

The scalar bi-spectrum during inflation and preheating in single field inflationary models

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Proliferation of inflationary models¹

5-dimensional assisted inflation	extended open inflation	late-time mild inflation	pre-Big-Bang inflation
anisotropic brane inflation	extended warm inflation	low-scale inflation	primary inflation
anomaly-induced inflation	extra dimensional inflation	low-scale supergravity inflation	primordial inflation
assisted inflation	F-term inflation	M-theory inflation	quasi-open inflation
assisted chaotic inflation	F-term hybrid inflation	mass inflation	quintessential inflation
boundary inflation	false vacuum inflation	massive chaotic inflation	R-invariant topological inflation
brane inflation	false vacuum chaotic inflation	moduli inflation	rapid asymmetric inflation
brane-assisted inflation	fast-roll inflation	multi-scalar inflation	running inflation
brane gas inflation	first order inflation	multiple inflation	scalar-tensor gravity inflation
brane-antibrane inflation	gauged inflation	multiple-field slow-roll inflation	scalar-tensor stochastic inflation
braneworld inflation	generalised inflation	multiple-stage inflation	Seiberg-Witten inflation
Brans-Dicke chaotic inflation	generalized assisted inflation	natural inflation	single-bubble open inflation
Brans-Dicke inflation	generalized slow-roll inflation	natural Chaotic inflation	spinodal inflation
bulky brane inflation	gravity driven inflation	natural double inflation	stable starobinsky-type inflation
chaotic hybrid inflation	Hagedorn inflation	natural supergravity inflation	steady-state eternal inflation
chaotic inflation	higher-curvature inflation	new inflation	steep inflation
chaotic new inflation	hybrid inflation	next-to-minimal supersymmetric hybrid inflation	stochastic inflation
D-brane inflation	hyperextended inflation	non-commutative inflation	string-forming open inflation
D-term inflation	induced gravity inflation	non-slow-roll inflation	successful D-term inflation
dilaton-driven inflation	induced gravity open inflation	nonminimal chaotic inflation	supergravity inflation
dilaton-driven brane inflation	intermediate inflation	old inflation	supernatural inflation
double inflation	inverted hybrid inflation	open hybrid inflation	superstring inflation
double D-term inflation	isocurvature inflation	open inflation	supersymmetric hybrid inflation
dual inflation	K inflation	oscillating inflation	supersymmetric inflation
dynamical inflation	kinetic inflation	polynomial chaotic inflation	supersymmetric topological inflation
dynamical SUSY inflation	lambda inflation	polynomial hybrid inflation	supersymmetric new inflation
eternal inflation	large field inflation	power-law inflation	synergistic warm inflation
extended inflation	late D-term inflation		TeV-scale hybrid inflation

A partial list of ever-increasing number of inflationary models!

¹From E. P. S. Shellard, *The future of cosmology: Observational and computational prospects*, in *The Future of Theoretical Physics and Cosmology*, Eds. G. W. Gibbons, E. P. S. Shellard and S. J. Rankin (Cambridge University Press, Cambridge, England, 2003).



Large non-Gaussianities and possible implications

- If one assumes the bi-spectrum to be of the local form, the WMAP 7-year data constrains the non-Gaussianity parameter f_{NL} to be 32 ± 21 , at 68% confidence level².

²E. Komatsu *et al.*, *Astrophys. J. Suppl.* **192**, 18 (2011).

³J. Maldacena, *JHEP* **05**, 013 (2003).

⁴See, for instance, X. Chen, R. Easther and E. A. Lim, *JCAP* **0706**, 023 (2007).

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- If ongoing missions such as Planck indeed detect a large level of non-Gaussianity as suggested by the above mean value of f_{NL} , then it can result in a substantial tightening in the constraints on the various inflationary models. For example, canonical scalar field models that lead to nearly scale invariant primordial spectra contain only a small amount of non-Gaussianity and, hence, will cease to be viable³.

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- However, it is known that primordial spectra with features can lead to reasonably large non-Gaussianities⁴. Therefore, if the non-Gaussianity parameter f_{NL} actually proves to be large, then either one has to reconcile with the fact that the primordial spectrum contains features or we have to turn our attention to non-canonical scalar field models such as, say, D brane inflation models⁵.

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Plan of the talk

- 1 Some essential remarks on the evaluation of the scalar power spectrum
- 2 The scalar bi-spectrum and the non-Gaussianity parameter – Definitions
- 3 The Maldacena formalism for evaluating the bi-spectrum
- 4 Procedure for the numerical evaluation of the bi-spectrum
- 5 Discriminating power of the non-Gaussianity parameter
- 6 Contributions to the bi-spectrum during preheating
- 7 Summary



This talk is based on

- J. Martin and L. Sriramkumar, *The scalar bi-spectrum in the Starobinsky model: The equilateral case*, JCAP **1201**, 008 (2012).
- D. K. Hazra, L. Sriramkumar and J. Martin, *On the discriminating power of f_{NL}* , arXiv:1201.0926v1 [astro-ph.CO].
- D. K. Hazra, J. Martin and L. Sriramkumar, *Scalar bi-spectrum during preheating in single field inflationary models*, Phys. Rev. D **86**, 063523 (2012).



A few words on the conventions and notations

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- ◆ We shall denote differentiation with respect to the cosmic and the conformal times t and η by an overdot and an overprime, respectively.
- ◆ Further, N shall denote the number of e-folds.



The curvature perturbation and the governing equation

On quantization, the operator corresponding to the curvature perturbation $\mathcal{R}(\eta, \mathbf{x})$ can be expressed as

$$\begin{aligned}\hat{\mathcal{R}}(\eta, \mathbf{x}) &= \int \frac{d^3 \mathbf{k}}{(2\pi)^{3/2}} \hat{\mathcal{R}}_{\mathbf{k}}(\eta) e^{i \mathbf{k} \cdot \mathbf{x}} \\ &= \int \frac{d^3 \mathbf{k}}{(2\pi)^{3/2}} \left[\hat{a}_{\mathbf{k}} f_{\mathbf{k}}(\eta) e^{i \mathbf{k} \cdot \mathbf{x}} + \hat{a}_{\mathbf{k}}^\dagger f_{\mathbf{k}}^*(\eta) e^{-i \mathbf{k} \cdot \mathbf{x}} \right],\end{aligned}$$

where $\hat{a}_{\mathbf{k}}$ and $\hat{a}_{\mathbf{k}}^\dagger$ are the usual creation and annihilation operators that satisfy the standard commutation relations.

The modes $f_{\mathbf{k}}$ are governed by the differential equation

$$f_{\mathbf{k}}'' + 2 \frac{z'}{z} f_{\mathbf{k}}' + k^2 f_{\mathbf{k}} = 0,$$

where $z = a M_{\text{Pl}} \sqrt{2 \epsilon_1}$, with $\epsilon_1 = -d \ln H / d \ln N$ being the first slow roll parameter.



The Bunch-Davies initial conditions

While studying the evolution of the curvature perturbation, it often proves to be more convenient to work in terms of the so-called Mukhanov-Sasaki variable $v_{\mathbf{k}}$, which is defined as $v_{\mathbf{k}} = z f_{\mathbf{k}}$. In terms of the variable $v_{\mathbf{k}}$, the above equation of motion for $f_{\mathbf{k}}$ reduces to the following simple form:

$$v_{\mathbf{k}}'' + \left(k^2 - \frac{z''}{z} \right) v_{\mathbf{k}} = 0.$$

The initial conditions on the perturbations are imposed when the modes are well inside the Hubble radius during inflation.

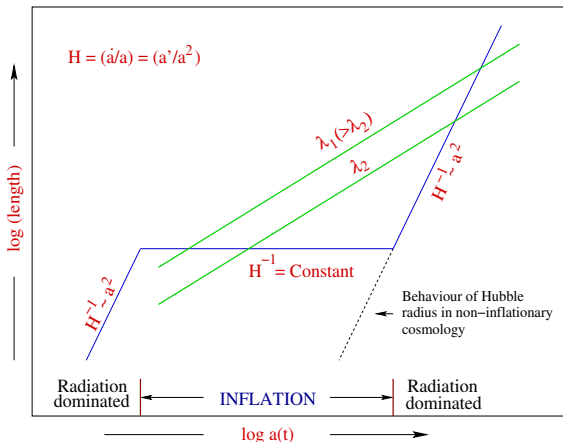
Usually, it is the so-called Bunch-Davies initial conditions that are imposed on the modes, which amounts to demanding that the Mukhanov-Sasaki variable $v_{\mathbf{k}}$ reduces to following Minkowski-like positive frequency form in the sub-Hubble limit⁶:

$$\lim_{k/(aH) \rightarrow \infty} v_{\mathbf{k}} = \frac{1}{\sqrt{2k}} e^{-ik\eta}.$$

⁶T. Bunch and P. C. W. Davies, Proc. Roy. Soc. Lond. A **360**, 117 (1978).



The behavior of modes during inflation



A schematic diagram illustrating the behavior of the physical wavelength $\lambda_p \propto a$ (the green lines) and the Hubble radius H^{-1} (the blue line) during inflation and the radiation dominated epochs⁷.

⁷See, for example, E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley Publishing Company, New York, 1990), Fig. 8.4.



The scalar power spectrum

The dimensionless scalar power spectrum $\mathcal{P}_s(k)$ is defined in terms of the correlation function of the Fourier modes of the curvature perturbation $\hat{\mathcal{R}}_k$ as follows:

$$\langle 0 | \hat{\mathcal{R}}_k(\eta) \hat{\mathcal{R}}_p(\eta) | 0 \rangle = \frac{(2\pi)^2}{2k^3} \mathcal{P}_s(k) \delta^{(3)}(\mathbf{k} + \mathbf{p}),$$

where $|0\rangle$ is the Bunch-Davies vacuum, defined as $\hat{a}_k|0\rangle = 0 \forall \mathbf{k}$.

In terms of the quantities f_k and v_k , the power spectrum is given by

$$\mathcal{P}_s(k) = \frac{k^3}{2\pi^2} |f_k|^2 = \frac{k^3}{2\pi^2} \left(\frac{|v_k|}{z} \right)^2$$

and, analytically, the spectrum is evaluated in the super-Hubble limit, *i.e.* when $k/(aH) \rightarrow 0$.

As is well known, numerically, the Bunch-Davies initial conditions are imposed on the modes when they are *well inside the Hubble radius*, and the power spectrum is evaluated at suitably late times when the modes are *sufficiently outside*⁸.

⁸See, for example, D. S. Salopek, J. R. Bond and J. M. Bardeen, *Phys. Rev. D* **40**, 1753 (1989); C. Ringeval, *Lect. Notes Phys.* **738**, 243 (2008).



The scalar bi-spectrum

The scalar bi-spectrum $\mathcal{B}_S(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ is related to the three point correlation function of the Fourier modes of the curvature perturbation, evaluated towards the end of inflation, say, at the conformal time η_e , as follows⁹:

$$\langle \hat{\mathcal{R}}_{\mathbf{k}_1}(\eta_e) \hat{\mathcal{R}}_{\mathbf{k}_2}(\eta_e) \hat{\mathcal{R}}_{\mathbf{k}_3}(\eta_e) \rangle = (2\pi)^3 \mathcal{B}_S(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3).$$

⁹D. Larson *et al.*, *Astrophys. J. Suppl.* **192**, 16 (2011);
E. Komatsu *et al.*, *Astrophys. J. Suppl.* **192**, 18 (2011).



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In our discussion below, for the sake of convenience, we shall set

$$\mathcal{B}_S(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = (2\pi)^{-9/2} G(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3).$$

⁹D. Larson *et al.*, *Astrophys. J. Suppl.* **192**, 16 (2011);
E. Komatsu *et al.*, *Astrophys. J. Suppl.* **192**, 18 (2011).



The non-Gaussianity parameter f_{NL}

The observationally relevant non-Gaussianity parameter f_{NL} is introduced through the relation¹⁰

$$\mathcal{R}(\eta, \mathbf{x}) = \mathcal{R}_{\text{G}}(\eta, \mathbf{x}) - \frac{3f_{\text{NL}}}{5} [\mathcal{R}_{\text{G}}^2(\eta, \mathbf{x}) - \langle \mathcal{R}_{\text{G}}^2(\eta, \mathbf{x}) \rangle],$$

where \mathcal{R}_{G} denotes the Gaussian quantity, and the factor of $3/5$ arises due to the relation between the Bardeen potential and the curvature perturbation during the matter dominated epoch.

Utilizing the above relation and Wick's theorem, one can arrive at the three point correlation function of the curvature perturbation in Fourier space in terms of the parameter f_{NL} . It is found to be

$$\begin{aligned} \langle \hat{\mathcal{R}}_{\mathbf{k}_1} \hat{\mathcal{R}}_{\mathbf{k}_2} \hat{\mathcal{R}}_{\mathbf{k}_3} \rangle &= -\frac{3f_{\text{NL}}}{10} (2\pi)^{5/2} \left(\frac{1}{k_1^3 k_2^3 k_3^3} \right) \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \\ &\times [k_1^3 \mathcal{P}_{\text{S}}(k_2) \mathcal{P}_{\text{S}}(k_3) + \text{two permutations}]. \end{aligned}$$

¹⁰E. Komatsu and D. N. Spergel, Phys. Rev. D **63**, 063002 (2001).



The relation between f_{NL} and the bi-spectrum

Upon making use of the above expression for the three point function of the curvature perturbation and the definition of the bi-spectrum, we can, in turn, arrive at the following relation¹¹:

$$\begin{aligned}
 f_{\text{NL}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) &= -\frac{10}{3} (2\pi)^{1/2} (k_1^3 k_2^3 k_3^3) \mathcal{B}_S(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \\
 &\quad \times [k_1^3 \mathcal{P}_S(k_2) \mathcal{P}_S(k_3) + \text{two permutations}]^{-1} \\
 &= -\frac{10}{3} \frac{1}{(2\pi)^4} (k_1^3 k_2^3 k_3^3) G(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \\
 &\quad \times [k_1^3 \mathcal{P}_S(k_2) \mathcal{P}_S(k_3) + \text{two permutations}]^{-1}.
 \end{aligned}$$

¹¹See, for instance, S. Hannestad, T. Haugbolle, P. R. Jarnhus and M. S. Sloth, JCAP **1006**, 001 (2010).



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 &\quad \times [k_1^3 \mathcal{P}_S(k_2) \mathcal{P}_S(k_3) + \text{two permutations}]^{-1} \\
 &= -\frac{10}{3} \frac{1}{(2\pi)^4} (k_1^3 k_2^3 k_3^3) G(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \\
 &\quad \times [k_1^3 \mathcal{P}_S(k_2) \mathcal{P}_S(k_3) + \text{two permutations}]^{-1}.
 \end{aligned}$$

Note that, in the equilateral limit, *i.e.* when $k_1 = k_2 = k_3$, this expression for f_{NL} simplifies to

$$f_{\text{NL}}^{\text{eq}}(k) = -\frac{10}{9} \frac{1}{(2\pi)^4} \frac{k^6 G(k)}{\mathcal{P}_S^2(k)}.$$

¹¹See, for instance, S. Hannestad, T. Haugbolle, P. R. Jarnhus and M. S. Sloth, JCAP **1006**, 001 (2010).



The action at the cubic order

It can be shown that, the third order term in the action describing the curvature perturbation is given by¹²

$$\begin{aligned} \mathcal{S}_3[\mathcal{R}] = & M_{\text{Pl}}^2 \int d\eta \int d^3\mathbf{x} \left[a^2 \epsilon_1^2 \mathcal{R} \mathcal{R}'^2 + a^2 \epsilon_1^2 \mathcal{R} (\partial\mathcal{R})^2 \right. \\ & - 2 a \epsilon_1 \mathcal{R}' (\partial^i \mathcal{R}) (\partial_i \chi) + \frac{a^2}{2} \epsilon_1 \epsilon_2' \mathcal{R}^2 \mathcal{R}' + \frac{\epsilon_1}{2} (\partial^i \mathcal{R}) (\partial_i \chi) (\partial^2 \chi) \\ & \left. + \frac{\epsilon_1}{4} (\partial^2 \mathcal{R}) (\partial\chi)^2 + \mathcal{F} \left(\frac{\delta\mathcal{L}_2}{\delta\mathcal{R}} \right) \right], \end{aligned}$$

where $\mathcal{F}(\delta\mathcal{L}_2/\delta\mathcal{R})$ denotes terms involving the variation of the second order action with respect to \mathcal{R} , while χ is related to the curvature perturbation \mathcal{R} through the relations

$$\Lambda = a \epsilon_1 \mathcal{R}' \quad \text{and} \quad \partial^2 \chi = \Lambda.$$

¹²J. Maldacena, JHEP **0305**, 013 (2003);

D. Seery and J. E. Lidsey, JCAP **0506**, 003 (2005);

X. Chen, M.-x. Huang, S. Kachru and G. Shiu, JCAP **0701**, 002 (2007).



Evaluating the bi-spectrum

At the leading order in the perturbations, one then finds that the three point correlation in Fourier space is described by the integral¹³

$$\begin{aligned} & \langle \hat{\mathcal{R}}_{\mathbf{k}_1}(\eta_e) \hat{\mathcal{R}}_{\mathbf{k}_2}(\eta_e) \hat{\mathcal{R}}_{\mathbf{k}_3}(\eta_e) \rangle \\ &= -i \int_{\eta_i}^{\eta_e} d\eta a(\eta) \left\langle \left[\hat{\mathcal{R}}_{\mathbf{k}_1}(\eta_e) \hat{\mathcal{R}}_{\mathbf{k}_2}(\eta_e) \hat{\mathcal{R}}_{\mathbf{k}_3}(\eta_e), \hat{H}_I(\eta) \right] \right\rangle, \end{aligned}$$

where \hat{H}_I is the Hamiltonian corresponding to the above third order action, while η_i denotes a sufficiently early time when the initial conditions are imposed on the modes, and η_e denotes a very late time, say, close to when inflation ends.

Note that, while the square brackets imply the commutation of the operators, the angular brackets denote the fact that the correlations are evaluated in the initial vacuum state (*viz.* the Bunch-Davies vacuum in the situation of our interest).

¹³See, for example, D. Seery and J. E. Lidsey, JCAP **0506**, 003 (2005); X. Chen, Adv. Astron. **2010**, 638979 (2010).



The resulting bi-spectrum

The quantity $G(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ evaluated towards the end of inflation at the conformal time $\eta = \eta_e$ can be written as¹⁴

$$\begin{aligned}
 G(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) &\equiv \sum_{C=1}^7 G_C(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \\
 &\equiv M_{\text{Pl}}^2 \sum_{C=1}^6 \left\{ [f_{\mathbf{k}_1}(\eta_e) f_{\mathbf{k}_2}(\eta_e) f_{\mathbf{k}_3}(\eta_e)] \mathcal{G}_C(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \right. \\
 &\quad \left. + [f_{\mathbf{k}_1}^*(\eta_e) f_{\mathbf{k}_2}^*(\eta_e) f_{\mathbf{k}_3}^*(\eta_e)] \mathcal{G}_C^*(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \right\} + G_7(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3),
 \end{aligned}$$

where the quantities $\mathcal{G}_C(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ with $C = (1, 6)$ correspond to the six terms in the interaction Hamiltonian.

The additional, seventh term $G_7(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ arises due to a field redefinition, and its contribution to $G(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ is given by

$$G_7(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = \frac{\epsilon_2(\eta_e)}{2} (|f_{\mathbf{k}_2}(\eta_e)|^2 |f_{\mathbf{k}_3}(\eta_e)|^2 + \text{two permutations}).$$

¹⁴J. Martin and L. Sriramkumar, JCAP **1201**, 008 (2012).



The integrals involved

The quantities $\mathcal{G}_C(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ with $C = (1, 6)$ are described by the integrals

$$\mathcal{G}_1(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = 2i \int_{\eta_i}^{\eta_e} d\eta a^2 \epsilon_1^2 (f_{\mathbf{k}_1}^* f_{\mathbf{k}_2}^* f_{\mathbf{k}_3}^* + \text{two permutations}),$$

$$\mathcal{G}_2(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = -2i (\mathbf{k}_1 \cdot \mathbf{k}_2 + \text{two permutations}) \int_{\eta_i}^{\eta_e} d\eta a^2 \epsilon_1^2 f_{\mathbf{k}_1}^* f_{\mathbf{k}_2}^* f_{\mathbf{k}_3}^*,$$

$$\mathcal{G}_3(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = -2i \int_{\eta_i}^{\eta_e} d\eta a^2 \epsilon_1^2 \left[\left(\frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{k_2^2} \right) f_{\mathbf{k}_1}^* f_{\mathbf{k}_2}^* f_{\mathbf{k}_3}^* + \text{five permutations} \right],$$

$$\mathcal{G}_4(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = i \int_{\eta_i}^{\eta_e} d\eta a^2 \epsilon_1 \epsilon_2' (f_{\mathbf{k}_1}^* f_{\mathbf{k}_2}^* f_{\mathbf{k}_3}^* + \text{two permutations}),$$

$$\mathcal{G}_5(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = \frac{i}{2} \int_{\eta_i}^{\eta_e} d\eta a^2 \epsilon_1^3 \left[\left(\frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{k_2^2} \right) f_{\mathbf{k}_1}^* f_{\mathbf{k}_2}^* f_{\mathbf{k}_3}^* + \text{five permutations} \right],$$

$$\mathcal{G}_6(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = \frac{i}{2} \int_{\eta_i}^{\eta_e} d\eta a^2 \epsilon_1^3 \left\{ \left[\frac{k_1^2 (\mathbf{k}_2 \cdot \mathbf{k}_3)}{k_2^2 k_3^2} \right] f_{\mathbf{k}_1}^* f_{\mathbf{k}_2}^* f_{\mathbf{k}_3}^* + \text{two permutations} \right\},$$

where ϵ_2 is the second slow roll parameter that is defined with respect to the first as follows: $\epsilon_2 = d \ln \epsilon_1 / dN$.



Evolution of $f_{\mathbf{k}}$ on super-Hubble scales

During inflation, when the modes are on super-Hubble scales, it is well known that the solution to $f_{\mathbf{k}}$ can be written as

$$f_{\mathbf{k}} \simeq A_{\mathbf{k}} + B_{\mathbf{k}} \int^{\eta} \frac{d\tilde{\eta}}{z^2(\tilde{\eta})},$$

where $A_{\mathbf{k}}$ and $B_{\mathbf{k}}$ are k -dependent constants which are determined by the initial conditions imposed on the modes in the sub-Hubble limit.

Therefore, on super-Hubble scales, the mode $f_{\mathbf{k}}$ simplifies to

$$f_{\mathbf{k}} \simeq A_{\mathbf{k}}.$$

Moreover, the leading non-zero contribution to its derivative is determined by the decaying mode, and is given by

$$f'_{\mathbf{k}} \simeq \frac{B_{\mathbf{k}}}{z^2} = \frac{\bar{B}_{\mathbf{k}}}{a^2 \epsilon_1},$$

where we have set $\bar{B}_{\mathbf{k}} = B_{\mathbf{k}}/(2M_{\text{Pl}}^2)$.



Splitting the integrals

To begin with, we shall divide each of the integrals $\mathcal{G}_C(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$, where $C = (1, 6)$, into two parts as follows:

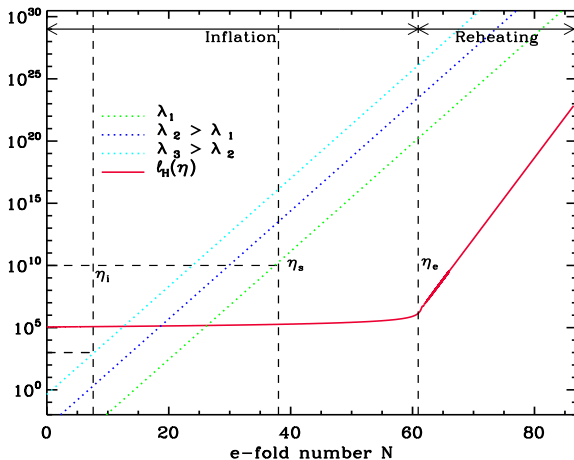
$$\mathcal{G}_C(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = \mathcal{G}_C^{\text{is}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) + \mathcal{G}_C^{\text{se}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3).$$

The integrals in the first term $\mathcal{G}_C^{\text{is}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ run from the earliest time (*i.e.* η_i) when the smallest of the three wavenumbers k_1 , k_2 and k_3 is sufficiently inside the Hubble radius [typically corresponding to $k/(aH) \simeq 100$] to the time (say, η_s) when the largest of the three wavenumbers is well outside the Hubble radius [say, when $k/(aH) \simeq 10^{-5}$].

Then, evidently, the second term $\mathcal{G}_C^{\text{se}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ will involve integrals which run from the latter time η_s to the end of inflation at η_e .



The various times of interest



The exact behavior of the physical wavelengths and the Hubble radius plotted as a function of the number of e-folds in the case of the archetypical quadratic potential, which allows us to illustrate the various times of our interest, *viz.* η_i , η_s and η_e .



Contributions due to the fourth and the seventh terms

Upon using the form of the mode $f_{\mathbf{k}}$ and its derivative on super-Hubble scales, the integral appearing in the fourth term can be trivially carried out with the result that the corresponding contribution to the bi-spectrum can be expressed as

$$G_4^{\text{se}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \simeq i M_{\text{Pl}}^2 [\epsilon_2(\eta_e) - \epsilon_2(\eta_s)] \\ \times [|A_{\mathbf{k}_1}|^2 |A_{\mathbf{k}_2}|^2 (A_{\mathbf{k}_3} \bar{B}_{\mathbf{k}_3}^* - A_{\mathbf{k}_3}^* \bar{B}_{\mathbf{k}_3}) + \text{two permutations}] .$$

The Wronskian corresponding to the equation of motion for $f_{\mathbf{k}}$ and the standard Bunch-Davies initial conditions can then be utilized to arrive at the following simpler expression:

$$G_4^{\text{se}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \simeq -\frac{1}{2} [\epsilon_2(\eta_e) - \epsilon_2(\eta_s)] [|A_{\mathbf{k}_1}|^2 |A_{\mathbf{k}_2}|^2 + \text{two permutations}] .$$

The first of these terms involving the value of ϵ_2 at η_e *exactly* cancels the contribution $G_7(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ (with $f_{\mathbf{k}}$ set to $A_{\mathbf{k}}$).

Note that the remaining term is essentially the same as the one due to the field redefinition, but which is now evaluated on super-Hubble scales (*i.e.* at η_s) rather than at the end of inflation.



The contribution due to the second term

Upon making use of the behavior of the mode $f_{\mathbf{k}}$ on super-Hubble scales in the corresponding integral, one obtains the contribution to the bi-spectrum due to $\mathcal{G}_2^{\text{se}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ to be

$$\begin{aligned} G_2^{\text{se}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = & -2i M_{\text{Pl}}^2 (\mathbf{k}_1 \cdot \mathbf{k}_2 + \text{two permutations}) \\ & \times |A_{\mathbf{k}_1}|^2 |A_{\mathbf{k}_2}|^2 |A_{\mathbf{k}_3}|^2 [I_2(\eta_e, \eta_s) - I_2^*(\eta_e, \eta_s)], \end{aligned}$$

where the quantity $I_2(\eta_e, \eta_s)$ is described by the integral

$$I_2(\eta_e, \eta_s) = \int_{\eta_s}^{\eta_e} d\eta a^2 \epsilon_1^2.$$

Note that, due to the quadratic dependence on the scale factor, actually, $I_2(\eta_e, \eta_s)$ is a rapidly growing quantity at late times.

However, the complete super-Hubble contribution to the bi-spectrum vanishes identically since the integral $I_2(\eta_e, \eta_s)$ is a purely real quantity¹⁵.

¹⁵D. K. Hazra, J. Martin and L. Sriramkumar, Phys. Rev. D **86**, 063523 (2012).



The contributions due to the remaining terms

On super-Hubble scales, one can easily show that the contributions due to the first and the third terms can be written as

$$G_1^{\text{ES}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) + G_3^{\text{ES}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = 2i M_{\text{Pl}}^2 \left[\left(1 - \frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{k_2^2} - \frac{\mathbf{k}_1 \cdot \mathbf{k}_3}{k_3^2} \right) |A_{\mathbf{k}_1}|^2 \right. \\ \times (A_{\mathbf{k}_2} \bar{B}_{\mathbf{k}_2}^* A_{\mathbf{k}_3} \bar{B}_{\mathbf{k}_3}^* - A_{\mathbf{k}_2}^* \bar{B}_{\mathbf{k}_2} A_{\mathbf{k}_3}^* \bar{B}_{\mathbf{k}_3}) \\ \left. + \text{two permutations} \right] I_{13}(\eta_e, \eta_s),$$

while the corresponding contributions due to the fifth and the sixth terms are given by

$$G_5^{\text{SE}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) + G_6^{\text{SE}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = \frac{i M_{\text{Pl}}^2}{2} \left[\left(\frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{k_2^2} + \frac{\mathbf{k}_1 \cdot \mathbf{k}_3}{k_3^2} + \frac{k_1^2 (\mathbf{k}_2 \cdot \mathbf{k}_3)}{k_2^2 k_3^2} \right) \right. \\ \times |A_{\mathbf{k}_1}|^2 (A_{\mathbf{k}_2} \bar{B}_{\mathbf{k}_2}^* A_{\mathbf{k}_3} \bar{B}_{\mathbf{k}_3}^* - A_{\mathbf{k}_2}^* \bar{B}_{\mathbf{k}_2} A_{\mathbf{k}_3}^* \bar{B}_{\mathbf{k}_3}) \\ \left. + \text{two permutations} \right] I_{56}(\eta_e, \eta_s),$$

where the quantities $I_{13}(\eta_e, \eta_s)$ and $I_{56}(\eta_e, \eta_s)$ are described by the integrals

$$I_{13}(\eta_e, \eta_s) = \int_{\eta_s}^{\eta_e} \frac{d\eta}{a^2} \quad \text{and} \quad I_{56}(\eta_e, \eta_s) = \int_{\eta_s}^{\eta_e} \frac{d\eta}{a^2} \epsilon_1.$$



The complete super-Hubble contribution to $f_{\text{NL}}^{\text{eq}}$

To arrive at the complete super-Hubble contribution to the non-Gaussianity parameter f_{NL} , let us restrict ourselves to the equilateral limit for simplicity.

In such a case, the sum of the super-Hubble contributions due to the first, the third, the fifth and the sixth terms to $f_{\text{NL}}^{\text{eq}}$ is found to be

$$f_{\text{NL}}^{\text{eq (se)}}(k) \simeq -\frac{5i M_{\text{Pl}}^2}{18} \left(\frac{A_{\mathbf{k}}^2 \bar{B}_{\mathbf{k}}^{*2} - A_{\mathbf{k}}^{*2} \bar{B}_{\mathbf{k}}^2}{|A_{\mathbf{k}}|^2} \right) \left[12 I_{13}(\eta_e, \eta_s) - \frac{9}{4} I_{56}(\eta_e, \eta_s) \right],$$

where we have made use of the fact that $f_{\mathbf{k}} \simeq A_{\mathbf{k}}$ at late times in order to arrive at the power spectrum.



An estimate of the super-Hubble contribution to f_{NL}^{eq}

Consider power law inflation of the form $a(\eta) = a_1 (\eta/\eta_1)^{\gamma+1}$, where a_1 and η_1 are constants, while γ is a free index. For such an expansion, the first slow roll parameter is a constant, and is given by $\epsilon_1 = (\gamma + 2)/(\gamma + 1)$.

In such a case, one can easily obtain that

$$f_{NL}^{eq(se)}(k) = \frac{5}{72\pi} \left[12 - \frac{9(\gamma+2)}{\gamma+1} \right] \Gamma^2 \left(\gamma + \frac{1}{2} \right) 2^{2\gamma+1} (2\gamma+1) (\gamma+2) \\ \times (\gamma+1)^{-2(\gamma+1)} \sin(2\pi\gamma) \left[1 - \frac{H_s}{H_e} e^{-3(N_e - N_s)} \right] \left(\frac{k}{a_s H_s} \right)^{-(2\gamma+1)}$$

and, in arriving at this expression, for convenience, we have set η_1 to be η_s .

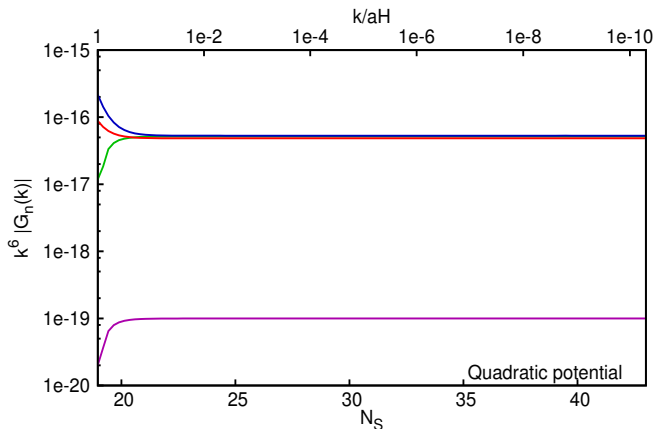
For $\gamma = -(2 + \epsilon)$, where $\epsilon \simeq 10^{-2}$, the above estimate for f_{NL} reduces to

$$f_{NL}^{eq(se)}(k) \lesssim -\frac{5\epsilon^2}{9} \left(\frac{k_s}{a_s H_s} \right)^3 \simeq -10^{-19},$$

where, in obtaining the final value, we have set $k_s/(a_s H_s) = 10^{-5}$.



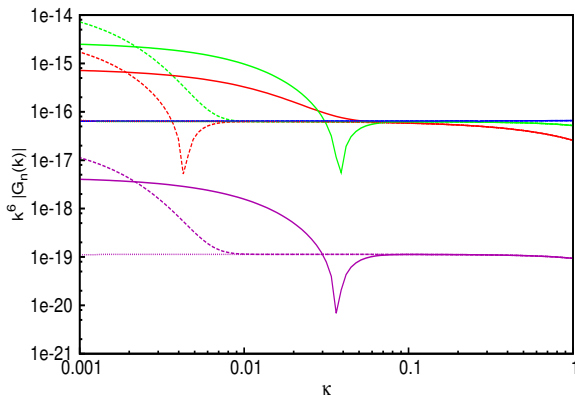
Convergence on the upper limit



Focusing on the equilateral limit, the quantities k^6 times the absolute values of $G_1 + G_3$ (in green), G_2 (in red), $G_4 + G_7$ (in blue) and $G_5 + G_6$ (in purple), evaluated numerically, have been plotted as a function of the upper limit of the integrals involved for a given mode in the case of the conventional, quadratic inflationary potential. Evidently, the integrals converge rapidly once the mode leaves the Hubble radius.



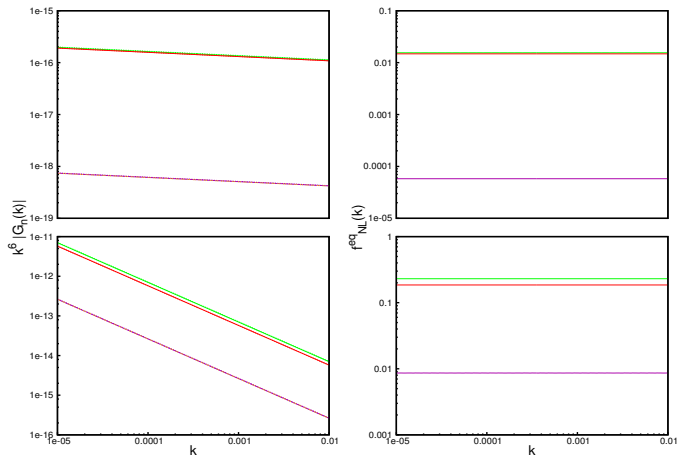
Implementation of the cut-off in the sub-Hubble limit



The various contributions to the bi-spectrum, obtained numerically, have been plotted (with the same choice of colors as in the previous figure) as a function of the cut-off parameter κ for a given mode and a fixed upper limit [corresponding to $k/(aH) = 10^{-5}$] in the case of the quadratic inflationary potential. The solid, dashed and the dotted lines correspond to integrating from $k/(aH)$ of 10^2 , 10^3 and 10^4 , respectively. Clearly, $\kappa = 0.1$ seems to be an optimal value.



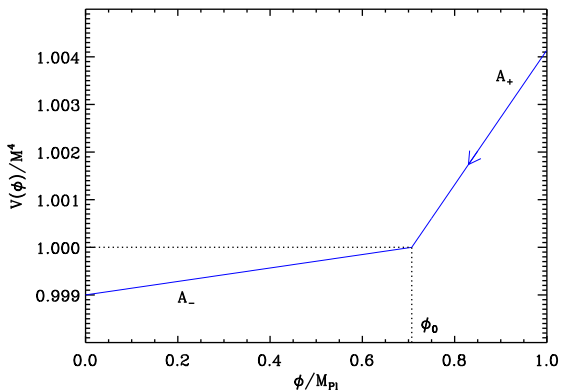
The spectral dependence in power law inflation



The different non-zero contributions to the bi-spectrum in the power law case (on the left) and the corresponding contributions to the non-Gaussianity parameter f_{NL}^{eq} (on the right), arrived at numerically, have been plotted as solid lines for two different values of γ ($\gamma = -2.02$ on top and $\gamma = -2.25$ below). The dots on the lines represent the spectral dependences arrived at from analytical arguments.



The Starobinsky model



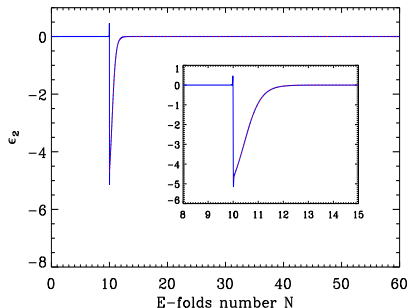
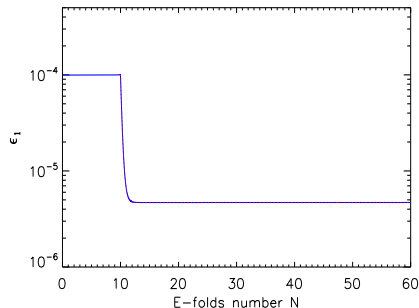
The Starobinsky model involves the canonical scalar field which is described by the potential¹⁶

$$V(\phi) = \begin{cases} V_0 + A_+ (\phi - \phi_0) & \text{for } \phi > \phi_0, \\ V_0 + A_- (\phi - \phi_0) & \text{for } \phi < \phi_0. \end{cases}$$

¹⁶A. A. Starobinsky, Sov. Phys. JETP Lett. **55**, 489 (1992).



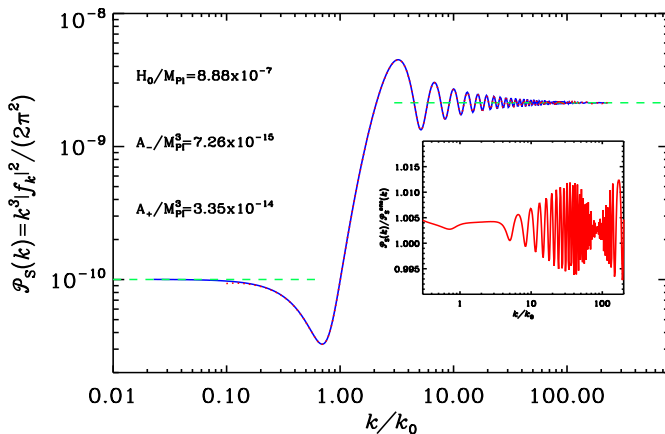
Evolution of the slow roll parameters



The evolution of the first (on the left) and the second (on the right) slow roll parameters ϵ_1 and ϵ_2 in the Starobinsky model. While the solid blue curves describe the numerical results, the dotted red curves (which lie right below the blue curves and hence not very evident!) represent the corresponding analytical expressions.



The scalar power spectrum in the Starobinsky model

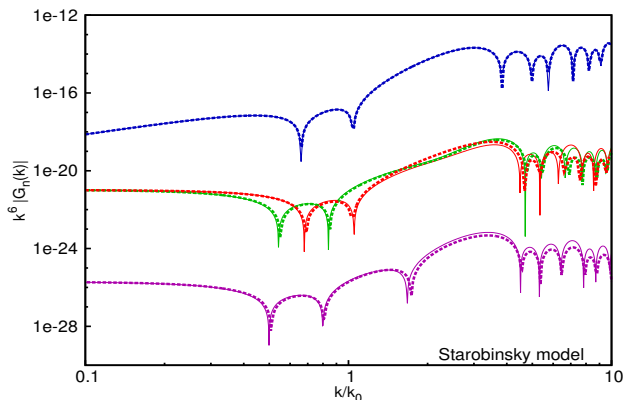


The scalar power spectrum in the Starobinsky model¹⁷. While the solid blue curve denotes the analytic result, the red dots represent the scalar power spectrum that has been obtained through a complete numerical integration of the background as well as the perturbations.

¹⁷ J. Martin and L. Sriramkumar, JCAP **1201**, 008 (2012).



Comparison in the case of the Starobinsky model



A comparison of the analytical expressions (the solid curves) with the corresponding numerical results (the dashed curves) in the case of the Starobinsky model. We have chosen the same set of colors to denote the different contributions to the bi-spectrum as in the earlier figures¹⁸.

¹⁸See, J. Martin and L. Sriramkumar, JCAP **1201**, 008 (2012);

In this context, also see, F. Arroja, A. E. Romano and M. Sasaki, Phys. Rev. D **84**, 123503 (2011);

F. Arroja and M. Sasaki, JCAP **1208**, 012 (2012).



Punctuated inflation

Punctuated inflation is a scenario wherein a brief period of rapid roll inflation or even a departure from inflation is sandwiched between two epochs of slow roll inflation¹⁹.

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- ¹⁹R. K. Jain, P. Chingangbam, J.-O. Gong, L. Sriramkumar and T. Souradeep, JCAP **0901**, 009 (2009);
R. K. Jain, P. Chingangbam, L. Sriramkumar and T. Souradeep, Phys. Rev. D **82**, 023509 (2010).
- ²⁰R. Allahverdi, K. Enqvist, J. Garcia-Bellido, A. Jokinen and A. Mazumdar, JCAP **0706**, 019 (2007).



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Such a scenario can be achieved in inflaton potentials such as²⁰

$$V(\phi) = (m^2/2) \phi^2 - \left(\sqrt{2\lambda(n-1)} m/n \right) \phi^n + (\lambda/4) \phi^{2(n-1)},$$

where $n > 2$ is an integer. This potential contains a point of inflection located at

$$\phi_0 = \left[\frac{2m^2}{(n-1)\lambda} \right]^{\frac{1}{2(n-2)}},$$

and it is the presence of this inflection point that admits punctuated inflation.

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These scenarios can lead to a sharp drop in power on large scales and result in an improved fit to the data at the low multipoles.

¹⁹R. K. Jain, P. Chingangbam, J.-O. Gong, L. Sriramkumar and T. Souradeep, JCAP **0901**, 009 (2009);
R. K. Jain, P. Chingangbam, L. Sriramkumar and T. Souradeep, Phys. Rev. D **82**, 023509 (2010).

²⁰R. Allahverdi, K. Enqvist, J. Garcia-Bellido, A. Jokinen and A. Mazumdar, JCAP **0706**, 019 (2007).



Inflaton potentials with a step

Given a potential $V(\phi)$, one can introduce the step in the following fashion²¹:

$$V_{\text{step}}(\phi) = V(\phi) \left[1 + \alpha \tanh \left(\frac{\phi - \phi_0}{\Delta\phi} \right) \right],$$

where, evidently, α , ϕ_0 and $\Delta\phi$ denote the height, the location, and the width of the step, respectively.

²¹J. A. Adams, B. Cresswell and R. Easther, Phys. Rev. D **64**, 123514 (2001).

²²L. Covi, J. Hamann, A. Melchiorri, A. Slosar and I. Sorbera, Phys. Rev. D **74**, 083509 (2006);
M. J. Mortonson, C. Dvorkin, H. V. Peiris and W. Hu, Phys. Rev. D **79**, 103519 (2009);
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Such a step in potentials $V(\phi)$ which otherwise only result in slow roll lead to oscillatory features in the scalar power spectrum that provide a better fit to the outliers near $\ell = 20$ and $\ell = 44$ ²².

²¹J. A. Adams, B. Cresswell and R. Easther, Phys. Rev. D **64**, 123514 (2001).

²²L. Covi, J. Hamann, A. Melchiorri, A. Slosar and I. Sorbera, Phys. Rev. D **74**, 083509 (2006);
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Oscillating inflation potentials

Potentials containing oscillatory terms are encountered in string theory. A popular example is the axion monodromy model, which is described by the potential²³

$$V(\phi) = \lambda \left[\phi + \alpha \cos \left(\frac{\phi}{\beta} + \delta \right) \right].$$

²³R. Flauger, L. McAllister, E. Pajer, A. Westphal and G. Xu, JCAP **1006**, 009 (2010).

²⁴M. Aich, D. K. Hazra, L. Sriramkumar and T. Souradeep, arXiv:1106.2798v2 [astro-ph.CO].

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Interestingly, such a potential leads to non-local features – *i.e.* a certain characteristic and repeated pattern that extends over a wide range of scales – in the primordial spectrum which result in an improved fit to the data²⁴.

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Interestingly, such a potential leads to non-local features – *i.e.* a certain characteristic and repeated pattern that extends over a wide range of scales – in the primordial spectrum which result in an improved fit to the data²⁴.

Another potential that has been considered in this context is the conventional quadratic potential which is superposed by sinusoidal oscillations as follows²⁵:

$$V(\phi) = \frac{1}{2} m^2 \phi^2 \left[1 + \alpha \sin \left(\frac{\phi}{\beta} + \delta \right) \right].$$

²³R. Flauger, L. McAllister, E. Pajer, A. Westphal and G. Xu, JCAP **1006**, 009 (2010).

²⁴M. Aich, D. K. Hazra, L. Sriramkumar and T. Souradeep, arXiv:1106.2798v2 [astro-ph.CO].

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The various models of interest

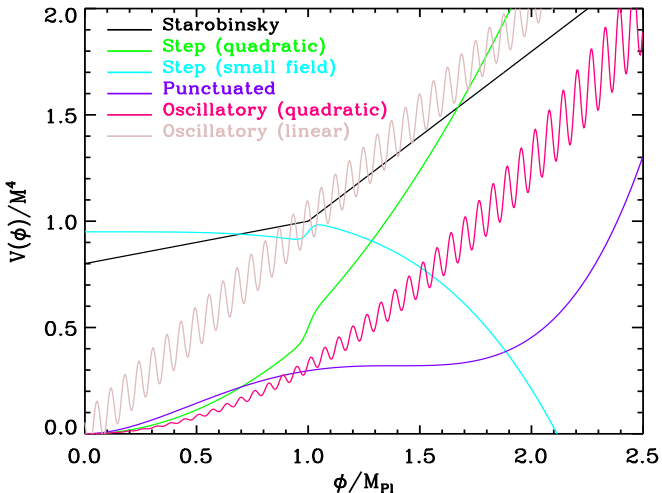
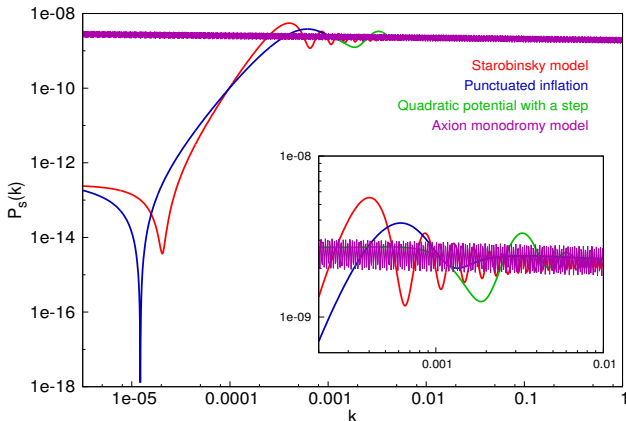


Illustration of the potentials in the different inflationary models of our interest



Inflationary models leading to features

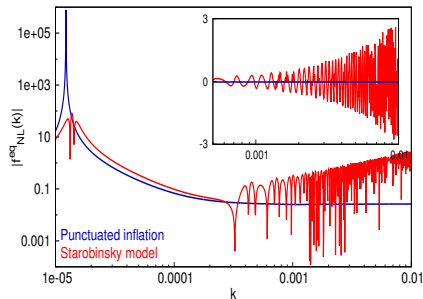
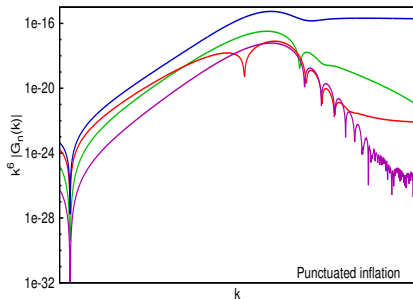


The scalar power spectra in the different inflationary models that lead to a better fit to the CMB data than the conventional power law spectrum²⁶.

²⁶R. K. Jain, P. Chingangbam, J.-O. Gong, L. Sriramkumar and T. Souradeep, JCAP **0901**, 009 (2009);
 D. K. Hazra, M. Aich, R. K. Jain, L. Sriramkumar and T. Souradeep, JCAP **1010**, 008 (2010);
 M. Aich, D. K. Hazra, L. Sriramkumar and T. Souradeep, arXiv:1106.2798v2 [astro-ph.CO].



f_{NL}^{eq} in punctuated inflation

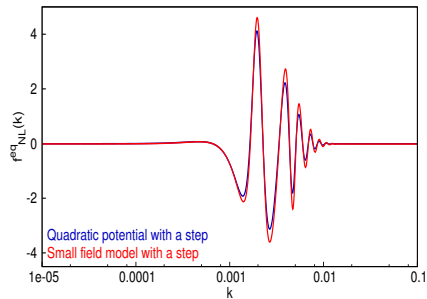
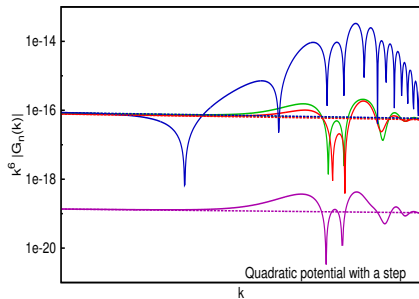


The contributions to the bi-spectrum due to the various terms (on the left), and the absolute value of f_{NL}^{eq} due to the dominant contribution (on the right), in the punctuated inflationary scenario²⁷. The absolute value of f_{NL}^{eq} in a Starobinsky model that closely resembles the power spectrum in punctuated inflation has also been displayed. The large difference in f_{NL}^{eq} between punctuated inflation and the Starobinsky model can be attributed to the considerable difference in the background dynamics.

²⁷ D. K. Hazra, L. Sriramkumar and J. Martin, arXiv:1201.0926v1 [astro-ph.CO].



f_{NL}^{eq} in models with a step



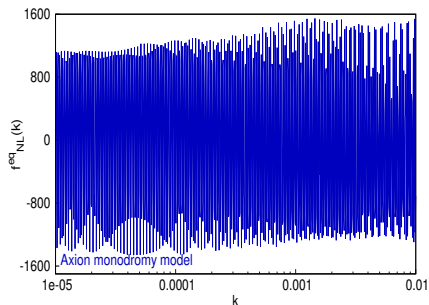
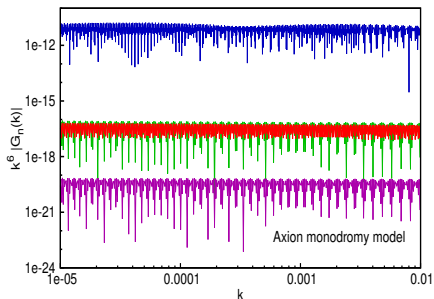
The contributions due to the various terms (on the left) and f_{NL}^{eq} due to the dominant contribution (on the right) when a step has been introduced in the popular chaotic inflationary model involving the quadratic potential²⁸. The f_{NL}^{eq} that arises in a small field model with a step has also been illustrated²⁹. The background dynamics in these two models are very similar, and hence they lead to almost the same f_{NL}^{eq} .

²⁸ X. Chen, R. Easther and E. A. Lim, JCAP **0706**, 023 (2007); JCAP **0804**, 010 (2008);
 P. Adshead, W. Hu, C. Dvorkin and H. V. Peiris, Phys. Rev. D **84**, 043519 (2011);
 P. Adshead, C. Dvorkin, W. Hu and E. A. Lim, Phys. Rev. D **85**, 023531 (2012).

²⁹ D. K. Hazra, L. Sriramkumar and J. Martin, arXiv:1201.0926v1 [astro-ph.CO].



f_{NL}^{eq} in the axion monodromy model



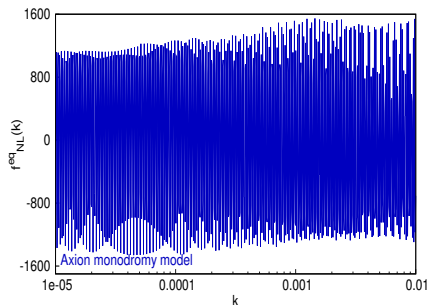
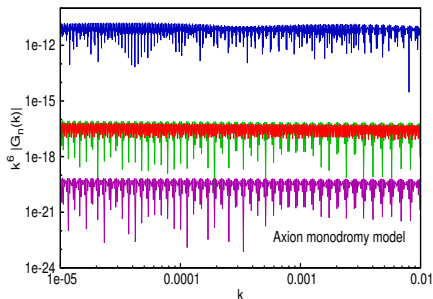
The contributions due to the various terms (on the left) and f_{NL}^{eq} due to the dominant contribution (on the right) in the axion monodromy model³⁰. The modulations in the potential give rise to a certain resonant behavior, leading to a large f_{NL}^{eq} ³¹.

³⁰D. K. Hazra, L. Sriramkumar and J. Martin, arXiv:1201.0926v1 [astro-ph.CO].

³¹S. Hannestad, T. Haugbolle, P. R. Jarnhus and M. S. Sloth, JCAP **1006**, 001 (2010);
R. Flauger and E. Pajer, JCAP **1101**, 017 (2011).



f_{NL}^{eq} in the axion monodromy model



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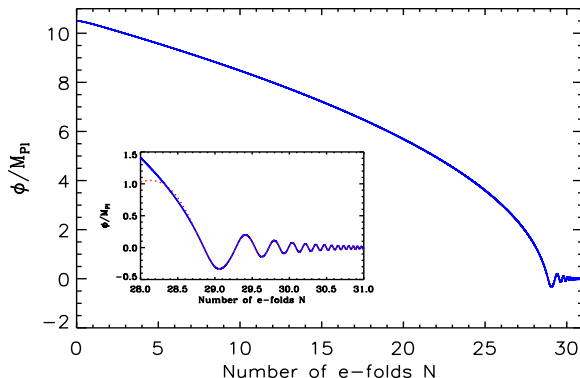
In contrast, the quadratic potential with superposed oscillations does not lead to such a large level of non-Gaussianity.

³⁰D. K. Hazra, L. Sriramkumar and J. Martin, arXiv:1201.0926v1 [astro-ph.CO].

³¹S. Hannestad, T. Haugbolle, P. R. Jarnhus and M. S. Sloth, JCAP **1006**, 001 (2010);
R. Flauger and E. Pajer, JCAP **1101**, 017 (2011).



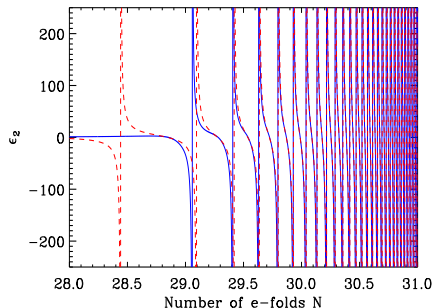
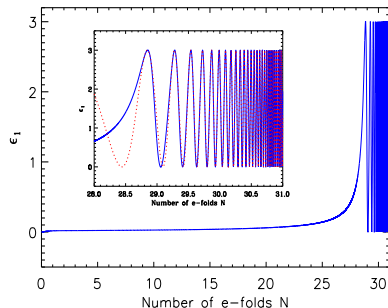
Behavior of the field in a quadratic potential



The behavior of the scalar field during the epochs of inflation and preheating have been plotted as a function of the number of e-folds for the case of the conventional chaotic inflationary model described by the quadratic potential. The blue curve denotes the numerical result, while the dotted red curve in the inset represents the analytical result.



Evolution of the slow roll parameters



The evolution of the first (on the left) and the second (on the right) slow roll parameters ϵ_1 and ϵ_2 as the field is oscillating about the quadratic minimum. As in the previous figure, the blue curves represent the numerical result, while the dashed red curves denote the analytical result during preheating. Note that, for the choice parameters and initial conditions that we have worked with, ϵ_1 turns unity at the e-fold of $N_e \simeq 28.3$, indicating the termination of inflation at the point.



The curvature perturbation during preheating

Though the modes of cosmological interest are well outside the Hubble radius at late times, the conventional super-Hubble solutions to the modes f_k during inflation do not *a priori* hold at the time of preheating.

This is due to the fact that, though $k/(aH)$ is small, because of the oscillating scalar field, the quantity z''/z itself can vanish during preheating. In fact, when the values of the parameters fall in certain domains known as the resonant bands, the modes display an instability³².

However, for the case of quadratic minima associated with mass, say, m , it can be shown that, the conventional, inflationary, super-Hubble solutions indeed apply provided the following two conditions are satisfied:

$$\left(\frac{k}{aH}\right)^2 \frac{H^2}{m^2} \ll 1 \quad \text{and} \quad \left(\frac{k}{aH}\right)^2 \frac{H}{3m} \ll 1.$$

Given that, $H < m$ immediately after inflation, it is evident that the first of the above two conditions will be satisfied if the second is³³.

³²F. Finelli and R. Brandenberger, *Phys. Rev. Lett.* **82**, 1362 (1999).

³³K. Jedamzik, M. Lemoine and J. Martin, *JCAP* **1009**, 034 (2010); *JCAP* **1004** 021 (2010).



Analytic solution during preheating

Up to the order k^2 , the dominant, super-Hubble, inflationary solution to the mode $f_{\mathbf{k}}$ is given by

$$f_{\mathbf{k}}(\eta) \simeq A_{\mathbf{k}} \left[1 - k^2 \int^{\eta} \frac{d\bar{\eta}}{z^2(\bar{\eta})} \int^{\bar{\eta}} d\tilde{\eta} z^2(\tilde{\eta}) \right].$$

The solutions for the background available when the potential around the minimum behaves quadratically allows us to actually evaluate the above integrals in a closed form.

We find that, during this epoch, the growing mode of the curvature perturbation can be written as

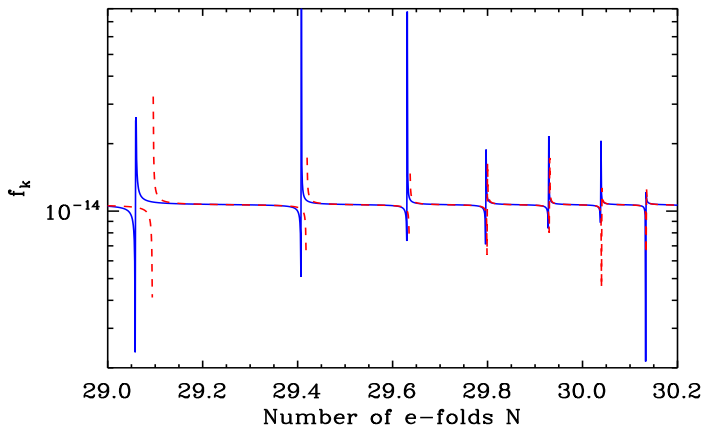
$$f_{\mathbf{k}} = A_{\mathbf{k}} \left[1 - \frac{1}{5} \left(\frac{k}{aH} \right)^2 \frac{H}{m} \tan(mt + \Delta) \right],$$

where Δ is a constant of integration³⁴.

³⁴R. Easther, R. Flauger and J. B. Gilmore, JCAP **1104**, 027 (2011).



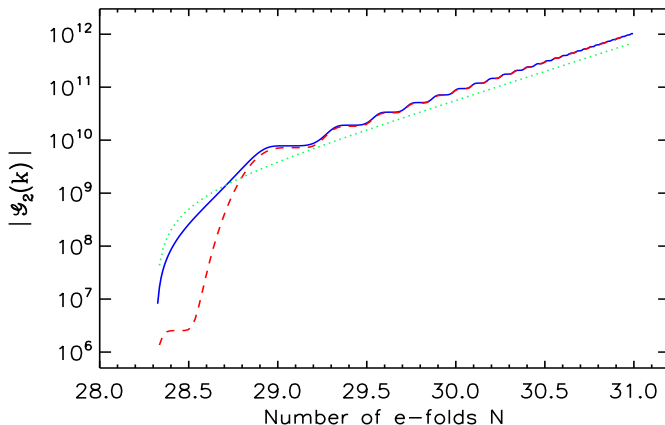
Comparison with the numerical result



The behavior of the curvature perturbation during preheating. The blue curve denotes the numerical result, while the dashed red curve represents the super-Hubble analytical solution. We have chosen to work with a very small scale mode that leaves the Hubble radius at about two e-folds before the end of inflation.



An illustration of the accuracy of the analytical result



The behavior of the quantity $\mathcal{G}_2(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ in the equilateral limit for a mode that leaves the Hubble radius at about 20 e-folds before the end of inflation. The blue curve represents the numerical result, while the dashed red curve denotes the analytical result³⁵.

³⁵D. K. Hazra, J. Martin and L. Sriramkumar, *Phys. Rev. D* **86**, 063523 (2012).



An estimate of the contribution to $f_{\text{NL}}^{\text{eq}}$ during preheating

Upon assuming inflation to be of the power law form, the contribution to the non-Gaussianity parameter f_{NL} during preheating can be obtained to be

$$\begin{aligned}
 f_{\text{NL}}^{\text{eq}}(k) &= \frac{115 \epsilon_1}{288 \pi} \Gamma^2 \left(\gamma + \frac{1}{2} \right) 2^{2\gamma+1} (2\gamma+1)^2 \sin(2\pi\gamma) |\gamma+1|^{-2(\gamma+1)} \\
 &\times \left[1 - e^{-3(N_f - N_e)/2} \right] \left[\left(\frac{\pi^2 g_*}{30} \right)^{-1/4} (1 + z_{\text{eq}})^{1/4} \frac{\rho_{\text{cri}}^{1/4}}{T_{\text{rh}}} \right]^{-(2\gamma+1)} \\
 &\times \left(\frac{k}{a_0 H_0} \right)^{-(2\gamma+1)},
 \end{aligned}$$

where g_* denotes the effective number of relativistic degrees of freedom at reheating, T_{rh} the reheating temperature and z_{eq} the redshift at the epoch of equality. Also, ρ_{cri} , a_0 and H_0 represent the critical energy density, the scale factor and the Hubble parameter today, respectively.

For a model with $\gamma \simeq -2$ and a reheating temperature of $T_{\text{rh}} \simeq 10^{10}$ GeV, one obtains that $f_{\text{NL}} \approx 10^{-60}$ for the modes of cosmological interest (*i.e.* for k such that $k/a_0 \simeq H_0$), a value which is completely unobservable³⁶.

³⁶D. K. Hazra, J. Martin and L. Sriramkumar, *Phys. Rev. D* **86**, 063523 (2012).



Summary

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- Further, we have shown that, in single field inflationary potentials with a quadratic minimum, the contributions to the bi-spectrum during preheating proves to be completely negligible.



Thank you for your attention