Effective Field Theory of
Multi-Field Inflation a la Weinberg

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map:

- keywords: Effective Field Theory, Inflation

single field	C. Cheung et al. JHEP, arXiv:0709.0293 The EFT of Inflation	S. Weinberg PRD, arXiv:0804.4291 EFT for Inflation
multi-field	L. Senatore and M. Zaldarriaga JHEP, arXiv:1009.2093 The EFT of Multifield Inflation	JCAP, arXiv:1203.2266 EFT of Multi-Field Inflation a la Weinberg

Effective Field Theory

- an _effective_ theory:

    - is true for a certain domain of energy.

- two cases:

    - as a part of a true theory for whole energy scales

        - using EFT to simplify calculations!

    - in lack of a complete theory for the energy scales of interests

        - using EFT since there is no other choice!

        - constructing EFT, by considering e.g. symmetry properties of the model.

EFT for Multi-Field Inflation

- the <u>most general form</u> of Lagrangian up to the 4th order derivatives:

  after simplifications:

$$\Big\{ b_1^{IJ}(\vec{\varphi})\Box\varphi_I\Box\varphi_J + b_2^{IJK}(\vec{\varphi})\nabla_\mu\varphi_I\nabla^\mu\varphi_J\Box\varphi_K + b_3^{IJKL}(\vec{\varphi})\nabla_\mu\varphi_I\nabla^\mu\varphi_J\nabla_\nu\varphi_K\nabla^\nu\varphi_L + b_4^{IJ}(\vec{\varphi})\nabla_\mu\varphi_I\nabla^\mu\varphi_J \quad (A1)$$

$$+ \; b_5(\vec{\varphi}) + b_6^{IJ}(\vec{\varphi})(\nabla^\mu\varphi_I)(\Box\nabla_\mu\varphi_J) + b_7^I(\vec{\varphi})(\nabla_\mu\nabla_\nu\varphi_I)^2 + b_8^{IJK}(\vec{\varphi})(\nabla_\mu\varphi_I)(\nabla_\nu\varphi_J)(\nabla^\mu\nabla^\nu\varphi_K) + b_9^I(\vec{\varphi})\nabla^\mu\Box\nabla_\mu\varphi_I$$

$$+ \; b_{10}^I(\vec{\varphi})\Box^2\varphi_I + b_{11}^I(\vec{\varphi})(\nabla^\mu)\nabla_\mu\Box\varphi_I + c_1^{IJ}(\vec{\varphi})R\nabla_\mu\varphi_I\nabla^\mu\varphi_J + c_2^{IJ}(\vec{\varphi})R^{\mu\nu}\nabla_\mu\varphi_I\nabla_\nu\varphi_J + c_3^I(\vec{\varphi})R\Box\varphi_I$$

$$+ \; c_4^I(\vec{\varphi})(\nabla^\mu R)(\nabla_\mu\varphi_I) + c_5(\vec{\varphi})\Box R + c_6^I(\vec{\varphi})R_{\mu\nu}\nabla^\mu\nabla^\nu\varphi_I + a_1(\vec{\varphi})R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + a_2(\vec{\varphi})R_{\mu\nu}R^{\mu\nu} + a_3(\vec{\varphi})R^2 + a_4(\vec{\varphi})R \Big\}$$

before simplifications!

perturbations:

perturbations in single field model (Weinberg's paper)

$$\mathcal{L} = \sqrt{g}\left[-\frac{M^2}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - M_P^2 U(\varphi) + f(\varphi)\left(g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu}\right)^2 \right]$$

$$= \bar{\mathcal{L}} - \frac{1}{2}a^3\left(M^2 + 4f(\bar\varphi)\dot{\bar\varphi}^2\right) \times \left(-\dot{\delta\varphi}^2 + a^{-2}(\vec\nabla\delta\varphi)^2\right)$$

$$+ 4a^3 f(\bar\varphi)\dot{\bar\varphi}^2\left(\dot{\delta\varphi}^2 + \dot{\delta\varphi}^3/\dot{\bar\varphi} - a^{-2}\dot{\delta\varphi}(\vec\nabla\delta\varphi)^2/\dot{\bar\varphi} + \frac{1}{4}\dot{\delta\varphi}^4/\dot{\bar\varphi}^2 - \frac{1}{2}a^{-2}\dot{\delta\varphi}^2(\vec\nabla\delta\varphi)^2/\dot{\bar\varphi}^2 + \frac{1}{4}a^{-4}(\vec\nabla\delta\varphi)^4/\dot{\bar\varphi}^2\right)$$

-- it is up to 4th order of perturbations automatically.

-- speed of sound ≠ 1

-- large non-Gaussianity?

-- speed of sound is constrained by validity of EFT!

perturbations:

- multi-field case:

$$a^{-3}\Delta\mathcal{L}^{(2)} = b_3^{IJKL}(\bar\varphi)\Big\{ \dot{\bar\varphi}_I\dot{\bar\varphi}_J\delta\dot\varphi_K\delta\dot\varphi_L + \dot{\bar\varphi}_I\dot{\bar\varphi}_K\delta\dot\varphi_J\delta\dot\varphi_L + \dot{\bar\varphi}_I\dot{\bar\varphi}_L\delta\dot\varphi_K\delta\dot\varphi_J + \dot{\bar\varphi}_K\dot{\bar\varphi}_J\delta\dot\varphi_I\delta\dot\varphi_L + \dot{\bar\varphi}_L\dot{\bar\varphi}_J\delta\dot\varphi_I\delta\dot\varphi_L + \dot{\bar\varphi}_K\dot{\bar\varphi}_L\delta\dot\varphi_I\delta\dot\varphi_J$$
$$- a^{-2}\dot{\bar\varphi}_I\dot{\bar\varphi}_J\partial_i\delta\varphi_K\partial^i\delta\varphi_L - a^{-2}\dot{\bar\varphi}_K\dot{\bar\varphi}_L\partial_i\delta\varphi_I\partial^i\delta\varphi_J\Big\} - \frac{M^2}{2}\delta^{IJ}\Big[-\delta\varphi_I\delta\varphi_J + a^{-2}\partial_i\delta\varphi_I\partial^i\delta\varphi_J\Big],$$

$$a^{-3}\Delta\mathcal{L}^{(3)} = b_3^{IJKL}(\bar\varphi)\Big\{ \dot{\bar\varphi}_I\delta\dot\varphi_J\delta\dot\varphi_K\delta\dot\varphi_L + \dot{\bar\varphi}_J\delta\dot\varphi_I\delta\dot\varphi_K\delta\dot\varphi_L + \dot{\bar\varphi}_K\delta\dot\varphi_L\delta\dot\varphi_I\delta\dot\varphi_J + \dot{\bar\varphi}_L\delta\dot\varphi_K\delta\dot\varphi_I\delta\dot\varphi_J$$
$$- a^{-2}\Big(\dot{\bar\varphi}_I\delta\dot\varphi_J\partial_i\delta\varphi_K\partial^i\delta\varphi_L + \dot{\bar\varphi}_J\delta\dot\varphi_I\partial_i\delta\varphi_K\partial^i\delta\varphi_L + \dot{\bar\varphi}_K\delta\dot\varphi_L\partial_i\delta\varphi_I\partial^i\delta\varphi_J + \dot{\bar\varphi}_L\delta\dot\varphi_K\partial_i\delta\varphi_I\partial^i\delta\varphi_J\Big)\Big\},$$

$$a^{-3}\Delta\mathcal{L}^{(4)} = b_3^{IJKL}(\bar\varphi)\Big\{ \delta\dot\varphi_I\delta\dot\varphi_J\delta\dot\varphi_K\delta\dot\varphi_L - a^{-2}\Big(\delta\dot\varphi_I\delta\dot\varphi_J\partial_i\delta\varphi_K\partial^i\delta\varphi_L + \delta\dot\varphi_K\delta\dot\varphi_L\partial_i\delta\varphi_I\partial^i\delta\varphi_J\Big)$$
$$+ a^{-4}\partial_i\delta\varphi_I\partial^i\delta\varphi_J\partial_j\delta\varphi_K\partial^j\delta\varphi_L\Big\}.$$

perturbations!

perturbations:

- two-field case:

$$a^{-3}\Delta\mathcal{L}^{(2)} = \dot{\delta\varphi}^2\left[\frac{M_1^2}{2} + 6g_1\dot{\bar{\varphi}}^2 + 3g_3\dot{\bar{\varphi}}\dot{\bar{\chi}} + (g_5 + g_6)\dot{\bar{\chi}}^2\right] + \dot{\delta\chi}^2\left[\frac{M_2^2}{2} + 6g_2\dot{\bar{\chi}}^2 + 3g_4\dot{\bar{\varphi}}\dot{\bar{\chi}} + (g_5 + g_6)\dot{\bar{\varphi}}^2\right]$$

$$+ \dot{\delta\varphi}\dot{\delta\chi}\left[3g_3\dot{\bar{\varphi}}^2 + 3g_4\dot{\bar{\chi}}^2 + 4(g_5 + g_6)\dot{\bar{\varphi}}\dot{\bar{\chi}}\right]$$

$$- a^{-2}\left(\partial_i\delta\varphi\partial^i\delta\varphi\left[\frac{M_1}{2} + 2g_1\dot{\bar{\varphi}}^2 + g_3\dot{\bar{\varphi}}\dot{\bar{\chi}} + g_5\dot{\bar{\chi}}^2\right] + \partial_i\delta\chi\partial^i\delta\chi\left[\frac{M_2}{2} + 2g_2\dot{\bar{\chi}}^2 + g_4\dot{\bar{\varphi}}\dot{\bar{\chi}} + g_5\dot{\bar{\varphi}}^2\right]\right.$$

$$\left. + \partial_i\delta\varphi\partial^i\delta\chi\left[g_3\dot{\bar{\varphi}}^2 + g_4\dot{\bar{\chi}}^2 + 2g_6\dot{\bar{\varphi}}\dot{\bar{\chi}}\right]\right),$$

second order perturbations!

…. and cubic and quartic terms!

. . . transition to adiabatic and entropy curvature perturbations!

perturbations: adiabatic & entropy modes

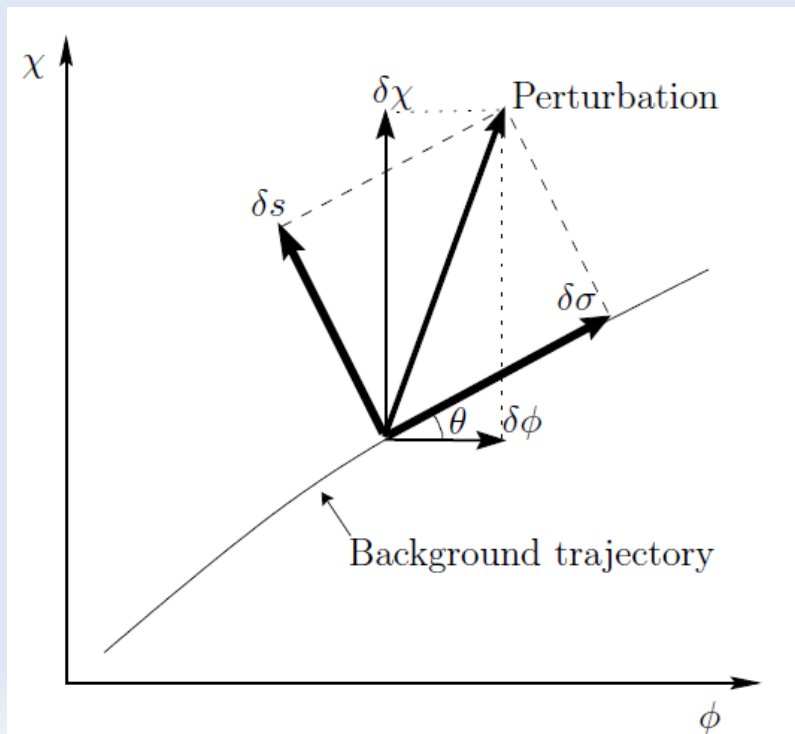adiabatic mode: entropy mode:

$$\delta\sigma \equiv \vec{T}.\vec{\delta}, \qquad\qquad\qquad \delta s \equiv \vec{N}.\vec{\delta}$$

$$\vec{\delta} \equiv (\delta\varphi, \delta\chi), \qquad \vec{T} = (\cos\theta, \sin\theta) \equiv (\dot{\varphi}/\dot{\sigma}, \dot{\chi}/\dot{\sigma}), \qquad \vec{N} \equiv (\sin\theta, -\cos\theta)$$

Gordon et al. arXiv:astro-ph/0009131

perturbations: adiabatic & entropy modes

$$4\dot{\sigma}(\vec{T}.\dot{\vec{\delta}})^3 \times \left[g_1 \cos^4\theta + g_2 \sin^4\theta + g_3 \cos^3\theta \sin\theta + g_4 \cos\theta \sin^3\theta + (g_5 + g_6)\cos^2\theta \sin^2\theta \right]$$

$$+ \ \dot{\sigma}(\vec{N}.\dot{\vec{\delta}})^3 \times$$

$$\left[+ g_4 \cos^4\theta + g_3 \sin^4\theta - 2\Big(2g_2 - (g_5 + g_6)\Big)\cos^3\theta \sin\theta + 2\Big(2g_1 - (g_5 + g_6)\Big)\cos\theta \sin^3\theta + 3(g_4 - g_3)\cos^2\theta \sin^2\theta \right]$$

$$+ \ 3\dot{\sigma}(\vec{T}.\dot{\vec{\delta}})^2(\vec{N}.\dot{\vec{\delta}}) \times$$

$$\left[+ g_3 \cos^4\theta + g_4 \sin^4\theta + 2\Big(2g_1 - (g_5 + g_6)\Big)\cos^3\theta \sin\theta - 2\Big(2g_2 - (g_5 + g_6)\Big)\cos\theta \sin^3\theta + 3(g_3 - g_4)\cos^2\theta \sin^2\theta \right]$$

$$+ \ 2\dot{\sigma}(\vec{T}.\dot{\vec{\delta}})(\vec{N}.\dot{\vec{\delta}})^2 \times$$

$$\left[(g_5 + g_6)\Big(\cos^4\theta + \sin^4\theta\Big) + 3(g_4 - g_3)\Big(\cos^3\theta \sin\theta - \cos\theta \sin^3\theta\Big) + \Big(3(g_1 + g_2) - 2(g_5 + g_6)\Big)\cos^2\theta \sin^2\theta \right]$$

$$(\dot{\delta\sigma} - \dot{\theta}\delta s)$$

$$(\dot{\delta s} + \dot{\theta}\delta\sigma)$$

note that just these two combinations appear in this formalism!

shape of non-Gaussianity

□ due to previous slide: for example:

$$(\vec{T}.\dot{\vec{\delta}})^3 = \delta\dot{\sigma}^3 - 3\dot{\theta}\delta\dot{\sigma}^2\delta s + 3\dot{\theta}^2\delta\dot{\sigma}\delta s^2 - \dot{\theta}^3\delta s^3$$

equilateral NG

in adiabatic mode

local NG

in entropy mode

--- in this formalism

the "Cosine" between different kinds of NG is fixed!

amplitude of NG

$$\frac{\mathcal{L}^{(3)}}{\mathcal{L}^{(2)}} = \frac{\{H^3, H^2\dot{\theta}, H\dot{\theta}^2, \dot{\theta}^3\}}{\{H^2, H\dot{\theta}, \dot{\theta}^2\}} \times \left(\frac{f(g_i)}{M^2}\dot{\sigma}^2\right) \times \frac{\delta\sigma}{\dot{\sigma}} = \frac{\{H^3, H^2\dot{\theta}, H\dot{\theta}^2, \dot{\theta}^3\}}{H \times \{H^2, H\dot{\theta}, \dot{\theta}^2\}} \times \left(\frac{f(g_i)}{M^2}\dot{\sigma}^2\right) \times \zeta$$

$$\dot{\theta} \gg H$$

$$f_{NL} \sim \frac{\mathcal{L}^{(3)}}{\mathcal{L}^{(2)}}\zeta^{-1} \sim \frac{\dot{\theta}}{H} \times \left(\frac{f(g_i)}{M^2}\dot{\sigma}^2\right)$$

-- validity condition of EFT i.e. $\dfrac{f(g_i)}{M^2}\dot{\sigma}^2 < 1$ constrains the amplitude of NG!
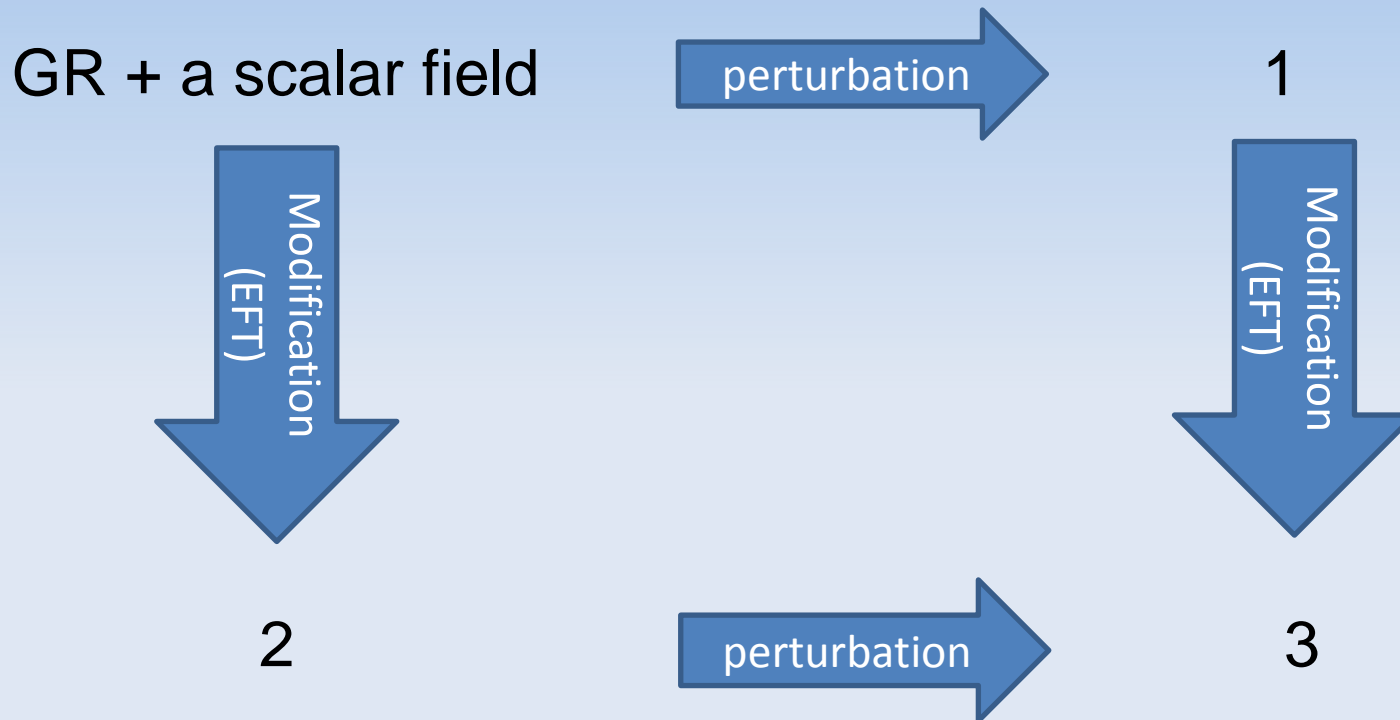
-- except if the curvature of classical (background) path be large!

-- or: if by a mechanism (e.g. Vainshtein) one can modify the validity condition of EFT!

compare with Senatore & Zaldarriaga

-- Senatore & Zaldarriaga model is based on Cheung et al.'s work!

-- in Cheung's work, EFT is constructed on perturbations' level!

    -- since their model is single field, the perturbation is associated to adiabatic mode!

-- so in Senatore & Z., the entropy modes are added into a base with already known adiabatic mode!

but

-- in our case we started with zeroth order term of perturbations.

    -- then defined the perturbations without any distinguishability between adiabatic and entropy modes!

compare with Senatore & Zaldarriaga

GR + a scalar field $\quad\longrightarrow$ perturbation $\quad\longrightarrow\quad$ 1

Modification (EFT) $\quad\downarrow\qquad\qquad\qquad\qquad\qquad$ Modification (EFT) $\quad\downarrow$

2 $\qquad\longrightarrow$ perturbation $\quad\longrightarrow\quad$ 3

Cheung et al. and Senatore & Z.: $\qquad 0 \longrightarrow 1 \longrightarrow 3$

Weinberg and this talk: $\qquad\qquad\quad 0 \longrightarrow 2 \longrightarrow 3$

compare with Senatore & Zaldarriaga

- so in Senatore & Zaldarriaga, the shift symmetry results in a Lagrangian similar to

$$
\begin{aligned}
a^{-3}\Delta\mathcal{L}^{(2)} =\ & \dot{\delta\varphi}^2\left[\frac{M_1^2}{2} + 6g_1\dot{\bar\varphi}^2 + 3g_3\dot{\bar\varphi}\dot{\bar\chi} + (g_5 + g_6)\dot{\bar\chi}^2\right] + \dot{\delta\chi}^2\left[\frac{M_2^2}{2} + 6g_2\dot{\bar\chi}^2 + 3g_4\dot{\bar\varphi}\dot{\bar\chi} + (g_5 + g_6)\dot{\bar\varphi}^2\right] \\
& + \dot{\delta\varphi}\dot{\delta\chi}\left[3g_3\dot{\bar\varphi}^2 + 3g_4\dot{\bar\chi}^2 + 4(g_5 + g_6)\dot{\bar\varphi}\dot{\bar\chi}\right] \\
& - a^{-2}\bigg(\partial_i\delta\varphi\partial^i\delta\varphi\left[\frac{M_1}{2} + 2g_1\dot{\bar\varphi}^2 + g_3\dot{\bar\varphi}\dot{\bar\chi} + g_5\dot{\bar\chi}^2\right] + \partial_i\delta\chi\partial^i\delta\chi\left[\frac{M_2}{2} + 2g_2\dot{\bar\chi}^2 + g_4\dot{\bar\varphi}\dot{\bar\chi} + g_5\dot{\bar\varphi}^2\right] \\
& + \partial_i\delta\varphi\partial^i\delta\chi\left[g_3\dot{\bar\varphi}^2 + g_4\dot{\bar\chi}^2 + 2g_6\dot{\bar\varphi}\dot{\bar\chi}\right]\bigg),
\end{aligned}
$$

$$\delta\varphi \to \delta\varphi + c_1 \text{ and } \delta\chi \to \delta\chi + c_2$$

i.e. there are just _derivatives_ of adiabatic and entropy perturbations!

but in our model the case is different!

compare with Senatore & Zaldarriaga

shift symmetry: $\delta\varphi \rightarrow \delta\varphi + c_1 \ \text{and} \ \delta\chi \rightarrow \delta\chi + c_2$

due to

$$\delta\sigma \equiv \vec{T}.\vec{\delta}, \qquad\qquad \delta s \equiv \vec{N}.\vec{\delta}$$

$$\vec{\delta} \equiv (\delta\varphi, \delta\chi), \qquad \vec{T} = (\cos\theta, \sin\theta) \equiv (\dot\varphi/\dot\sigma, \dot\chi/\dot\sigma), \qquad \vec{N} \equiv (\sin\theta, -\cos\theta)$$

results in

$$\delta\sigma \rightarrow \delta\sigma + (c_1 \cos\theta + c_2 \sin\theta)$$
$$\delta s \rightarrow \delta s + (c_1 \sin\theta - c_2 \cos\theta)$$

which causes a new symmetry for adiabatic and entropy modes:

$$\dot{\delta\sigma} - \dot\theta\delta s \rightarrow \dot{\delta\sigma} - \dot\theta\delta s \quad \longleftarrow \quad \vec{T}.\dot{\vec{\delta}}$$
$$\dot{\delta s} + \dot\theta\delta\sigma \rightarrow \dot{\delta s} + \dot\theta\delta\sigma \quad \longleftarrow \quad \vec{N}.\dot{\vec{\delta}}$$

conclusions

-- this model does not predict a large non-Gaussianity except:

    -- for a highly curved classical path in phase-space!

    -- or if a shielding mechanism allows large first correction term in EFT.

-- different shapes of non-Gaussianity are correlated!

-- in contrast to Senatore & Zaldarriaga, we suggest EFT for multi-filed inflation should be constructed as

$$\Delta \mathcal{L} \propto \sum c_{n_0, n_1, \ldots, n_N} \left(\vec{T}.\dot{\vec{\delta}} \right)^{n_0} \left(\vec{N}_1.\dot{\vec{\delta}} \right)^{n_1} \left(\vec{N}_2.\dot{\vec{\delta}} \right)^{n_2} \ldots \left(\vec{N}_N.\dot{\vec{\delta}} \right)^{n_N}$$

thank you!