Magnetic fields from inflation

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Lorenzo Sorbo and CC, in preparation

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- magnetic fields of microGauss amplitude observed in galaxies, clusters and high redshift objects $(z < 4)$

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$$B_{\rm Mpc} > 6 \cdot 10^{-18} {\rm G}$$

Vovk et al 1112.2534

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$$(B_\lambda \propto \lambda^{-1/2})$$

- structure formation amplifies the field: required seed value

$B \sim 10^{-9} {\rm G}$ structure collapse (clusters)

$B \sim 10^{-21} {\rm G}$ galactic dynamo (challenged by high z observations)

AT WHAT SCALE?

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  - observations in all structures and at high redshift
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- current limits on primordial magnetic fields on Mpc scale:

$$6 \cdot 10^{-18}\,\mathrm{G} < B_{\mathrm{Mpc}} < 3.4 \cdot 10^{-9}\,\mathrm{G}$$

IGM cascades | CMB (Planck 2013) conservative

Primordial magnetic fields

- more than 100 proposed generation mechanisms but no preferred one

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CAUSAL :

- phase transitions, MHD turbulence, charge separation + vorticity...
- small correlation length (maximum horizon size) and blue spectrum

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NON CAUSAL :

- inflation
- generation at all scales, model dependent

MF generated during inflation

$$\mathcal{S} = \int d^4x \sqrt{-g} \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right)$$

need to break conformal invariance
otherwise no amplification of vacuum fluctuations

Ratra model for MF generation

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simplest model:

$$\mathcal{S} = \int d^4x\sqrt{-g}\left(-\frac{f^2(\phi)}{4}F_{\mu\nu}F^{\mu\nu}\right)$$

- $F_{\mu\nu}$ test field that does not change the background evolution
- inflation with $H \simeq \text{const}$

Turner and Widrow 1988
Ratra 1992

Ratra model for MF generation

- assume a model for the function

$$f(\phi) \to f(\tau) = a(\tau)^n = \left(-\frac{1}{H\tau}\right)^n$$

$$(a_{\rm end} = 1)$$

Martin and Yokoyama 0711.4307
Demozzi et al 0907.1030

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- assume a model for the function

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- quantise the gauge field

$$A_i(\mathbf{x},\tau) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \sum_{\sigma=\pm} \epsilon_i^\sigma(\mathbf{k})\, e^{i\mathbf{k}\cdot\mathbf{x}} [A_\sigma(\mathbf{k},\tau)\hat{a}_\sigma(\mathbf{k}) + A_\sigma^*(-\mathbf{k},\tau)\, \hat{a}_\sigma^\dagger(-\mathbf{k})]$$

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- equation of motion for the helicity modes (canonically normalised) $\mathcal{A}_\sigma = f A_\sigma$

$$\ddot{\mathcal{A}}_\sigma + \left(k^2 - \frac{n(n+1)}{\tau^2}\right)\mathcal{A}_\sigma = 0$$

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amplification at large scales

$-k\tau \ll 1$

same equation for both helicities

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- solve imposing vacuum solution for $-k\tau \to \infty$ $\quad \mathcal{A}_\sigma = \frac{e^{-ik\tau}}{\sqrt{2k}}$

Ratra model for MF generation

- find the solutions at large scales $-k\tau \ll 1$

$$\mathcal{A} = \frac{1}{\sqrt{k}}[c_1(k)(-k\tau)^n + c_2(k)(-k\tau)^{n+1}]$$

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- calculate the power spectra of the electric and magnetic fields at the end of inflation

$$\frac{\mathrm{d}\rho_B}{\mathrm{d}\ln k} \simeq k^5\,|\mathcal{A}|^2 \qquad\qquad \frac{\mathrm{d}\rho_E}{\mathrm{d}\ln k} \simeq k^3\,f^2\left|\left(\frac{\mathcal{A}}{f}\right)'\right|^2$$

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- after reheating, conductivity in the universe is very large : E-field dissipates away and B-field stays

(verify that reheating does not modify the spectra at very large scales)
Martin and Yokoyama 0711.4307

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- after reheating, conductivity in the universe is very large : E-field dissipates away and B-field stays

- evolve the field until today according to $B \propto a^{-2}$

(magnetic flux is frozen at large scales due to very high conductivity)

Ratra model for MF generation

- get the value at the reference (cosmological) scale of 1 Mpc

$$B_\lambda = \sqrt{\frac{\mathrm{d}\rho_B}{\mathrm{d}\ln k}}\Bigg|_{k=\frac{2\pi}{\lambda}} \left(\frac{a_{\mathrm{end}}}{a_0}\right)^2 = \left(\frac{H}{a_0}\right)^2 \begin{cases} (H\lambda)^{-n-3} & n < -1/2 \\ (H\lambda)^{n-2} & n > -1/2 \end{cases}$$

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- two interesting regimes: scale invariant spectrum $(H\lambda \gg 1)$

$$\left.\begin{array}{l} n = -3 \\ n = 2 \end{array}\right\} \qquad B_{\rm Mpc} \simeq 10^{-5} \left(\frac{T_{\rm reh}}{M_{\rm pl}}\right)^2 \text{Gauss}$$

high values of B-field at large scales for high scale inflation!

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HOWEVER THERE ARE CONSTRAINTS

Ratra model for MF generation

- avoid back-reaction of the electric field energy density on the background:

$$\frac{\mathrm{d}\rho_E}{\mathrm{d}\ln k} = H^4 \begin{cases} (k/H)^{2n+4} & n < 1/2 \\ (k/H)^{6-2n} & n > 1/2 \end{cases}$$

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- avoid strong coupling of the theory : Demozzi et al 0907.1030

$$-\frac{f^2}{4} F_{\mu\nu} F^{\mu\nu} \rightarrow -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\psi}\gamma^\mu \left(\partial_\mu + i\frac{A_\mu}{f} \right) \psi$$

$$f \geq 1 \quad \rightarrow \quad \boxed{n < 0}$$

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- the spectrum of the MF is blue : $(k/H)^{n+3}$

Ratra model for MF generation

$$B_{\rm Mpc} \propto \left(\frac{M_{\rm pl}}{H}\right)^{n+2} {\rm G} < 10^{-65}\,{\rm G}$$

possible ways to save this model and explain MF lower bound in IGM (complicated)

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1. back reaction : magnetogenesis active only when O(Mpc) scales exit the horizon (the spectrum can be more red)
2. dilution in the radiation era: stiff fluid phase before reheating
3. amplitude of the MF: lower the scale of inflation (10 MeV)

Ferreira et al 1305.7151

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1. back reaction : produce some MF during inflation and some during reheating
2. amplitude of the MF: lower the scale of inflation (50 MeV)

Kobayashi 1403.5168

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2. dilution in the ... eheating
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probably ruled out:
too low-scale inflation for BICEP2

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Kobayashi 1403.5168

Axial coupling

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Anber and Sorbo astro-ph/0606534
Durrer et al 1005.5322

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- equation of motion in this case:

$$\ddot{\mathcal{A}}_\sigma + \left(k^2 - \sigma\, f_N \frac{k}{\tau} \right) \mathcal{A}_\sigma = 0$$

coupling $$f_N = \frac{f' \dot{\phi}}{\mathcal{H}} \simeq \text{const}$$

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only left handed helicity is amplified : parity violation

amplification around horizon crossing

$$-k\tau \lesssim \sigma\, f_N$$

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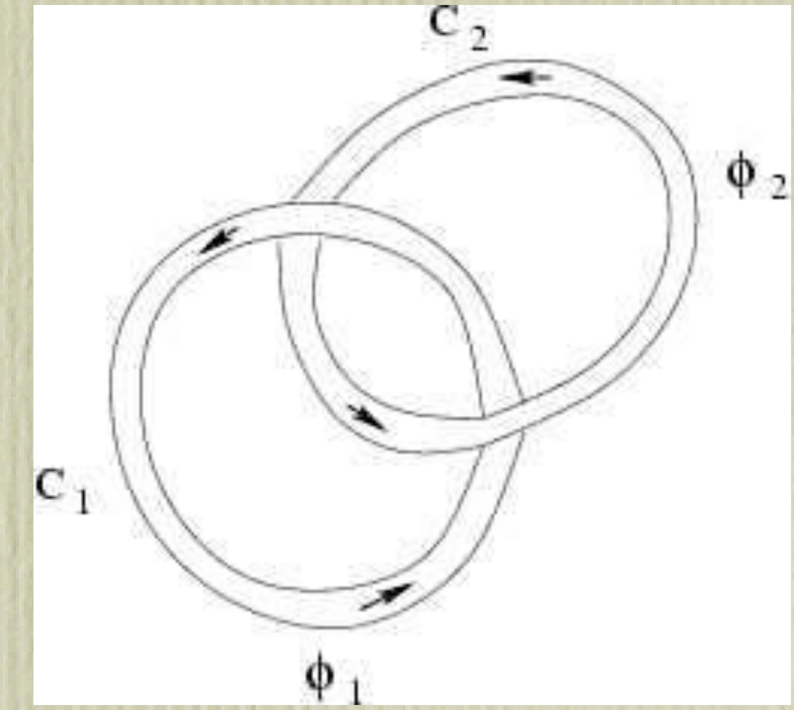
$$\mathcal{A}_\sigma(-k\tau \ll 1) \propto \frac{\exp(-\sigma\, f_N)}{\sqrt{2k}}$$

the magnetic field arising from this mechanism is helical!

Evolution of helical magnetic field

$$H = \frac{1}{V}\int_V d^3x\, \mathbf{A}\cdot\mathbf{B} \simeq B^2 L$$



Evolution of helical magnetic field

$$H = \frac{1}{V}\int_V d^3x\,\mathbf{A}\cdot\mathbf{B} \simeq B^2 L$$

in a turbulent medium, helicity is conserved if the conductivity is high

$$\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{1}{\sigma_c}\int_V d^3x\,\mathbf{B}\cdot(\nabla\times\mathbf{B}) \to 0 \qquad\qquad \sigma_c \to \infty$$

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DURING A TURBULENT PHASE:

magnetic power dissipated at small scales by viscosity in the primordial fluid → some must be transferred at large scales to conserve helicity

MF EVOLVES THROUGH INVERSE CASCADE

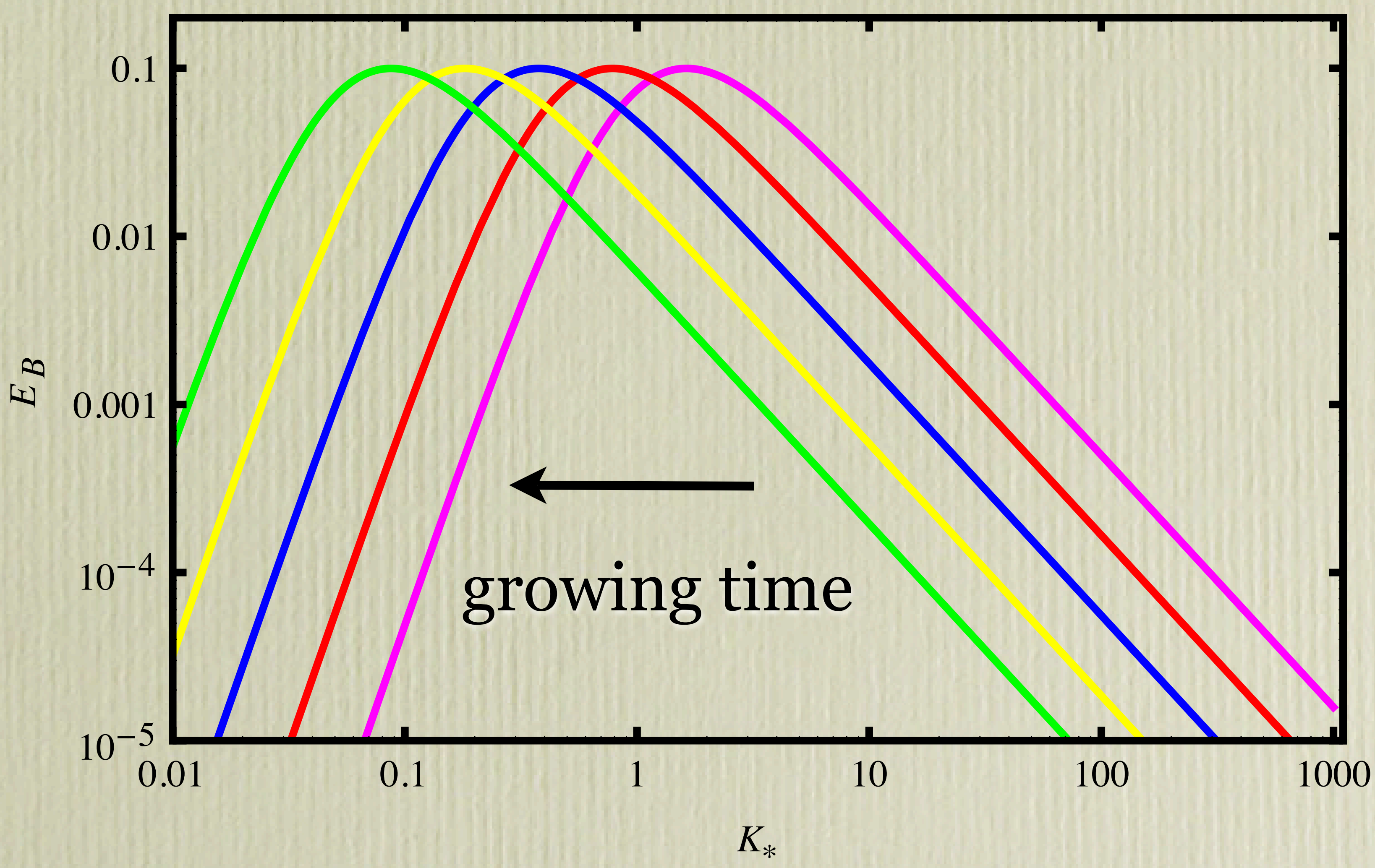
Banerjee and Jedamzik astro-ph/0410032, Durrer and Neronov 1303.7121

Evolution of helical magnetic field

magnetic energy is transferred to larger scales and correlation scale grows

$$H = B^2 L = \text{const} \qquad \begin{cases} B(\tau) \propto \tau^{-1/3} \\ L(\tau) \propto \tau^{2/3} \end{cases}$$

spectral index at large scales constant



ENERGY TRANSFERRED WHERE WE NEED IT!

Evolution of helical magnetic field

are we in a turbulent phase after inflation?
study the system of MHD equations

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} + \frac{\mathbf{B}\times(\nabla\times\mathbf{B})}{\rho+p} = \begin{cases} \nu\left[\Delta\mathbf{v} + \frac{1}{3}\nabla(\nabla\cdot\mathbf{v})\right] & (1) \\ -\alpha\,\mathbf{v} & (2) \end{cases}$$

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla\times(\mathbf{v}\times\mathbf{B}) = \frac{\Delta\mathbf{B}}{\sigma_c}$$

- MF sources fluid motions and vice-versa : MF can induce turbulence
- equipartition : $$\frac{B^2}{\rho+p} \simeq v^2$$

Banerjee and Jedamzik astro-ph/0410032, Durrer and Neronov 1303.7121

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- case (1): kinetic viscosity $\nu = \ell_{\rm mfp}$
- the system is turbulent on the scale of the flow if the advective term is larger than the viscous term

$$Re = \frac{v\,L}{\nu} \gg 1$$

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- case (1): kinetic viscosity $\quad \nu = \ell_{\mathrm{mfp}}$
- kinetic diffusion is more important than magnetic diffusion if

$$ReM = v\,L\,\sigma_c \qquad\qquad P = \frac{ReM}{Re} = \sigma_c\,\nu \gg 1$$

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- when kinetic viscosity is important : the velocity field dissipates away

$$Re \lesssim 1$$

- the magnetic field decouples from the flow and stays frozen-in

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- case (2) : free-streaming phase $\ell_{\rm mfp} > L$
- drag coefficient $\alpha \simeq \ell_{\rm mfp}^{-1}$
- the system goes back to turbulence when $ReD = \frac{v}{L\alpha} \gg 1$

Evolution of helical magnetic field

- P>>1 : the system starts turbulent, viscosity : neutrinos
- neutrino mfp grows, the system become viscous, MF is conserved
- neutrino mfp keeps growing, reaches the scale of the flow, fs phase
- the system can go back to turbulence before neutrino decoupling or not
- photons determine the viscosity after neutrino decoupling, their mfp increases a lot at e+ e- annihilation
- photon mfp large, the system is viscous, MF is conserved
- photon mfp keeps growing, reaches the scale of the flow, fs phase
- the free-streaming phase ends at recombination (drag coefficient to zero)
- further evolution in the matter era is effectively frozen-in

Axial coupling

$$\mathcal{S} = \int d^4x \sqrt{-g} \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{f^2(\phi)}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} \right)$$

- equation of motion in this case:

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- exponential amplification at horizon crossing and saturation at large scales

$$\mathcal{A}_\sigma(-k\tau \ll 1) \propto \frac{\exp(-\sigma\, f_N)}{\sqrt{2k}}$$

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Axial coupling

- magnetic field power spectrum

$$\left.\frac{\mathrm{d}\rho_B}{\mathrm{d}\ln k}\right|_{\mathrm{end}} = H^4\, e^{2\pi\, f_N} \left(\frac{k}{H}\right)^4$$

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but the spectrum is blue

and the inverse cascade is not enough to save the model

$$B_{\mathrm{Mpc}} = 10^{-43} e^{\pi f_N} \left(\frac{T_{\mathrm{reh}}}{10^{14}\mathrm{GeV}}\right)^{9/11} \mathrm{Gauss}$$

Durrer et al 1005.5322

Problems and possible solutions

in the Ratra model the spectrum can vary (parameter n) but the amplitude is fixed

in the axial model the amplitude can vary (parameter ξ) but the spectrum is fixed and it is too blue (k^2)

→ combine the two models

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very low scale inflation is required to enhance the MF amplitude:
problem with BICEP2

→ the gauge field sources the tensor perturbations

Helical Ratra magnetogenesis

$$\mathcal{L} = f^2(\tau)\,\left(-\frac{1}{4}F_{\mu\nu}\,F^{\mu\nu} + \frac{\gamma}{8}F_{\mu\nu}\,\tilde{F}^{\mu\nu}\right)$$

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$$f(\tau) = \left(-\frac{1}{H\tau}\right)^n$$

$$-2 < n < -1$$

1. avoid strong coupling
2. avoid strong back-reaction by the EF
3. get the most red spectrum possible

Helical Ratra magnetogenesis

$$\mathcal{L} = f^2(\tau)\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\gamma}{8}F_{\mu\nu}\tilde{F}^{\mu\nu}\right)$$

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$$\gamma = -\frac{\xi}{n}$$

axial coupling to get an helical MF
and amplify large scales
(turns out to be of order 10)

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- equation of motion in this case :

$$\ddot{\mathcal{A}}_\sigma + \left(k^2 + 2\,\sigma\,\xi\,\frac{k}{\tau} - \frac{n\,\left(n+1\right)}{\tau^2}\right)\mathcal{A}_\sigma = 0$$

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amplification at super-horizon scales
gives n-dependent spectral index to
the power spectrum

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- solution at large scales : $-k\tau \ll 1/\xi$

$$\mathcal{A}_\sigma \simeq \sqrt{-\frac{\tau}{2\,\pi}}\, e^{\pi\xi}\, \Gamma\left(|2n+1|\right)\, |2\,\xi\,k\,\tau|^{-|n+1/2|}$$

Helical Ratra magnetogenesis

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vector field during inflation induces METRIC PERTURBATIONS that must be smaller than the observed ones

stronger constraints comes from TENSOR modes

$$ds^2 = -dt^2 + a^2(t)[(\delta_{ij} + h_{ij})dx^i dx^j]$$

$$({h_i}^i = {h_i}^j{}_{|j} = 0)$$

since we have a chiral source, decompose into helicity modes

$$h_{ij} = h_L(\mathbf{k}, \tau)\epsilon^L_{ij}(\mathbf{k}) + h_R(\mathbf{k}, \tau)\epsilon^R_{ij}(\mathbf{k})$$

Tensor mode spectrum from the gauge field

GWs are sourced by the tensor part of the energy momentum tensor of the EM field :

$$\ddot{h}_\sigma + 2\,\frac{\dot{a}}{a}\,\dot{h}_\sigma + k^2 h_\sigma = \frac{2}{M_{\rm Pl}^2}\,\Pi_\sigma{}^{ij}(\mathbf{k})\,T_{ij}^{EM}(\mathbf{k})$$

projector that extracts the tensor mode

$$-\left(\frac{f(\tau)}{a}\right)^2 \dot{A}_i(\mathbf{x},\tau)\dot{A}_j(\mathbf{x},\tau)$$

contribution from the EF is the dominant one

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$$h_\sigma(\mathbf{k},\tau) = \frac{2}{M_{\rm pl}^2}\int dt\,G_k(\tau,t)\,T_\sigma(\mathbf{k},t)$$

GW spectrum :

$$\langle h_\sigma(\mathbf{k},\tau)h_\sigma(\mathbf{q},\tau)\rangle = \frac{4}{M_{\rm pl}^4}\int dt\,G_k(\tau,t)\int dt'\,G_q(\tau,{}')\,\langle T_\sigma(\mathbf{k},t)T_\sigma(\mathbf{q},t')\rangle$$

Tensor mode spectrum from the gauge field

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assume that BICEP2 is correct, in order to determine
the energy scale of inflation in this model

$$\frac{H}{M_{\rm Pl}}(n,\xi) = \left(\frac{r\mathcal{P}_\zeta}{F(n)}\frac{\xi^6}{e^{4\pi\xi}}\right)^{1/4}$$

$$r = 0.2$$

$$\mathcal{P}_\zeta = 2.5\cdot 10^{-9}$$

the Hubble rate is tunable:
we can get low scale inflation with large value of
the tensor to scalar ratio

good for the MF amplitude!

Helical Ratra magnetogenesis

- magnetic field spectrum at the end of inflation :

$$\left.\frac{\mathrm{d}\rho_B}{\mathrm{d}\ln k}\right|_{\mathrm{end}} = H^4\, e^{2\pi\xi}\, \xi^{2n+1} \left(\frac{k}{H}\right)^{2n+6}$$

- magnetic field correlation scale :

$$L_{\mathrm{end}} = G(n)\,\frac{\xi}{H}$$

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- evolve in time starting from these initial conditions :

  1. turbulent phase until 20 GeV < T < 0.1 GeV (inverse cascade)

  2. viscous phase followed by free-streaming phase until neutrino decoupling

  3. again, viscous phase followed by free-streaming phase until photon decoupling

  4. frozen-in evolution in the matter era

Helical Ratra magnetogenesis

- get the value of the magnetic field today at the correlation scale as a function of $B_L(n, \xi)$

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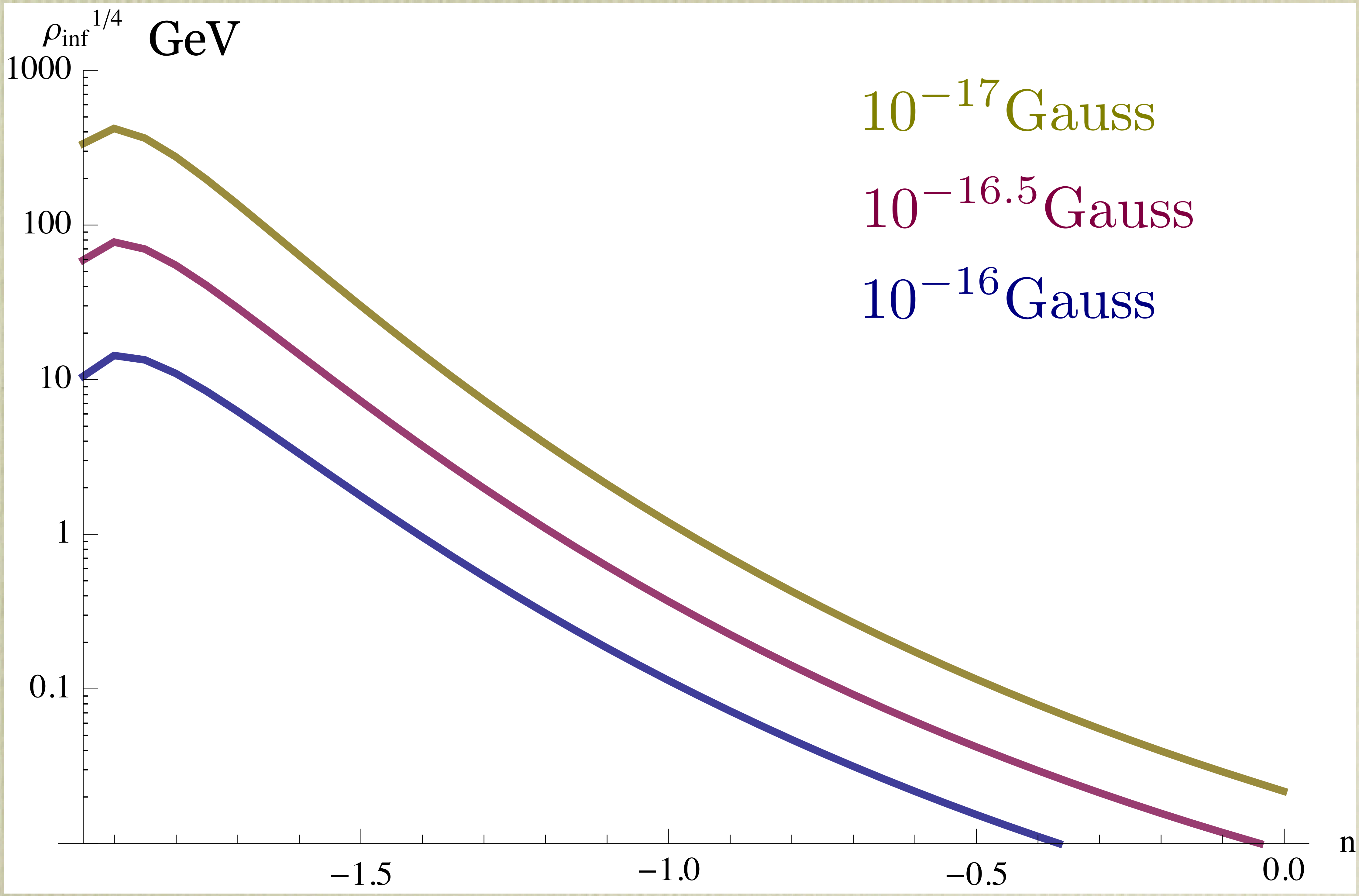
- get the energy scale of inflation for which this is satisfied as a function of n

(ξ is fixed by the required MF amplitude)

Helical Ratra magnetogenesis



it is possible to generate large enough MF with not too low scale inflation

Conclusions

- magnetogenesis during inflation is difficult, but it has the advantage that the field can be generated also at very large scales
- models constrained by back-reaction, strong coupling, BICEP2
- not too complicated model that works :
  avoid back reaction and strong coupling constraints, and explain BICEP2 result with low scale inflation
- parity violation is fundamental : helical field, evolving through inverse cascade in the first stages of its evolution
- if BICEP2 turns out to be dust and the tensor to scalar ratio can be smaller, the MF can be larger
- the NG constraints is expected to be smaller than the GW one
  (in progress)