I-Love-Q

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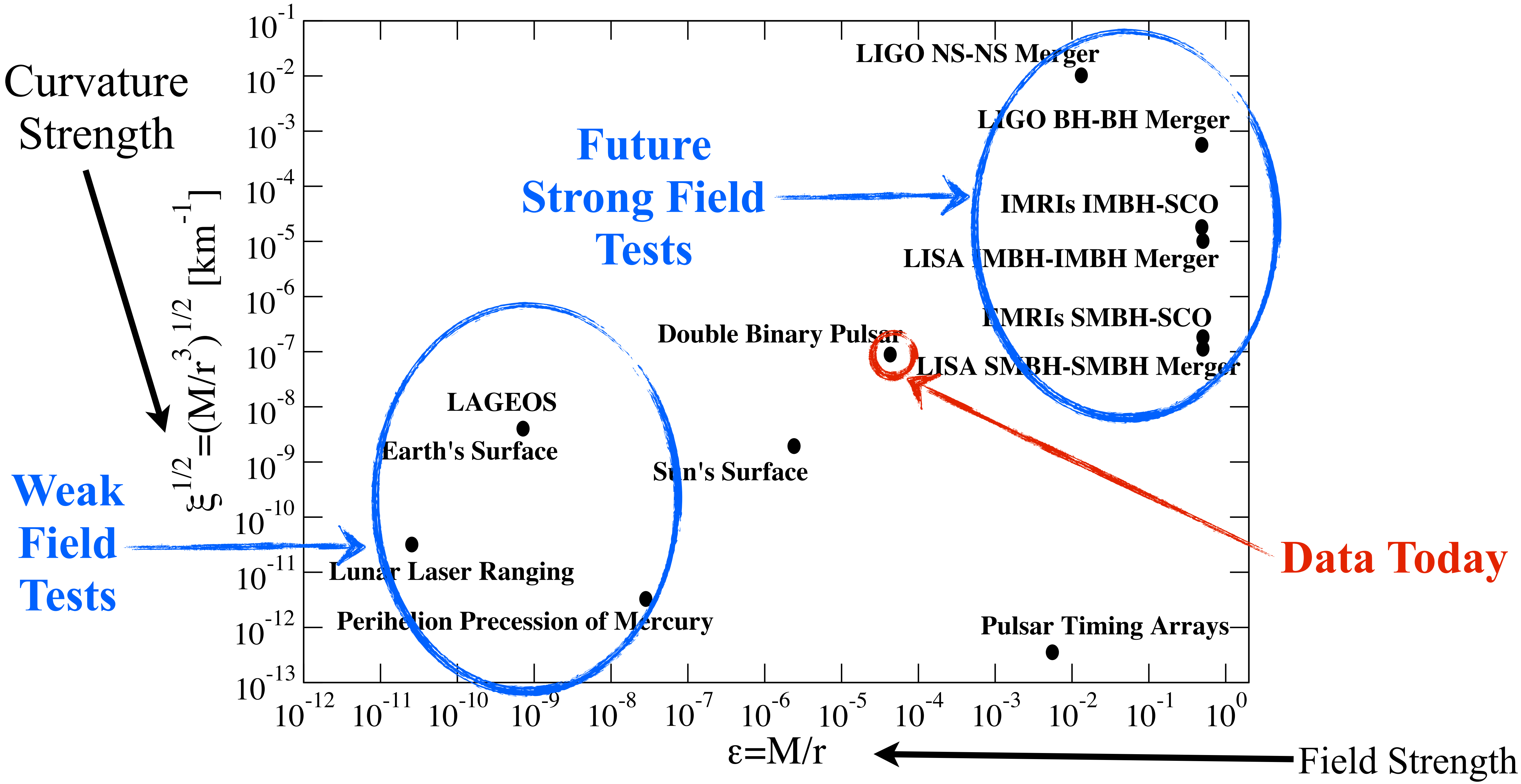
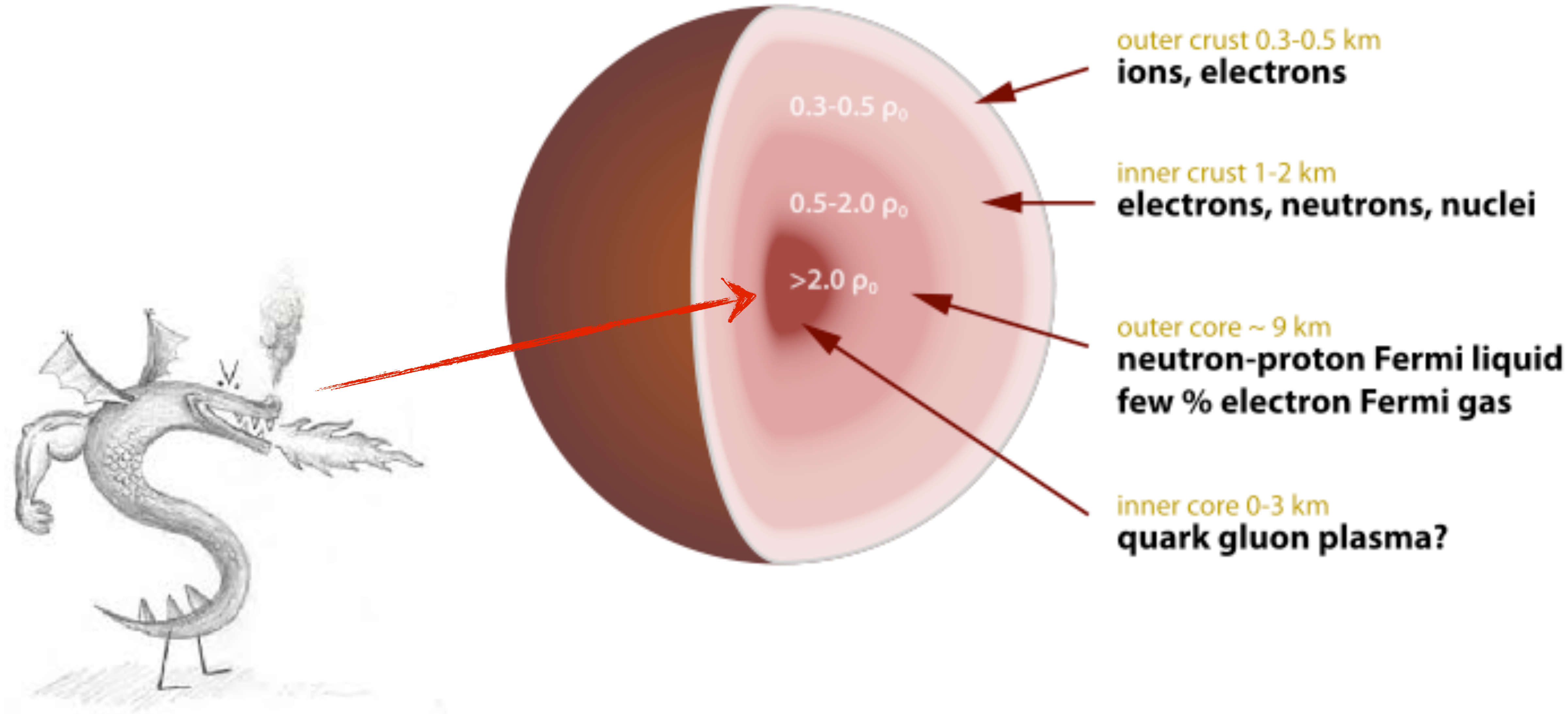
A Laboratory for the Extreme...



NSs can probe the non-linear, weakly-dynamical regime

[Will, Liv. Rev., 2005, Psaltis, Liv. Rev., 2008, Siemens & Yunes, Liv. Rev. 2012 to appear,]

But Dragons are Scary...



How can we do any physics if we do not understand the interior?

Roadmap

A Relativist in the Neutron Star's Court

I-Love-Q

Applications

Observational Astrophysics

Gravitational Waves

Experimental Relativity

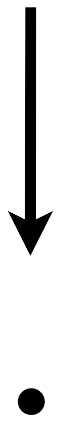
A Relativist in a Neutron Star's Court

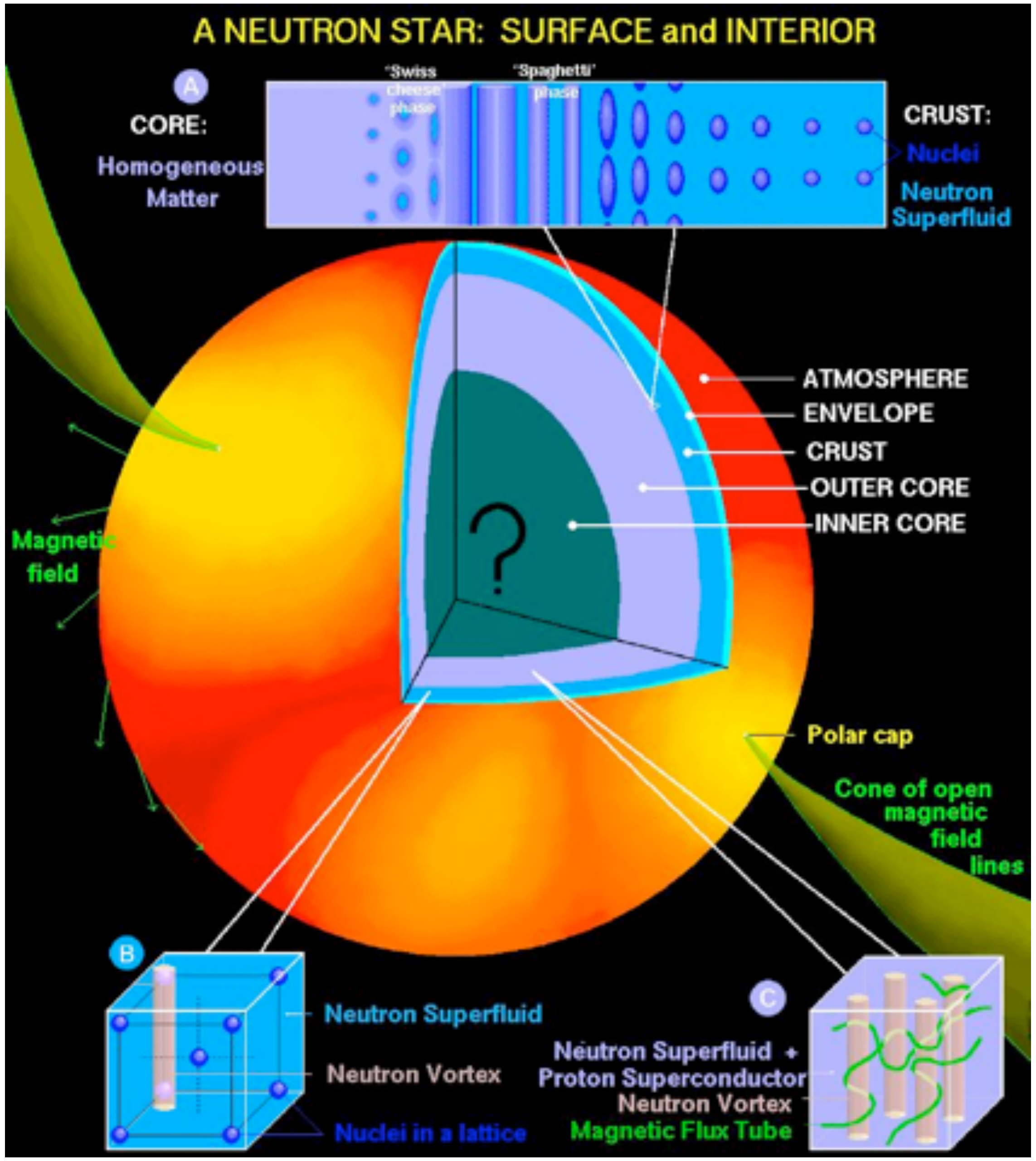
You say Tomato, I say point particle

For a Relativist

For a real astrophysicist

$$T^{\alpha\beta}(x) = m \int_\gamma \frac{g^\alpha{}_\mu(x,z) g^\beta{}_\nu(x,z) \dot{z}^\mu \dot{z}^\nu}{\sqrt{-g_{\mu\nu}\dot{z}^\mu \dot{z}^\nu}} \delta_4(x,z) d\lambda$$





Relativistic Stellar Structure

Einstein Equations

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

Defines the theory of gravity

Equation of Motion

$$\nabla^{\mu} T_{\mu\nu} = 0$$

Ensures theory is gauge invariant

Equation of State

$$p = p(\rho)$$

Describe Neutron Star Interior

$$g_{\mu\nu} = g_{\mu\nu}(t, r, \theta, \phi)$$

Physical Properties of Neutron Stars

The Method to our Madness...

$$g_{\mu\nu}(\not{t}, r, \theta, \not{\phi})$$

$$p = p(\not{r}, \rho, \not{\Omega}, \not{T}, \not{E}, \not{B}, \ldots)$$

Axisymmetric spacetime.

Stationary spacetime.

Slow rotation of neutron star:
Expand in J << M^2.

Uniform rotation.

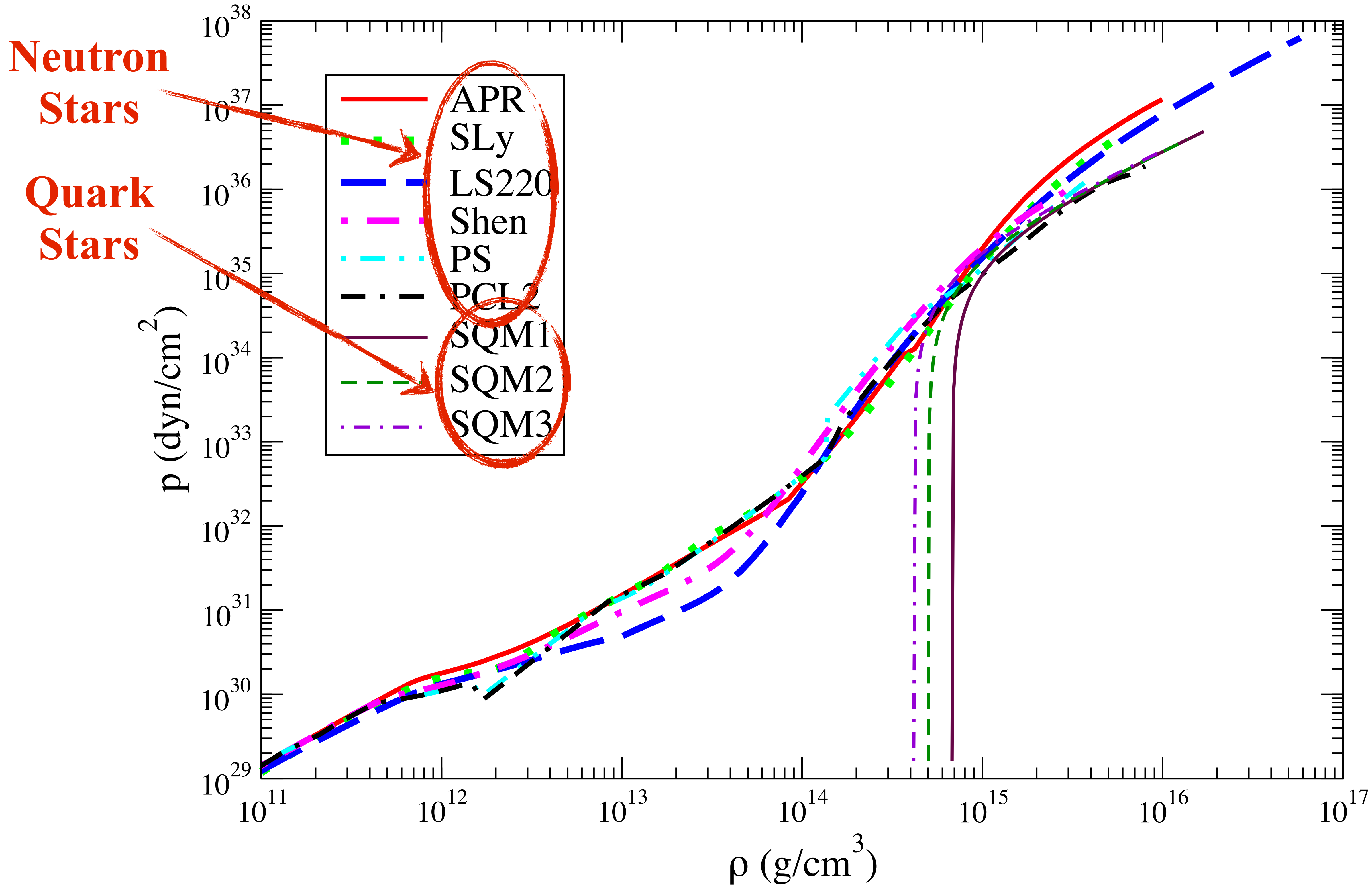
Cold Neutron Star.

Uncharged Neutron Star.

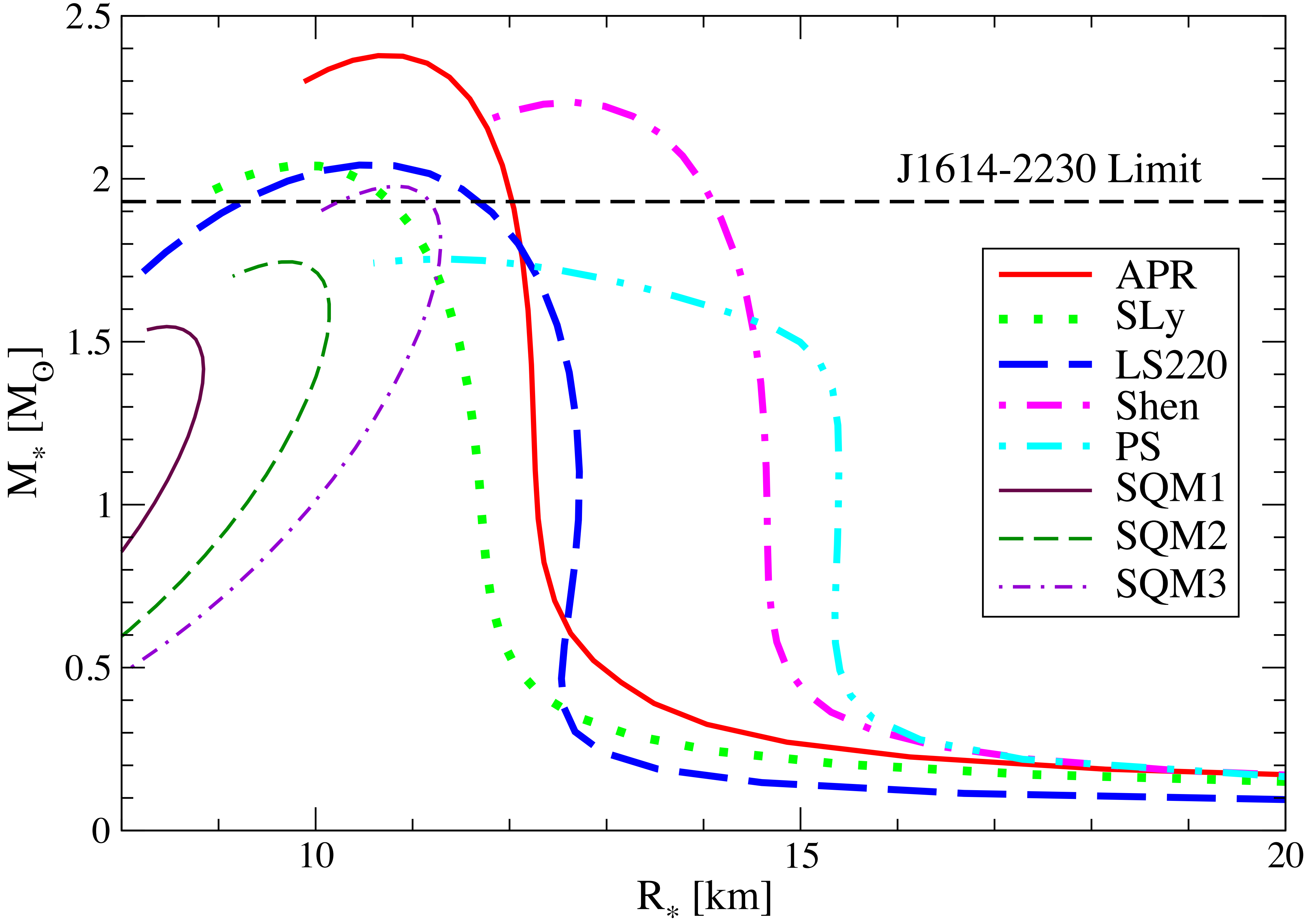
Isotropic pressure.

Small deformation:
Expand in M/b<<1

Equations of State



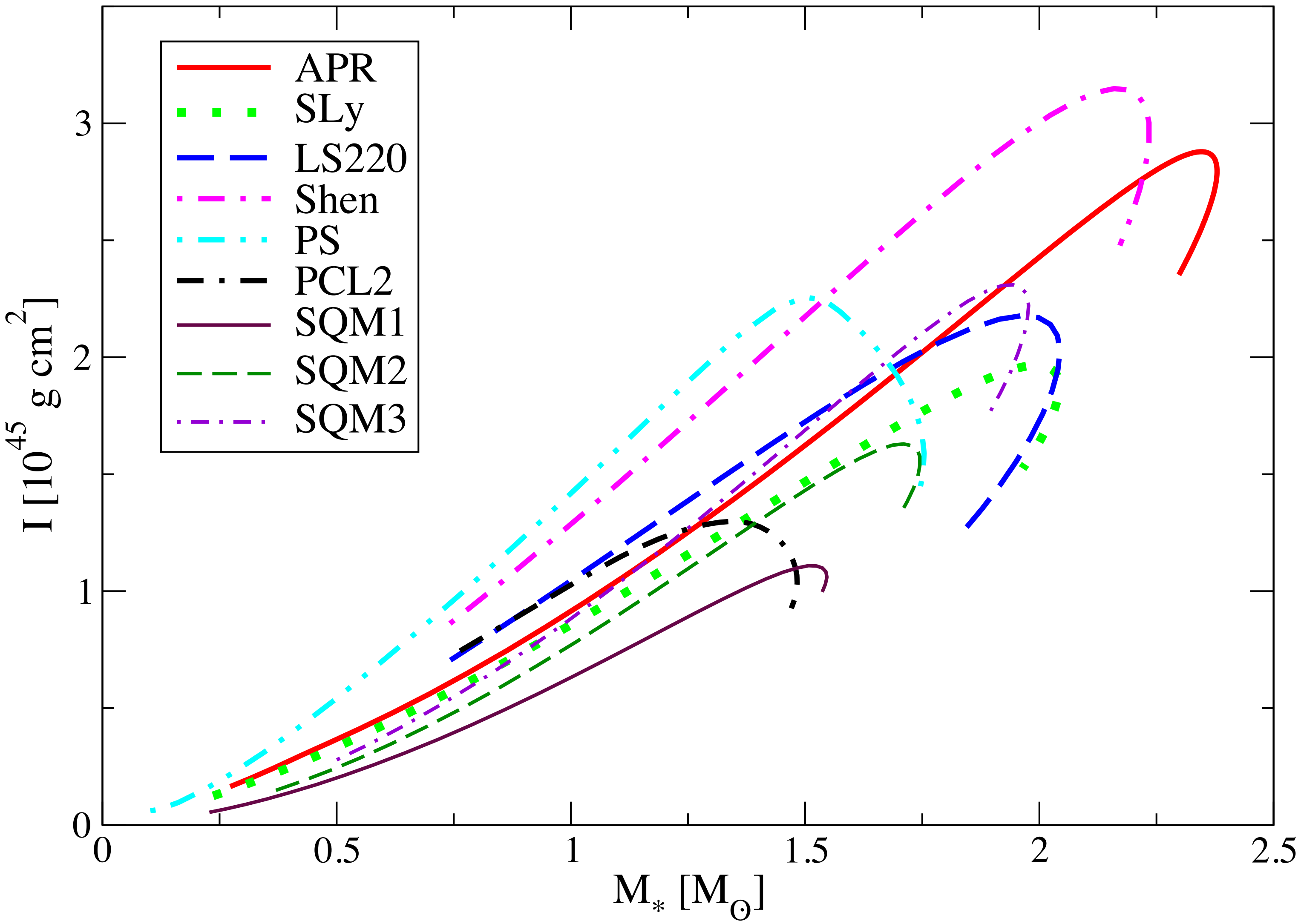
Mass-Radius Curves



Different Eos = Different Star

Moment of Inertia

Resistance to a change in angular velocity. ← $I \equiv \frac{J}{\Omega}$ → Spin Angular Momentum (J), Spin Angular Velocity (Ω)

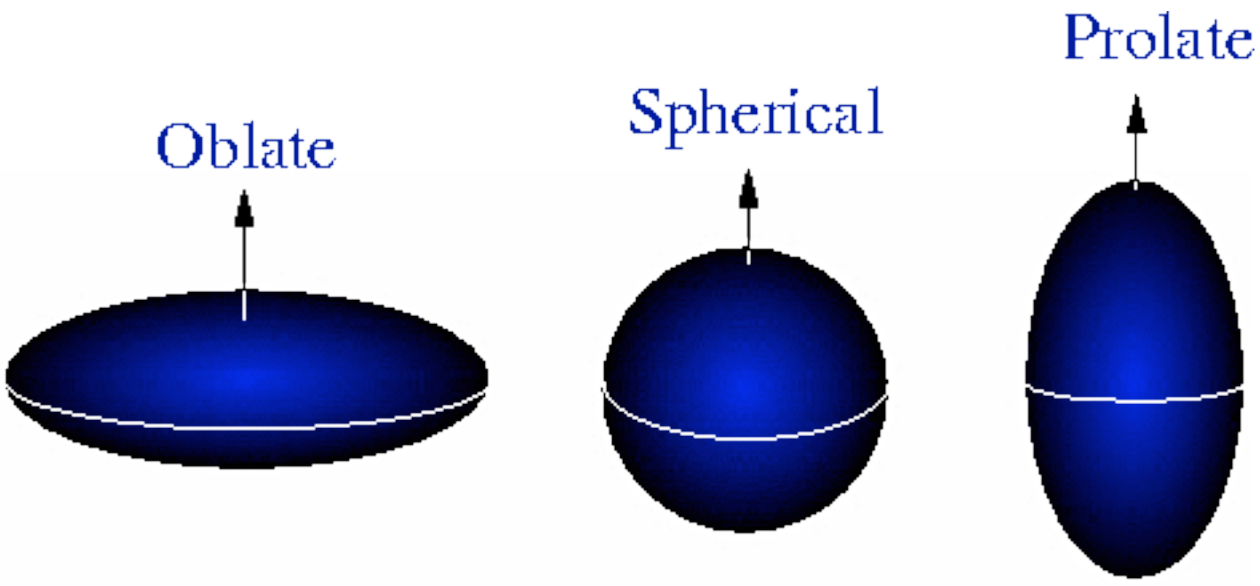


No Universality in the I-M relation.

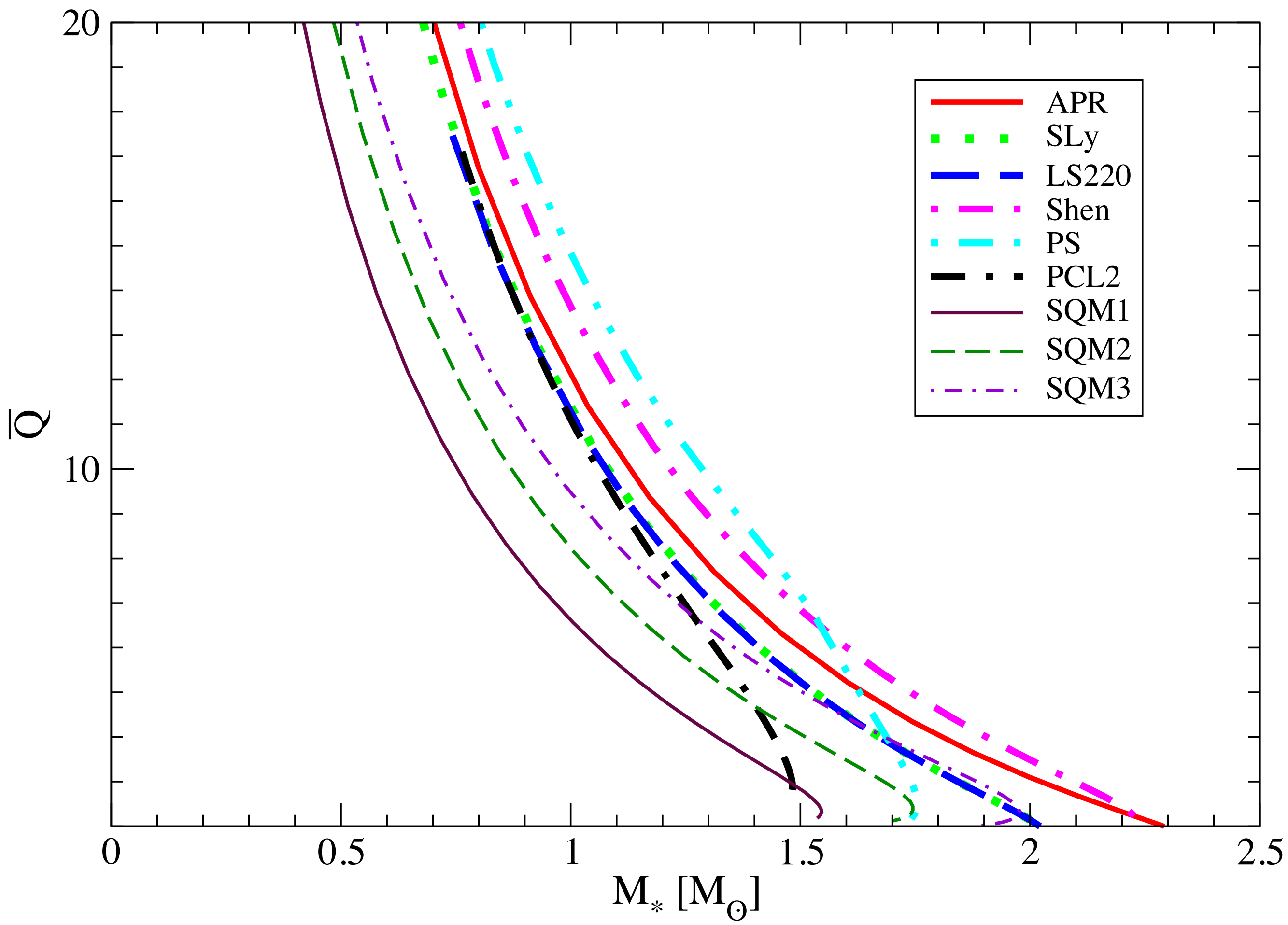
Quadrupole Moment

$$V(x) = -\frac{M}{r} - Q\frac{P_2(\cos\theta)}{r^3} + \mathcal{O}\left(\frac{1}{r^4}\right)$$

Potential ← (equation) → Far-Field Expansion



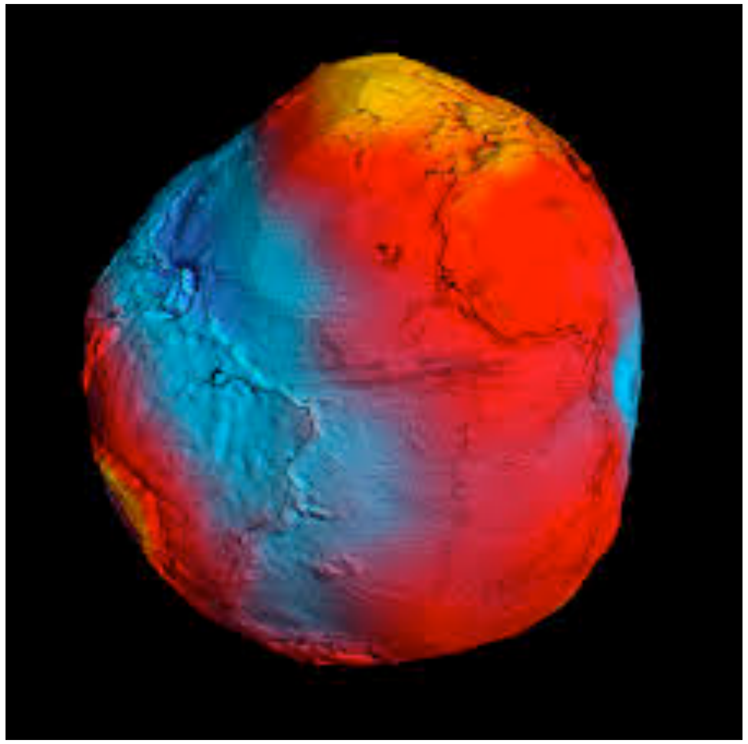
$$Q \sim \frac{J^2}{M} \sim \frac{I^2\Omega^2}{M} \sim R^5\Omega^2$$



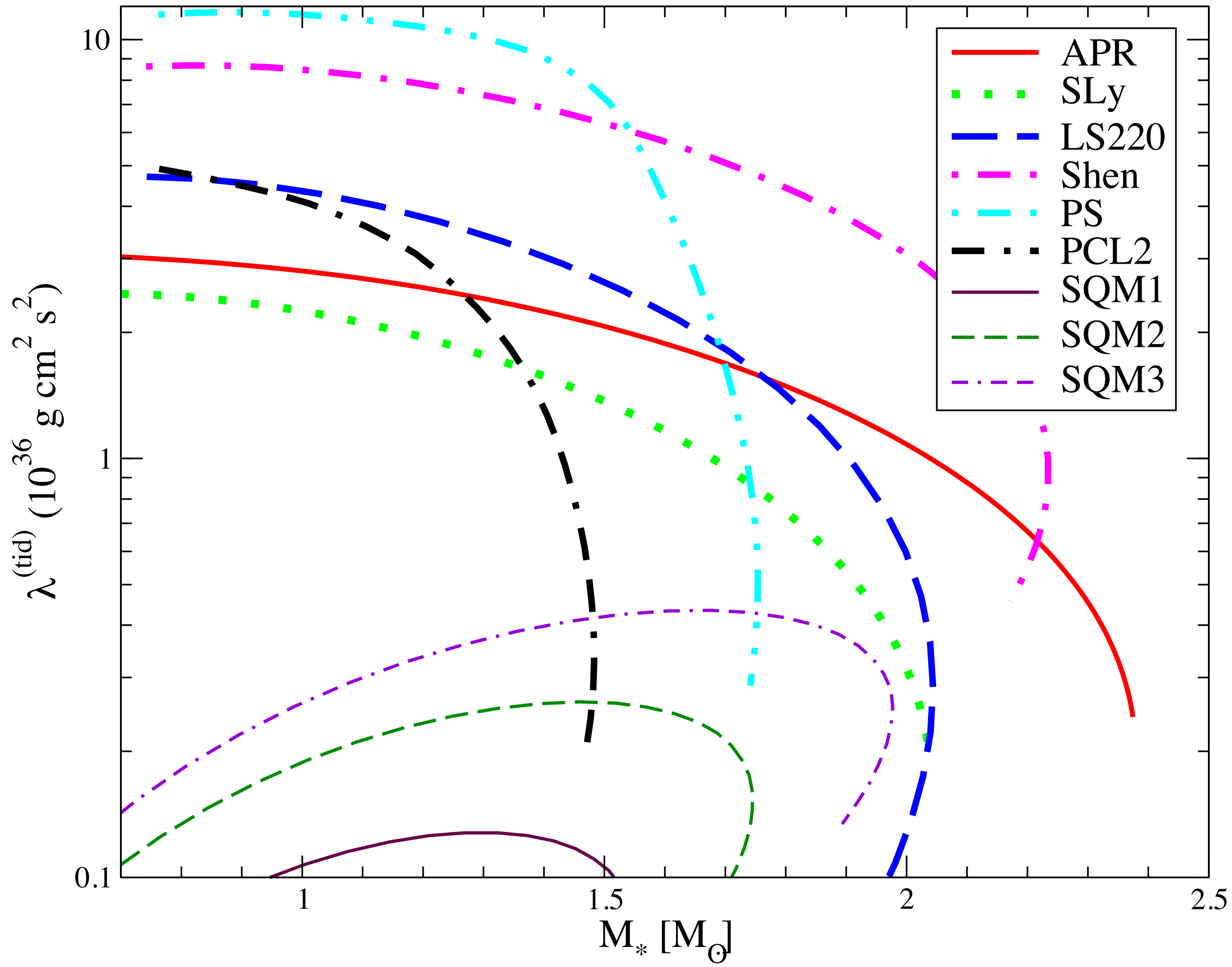
No Universality in the Q-M relation.

Tidal Love Number

Rigidity and Susceptibility of a body's shape to change in response to a tidal potential ← $\lambda = \frac{Q}{\mathcal{E}}$ → Quadrupole Moment, Disturbing Tidal Potential



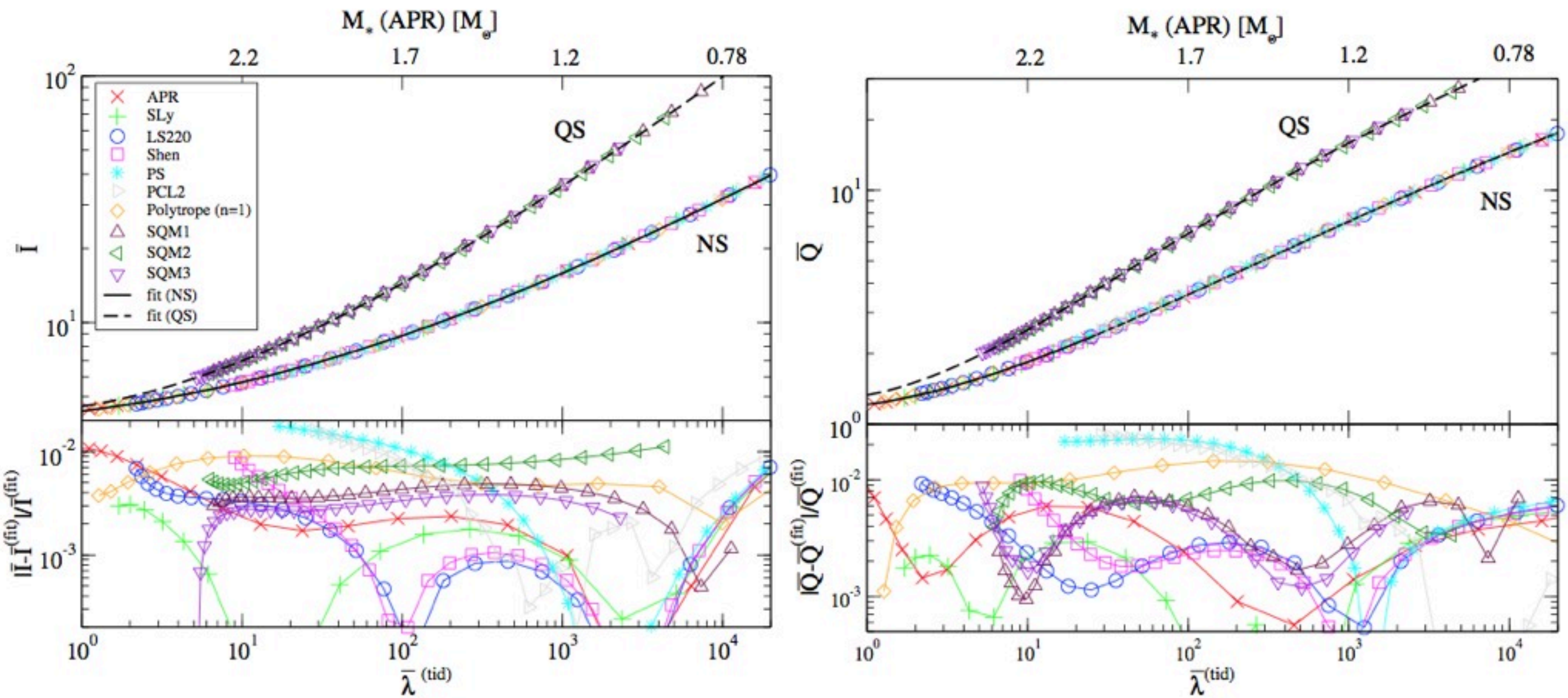
$$\lambda^{(rot)} \sim \frac{R^5\Omega^2}{\Omega^2} \sim R^5$$



No Universality in the I-M relation.

I-Love-Q

Universality is Ubiquitous...

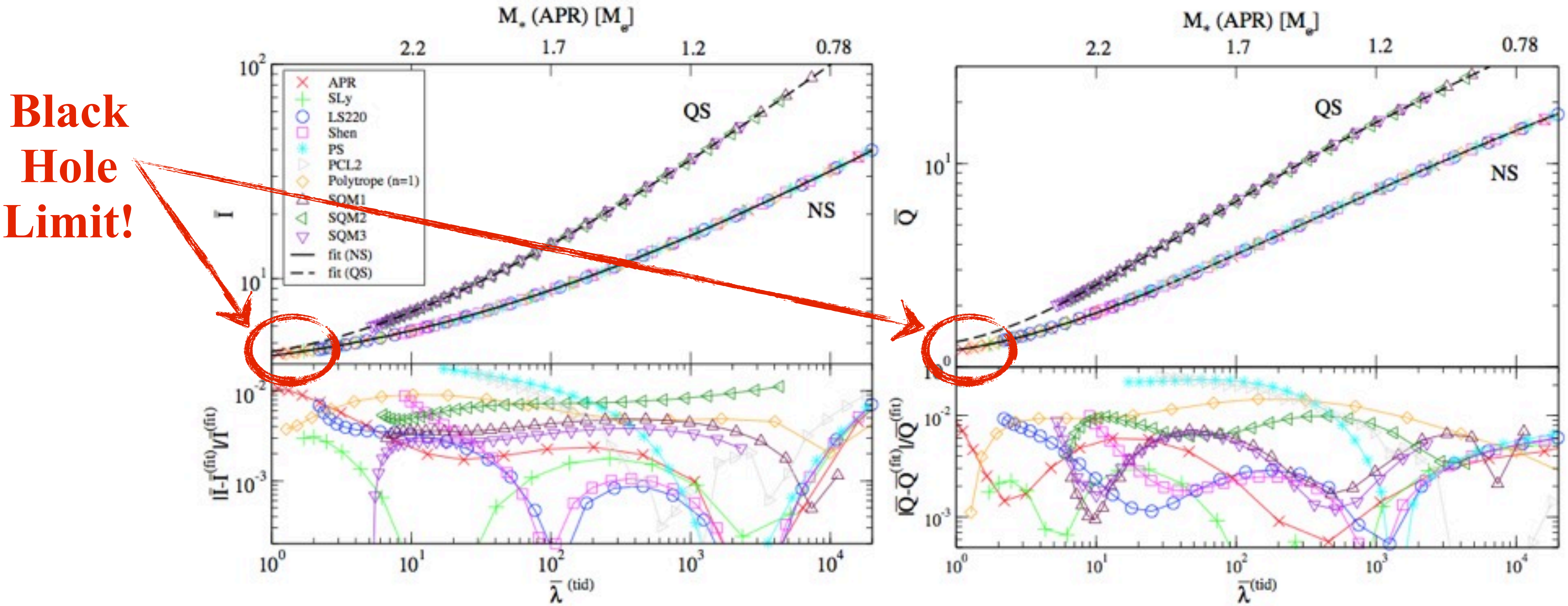


The moment of inertia, quadrupole moment and Love number satisfy Universal, EoS-independent relations!

$$\bar{Q} = \frac{Q}{M^3\chi^2} \quad \bar{\lambda} = \frac{\lambda}{M^5} \quad \bar{I} = \frac{I}{M^3}$$

Why I-Love-Q?

We don't know...but



Black Hole no-hair theorems require: $\bar{I} \to 4, \bar{\lambda} \to 0, \bar{Q} \to 1$

Could there be an effective no-hair theorem for neutron stars?

For example:

$$\bar{I} = \frac{1}{M^3}\frac{J}{\Omega} = \frac{1}{M^3}\frac{Ma}{a/(4M^2)} = 4$$

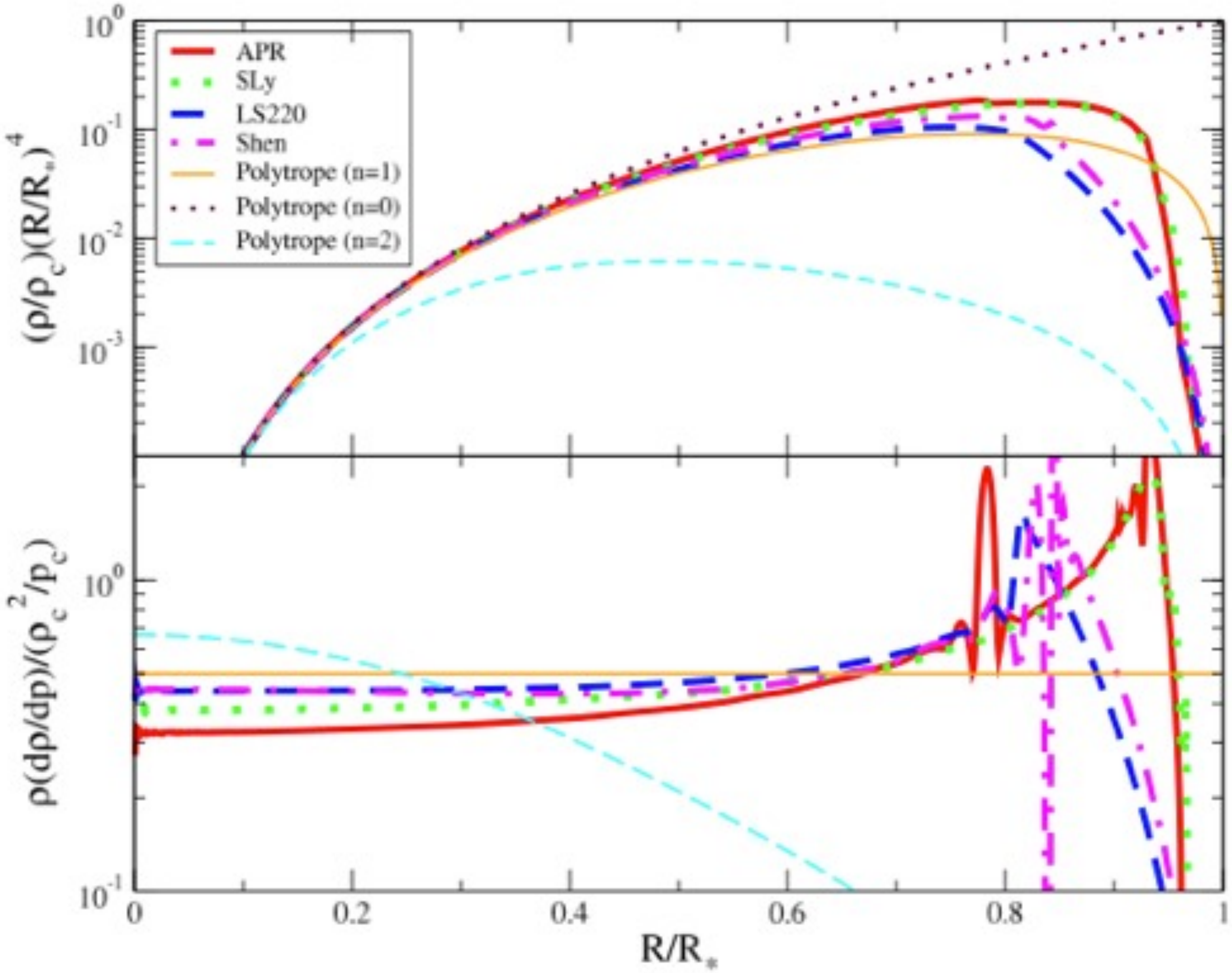
Let's look at the math...

Consider the Newtonian definition of I and Q:

$$I^{\mathrm{N}} \propto \int_0^{R_*} \frac{\rho}{\rho_c} \left(\frac{R}{R_*} \right)^4 \propto Q^{\mathrm{N}}$$

$$\lambda^{\mathrm{N}} \propto \frac{h_2'}{h_2}$$

$$h_2'' + \frac{2}{R} h_2' + \left(\rho \frac{d\rho}{dp} - \frac{6}{R^2} \right) = 0$$



I-Love-Q depend most sensitively on the neutron star outer layers !

Applications

Observational Astrophysics

Spin-Orbit coupling induces (geodetic) precession of the orbital plane. $\longrightarrow$ $$\frac{d\vec{L}}{dt} \propto \vec{S}_A \times \vec{L},$$

Precession induces a change in the inclination angle. $\longrightarrow$ $$\frac{d\iota}{dt} \propto \vec{S}_A \times \vec{L} \propto I$$

Change in inclination forces pulsar beam to sweep in and out. $\longrightarrow$ Measurement of Moment of Inertia (10%)

A measurement of any single member of the I-Love-Q trio, automatically provides information about the other two.

Gravitational Wave Astrophysics

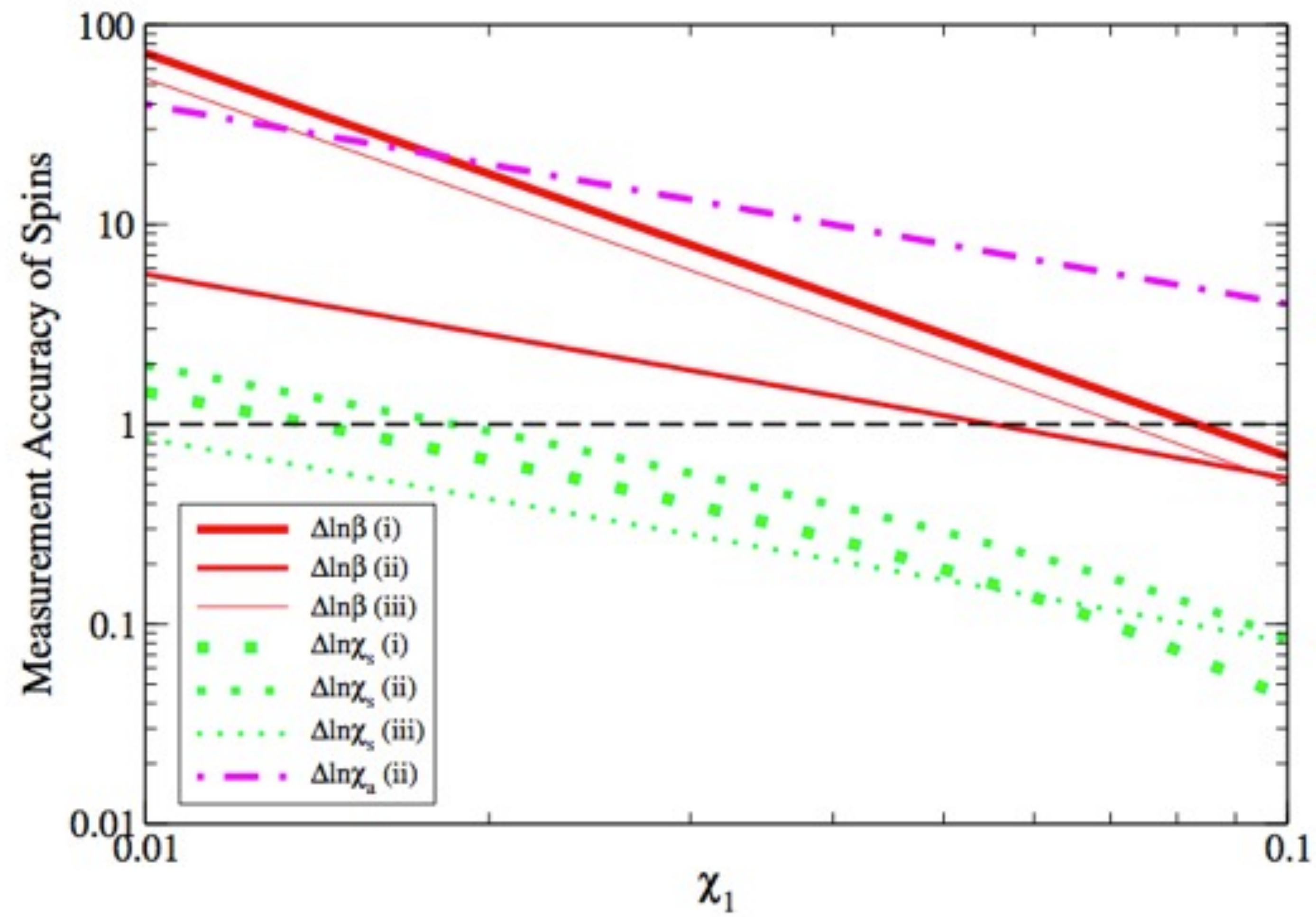
Fourier Transform of Response Function ← $\tilde{h}(f) = A(f)e^{i\Psi(f)}$ → Fourier Phase, Fourier Amplitude

Degeneracy between Si and Q in the Fourier Phase!!

$$\Psi(f) = \ldots + \Psi_0 f^{-5/3}\left[1 + \ldots + \Psi_4(Q, S_1, S_2) f^{4/3} + \ldots + \Psi_{10}(\lambda) f^{10/3}\right]$$

Use Love-Q relation to rewrite Q as a function of the Love number

Degeneracy is broken and we can now measure the spins



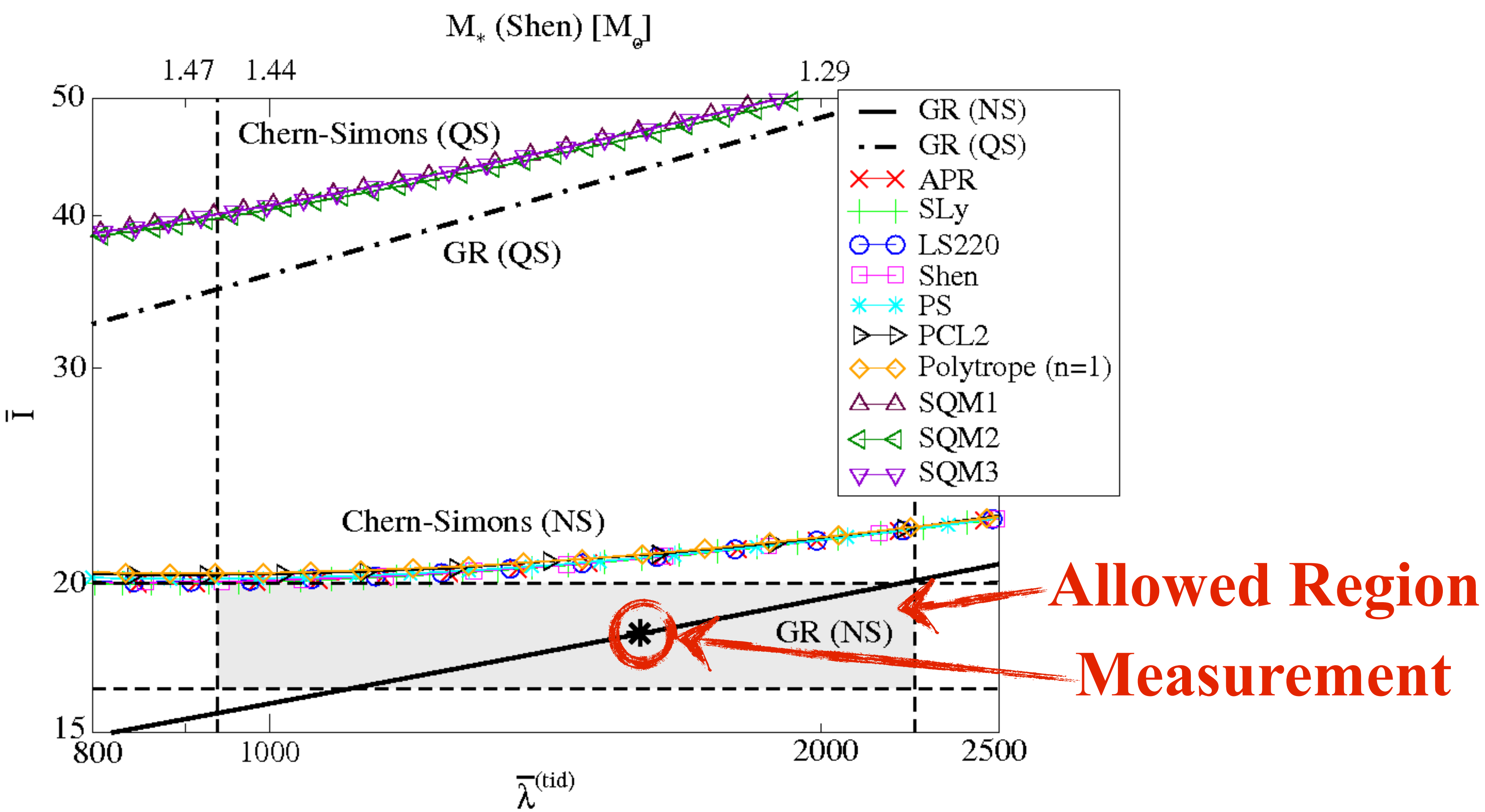
Experimental Relativity

Imagine a 10% measurement of I with a double binary pulsar and a 60% measurement of Love with gravitational waves

Two Cases:

(i) CS coupling small enough so that CS I-Love curve for NSs goes through allowed region.

(ii) CS coupling bounded such that CS I-Love curve for QSs goes through allowed region.



I-Love-Q relations allow for EoS independent and model-independent tests of General Relativity

What does it all Mean?

The Universal I-Love-Q relations can help us learn about neutron stars, extract information from gravitational waves and test General Relativity.

Why do they hold? No hair theorem?

Are there other Universal Relations?

Do they hold for rapidly rotating, hot, neutron stars or magnetars?

What about superfluidity and superconductivity?

Maybe Dragons Aren't That Scary...