Rethinking the link between matter and geometry

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Outline

- Introduction
- Background : intrinsic decoupling in scalar-tensor theories with universal matter coupling
  - Action, field equations & phenomenology
- The new proposal
  - Action, field equations & an alternative formulation
- Discussion
  - Good things and questions
- Conclusion

Equivalence principle

<u>The happiest thought of my life</u>

« I was sitting in a chair in the pattent office at Bern when all of a sudden a thought occurred to me : 'If a person falls freely he will not feel his own weight.' I was startled. This simple thought made a deep impression on me. It impelled me toward a theory of gravitation »

Albert Einstein

Equivalence principle

Gravity is not a force

Gravity is inertia **BUT** in a curved space-time

Explains the equivalence between inertial and gravitational masses

GR Lagrangian formulation

Ricci scalar

$$S = \frac{1}{c} \int \left(\frac{R(g)}{2\kappa} + L_m \right) \sqrt{|g|}\, d^4 x \qquad \kappa = \frac{8\pi G}{c^4}$$

Naive classical assumption for (dust) matter

$$L_m^{OS} = -\rho = -\sum_A m_A c^2 \left(u^0 \sqrt{-g} \right)^{-1} \delta^{(3)}(\vec{x} - \vec{x}_A)$$

$$d m_A / d\tau = 0$$

$$S = \frac{1}{c} \int \frac{R(g)}{2\kappa} \sqrt{|g|}\, d^4 x - c^2 \sum_A \int_A m_A\, d\tau$$

→ **EIH equation of motion**

EIH=Einstein-Infeld-Hoffmann

GR and SM Lagrangian formulation

quarks and leptons

EM

Strong nuclear

$$S=\frac{1}{c}\int\left(\frac{R(g)}{2\kappa}+L_m(g,A,W,G,H,f)\right)\sqrt{|g|}\,d^4x$$

Higgs

Weak nuclear

Standard model particle Lagrangian

And constants…

GR Lagrangian formulation

$$S = \frac{1}{c} \int \left(\frac{R(g)}{2\kappa} + L_m \right) \sqrt{|g|}\, d^4 x$$

Action densities « glued together additively »
in Einstein's words

<u>Meaning</u> : Matter and geometry can be described separately → they are independent by nature

GR tests

- Weak field
  - Solar system (bending of light, perihelion advance, gravitational redshift, etc.)
  - Binary pulsars decay orbit
  - Lensing (weak and strong)
  - Observation of gravitational waves
- Non-linear regimes
  - Cosmology – standard ΛCDM model
  - Black holes
  - Generation of gravitational waves

GR issues

- Mach's Principle

Mach's principle

« it would be a remarkable coincidence if the inertial frame [...] just happened to be the reference frame in which typical stars are at rest »

Steven Weinberg

« [In modern words, Mach's idea is that] inertial forces observed locally in an accelerated laboratory may be interpreted as gravitational effects having their origin in distant matter accelerated relative to the laboratory »

Carl Brans

Mach's principle

The existence of vacuum solutions in GR show that the expression of Mach's principle is <u>at best</u> **imperfect** in **GR**.

See for instance [Brans & Dicke, *Phys. Rev.* 1961]
Or [Weinberg, *Rev. Mod. Phys.* 1989]
Or [Pais, *Subttle is the Lord: The science and the life of Albert Einstein*, 1982]

GR issues

- Mach's Principle
- Dark energy, dark matter and inflation

  (but solution could come from particle sector)

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- Quantum gravity

  (but GR could be assymptotically safe or need other non-perturbative technics like in LQG,...)

GR issues

- Mach's Principle
- Dark energy, dark matter and inflation

  (but solution could come from particle sector)
- Quantum gravity

  (but GR could be assymptotically safe or need other non-perturbative technics like in LQG,...)
- Unification

  (but current paradigm : GR is not a force
  → a priori no reason to be unified with gauge theories)

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Intrinsic decoupling from universal scalar-matter coupling

General class :

$$S=\frac{1}{c}\int\left[\frac{1}{2\kappa}\left(\phi R-\frac{\omega(\phi)}{\phi}(\partial_\sigma\phi)^2\right)+f(\phi)L_m(g,\Psi)\right]\sqrt{|g|}\,d^4x$$

Field equations

Simplification for this talk

$$S=\frac{1}{c}\int\left[\frac{1}{2\kappa}\left(\phi R-\frac{\omega(\phi)}{\phi}(\partial_\sigma\phi)^2\right)+f(\phi)L_m(g,\Psi)\right]\sqrt{|g|}\,d^4x$$

$$R_{\alpha\beta}-\frac{1}{2}g_{\alpha\beta}R=\kappa\frac{f(\phi)}{\phi}T_{\alpha\beta}+\frac{1}{\phi}\left[\nabla_\alpha\nabla_\beta-g_{\alpha\beta}\nabla^2\right]\phi+\frac{\omega}{\phi^2}\left[\partial_\alpha\phi\,\partial_\beta\phi-\frac{1}{2}g_{\alpha\beta}(\partial\phi)^2\right]$$

$$\nabla_\sigma T^{\alpha\sigma}=\left(L_m^{OS}g^{\alpha\sigma}-T^{\alpha\sigma}\right)\partial_\sigma\ln f(\phi)$$

$$\frac{2\omega+3}{\phi}\nabla^2\phi=\kappa\left(\frac{f(\phi)}{\phi}T-2f'(\phi)L_m^{OS}\right)$$

Special cases with decoupling

$$\frac{2\omega+3}{\phi}\nabla^2\phi=\kappa\left(\frac{f(\phi)}{\phi}T-2f'(\phi)L_m^{OS}\right)$$

Same naive classical assumption for (dust) matter as in GR

$$L_m^{OS}=-\rho=-\sum_A m_A c^2\left(u^0\sqrt{-g}\right)^{-1}\delta^{(3)}(\vec{x}-\vec{x}_A)$$

$$\Rightarrow \quad L_m^{OS}=T \qquad dm_A/d\tau=0$$

$$\frac{\phi f'(\phi)}{f(\phi)}=\frac{1}{2} \quad \Rightarrow \quad \nabla^2\phi=0$$

Special cases with decoupling

In other words :

$$S = \frac{1}{c} \int \left[\frac{1}{2\kappa} \left(\phi R - \frac{\omega}{\phi} (\partial_\sigma \phi)^2 \right) + \sqrt{\phi} L_m(g, \Psi) \right] \sqrt{|g|} d^4 x$$

and $$L_m^{OS} = -\rho = -\sum_A m_A c^2 \left(u^0 \sqrt{-g} \right)^{-1} \delta^{(3)}(\vec{x} - \vec{x}_A)$$

Total decoupling

(Not entirely correct)

$$\nabla^2 \phi = 0 \quad \rightarrow \quad \phi = \phi_C \quad \rightarrow \quad R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = \kappa \frac{f(\phi_c)}{\phi_c} T_{\alpha\beta}$$

(but gives the idea)

The scalar field is not sourced → one recovers GR for dust fields

Subtelty

So far, one used a naive classical view of (dust) matter

$$S = \int h\, L_m \sqrt{|g|}\, d^4 x \quad \Longrightarrow \quad \begin{cases} S = \sum_A \int_A h\, m_A\, d\tau \\ \dfrac{d\, m_A}{d\tau} = 0 \end{cases}$$

<u>Is this correct</u> ??

Standard model Lagrangian
< 100 GeV → SU(3)xU(1)

$$h\,L_m = h\left(\sum_i \bar{\psi}_i\left(i\,D - m_i\right)\psi_i - \frac{1}{4\alpha^2}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4\alpha_3^2}F^{a\mu\nu}F^a_{\mu\nu}\right)$$

How does this can reduce on-shell to

$$S_m = \sum_A \int_A h\,m_A\,d\tau$$

???

Standard model Lagrangian
< 100 GeV → SU(3)xU(1)

$$L_m = \sum_i \bar{\psi}_i (i D - m_i) \psi_i - \frac{1}{4\alpha^2} F^{\mu\nu} F_{\mu\nu} - \frac{1}{4\alpha_3^2} F^{a\mu\nu} F^a_{\mu\nu}$$

Invariant under conformal transformations

Classicaly, do not contribute to the mass of particles (since traceless)

If that was true, we could explain around 1% of atomic mass

One must take into account quantum effects : trace anomalies

$$h\,T_m^{nucl.} = h\left(-\sum_i m_i\left(1+\gamma_{m_i}\right)\bar{\psi}_i\,\psi_i - \frac{\beta(\alpha)}{2\,\alpha^3}F^{\mu\nu}F_{\mu\nu} - \frac{\beta_3(\alpha_3)}{2\,\alpha_3^3}F^{a\,\mu\nu}F^{a}_{\mu\nu}\right)$$

Scale invariance "broken" by running of coupling constants → <u>Contribute to particles' mass</u> By virtue of *Noether's theorem*

Trace anomalies are responsible for around 99% of nuclear mass

Related to (for instance) [Gasser & Leutwyler, *Phys. Rep.* 1982], [Damour & Donoghue, *Phys. Rev. D* 2010] and [Nitti & Piazza, *Phys. Rev. D*, 2012]

Considering both QED and QCD trace anomalies

$$h\,T_m^{nucl.} = h\left(-\sum_i m_i(1+\gamma_{m_i})\bar{\psi}_i\psi_i - \frac{\beta(\alpha)}{2\alpha^3}F^{\mu\nu}F_{\mu\nu} - \frac{\beta_3(\alpha_3)}{2\alpha_3^3}F^{a\mu\nu}F^a_{\mu\nu}\right)$$

$$S_m = \int h\,T\sqrt{|g|}\,d^4x \;\Rightarrow\; S_m = \sum_A \int_A h\,m_A\,d\tau$$

[Hui and Nicolis, *Phys. Lett.* 2010]

$$d\,m_A/d\tau = 0$$

One recovers the naive classical universal coupling

Possibly up to Planck suppressed terms

[Armendariz-Picon & Penco, *Phys. Rev. D* 2012]

However

$h\,T_m^{nucl.}$ not quite $h\,L_m^{nucl.}$

$$S = \int h\,L_m \sqrt{|g|}\,d^4x \;\overset{?}{\Longrightarrow}\; \begin{cases} S = \sum_A \int_A h\,m_A\,d\tau \\ \dfrac{d\,m_A}{d\,\tau} = 0 \end{cases}$$

<u>Is this correct</u> ??

Could be, but still working on it

(btw, seek help with effective QCD and QED Lagrangians)

Reminder of the decoupling

$$S=\frac{1}{c}\int\left[\frac{1}{2\kappa}\left(\phi R-\frac{\omega}{\phi}(\partial_\sigma\phi)^2\right)+\sqrt{\phi}L_m(g,\Psi)\right]\sqrt{|g|}d^4x$$

and $$L_m^{OS}=-\rho=-\sum_A m_A c^2\left(u^0\sqrt{-g}\right)^{-1}\delta^{(3)}(\vec{x}-\vec{x}_A)$$

$$S_m=-c^2\sum_A\int_A\sqrt{\phi}\,m_A\,d\tau$$

Total decoupling $\quad d\,m_A/d\tau=0$

(Not entirely correct)

$$\nabla^2\phi=0 \quad\rightarrow\quad \phi=\phi_C \quad\rightarrow\quad R_{\alpha\beta}-\frac{1}{2}g_{\alpha\beta}R=\kappa\frac{f(\phi_c)}{\phi_c}T_{\alpha\beta}$$

(but gives the idea)

The scalar field is not sourced → one recovers GR for dust fields

What happens for pressureful fluids ?

$$S=\int\left[\frac{1}{2\kappa}\left(\phi R-\frac{\omega}{\phi}(\partial_\sigma\phi)^2\right)+\sqrt{\phi}\,L_m(g,\Psi)\right]\sqrt{|g|}\,d^4x$$

$$\frac{2\omega+3}{\phi}\nabla^2\phi=\kappa\frac{1}{\sqrt{\phi}}\left(T-L_m^{OS}\right)$$

What is the on-shell Lagrangian for a pressureful fluid ?

What happens for pressureful fluids ?

For a barotropic fluid, one can show that :

$$\left(P = P(\rho)\right)$$

$$\Rightarrow \left(L_m^{OS} = L_m^{OS}(\rho)\right)$$

$$T_{\alpha\beta} = -\rho \frac{d\, L_m^{OS}}{d\,\rho} U_\alpha U_\beta + \left(L_m^{OS} - \rho \frac{d\, L_m^{OS}}{d\,\rho} \right) g_{\alpha\beta}$$

Equating to perfect fluid stress-energy tensor $T_{\alpha\beta} = (\epsilon + P) U_\alpha U_\beta + P\, g_{\alpha\beta}$

One gets

$$\left. \begin{aligned} L_m^{OS} &= -\epsilon \\ \frac{d\, L_m^{OS}}{d\,\rho} &= \frac{-\epsilon + P}{\rho} \end{aligned} \right\} \Rightarrow \boxed{L_m^{OS} = -c^2 \rho \left(1 + \int \frac{P}{c^2 \rho} d\,\rho \right)}$$

[Minazzoli & Harko, *Phys. Rev. D* 2012]

What happens for pressureful fluids ?

$$\frac{2\omega+3}{\phi}\nabla^2\phi=\kappa\frac{1}{\sqrt{\phi}}\left(T-L_m^{OS}\right)$$

$$L_m^{OS}=-\epsilon=-c^2\rho\left(1+\int\frac{P}{c^2\rho}d\rho\right)$$

$$T=-\epsilon+3P$$

$$\frac{2\omega+3}{\phi}\nabla^2\phi=\kappa\frac{3P}{\sqrt{\phi}}$$

Scalar field sourced by pressure → « pressuron »

[Minazzoli & Hees, *Phys. Rev. D* 2013], [Minazzoli & Hees, *Phys. Rev. D* 2014]

Warning !

$$L_m^{OS} = -\epsilon = -c^2\rho\left(1 + \int \frac{P}{c^2\rho} d\rho\right)$$

$$\frac{2\omega+3}{\phi}\nabla^2\phi = \kappa\frac{3P}{\sqrt{\phi}}$$

This must be recovered from microphysics before being trusted

(macroscopic derivation <u>only</u> indicates that it is plausible)

Solar system phenomenology

Post-Newtonian parameters : $\gamma = \gamma_{GR} = 1$ $\beta = \beta_{GR} = 1$ $\forall\, \omega(\phi)$

Newtonian potential modified at the post-Newtonian level :

$$U = U_{GR} - \frac{1}{c^2} \frac{3G}{2\omega_0 + 3} \int \frac{P(\vec{x}\,')\, d^3 x'}{|\vec{x} - \vec{x}\,'|}$$

Point particle equation of motion modifed :

$$u^\sigma \nabla_\sigma u^\mu = -\frac{1}{2}\left(g^{\mu\sigma} + u^\mu u^\sigma\right) \partial_\sigma \ln \phi$$

Turns out, both modification cancel out !!! $\boxed{\vec{a} = \vec{a}_{GR}}$

[Minazzoli & Hees, *Phys. Rev. D* 2013]

Solar system phenomenology

- No Nordvedt effect at 1.5 PN
- Same trajectories for light until c^{-4} (not detectable)
- Gravitationnal redshift differs from GR !

$$\frac{\Delta\nu}{\nu}_{Pressuron} - \frac{\Delta\nu}{\nu}_{GR} = -\frac{3}{2\omega_0+3}\left\langle \frac{P}{c^2\rho} \right\rangle$$

$$\left\langle \frac{P}{c^2\rho} \right\rangle \approx 10^{-6}$$ for the Earth

ACES intends to test gravitational redshift at the 10^{-6} level
STE-QUEST...10^{-7}

Pressuron : in other words

$$S=\int d^4x\sqrt{-g}\left(h^2 R-Z(h)(\partial h)^2\right)+\sum_A m_A\int h\, d\sigma_A$$

$$\tilde{g}_{\alpha\beta}=h^2 g_{\alpha\beta}$$

$$S=\int d^4x\sqrt{-\tilde{g}}\left(\tilde{R}-\tilde{Z}(h)(\tilde{\partial} h)^2\right)+\sum_A m_A\int d\tilde{\sigma}_A$$

Seems like « Weyled » general relativity

[Deruelle & Sasaki, 2010]

<u>But</u>

not quite (due to free fields)

$$S^{FF}=-\int d^4x\sqrt{-\tilde{g}}\,h\frac{\tilde{F}^2}{4\alpha^2}=-\int d^4x\sqrt{-g}\,h\frac{F^2}{4\alpha^2}$$

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Letter from Einstein to Weyl

« *Ultimately it must turn out that action densities <u>must not be glued together additively</u>. I too, concocted various things, but time and again I sank my head in resignation.* »

Albert Einstein, 1918

[Pais, *Subttle is the Lord: The science and the life of Albert Einstein*, 1982]

Reminder GR action :

$$S = \int \left(\frac{R}{2\,\kappa} + L_m \right) \sqrt{|g|}\, d^4 x$$

Question

Is there another way to glue together the action densities of matter and geometry that gives a world similar (enough) to the one we live in ?

Question

Is there another way to glue together the action densities of matter and geometry that gives a world similar (enough) to the one we live in ?

Answer : **Could be !** My colleagues and I seem to have found one of such possibilities

[Ludwig, Minazzoli & Capozziello, *Phys. Lett. B* 2015]

A new way to glue matter to geometry

$$S = -\frac{\kappa}{2c} \int \frac{L_m^2}{R} \sqrt{|g|} d^4 x$$

Instead of :

$$S = \frac{1}{c} \int \left(\frac{R}{2\kappa} + L_m \right) \sqrt{|g|} d^4 x$$

<u>Multiplicative coupling meaning</u> : matter and geometry are fundamentally inseparable.

One cannot exist without the other !

Field equations

$$S = -\frac{\kappa}{2c}\int \frac{L_m^2}{R}\sqrt{|g|}\,d^4x$$

$$R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R = -\frac{R}{L_m}T_{\alpha\beta} + \frac{R^2}{L_m^2}\left[\nabla_\alpha\nabla_\beta - g_{\alpha\beta}\nabla^2\right]\frac{L_m^2}{R^2}$$

$$3\frac{R^2}{L_m^2}\nabla^2\frac{L_m^2}{R^2} = R - \frac{R}{L_m}T$$

$$R_{\alpha\beta}-\frac{1}{2}g_{\alpha\beta}R=-\frac{R}{L_m}T_{\alpha\beta}+\frac{R^2}{L_m^2}\left[\nabla_\alpha\nabla_\beta-g_{\alpha\beta}\nabla^2\right]\frac{L_m^2}{R^2}$$

$$3\frac{R^2}{L_m^2}\nabla^2\frac{L_m^2}{R^2}=R-\frac{R}{L_m}T$$

Defining : $h\stackrel{\text{def}}{=}-\kappa\frac{L_m}{R}$ the field equations write :

$$R_{\alpha\beta}-\frac{1}{2}g_{\alpha\beta}R=\frac{\kappa}{h}T_{\alpha\beta}+\frac{1}{h^2}\left[\nabla_\alpha\nabla_\beta-g_{\alpha\beta}\nabla^2\right]h^2$$

$$\frac{3}{h^2}\nabla^2h^2=\frac{\kappa}{h}\left(T-L_m\right)$$

Alternative form

$$R_{\alpha\beta}-\frac{1}{2}g_{\alpha\beta}R=\frac{\kappa}{h}T_{\alpha\beta}+\frac{1}{h^2}\left[\nabla_\alpha\nabla_\beta-g_{\alpha\beta}\nabla^2\right]h^2$$

$$\frac{3}{h^2}\nabla^2 h^2=\frac{\kappa}{h}\left(T-L_m\right)$$

Those field equations can be recovered by another (more usual) action density :

$$S=\frac{1}{c}\int\left(\frac{h^2 R}{2\kappa}+h\,L_m\right)\sqrt{|g|}\,d^4x$$

Alternative form

$$S = \frac{1}{c} \int \left(\frac{h^2 R}{2\kappa} + h L_m \right) \sqrt{|g|}\, d^4 x$$

$$S = \frac{1}{c} \int \left(\frac{\phi R}{2\kappa} + \sqrt{\phi} L_m \right) \sqrt{|g|}\, d^4 x$$

The alternative form is nothing but a pressuron action without kinetic term

[Ludwig, Minazzoli & Capozziello, *Phys. Lett. B* 2015]

Reminder Pressuron :

$$S = \frac{1}{c} \int \left[\frac{1}{2\kappa} \left(\phi R - \frac{\omega(\phi)}{\phi} (\partial_\sigma \phi)^2 \right) + \sqrt{\phi} L_m(g, \Psi) \right] \sqrt{|g|}\, d^4 x$$

[Minazzoli & Hees, *Phys. Rev. D* 2013]

Conclusion about this theory

$$S = -\frac{\kappa}{2c} \int \frac{L_m^2}{R} \sqrt{|g|} d^4 x$$

$$\Updownarrow$$

$$S = \frac{1}{c} \int \left(\frac{h^2 R}{2\kappa} + h L_m \right) \sqrt{|g|} d^4 x$$

The new theory effectively reduces to a special case of the previous theory that seems to reduce to GR in the dust limit

[Ludwig, Minazzoli & Capozziello, *Phys. Lett. B* 2015]

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Good points of the theory

- May recover GR in regimes where it is tested
- Matter and geometry become inseparable
  - Therefore, satisfies stronger version of Mach's principle than usual scalar-tensor theories

[Dicke, *Phys. Rev.* 1962]

- Satisfy Occam's razor principle of economy

$$S = -\frac{\kappa}{2c} \int \frac{L_m^2}{R} \sqrt{|g|} d^4 x$$

is as simple as
(no new ingredient)

$$S = \frac{1}{c} \int \left(\frac{R}{2\kappa} + L_m \right) \sqrt{|g|} d^4 x$$

One just changed the way matter and geometry are glued together !

Issues/open questions

- Cosmology
  - No place for a cosmological constant → how to get the phenomenology of dark energy without spoiling the correct solar system phenomenology ?
  - Radiation era physics needs careful study [Minazzoli, *Phys. Lett. B* 2014]
  - Very high density era ? (horizon problem ?)
- Strong field ?
  - What is the role of pressure in relativistic regimes ?

    → should we expect deviation from GR ? when ?
  - Binaries' coalescence
    - e.g. What is the sensitivity of black holes? Neutron stars?

Issues/open questions

- Radiative corrections should <u>not</u> be studied in Minkowskian backgrounds <u>**by definition**</u>

  (→ vacuum energy issue ?)

- How to tackle whole quantification when only multiplicative coupling between matter and geometry ?
- Effective Lagrangian should be worked out explicitely for nuclear matter!
- Pressureful limit (for general pressuron) should be recovered from microphysics
- Etc.

Final words with the letter from Einstein to Weyl

« *Ultimately it must turn out that action densities <u>must not be glued together additively</u>.* »

Albert Einstein, 1918

$$S=-\frac{\kappa}{2c}\int\frac{L_m^2}{R}\sqrt{|g|}\,d^4x$$ to be compared to $$S=\frac{1}{c}\int\left(\frac{R}{2\kappa}+L_m\right)\sqrt{|g|}\,d^4x$$

One just changed the way matter and geometry are glued together !

<u>Thank you for your attention</u> !!!

Additional slides

Pressuron

$$S=\int d^4x\sqrt{-g}\left(h^2R-Z(h)(\partial h)^2\right)+\sum_A m_A\int h\,d\sigma_A$$

$$\tilde{g}_{\alpha\beta}=h^2 g_{\alpha\beta}$$

$$S=\int d^4x\sqrt{-\tilde{g}}\left(\tilde{R}-\tilde{Z}(h)(\tilde{\partial} h)^2\right)+\sum_A m_A\int d\tilde{\sigma}_A$$

Seems like « Weyled » general relativity

<u>But</u>

not quite (due to free fields)

$$S^{FF}=-\int d^4x\sqrt{-\tilde{g}}\,h\,\frac{\tilde{F}^2}{4\alpha^2}=-\int d^4x\sqrt{-g}\,h\,\frac{F^2}{4\alpha^2}$$

Brans-Dicke :

$$S=\int d^4x\sqrt{-g}\left(h^2R-Z(h)(\partial h)^2\right)+\sum_A\int m_A\,d\tau_A$$

$$S=\int d^4x\sqrt{-\tilde{g}}\left(\tilde{R}-\tilde{Z}(h)(\tilde{\partial} h)^2\right)+\sum_A\int \frac{m_A}{h}\,d\tilde{\tau}_A$$

$$S^{FF}=-\int d^4x\sqrt{-\tilde{g}}\,\frac{\tilde{F}^2}{4\alpha^2}=-\int d^4x\sqrt{-g}\,\frac{F^2}{4\alpha^2}$$

$$\tilde{g}_{\alpha\beta}=h^2g_{\alpha\beta}$$

Pressuron :

$$S=\int d^4x\sqrt{-g}\left(h^2R-Z(h)(\partial h)^2\right)+\sum_A\int h\,m_A\,d\tau_A$$

$$S=\int d^4x\sqrt{-\tilde{g}}\left(\tilde{R}-\tilde{Z}(h)(\tilde{\partial} h)^2\right)+\sum_A\int m_A\,d\tilde{\tau}_A$$

$$S^{FF}=-\int d^4x\sqrt{-\tilde{g}}\,h\,\frac{\tilde{F}^2}{4\alpha^2}=-\int d^4x\sqrt{-g}\,h\,\frac{F^2}{4\alpha^2}$$

General relativity :

$$S=\int d^4x\sqrt{-g}\left(h^2R-Z(h)(\partial h)^2\right)+\sum_A\int h\,m_A\,d\tau_A$$

$$S=\int d^4x\sqrt{-\tilde{g}}\left(\tilde{R}-\tilde{Z}(h)(\tilde{\partial}h)^2\right)+\sum_A\int m_A\,d\tilde{\tau}_A$$

$$S^{FF}=-\int d^4x\sqrt{-\tilde{g}}\frac{\tilde{F}^2}{4\alpha^2}=-\int d^4x\sqrt{-g}\frac{F^2}{4\alpha^2}$$

$$\tilde{g}_{\alpha\beta}=h^2g_{\alpha\beta}$$

Pressuron :

$$S=\int d^4x\sqrt{-g}\left(h^2R-Z(h)(\partial h)^2\right)+\sum_A\int h\,m_A\,d\tau_A$$

$$S=\int d^4x\sqrt{-\tilde{g}}\left(\tilde{R}-\tilde{Z}(h)(\tilde{\partial}h)^2\right)+\sum_A\int m_A\,d\tilde{\tau}_A$$

$$S^{FF}=-\int d^4x\sqrt{-\tilde{g}}\,h\frac{\tilde{F}^2}{4\alpha^2}=-\int d^4x\sqrt{-g}\,h\frac{F^2}{4\alpha^2}$$

Coupling model of Damour and Donoghue (universal case)

$$L_{\text{int}} = h\,T_m^{nucl.} = h\left(-\sum_i m_i(1+\gamma_{m_i})\,\bar{\psi}_i\psi_i - \frac{\beta_3(\alpha_3)}{2\alpha_3^3}F^{a\mu\nu}F^a_{\mu\nu}\right)$$

+ Free-field : $h\,L_m^{FF} = \frac{h}{4\alpha^2}F^{\mu\nu}F_{\mu\nu}$

PHYSICAL REVIEW D **82**, 084033 (2010)

$$\mathcal{L}_{\text{int}\phi} = \kappa\phi\left[+\frac{d_e}{4e^2}F_{\mu\nu}F^{\mu\nu} - \frac{d_g\beta_3}{2g_3}F^A_{\mu\nu}F^{A\mu\nu} - \sum_{i=e,u,d}(d_{m_i} + \gamma_{m_i}d_g)m_i\bar{\psi}_i\psi_i\right]. \quad (12)$$

$h = \kappa\phi\,d_g$ $\quad$ $d_e = d_{m_i} = d_g$

Coupling model of Damour and Donoghue (universal case)

$$L_{\text{int}} = h\,T_m^{nucl.} = h\left(-\sum_i m_i\left(1+\gamma_{m_i}\right)\bar{\psi}_i\psi_i - \frac{\beta_3(\alpha_3)}{2\alpha_3^3}F^{a\mu\nu}F^a_{\mu\nu}\right)$$

Universal coupling to trace anomaly accounting for low energy (<<100GeV) standard model

$$h\,T_m^{nucl.} = h\left(-\sum_i m_i\left(1+\gamma_{m_i}\right)\bar{\psi}_i\psi_i - \frac{\beta(\alpha)}{2\alpha^3}F^{\mu\nu}F_{\mu\nu} - \frac{\beta_3(\alpha_3)}{2\alpha_3^3}F^{a\mu\nu}F^a_{\mu\nu}\right)$$

But again

$h\,T_m^{nucl.}$ not quite $h\,L_m^{nucl.}$ → Still some work to do !