Effective field theory methods and the post-Newtonian framework

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Outline

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1. Effective Field Theory methods
   - Introduction
   - An Example of EFT at work
2. Binary inspiral and the PN approximation
   - EFT applied to 2-body systems
   - Algorithm for computing PN-Hamiltonian dynamics
   - The dissipative sector
3. Binary inspiral phenomenology
   - Application to GW's
   - Fundamental gravity tests

EFT principles: known fundamental theory

- Fundamental theory known:
effects of short distance physics r_s (heavy d.o.f. Λ)
on large distance physics $r \gg r_s$ (light modes $\omega \ll \Lambda$)

$$\begin{aligned} \exp\left(iS_{eff}[\phi]\right) &= \int \mathcal{D}\Phi(x) e^{iS[\phi,\Phi]} \\ S_{eff} &= \sum_i c_i \int d^d x\, O_i(x) \end{aligned}$$

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Wilson Coefficients
$c_i(\mu = \Lambda) \sim \Lambda^{\Delta - d}$

local operators of $\phi(x)$
mass dim. Δ: Decoupling

Renormalize existing coefficients and generates new ones
Dependence of large scale theory on small scale r given by simple power counting rule

- Hamiltonian dynamics in 2-body problem
- self-force in 2-body problem

EFT principles: unknown fundamental theory

- Fundamental theory unknown:
large scale effective Lagrangean can be expanded in terms of local operators

$$\downarrow$$

write down the most general set of operators consistent with long scale system symmetries with unknown coefficients.

$$\downarrow$$

Example: finite size effects in gravitational coupling of isolated bodies

EFT for isolated compact object

Fundamental

- Fundamental gravitational fields
- Fundamental coupling to particle world line

Effective

- List generic operators coupled to particle world-line
- Diffeomorphism invariance

$$S_{pp-fun} = -\sum_i m_i \int d\tau$$

Integrating out

$$\begin{aligned} S_{pp-eff} &= -m\int d\tau + c_R\int d\tau R + c_V\int d\tau R_{\mu\nu}\dot{x}^\mu\dot{x}^\nu + \\ & c_E\int d\tau E_{\mu\nu}E^{\mu\nu} + c_B\int d\tau B_{\mu\nu}B^{\mu\nu} + \ldots \end{aligned}$$

$$E_{\mu\nu} \equiv R_{\mu\alpha\nu\beta}\dot{x}^\mu\dot{x}^\nu \qquad B_{\mu\nu} = \epsilon_{\mu\rho\sigma\alpha}R^{\rho\sigma}_{\nu\beta}\dot{x}^\alpha\dot{x}^\beta$$

EFT for isolated compact object

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EFT applications

- Cosmology
  Generic gravity Lagrangean invariant under spatial rotations (time-dependent space diffeomorphisms)
  Short vs. Large
  inflaton fluctuation vs. Hubble scale of the background

  See P. Cheung et al. 2007

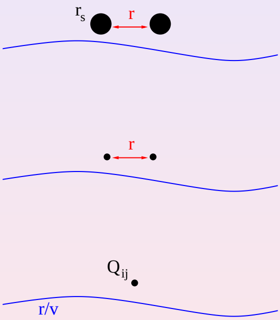
- Hydrodynamics
  Derivative expansions:
  Short vs. Large
  Field time derivative vs. mean free time
  Field space derivatives vs. mean free length

  See Dubovsky et al. 2011

Outline

Different scales in EFT

- Very short distance $\lesssim r_s$ negligible up to 5PN (effacement principle)
- Short distance: potential gravitons $k_\mu \sim (v/r, 1/r)$
- Long distance: GW's $k_\mu \sim (v/r, v/r)$ coupled to point particles with moments

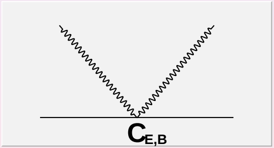


Matching example

Cross section for graviton scattering by a single black hole:

$$\sigma_{fund-BH} = r_s^2 f(r_s\omega) \sim \ldots r_s^6\omega^4 + \ldots r_s^{10}\omega^8 \ldots$$

Effective contribution to the amplitude:



$$\frac{c_{E,B}}{m_{Pl}^2}\omega^4$$

$$\sigma_{EFT-BH} \sim \ldots + r_s \frac{c_{E,B}}{m_{Pl}^2}\omega^4 + \ldots + \frac{c_{E,B}^2}{m_{Pl}^4}\omega^8 \implies c_E \propto m_{Pl}^2 r_s^5$$

The Transverse-Traceless gauge

$h_{\mu\nu}$ includes

1. 4 gauge degrees of freedom
2. 2 physical, radiative degrees of freedom
3. 4 physical, non-radiative degrees of freedom

1&3 propagate with "the speed of thought" (Eddington '22)
After fixing the diffeomorphism invariance:

$$h_{\mu\nu} = \begin{pmatrix} -2\Phi & \Xi_i \\ \Xi_i & h^{TT}_{ij} + \theta\delta_{ij} \end{pmatrix}$$

$\partial_i \Xi^i = h^{TT}_{ij}\delta^{ij} = \partial^i h_{ij} = 0$: 6 degrees of freedom left, 4 eaten by gauge fixing
Einstein eq's:

$$\begin{aligned} \nabla^2\Phi = \nabla^2\Xi_i = \nabla^2\Theta &= 0 \\ \Box h^{TT}_{ij} &= 0 \end{aligned}$$

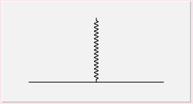
Conservative dynamics

$$\exp\left[iS_{eff}(x_a)\right] = \int \mathcal{D}h(x) \exp\left[iS_{EH}(h) + iS_{pp}(h, x_a)\right]$$

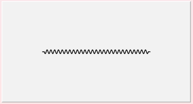
$$\begin{aligned} S_{pp} &= -\frac{m}{m_{Pl}} \int dt \left(h_{00}/2 + v_i h_{0i} + v^i v^j h_{ij}/2 + \ldots\right) \\ S_{EH} &= \int d^4x (\partial_i h)^2 - (\partial_t h)^2 + \frac{h(\partial h)^2}{m_{Pl}} + \ldots \end{aligned}$$

Power counting to integrate out potential gravitons

h-M Vertex: $\sim dt\, d^3k \frac{m}{M_{Pl}}$

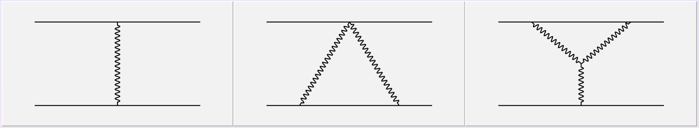


Propagator: $\delta(t)\frac{1}{k^2}\delta^{(3)}(k)$



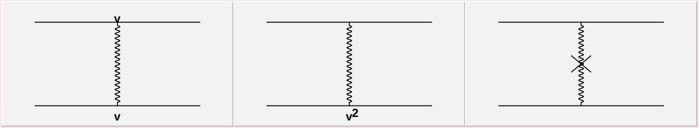
$\ldots$

The 1PN potential



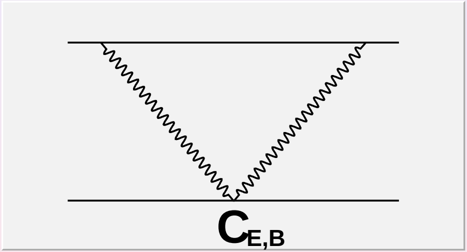
Scaling: L $\qquad\qquad Lv^2$

Using virial theorem $v^2 \sim G_N M/r$



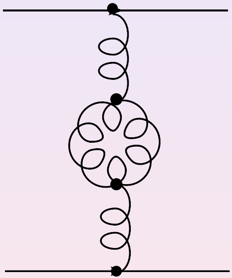
$$V = -\frac{Gm_1m_2}{2r}\left[1 - \frac{G_N m_1}{2r} + \frac{3}{2}(v_1^2) - \frac{7}{2}v_1 v_2 - \frac{1}{2}v_1\hat{r}v_2\hat{r}\right] + 1 \leftrightarrow 2$$

Finite size effects enters at 5PN



Lv^{10}

Quantum corrections are irrelevant



v^3

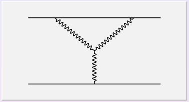
"Usual" rule for quantum weight $\hbar^{I-V} = \hbar^{L-1}$

What's new to EFT in gravity?

The '+'s with respect to standard approach

- Built-in regularization mechanism enable to treat divergencies (though dimensional regularization used also in traditional PN computation)
- Systematic use of Feynman diagram with manifest power counting rule, enabling the construction of automatized algorithms
- Effective 2-body action is produced without the need to solve for the metric
- recast old problems in a field theory language

The 3PN computation automatized

Topologies 

Graphs — v and time derivative-insertions

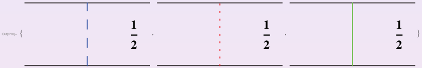
Amplitudes
$$A = G_N m_i v_i \int d^d k\, d^d k_1 \frac{1}{k^2 (k-k_1)^2} \cdots$$

Evaluation — Analytic integral in a database

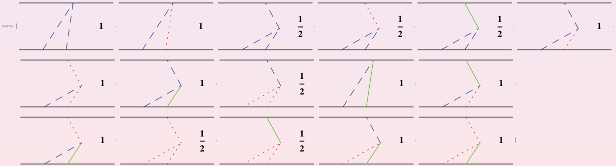
S. Foffa & RS PRD 2011

Feynman diagrams at 3PN order

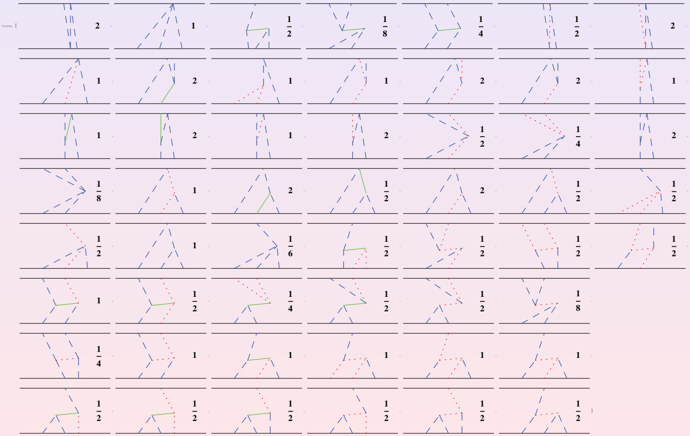
$G_N v^6$



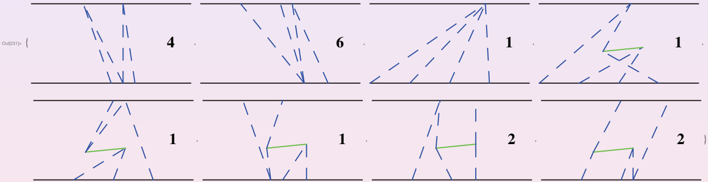
$G_N^2 v^4$



Feynman diagrams at 3PN order: $G_N^3 v^2$



Feynman diagrams at 3PN order: G_N^4



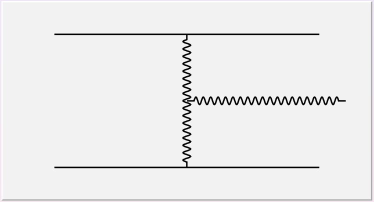
Final result matches previous derivation of 3PN Hamiltonian,
see eq. (174) of Blanchet's Living Review on Relativity

The 4 PN status

- 3 graphs @ $G_N v^8$ order
- 23 @ $G_N^2 v^6$
- 202 @ $G_N^3 v^4$
- 307 @ $G_N^4 v^2$
- 50 @ G_N^5

The dissipative sector

Coupling gravitational waves in all possible ways to the composite systems e.g.

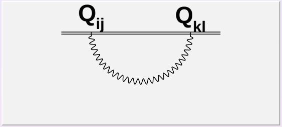


$+\ldots$

$$S_{EFT-diss} = -\frac{m}{m_{Pl}}\int h_{00} - \left[\frac{1}{2}\sum_a m_a v_a^2 - \frac{G_N m_1 m_2}{r}\right]\frac{h_{00}}{2m_{Pl}} - \frac{1}{2m_{Pl}}\epsilon_{ijk}L_k\partial_j h_{0i} + \frac{1}{2m_{Pl}}\sum_a m_a x_i x_j R_{0i0j}$$

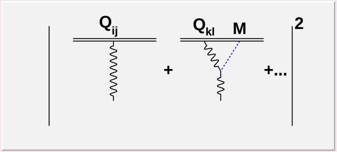
Radiation reaction

Radiation emitted and absorbed



Effective action modified:

Imaginary part $\longrightarrow$ power loss



Optical theorem

(Goldberger and Ross PRD '10)

Real part $\longrightarrow$ modifies e.o.m.

(Galley a Tiglio PRD '09)

Conservative part of the self force

- At leading order the SF affects e.o.m. at 2.5PN order

Burke-Thorne radiation reaction

$$\Delta^{(SF)}\ddot{x}_{ai}(t) = \frac{2G_N}{5}x_{aj}(t)Q^{(5)}_{ij}(t) - \frac{8}{5}G_N^2 M x_{aj}\int_{-\infty}^{t} dt' Q^{(7)}_{ij}(t')\log\left[\frac{(t-t')}{T}\right]$$

relative 1.5PN tail correction

- Conservative part associated with tail integral

$$\Delta^{(SF)}\ddot{x}_{ai}(t) = \frac{8G_N^2 M}{5}x_{aj}(t)Q^{(6)}_{ij}(t)\log\left(\frac{r}{\lambda}\right)$$

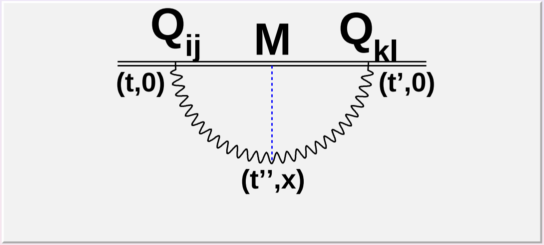
Gravitational radiation emitted, scattered, and absorbed.

L. Blanchet and T. Damour PRD '88

L. Blanchet, S.L. Detweiler, A. Le Tiec, B. F. Whiting PRD '10

Real part of the self-force diagram

Radiation emitted, scattered and absorbed



$$
\begin{aligned}
iS_{eff} \quad \propto \quad & G_N^2 M \int dt\, Q^{(2)}_{-ij}(t) \int dt' Q^{(2)}_{+ij}(t') \times \\
& \int dt'' d^3x\, \partial_t^2 G_R(t-t'',x) G_R(t''-t',x) \frac{1}{r}
\end{aligned}
$$

Regularization and renormalization

$$
\begin{aligned}
iS_{eff} \quad &= \quad -i\frac{4G_N^2 M}{5}\int dt\, Q_{-ij}(t) \times \\
&\left\{ -Q^{(6)}_{+ij}(t')\log(t-t')\Big|_{-\infty}^{t} + \int_{-\infty}^{t} dt' Q^{(7)}_{+ij}(t')\log\left[(t-t')\mu\right] \right\}
\end{aligned}
$$

Arbitrary scale μ, renormalized multipole

$$
Q_{ij}^{(Bare)}(\omega) = Z(\omega,\mu)Q_{ij}^{(Ren)}(\omega,\mu)
$$

With $\mu \to 1/r$ and using

$$
\log[(t-t')\mu] = \log\left(\frac{t-t'}{\lambda}\right) - \log\left(\frac{r}{\lambda}\right)
$$

$$
\Delta^{(QQM)}_{cons}\ddot{x}_{ai} = \frac{8}{5} x_{aj} Q^{(6)}_{ij}\log(r/\lambda)
$$

Renormalization in Fourier space

Classical renormalization from UV effect to real part of S_{eff}

$$S_{eff}^{(R)} = -\frac{G_N}{5}\int_{-\infty}^{\infty} dk\, Q_{-ij}(k)Q_{+ij}(-k)\left[(-ik)^5 + 4G_N M\,(-ik)^6\left(\log^{(UV)}\left(\frac{k^2}{\mu^2}\right)+c\right)+\ldots\right]$$

vs. imaginary part computed via optical theorem

$$S_{eff}^{(I)} = \frac{G_N}{10}\int_0^{\infty} dk\, Q_{ij}(k)Q_{ij}(-k)(-ik)^5\left\{1 + 2\pi G_N Mk + (G_N Mk)^2\left[-\frac{214}{105}\log^{(UV)}\left(\frac{k^2}{\mu^2}\right)+c'\right]+\ldots\right\}$$

found in Goldberger and Ross PRD '10

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GW detection

Inspiral

Virial relation:

$$v \equiv (G_N M \pi f_{GW})^{1/3} \qquad \nu = \frac{m_1 m_2}{(m_1+m_2)^2}$$

$$E(v) = -\frac{1}{2}\nu M v^2 \left(1 + \#(\nu)v^2 + \#(\nu)v^4 + \dots\right)$$

$$P(v) \equiv -\frac{dE}{dt} = \frac{32}{5G_N} v^{10} \left(1 + \#(\nu)v^2 + \#(\nu)v^3 + \dots\right)$$

E(v)(P(v)) known up to 3(3.5)PN

$$\frac{1}{2\pi}\phi(T) = \frac{1}{2\pi}\int^T \omega(t)dt = -\int^{v(T)} \frac{\omega(v)dE/dv}{P(v)} dv$$

$$\sim \int \left(1 + \#(\nu)v^2 + \dots + \#(\nu)v^6 + \dots\right)\frac{dv}{v^6}$$

GW detection

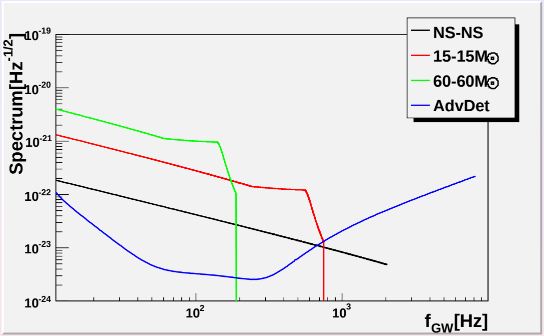
$$N_{cycles} \simeq 1.6 \cdot 10^4 \left(\frac{10\mathrm{Hz}}{f_{min}}\right)^{5/3} \left(\frac{1.2 M_\odot}{M_c}\right)^{5/3}$$

Sensitivity $\propto M_c^{5/3}\sqrt{N_{cycles}} \propto M_c^{5/6}$

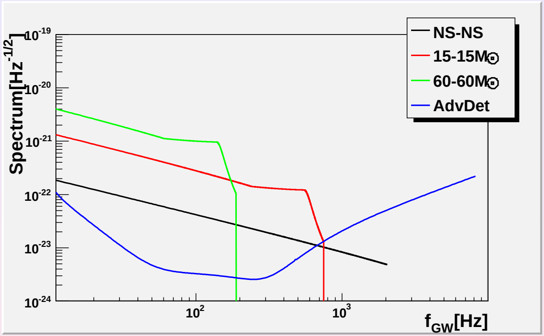
$f_{Max} \propto M^{-1}$, $M_c \equiv (m_1 m_2)^{3/5}(m_1+m_2)^{2/5}$

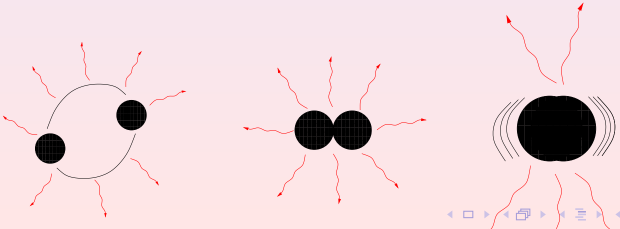
Important to know the phase at $O(1)$ when taking correlation of detector's output and model waveform

Detector sensitivity



Detector sensitivity





Observational rate estimates

LIGO/Virgo Advanced Observatories will detect

	NS-NS	$10\ M_\odot$ BH-BH
Distance (Mpc)	$300\mathrm{Mpc}$	$1\mathrm{GPc}$
Rates $\mathrm{MWEG}^{-1}\mathrm{Myear}^{-1}$	$1 \div 10^3$	$4 \cdot 10^{-2} \div 100$

$$N = 0.011 \times \frac{4}{3}\pi \left(\frac{D_H}{2.26 Mpc}\right)^3 \mathrm{MWEG}$$

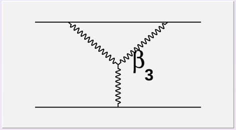
Best case:

$$r_{NS-NS} \sim 400\mathrm{yr}^{-1} \qquad r_{BH-BH} \sim 10^3\mathrm{yr}^{-1}$$

I. Mandell et al. PRD 2010

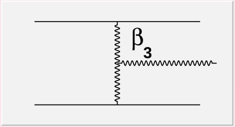
Graviton self-interaction vertices

- Conservative dynamics



$$V \supset \beta_3 \frac{G_N^2 m_1 m_2 (m_1 + m_2)}{r^2}$$

- Emission



$$L_{pp} \supset h_{ij} \beta_3 (\nu M x_i \ddot{x}_j)$$

Example of tagging of fundamental physics effects
β_3 is not a viable modification of General Relativity

Bound of soelf-interaction triple vertex

At present the binary pulsars give best constraint on non-conservative effect from β_3

$$\dot{P}_{\beta_3} = \dot{P}_{GR}(1 + c\beta_3) \qquad c \simeq 3.21$$

Given that $\frac{\dot{P}_{obs}}{\dot{P}_{GR}} - 1 \simeq 0.1\% \implies \beta_3 = (4.0 \pm 6.4) \cdot 10^{-4}$
Conservative effect of β_3 already constrained by Lunar Laser Ranging, as @ 1PN

$$\beta_3 = \beta_{PPN} < 2 \cdot 10^{-4}$$

Cannella et al. '09

Bayesian analysis of GR vs. modGR

Searching for waveforms whose phase is modified at any PPN waveforms

$$\begin{aligned}\phi(t) &= \phi_N(t)\left[1+\phi_1(t)(1+\delta_1)+\phi_{1.5}(1+\delta\phi_{1.5})\right.\\ &\quad \left.+\phi_2(1+\delta\phi_2)\right]\end{aligned}$$

and injecting fake signals with

$$\phi_{inj}(t)=\phi_{GR}(t)+\phi_N(t)\delta\phi_A(t)$$

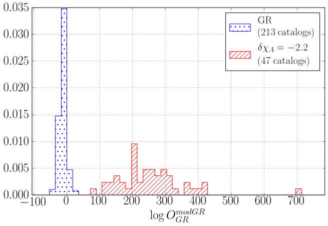
Li et al. 2011

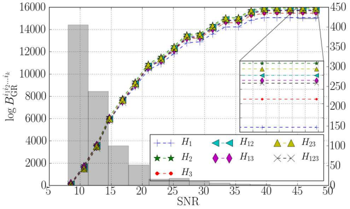
Baysian analysis of GR vs. modGR

$$
\begin{aligned}
O_i &= P(H_i|d) \\
&= P(H_i)\frac{P(d|H_i)}{P(d)}
\end{aligned}
$$

$$
\begin{aligned}
O_{GR}^{mGR} &= \frac{O_{mGR}}{O_{GR}} \\
&\propto \frac{P(d|H_{mGR})}{P(d|H_{GR})}
\end{aligned}
$$

(1 catalog = 15 sources)





Conclusions

- EFT is powerful, PN computation are catching up with the traditional methods
- Universal tools applicable to problems exhibiting clear scale separation