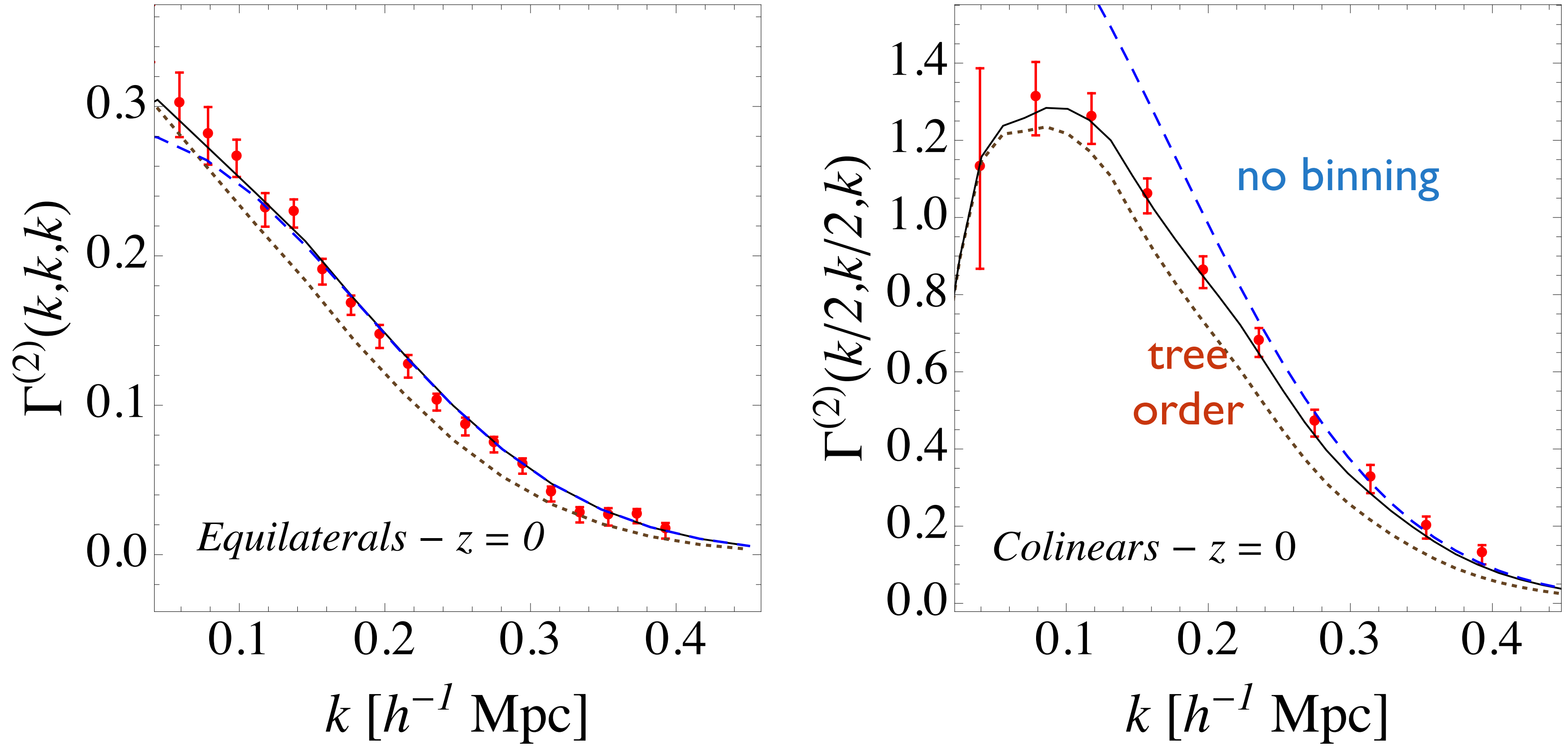




Comparison with numerical simulations at tree and one-loop order for the 3-point propagator

FB, Crocce, Scoccimarro '12





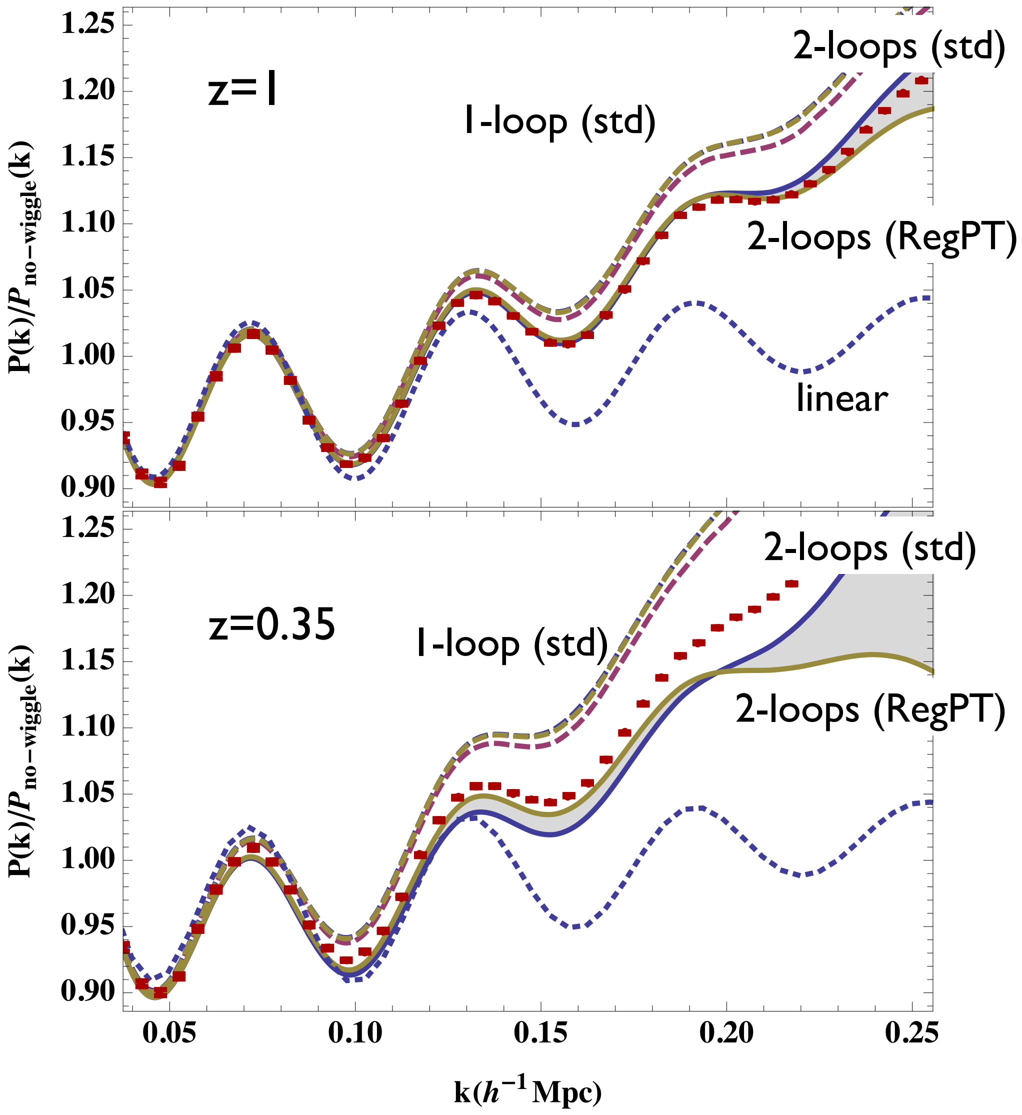
no binning

tree order

Power spectra up to 1-loop and 2-loop order

Taruya , FB, Nishimichi, Codis '12 Crocce, Scocimarro, FB, '12

1st computation of 2-loop order effects in Okamura, Taruya, Matsubara, '11



- Public codes for fast computations of power spectra at 2-loop order are now available.

  `http://maia.ice.cat/crocce/mptbreeze/`

  `http://www-utap.phys.s.u-tokyo.ac.jp/~ataruya/regpt_code.html`

- Theoretical predictions are within 1% accuracy.

Equal-time spectra in the eikonal approximation

Consequence 1: multi-spectra are independent on the large-scale adiabatic modes (in the eikonal limit)

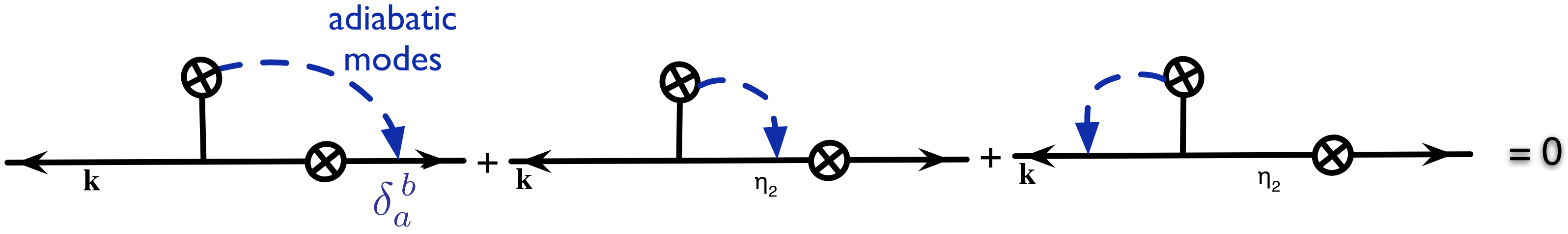
FB, Van de Rijt, Vernizzi, '12

This is a direct consequence of the functional dependance on the large-scale adiabatic displacement field.

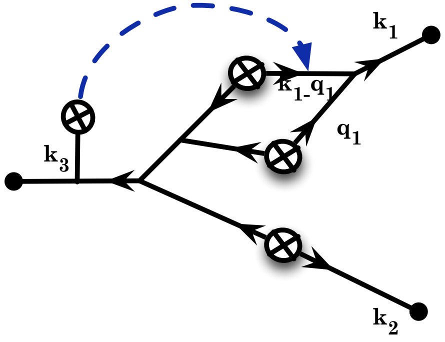
$$\psi_a(\mathbf{k},\eta;\Xi^{\rm adiab.}) = \xi_a{}^b(\mathbf{k},\eta,\eta_0;\Xi^{\rm adiab.})\psi_b(\eta_0)$$

$$\xi_a{}^b(\mathbf{k},\eta,\eta_0;\Xi^{\rm adiab.}) = g_a{}^b(\eta,\eta_0)\exp\left({\rm i}\int_{\eta_0}^{\eta}{\rm d}\eta'\ \mathbf{k}.\mathbf{v}^{\rm adiab.}(\eta')\right)$$

*Consequence 2: multi-spectra are independent on the large-scale adiabatic modes at any order in **standard** Perturbation Theory*



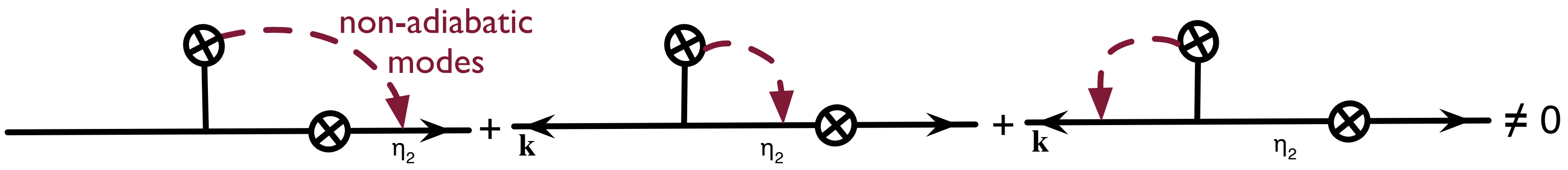
One-loop correction to power spectrum
or ... any poly-spectrum at any loop order



But not necessarily so for all PT schemes...

What is true for adiabatic modes is not true for non-adiabatic modes!

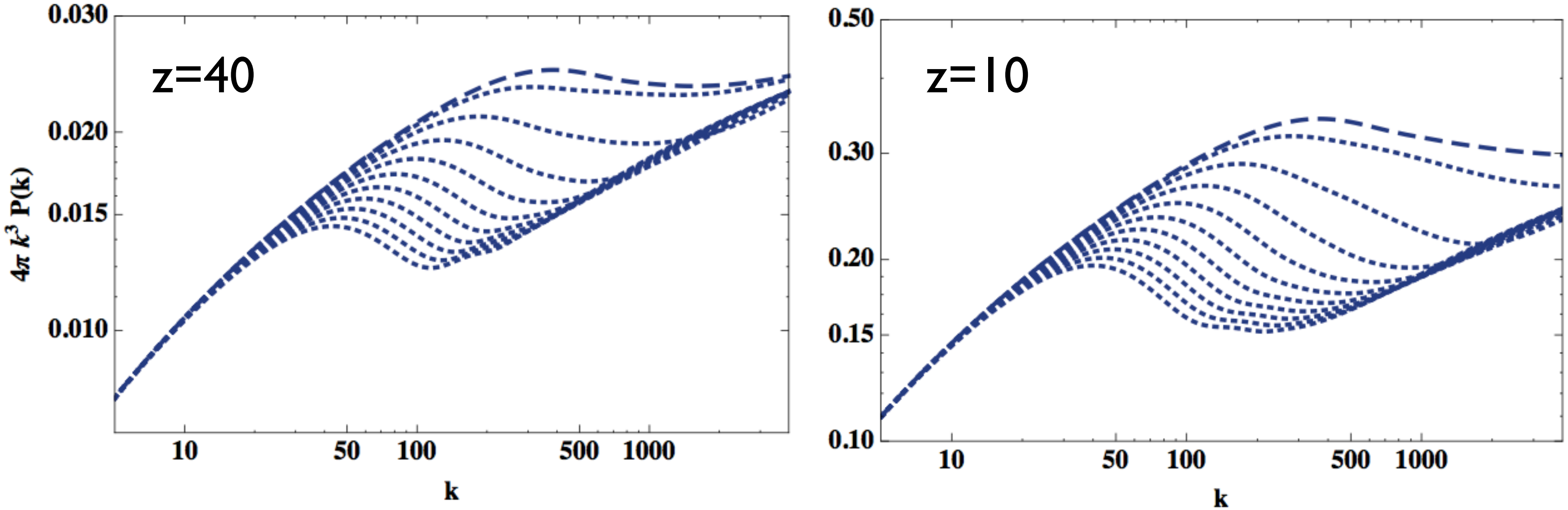
FB, Van de Rijt, Vernizzi, '12 in prep.



Resulting power spectrum in the eikonal limit (beyond one-loop results)

$$P_\delta(\mathbf{k};\Xi^{\rm iso.}) = {\xi_1}^a(\mathbf{k},\eta,\eta_0;\Xi^{\rm iso.})\,{\xi_1}^b(\mathbf{k},\eta,\eta_0;\Xi^{\rm iso.})\,P^{\rm init.}_{ab}(k,\eta_0)$$

modes mainly produced at horizon scale at decoupling



Formation of first structures is modulated and anisotropic

D. Tseliakhovich and C. Hirata, PRD, '10

Bad news for biasing: Galaxy formation is potentially modulated by large scale velocity modes (at 100-10 Mpc scales).

Dalal, Pen, Seljak '10

Yoo, Dalal, Seljak '11

In general however non-adiabatic modes have very little (totally negligible ?) impact on modes of interest here.

Somogyi & Smith 2010

FB, Van de Rijt, Vernizzi 2011

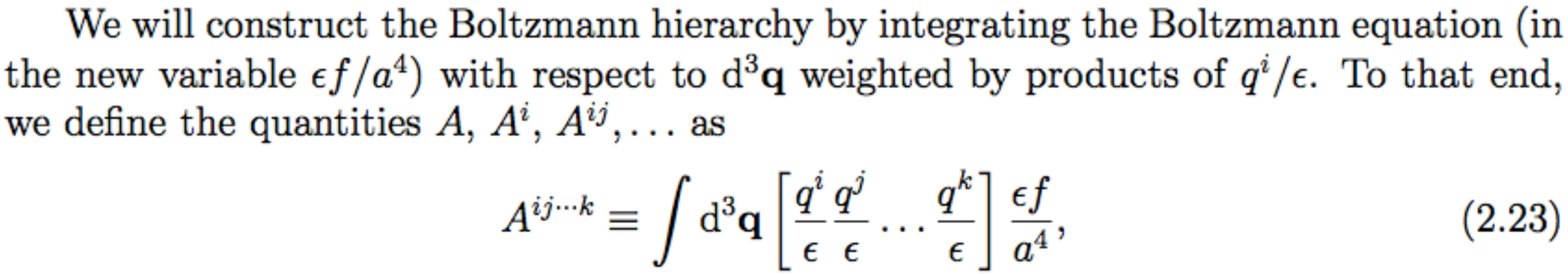
Enriching the content of the universe is likely to induce similar effects beyond linear theory results. This is potentially the case for massive neutrinos (whose velocities differ from the velocity of the cold dark matter component). The full non-linear hierarchy of equations in case of massive neutrinos is now known. We have started to investigate the impact of non-adiabatic modes .

PhD thesis of Nicolas van de Rijt '12

Enlarging the fluid content of the universe

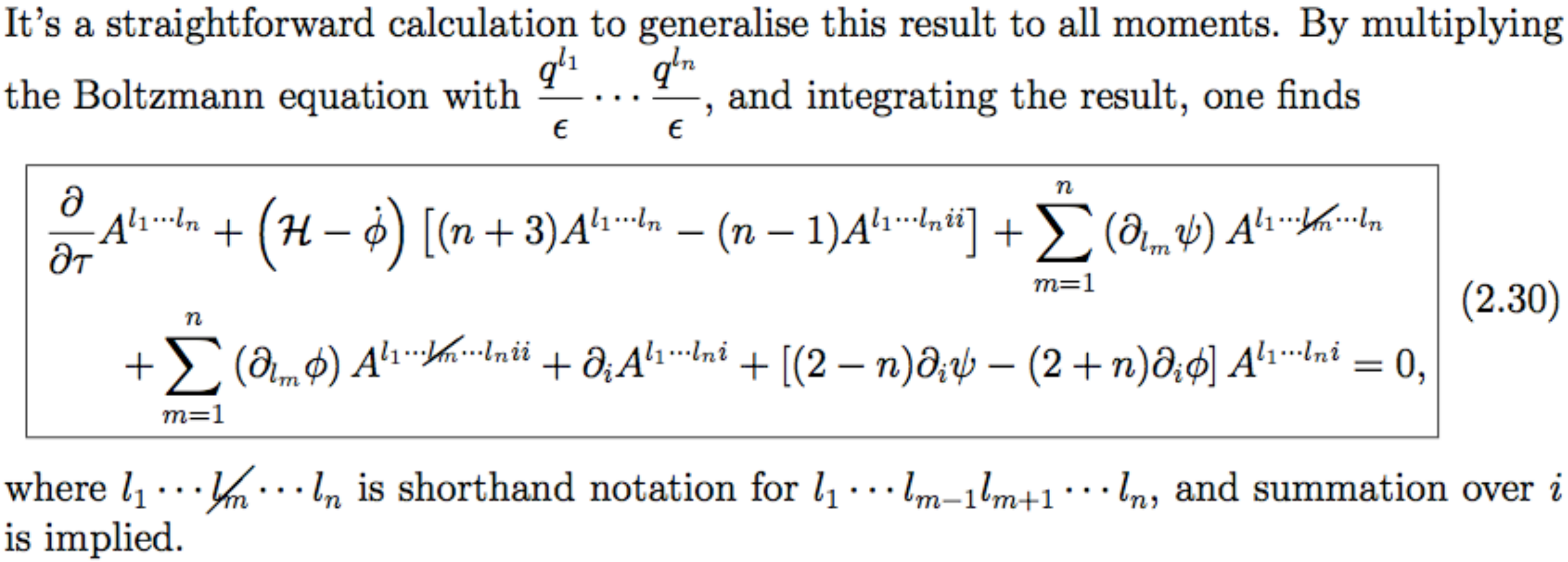
With relativistic species - relativistic neutrinos

PhD thesis of Nicolas van de Rijt '12



We will construct the Boltzmann hierarchy by integrating the Boltzmann equation (in the new variable $\epsilon f/a^4$) with respect to $\mathrm{d}^3\mathbf{q}$ weighted by products of q^i/ϵ. To that end, we define the quantities A, A^i, $A^{ij}, \ldots$ as

$$A^{ij\cdots k} \equiv \int \mathrm{d}^3\mathbf{q} \left[\frac{q^i}{\epsilon}\frac{q^j}{\epsilon}\cdots\frac{q^k}{\epsilon}\right]\frac{\epsilon f}{a^4}, \tag{2.23}$$



It's a straightforward calculation to generalise this result to all moments. By multiplying the Boltzmann equation with $\frac{q^{l_1}}{\epsilon}\cdots\frac{q^{l_n}}{\epsilon}$, and integrating the result, one finds

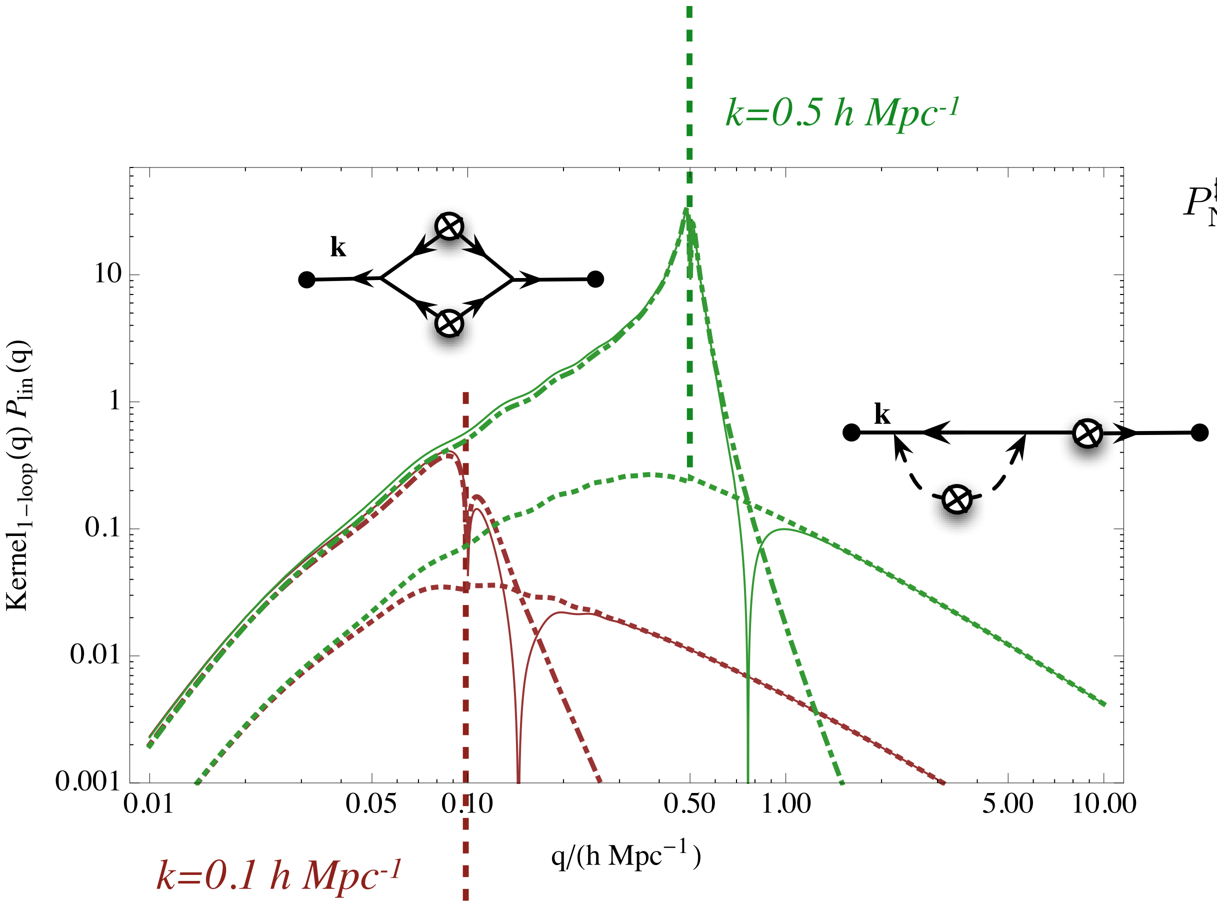
$$\boxed{\begin{aligned}&\frac{\partial}{\partial\tau}A^{l_1\cdots l_n} + \left(\mathcal{H}-\dot{\phi}\right)\left[(n+3)A^{l_1\cdots l_n} - (n-1)A^{l_1\cdots l_n ii}\right] + \sum_{m=1}^{n}\left(\partial_{l_m}\psi\right)A^{l_1\cdots\cancel{l_m}\cdots l_n}\\ &\quad+\sum_{m=1}^{n}\left(\partial_{l_m}\phi\right)A^{l_1\cdots\cancel{l_m}\cdots l_n ii} + \partial_i A^{l_1\cdots l_n i} + \left[(2-n)\partial_i\psi - (2+n)\partial_i\phi\right]A^{l_1\cdots l_n i} = 0,\end{aligned}} \tag{2.30}$$

where $l_1\cdots\cancel{l_m}\cdots l_n$ is shorthand notation for $l_1\cdots l_{m-1}l_{m+1}\cdots l_n$, and summation over i is implied.

The "UV" domain and the Galilean invariance

Kernels in Perturbation Theory calculations

FB, Taruya, Nishimichi, '12



$$P_{\rm NL}^{\sharp-loop}(k) = \int \frac{{\rm d}q}{q}\ K^{\sharp-loop}(k,q)\ P_{\rm lin.}(q)$$

Expression of the density kernel for the propagator at 1-loop order

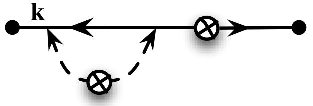
$$f(k,q) \;=\; \frac{3\left(2k^2+7q^2\right)\left(k^2-q^2\right)^3 \log\left[\frac{(k-q)^2}{(k+q)^2}\right] + 4\left(6k^7q - 79k^5q^3 + 50k^3q^5 - 21kq^7\right)}{2016k^3q^5}$$

Kernels for the 2-point propagators at *p*-loop order

$$P_{\mathrm{NL}}^{\sharp-loop}(k) = \int \frac{\mathrm{d}q}{q}\; K^{\sharp-loop}(k,q)\; P_{\mathrm{lin.}}(q)$$

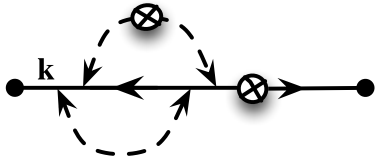
Convergence properties

1-loop



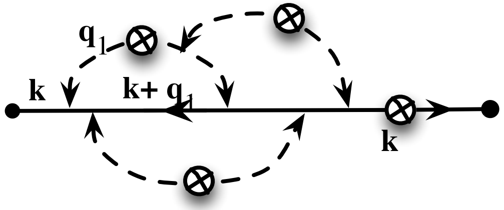
$n_s < -1$

2-loop

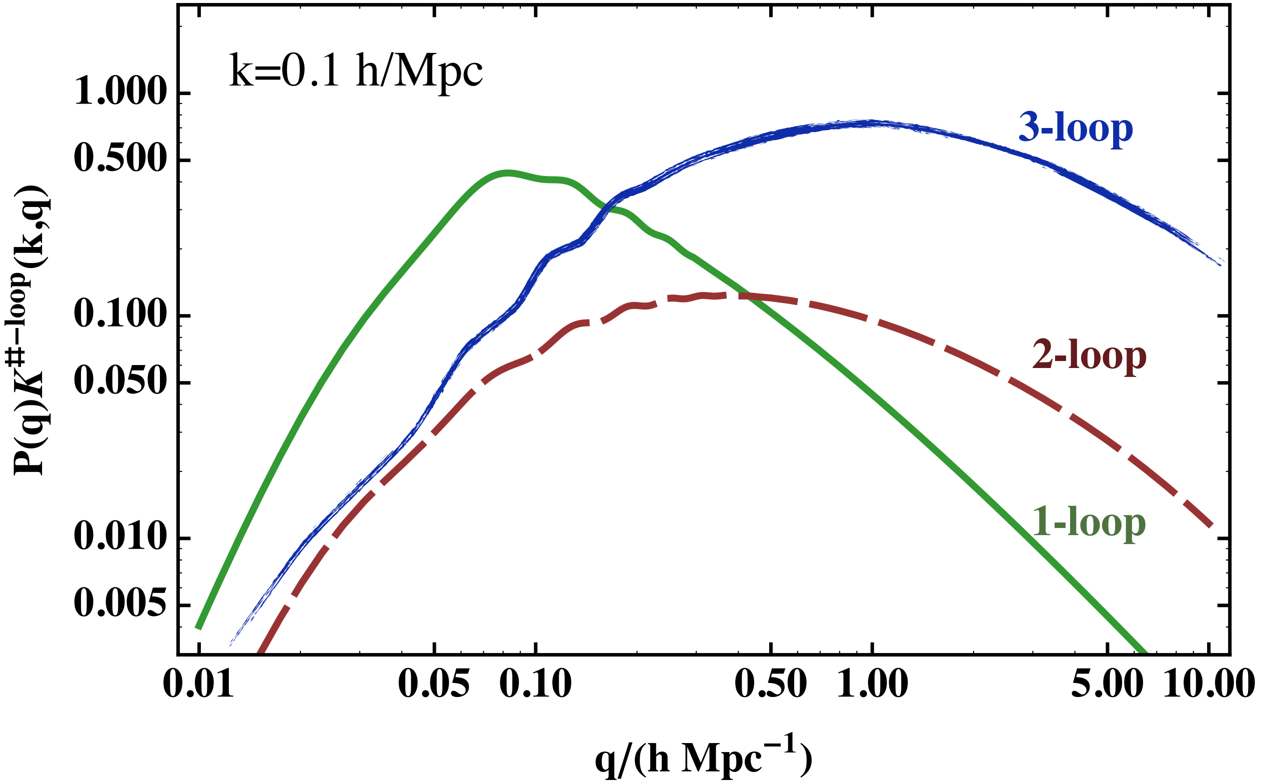


$n_s < -2$

3-loop



$n_s < -2.33$



Should it be regularized or taken into account with Effective Theory approaches?

Pietroni et al. '11, Carrasco et al. '12

- UV shape of kernels is key to the validity of PT calculations and comparison with numerical simulations
- It comes from the IR behavior of coupling functions

$$\frac{\partial}{\partial\eta}\Phi_a(\mathbf{k},\eta)+\Omega_{ab}(\eta)\Phi_b(\mathbf{k},\eta)=\gamma_{abc}(\mathbf{k}_1,\mathbf{k}_2)\Phi_b(\mathbf{k}_1)\Phi_c(\mathbf{k}_2)$$

$$\gamma_{abc}(k_1,k_2)=\begin{pmatrix} \left\{0,\frac{(k_1+k_2).k_2}{2k_2.k_2}\right\} & \left\{\frac{(k_1+k_2).k_1}{2k_1.k_1},0\right\} \\ \{0,0\} & \left\{0,\frac{k_1.k_2(k_1+k_2).(k_1+k_2)}{2k_1.k_1k_2.k_2}\right\} \end{pmatrix}$$

▶ $$\gamma_{abc}(\mathbf{q},\mathbf{k}-\mathbf{q})\ \sim\ k^2/q^2$$

and power counting

$$\left[\frac{1}{q^2}\right]\left[q^3\ P_{\text{linear}}(q)\right]^{\sharp\ \text{loops}}$$

- UV regularization seems necessary (starting at 2-loop order and for z < 0.5): it is not cleat if it can be obtained from re-summations of contributing diagrams or from extra physical effects (in particular shell crossings, etc...)

- *Modified gravity models alter the coupling structure and therefore might change the converging properties of theory. This is suggested by preliminary results obtained in some classes of modified gravity models (with a dynamical dilaton field with Damour-Polyakov mechanism for instance).*

- Something to learn from these results for the backreaction problem, that is the impact of the small scale structure on the large ones.

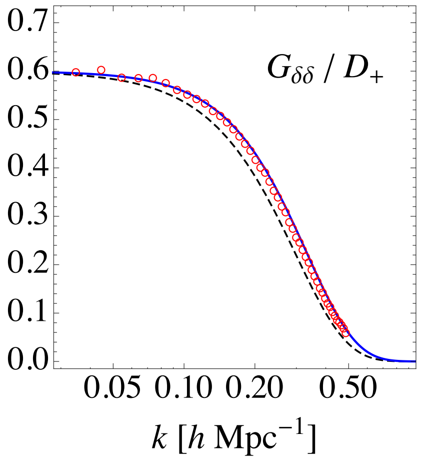
Conclusions

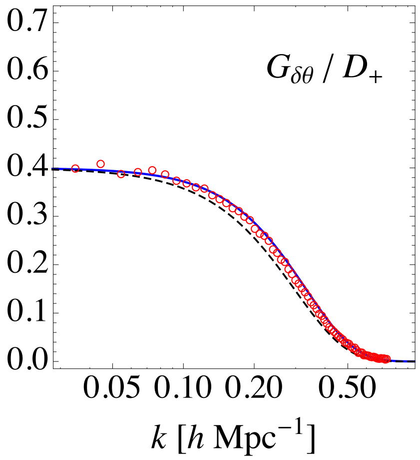
Two-loop calculations can now be done routinely (and very rapidly)

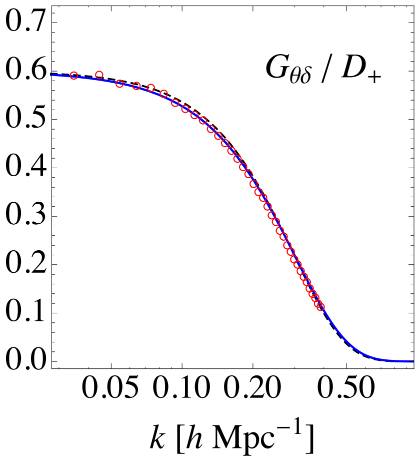
- Public codes for fast computations of power spectra at 2-loop order are now available. Codes take a few seconds to compute power spectra.

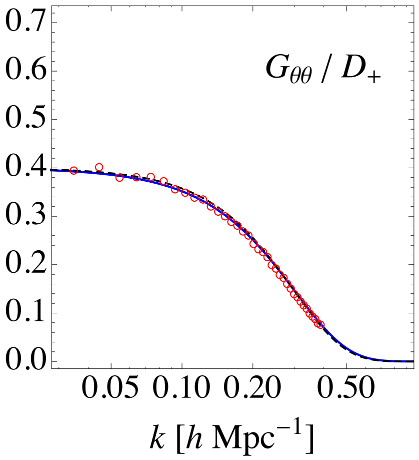
  `http://maia.ice.cat/crocce/mptbreeze/`
  `http://www-utap.phys.s.u-tokyo.ac.jp/~ataruya/regpt_code.html`

- So far performances are focused on mild values of k for the density field. Theoretical predictions are within 1% accuracy.
- Extensions to velocity components are under construction with the same methods.









Conclusions

Large-Scale Structure studies offer new opportunities for precision cosmology calculations;

An interesting playground for field theory calculations

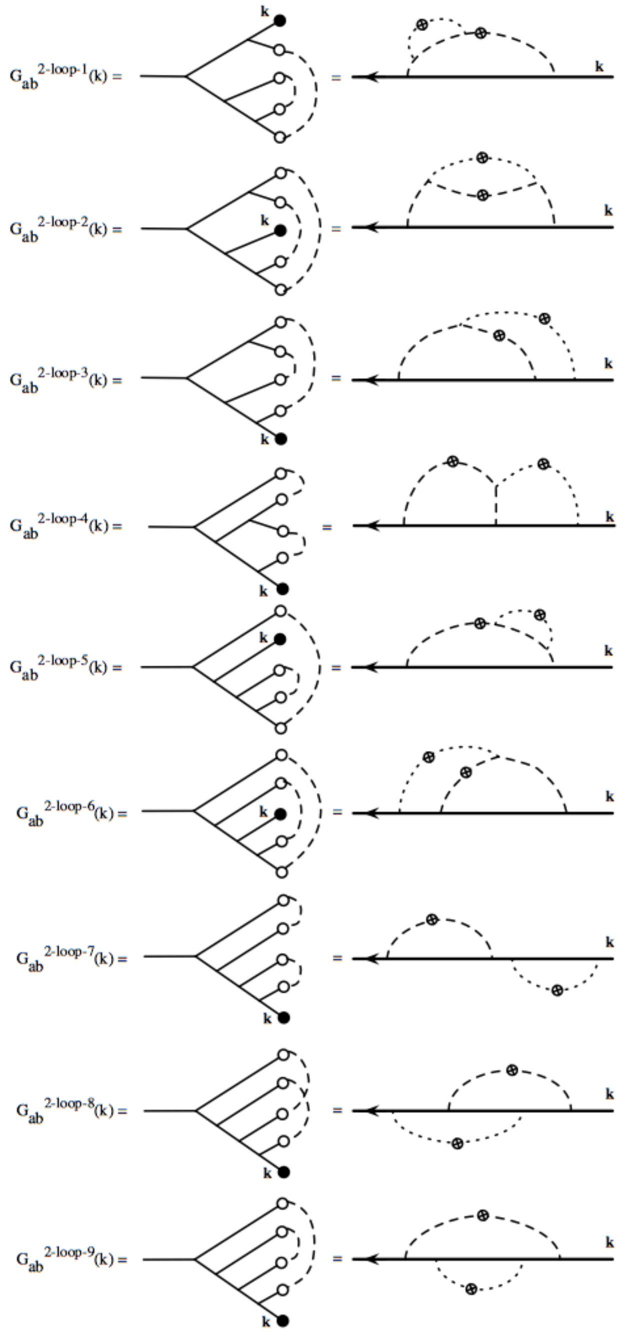


FIG. 4: Diagrams contributing to the two-loop expression of the propagators.