

Spontaneous scalarization: a promising avenue for gravitational-wave astronomy

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arXiv:1602.????? (hopefully soon) with U. Sperhake, C. Ott

arXiv:1505.07462 (**CQG** 32:204001) with M. Horbatsch, H. Silva, P. Pani,
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Outline

1. **Why** scalar-tensor theory?
2. Action, equations, **numerics**
3. **Core collapse** and GW breathing mode
4. **Multi-scalars** and target space



Why testing GR?

~1919:

Journalist: “Herr Einstein, what if the theory turned out to be wrong?”

Einstein: “I would feel sorry for the dear Lord. The theory is correct.”



Theory:

- Where is quantum mechanics?
- Are there really singularities around?

Puzzling observations:

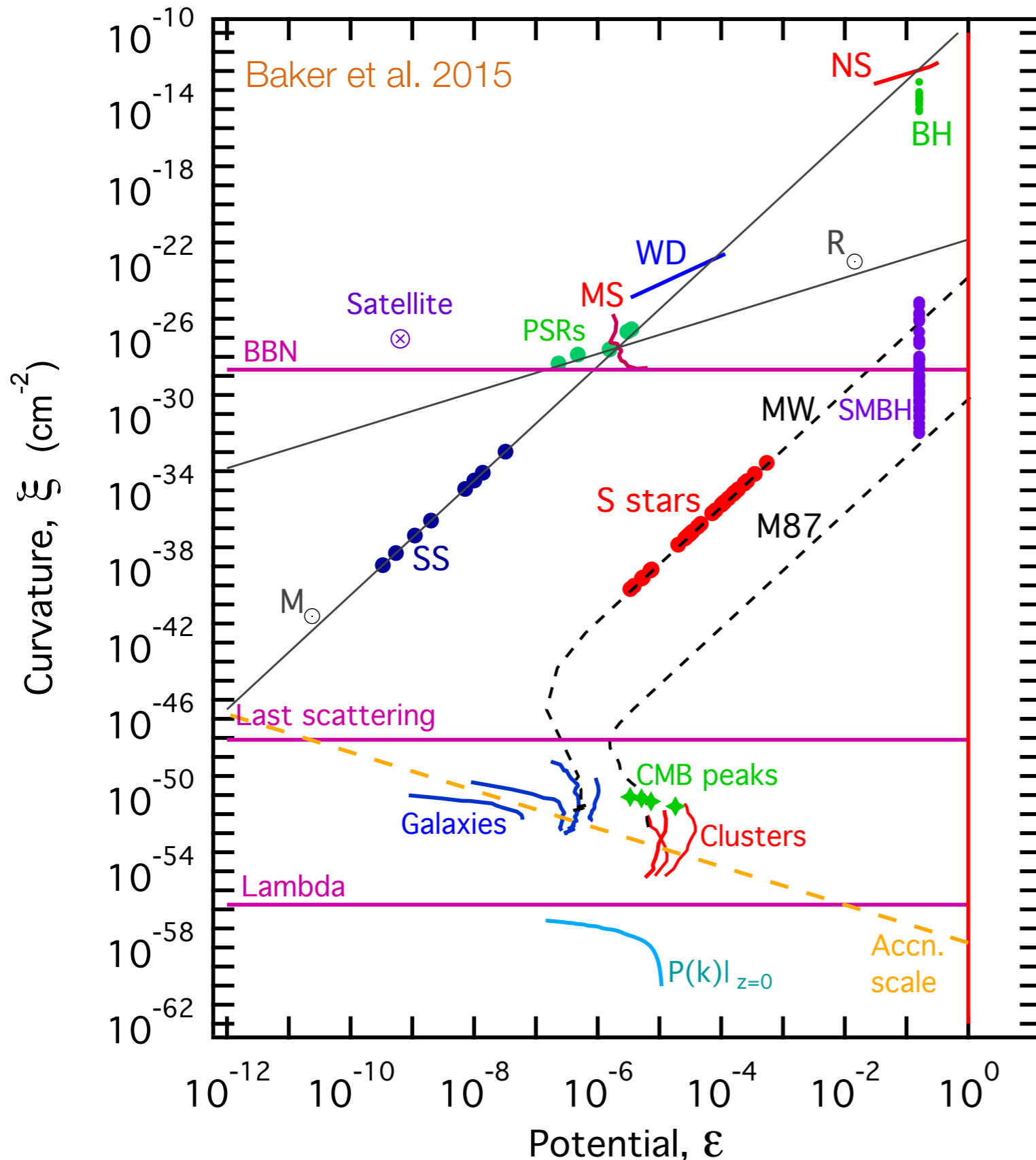
- Dark energy makes up most of the Universe
- Why is Lambda so small?

Tests

- GR is extremely well tested “in between” these two regimes $1 \text{ mm} \lesssim L \lesssim 1 \text{ AU}$

Extreme challenge for theorists

Strong-field test of gravity



Gravitational field

$$\epsilon \equiv \frac{GM}{rc^2}$$

Curvature (Kretschmann)

$$\xi = (R^{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta})^{1/2} = \sqrt{48} \frac{GM}{r^3 c^2}$$

For a compass to navigate this alternative theory business: [Berti et al. 2015](#)

Why scalar-tensor theory?

Damour and Esposito-Farese 1992

- Modifications of GR from high-energy theory often lead to the introduction of **additional degrees of freedom** (Lovelock theorem)
see eg. [Sotiriou et al 2007](#)
- **(Multi-)Scalar-tensor theories:** gravity mediated by the metric and additional scalar field(s)
- Some high-energy theories predict GR + scalars as their low-energy limit.
cf e.g. review by [Will 2014](#)

Complicated enough

to introduce testable modifications
(e.g. the Eddington PPN parameters)

Simple enough

to work out predictions (and even
do full numerical simulations)



A tale of two formulations

Jordan frame

$$S = \int dx^4 \sqrt{-g} \left[\frac{F(\phi)}{16\pi} R - \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi) (\partial_\nu \phi) - V(\phi) \right] + S_m(\psi_m, g_{\mu\nu})$$

The most general action...

1. single scalar field coupled non-minimally
2. invariant under space-time diffeomorphisms
3. at most two space-time derivatives
4. satisfy the Weak Equivalence Principle (WEP)

Damour and Esposito-Farese 1992

Moreover...

1. Vanishing potential $V(\phi) = 0$
2. **Coupling function** $F = F(\phi)$
3. Note the WEP!

Conformal transformation: $\bar{g}_{\mu\nu} = F g_{\mu\nu}$

Einstein frame

$$S = \frac{1}{16\pi} \int dx^4 \sqrt{-\bar{g}} \left[\bar{R} - 2\bar{g}^{\mu\nu} (\partial_\mu \varphi) (\partial_\nu \varphi) \right] + S_m[\psi_m, \bar{g}_{\mu\nu} / F]$$

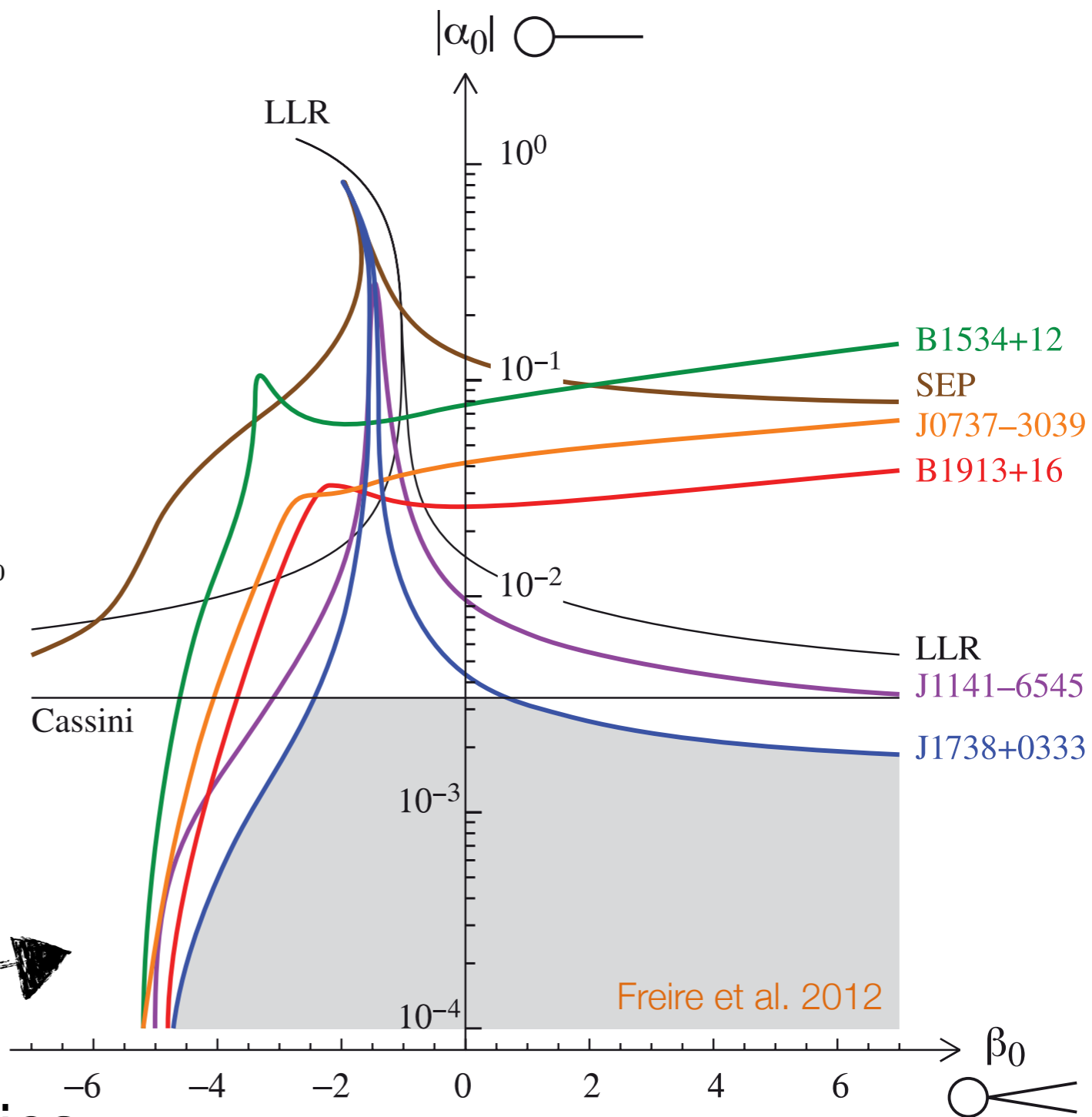
Dominant corrections to GR

1PN corrections in ST theories depend on two parameters only

$$\varphi_0 = \lim_{r \rightarrow \infty} \varphi$$

$$\alpha_0 = -\frac{1}{2} \left. \frac{\partial \log F}{\partial \varphi} \right|_{\varphi=\varphi_0} \quad \beta_0 = -\frac{1}{2} \left. \frac{\partial^2 \log F}{\partial \varphi^2} \right|_{\varphi=\varphi_0}$$

Experimental constraints

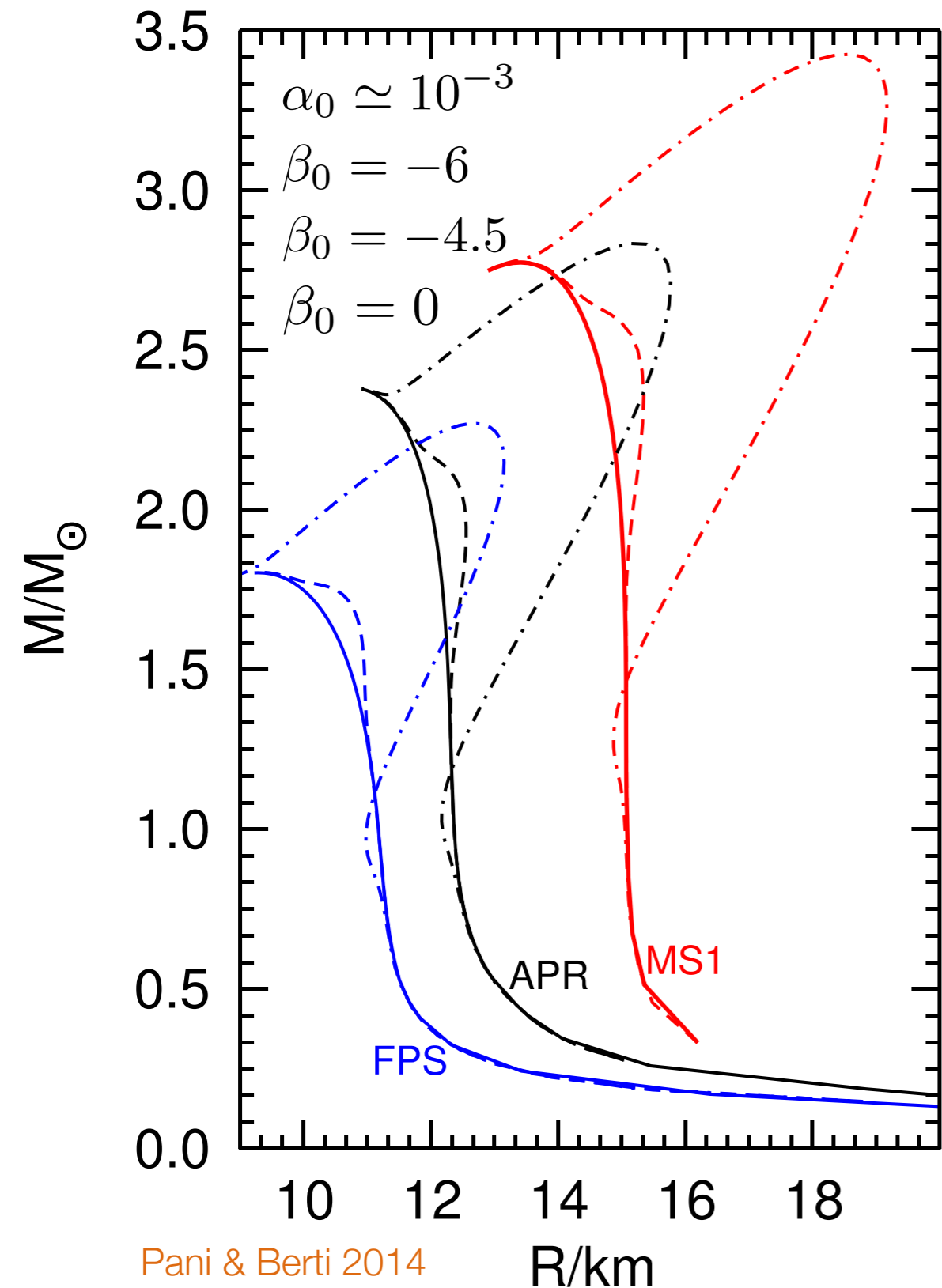


Let's take this subset of ST theories

coupling function $F = \exp \left[-2\alpha_0(\varphi - \varphi_0) - \beta_0(\varphi - \varphi_0)^2 \right]$

Spontaneous scalarization

Damour and Esposito-Farese 1993, 1996



Perturbative corrections enters as...

$$\alpha_0^2 \times \left[\lambda_0 + \lambda_1 \frac{Gm}{Rc^2} + \lambda_2 \left(\frac{Gm}{Rc^2} \right)^2 + \dots \right]$$

If $\alpha_0 \sim 0$ the theory is ***perturbative*** equivalent to GR, but if

$$\frac{Gm}{Rc^2} \sim 0.2 \quad (\text{so, neutron stars!})$$

**Strong-field
non-linearities!**

Fundamental threshold:

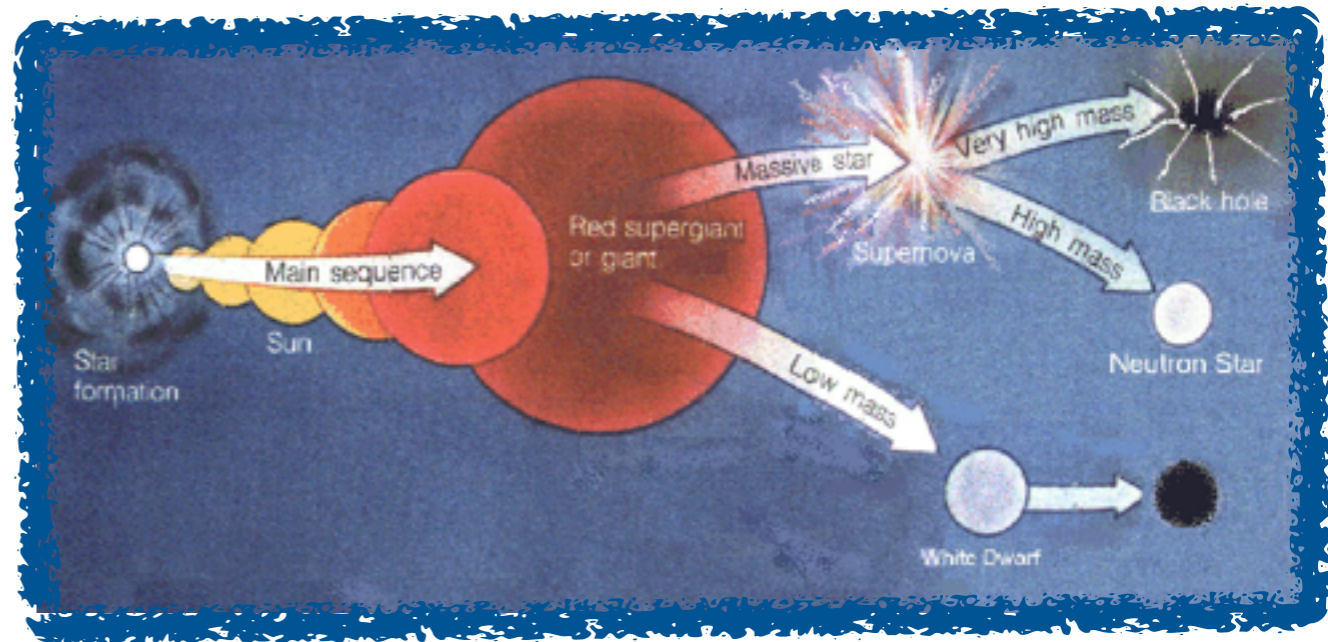
Novak 1998, Harada 1998

$$\beta_0 \lesssim -4.35$$

Core collapse in a nutshell

- End of star's life: iron core supported by **degenerate pressure of rel. electrons.**
- Collapse, outgoing shock, Type II supernova
- Core is left behind as a **neutron star**
- Accretion: **BH formation**

Type II SN are as luminous as entire galaxies



**Are non-trivial
scalar-field
profiles excited
following core-
collapse?
How about GWs?**

Hydrodynamics in ST theories

Jordan frame

$$S = \int dx^4 \sqrt{-g} \left[\frac{F(\phi)}{16\pi} R - \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi) (\partial_\nu \phi) - V(\phi) \right] + S_m(\psi_m, g_{\mu\nu})$$

$$G_{\mu\nu} = \frac{8\pi}{F} (T_{\mu\nu}^F + T_{\mu\nu}^\phi + T_{\mu\nu})$$
$$T_{\mu\nu}^F = \frac{1}{8\pi} (\nabla_\mu \nabla_\nu F - g_{\mu\nu} \nabla^\rho \nabla_\rho F)$$
$$T_{\mu\nu}^\phi = \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} \partial^\nu \phi \partial_\nu \phi$$

Einstein equations

$$\nabla^\rho \nabla_\rho \phi = -\frac{1}{16\pi} F_{,\phi} R$$

Wave equation

Radial gauge

Perfect fluid

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -\alpha^2 dt^2 + X^2 dr^2 + \frac{r^2}{F} d\Omega^2$$

$$T_{\alpha\beta} = \rho h u_\alpha u_\beta + P g_{\alpha\beta} \quad u^\mu = \frac{1}{\sqrt{1-v^2}} \left[\frac{1}{\alpha}, \frac{v}{X}, 0, 0 \right]$$

1. Curvature equations

Constraints for the enclosed mass and metric potential

2. Scalar-field wave equations

3 first-order PDEs

second-order finite differences + outgoing boundary condition


3. Matter equations

ρ : mass density
 h : enthalpy
 v : radial velocity

**Primitive
variables**

**Conserved
variables**

$$D = \frac{\rho X}{F \sqrt{F} \sqrt{1 - v^2}},$$
$$S^r = \frac{\rho h v}{F^2 (1 - v^2)},$$
$$\tau = \frac{\rho h}{F^2 (1 - v^2)} - \frac{P}{F^2} - D$$


$$\partial_t \mathbf{U} + \frac{1}{r^2} \partial_r \left[r^2 \frac{\alpha}{X} \mathbf{f}(\mathbf{U}) \right] = \mathbf{s}(\mathbf{U})$$

Integrated with high-resolution shock-capturing schemes

Code built on top of GR1D O'Connor & Ott 2010

Mimicking nuclear physics

We need an **equation of state** to close the system

Hybrid EOS:

Janka, Zwerger, Moenchmeyer 1993
Zwerger, Mueller 1997

$$P = P_c + P_{\text{th}}$$

$$P_c = \begin{cases} K_1 \rho^{\Gamma_1} & \text{if } \rho \leq \rho_{\text{nuc}} \\ K_2 \rho^{\Gamma_2} & \text{if } \rho > \rho_{\text{nuc}} \end{cases}$$

$$P_{\text{th}} = (\Gamma_{\text{th}} - 1) \rho \epsilon_{\text{th}}$$
$$\epsilon_{\text{th}} = \epsilon - \epsilon_c$$

Piecewise polytropic

- Iron core collapse

$$\Gamma_1 \lesssim 4/3$$

- Stiffening at nuclear densities

$$\Gamma_2 \simeq 2.5 - 3$$

Ideal gas

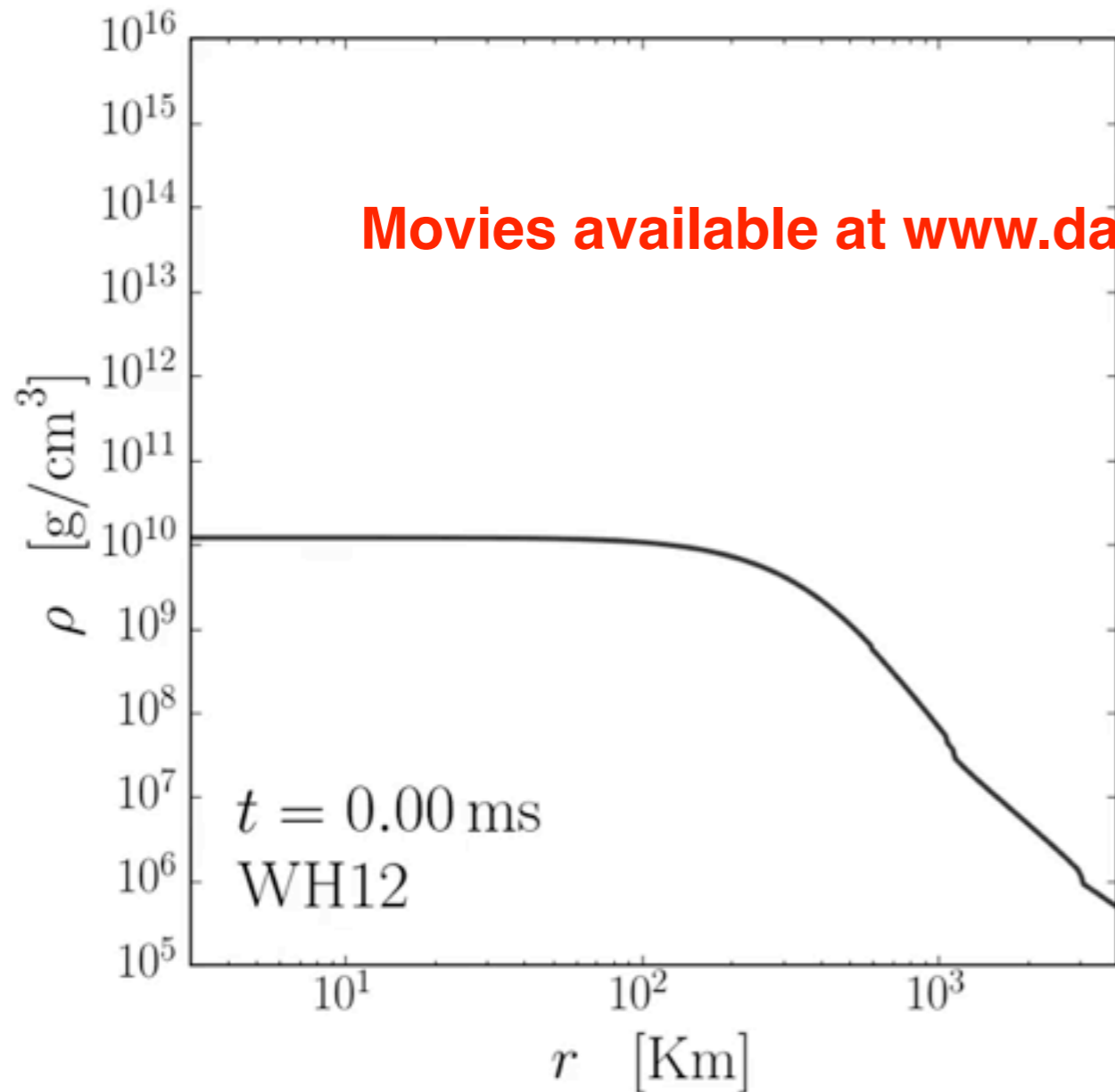
- Response of the heated post-shock material

$$4/3 < \Gamma_{\text{th}} < 5/3$$

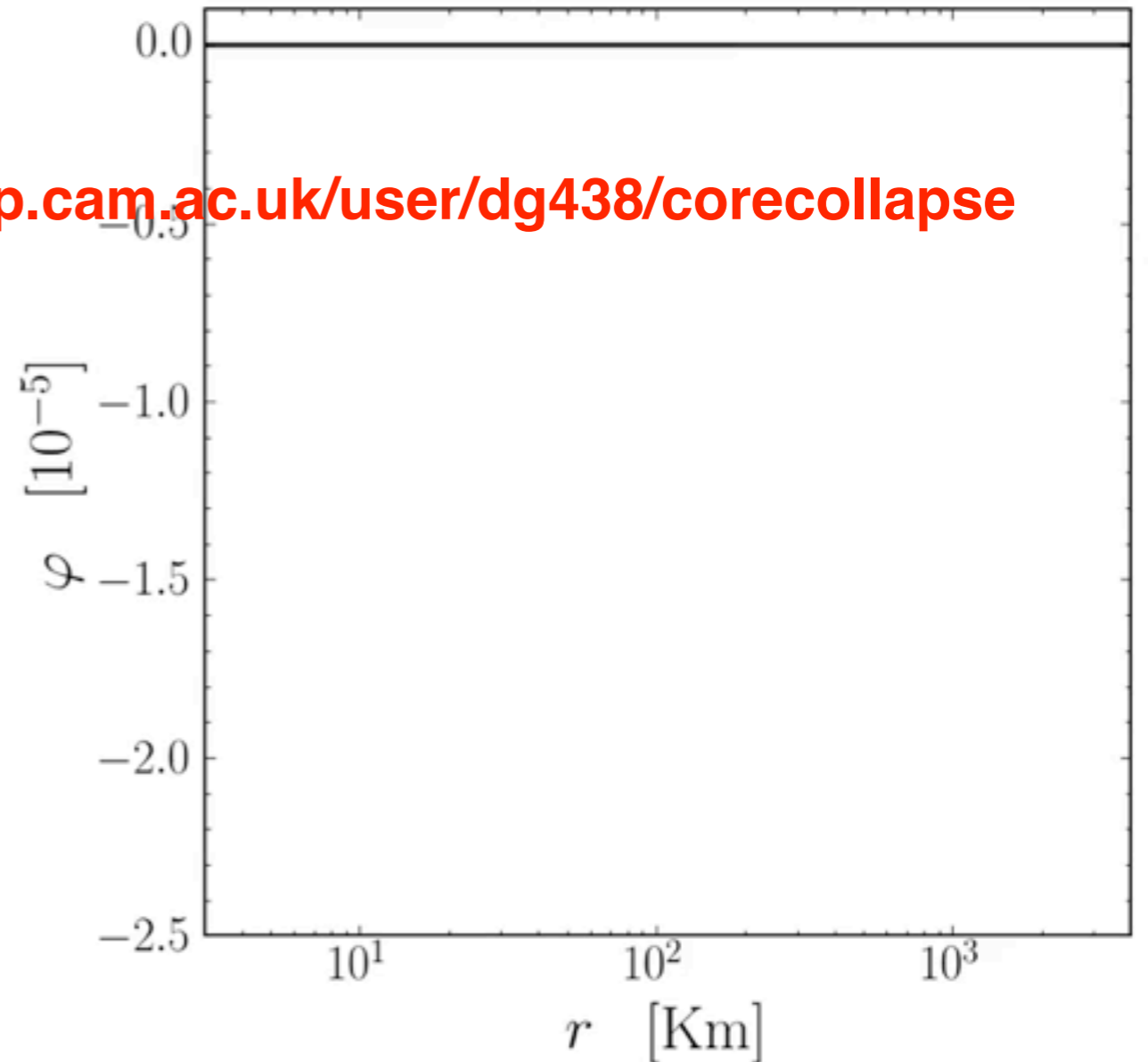
Tested against finite-temperature EOS: [Dimmelmeier et al. 2007, 2008](#)

Collapse, bounce, shock... and NS

Mass density



Scalar field



Movies available at www.damtp.cam.ac.uk/user/dg438/corecollapse

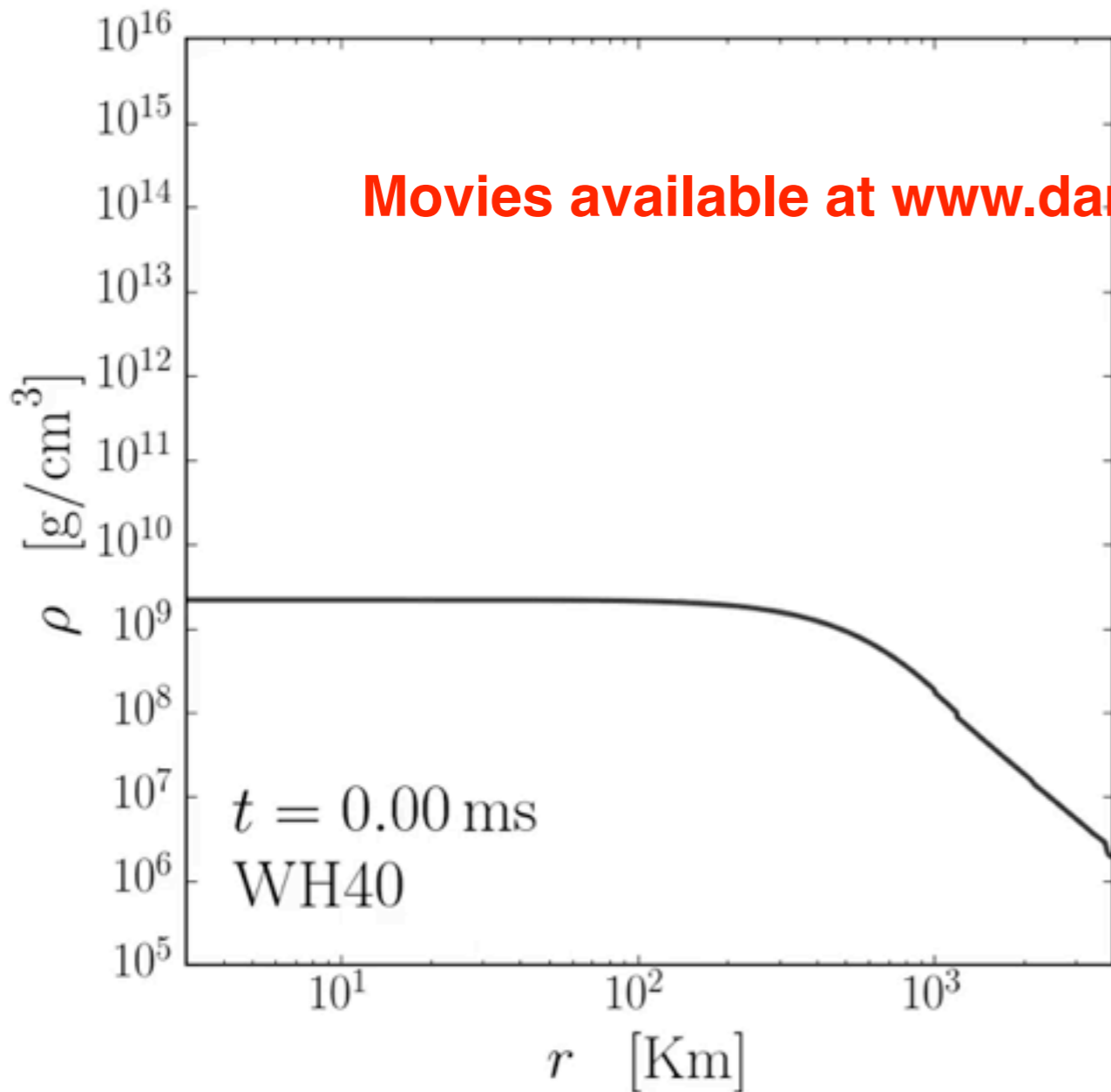
Initial profile: realistic SN progenitor $M_{\text{ZAMS}} = 12M_{\odot}$ Woosley & Heger 2007

ST theory $\alpha_0 = 10^{-4}$ $\beta_0 = -4.35$

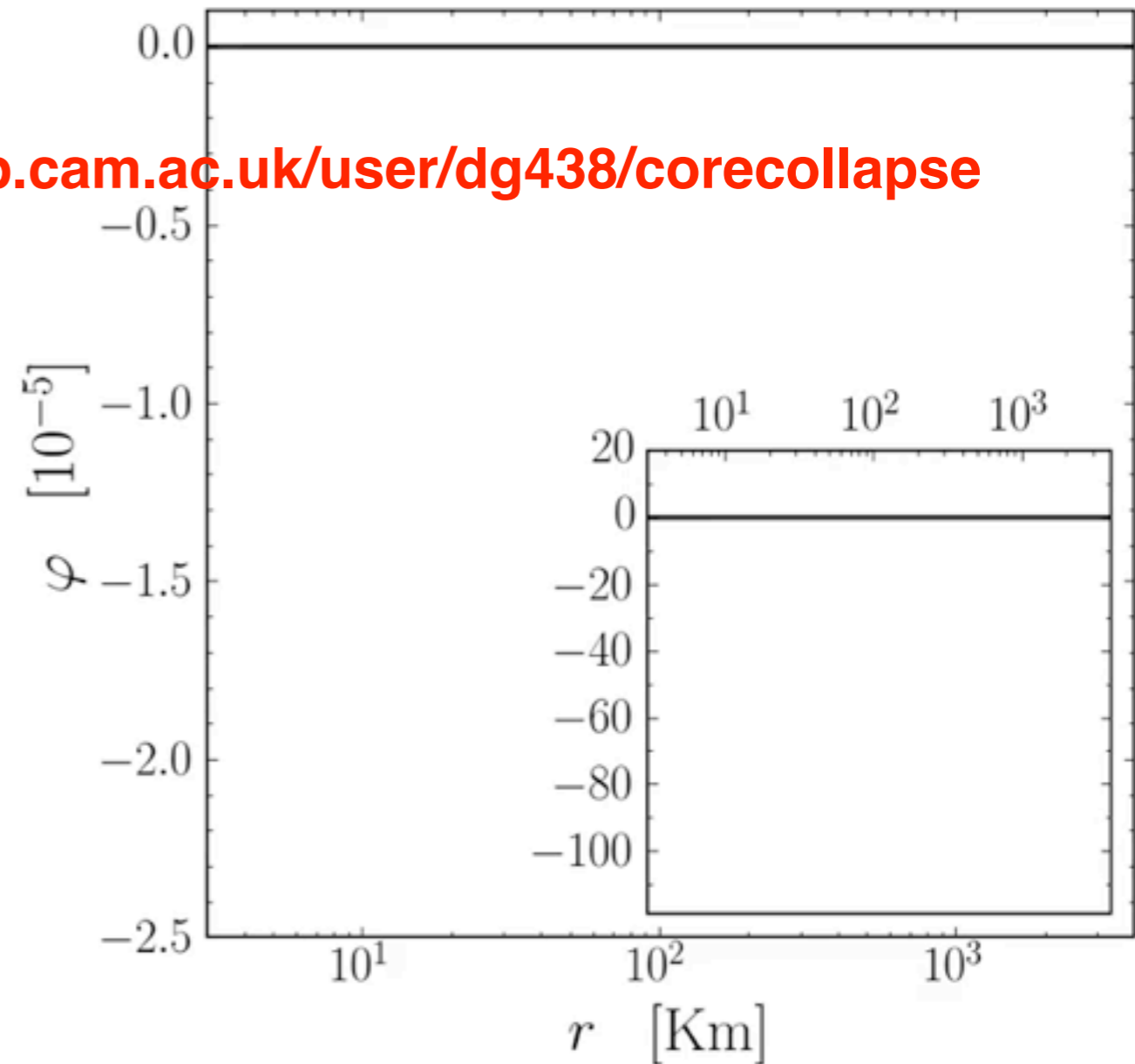
First collapse of realistic massive star through bounce in ST theory

Collapse, bounce, shock... and BH

Mass density



Scalar field



Movies available at www.damtp.cam.ac.uk/user/dg438/corecollapse

Initial profile: realistic SN progenitor $M_{\text{ZAMS}} = 40M_{\odot}$

Woosley & Heger 2007

ST theory $\alpha_0 = 10^{-4}$ $\beta_0 = -4.35$

First collapse of realistic massive star through bounce in ST theory

Proof of convergence

$$\alpha_0 = 10^{-4}$$
$$\beta_0 = -4.5$$

Self-convergence factor

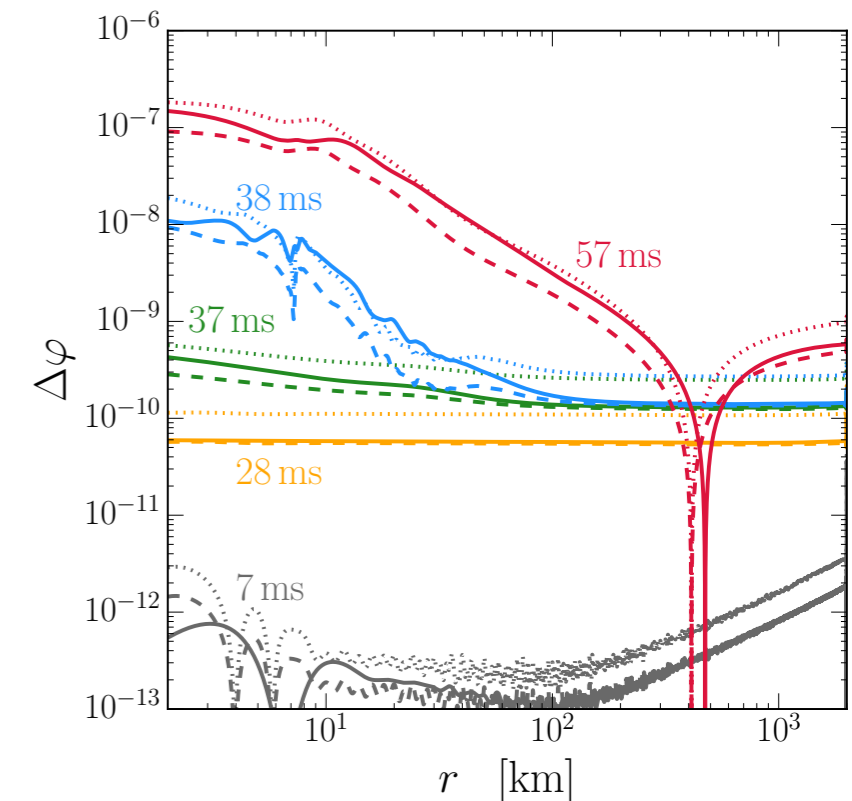
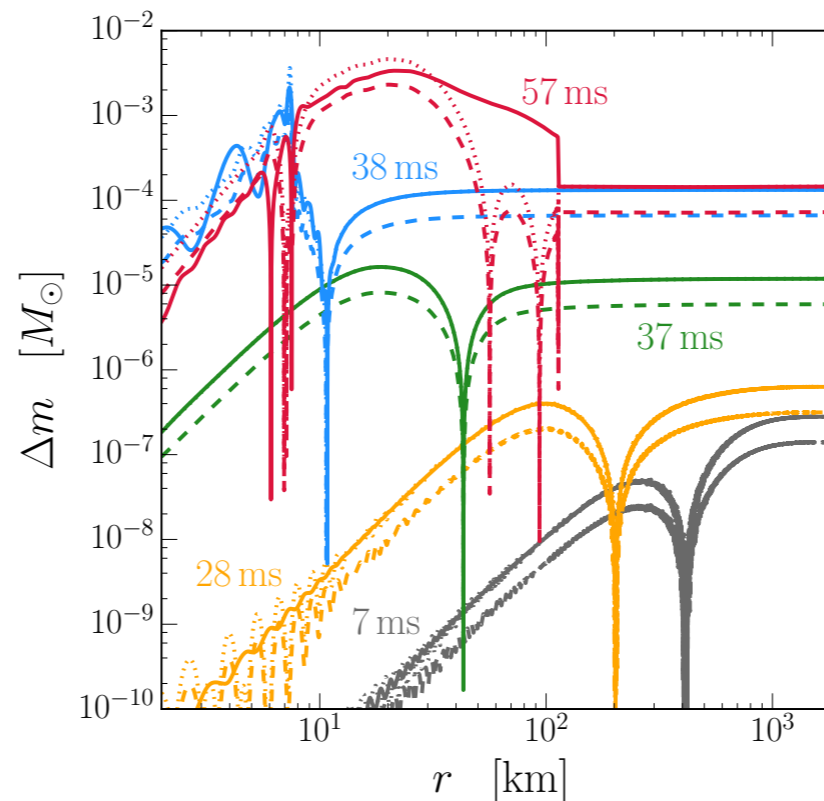
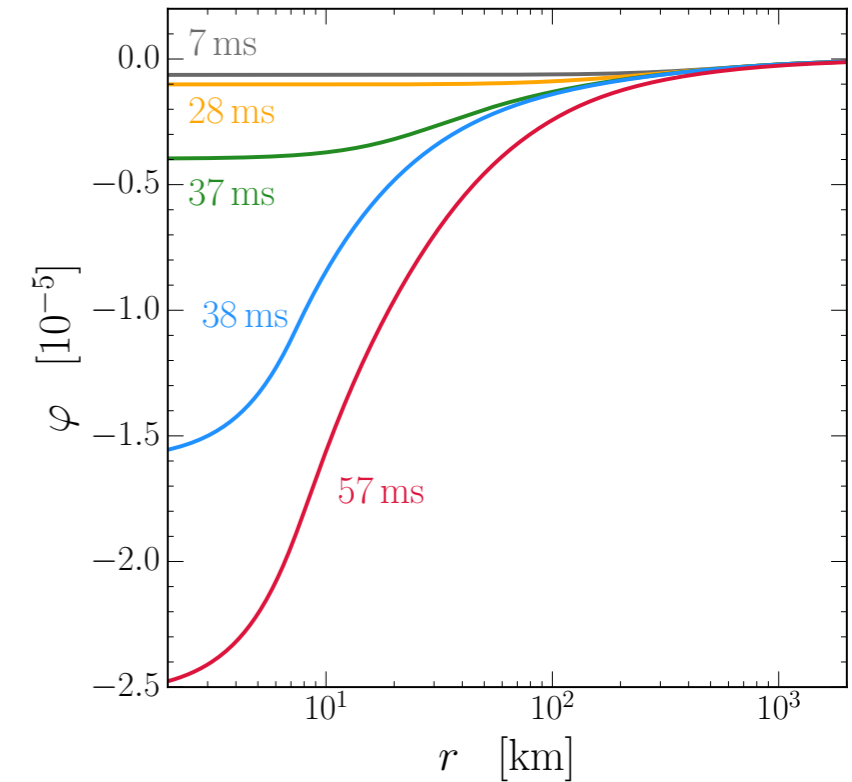
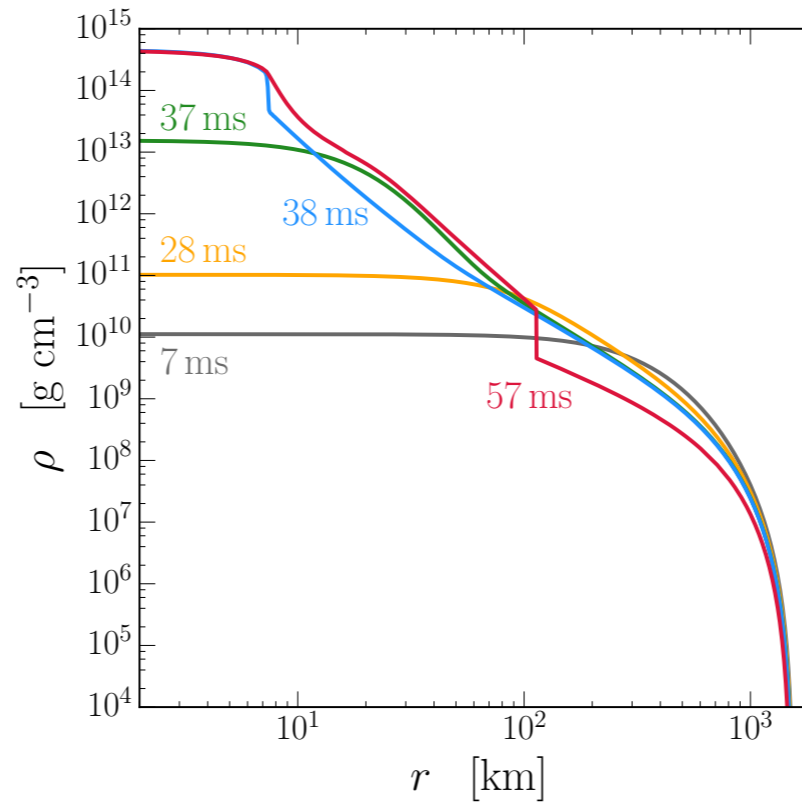
$$Q = \frac{q_1 - q_2}{q_2 - q_3} = \frac{(dr_1)^n - (dr_2)^n}{(dr_2)^n - (dr_3)^n}$$

n: numerical scheme

- PREshock $n=2$
- POSTshock $n=1$

1st order

2nd order



Breathing mode

In ST theories there are GWs in spherical symmetry

Damour and Esposito-Farese 1992

$$h(t) = \frac{2}{D} \alpha_0 r (\varphi - \varphi_0)$$

Time domain waveform

Distance

Extraction radius

Coupling with the detector

Scalar field



Microphysics

Hybrid EOS:

$$P = P_c + P_{\text{th}}$$

$$P_c = \begin{cases} K_1 \rho^{\Gamma_1} & \text{if } \rho \leq \rho_{\text{nuc}} \\ K_2 \rho^{\Gamma_2} & \text{if } \rho > \rho_{\text{nuc}} \end{cases}$$

$$P_{\text{th}} = (\Gamma_{\text{th}} - 1) \rho \epsilon_{\text{th}}$$

(o) Fast relaxation

initial profile is GR, not ST!

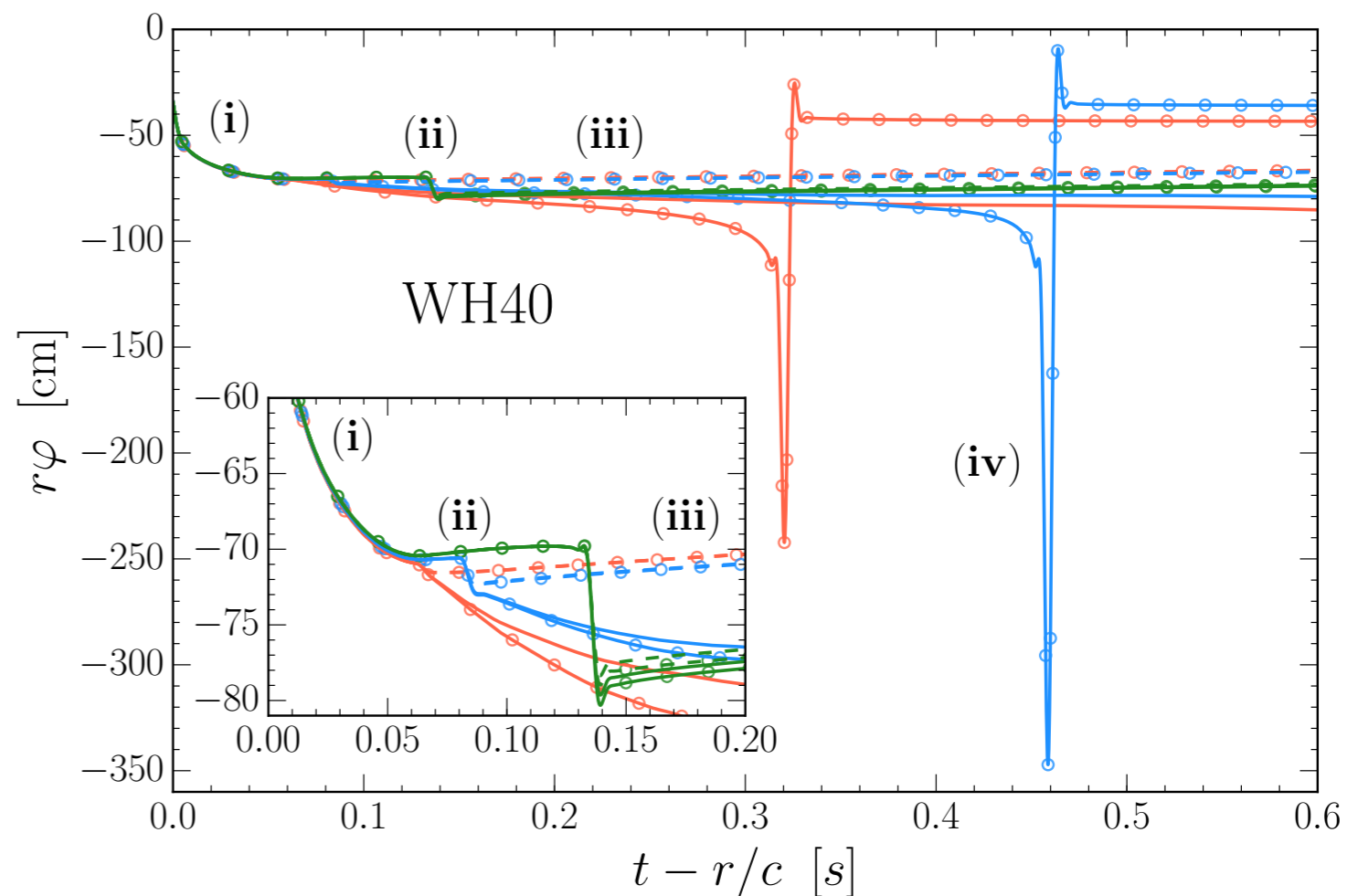
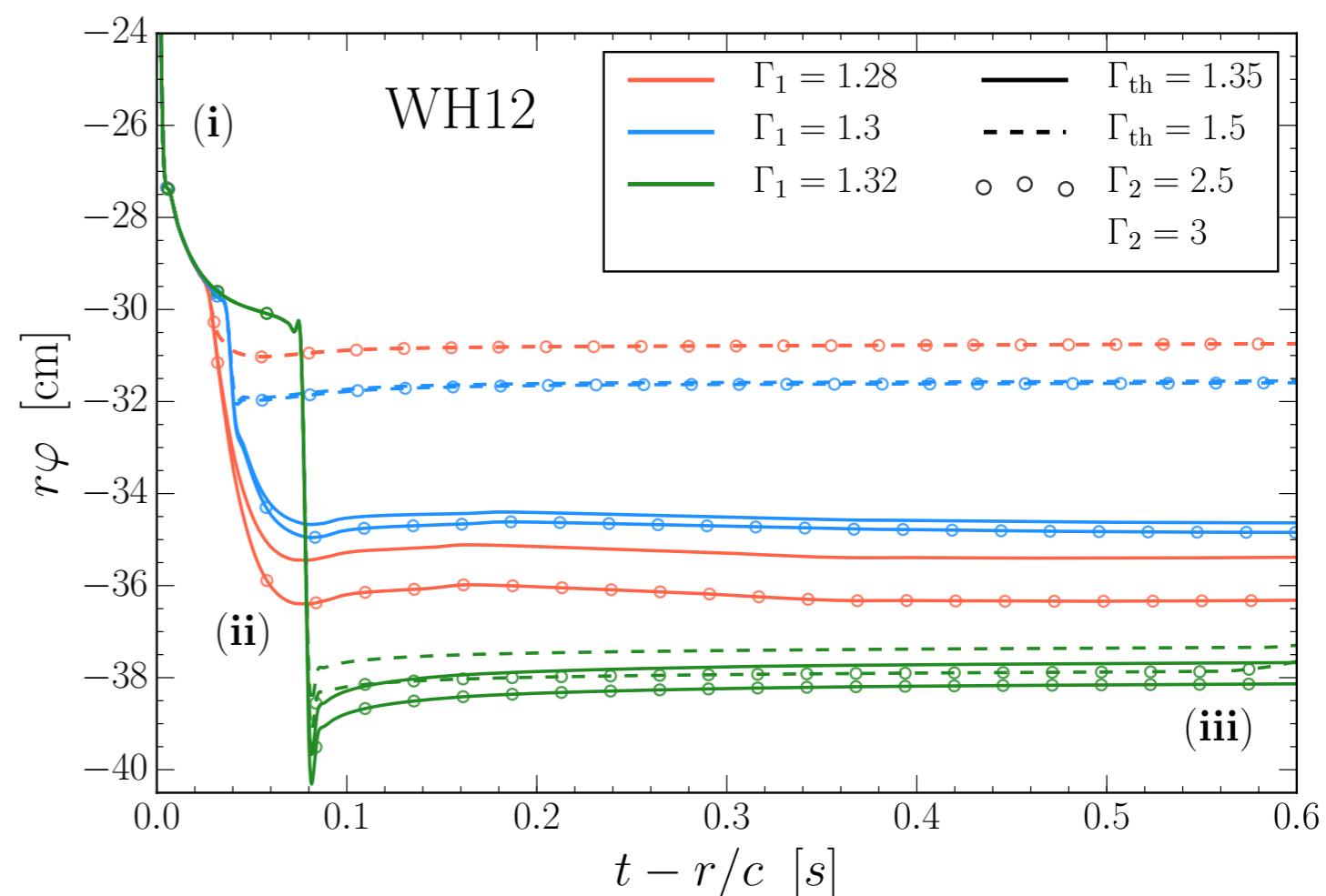
(i) Collapse

(ii) Bounce!

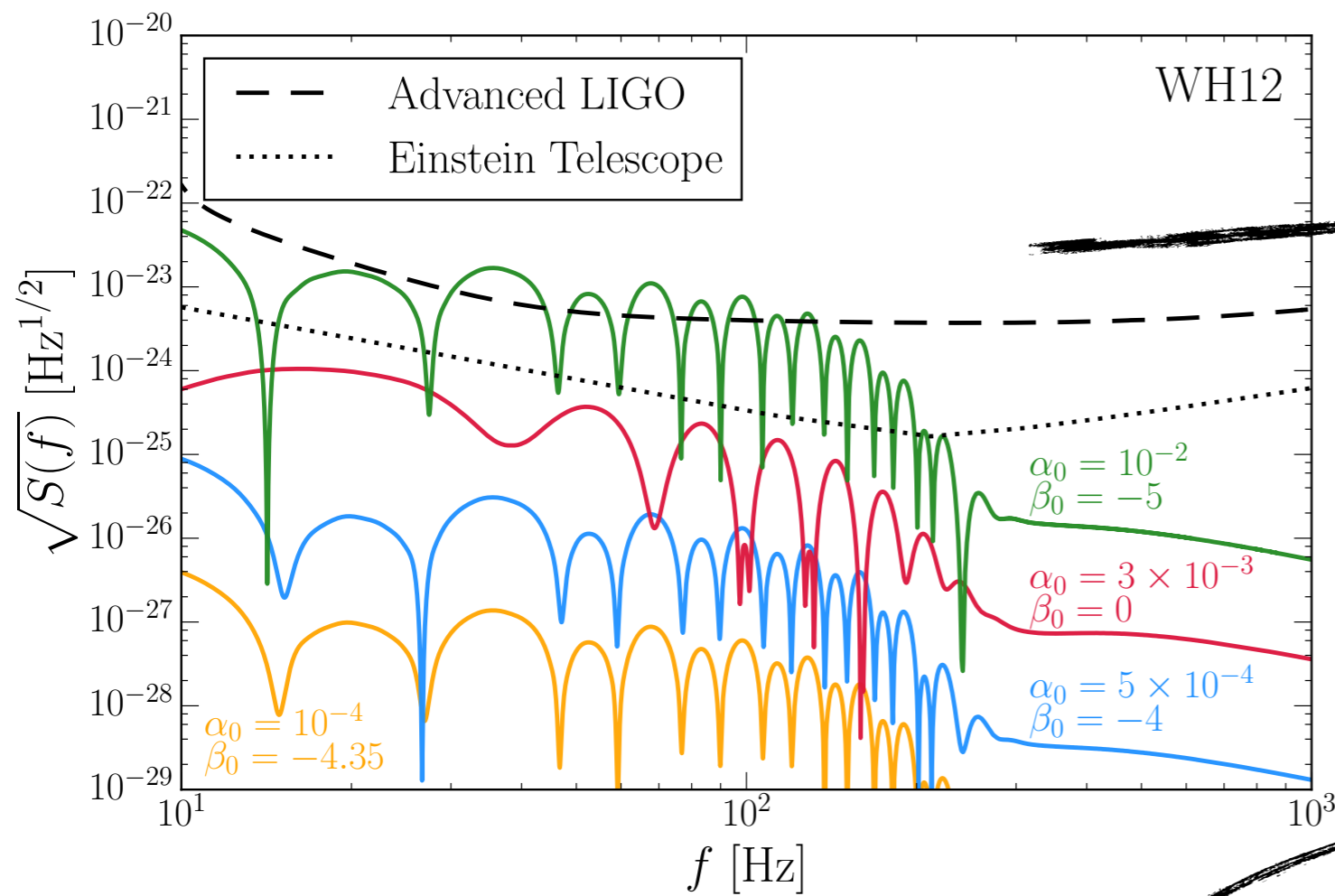
(iii) Core forms a NS

(iv) Collapse to a BH

$$\alpha_0 = 10^{-4} \quad \beta_0 = -4.35$$



Monopolar GWs



$$M_{\text{ZAMS}} = 12M_{\odot}$$

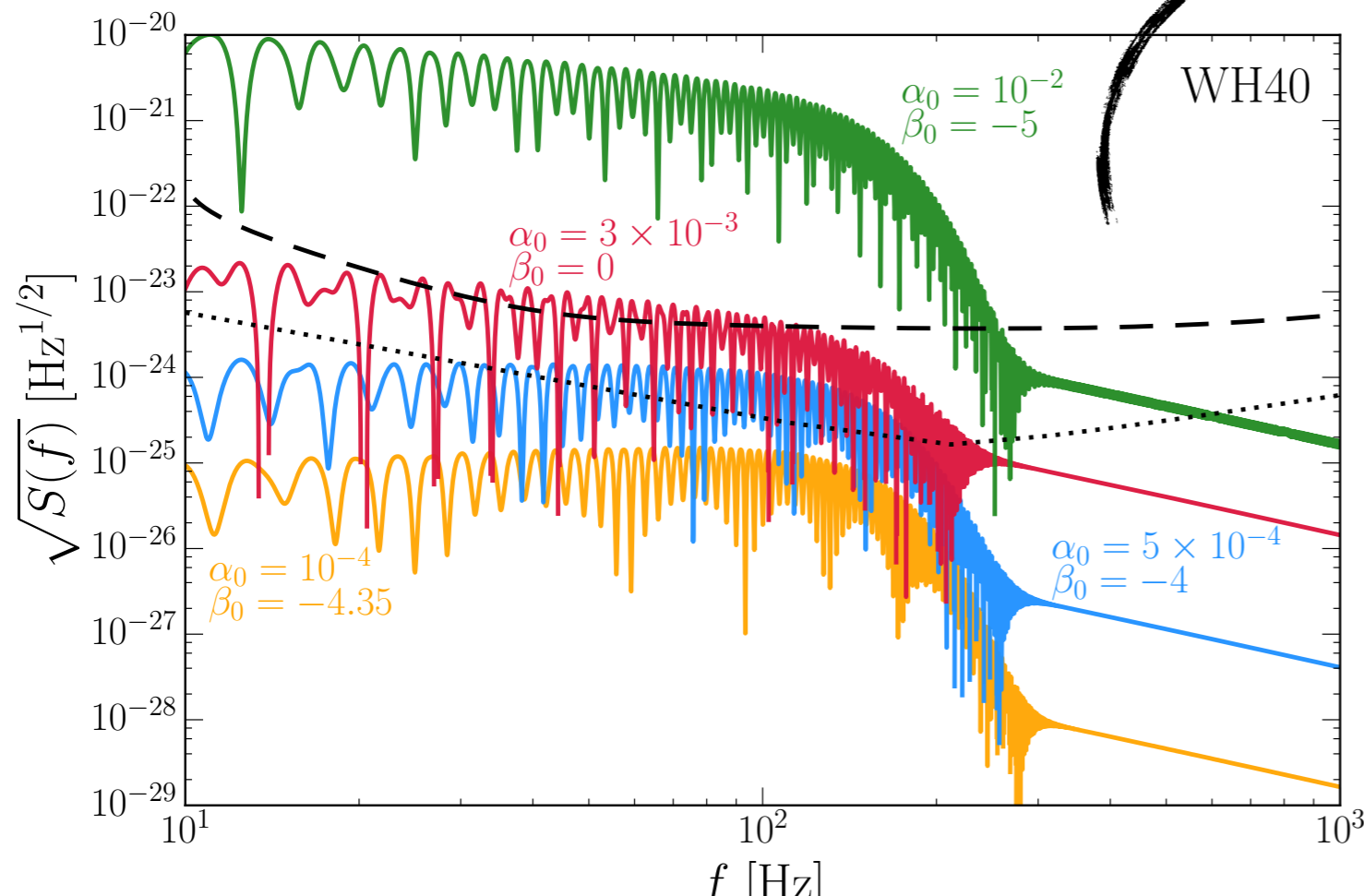
Can't get enough compactness for spontaneous scalarization

- We need better microphysics!
Neutrino transfer and cooling?

$$M_{\text{ZAMS}} = 40M_{\odot}$$

Powerful signal from BH formation

- Is transition to spontaneously scalarized star important?



Marginal detectability with Ad. LIGO for galactic sources

Source in the Milky Way: $D = 10$ kpc

One, two, many.

Tensor-multi-scalar theories

$$S = \frac{1}{4\pi G_\star} \int d^4x \sqrt{-g} \left[\frac{R}{4} - \frac{1}{2} g^{\mu\nu} \gamma_{AB}(\varphi) \partial_\mu \varphi^A \partial_\nu \varphi^B - V(\varphi) \right] + S_m[A^2(\varphi) g_{\mu\nu}; \Psi]$$

One example: **maximally symmetric two fields**

$$S = \frac{1}{4\pi G_\star} \int d^4x \sqrt{-g} \left[\frac{R}{4} - g^{\mu\nu} \gamma(\varphi, \bar{\varphi}) \nabla_\mu \bar{\varphi} \nabla_\nu \varphi - V(\varphi, \bar{\varphi}) \right] + S_m[A^2(\varphi, \bar{\varphi}) g_{\mu\nu}; \Psi]$$

$$\gamma(\varphi, \bar{\varphi}) = \frac{1}{2} \left(1 + \frac{\bar{\varphi}\varphi}{4r^2} \right)^{-2}$$

Curvature radius

$$\log A(\varphi, \bar{\varphi}) = \alpha^* \varphi + \bar{\alpha}^* \bar{\varphi} + \frac{1}{2} \beta_0 \varphi \bar{\varphi} + \frac{1}{4} \beta_1^* \varphi^2 + \frac{1}{4} \bar{\beta}_1^* \bar{\varphi}^2$$

Coupling function

Interactions between the fields: **target space**

Basically unconstrained! Genuine two-field physics?

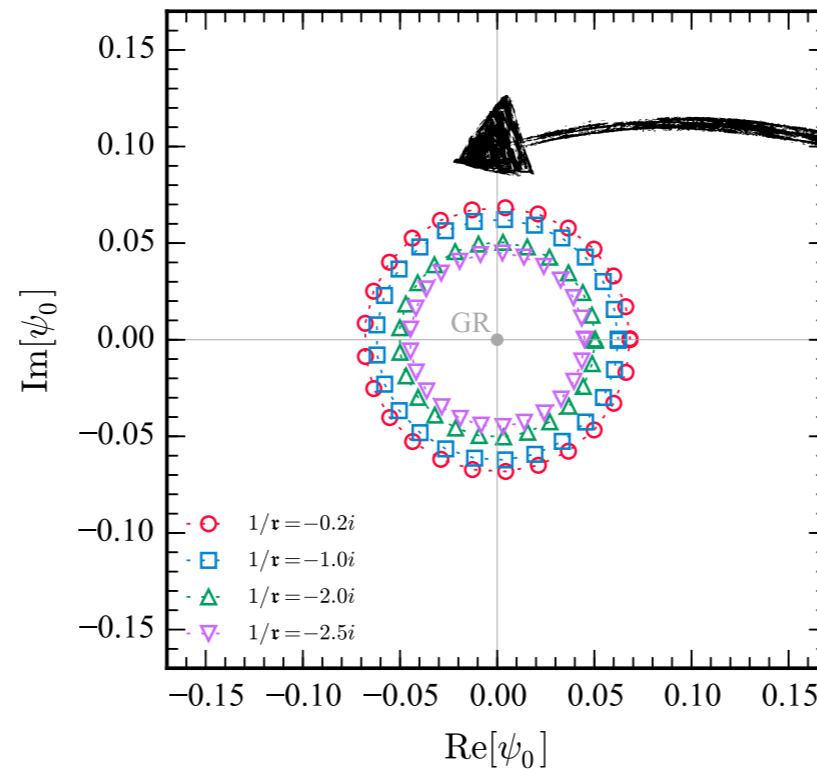
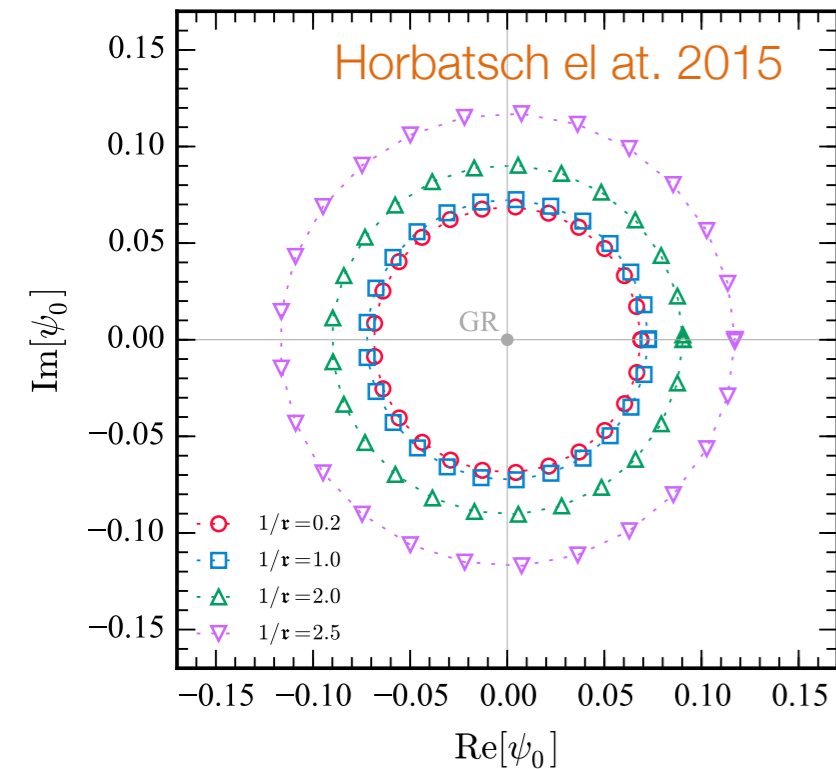
Curvature radius

$$\log A(\psi, \bar{\psi}) = \alpha\psi + \bar{\alpha}\bar{\psi} + \frac{1}{2}\beta_0\psi\bar{\psi} + \frac{1}{4}\beta_1\psi^2 + \frac{1}{4}\beta_1\bar{\psi}^2$$

$$\alpha = 0 \quad \beta_1 = 0$$

Rotation symmetry

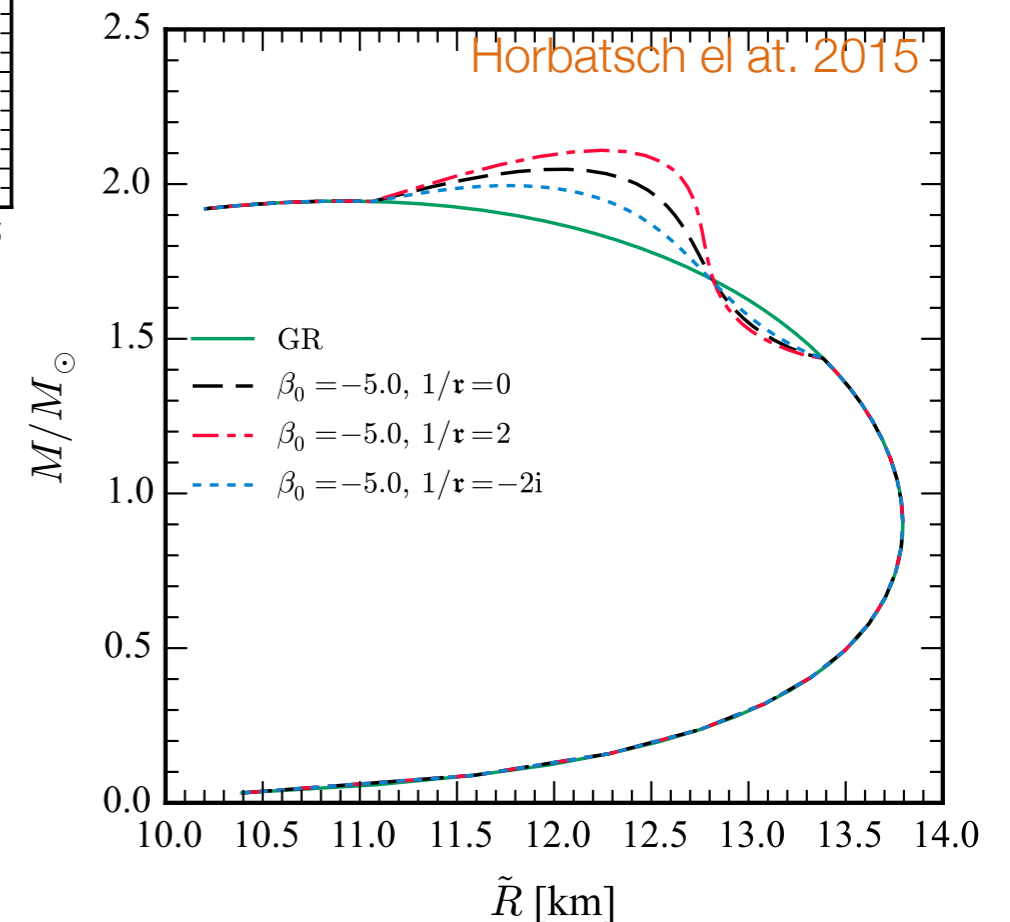
$$A(\psi, \bar{\psi}) = \exp\left(\frac{1}{2}\beta_0\psi\bar{\psi}\right)$$



Central value of the scalar field just a circle...

Neutron stars mass/radius

Neutron stars feel the target space!



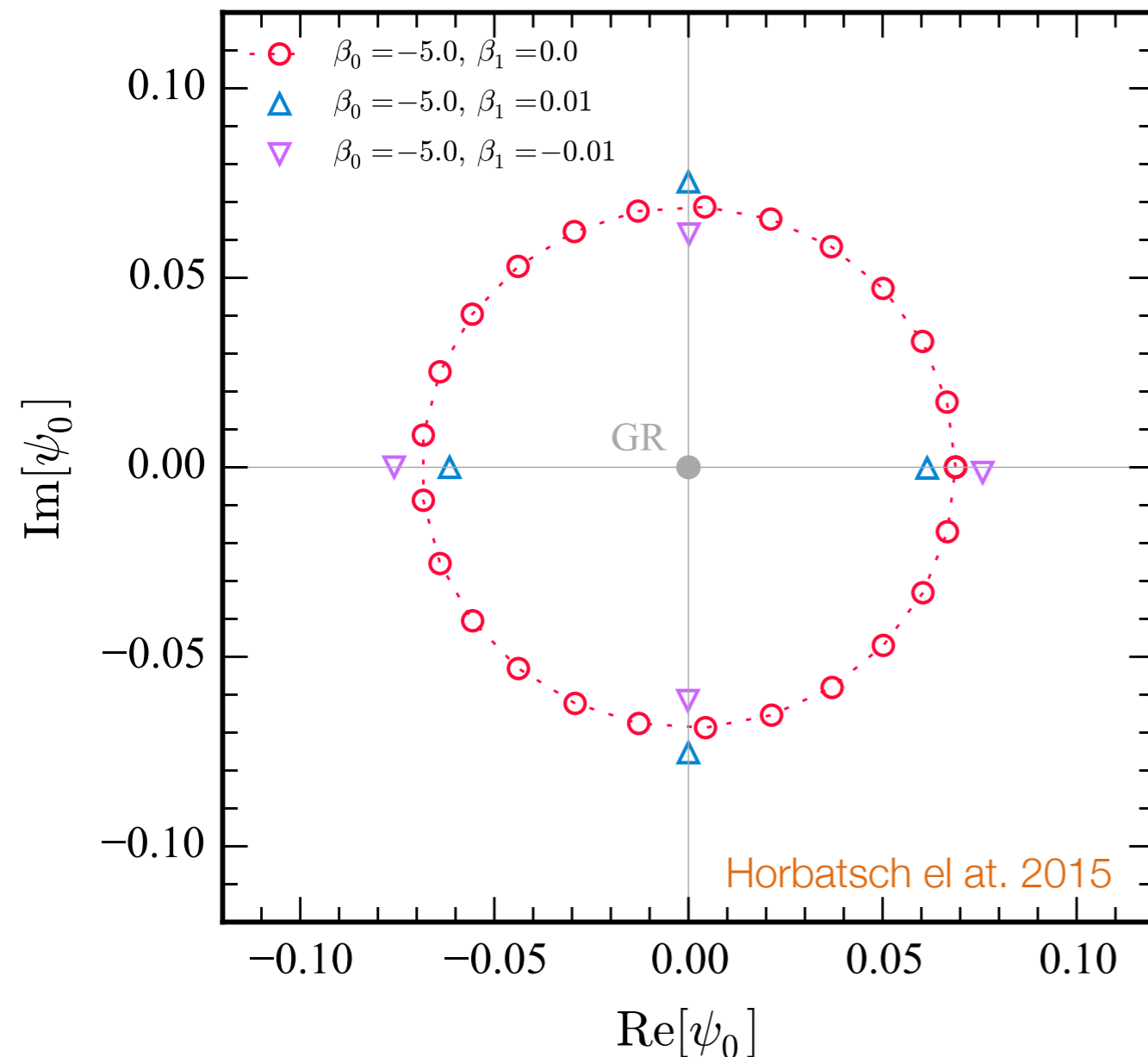
Breaking the symmetry

$$\log A(\psi, \bar{\psi}) = \alpha\psi + \bar{\alpha}\bar{\psi} + \frac{1}{2}\beta_0\psi\bar{\psi} + \frac{1}{4}\beta_1\psi^2 + \frac{1}{4}\beta_1\bar{\psi}^2$$

$$\alpha = 0 \quad \beta_1 \neq 0$$

~~Rotation symmetry~~

$$A(\psi, \bar{\psi}) = \exp\left(\frac{1}{2}\beta_0\psi\bar{\psi}\right)$$



**Central value of
the scalar field**

The circle collapses to a cross...
even for arbitrarily small β_1

Not yet understood...

genuine biscalarization found
only for $\alpha \neq 0$

Outline

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Soon!

[arXiv:1505.07462](https://arxiv.org/abs/1505.07462)

