Lorentz violation in gravity: why, how and where

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Why Lorentz Violation

Lorentz invariance (LI) is a key ingredient of
Particle Physics, **Gravity**, **Dark Sector**

such a fundamental principle should be questioned by studying viable alternatives

Bounds: $\lesssim 10^{-20}$ $\lesssim 10^{-7}$ $\lesssim 10^{-2}$

other benefits of studying violative of LI in gravity:

- Improve the UV properties of GR
- New ideas for black hole thermodynamics
- New ideas for cosmic acceleration
- Interesting (testable) phenomenology

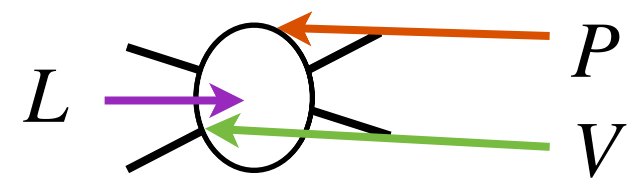
How I: Hořava Gravity in a Nutshell

Hořava 09

Lifshitz scalar (**LV:** no extra poles, no ghosts)

$$\mathcal{L} = \phi \left[\partial_0^2 - \left(\frac{-\Delta}{M_\star^2}\right)^z \Delta\right] \left\{\phi + \sum_n a_n \left(\frac{\phi}{M_P}\right)^n\right\}$$

Power counting for amplitudes



$$I \sim \left(\int^{\Lambda_0} \mathrm{d}\omega \int^{\Lambda_i} \mathrm{d}^3 k_i\right)^L \left(\frac{1}{\omega^2 - \bar{\mathrm{k}}^2 \left(\frac{\bar{\mathrm{k}}^2}{M_\star^2}\right)^z + i\epsilon}\right)^{P-V} \sim \Lambda_i^{(2-z)L+2(z+1)}$$

$\uparrow$

$\Lambda_0 \sim \Lambda_i^{z+1}$

- $z = 0$ (LI/GR):$\sim \Lambda_i^{2(L+1)}$ grows with ***L*** !
- $z = 2$ (LV) $\sim \Lambda_i^6$ fixed !

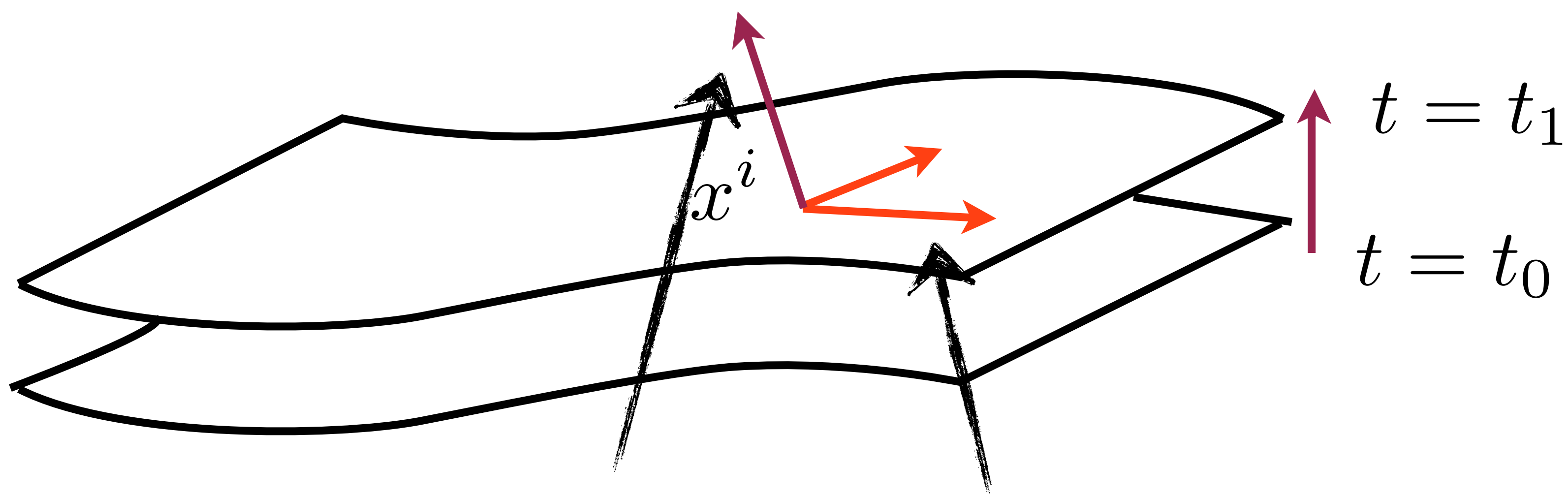
\# of counterterms may be finite

How I: Hořava Gravity in a Nutshell

For gravity this is more involved:

Hořava 09

preferred **foliation** of space-time



$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = N^2 \mathrm{d}t^2 - \gamma_{ij}(\mathrm{d}x^i + N^i \mathrm{d}t)(\mathrm{d}x^j + N^j \mathrm{d}t)$$

Broken diffeomorphisms: new group of covariance

$$x^i \mapsto \tilde{x}^i(x^j, t) \qquad t \mapsto \tilde{t}(t)$$

FDiff: Foliation preserving Diff

Extra (gapless?) polarization expected

How I: Hořava Gravity in a Nutshell

Hořava 09
Blas, Pujolàs, Sibiryakov 09

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = N^2\mathrm{d}t^2 - \gamma_{ij}(\mathrm{d}x^i + N^i\mathrm{d}t)(\mathrm{d}x^j + N^j\mathrm{d}t)$$

Covariant objects under FDiff

$$K_{ij} \sim \frac{\partial \gamma_{ij}}{\partial t} \sim \omega\, \gamma_{ij} \qquad {}^{(3)}R^i{}_{jkl} \sim \mathrm{k}^2 \gamma_{ij} \qquad a_i \equiv \frac{\partial_i N}{N} \sim \mathrm{k}_i\, \phi$$

GR Lagrangian extended to

$$\mathcal{L} = M_P^2 N\sqrt{\gamma}\Big(\underbrace{K_{ij}K^{ij} - (1-\lambda')(\gamma_{ij}K^{ij})^2}_{\partial_0^2} - (1-\beta')^{(3)}R + \alpha'\, a_i a^i ... + \frac{\Delta^{2\,(3)}R}{M_\star^4}\Big)$$

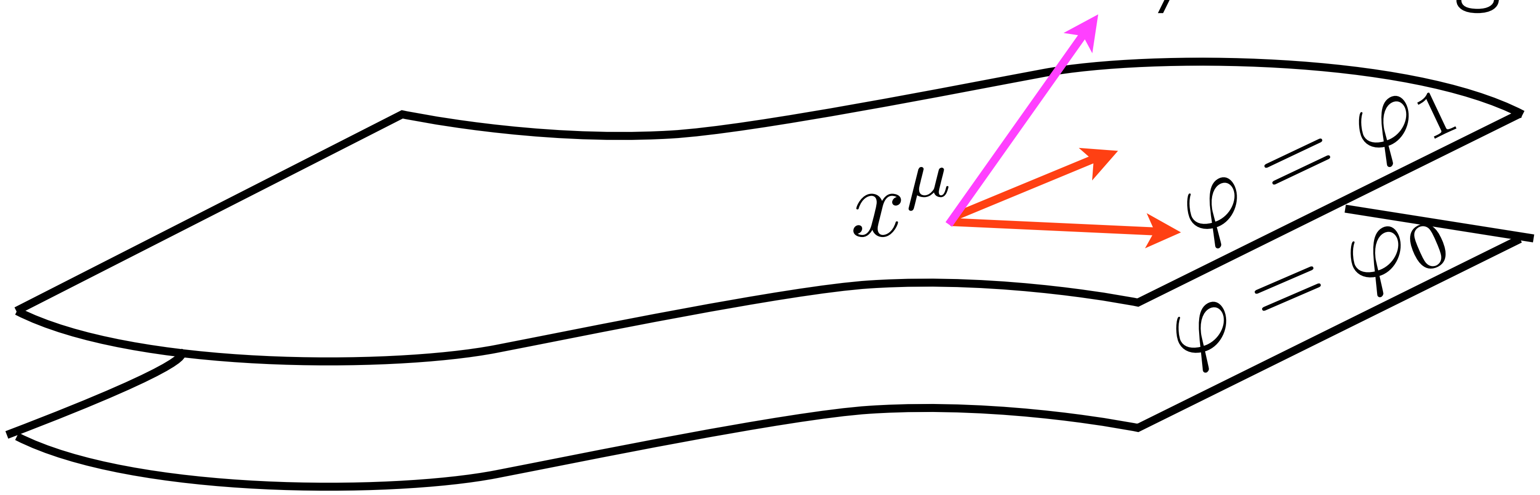
Low energy (IR)	Renormalizability
	Finite # of counterterms

(Naive) GR limit: $\lambda' = \beta' = \alpha' = 0$

How II: Khronometric Theory

Blas, Pujolàs, Sibiryakov 09

Diff invariance restored by adding a compensator: φ



φ **khronon** $\chi\rho\acute{o}\nu o\varsigma$

Diff

$\varphi \mapsto f(\varphi)$ ➡ $u_\mu \equiv \dfrac{\partial_\mu \varphi}{\sqrt{\partial_\alpha \varphi \partial^\alpha \varphi}}$

$$\mathcal{K}_{\mu\nu} \equiv \left(\delta_\mu^\sigma - u^\sigma u_\mu\right) \nabla_\sigma u_\nu$$

$$\mathcal{L}_{EH} + M_P^2 \sqrt{-g} \left(\lambda \left(\nabla^\mu u_\mu\right)^2 + \alpha \left(u^\nu \nabla_\nu u_\mu\right)^2 + \beta \nabla_\mu u_\nu \nabla^\nu u^\mu \right)$$

$+O(1/M_\star)$

- Unitary gauge

$\varphi = t$

➡ $x^i \mapsto \tilde{x}^i(x^j, t)$

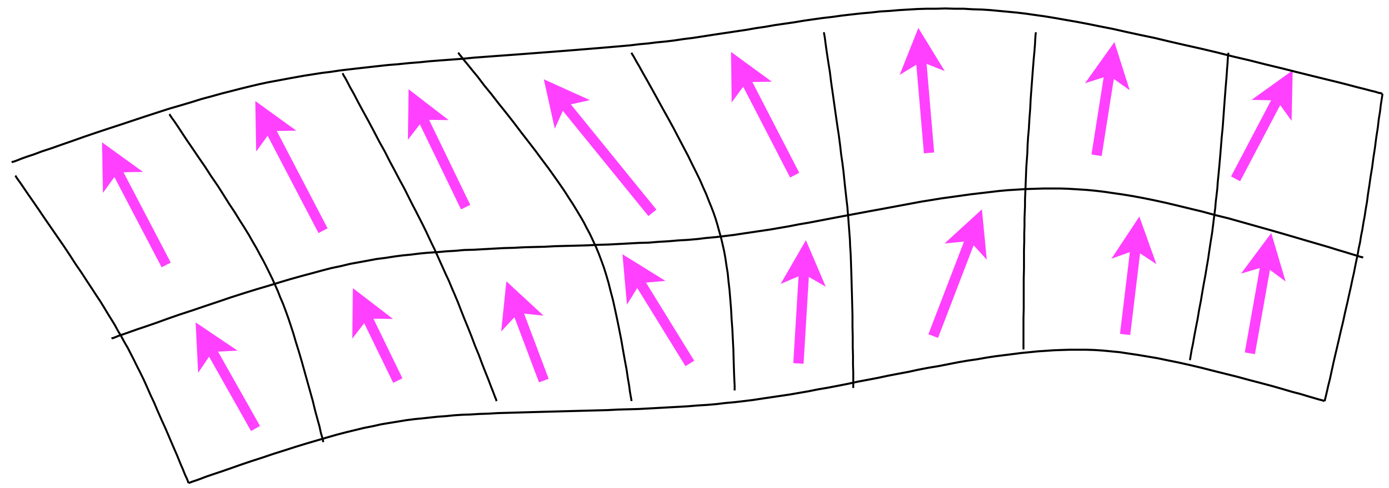
$t \mapsto \tilde{t}(t)$

K_{ij}

$\mathcal{L}$

How III: Generic

Space-time filled by a preferred **time** direction associated to a time-like unit vector u_μ

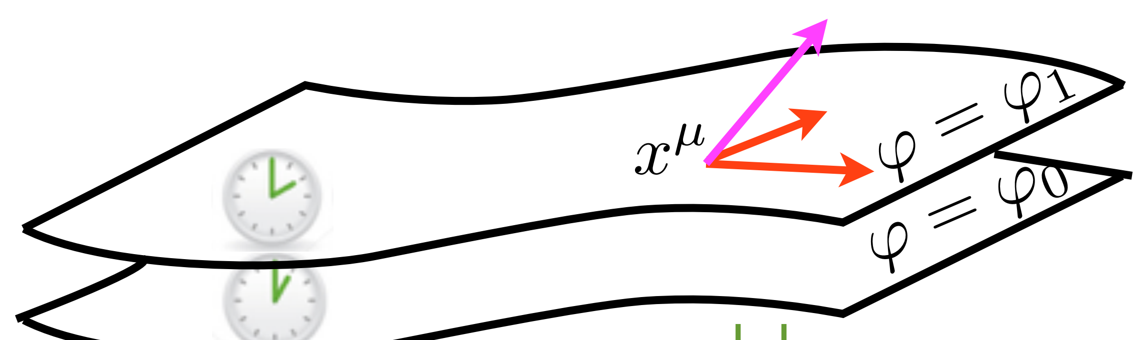


Generic: **Einstein-æther**	Hypersurface orthogonal: **Khronometric**

Jacobson, Mattingly 01

$$u_\mu u^\mu = 1$$

Scalar-vector



$$u_\mu \equiv \frac{\partial_\mu \varphi}{\sqrt{\partial_\alpha \varphi \partial^\alpha \varphi}}$$

khronon ($\partial_\mu \varphi$)

Gravitational Lagrangian (IR)

Ingredients: $u_\mu \; , \; g_{\mu\nu}$

Khronometric $u_\mu \equiv \frac{\partial_\mu \varphi}{\sqrt{\partial_\sigma \ldots}}$

$$\mathcal{L}_{\chi GR} = \mathcal{L}_{EH} + M_P^2 \sqrt{-g} \Big(\lambda \, (\nabla^\mu \ldots)^2 + \beta \nabla_\mu u_\nu \nabla^\nu u^\mu \Big)$$

- massless spin 2 graviton $\omega^2 \ldots \frac{1}{\ldots}$
- ... massless scalar $\omega^2 = c_\chi^2 \, \mathrm{k}^2$,

Possible UV completion: Hořava gravity $M_\star < \Lambda_{IR}$

EFTs with cut-off $\Lambda_{IR} \sim \sqrt{\alpha} \, M_P$

Einstein-æth... ... u_μ): extra term $\gamma \nabla_\mu u_\nu \nabla^\mu u^\nu$

Jacobson, Mattingly 01

$$u_\mu u^\mu = 1$$

- extra vector polarizations $u_\mu = \bar{u}_\mu + \delta u_\mu$

Where:

GR is modified in UV and IR,

there may be traces

of LV everywhere!

Matter Lagrangian (& Tests)

Ingredients: $u_\mu\ ,\ g_{\mu\nu}$ + SM Fields + DM + DE

$$\mathcal{L}_m = \mathcal{L}_{LI}(\mathrm{SM},\mathrm{DM},\mathrm{DE},g_{\mu\nu}) + \kappa_{SM}\ \mathcal{L}_{LV}(\mathrm{SM},g_{\mu\nu},u_\mu)$$
$$+\kappa_{DM}\ \mathcal{L}_{LV}(\mathrm{DM},g_{\mu\nu},u_\mu) + \kappa_{DE}\ \mathcal{L}_{LV}(\mathrm{DE},g_{\mu\nu},u_\mu)$$

SM: e.g. $\bar{\psi}\, u^\mu u^\nu \gamma_\mu \partial_\nu \psi$ ➡ $\omega_\psi^2 = m_\psi^2 + c_\psi^2 \mathrm{k}^2$

$|1 - c_{p,n}/c_\gamma| < 10^{-22}$ Dynamical explanation?

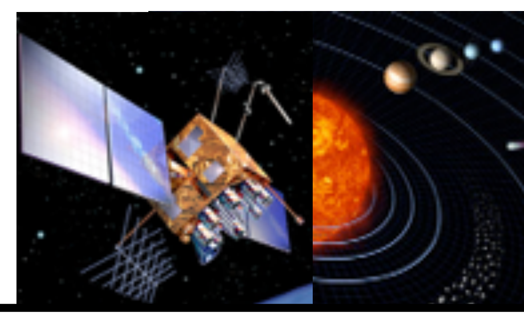
Kostelecky, Liberati, Mattingly, ... In the following $\kappa_{SM} = 0$

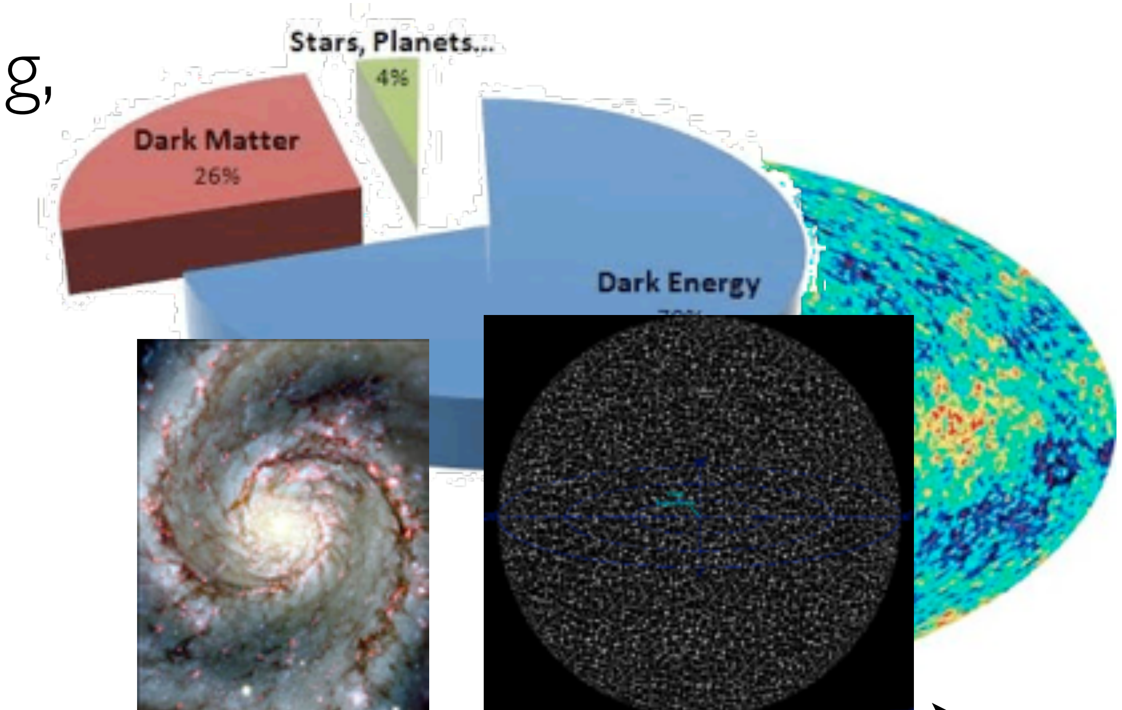
DM, DE: κ_{DM}, κ_{DE}? to be answered by cosmology

Tests of Gravity

Atomic interferometry
Torsion balance

Nordtvedt effect,
Light deviation
Lunar laser ranging,
Planets (radar)
Satellites
(Probe B, GPS)





10^{-6} 10^{11} 10^{26} m

UV modifications

$$\phi = -\frac{G_N M}{r}\left(1 + a e^{-M_\star r}\right)$$

$$M_\star > 0.1 \text{ eV}$$

Huge improvement may come from detection of primordial GW (?)

Solar system, GWs, cosmology,...

Einstein-æther or **Kh** theory

$$\mathcal{L}_{EH} + \sqrt{-g}\left(\lambda\left(\nabla^\mu u_\mu\right)^2 + \alpha\left(u^\nu \nabla_\nu u_\mu\right)^2 + \beta \nabla_\mu u_\nu \nabla^\nu u^\mu\right)$$

$$\kappa_{DM}, \kappa_{DE}?$$

Theoretical & Solar System Constraints (Kh)

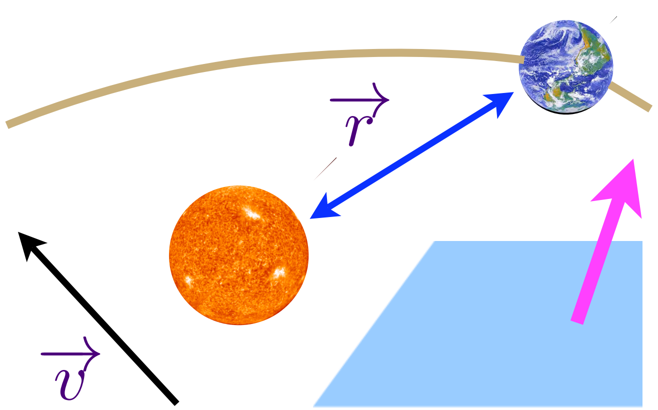
Theoretical

stability, no ghosts $\quad c_\chi^2 > 0, \quad c_t^2 > 0, \quad 0 < \alpha < 2$

no gravitational Cherenkov $\quad c_t^2 \geq 1, \; c_\chi^2 \geq 1$

Solar system

once $\kappa_{SM} \lesssim 10^{-20}$ imposed: WEP satisfied



$\vec{v} \sim 10^{-2}$

$$h_{00} = -2G_N \frac{M}{r}\left(1 - \frac{(\alpha_1^{PPN} - \alpha_2^{PPN})v^2}{2} - \frac{\alpha_2^{PPN}}{2}\frac{(x^i v^i)^2}{r^2}\right)$$

$$h_{0i} = \frac{\alpha_1^{PPN}}{2} G_N \frac{m}{r} v^i$$

Will 05

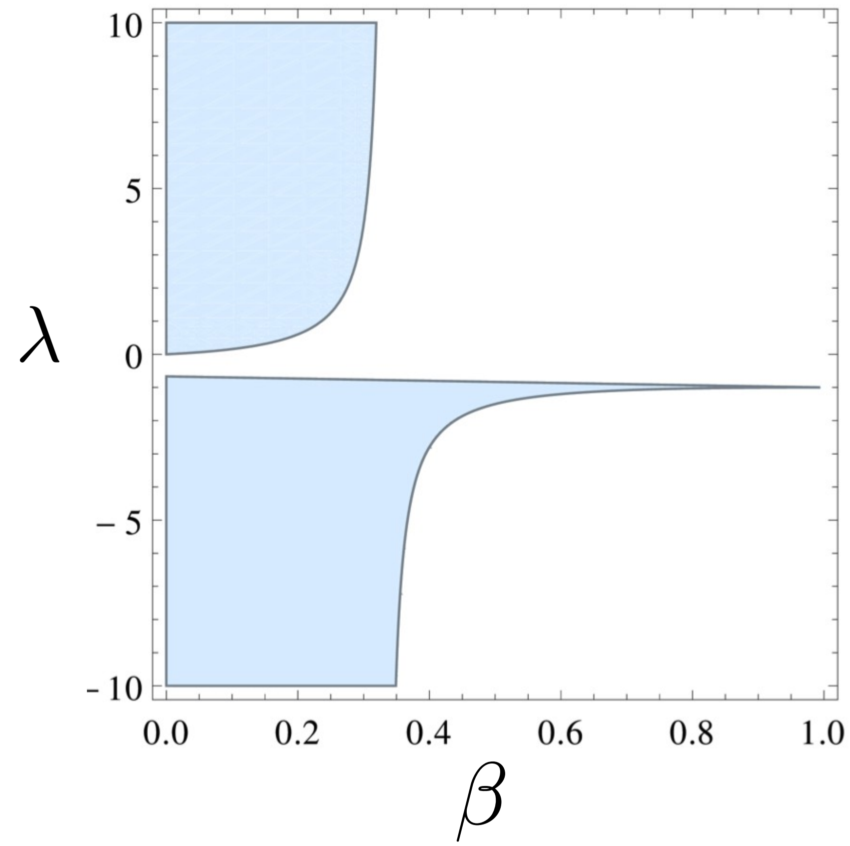
$$\alpha_1^{PPN} = -4(\alpha - 2\beta) \qquad \lesssim 10^{-4}$$

$$G_N \equiv \frac{1}{8\pi M_P^2 (1 - \alpha/2)} \qquad \alpha_2^{PPN} = \frac{(\alpha - 2\beta)(\alpha - \lambda - 3\beta)}{2(\lambda + \beta)} \qquad \lesssim 10^{-7}$$

$\alpha = 2\beta$ identical to GR in the Solar System!

Gravitational Radiation (Kh)

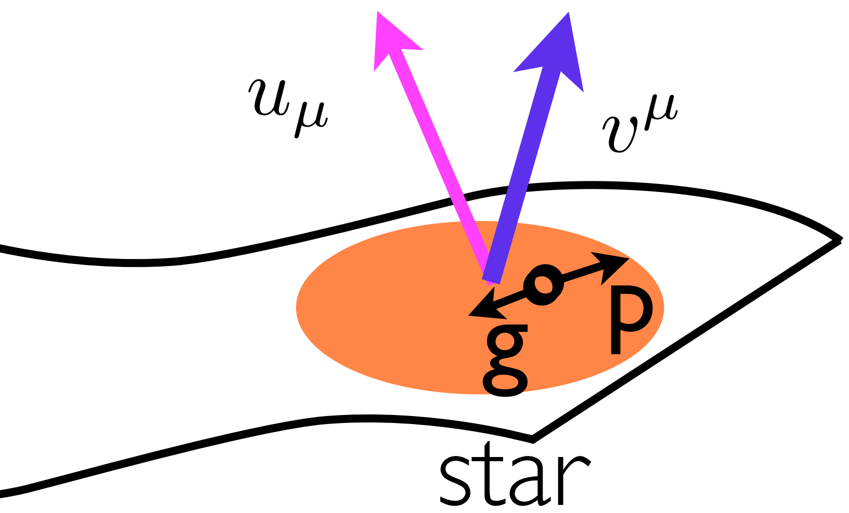
TH & Solar System constraints leave 2 free parameters



derived from situations with **weak** gravitational fields

GWs tests improve both aspects!

Expected Astrophysical Effects



Matter forces are not modified

Gravitation modified (coupling between gravitons and æther)

Violation of **strong equivalence principle (SEP)**
(Nordtvedt effect)

effectively, for **strong gravity** regimes
this produces a coupling matter-æther for point particles!

$$S_{pp} = -\tilde{m} \int \mathrm{d}s \quad \Rightarrow \quad S_{pp} = -\tilde{m} \int \mathrm{d}s \, f(u_\mu v^\mu)$$

the **orbital** equations depend on $u_\mu v^\mu$

Orbital effects: PN analysis

$$S_{pp} = -\tilde{m} \int \mathrm{d}s\, f(u_\mu v^\mu)$$

Slowly moving star $v^i \ll 1$

$$S_{ppA} = -\tilde{m}_A \int ds_A \left[(1 + \sigma_A (1 - u_\mu v^\mu) + O(u_\mu v^\mu - 1)^2 \right]$$

sensitivity: encapsulates the strong-field effects

Newtonian limit

Foster 07

$$\dot{v}^i_A = \sum_{B \neq A} \frac{-\mathcal{G}_{AB} m_B}{r^3_{AB}} r^i_{AB}$$

$$\mathcal{G}_{AB} \equiv \frac{G_N}{(1+\sigma_A)(1+\sigma_B)}$$

strong field G_N

$$m_A \equiv \tilde{m}_A (1 + \sigma_A)$$

active masses

$$P_i = m_1 v^i_1 + m_2 v^i_2$$ conserved momentum

Dipolar radiation

SEP violation : dipolar radiation expected
(similar phenomenon in scalar-tensor)

Eardley, Will 70s
Damour, Esposito-Farese 92

$$h \sim \frac{G}{c^3}\frac{d}{dt}\frac{\Sigma_i}{r} \sim \frac{G}{c^3}\frac{\tilde{P}_i}{r} \quad , \quad \Sigma_i \equiv \int d^3x \rho\, x^i$$

SEP violated: the conserved momentum does not correspond to $\tilde{P}_i = \tilde{m}_1 v_1^i + \tilde{m}_2 v_2^i$

$$\dot{h} \sim \frac{G}{c^3}\frac{\dot{\tilde{P}}_i}{r}$$

The **dipole mode** can be seen in interferometers or in the evolution of binaries

$$\dot{\mathcal{E}} = -G\Big\langle \frac{\mathcal{A}_1}{5}\dddot{Q}_{ij}\dddot{Q}_{ij} + \mathcal{B}_1 \dddot{I}\,\dddot{I} + \mathcal{C}\dot{\Sigma}_i\dot{\Sigma}_i + \ldots\Big\rangle$$

GR: $\mathcal{A}_1 = 1,\ \mathcal{B}_1 = \mathcal{C} = 0$

Computing the sensitivities

Yagi, Blas, Yunes, Barausse 13

Matching of real solution to the effective one

$$S_{ppA} = -\tilde{m}_A \int ds_A \left[(1 + \sigma_A(1 - u_\mu v^\mu) + O(u_\mu v^\mu - 1)^2\right]$$

Slowly moving star: $v^i \ll 1$ (velocity wrt æther)

Far-away from the star

$$g_{00} = 1 - \frac{1}{c^2}\frac{2G_N\tilde{m}_1}{r_1} + \frac{1}{c^4}\left[\frac{2G_N^2\tilde{m}_1^2}{r_1^2} - \frac{3G_N\tilde{m}_1}{r_1}v_1^2\left(1+\sigma_1\right)\right],$$

$$g_{0i} = -\frac{1}{c^3}\left[B_1^- \frac{G_N\tilde{m}_1}{r_1}v_1^i + B_1^+\frac{G_N\tilde{m}_1}{r_1}v_1^j\hat{r}_1^j\hat{r}_1^i\right], \quad g_{ij} = -\left(1 + \frac{1}{c^2}\frac{2G_N\tilde{m}_1}{r_1}\right)\delta_{ij}$$

From the **real** system in this approximation

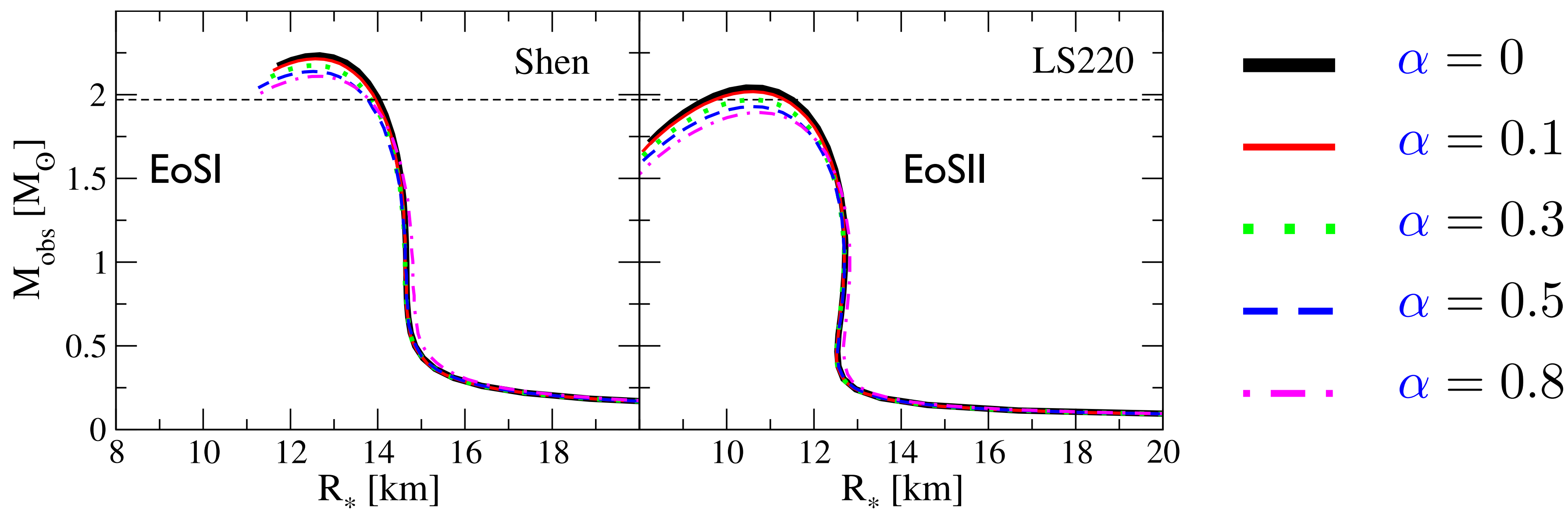
$$ds^2 = e^{\nu(r)}dt^2 - e^{\mu(r)}dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$
$$+ 2vV(r,\theta)dtdr + 2vrS(r,\theta)dtd\theta + \mathcal{O}(v^2),$$

$$u_\mu = e^{\nu(r)/2}\delta_\mu^t + vW(r,\theta)\delta_\mu^r + \mathcal{O}(v^2)$$

Neutron Stars Results

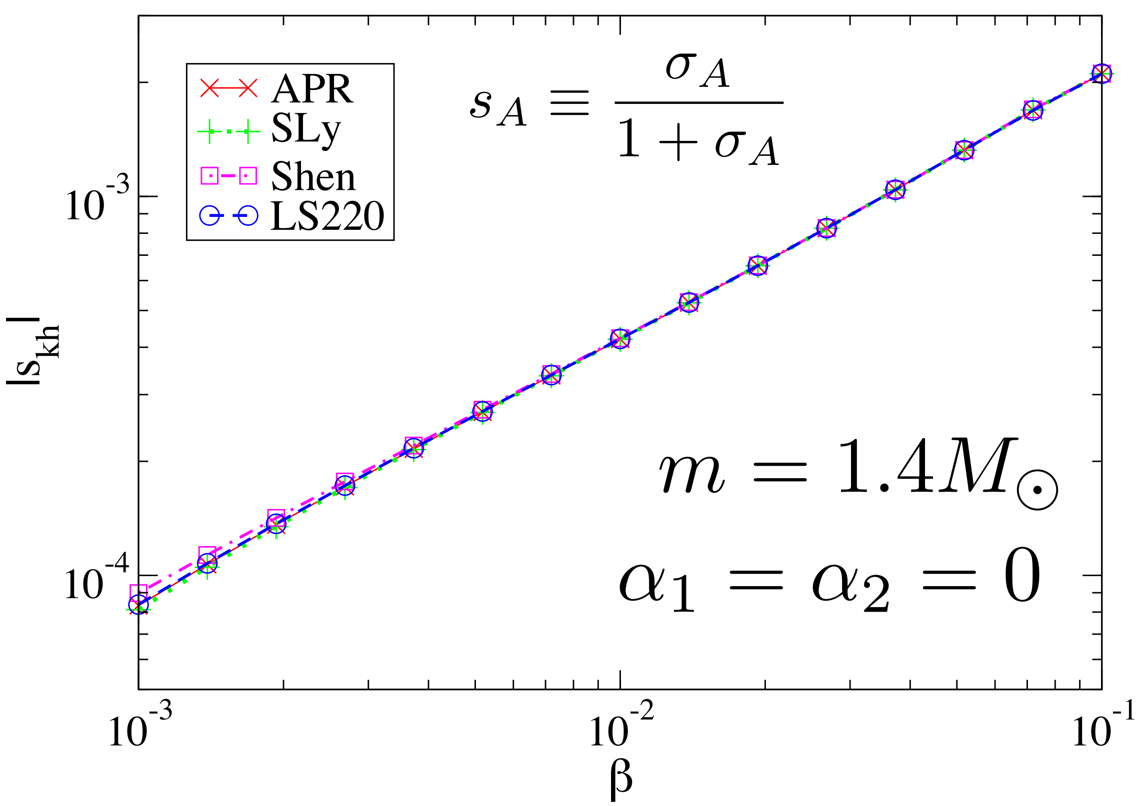
Yagi, Blas, Yunes, Barausse 13

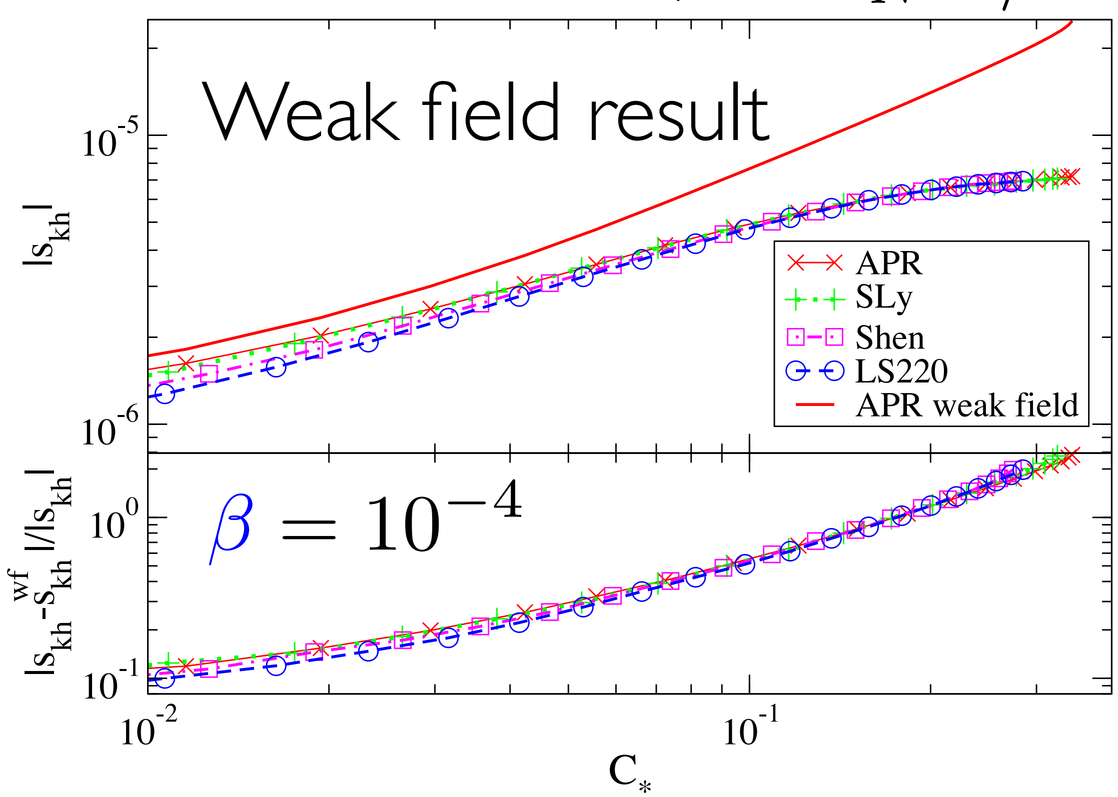
At $O(v^0)$: modified **TOV**



At $O(v)$: sensitivities

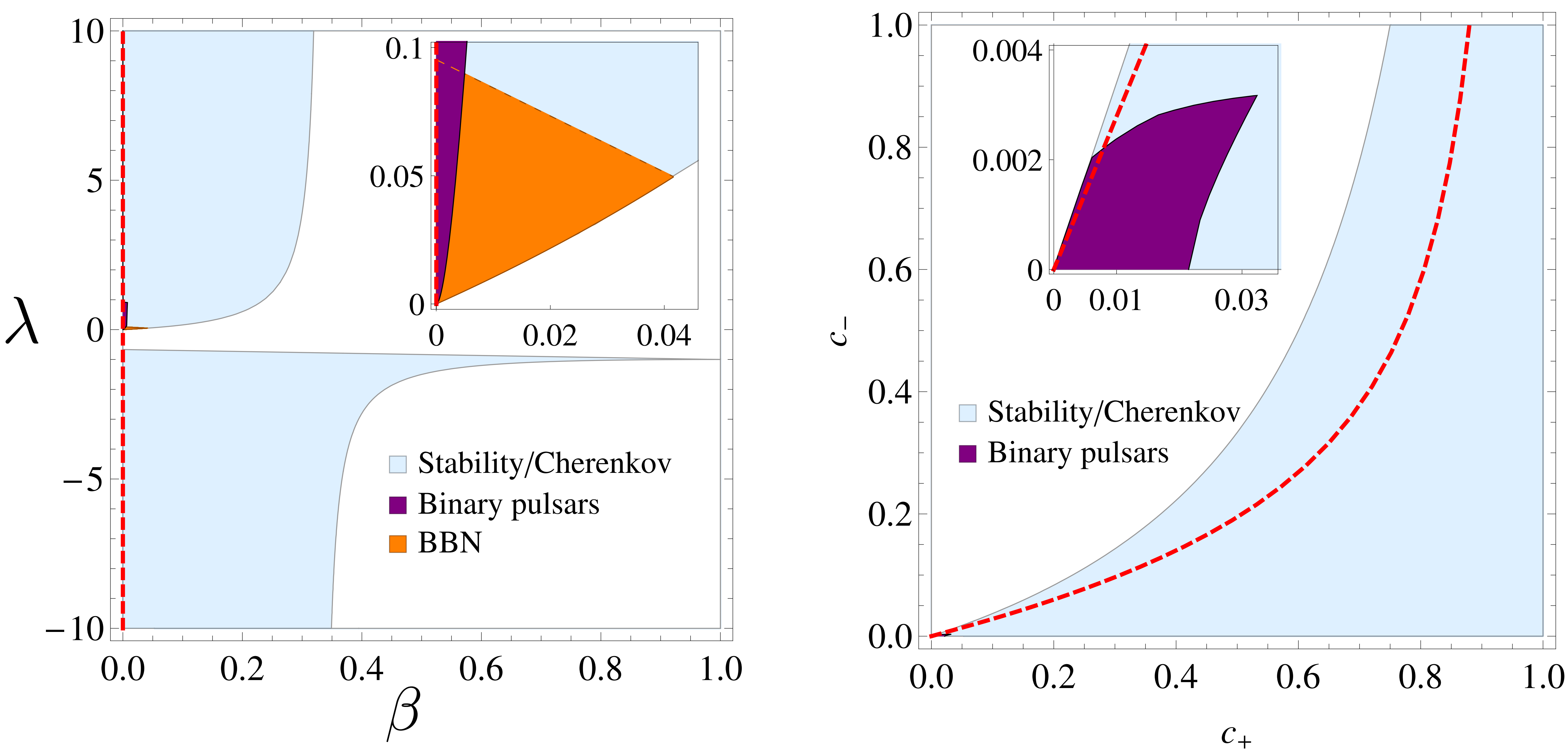
$C_* \equiv G_N m/R$





Gravitational Radiation (Kh & E-æ)

Yagi, Blas, Yunes, Barausse 13



Combined constraints from WD-NS and NS-NS systems
PSR J1141-6545, PSR J0348+0432, PSR J0737-3039

(Solar system constraints enforced)

LV in Cosmology

$$\mathcal{L}_{\chi GR} = \mathcal{L}_{EH} + M_P^2\sqrt{-g}\left(\lambda\left(\nabla^\mu u_\mu\right)^2 + \alpha\left(u^\nu\nabla_\nu u_\mu\right)^2 + \beta\nabla_\mu u_\nu\nabla^\nu u^\mu\right)$$

$$\mathcal{L}_m = \mathcal{L}_{LI}(\mathrm{SM}, \mathrm{DM}, \Lambda, g_{\mu\nu}) + \kappa_{SM}\,\mathcal{L}_{LV}(\mathrm{SM}, g_{\mu\nu}, u_\mu)$$
$$+\kappa_{DM}\,\mathcal{L}_{LV}(\mathrm{DM}, g_{\mu\nu}, u_\mu) + \kappa_{DE}\,\mathcal{L}_{LV}(\mathrm{DE}, g_{\mu\nu}, u_\mu)$$

- **MOND**
  Blanchet, Marsat 11
- **Natural dark energy**
  Blas, Sibiryakov 11

LV effects from the coupling to $u_\mu = \bar{u}_\mu + \delta u_\mu$:

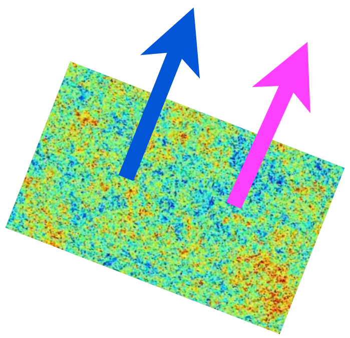
(i) the background $\bar{u}_\mu$ modifies the inertial mass
(ii) new interaction from δu_μ

DM & gravitons gravitate differently:
no equivalence principle and enhanced collapse!

Relativistic Cosmology: Background

$$G_{\mu\nu} = \frac{1}{M_P^2} T^m_{\mu\nu} + \frac{1}{M_P^2} T^{DM}_{\mu\nu} + \frac{1}{M_P^2} T^{aether}_{\mu\nu} + \frac{1}{M_P^2} T^{\Lambda}_{\mu\nu}$$

Background: Homogeneous and isotropic (preferred foliation aligned with CMB frame)



$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = \mathrm{d}t^2 - a(t)^2 \mathrm{d}x^i \mathrm{d}x^i$$

$$u_\mu = (u_0(t), 0, 0, 0) = v_\mu \quad , \quad \rho(t)$$

Friedmann equations almost not modified!

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_c}{3}\rho_m$$

$$G_c = \frac{1}{8\pi M_P^2[1 + 3\lambda/2 + \beta/2]}$$

From BBN ($^4\mathrm{He}$ abundance)

$$G_c = G_N + O(.01)$$

$$G_N = \frac{1}{8\pi M_P^2(1 - \alpha/2)}$$

Carroll, Lim 04

LV effects in Perturbations

$\kappa_{DM} = 0$

Kobayashi, Urakawa, Yamaguchi 10

$$\mathrm{d}s^2 = a(t)^2 \left[(1+2\phi)\mathrm{d}t^2 - \delta_{ij}(1-2\psi)\mathrm{d}x^i \mathrm{d}x^j\right]$$

- Faster Jeans instability: DM dom, subhorizon

$$\frac{k^2\phi}{a^2} = \frac{3H^2(1+\beta/2+3\lambda/2)}{2(1-\alpha/2)}\delta = \frac{3G_N}{2G_c}H^2\delta \quad ; \quad \delta'' + 2H\delta' = -\frac{k^2\phi}{a^2}$$

$$\delta \sim t^{-1+\sqrt{1+24\frac{G_N}{G_c}}}$$

+ Solar system constraints (**Kh**) $\alpha = 2\beta$

$$\frac{G_N}{G_c} - 1 = \frac{3(\beta+\lambda)}{2} + O(2) > 0$$

- Anisotropic stress

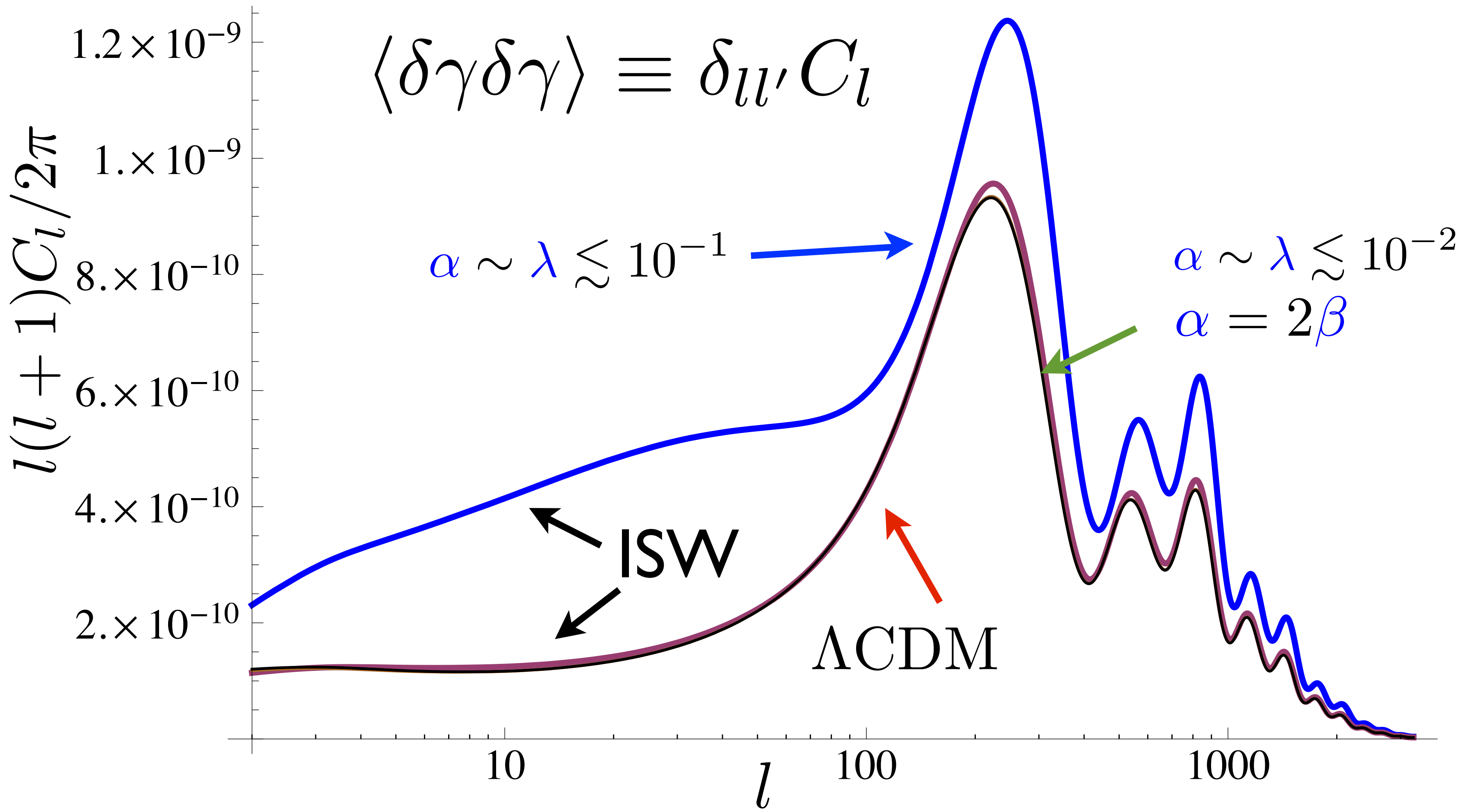
$$\phi - \psi = O(\beta)$$

Cosmic Microwave Background

Audren, Blas, Lesgourgues, Sibiryakov 13

$$\delta_\gamma'' + k^2 c_s^2 \delta_\gamma = -\frac{4k^2}{3}\psi + \ldots \qquad k^2\psi \sim \frac{G_N}{G_c}\delta_\gamma \;\Rightarrow\; c_s^{eff}$$

Shift of the peaks, change of zero point of oscillation and amplitude



http://class-code.net

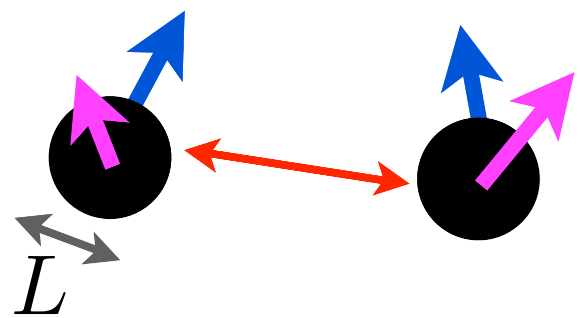
Dark matter: 5th force

$$S_{pp} = -\tilde{m}\int \mathrm{d}s\, f(u_\mu v^\mu)$$

$$S = M_P^2 \int \mathrm{d}^4x \left[\phi\Delta\phi + \frac{\alpha}{2}\delta u^i \Delta \delta u^i\right] + \int \mathrm{d}^4x\, \rho \left[\frac{(v^i)^2}{2} - \phi - Y(\delta u^i - v^i)^2\right]$$

Potential for DM and aether: ρY

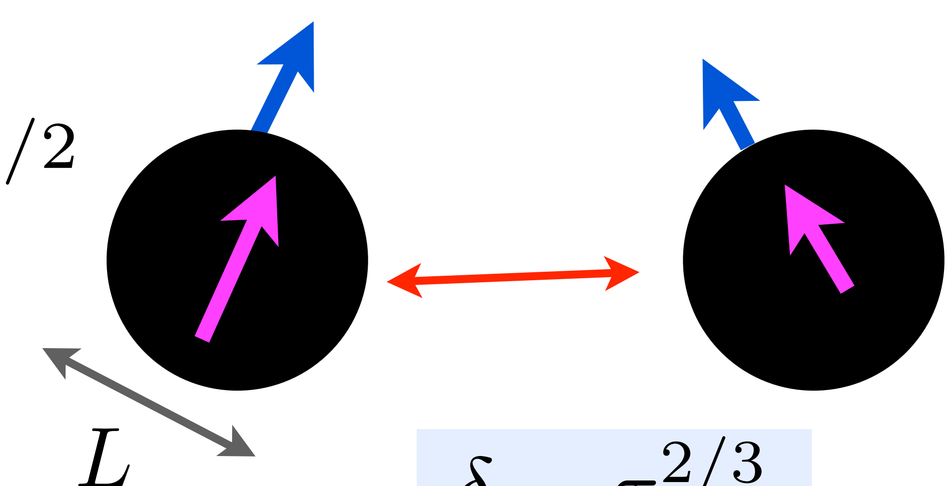
(i) $L \ll \left(\frac{\alpha M_P^2}{\rho Y}\right)^{1/2}$



$$F = \frac{F_N}{1-Y} \qquad Y > 0$$

Faster Jeans instability: $\delta \sim \tau^\gamma, \quad \gamma = \frac{1}{6}\left[-1 + \sqrt{\frac{25-Y}{1-Y}}\right]$

(ii) $L \gg \left(\frac{\alpha M_P^2}{\rho Y}\right)^{1/2}$



$\delta \sim \tau^{2/3}$

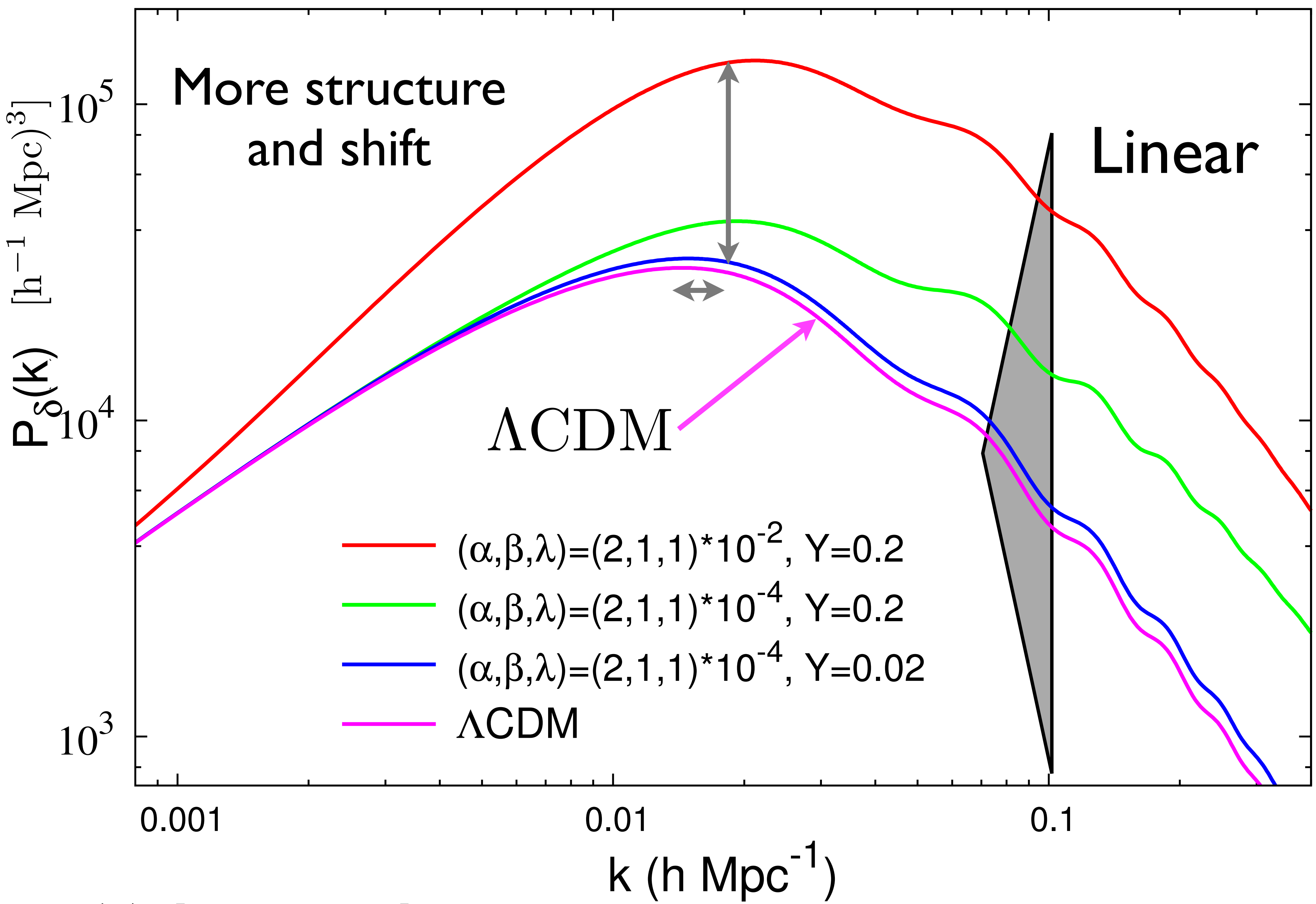
$$F = F_N$$

Screening by alignment

Matter Power Spectrum

$$\langle\delta(k)\delta(k')\rangle \equiv \delta^{(3)}(k+k')P(k)k^3$$

Blas, Ivanov, Sibiryakov 12



http://class-code.net

Cosmological Constraints (Kh)

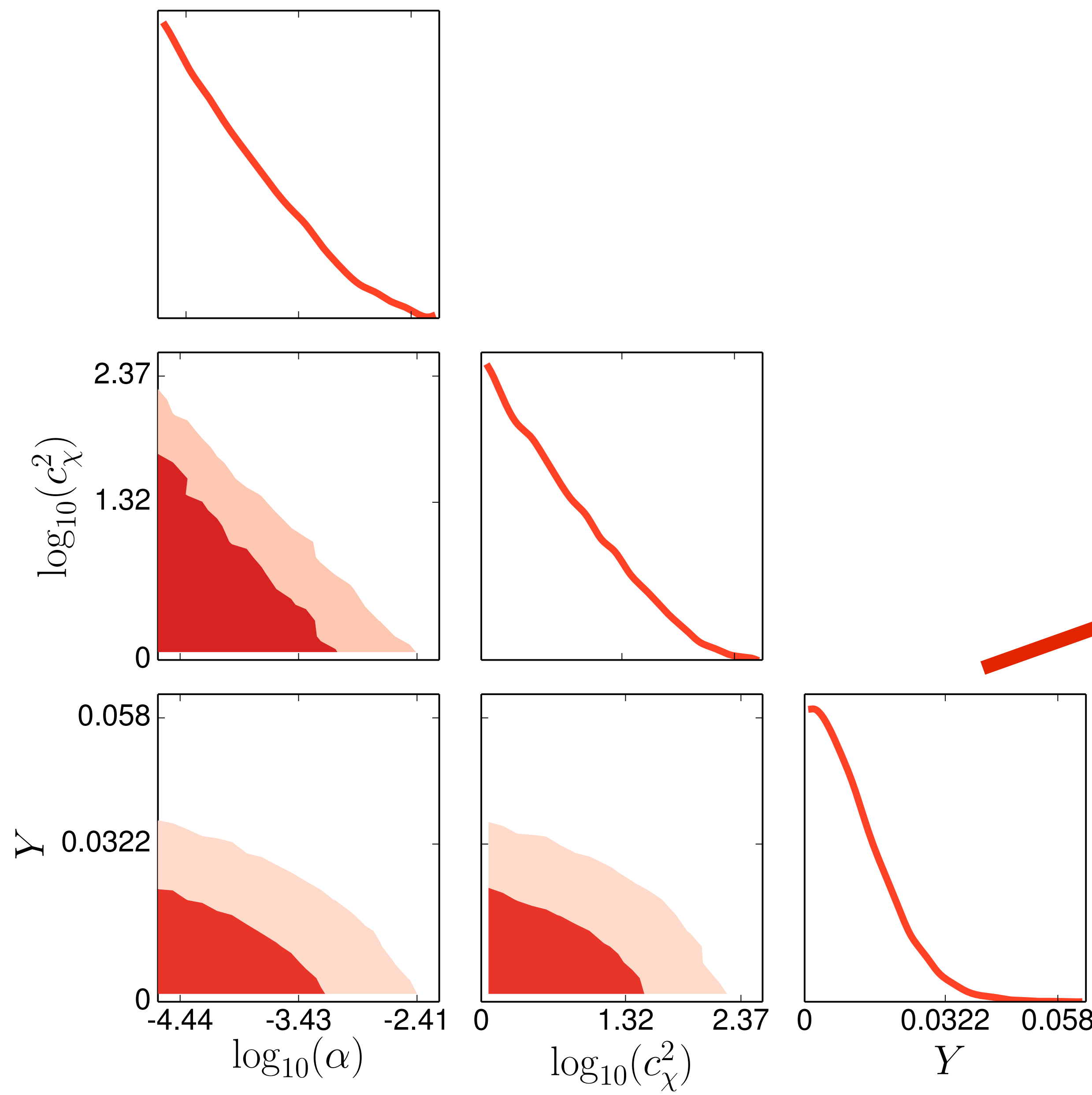
Audren, Blas, Ivanov, Lesgourgues, Sibiryakov *to appear*

Planck, SPT, WiggleZ

$$\alpha = 2\beta$$

$$c_\chi^2 = \frac{\beta + \lambda}{\alpha}$$

$$Y \equiv \kappa_{CDM}$$



First bound on LI of DM

http://montepython.net/

Conclusions

- Exploring Lorentz violation yields a rich phenomenology with strong theoretical motivations (effective or fundamental)
- Lorentz violation modifies gravity at every scale (extra massless d.o.f. $\varphi = t + \chi$)
- Tests in the gravitational sector

  Short distance modifications: $M_\star > 0.1\ \mathrm{eV}$

  Solar system tests: $\alpha_1^{PPN} \lesssim 10^{-4}$ $\alpha_2^{PPN} \lesssim 10^{-7}$ ⇒ $\alpha = 2\beta$

  GW (strong fields): $\beta,\ \lambda \lesssim O(.01)$
- Cosmological constraints (background and perturbations): growth rate + anisotropic stress + screening
- Effects on the CMB and matter power spectrum

$$\beta,\ \lambda \lesssim O(.01) \qquad \kappa_{DM} \lesssim O(.01)$$

Next Challenges

- **Non-linear cosmology**

- Better UV properties than GR (e.g. Hořava gravity)
  - Performing a 1-loop calculation (so far only scaling arguments)
  - Black hole (singularities & thermod)
  - Early universe (inflation?)

- Making Lorentz Invariance emergent in the IR
  - RG flow — Nielsen, Picek 83 — Bednik, Pujolàs, Sibiryakov 13
  - SUSY — Groot Nibbelink, Pospelov, 04
  - M_P suppression — Pospelov, Shang 10