A Panorama of Modified Gravity

Philippe Brax
IPhT Saclay
(associate researcher IAP)

The Universe accelerates: why?

Maybe a landscape of Universes?

Or not?

The acceleration of the Universe could also be due to many mundane causes:

A calculable cosmological constant and/or vacuum energy

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -8\pi G_N \Lambda^4 g_{\mu\nu}$$

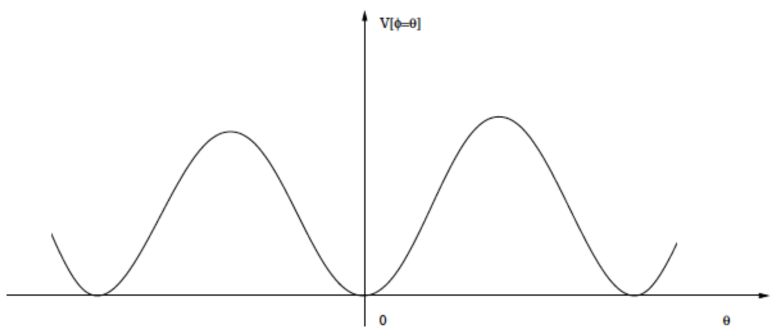
A modification of General Relativity (GR)

$$F(R_{\mu\nu}, g_{\mu\nu} \ldots) = 8\pi G_N T_{\mu\nu}$$

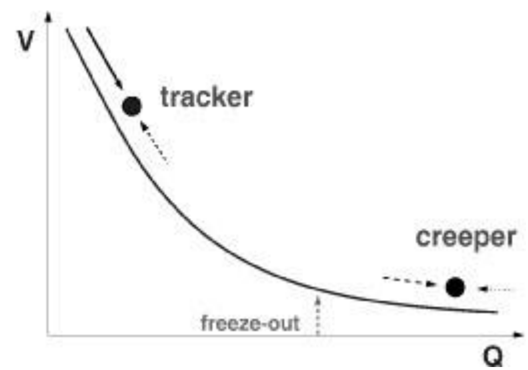
The cosmological dynamics of fields (new matter)

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G_N T^{\text{new}}_{\mu\nu}$$

DARK
ENERGY



Thawing



Freezing

A Simple Example: Pseudo-Goldstone Models

Global U(1) symmetry broken at scale f:

$$\Phi = f e^{i\varphi/\sqrt{2}f}$$

Symmetry broken by small term:

$$\mathcal{L}_{\text{breaking}} = \mu^4 \frac{\Phi}{2f} + cc$$

Low energy potential:

$$V(\varphi) = \mu^4 \cos \frac{\varphi}{\sqrt{2}f}$$

Cosmologically, if initially the field is small compared to f, it is frozen there until its mass becomes larger than the Hubble rate. Tuning this event to be in the recent past of the Universe and requiring that the dark energy is due to the low energy potential implies:

$$\mu \sim \Lambda, \quad f \sim m_{\text{Pl}}$$

A major drawback of these models is that the phase transition must be at the Planck scale, this requires embedding this model in theories dealing with quantum gravity… maybe embedding it in string theory although no global symmetries there…

On the other hand, this model is useful to see that low energy dark energy fields couple to matter:

$$\mathcal{L}_{\rm int} = \frac{\partial\Phi\partial\bar{\Phi}}{f^3}\bar{\psi}\psi \longrightarrow \boxed{\mathcal{L}_{\rm int} \sim \frac{(\partial\phi)^2}{f^3}\bar{\psi}\psi}$$

The derivative interaction implies no effect on static tests of gravity vs fifth forces. This an example of a disformal coupling to matter.

This is one example of coupling to matter, we will also consider conformal couplings which are tightly constrained.

Modified gravity

Massive Gravity

Mass term for a graviton

The simplest modification is massive gravity (Pauli-Fierz):

$$\delta\mathcal{L} = \frac{m_G^2}{4}(h^{\mu\nu}h_{\mu\nu} - h^2)$$

Pauli-Fierz gravity is ghost free (negative kinetic energy terms) . Unfortunately, a massive graviton carries 5 polarisations when a massless one has only two polarisations. In the presence of matter, the graviton wave function takes the form:

$$h_{\mu\nu} = \frac{8\pi G_N}{p^2}(T_{\mu\nu} - \frac{1}{2}T\eta_{\mu\nu}) \to h_{\mu\nu} = \frac{8\pi G_N}{p^2 + m_G^2}(T_{\mu\nu} - \frac{1}{3}T\eta_{\mu\nu})$$

The massless limit does not give GR! (van Dam-Velman-Zakharov discontinuity). The extra polarization is lethal. Solved by Vainshtein mechanism (non-linearity).

Ghost in curved space-time!

Bimetric Gravity

One way to cure these problems is to consider a non-linear version of massive gravity with two dynamical metric:

$$S = \int d^4x (e_1 \frac{R_1}{16\pi G_N} + e_2 \frac{R_2}{16\pi G_N} + \Lambda^4 \sum_{ijkl} m_{ijkl} \epsilon_{abcd} \epsilon^{\mu\nu\rho\sigma} e^{ai}_{\mu} e^{bj}_{\nu} e^{ck}_{\rho} e^{dl}_{\sigma})$$

where the graviton mass is of order: $$m_g^2 \sim \frac{\Lambda^4}{m_{\rm Pl}^2}$$

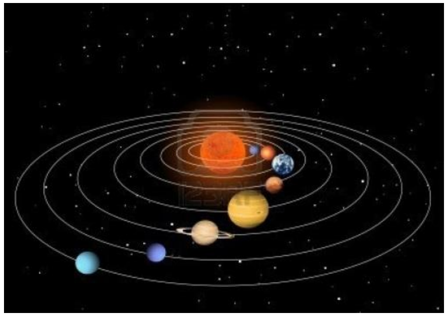
What we have learnt in the last ten years is that such extensions involve light scalar fields, why?:

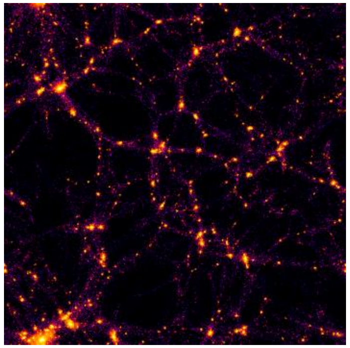
Massive gravitons have a scalar part

$$5 = 2 + 2 + 1$$

Their interaction with matter generates fifth forces which would have been seen in the laboratory and the solar system.

GRAVITY ACTS ON ALL SCALES





Looking for extensions of General Relativity valid from small to large scales.

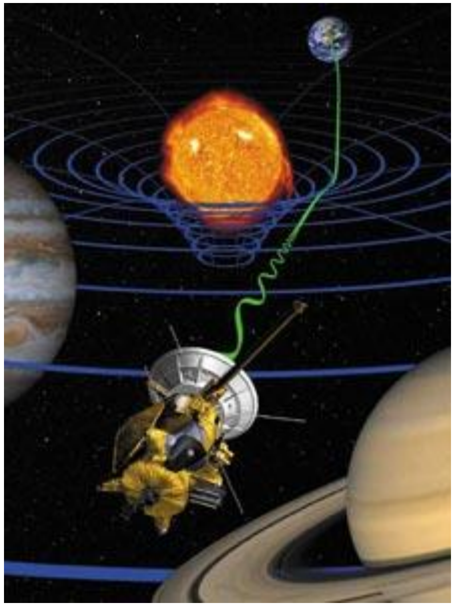
In general, these theories require a fine-tuning of the cosmological constant but have unexpected field theoretic properties which go beyond the usual framework of field theory: irrelevant operators dominate, UV completions may not be required (classicalisation)… The most general form of these theories is known in some cases (Horndeski for one scalar) but one must go into the details of the models to make them work from the solar system (or the laboratory) to large cosmological scales.

This approach is drastically different from the "effective dark energy field theory" method as we are motivated by the compatibility between the local gravitational observations and the physics on the largest cosmological scales:

not just parameterising all the possible extensions of Λ-CDM at the linear level.

Deviations from Newton's law are parametrised by:

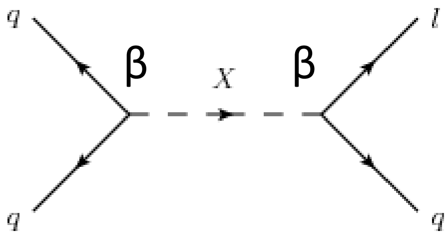
$$\phi_N = -\frac{G_N}{r}(1 + 2\beta^2 e^{-r/\lambda})$$



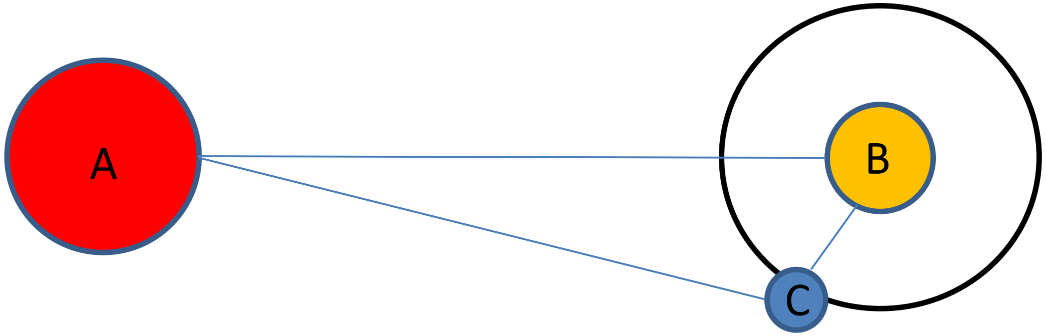
Bertotti et al. (2004)

For large range forces with large λ, the tightest constraint on the coupling β comes from the Cassini probe measuring the Shapiro effect (time delay):

$$\beta^2 \leq 4 \cdot 10^{-5}$$



VIOLATION OF THE STRONG EQUIVALENCE PRINCIPLE



$$\eta_{BC} \equiv \left| \frac{a_C - a_B}{a_C + a_B} \right|$$

$$\eta_{\text{moon}-\text{earth}} \leq 10^{-13}$$

Lunar ranging experiment

Tight bound on the perihelion advance of the moon too (see later)

How do we hide light scalar fields ?

SCREENING

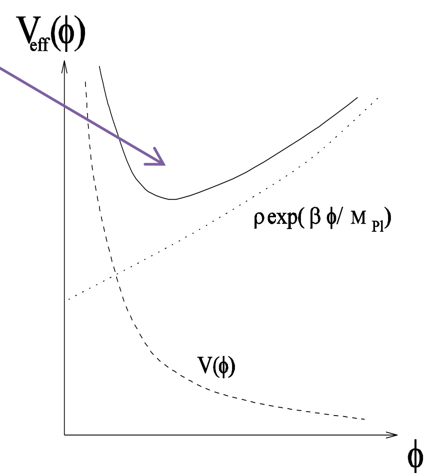
The effect of the environment

When conformally coupled to matter, scalar fields have a matter dependent effective potential

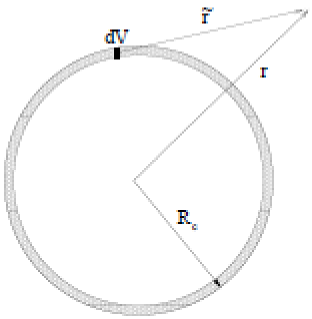
$$V_{eff}(\phi) = V(\phi) + \rho_m (A(\phi) - 1)$$

Environment dependent minimum

$$\beta(\phi) = m_{\rm Pl} \frac{d \ln A(\phi)}{d\phi}$$



Chameleon



The field generated from deep inside is Yukawa suppressed. Only a thin shell radiates outside the body. Hence suppressed scalar contribution to the fifth force.

One example: f(R) gravity

$$S_{\rm MG} = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}f(R)$$

f(R) is totally equivalent to a field theory with gravity and a scalar

$$\boxed{S = \int d^4x\sqrt{-g}\left(\frac{1}{16\pi G_N}R - \frac{1}{2}(\partial\phi)^2 - V(\phi) + \mathcal{L}_m(\psi_m, e^{2\phi/\sqrt{6}m_{\rm Pl}}g_{\mu\nu})\right)}$$

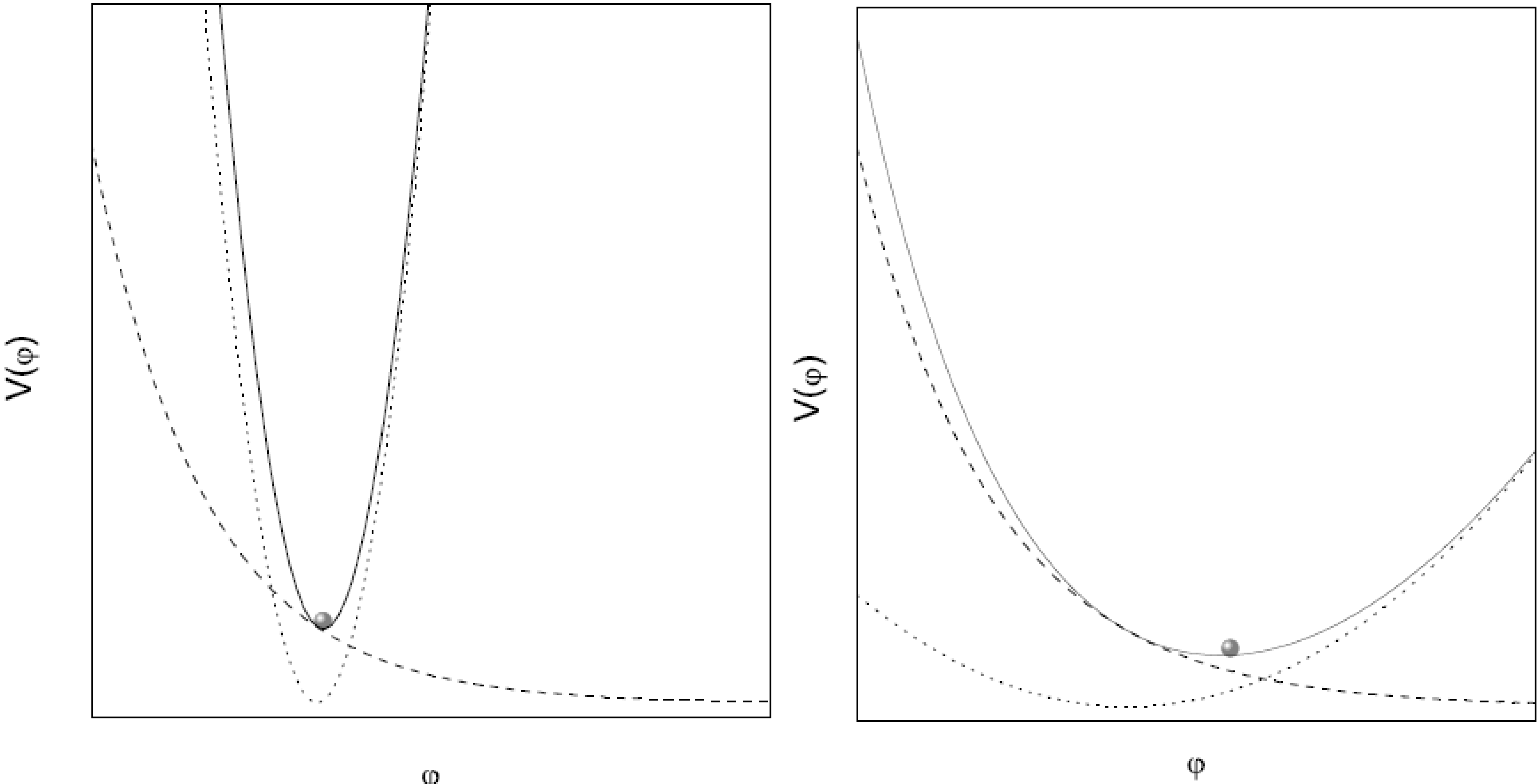
The potential V is directly related to f(R)

Crucial coupling between matter and the scalar field

$$V(\phi) = m_{\rm Pl}^2\frac{Rf' - f}{2f'^2},\ \ f' = e^{-2\phi/\sqrt{6}m_{\rm Pl}}$$

Would be ruled out if no chameleon effect

Damour-Polyakov mechanism



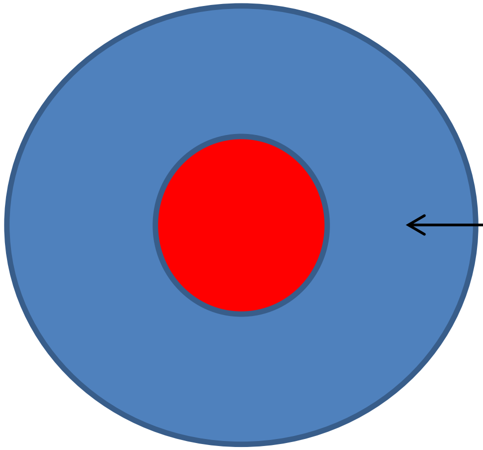
Dilaton

$$V(\phi) = V_0 e^{-\phi/m_{\rm Pl}}, \quad A(\phi) = 1 + \frac{A_2}{2m_{\rm Pl}^2}(\phi - \phi_\star)^2$$

Another simple example: the CUBIC GALILEON

$$\mathcal{L}_\phi = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2\Lambda^3}(\partial\phi)^2\partial^2\phi + \frac{\beta\phi}{M_P}T\ .$$

$\Lambda^3 = m^2 m_{\rm Pl}$ m graviton mass



$$\frac{F_\phi}{F_N} = 2\beta^2\left(\frac{r}{R_V}\right)^{3/2}\ .$$

Well inside the Vainshtein radius, Newtonian gravity is restored. Well outside gravity is modified.

The Vainshtein radius is very large for stars, typically 0.1 kpc for the sun, and a mass for the graviton of the order of the Hubble rate .

K-mouflage models

$$S = \int d^4x\sqrt{-g}(\frac{R}{16\pi G_N} + M^4 K(\chi)) + S_m(\psi, A^2(\phi)g_{\mu\nu}) \qquad \chi = -\frac{(\partial\phi)^2}{2M^4}$$

M is the dark energy scale for cosmologically interesting models. Examples that one may consider:

$$K(\chi) = -1 + \chi + K_0\chi^m$$

Particles have modified trajectories compared to Newton's law in this background:

$$\frac{d^2r}{dt^2} = -\frac{G_N m}{r^2}(1 + \frac{2\beta^2}{K'})$$

Screening happens inside the K-mouflage radius where K'>>1. Still the tiny deviations depend on r and can lead to an anomalous perihelion in the earth-moon system.

In fact, around a background configuration and in the presence of matter, the Lagrangian of such extensions can be linearised and the main screening mechanisms can be schematically distinguished :

$$\mathcal{L} \supset \boxed{-\frac{Z(\phi_0)}{2}}(\partial\delta\phi)^2 - \frac{m^2(\phi_0)}{2}\delta\phi^2 + \frac{\beta(\phi_0)}{M_P}\delta\phi\delta T \ ,$$

The Vainshtein mechanism reduces the coupling by increasing Z. The K-mouflage mechanism has the same effect … while the Damour-Polyakov mechanism suppresses β and the chameleon property increases m.

The Vainshtein and K-mouflage mechanisms can be easily analysed:

Effective Newtonian potential:

$$\Psi = (1 + \frac{2\beta^2(\phi)}{Z(\phi)})\Phi_N$$

For theories with second order equations of motion:

$$Z(\phi) = 1 + a(\phi) L^2 \frac{D^\mu D_\mu \phi}{m_{\rm Pl}} + b(\phi) \frac{(\partial\phi)^2}{M^4} + \dots$$

Vainshtein

K-mouflage

$$M^4 \sim 3 H_0^2 m_{\rm Pl}^2, \quad L \sim H_0^{-1}$$

Cosmological choice

Vainshtein

Newtonian gravity retrieved when the curvature is large enough:

$$\nabla^2 \Phi_N \geq \frac{1}{2\beta L^2}$$

On large cosmological scales, this tells us that overdensities such as galaxy clusters are screened :

$$\delta \geq \frac{1}{3\Omega_{m0}\beta}$$

On small scales (solar system, galaxies) screening only occurs within the Vainshtein radius:

$$R_V = \left(\frac{3\beta m L^2}{4\pi m_{\rm Pl}^2}\right)^{1/3}$$

K-mouflage

Newtonian gravity retrieved when the gravitational acceleration is large enough:

$$|\nabla\Phi_N| \geq \frac{M^2}{2\beta m_{\rm Pl}}$$

On large cosmological scales, this tells us that overdensities such as galaxy clusters are not screened :

$$\frac{k}{H_0} \leq \beta\delta$$

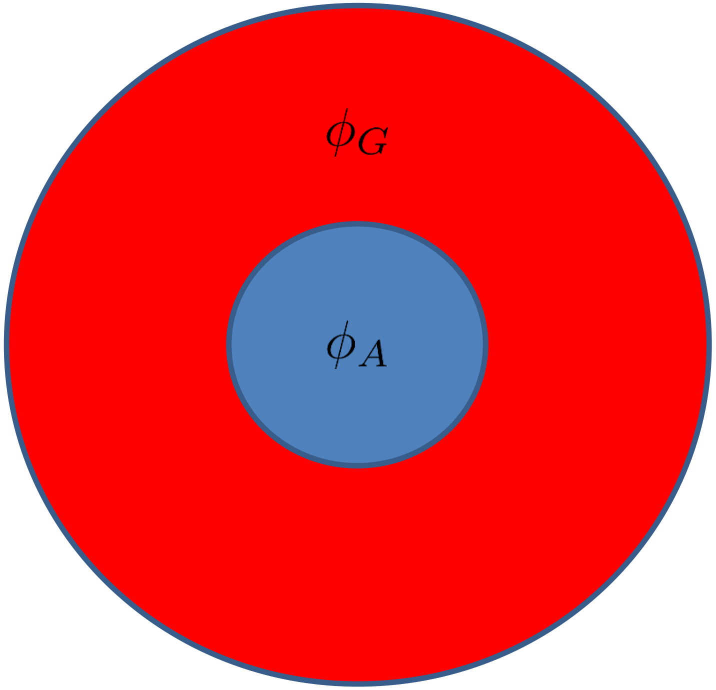
On small scales (solar system, galaxies) screening only occurs within the K-mouflage radius:

$$R_K = \left(\frac{\beta m}{4\pi m_{\rm Pl} M^2}\right)^{1/2}$$

Dwarf galaxies are not screened.

Chameleons:

The screening criterion for an object BLUE embedded in a larger region RED expresses the fact that the Newtonian potential of an object must be larger than the variation of the field:



Scalar charge: $$Q_A = \frac{|\phi_G - \phi_A|}{2 m_{\rm Pl} \Phi_A}$$

$$\boxed{Q_A \leq \beta_G}$$

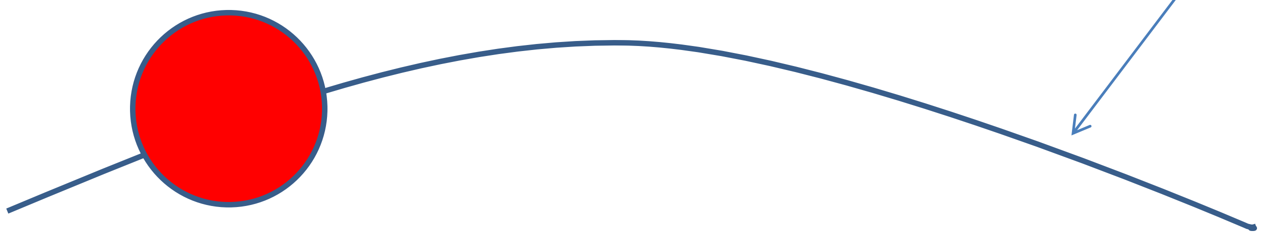
Self screening: large Newton potential

Blanket screening: due to the environment G

Φ_A Newton's potential at the surface

Summary: the motion of massive objects in modified gravity theories:

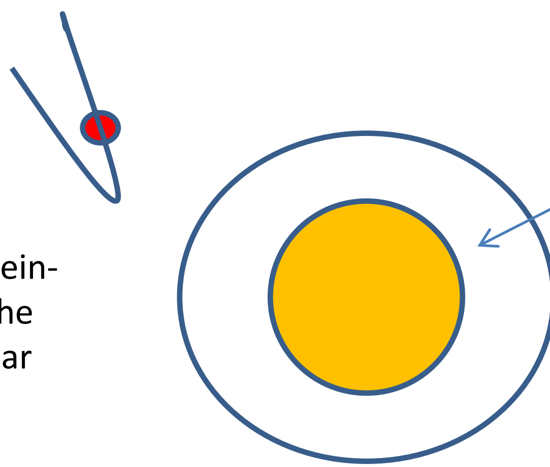
Trajectories due to gravity + scalar



$$\ddot{X} = -\nabla \Phi_N - Q \nabla \phi_{\text{ext}}$$

Chameleons: $Q \ll \beta$

Unlike chameleons et al., K-mouflage and Vainshtein do not affect the charge Q: $Q = \beta$



$\nabla \phi_{\text{ext}} \ll 2\beta \nabla \Phi_N$

Outside the Vainshtein-Kmouflage radius, the object feels the scalar force

$\nabla \phi_{\text{ext}} = 2\beta \nabla \Phi_N$ outside the Vainshtein-Kmouflage radius

One can also change the type of coupling to matter!

Matter couples to a metric which can differ from the Einstein metric involved in the Einstein-Hilbert term with a constant Newton constant:

$$S = \int d^4x \sqrt{-g}\frac{R(g)}{16\pi G_N} + S_m(\psi^i, \tilde{g}_{\mu\nu})$$

Bekenstein (1992) showed that causality and the weak equivalence principle restricts the form of the auxiliary metric:

$$\tilde{g}_{\mu\nu} = A(\phi, X)g_{\mu\nu} + B(\phi, X)\partial_\mu\phi\partial_\nu\phi$$

$$X = -\frac{1}{2}(\partial\phi)^2$$

What is the physics associated with the disformal coupling B(ϕ,X) ?

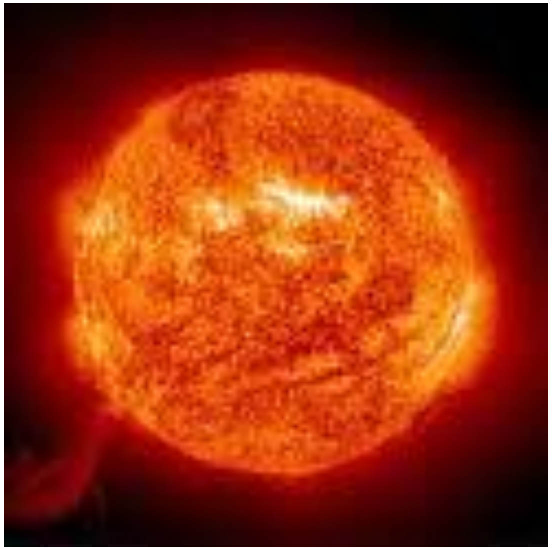
Expanding the effect of the disformal coupling to leading order, we obtain a direct coupling with the energy-momentum tensor:

$$\mathcal{L} \supset \frac{1}{M^4}\partial_\mu\phi\partial_\nu\phi T^{\mu\nu}$$

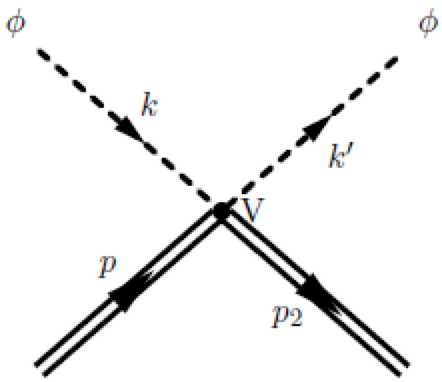
Gravity tests involving static objects are not affected by the disformal coupling: SCREENING

$$T^{00} = \rho \quad \rightarrow \quad \frac{\dot{\phi}^2}{M^4}\rho \equiv 0$$

As the axions, the disformally coupled scalars can have an effect at high density and high temperature inside the inferno at the core of stars. From the gentle burning of main sequence stars, to helium burning stars and then the explosion of supernovae, the processes involve higher energies (hence shorter distances) and electromagnetic to strong interaction processes.

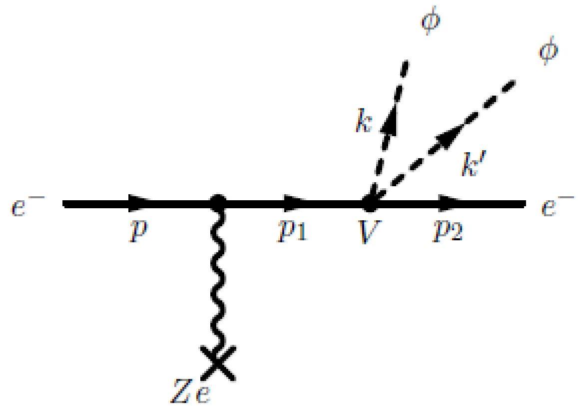


Scalars increase the burning rate provided they can escape the star itself. This can be hampered if the scalar have a mean free path due to their interactions with nucleons which is smaller than the size of the star. The mean free path is always large for the values of M compatible with the absence of too much burning inside a star.



$$l = \frac{16\pi(N+Z)}{Z}\frac{M^8}{\rho T^4 m_p}$$

Bremsstrahlung is one of the most common processes in stars. Here two scalars can be emitted from one electron in the electric field of a nucleus.



$$\epsilon = \frac{Z^2\alpha^2 m_e}{210\pi^3 A m_p}\rho\left(\frac{T}{M}\right)^8 \frac{1}{(2\pi m_e T)^{3/2}} g\left(\frac{m_D^2}{2m_e T}\right),$$

$$g(x) = \int_0^\infty du \frac{u^9 e^{-u}}{(u+x)^2}.$$

The strongest constraint comes from stars for which helium burns: $\epsilon_{HB} \leq 10\ \mathrm{erg/s\cdot g}$

$M \geq 173\ \mathrm{MeV}$

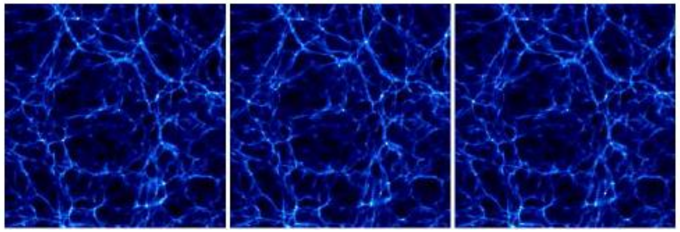
Are there distinguishable features of the screening mechanisms?

At the background level?

Examples: the Sandage test +gravitational waves

At the perturbative level?

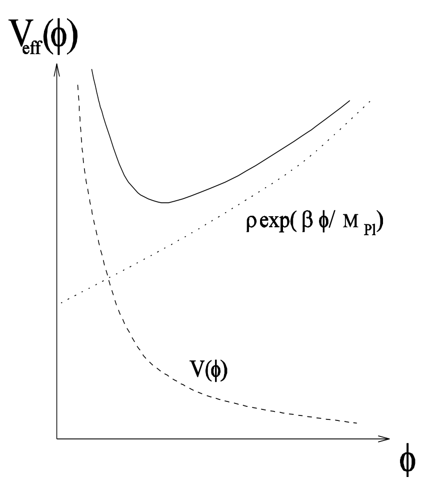
Linear scales? Non-linear?



Chameleons in more detail:

All the chameleon and Damour-Polyakov models can be entirely characterised by 2 density dependent functions. The non-linear potential and coupling of the model can be reconstructed using:

$$\begin{aligned}\phi(a) &= \phi_i + \frac{3}{m_{\rm Pl}}\int_{a_i}^{a} da \frac{\beta(a)\rho(a)}{a m^2(a)} \\ V(a) &= V_i - \frac{3}{m_{\rm Pl}}\int_{a_i}^{a} da \frac{\beta^2(a)\rho^2(a)}{a m^2(a)}\end{aligned}$$

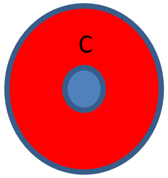


tomography

$$\begin{matrix} m(a) \\ \beta(a) \end{matrix} \longleftrightarrow \begin{matrix} m(\rho) \\ \beta(\rho) \end{matrix}$$

$$\rho(a) = \frac{\rho_0}{a^3}$$

The Milky Way must be screened



If no effects on the dynamics of satellite galaxies :

$$\phi_C \leq 2\beta_C m_{\rm Pl}\Phi_G$$

ϕ_C

β_C

The environment C is the cosmological background if the local cluster is not screened.

It is the local cluster if it is screened.

This gives a bound depending on the mass and coupling

$$a_C = 1$$

Self screening

$$a_C = (200)^{-1/3}$$

Blanket screening

$$9\Omega_{m0}H_0^2 \int_{a_G}^{a_C} da \frac{\beta(a)}{a^4 m^2(a)} \leq 2\beta_C \Phi_G$$

$$\Phi_G \sim 10^{-6} \qquad a_G \sim 10^{-2}$$

For chameleons and dilatons:

3/2<r<3, s=0 : chameleons

$$\beta(a) = \beta_0 a^{-s}, \quad m(a) = m_0 a^{-r}$$

r=3/2, s=-3: dilaton

Self-screening of the Milky Way:

$$\frac{m_0^2}{H_0^2} \geq \frac{9\Omega_{m0}}{2(2r-3-s)} 10^6$$

This bounds the range of the scalar interaction to be less than a few Mpc's on cosmological scales

This also constrains the background to coincide with the concordance model:

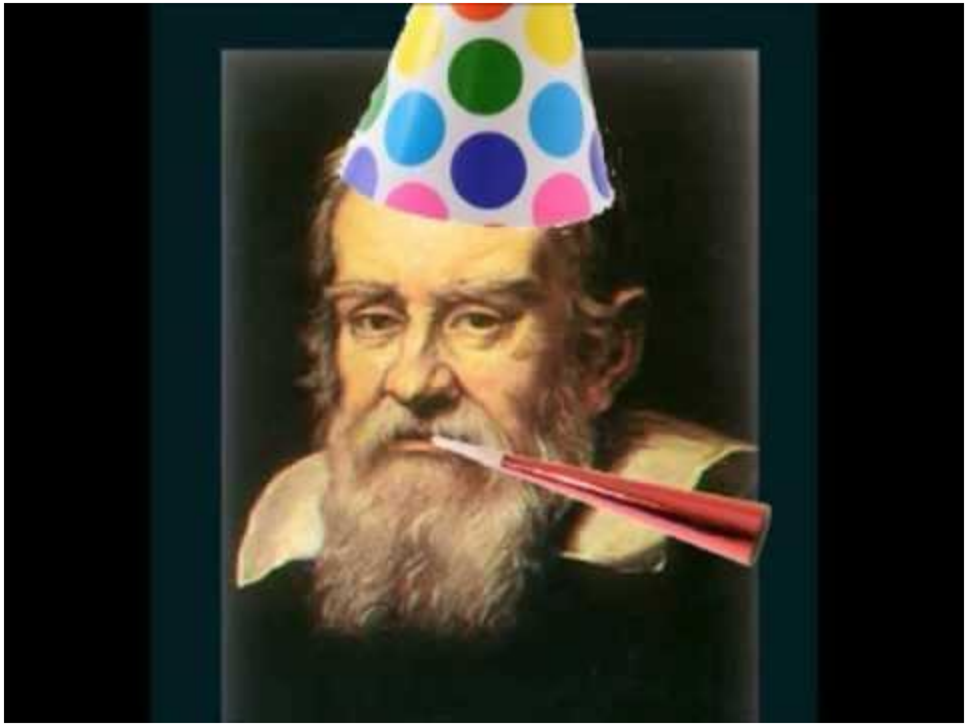
$$\omega + 1 = \mathcal{O}(\frac{H_0^2}{m_0^2}) \leq 10^{-6}$$

This applies to the popular Hu-Sawicki model which serves as a template for a lot of works:

$$f(R) = R - \Lambda_0 - \frac{f_{R_0}}{n}\frac{R_0^{n+1}}{R^n}$$

$$\beta = \frac{1}{\sqrt{6}},\ m_0 = H_0\left(\frac{4\Omega_{\Lambda 0} + \Omega_{m0}}{(n+1)|f_{R0}|}\right)^{1/2},\ m = m_0\left(\frac{4\Omega_{\Lambda 0} + \Omega_{m0}a^{-3}}{4\Omega_{\Lambda 0} + \Omega_{m0}}\right)^{(n+2)/2}$$

More about Galileons:



The Galileon models are motivated by the requirement that the effective scalar field theory is invariant under a symmetry which prevents the existence of a potential term generalising a cosmological constant. The aim is to get self-accelerated Universes without a cosmological constant. The acceleration of the Universe would have to be self-generated dynamically:

$$\partial_\mu \phi \to \partial_\mu \phi + c_\mu$$

This is a Galilean invariance of the action. The scalar Lagrangian involves only a small number of building blocks:

$$\mathcal{L} = K(\phi, X) - G_3(\phi, X) D^2\phi + G_4(\phi, X) R + G_{4,X}((D^2\phi)^2 - (D_\mu D_\nu \phi)^2)$$
$$-\frac{1}{6} G_{5X}((D^2\phi)^3 - 3D^2\phi(D_\mu D_\nu \phi)^2 + 2D^\mu D_\alpha \phi D^\alpha D_\beta \phi D^\beta D_\mu \phi)$$

where X is the kinetic term of the scalar field. The scalar field ϕ is screened by the Vainshtein mechanism.

The coupling functions are not arbitrary but specified by 4 real coefficients:

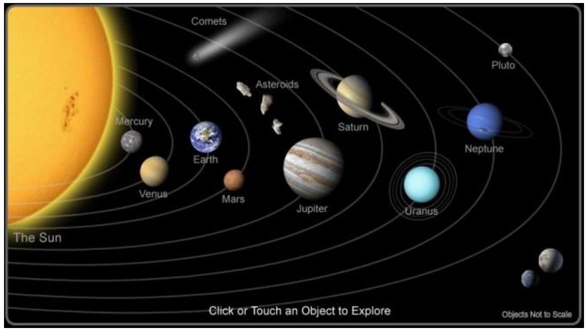
$$K(\phi, X) = c_2 X,\; G_3(\phi, X) = -\frac{2c_3}{\Lambda^3} X,\; G_4(\phi, X) = \frac{2c_4}{\Lambda^6} X^2,\; G_5(\phi, X) = -\frac{6c_5}{\Lambda^9} X^2$$

The simplest example: the cubic Galileon. The scale Λ is the strong coupling scale of the model. These models have a cosmological interest when:

$$\Lambda^3 = m_{\rm Pl} H_0^2$$

Screening for Galileon theories happens inside the Vainshtein radius. Still the tiny deviations depends on r and can lead to an anomalous perihelion in the earth-moon system.

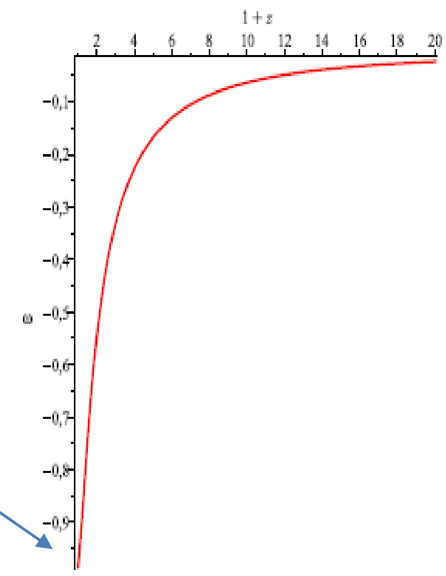
$$|\delta\theta| = \pi r \frac{d}{dr}(r^2 \frac{d}{dr}(\frac{\epsilon}{r})) \leq 10^{-11}$$



$$\frac{\beta^{1/3}\Lambda}{c_3^{1/3}} \leq 1.6\ 10^{-22}\ \text{GeV}$$

This forces us to consider cosmological Galileons for couplings to matter of order one

Equation of state -1 now



This requires tuning of some of the parameters of the models

The Sandage Test

The Sandage effect:

$$\frac{\Delta v}{c} \equiv \frac{\Delta z}{z+1} = \frac{\Delta t}{1+z}((1+z)H_0 - H(z))$$

An observer measuring the variation of the redshift of a distant source at two different times will see a drift which can be interpreted as a Doppler effect due to the « spectroscopic velocity ». This is essentially a fancy way of doing cosmography.

The spectroscopic velocities can be calculated for typical models subject to the three screening mechanisms:

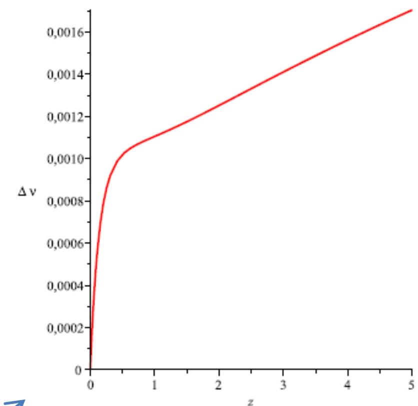
F(R) and
dilatons

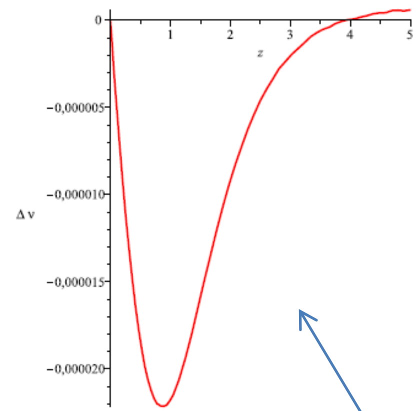
Cubic K-
mouflag
e

Quartic
Galileons

For models with the chameleon or Damour-Polyakov screening mechanisms, here f(R) in the large curvature limit and dilatons, the spectroscopic velocities differ by a negligible amount from Λ-CDM. This is entirely due to the stringent constraints on the cosmological mass of the scalar field from solar system tests. Not detectable.

$$\frac{m_0}{H_0} \geq 10^3$$

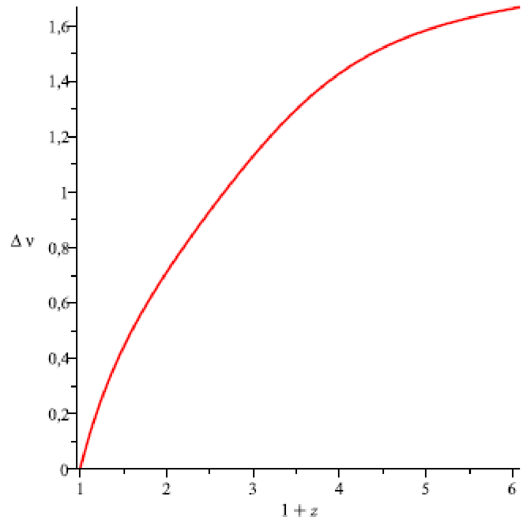




Dilaton coupled to matter.

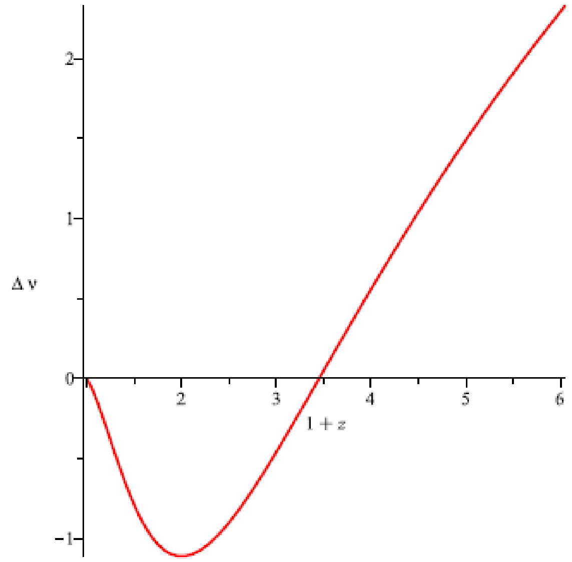
Large curvature f(R)

For K-mouflage models, here the cubic model, the late time evolution of the field is unscreened as the screening effects only take place earlier in the matter and radiation eras. This implies a large deviation from Λ-CDM at the cm/s level despite the constraint on β from solar system tests.



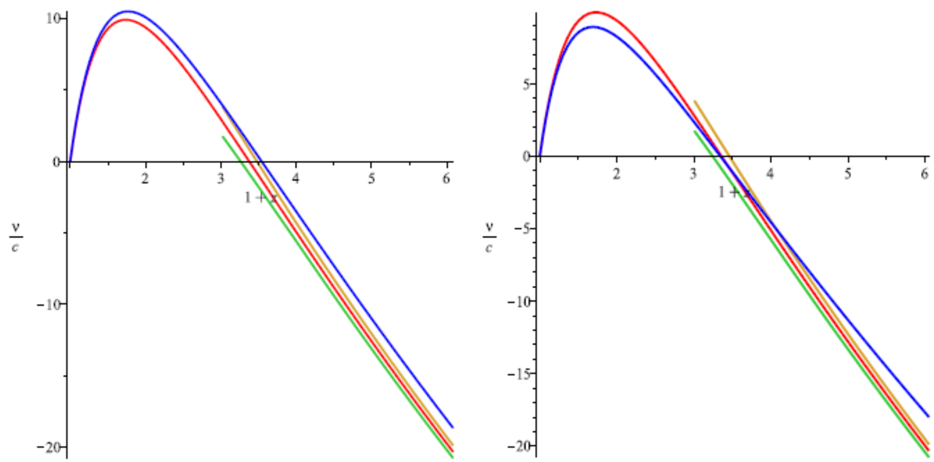
The variation could even be larger for models behaving at late time like free scalar fields.

Galileons, here the quartic model with an equation of state of -1 now, evolve also in a rapid fashion in the late time Universe. This entails a variation of the spectroscopic velocity at the cm/s level.



We have focused on Galileons for which Minkowski space is a stable solution, i.e. the ones embeddable into 5d branes models with positive tension branes.

The precision of forthcoming experiments such as HIRES-El can be estimated for quasar absorption system at redshifts z>2, depending essentially on the number of years of observations and the number of objects. For 30 years of tracking and 100 objects, K-mouflage (left) and Galileons (right) deviate at 2-sigma for z>4.



Cubic K-mouflage

Quartic Galileons

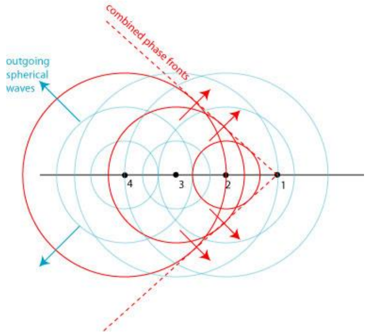
$$\sigma = 1.35(\frac{N_{QSO}}{30})^{-1}(\frac{S/N}{2370})^{-1})(1+z)^{-1.7}$$

Another possible effect of modified gravity is… to change the speed of gravitons compared to the speed of light.

Severe constraints must be taken into account:

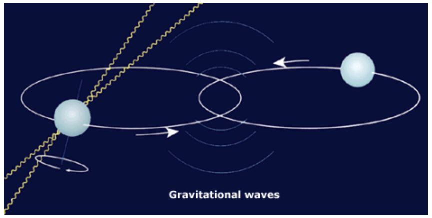
If the gravity waves travel slower than the speed of light, relativistic cosmic rays may emit gravitons by the Cerenkov effect. This would suppress them heavily unless:

$$1 - \frac{c_T}{c} \leq 10^{-17}$$



If the gravity waves travel faster than the speed of light, this would affect the period of binary pulsars unless:

$$\frac{c_T}{c} - 1 \leq 10^{-2}$$



Modified gravity models can have effects on the speed of gravity waves when the term Einstein-Hilbert term is modified in the significant way:

$$G_{4,X} \neq 0$$

Typically in astrophysical situations, we are interested in the emission of spherical waves in a time-dependent cosmological background. In this context the wave equation for gravitons takes the form:

$$\omega^2(G_4 - G_{4,X}\dot{\phi}^2) - 2\omega k\dot{\phi}\partial_r\phi - k^2(G_4 + G_{4,X}(\partial_r\phi)^2) = 0$$

The speed of gravity wave is “screened”, i.e. hardly modified, if the terms in $G_{4,X}$ can be neglected. Could it be that the “Vainshtein mechanism” plays a role here too and screens the speed of gravity waves to a level such that the tight bounds from the Cerenkov effect and the timing of pulsars could be satisfied?

Inside the Vainshtein radius of quartic Galileons, IF spatial gradients are larger than time derivatives:

$$c_T^2 = 1 - \frac{2XG_{4,X}}{G_4}$$

Where the gradient is essentially constant… and very small… implying that the speed of gravity waves would be screened…

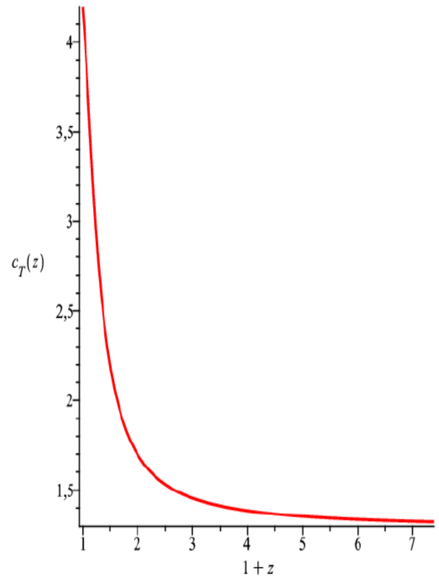
$$X = -\frac{\Lambda^4}{2}\left(\frac{c_{0b}M}{8\pi m_{\rm Pl}c_4}\right)^{2/3}, \quad |\Delta c_T^2| \leq 10^{-30}$$

Unfortunately, the time derivatives are smaller than the gradients when:

$$R_V H_0 \gg 1$$

This is violated for masses of around one solar mass! And gravity waves are not screened.

$$R_V H_0 \sim 10^{-7}$$



For Galileon models with an equation of state of -1 now, the speed of gravity waves is way too large… Can be cured using only cubic Galileons… but they have an equation of state -1 only when c2 <0 (Minkowski unstable)…

Difficult to distinguish models at the background level… how about

Large Scale Structure?

The chameleons modify the growth of structure in a scale and time dependent way.

Galileons only modify the growth in a time-dependent way. All structures up to clusters are screened so reducing possible effects.

K-mouflage modifies growth in a time-dependent way. Galaxy clusters are not screened so effect on the number of large clusters.

Chameleons:

The cosmological background evolves like in the concordance model. The main difference coming from the modification of gravity arises at the perturbation level where the Cold Dark Matter density contrast evolves like:

$$\delta'' + \mathcal{H}\delta' - \frac{3}{2}\Omega_m\mathcal{H}^2(1+\frac{2\beta^2}{1+\frac{m^2a^2}{k^2}})\delta = 0$$

Modified gravity

The new factor in the bracket is due to a modification of gravity depending on the comoving scale k. Most effects in the quasi-linear regime around Mpc scales, i.e. requires N-body or sem-analytical methods to go beyond linear perturbation theory.

This is now available for most models.

The growth of structures depends on the comoving Compton length:

$$\lambda_c = \frac{1}{ma}$$

Gravity acts in an usual way for scales larger than the Compton length (matter era)

$$\delta \sim a$$

Gravity is modified inside the Compton length with MORE growth (matter era):

$$\delta \sim a^{\frac{\nu}{2}}, \quad \nu = \frac{-1 + \sqrt{1 + 24(1 + 2\beta_c^2)}}{2}$$

When the coupling is a function of the scalar field, the growth is not power-like but still anomalous growth.

Lensing is also affected:

$$\Phi_{\mathrm{WL}} = \Phi_N$$

No modification of the lensing potential

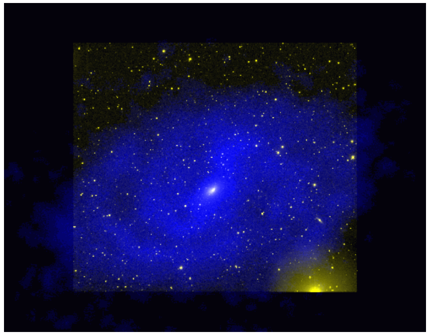
But the density contrast is modified and the effective Newton constant varies with time:

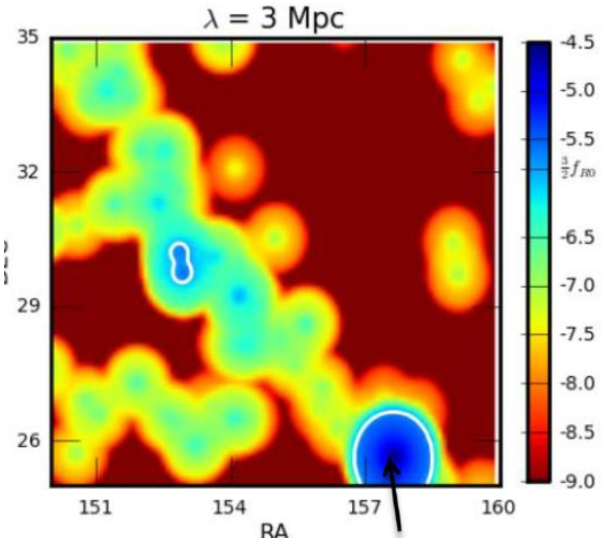
$$\Delta\Phi_N = 4\pi \bar{A}(t) G_N a^2 \delta\rho$$

Growth and lensing are modified in all these models. Still linear scales more reliable. Comparison with available data of large scale structure and lensing has been performed giving constraints on f(R) and other models such as dilatons which are loosely restricted. For f(R), the cosmological bounds are not as good as the bounds from astrophysics but subject to fewer uncertainties.

Astrophysical tests:

Motion of screened stars different from unscreened HI gas in unscreened dwarf galaxies.



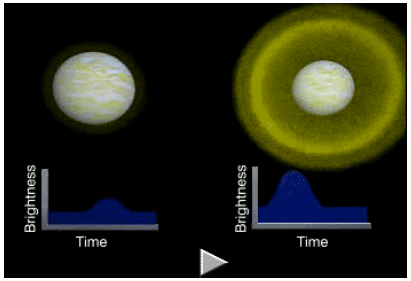


$\xi = 10^{-6}$

screened

Distance indicators for cepheids and TRGB stars in screened and unscreened dwarf galaxies are different as their luminosities vary

SDSS catalogue, within 200 Mpc, scalar range 1 Mpc

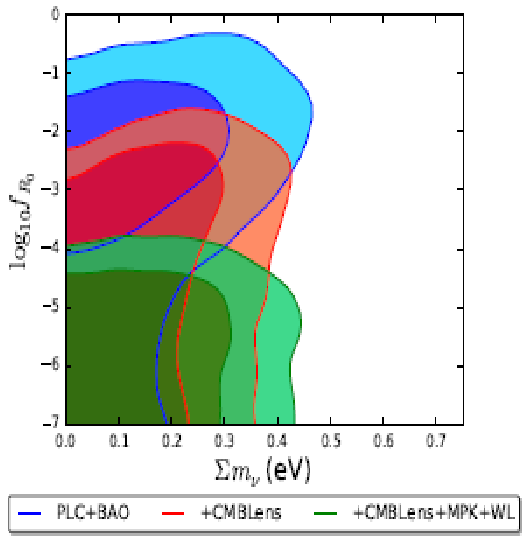


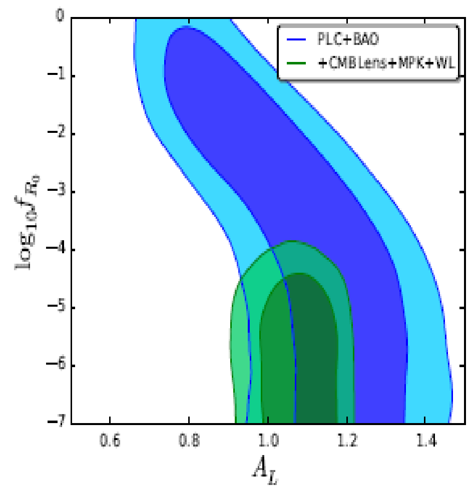
$\Phi_N \sim 10^{-7}$

No effects measured so far: bound on the range of the scalar interaction.

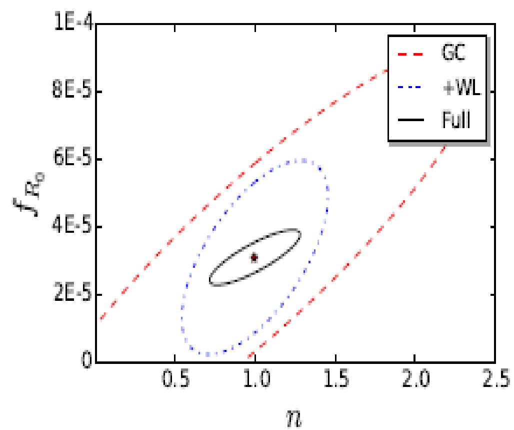
$$|f_{R_0} \leq 10^{-7}$$

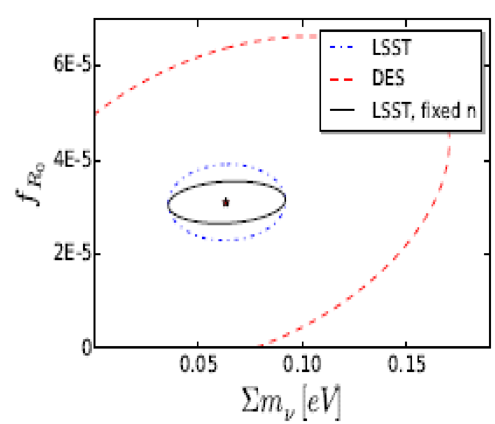
The latest contours on f(R) from cosmological observables



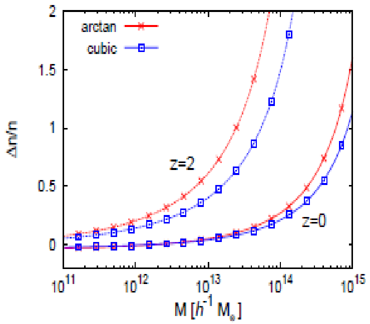


$f(R)$+A_L, fixed $\sum m_\nu$		$f(R)$+A_L, varying $\sum m_\nu$		
f_{R_0}	A_L	f_{R_0}	A_L	$\sum m_\nu$
$3\,(8) \times 10^{-5}$	$1.08^{+0.07\,(0.12)}_{-0.05\,(0.13)}$	$0.4\,(1.0) \times 10^{-4}$	$1.11^{+0.10\,(0.16)}_{-0.06\,(0.15)}$	$0.30\,(0.38)$





One of the main stumbling blocks of all these analyses is the difficulty of finding clear features which could distinguish models. One of those would be the fact that the physics of large clusters is largely affected for K-mouflage models passing the solar system tests.



More clusters for K-mouflage models on large scales

Much more analytical and numerical work will be required to chart the different modified gravity models which are compatible with local gravity tests.

Conclusions

Light scalar fields could be what remains from massive gravity, string theory …. They need to be screened in the local environment otherwise tight bounds would be violated.

This prompts one to study the screening mechanism from a bottom-up approach irrespective of their UV completion, if ever needed.

For conformally coupled scalars, there are only three main mechanisms: Vainshtein, K-mouflage and chameleon+Damour-Polyakov.

Must find distinguishing features.