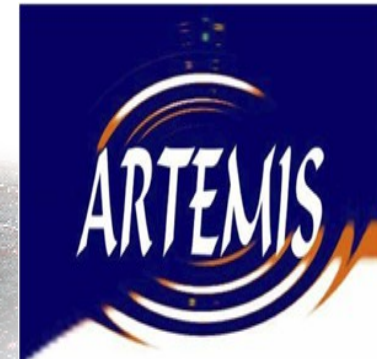


Rethinking the link between matter and geometry

Olivier Minazzoli

Centre Scientifique de Monaco et
Artemis, Université Côte d'Azur, Observatoire Côte
d'Azur



Outline

- Introduction
- Background : intrinsic decoupling in scalar-tensor theories with universal matter coupling
 - Action, field equations & phenomenology
- The new proposal
 - Action, field equations & an alternative formulation
- Discussion
 - Good things and questions
- Conclusion

Equivalence principle

The happiest thought of my life

« I was sitting in a chair in the patent office at Bern when all of a sudden a thought occurred to me : 'If a person falls freely he will not feel his own weight.' I was startled. This simple thought made a deep impression on me. It impelled me toward a theory of gravitation »

Albert Einstein

Equivalence principle

Gravity is not a force

Gravity is inertia **BUT** in a curved space-time

Explains the equivalence between inertial and gravitational masses

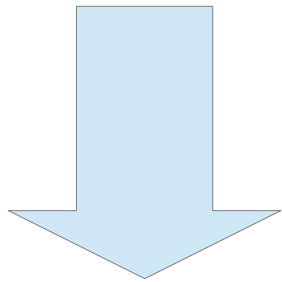
GR Lagrangian formulation

Ricci scalar

$$S = \frac{1}{c} \int \left(\frac{R(g)}{2\kappa} + L_m \right) \sqrt{|g|} d^4 x$$

$$\kappa = \frac{8\pi G}{c^4}$$

Naive classical assumption for (dust) matter



$$L_m^{OS} = -\rho = -\sum_A m_A c^2 \left(u^0 \sqrt{-g} \right)^{-1} \delta^{(3)}(\vec{x} - \vec{x}_A)$$

$$d m_A / d \tau = 0$$

$$S = \frac{1}{c} \int \frac{R(g)}{2\kappa} \sqrt{|g|} d^4 x - c^2 \sum_A \int_A m_A d\tau$$

→ **EIH equation of motion**

EIH=Einstein-Infeld-Hoffmann

GR and SM Lagrangian formulation

$$S = \frac{1}{c} \int \left(\frac{R(g)}{2\kappa} + L_m(g, A, W, G, H, f) \right) \sqrt{|g|} d^4x$$

The diagram illustrates the decomposition of the Lagrangian L_m in the action S . The action is given by $S = \frac{1}{c} \int \left(\frac{R(g)}{2\kappa} + L_m(g, A, W, G, H, f) \right) \sqrt{|g|} d^4x$. The term L_m is associated with the Standard Model particle Lagrangian. The diagram shows arrows pointing from labels to the fields in L_m :

- EM points to A
- Strong nuclear points to G
- Weak nuclear points to W
- Higgs points to H
- quarks and leptons points to f

Standard model
particle Lagrangian

And constants...

GR Lagrangian formulation

$$S = \frac{1}{c} \int \left(\frac{R(g)}{2\kappa} + L_m \right) \sqrt{|g|} d^4 x$$

Action densities « glued together additively »
in Einstein's words

Meaning : Matter and geometry can be described separately → they are independent by nature

GR tests

- Weak field
 - Solar system (bending of light, perihelion advance, gravitational redshift, etc.)
 - Binary pulsars decay orbit
 - Lensing (weak and strong)
 - Observation of gravitational waves
- Non-linear regimes
 - Cosmology – standard Λ CDM model
 - Black holes
 - Generation of gravitational waves

GR issues

- Mach's Principle

Mach's principle

« it would be a remarkable coincidence if the inertial frame [...] just happened to be the reference frame in which typical stars are at rest »

Steven Weinberg

« [In modern words, Mach's idea is that] inertial forces observed locally in an accelerated laboratory may be interpreted as gravitational effects having their origin in distant matter accelerated relative to the laboratory »

Carl Brans

Mach's principle

The existence of vacuum solutions in GR show that the expression of Mach's principle is at best imperfect in **GR**.

See for instance [Brans & Dicke, *Phys. Rev.* 1961]

Or [Weinberg, *Rev. Mod. Phys.* 1989]

Or [Pais, *Subtle is the Lord: The science and the life of Albert Einstein*, 1982]

GR issues

- Mach's Principle
- Dark energy, dark matter and inflation
(but solution could come from particle sector)

GR issues

- Mach's Principle
- Dark energy, dark matter and inflation
(but solution could come from particle sector)
- Quantum gravity
(but GR could be asymptotically safe or need other non-perturbative technics like in LQG,...)

GR issues

- Mach's Principle
- Dark energy, dark matter and inflation
(but solution could come from particle sector)
- Quantum gravity
(but GR could be asymptotically safe or need other non-perturbative technics like in LQG,...)
- Unification
(but current paradigm : GR is not a force
→ a priori no reason to be unified with gauge theories)

Outline

- Introduction
- **Background : intrinsic decoupling in scalar-tensor theories with universal matter coupling**
 - **Action, field equations & phenomenology**
- The new proposal
 - Action, field equations & an alternative formulation
- Discussion
 - Good things and questions
- Conclusion

Intrinsic decoupling from universal scalar-matter coupling

General class :

$$S = \frac{1}{c} \int \left[\frac{1}{2\kappa} \left(\phi R - \frac{\omega(\phi)}{\phi} (\partial_\sigma \phi)^2 \right) + f(\phi) L_m(g, \Psi) \right] \sqrt{|g|} d^4 x$$

Field equations

Simplification for this talk

$$S = \frac{1}{c} \int \left[\frac{1}{2\kappa} \left(\phi R - \frac{\omega(\phi)}{\phi} (\partial_\sigma \phi)^2 \right) + f(\phi) L_m(g, \Psi) \right] \sqrt{|g|} d^4 x$$

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = \kappa \frac{f(\phi)}{\phi} T_{\alpha\beta} + \frac{1}{\phi} \left[\nabla_\alpha \nabla_\beta - g_{\alpha\beta} \nabla^2 \right] \phi$$

$$+ \frac{\omega}{\phi^2} \left[\partial_\alpha \phi \partial_\beta \phi - \frac{1}{2} g_{\alpha\beta} (\partial \phi)^2 \right]$$

$$\nabla_\sigma T^{\alpha\sigma} = (L_m^{OS} g^{\alpha\sigma} - T^{\alpha\sigma}) \partial_\sigma \ln f(\phi)$$

$$\frac{2\omega + 3}{\phi} \nabla^2 \phi = \kappa \left(\frac{f(\phi)}{\phi} T - 2 f'(\phi) L_m^{OS} \right)$$

Special cases with decoupling

$$\frac{2\omega + 3}{\phi} \nabla^2 \phi = \kappa \left(\frac{f(\phi)}{\phi} T - 2 f'(\phi) L_m^{OS} \right)$$

Same naive classical assumption for (dust) matter as in GR

$$L_m^{OS} = -\rho = -\sum_A m_A c^2 \left(u^0 \sqrt{-g} \right)^{-1} \delta^{(3)}(\vec{x} - \vec{x}_A)$$



$$L_m^{OS} = T$$

$$d m_A / d \tau = 0$$

$$\frac{\phi f'(\phi)}{f(\phi)} = \frac{1}{2}$$



$$\nabla^2 \phi = 0$$

Special cases with decoupling

In other words :

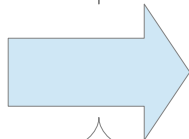
$$S = \frac{1}{c} \int \left[\frac{1}{2\kappa} \left(\phi R - \frac{\omega}{\phi} (\partial_\sigma \phi)^2 \right) + \sqrt{\phi} L_m(g, \Psi) \right] \sqrt{|g|} d^4 x$$

and $L_m^{OS} = -\rho = -\sum_A m_A c^2 \left(u^0 \sqrt{-g} \right)^{-1} \delta^{(3)}(\vec{x} - \vec{x}_A)$

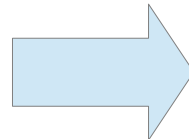
Total decoupling

(Not entirely correct)

$$\nabla^2 \phi = 0$$



$$\phi = \phi_c$$



$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = \kappa \frac{f(\phi_c)}{\phi_c} T_{\alpha\beta}$$

(but gives the idea)

The scalar field is not sourced → one recovers GR for dust fields

Subtlety

So far, one used a naive classical view of (dust) matter

$$S = \int h L_m \sqrt{|g|} d^4 x \quad \longrightarrow \quad \left\{ \begin{array}{l} S = \sum_A \int_A h m_A d\tau \\ \frac{d m_A}{d\tau} = 0 \end{array} \right.$$

Is this correct ??

Standard model Lagrangian < 100 GeV \rightarrow SU(3)xU(1)

$$h L_m = h \left(\sum_i \bar{\psi}_i (i D - m_i) \psi_i - \frac{1}{4 \alpha^2} F^{\mu\nu} F_{\mu\nu} - \frac{1}{4 \alpha_3^2} F^{a\mu\nu} F_{\mu\nu}^a \right)$$

How does this can reduce on-shell to

$$S_m = \sum_A \int_A h m_A d\tau$$

???

Standard model Lagrangian < 100 GeV \rightarrow SU(3)xU(1)

$$L_m = \sum_i \bar{\psi}_i (i D - m_i) \psi_i - \frac{1}{4\alpha^2} F^{\mu\nu} F_{\mu\nu} - \frac{1}{4\alpha_3^2} F^{a\mu\nu} F_{\mu\nu}^a$$

Invariant under conformal transformations



Classically, do not contribute to the mass of particles (since traceless)

If that was true, we could explain around 1% of atomic mass

One must take into account quantum effects : trace anomalies

$$h T_m^{nucl.} = h \left(- \sum_i m_i (1 + \gamma_{m_i}) \bar{\psi}_i \psi_i - \frac{\beta(\alpha)}{2\alpha^3} F^{\mu\nu} F_{\mu\nu} - \frac{\beta_3(\alpha_3)}{2\alpha_3^3} F^{a\mu\nu} F_{a\mu\nu} \right)$$

Scale invariance "broken" by running of coupling constants



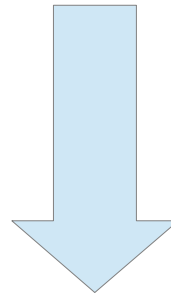
Contribute to particles' mass
By virtue of
Noether's theorem

Trace anomalies are responsible for around 99% of nuclear mass

Related to (for instance) [Gasser & Leutwyler, *Phys. Rep.* 1982], [Damour & Donoghue, *Phys. Rev. D* 2010] and [Nitti & Piazza, *Phys. Rev. D*, 2012]

Considering both QED and QCD trace anomalies

$$h T_m^{nucl.} = h \left(-\sum_i m_i (1 + \gamma_{m_i}) \bar{\psi}_i \psi_i - \frac{\beta(\alpha)}{2\alpha^3} F^{\mu\nu} F_{\mu\nu} - \frac{\beta_3(\alpha_3)}{2\alpha_3^3} F^{a\mu\nu} F_{\mu\nu}^a \right)$$



$$S_m = \int h T \sqrt{|g|} d^4 x \quad \longrightarrow \quad S_m = \sum_A \int_A h m_A d\tau$$

[Hui and Nicolis, *Phys. Lett.* 2010]

$$d m_A / d \tau = 0$$

One recovers the naive classical universal coupling

Possibly up to Planck suppressed terms

[Armendariz-Picon & Penco, *Phys. Rev. D* 2012]

However

$$\boxed{h T_m^{nucl.}} \text{ not quite } \boxed{h L_m^{nucl.}}$$

$$S = \int h L_m \sqrt{|g|} d^4 x \quad \xrightarrow{\quad ? \quad} \quad \left\{ \begin{array}{l} S = \sum_A \int_A h m_A d \tau \\ \frac{d m_A}{d \tau} = 0 \end{array} \right.$$

Is this correct ??

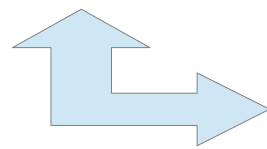
Could be, but still working on it

(btw, seek help with effective QCD and QED Lagrangians)

Reminder of the decoupling

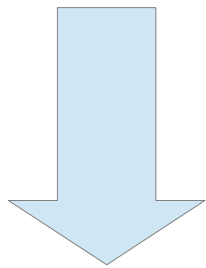
$$S = \frac{1}{c} \int \left[\frac{1}{2\kappa} \left(\phi R - \frac{\omega}{\phi} (\partial_\sigma \phi)^2 \right) + \sqrt{\phi} L_m(g, \Psi) \right] \sqrt{|g|} d^4 x$$

and $L_m^{OS} = -\rho = -\sum_A m_A c^2 \left(u^0 \sqrt{-g} \right)^{-1} \delta^{(3)}(\vec{x} - \vec{x}_A)$



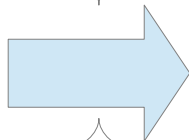
$$S_m = -c^2 \sum_A \int_A \sqrt{\phi} m_A d\tau$$

Total decoupling $dm_A/d\tau = 0$

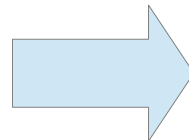


(Not entirely correct)

$$\nabla^2 \phi = 0$$



$$\phi = \phi_c$$



$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = \kappa \frac{f(\phi_c)}{\phi_c} T_{\alpha\beta}$$

(but gives the idea)

The scalar field is not sourced \rightarrow one recovers GR for dust fields

What happens for pressureful fluids ?

$$S = \int \left[\frac{1}{2\kappa} \left(\phi R - \frac{\omega}{\phi} (\partial_\sigma \phi)^2 \right) + \sqrt{\phi} L_m(g, \Psi) \right] \sqrt{|g|} d^4 x$$

$$\frac{2\omega + 3}{\phi} \nabla^2 \phi = \kappa \frac{1}{\sqrt{\phi}} (T - L_m^{OS})$$

What is the on-shell Lagrangian for a pressureful fluid ?

What happens for pressureful fluids ?

For a barotropic fluid, one can show that :

$$\begin{aligned} & (P = P(\rho)) \\ & \downarrow \\ & (L_m^{OS} = L_m^{OS}(\rho)) \end{aligned}$$

$$T_{\alpha\beta} = -\rho \frac{d L_m^{OS}}{d \rho} U_\alpha U_\beta + \left(L_m^{OS} - \rho \frac{d L_m^{OS}}{d \rho} \right) g_{\alpha\beta}$$

Equating to perfect fluid stress-energy tensor $T_{\alpha\beta} = (\epsilon + P) U_\alpha U_\beta + P g_{\alpha\beta}$

One gets

$$\left. \begin{aligned} L_m^{OS} &= -\epsilon \\ \frac{d L_m^{OS}}{d \rho} &= \frac{-\epsilon + P}{\rho} \end{aligned} \right\}$$

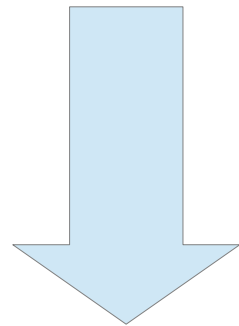


$$L_m^{OS} = -c^2 \rho \left(1 + \int \frac{P}{c^2 \rho} d \rho \right)$$

[Minazzoli & Harko, *Phys. Rev. D* 2012]

What happens for pressureful fluids ?

$$\frac{2\omega + 3}{\phi} \nabla^2 \phi = \kappa \frac{1}{\sqrt{\phi}} (T - L_m^{OS})$$



$$L_m^{OS} = -\epsilon = -c^2 \rho \left(1 + \int \frac{P}{c^2 \rho} d\rho \right)$$
$$T = -\epsilon + 3P$$

$$\frac{2\omega + 3}{\phi} \nabla^2 \phi = \kappa \frac{3P}{\sqrt{\phi}}$$

Scalar field sourced by pressure → « pressuron »

Warning !

$$L_m^{OS} = -\epsilon = -c^2 \rho \left(1 + \int \frac{P}{c^2 \rho} d\rho \right)$$

$$\frac{2\omega + 3}{\phi} \nabla^2 \phi = \kappa \frac{3P}{\sqrt{\phi}}$$

This must be recovered from microphysics
before being trusted

(macroscopic derivation only indicates that it is
plausible)

Solar system phenomenology

Post-Newtonian parameters : $\gamma = \gamma_{GR} = 1$ $\beta = \beta_{GR} = 1$ $\forall \omega(\phi)$

Newtonian potential modified at the post-Newtonian level :

$$U = U_{GR} - \frac{1}{c^2} \frac{3G}{2\omega_0 + 3} \int \frac{P(\vec{x}') d^3 x'}{|\vec{x} - \vec{x}'|}$$

Point particle equation of motion modified :

$$u^\sigma \nabla_\sigma u^\mu = -\frac{1}{2} (g^{\mu\sigma} + u^\mu u^\sigma) \partial_\sigma \ln \phi$$

Turns out, both modification cancel out !!!

$$\vec{a} = \vec{a}_{GR}$$

[Minazzoli & Hees, *Phys. Rev. D* 2013]

Solar system phenomenology

- No Nordvedt effect at 1.5 PN
- Same trajectories for light until c^{-4} (not detectable)
- Gravitational redshift differs from GR !

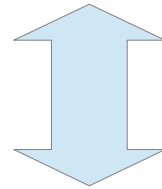
$$\frac{\Delta \nu}{\nu} \text{Pressuron} - \frac{\Delta \nu}{\nu} \text{GR} = -\frac{3}{2\omega_0 + 3} \left\langle \frac{P}{c^2 \rho} \right\rangle$$

$$\left\langle \frac{P}{c^2 \rho} \right\rangle \approx 10^{-6} \quad \text{for the Earth}$$

ACES intends to test gravitational redshift at the 10^{-6} level
 STE-QUEST..... 10^{-7}

Pressuron : in other words

$$S = \int d^4 x \sqrt{-g} \left(h^2 R - Z(h) (\partial h)^2 \right) + \sum_A m_A \int h d\sigma_A$$



$$\tilde{g}_{\alpha\beta} = h^2 g_{\alpha\beta}$$

$$S = \int d^4 x \sqrt{-\tilde{g}} \left(\tilde{R} - \tilde{Z}(h) (\tilde{\partial} h)^2 \right) + \sum_A m_A \int d\tilde{\sigma}_A$$

Seems like « Weyled » general relativity

[Deruelle & Sasaki, 2010] ←

But

not quite (due to free fields)

$$S^{FF} = - \int d^4 x \sqrt{-\tilde{g}} h \frac{\tilde{F}^2}{4\alpha^2} = - \int d^4 x \sqrt{-g} h \frac{F^2}{4\alpha^2}$$

Outline

- ~~Introduction~~
- ~~Background : intrinsic decoupling in scalar-tensor theories with universal matter coupling~~
 - ~~Action, field equations & phenomenology~~
- **The new proposal**
 - **Action, field equations & an alternative formulation**
- Discussion
 - Good things and questions
- Conclusion

Letter from Einstein to Weyl

« *Ultimately it must turn out that action densities must not be glued together additively. I too, concocted various things, but time and again I sank my head in resignation. »*

Albert Einstein, 1918

[Pais, *Subtle is the Lord: The science and the life of Albert Einstein*, 1982]

Reminder GR action :

$$S = \int \left(\frac{R}{2\kappa} + L_m \right) \sqrt{|g|} d^4 x$$

Question

Is there another way to glue together the action densities of matter and geometry that gives a world similar (enough) to the one we live in ?

Question

Is there another way to glue together the action densities of matter and geometry that gives a world similar (enough) to the one we live in ?

Answer : **Could be !** My colleagues and I seem to have found one of such possibilities

A new way to glue matter to geometry

$$S = -\frac{\kappa}{2c} \int \frac{L_m^2}{R} \sqrt{|g|} d^4 x$$

Instead of :

$$S = \frac{1}{c} \int \left(\frac{R}{2\kappa} + L_m \right) \sqrt{|g|} d^4 x$$

Multiplicative coupling meaning : matter and geometry are fundamentally inseparable.

One cannot exist without the other !

Field equations

$$S = -\frac{\kappa}{2c} \int \frac{L_m^2}{R} \sqrt{|g|} d^4 x$$

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = -\frac{R}{L_m} T_{\alpha\beta} + \frac{R^2}{L_m^2} \left[\nabla_{\alpha} \nabla_{\beta} - g_{\alpha\beta} \nabla^2 \right] \frac{L_m^2}{R^2}$$

$$3 \frac{R^2}{L_m^2} \nabla^2 \frac{L_m^2}{R^2} = R - \frac{R}{L_m} T$$

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = -\frac{R}{L_m} T_{\alpha\beta} + \frac{R^2}{L_m^2} \left[\nabla_{\alpha} \nabla_{\beta} - g_{\alpha\beta} \nabla^2 \right] \frac{L_m^2}{R^2}$$

$$3 \frac{R^2}{L_m^2} \nabla^2 \frac{L_m^2}{R^2} = R - \frac{R}{L_m} T$$

Defining : $h \stackrel{\text{def}}{=} -\kappa \frac{L_m}{R}$ the field equations write :

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = \frac{\kappa}{h} T_{\alpha\beta} + \frac{1}{h^2} \left[\nabla_{\alpha} \nabla_{\beta} - g_{\alpha\beta} \nabla^2 \right] h^2$$

$$\frac{3}{h^2} \nabla^2 h^2 = \frac{\kappa}{h} (T - L_m)$$

Alternative form

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = \frac{\kappa}{h} T_{\alpha\beta} + \frac{1}{h^2} \left[\nabla_{\alpha} \nabla_{\beta} - g_{\alpha\beta} \nabla^2 \right] h^2$$

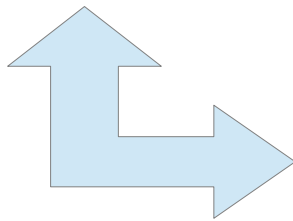
$$\frac{3}{h^2} \nabla^2 h^2 = \frac{\kappa}{h} (T - L_m)$$

Those field equations can be recovered by another
(more usual) action density :

$$S = \frac{1}{c} \int \left(\frac{h^2 R}{2\kappa} + h L_m \right) \sqrt{|g|} d^4 x$$

Alternative form

$$S = \frac{1}{c} \int \left(\frac{h^2 R}{2\kappa} + h L_m \right) \sqrt{|g|} d^4 x$$



$$S = \frac{1}{c} \int \left(\frac{\phi R}{2\kappa} + \sqrt{\bar{\phi}} L_m \right) \sqrt{|g|} d^4 x$$

The alternative form is nothing but a pressuron action without kinetic term

[Ludwig, Minazzoli & Capozziello, *Phys. Lett. B* 2015]

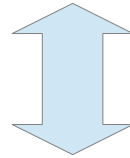
Reminder
Pressuron :

$$S = \frac{1}{c} \int \left[\frac{1}{2\kappa} \left(\phi R - \frac{\omega(\phi)}{\phi} (\partial_\sigma \phi)^2 \right) + \sqrt{\bar{\phi}} L_m(g, \Psi) \right] \sqrt{|g|} d^4 x$$

[Minazzoli & Hees, *Phys. Rev. D* 2013]

Conclusion about this theory

$$S = -\frac{\kappa}{2c} \int \frac{L_m^2}{R} \sqrt{|g|} d^4 x$$



$$S = \frac{1}{c} \int \left(\frac{h^2 R}{2\kappa} + h L_m \right) \sqrt{|g|} d^4 x$$

The new theory effectively reduces to a special case of the previous theory that seems to reduce to GR in the dust limit

Outline

- **Introduction**
- ~~Background : intrinsic decoupling in scalar-tensor theories with universal matter coupling~~
 - ~~Action, field equations & phenomenology~~
- ~~The new proposal~~
 - ~~Action, field equations & an alternative formulation~~
- **Discussion**
 - **Good things and questions**
- **Conclusion**

Good points of the theory

- May recover GR in regimes where it is tested
- Matter and geometry become inseparable
 - Therefore, satisfies stronger version of Mach's principle than usual scalar-tensor theories

[Dicke, *Phys. Rev.* 1962]

- Satisfy Occam's razor principle of economy

$$S = -\frac{\kappa}{2c} \int \frac{L_m^2}{R} \sqrt{|g|} d^4 x$$

is as simple as
(no new ingredient)

$$S = \frac{1}{c} \int \left(\frac{R}{2\kappa} + L_m \right) \sqrt{|g|} d^4 x$$

One just changed the way matter and geometry are glued together !

Issues/open questions

- Cosmology
 - No place for a cosmological constant → how to get the phenomenology of dark energy without spoiling the correct solar system phenomenology ?
 - Radiation era physics needs careful study ^[Minazzoli, *Phys. Lett. B* 2014]
 - Very high density era ? (horizon problem ?)
- Strong field ?
 - What is the role of pressure in relativistic regimes ?
→ should we expect deviation from GR ? when ?
 - Binaries' coalescence
 - e.g. What is the sensitivity of black holes? Neutron stars?

Issues/open questions

- Radiative corrections should not be studied in Minkowskian backgrounds **by definition**
(→ vacuum energy issue ?)
- How to tackle whole quantification when only multiplicative coupling between matter and geometry ?
- Effective Lagrangian should be worked out explicitly for nuclear matter!
- Pressureful limit (for general pressuron) should be recovered from microphysics
- Etc.

Final words with the letter from Einstein to Weyl

« *Ultimately it must turn out that action densities
must not be glued together additively. »*

Albert Einstein, 1918

$$S = -\frac{\kappa}{2c} \int \frac{L_m^2}{R} \sqrt{|g|} d^4 x$$

to be compared to

$$S = \frac{1}{c} \int \left(\frac{R}{2\kappa} + L_m \right) \sqrt{|g|} d^4 x$$

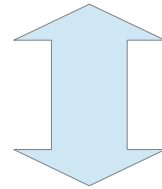
One just changed the way matter and geometry are glued together !

Thank you for your attention !!!

Additional slides

Pressuron

$$S = \int d^4 x \sqrt{-g} \left(h^2 R - Z(h) (\partial h)^2 \right) + \sum_A m_A \int h d\sigma_A$$



$$\tilde{g}_{\alpha\beta} = h^2 g_{\alpha\beta}$$

$$S = \int d^4 x \sqrt{-\tilde{g}} \left(\tilde{R} - \tilde{Z}(h) (\tilde{\partial} h)^2 \right) + \sum_A m_A \int d\tilde{\sigma}_A$$

Seems like « Weyled » general relativity

But

not quite (due to free fields)

$$S^{FF} = - \int d^4 x \sqrt{-\tilde{g}} h \frac{\tilde{F}^2}{4\alpha^2} = - \int d^4 x \sqrt{-g} h \frac{F^2}{4\alpha^2}$$

Brans-Dicke :

$$S = \int d^4 x \sqrt{-g} \left(h^2 R - Z(h) (\partial h)^2 \right) + \sum_A \int m_A d\tau_A$$

$$S = \int d^4 x \sqrt{-\tilde{g}} \left(\tilde{R} - \tilde{Z}(h) (\tilde{\partial} h)^2 \right) + \sum_A \int \frac{m_A}{h} d\tilde{\tau}_A$$

$$S^{FF} = - \int d^4 x \sqrt{-\tilde{g}} \frac{\tilde{F}^2}{4\alpha^2} = - \int d^4 x \sqrt{-g} \frac{F^2}{4\alpha^2}$$

Pressuron :

$$\tilde{g}_{\alpha\beta} = h^2 g_{\alpha\beta}$$

$$S = \int d^4 x \sqrt{-g} \left(h^2 R - Z(h) (\partial h)^2 \right) + \sum_A \int h m_A d\tau_A$$

$$S = \int d^4 x \sqrt{-\tilde{g}} \left(\tilde{R} - \tilde{Z}(h) (\tilde{\partial} h)^2 \right) + \sum_A \int m_A d\tilde{\tau}_A$$

$$S^{FF} = - \int d^4 x \sqrt{-\tilde{g}} h \frac{\tilde{F}^2}{4\alpha^2} = - \int d^4 x \sqrt{-g} h \frac{F^2}{4\alpha^2}$$

General relativity :

$$S = \int d^4 x \sqrt{-g} \left(h^2 R - Z(h) (\partial h)^2 \right) + \sum_A \int h m_A d\tau_A$$

$$S = \int d^4 x \sqrt{-\tilde{g}} \left(\tilde{R} - \tilde{Z}(h) (\tilde{\partial} h)^2 \right) + \sum_A \int m_A d\tilde{\tau}_A$$

$$S^{FF} = - \int d^4 x \sqrt{-\tilde{g}} \frac{\tilde{F}^2}{4\alpha^2} = - \int d^4 x \sqrt{-g} \frac{F^2}{4\alpha^2}$$

Pressuron :

$$\tilde{g}_{\alpha\beta} = h^2 g_{\alpha\beta}$$

$$S = \int d^4 x \sqrt{-g} \left(h^2 R - Z(h) (\partial h)^2 \right) + \sum_A \int h m_A d\tau_A$$

$$S = \int d^4 x \sqrt{-\tilde{g}} \left(\tilde{R} - \tilde{Z}(h) (\tilde{\partial} h)^2 \right) + \sum_A \int m_A d\tilde{\tau}_A$$

$$S^{FF} = - \int d^4 x \sqrt{-\tilde{g}} h \frac{\tilde{F}^2}{4\alpha^2} = - \int d^4 x \sqrt{-g} h \frac{F^2}{4\alpha^2}$$

Coupling model of Damour and Donoghue (universal case)

$$L_{\text{int}} = h T_m^{\text{nucl.}} = h \left(- \sum_i m_i (1 + \gamma_{m_i}) \bar{\psi}_i \psi_i - \frac{\beta_3(\alpha_3)}{2\alpha_3^3} F^{a\mu\nu} F_{\mu\nu}^a \right)$$

+ Free-field : $h L_m^{\text{FF}} = \frac{h}{4\alpha^2} F^{\mu\nu} F_{\mu\nu}$

PHYSICAL REVIEW D **82**, 084033 (2010)

$$\mathcal{L}_{\text{int}\phi} = \kappa\phi \left[+ \frac{d_e}{4e^2} F_{\mu\nu} F^{\mu\nu} - \frac{d_g \beta_3}{2g_3} F_{\mu\nu}^A F^{A\mu\nu} - \sum_{i=e,u,d} (d_{m_i} + \gamma_{m_i} d_g) m_i \bar{\psi}_i \psi_i \right]. \quad (12)$$

$$h = \kappa\phi d_g \quad d_e = d_{m_i} = d_g$$

Coupling model of Damour and Donoghue (universal case)

$$L_{\text{int}} = h T_m^{\text{nucl.}} = h \left(- \sum_i m_i (1 + \gamma_{m_i}) \bar{\psi}_i \psi_i - \frac{\beta_3(\alpha_3)}{2\alpha_3^3} F^{a\mu\nu} F_{\mu\nu}^a \right)$$

Universal coupling to trace anomaly accounting for low energy
(<<100GeV) standard model

$$h T_m^{\text{nucl.}} = h \left(- \sum_i m_i (1 + \gamma_{m_i}) \bar{\psi}_i \psi_i - \frac{\beta(\alpha)}{2\alpha^3} F^{\mu\nu} F_{\mu\nu} - \frac{\beta_3(\alpha_3)}{2\alpha_3^3} F^{a\mu\nu} F_{\mu\nu}^a \right)$$

But again

$$h T_m^{\text{nucl.}}$$

not quite

$$h L_m^{\text{nucl.}}$$



Still some work to do !