Spacetime curvature and Higgs stability before and after inflation

arXiv:1407.3141 (PRL 113, 211102);
arXiv:1506.04065

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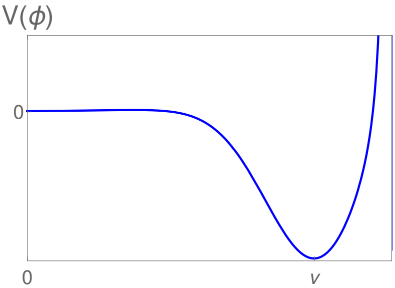
Outline

Standard Model Higgs potential



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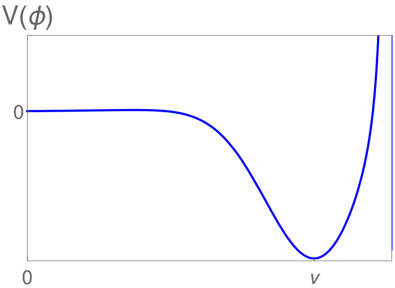

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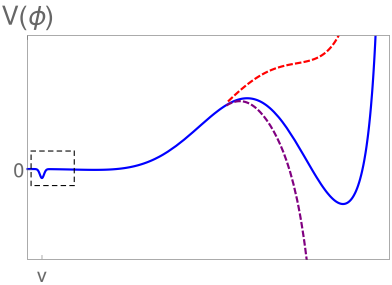

- Sensitive to M_h and M_t

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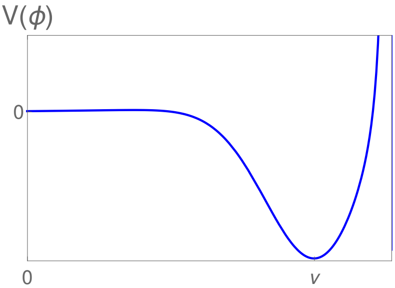

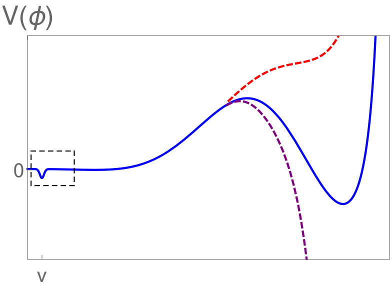

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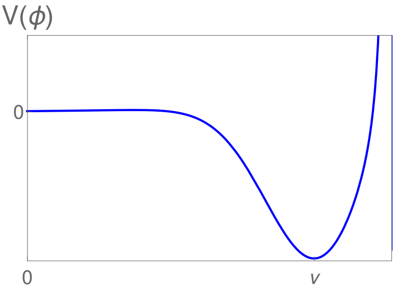

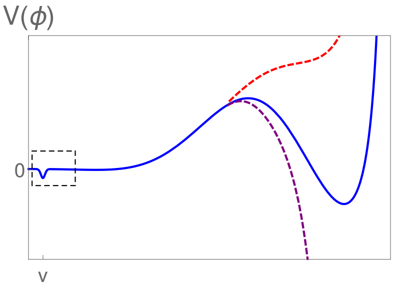

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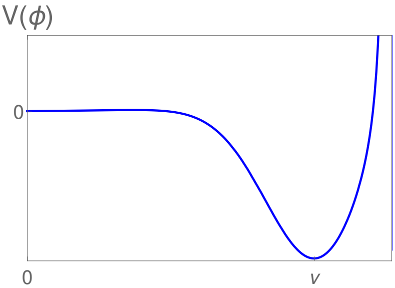

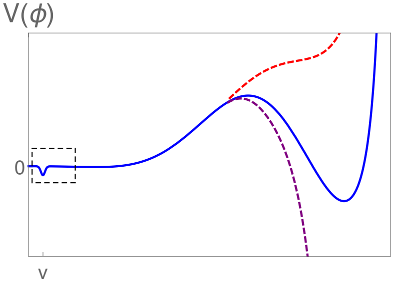

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- Is this also true for the early Universe (inflation, reheating)?
- New physics needed to stabilize the vacuum?

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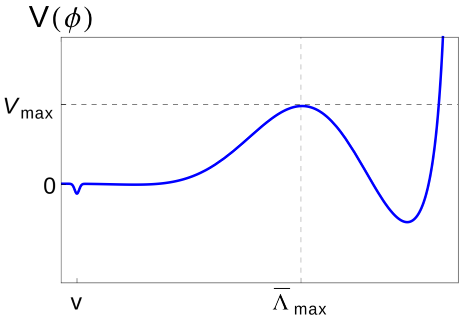
Inflation and the Standard Model

- In principle we can assume the SM to be valid
  - Energy-density is dominated by decoupled physics
- Inflation induces fluctuations to the Higgs field $\Delta\phi \sim H$
  - Important if $\overline{\Lambda}_{\rm max} \lesssim H$
  - State of the art calculations [2]: $\overline{\Lambda}_{\rm max} \sim 10^{11}\rm GeV$



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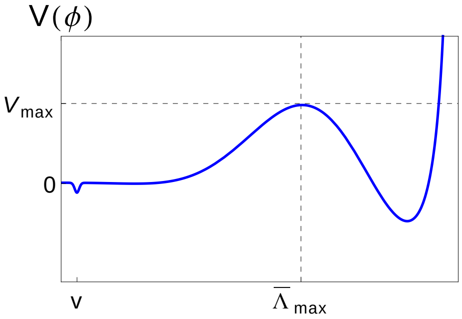

BICEP2/Keck/Planck

$H \lesssim 10^{14}\mathrm{GeV}$

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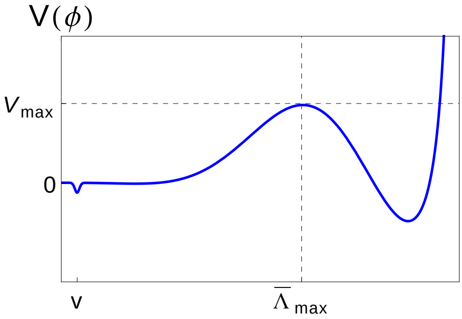

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BICEP2:

$H \sim 10^{3}\overline{\Lambda}_{\max}$

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Potential during inflation

- Fluctuations may be treated as stochastic variables [3]
- Probability density $P(t, \phi)$ from the Fokker-Planck equation

$$\dot{P}(t,\phi) = \frac{1}{3H}\frac{\partial}{\partial\phi}\left[P(t,\phi)\bar{V}'_{\rm eff}(\phi)\right] + \frac{H^3}{8\pi^2}\frac{\partial^2}{\partial\phi^2}P(t,\phi)$$

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$$V_{\text{eff}}(\phi) = \underbrace{\frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4}_{\text{classical}} + \underbrace{\frac{M(\phi)^4}{64\pi^2}\left[\log\overbrace{\left(\frac{M(\phi)^2}{\mu^2}\right)}^{\text{effective mass}} - \frac{3}{2}\right]}_{\text{quantum}}$$

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- μ is the *renormalization scale*

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Potential during inflation II

Effective potential for the SM Higgs

$$V_{\text{eff}}(\phi) = -\frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4 + \sum_{i=1}^{5}\frac{n_i}{64\pi^2}M_i^4(\phi)\left[\log\frac{M_i^2(\phi)}{\mu^2} - c_i\right]$$

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Φ	i	n_i	κ_i	κ_i'	c_i
$W^\pm$	1	6	$g^2/4$	0	$5/6$
Z^0	2	3	$(g^2+g'^2)/4$	0	$5/6$
t	3	-12	$y_t^2/2$	0	$3/2$
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- Explicit ependence on μ problematic

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- However, the physical result must not depend on μ:

$$\frac{d}{d\mu}V_{\text{eff}}(\phi) = 0 \quad \Leftrightarrow \quad \left\{\mu\frac{\partial}{\partial\mu} + \beta_\lambda\frac{\partial}{\partial\lambda} + \gamma_\phi\phi\frac{\partial}{\partial\phi}\right\}V_{\text{eff}}(\phi) = 0$$

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- Leads to *running parameters*
- We can choose μ arbitrarily [4]

optimal choice, if $\phi \gg m$

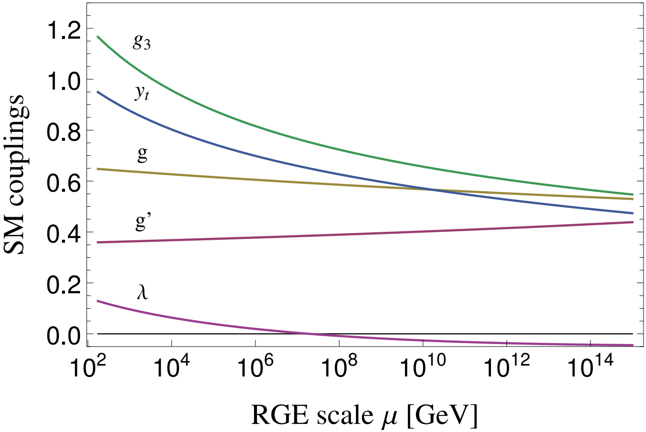
$$\mu \sim \phi$$

No large logarithms!

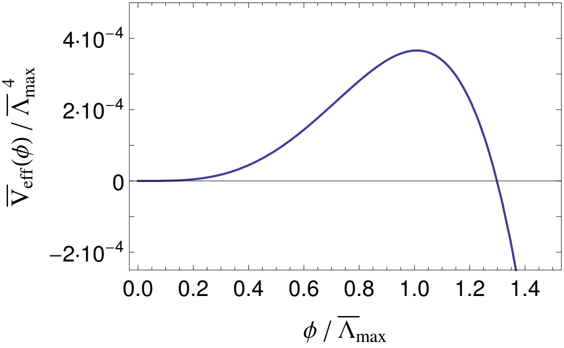
$$\Rightarrow \quad V_{\text{eff}}(\phi) \approx \frac{\lambda(\phi)}{4}\phi^4$$

[4] Ford et. al. (1993)

SM running (1-loop)



Stability results (Minkowski)



- For large H $(\sim 10^3\overline{\Lambda}_{\rm max})$, the SM is not stable [5]

[5] Kobakhidze & Spencer-Smith (2014); Hook et. al. (2014); Fairbairn & Hogan (2014); Enqvist, Meriniemi & Nurmi (2014); Zurek, Kearney & Yoo (2015)

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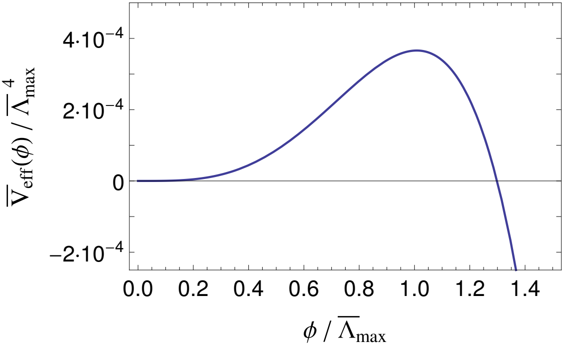

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Does including spacetime curvature in the quantum calculation change this?

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  - No loops with $g^{\mu\nu}$
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$$W^2 = \frac{\mathbf{k}^2}{a(t)^2} + M(\phi)^2 + \left(\xi - \frac{1}{6}\right)R + \mathcal{O}(\mathbf{k}^{-2})$$

QFT in curved space

- We can now repeat the stability calculation in FRW
  - All effective masses acquire shifts $\propto R$ [6]

$$\mu^2 \sim \phi^2 + R$$

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$\Rightarrow$ Spacetime curvature induces running of the constants ! [7]

- Also, include a non-minimal ξ-term in the Lagrangian
  - Generated by loops in curved space
  - Virtually unbounded by the LHC, $\xi_{\rm EW} < 10^{15}$ [8]

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$$\Rightarrow \quad V_{\rm eff}(\phi) \approx \frac{\lambda(\mu)}{4}\phi^4 + \frac{\xi(\mu)}{2}R\phi^2$$

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1-loop Effective potential in curved space

$$V_{\rm eff}(\phi,R) = -\frac{1}{2}m^2(\mu)\phi^2 + \frac{1}{2}\xi(\mu)R\phi^2 + \frac{1}{4}\lambda(\mu)\phi^4$$

$$+\sum_{i=1}^{9}\frac{n_i}{64\pi^2}M_i^4(\phi)\left[\log\frac{|M_i^2(\phi)|}{\mu^2} - c_i\right] \quad ; \quad \begin{array}{l} M_i^2(\phi) = \kappa_i\phi^2 - \kappa_i' + \theta_i R \\ \mu^2 = \phi^2 + R \end{array}$$

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t	7	-12	$y_t^2/2$	0	$1/12$	$3/2$
ϕ	8	1	3λ	m^2	$\xi-1/6$	$3/2$
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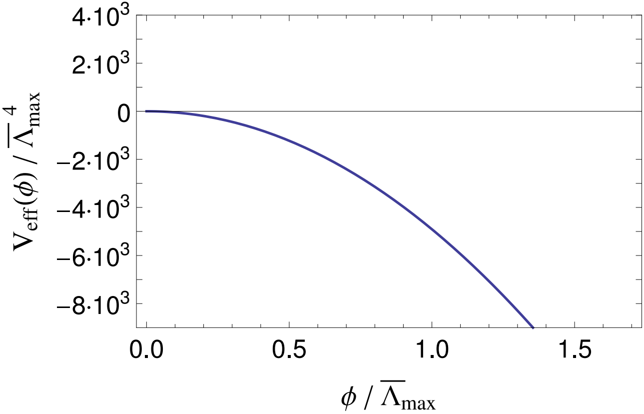

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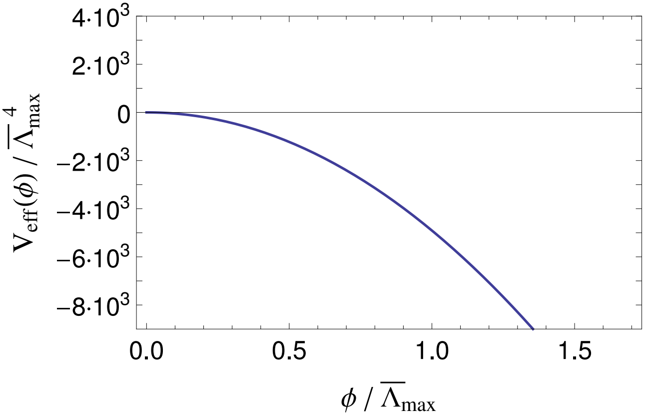

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- What causes this?

Stability results (curved space) II

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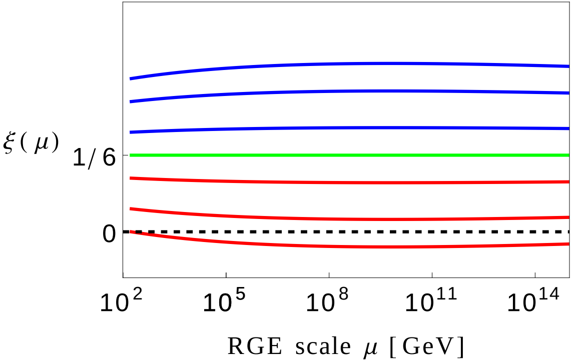
- ξ Can become positive or negative depending on $\xi_{\rm EW}$



ξ_{EW}

$0,\ 0.05,\ 0.12,\ 1/6,\ 0.22,\ 0.28,\ 0.33$

Stability results (curved space) III

- Now choosing $\xi_{\rm EW} = 0.1$ [9]

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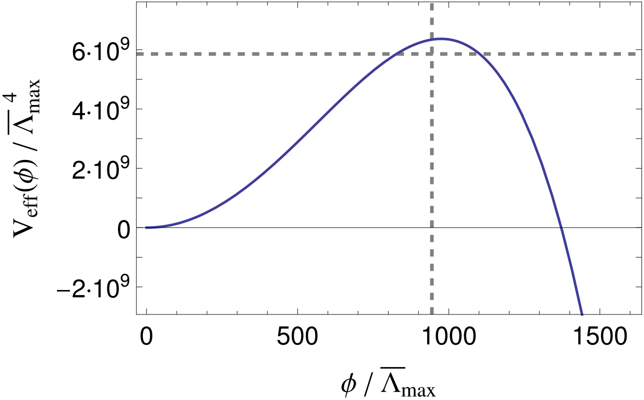

$$\Lambda_{max} \simeq H \left(\frac{12\xi}{|\lambda|}\right)^{1/2}$$

$$V_{max}^{1/4} \simeq H \frac{(6\xi)^{1/2}}{|\lambda|^{1/4}}$$

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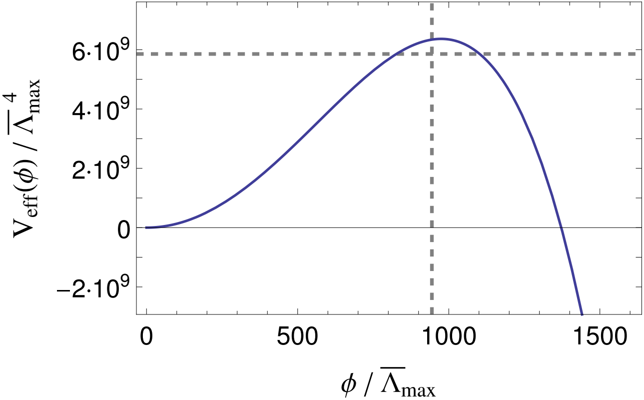

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$$P \sim \exp\left[-8\pi^2 \left(V_{\rm max}/3H^4 \right) \right] \quad \Rightarrow \quad \text{Stable!}$$

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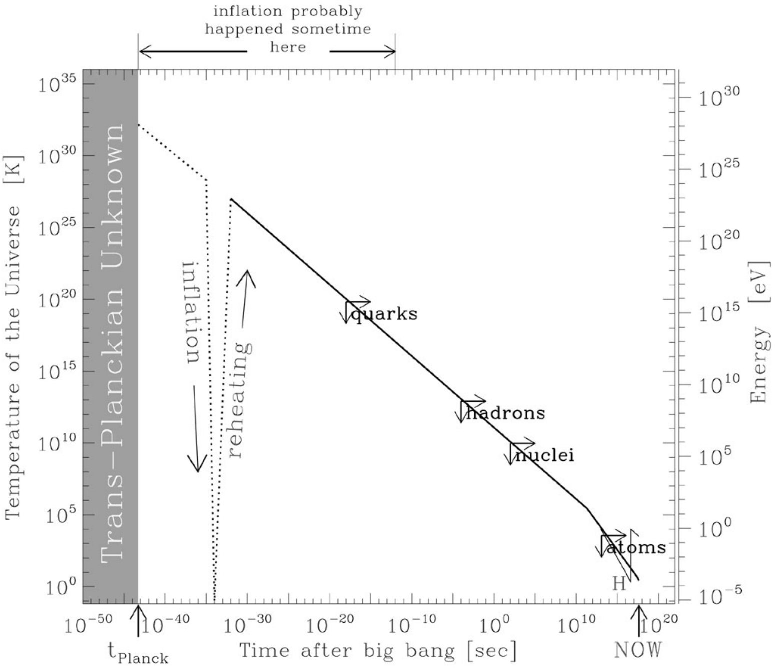

inflation probably happened sometime here
Trans−Planckian Unknown
inflation
reheating
quarks
hadrons
nuclei
atoms
H
Temperature of the Universe [K]
Energy [eV]
Time after big bang [sec]
t_{Planck}
NOW
10^{35}
10^{30}
10^{25}
10^{20}
10^{15}
10^{10}
10^{5}
10^{0}
10^{-50}
10^{-40}
10^{-30}
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Reheating

- Equation of state $w = p/\rho$ changes, $w_{\rm inf} = -1 \rightarrow w_{\rm reh}$
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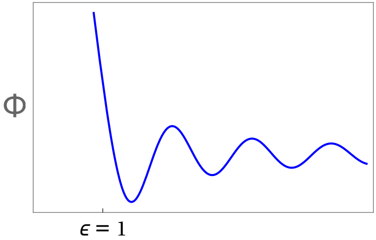
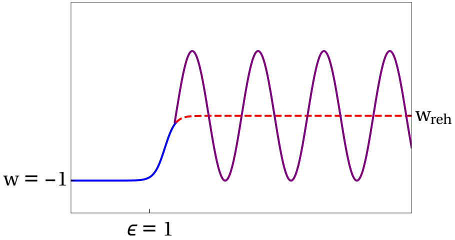
$\Rightarrow$ New stability constraints !

[10] Kofman, Linde & Starobinsky (1997)

Reheating II





- ξR acts as a large mass for $\xi \gtrsim 1/6$, (no stability issues during inflation)

$$f''(\eta) + \left[\mathbf{k}^2 + m^2_{\rm curv}\right] f(\eta) = 0\,; \quad m^2_{\rm curv} = \left(\xi - \frac{1}{6}\right) a^2 R$$

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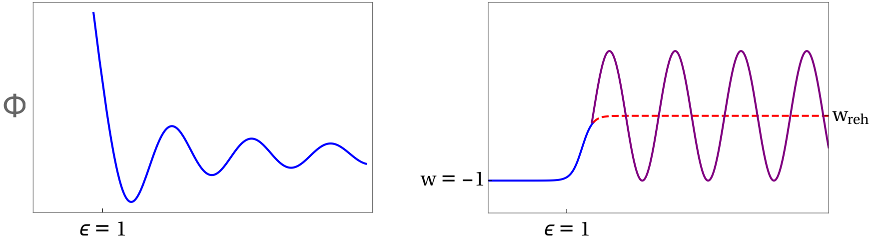

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$$f''(\eta) + \left[\mathbf{k}^2 + m_{\rm curv}^2\right] f(\eta) = 0\,; \quad m_{\rm curv}^2 = \left(\xi - \frac{1}{6}\right) a^2 R$$

- Change in curvature $\Delta R = 3(1 - 3\Delta w)H^2$

A rapidly changing "mass" may result in a large fluctuation

Fluctuation from a drop in mass

- We can model a changing mass with $\tanh(\pm\infty) = \pm 1$ [11]

$$m^2(\eta) = \frac{m_{\rm in}^2 + m_{\rm out}^2}{2} - \frac{m_{\rm in}^2 - m_{\rm out}^2}{2} \tanh \nu\eta.$$

- Exitations quantified by the *occupation number*, $n_{\mathbf{k}} = a_{\mathbf{k}}^\dagger a_{\mathbf{k}}$

$$n_{\mathbf{k}} = \frac{\sinh^2\left[\pi\left(\omega_{\rm out} - \omega_{\rm in}\right)/(2\nu)\right]}{\sinh(\pi\omega_{\rm in}/\nu)\sinh(\pi\omega_{\rm out}/\nu)}\,; \qquad \omega^2 = \mathbf{k}^2 + m^2$$

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- In the radiation-dominated case ($w_{\rm reh} = 1/3$) for a fast transition and $\mathbf{k} \to 0$

$$n_{\mathbf{k}} \sim \sqrt{3\xi}\frac{H}{|\mathbf{k}|}$$

For modes with $\mathbf{k} < aH$

$$\Rightarrow \quad \Delta\phi^2 = a\sqrt{3\xi}\left(\frac{H}{2\pi}\right)^2$$

- Potentially a large effect, $\Delta\phi \gtrsim \Lambda_I$

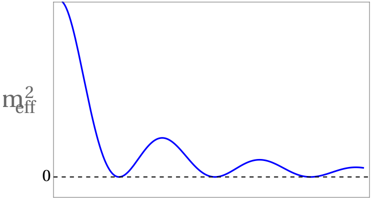
[11] Bernard & Duncan (1977)

Oscillating mass (example)

- In general the inflaton Φ oscillates during reheating

$$V_{\rm inf}(\Phi) \sim \frac{m_{\rm inf}^2}{2}\Phi^2 \quad \Rightarrow \quad \Phi \sim \Phi_0 \cos\left(t\, m_{\rm inf}\right)$$

- For example for a coupling $\mathcal{L}_{\rm int} \propto g\Phi^2\phi^2$



Oscillating mass for Higgs

$$m^2_{\rm eff} \sim g\Phi_0^2 \cos^2(t\, m_{\rm inf})$$

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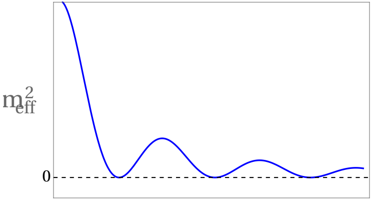


Oscillating mass for Higgs

$$m_{\rm eff}^2 \sim g\Phi_0^2\cos^2(t\, m_{\rm inf})$$

- *Parametric resonance* via the Mathieu equation

$$\frac{d^2 f(z)}{dz^2} + \left[A_{\mathbf{k}} - 2q\cos(2z)\right] f(z) = 0, \qquad z = t\, m_{\rm inf}$$

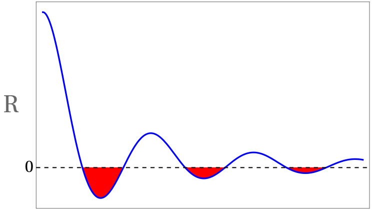
$\Rightarrow$ Exponential amplification, $n_{\mathbf{k}}(N) \propto \exp\{\mu_{\mathbf{k}} N\}$
  - May result in a very large fluctuation [12]

[12] Kofman, Linde & Starobinsky (1997)

Oscillating R

- The curvature also oscillates during reheating

$$R = \frac{1}{M_{\rm pl}^2}\left[4V_{\rm inf}(\Phi) - \left(\frac{d\Phi}{dt}\right)^2\right]$$



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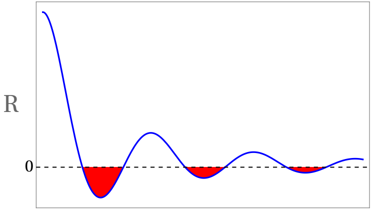

Curvature mass ξR oscillates to negative values

- *Tachyonic resonance* [13]
- Oscillations of R via ξ provide efficient reheating
  - *Geometric reheating* [14]

[13] Kofman et. al. (2006)
[14] Bassett et. al. (1997)

Fluctuations from parametric resonance

- Generically a resonance gives large fluctuations
  - $\Rightarrow$ May result in instabilities !
- After *one* oscillation, with $\xi \gtrsim 1$ and $\mathbf{k} < aH$

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Superhorizon modes, $\mathbf{k} < aH$

$$\Rightarrow \quad \Delta\phi^2 \sim \left(\frac{H}{2\pi}\right)^2 \frac{\exp\left\{\sqrt{\xi}\frac{2\Phi_0}{M_{\rm pl}}\right\}}{\sqrt{\xi}}$$

- Potentially a huge effect

- For a large H one easily generates $\Delta\phi \gg \Lambda_I$
- However, the resonance may be quickly shut off by backreaction

Constraints from backreaction

- So far we have neglected self-interactions and assumed $\rho_{\text{Higgs}} \ll \rho_{\text{inf}}$
  - For a large $\langle\hat{\phi}^2\rangle$ this may not be true

Self-interactions

$$\lambda\langle\hat{\phi}^2\rangle \ll \xi R$$

Gravity

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Gravity

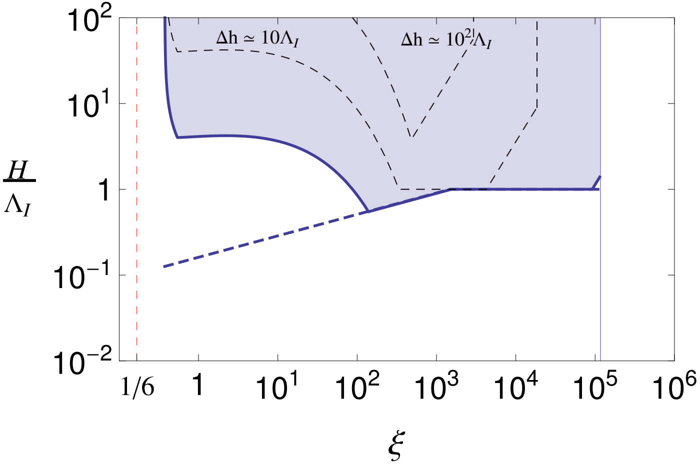
$$\rho_{\rm Higgs} \ll 3M_{\rm pl}H^2$$

- Importantly, $\lambda(H) \simeq \lambda_0\,{\rm sign}(\Lambda_I - H)$

$\Rightarrow$ For a large H self-interactions *amplify* the fluctuation

A wide range of parameters result in a large fluctuation !

Stability results, reheating



$\Rightarrow$ For $H \gtrsim \Lambda_I \sim 10^{11}$GeV, ξ is constrained to be $\sim 1/6$

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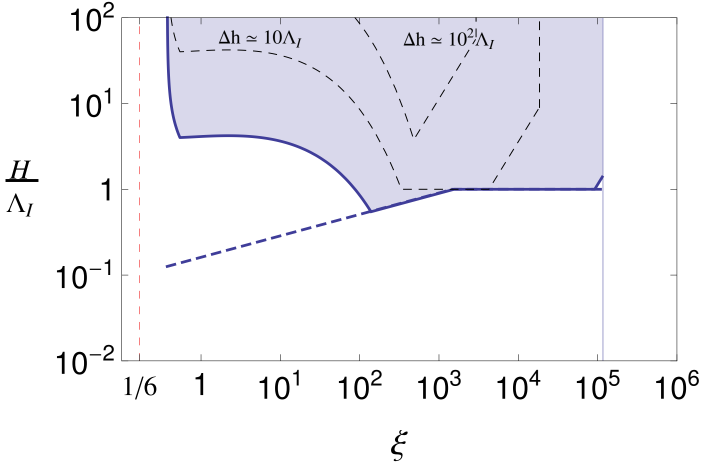

$\Rightarrow$ For $H \gtrsim \Lambda_I \sim 10^{11}$GeV, ξ is constrained to be $\sim 1/6$

- Direct couplings to the inflaton may also be problematic !

Outline

Conclusions

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- For a large H, curvature significantly effects the early universe SM instability
  - Running of couplings from H
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Thank You!

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