Random walks in the sky: analytical models of Large Scale Structure

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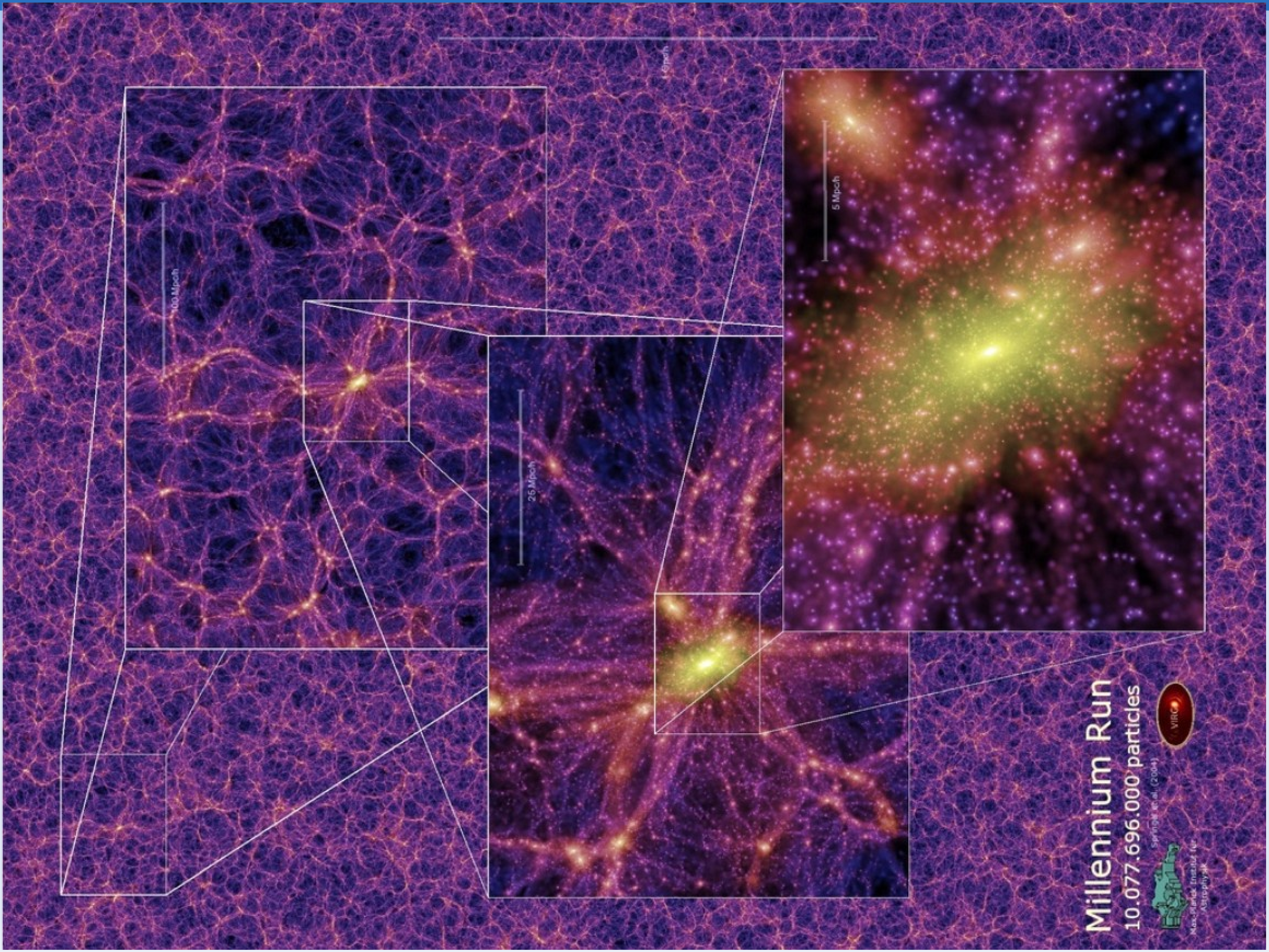
based on works with R. Sheth, A.Paranjape, T. Lazeyras, V. Desjacques, C.Pichon, C.Cadiou, S.Codis, K.Kralijc, Y.Dubois

Institut d’Astrophysique de Paris, September 2017

Really??



Really? Yes, for many reasons

- Understand N-body simulations
- Can't run a simulation for every choice of cosmological parameters!
- Explore non-standard cosmologies
- Huge degeneracy in parameter space: study deviation from universality
- Physically motivated fitting formulae (esp. for halo bias!)
- Improve data analysis

Why analytically?

One might wonder why we put effort into approximate descriptions of cosmic structure formation given the tremendous recent and promised advances in computing power. Surely the not very distant future will bring computations of arbitrarily large simulation volumes with arbitrarily high resolution using arbitrarily adaptive hydrodynamical and N-body techniques. That will be so. But even so, we need a physical language to discuss the outcomes.

(Bond & Myers 96)

Which models?

- Excursion set theory.
  Halos are patches with high enough initial mean density to recollapse by today.
- Theory of peaks.
  Halos are peaks of the initial density field smoothed on their mass scale
- Peak-patch models. Excursion set peaks.
  Combination of the above, with more sophisticated models of collapse
- Models of halo motion. Patches have center of mass acceleration
- Models of bias. Should follow consistently

Spherical Collapse

- Spherical evolution sensitive only to the total mass M inside the shell, not to the inner density profile
- That is, only mean initial overdensity within R matters:

$$\delta_{R,in}(\mathbf{x}) \equiv \frac{1}{V_R}\int_{|\mathbf{y}|<R} \mathrm{d}^3y\, \delta_{in}(\mathbf{x}+\mathbf{y}) \qquad \delta_{in}(\mathbf{x}) \equiv \frac{\rho_{in}(\mathbf{x})-\bar{\rho}_{in}}{\bar{\rho}_{in}}$$

- A shell of radius R containing $M = \bar{\rho}(4\pi/3)R^3$ collapses at $\mathbf{x}$ by z if

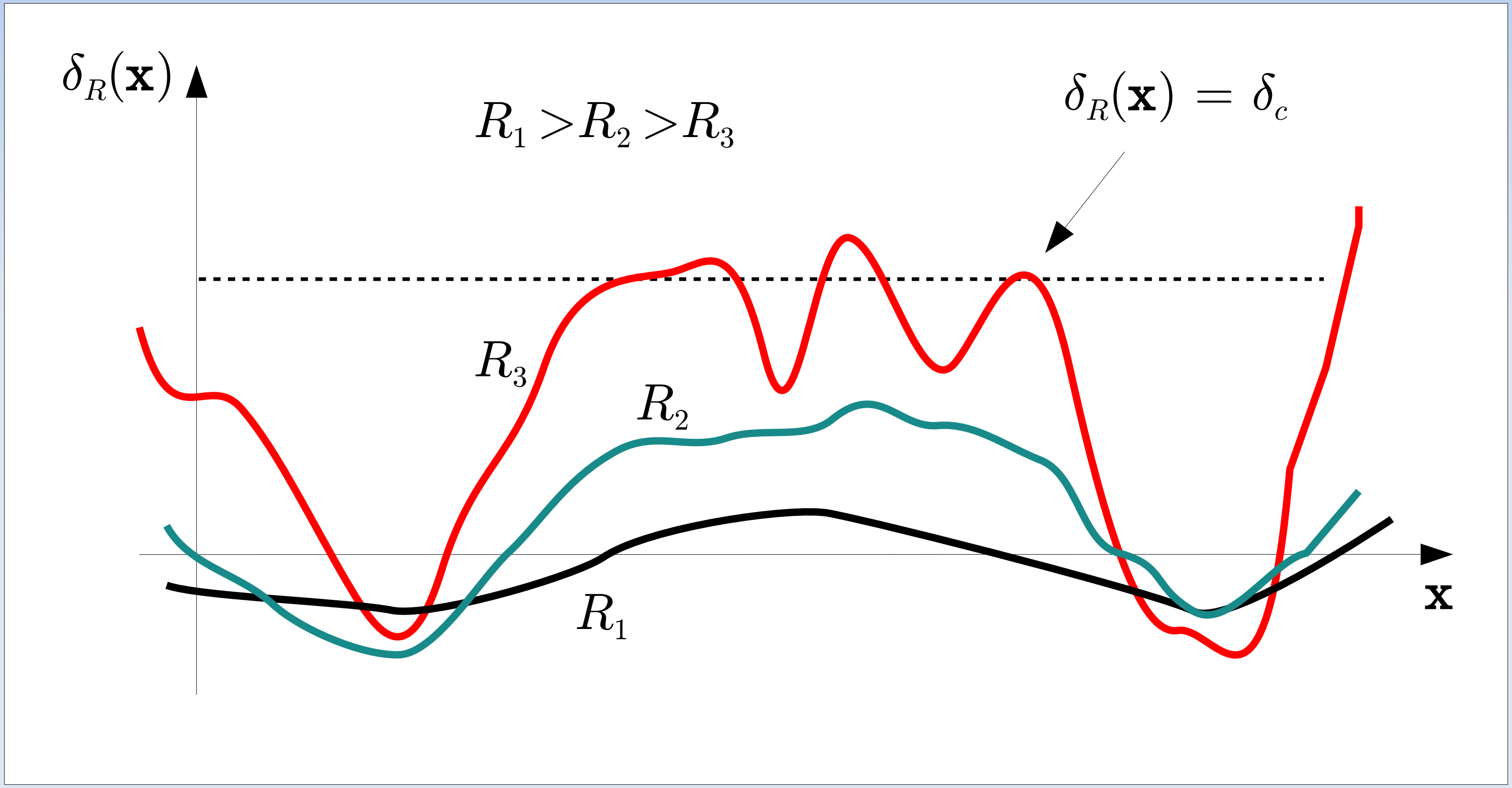
$$\delta_R(z,\mathbf{x}) \equiv D(z)\delta_{R,in}(\mathbf{x})/D(z_{in}) \gtrsim 1.69 \equiv \delta_c$$

- M sets the smoothing scale, the filter MUST be TopHat in real space
- In Fourier space, smoothing the linear field at $z = 0$:

$$\delta_R(\mathbf{x}) \equiv \int \frac{\mathrm{d}^3k}{(2\pi)^3}\, e^{i\mathbf{k}\cdot\mathbf{x}}\, \frac{3j_1(kR)}{kR}\, \delta(\mathbf{k}) \geq \frac{\delta_c}{D(z)}$$

Finding proto-halos

- The INITIAL density field varies with position AND smoothing scale

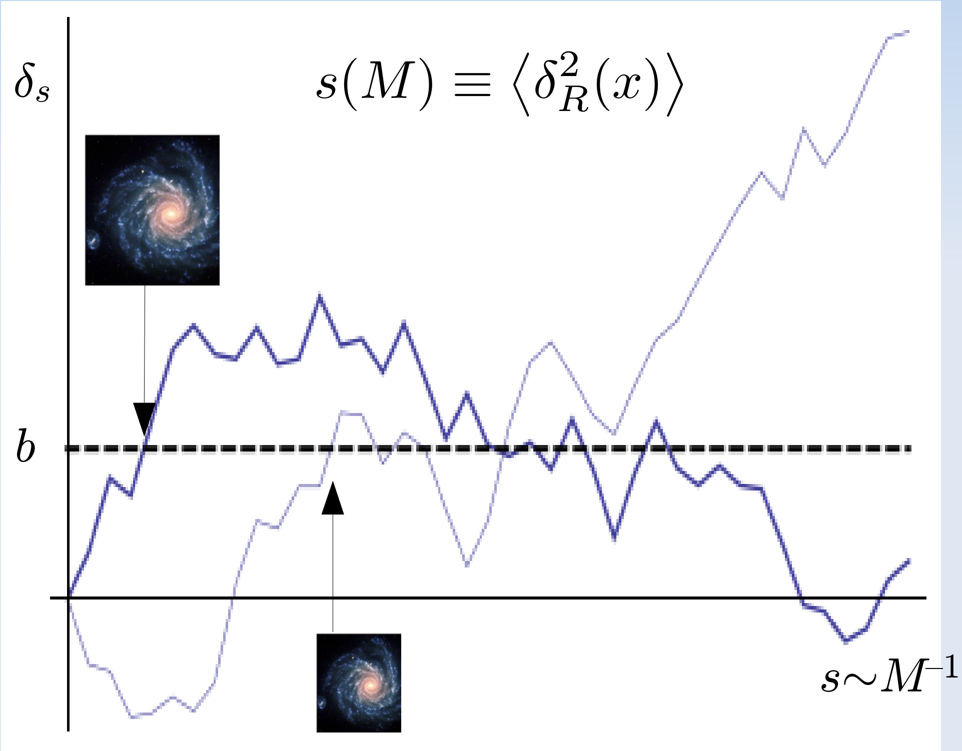


- First crossing fixes R and $\mathbf{x}$: size and position of the proto-halo
- Mass conservation. Final mass is $M = \rho_{\rm bkgd} 4\pi R^3/3$

Excursion set theory

- At each position $\mathbf{x}$, $\delta_R(\mathbf{x})$ follows a different random walk as R changes
- But the walks are not Markovian: steps correlate with each other
- True for any compact filter (include all Fourier modes)

FIRST PASSAGE of random walks w/ CORRELATED steps



- Abundance $n_h(M)$ $\longleftrightarrow$ first crossing probability $f(s)$ at scale $s(M)$
- But $f(s)$ is not known: need better maths

Excursion set theory

Halos as patches in the initial conditions that:

- are dense “enough” to have formed by today (“enough” is the initial overdensity of spherical collapse, for now...)
- are not contained in larger patches of the same density (“no cloud-in-cloud”)
- mimicked by random walks in mean density space reaching a critical threshold
- Abundance: propto first-passage pdf $f(s)$ with correlated steps
- Formally: $$f(s) \equiv \lim_{\Delta s \to 0} \frac{1}{\Delta s} \left\langle \vartheta(\delta_N - \delta_c) \prod_{i}^{N-1} \vartheta(\delta_c - \delta_i) \right\rangle$$

$$s(R) \equiv \langle \delta_R^2(x) \rangle = \int \mathrm{d}k \frac{k^2 P(k)}{2\pi^2} W^2(kR) \equiv \sigma^2(M)$$

First crossing distribution

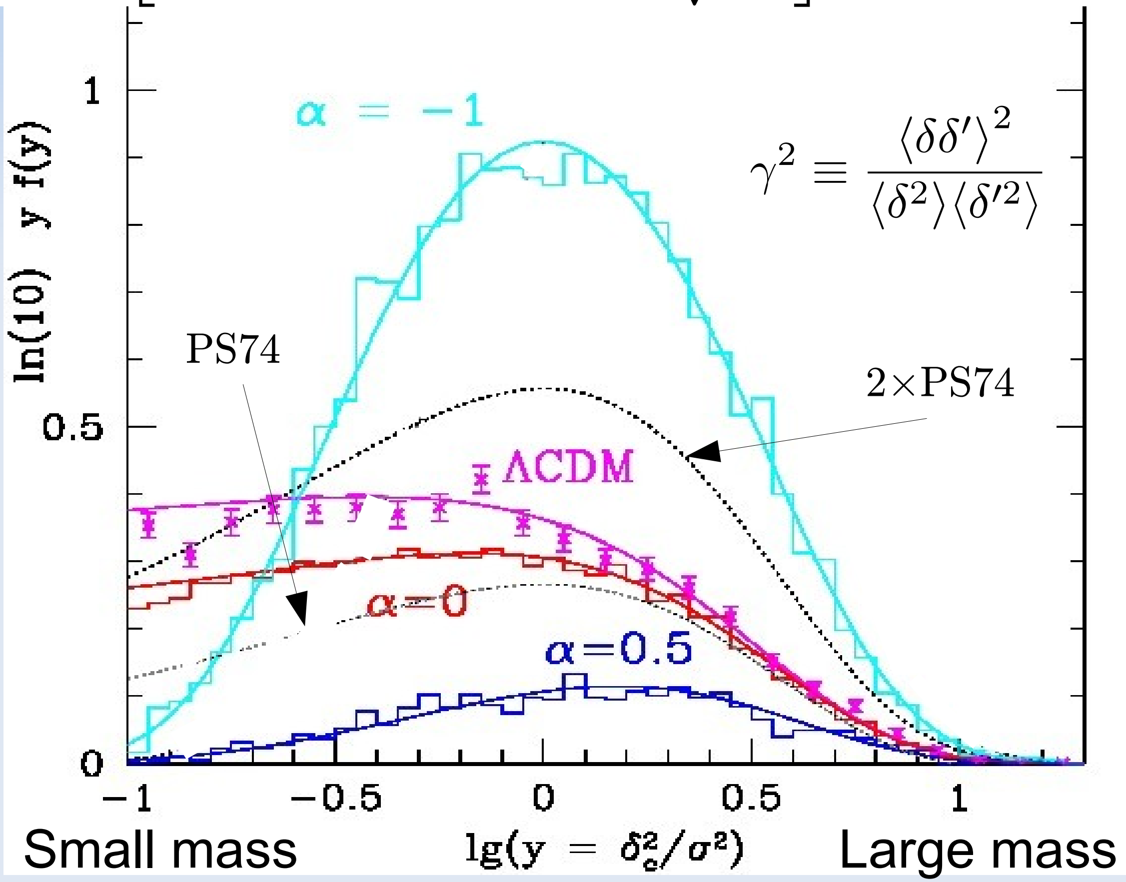
- Probability of ANY crossing at s **Press & Schechter (1974)**
- Want **FIRST** crossing to avoid cloud-in-cloud: $\delta_s > b(s)$ but $\delta_S < b(S)$ for $S < s$. Solved for Gaussian uncorrelated steps with constant/linear barrier **Bond et al. (1991)** **Jedamzik (1995); Sheth(1998)**
- May treat correlations as perturbations **Maggiore & Riotto (2010)** **Corasaniti & Achitouv (2011)**
- However: correlations make cloud-in-cloud less likely (less zig-zags) **Paranjape, Lam & Sheth (2011)**
- Can relax **FIRST** into **UPWARDS**: $\delta_s = b(s)$, $\delta'_s \equiv \mathrm{d}\delta/\mathrm{d}s \geq \mathrm{d}b/\mathrm{d}s$

$$\frac{M}{\bar{\rho}}\frac{\mathrm{d}n_h}{\mathrm{d}M} = \left|\frac{\mathrm{d}s}{\mathrm{d}M}\right| f_{\rm up}(s) = -\frac{\mathrm{d}s}{\mathrm{d}M}\int_{b'}^{\infty}\mathrm{d}\delta'_s(\delta'_s - b')\,p(\delta'_s, \delta = b)$$

MM & Sheth (2012)

Upcrossing distribution

$$f_{\mathrm{up}}(s) = f_{\mathrm{PS}}(s)\left[\frac{1+\mathrm{erf}(X/\sqrt{2})}{2} + \frac{e^{-X/2}}{X\sqrt{2\pi}}\right], \qquad X^2 \equiv \frac{\gamma^2\delta_c^2}{(1-\gamma^2)s}$$

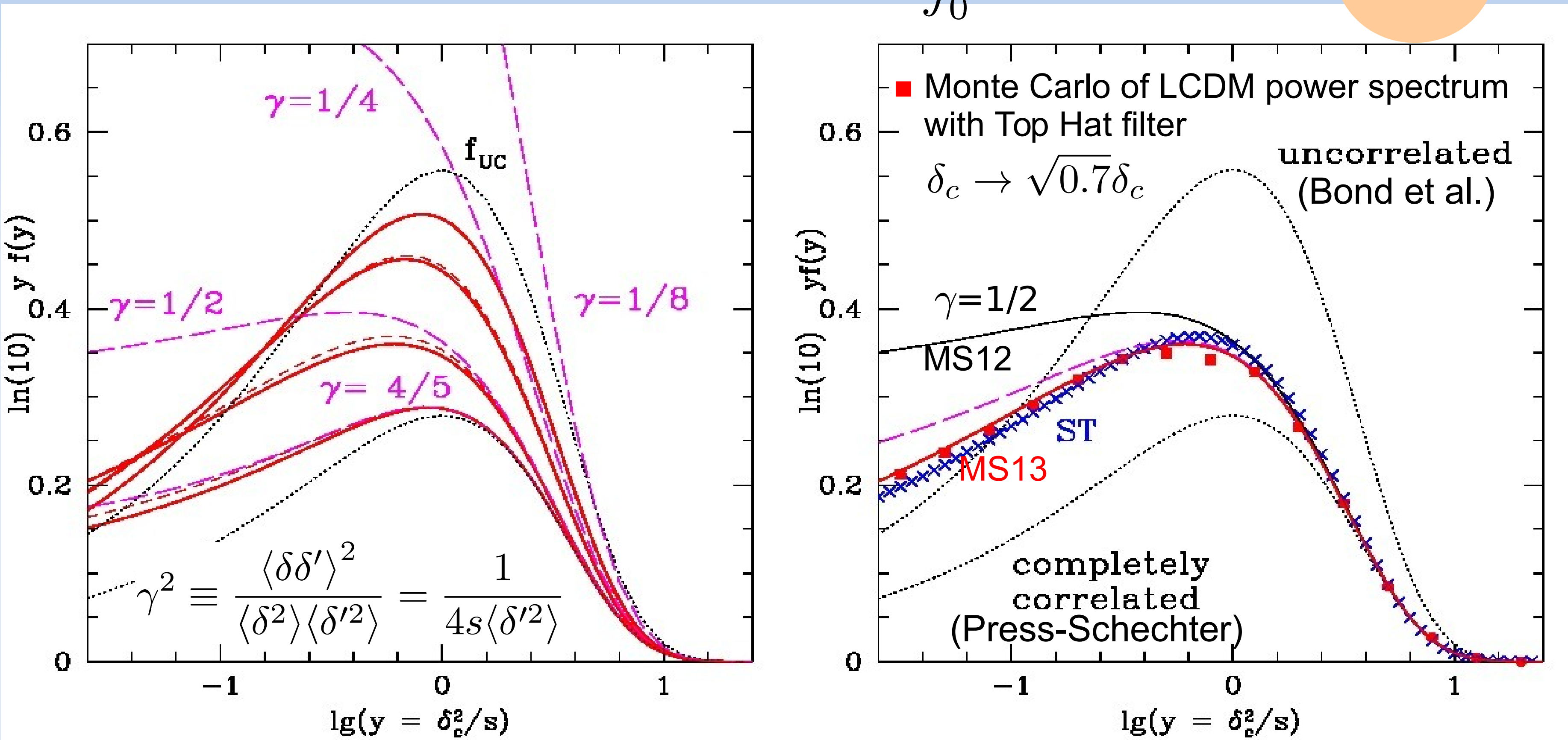


MM & Sheth (2012)

Compare with Monte Carlo walks (histograms) with various power spectra and barrier $b = \delta_c + \alpha s$. Dotted lines are PS74 and twice PS74 (uncorrelated steps)

Solution by back substitution

- An even better approx: $$p(\delta_s \geq b) = \int_0^s \mathrm{d}S f(S) p(\delta_s \geq b|\mathrm{up}, S)$$

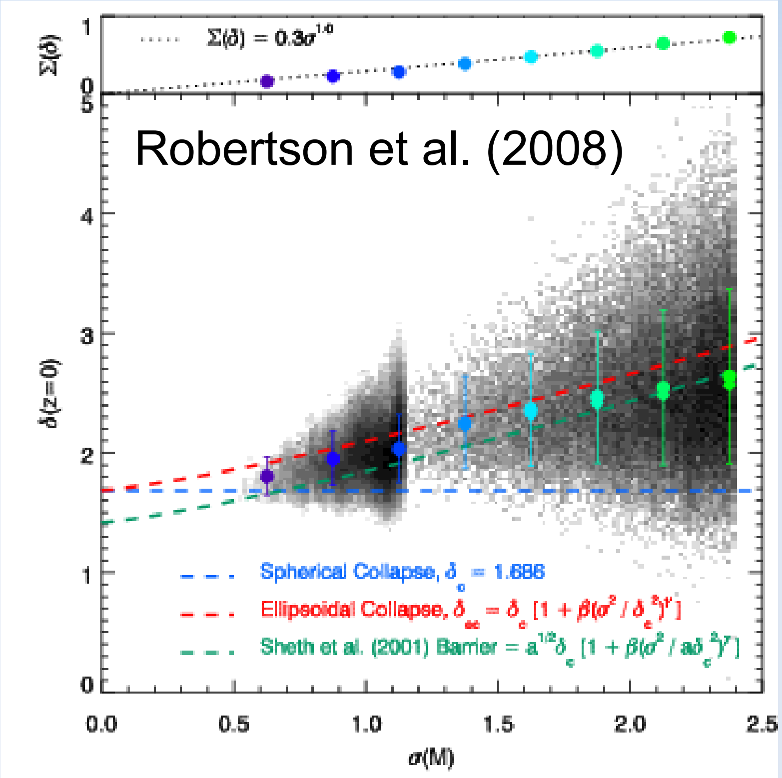


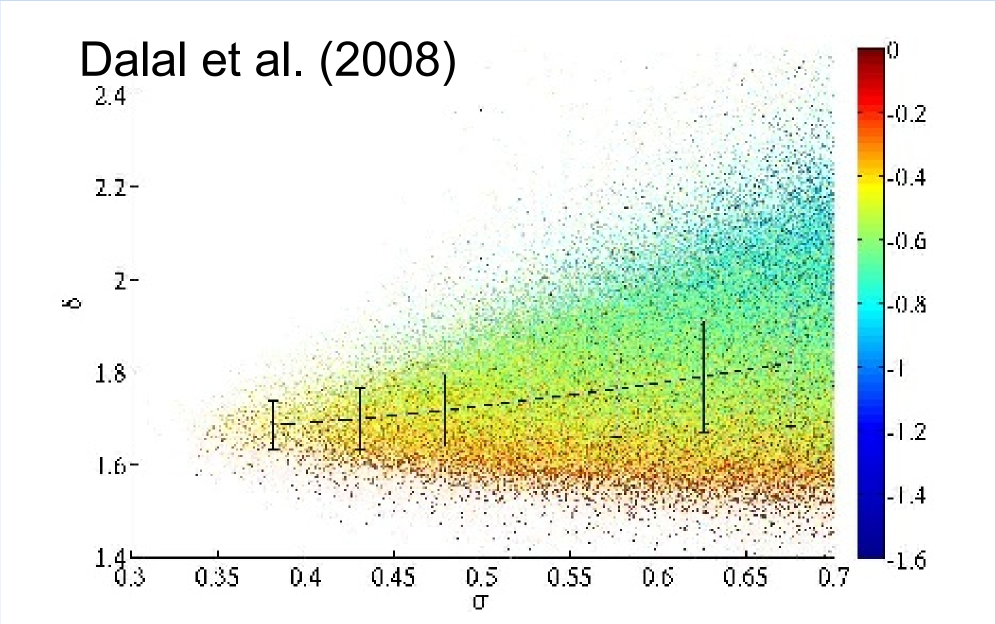
MM & Sheth (2013)

- Upcrossing captures $f(s)$ for all $P(k)$, filters and barriers. Yet, the mass function works only if $\delta_c \to .84\ \delta_c$. There is a flaw in the ansatz!

The critical density

- At small mass, barrier b becomes “stochastic” (other variables play a role, e.g. shear, shape, velocity dispersion) and scale-dependent



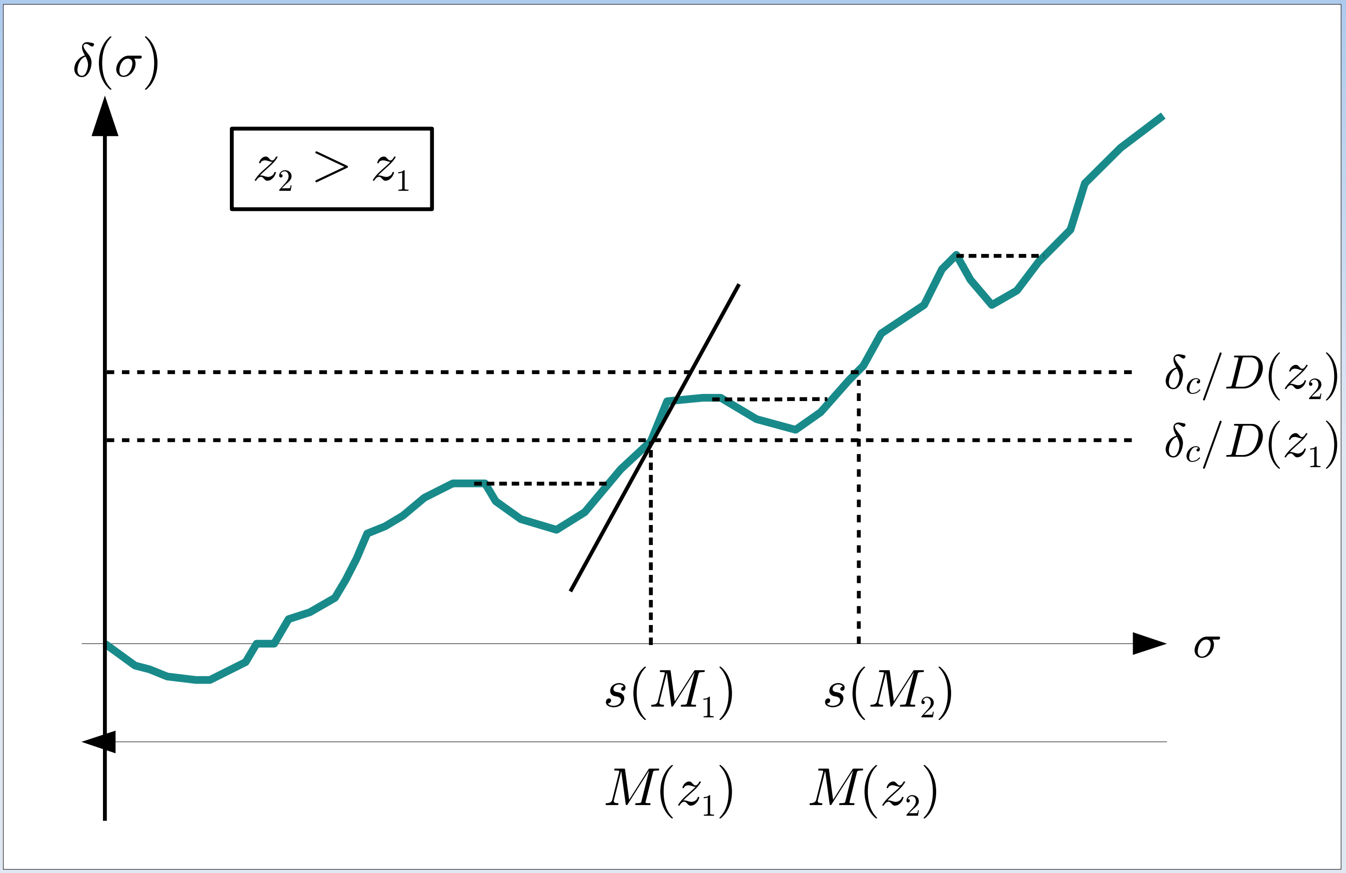


- Halos do not form at random locations, but at density peaks. Should do excursion sets there.

Assembly bias

- Assembly bias (“there’s more to a halo than its mass...”) is somewhat of an obvious statement. The opposite would be surprising!
- Surprisingly difficult to find the optimal variables to parametrize it because of strong statistical correlation
- Most quantities have unexpected behaviors in some regime
- Because halos are not isolated, their position in the cosmic web is an obvious candidate
- Observationally relevant: surveys (VIPERS, COSMOS, GAMA) find galaxies in different color bins at different distance to the cosmic web
- But… what is color? For DM halos, can play with accretion rate and formation time

Formation history



- As threshold drops with z, first crossing moves/jumps to larger M
- Continuous growth of M is accretion, finite jumps are mergers. Whole formation history $M(z)$ in the trajectory. Slope gives accretion rate.

Lacey and Cole (1993)

Accretion rate and formation time

- Following the first-crossing scale at all z gives $M(z)$:

$$\delta(\sigma(M(z))) = \frac{\delta_c}{D(z)}$$

- Differentiating w.r.t. z gives $\mathrm{d}M/\mathrm{d}z$:

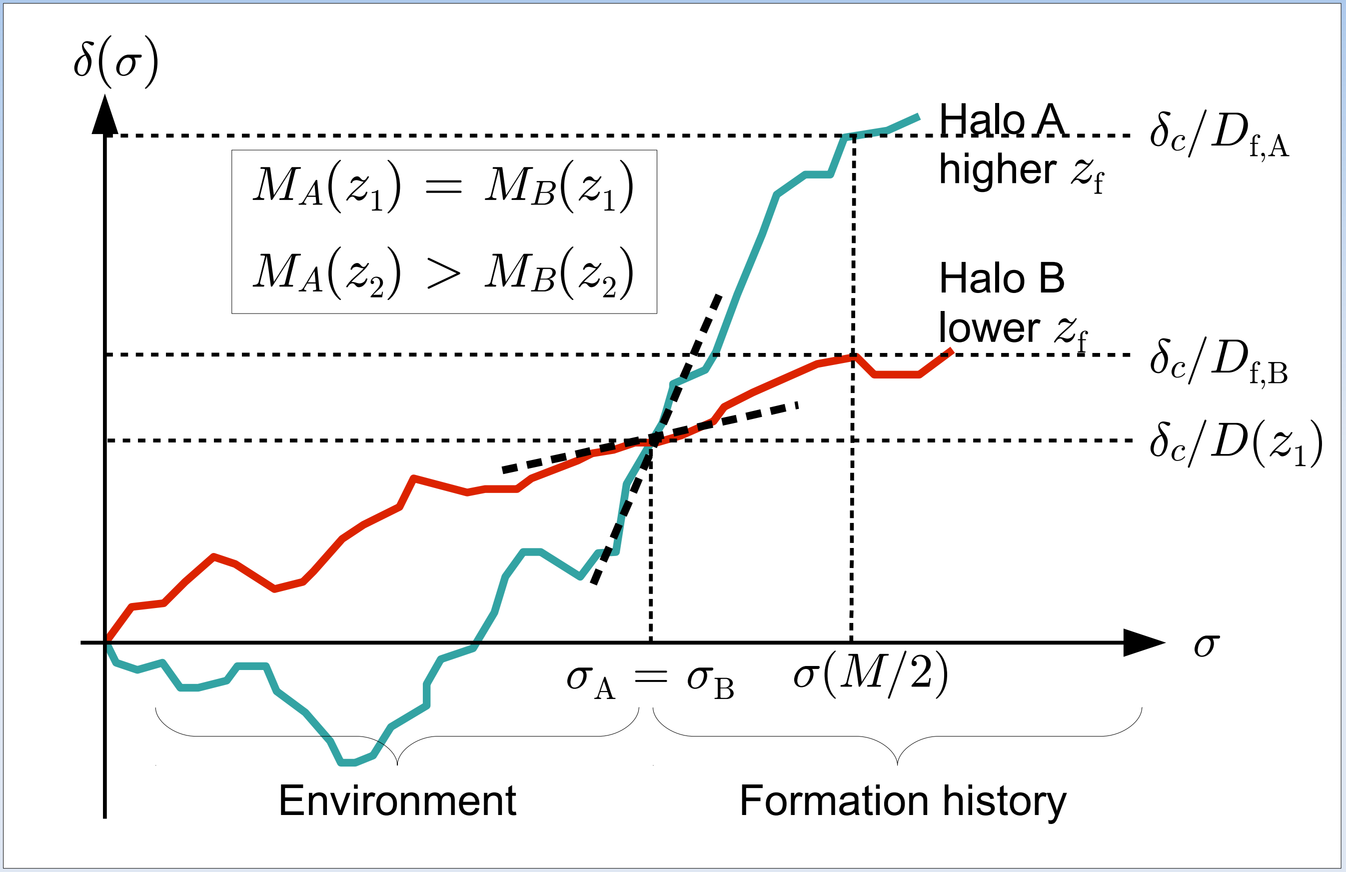
$$\frac{\mathrm{d}\sigma}{\mathrm{d}M}\frac{\mathrm{d}M}{\mathrm{d}z} = -\frac{\delta_c}{\delta'(\sigma)D^2}\frac{\mathrm{d}D}{\mathrm{d}z}$$

  can be done for correlated steps! Accretion rate fixes the slope.

- The height at $\sigma(M/2)$ gives the value of $\delta_c/D(z_\mathrm{f})$:

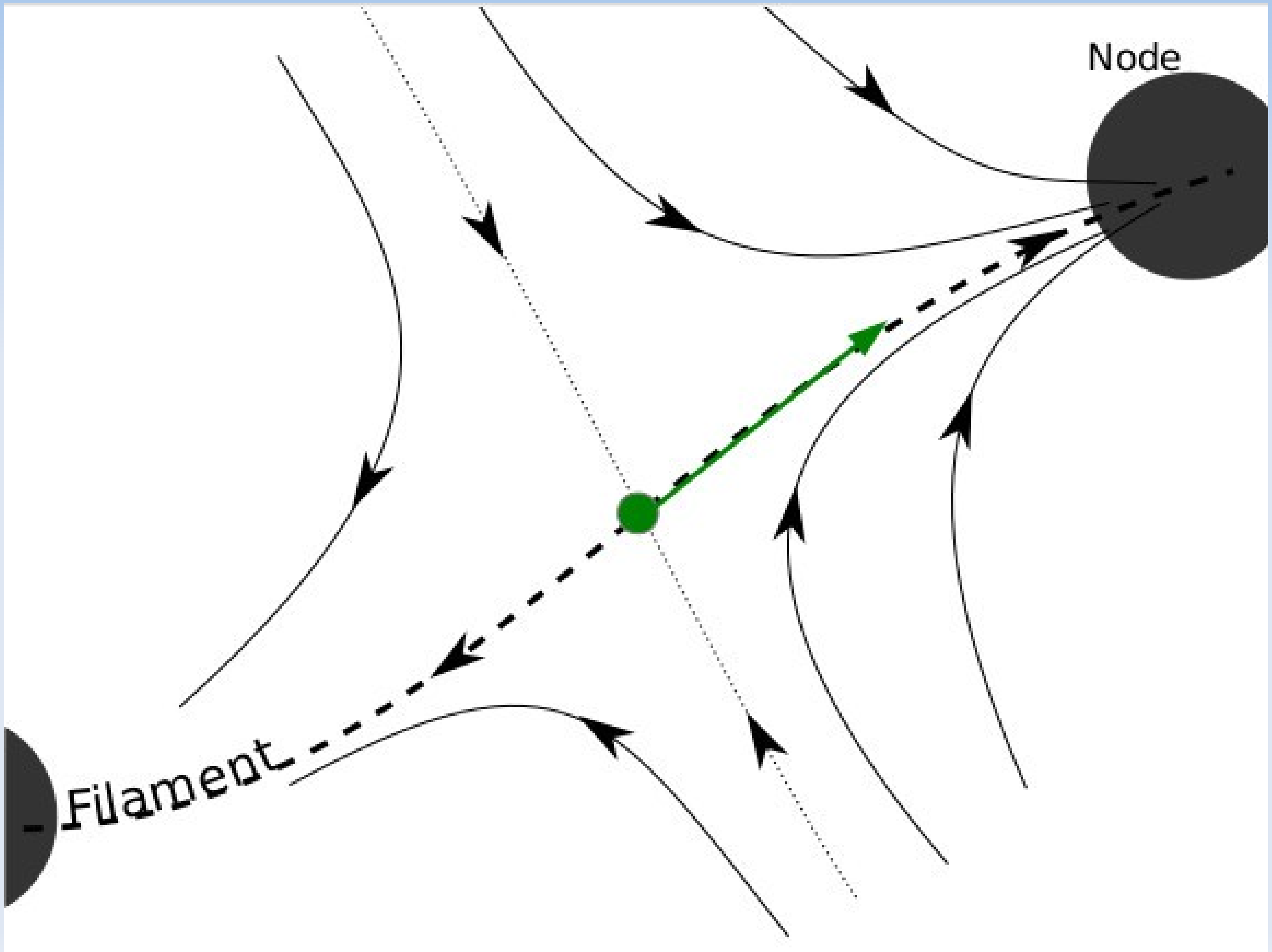
$$D_\mathrm{f} \equiv D(z_\mathrm{f}) = \frac{\delta_c}{\delta(\sigma(M/2))}$$

Accretion rate and formation time



- Same mass at z_1, but σ_A varies less with z : slower accretion. At $\sigma(M/2)$ halo A crosses a higher threshold : forming earlier
- But sharp turns are unlikely: B prefers denser environment than A (not so for uncorrelated steps). Assembly bias! **Dalal et al. (2008)**

Saddle point of the potential



Saddle point of the potential

- Mean potential in sphere of radius R_s

$$\phi_s = -\int \frac{\mathrm{d}^3k}{(2\pi)^3}\,\frac{\delta(\mathbf{k})}{k^2}\,\frac{3j_1(kR_s)}{kR_s}$$

- No center-of-mass motion:

$$g_i \equiv -\nabla_i\phi_s = 0$$

- One neg. eigenvalue of shear:

$$q_{ij} \equiv \frac{\nabla_i\nabla_j\phi_s}{\sigma^2(R_s)} = \frac{\delta_{ij}}{3}\nu_s + \bar{q}_{ij}$$

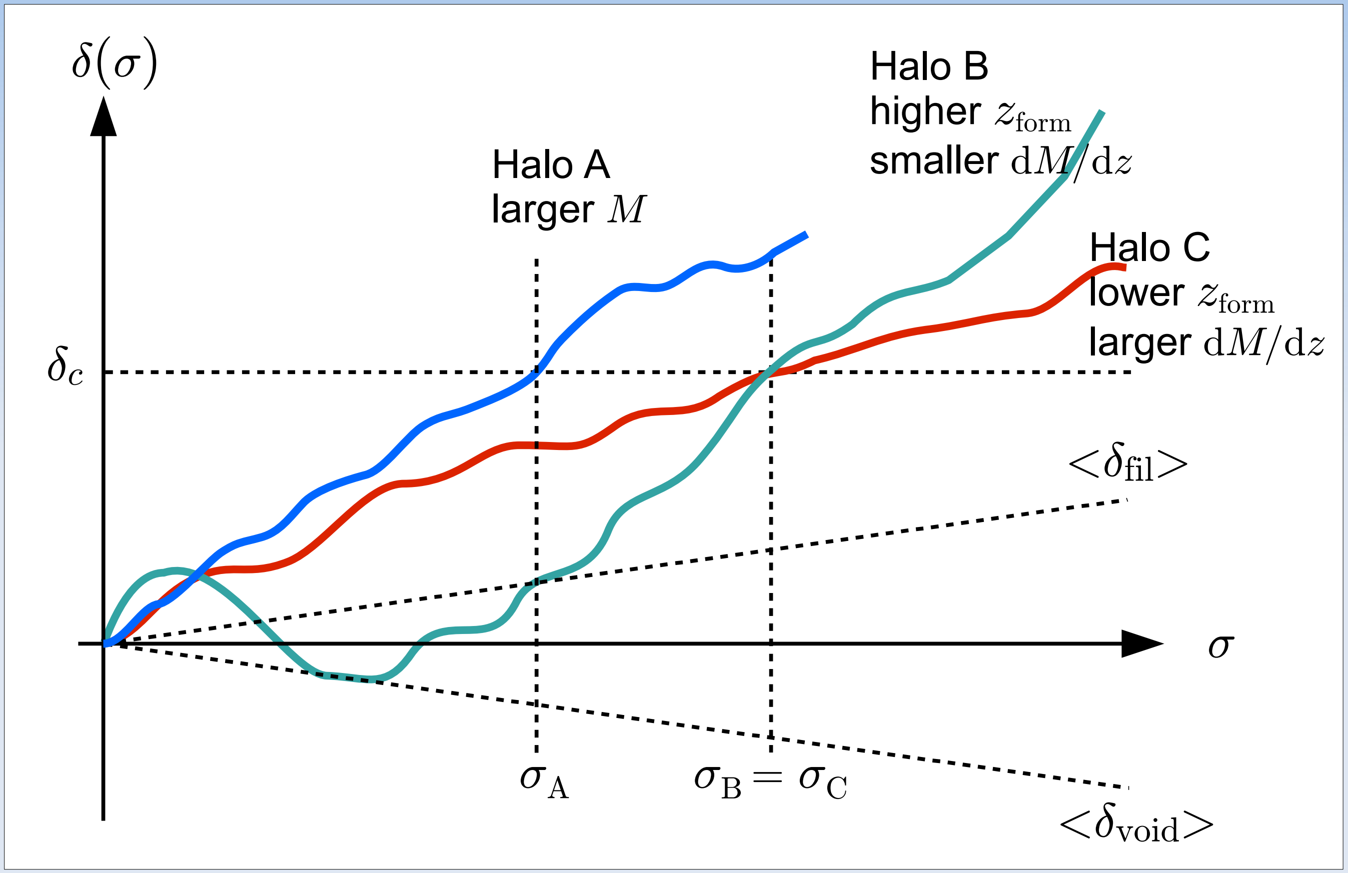
- Anisotropic conditional mean density (at finite distance):

$$\langle\delta(\mathbf{r})|\mathrm{saddle}\rangle = \xi_{00}(r)\nu_s + \cancel{3\xi_{11}(r)\frac{r}{R_\star}\hat{r}_i g_i} - 5\xi_{20}(r)\frac{3\hat{r}_i\bar{q}_{ij}\hat{r}_j}{2}\,,$$

anisotropy

- Saddles of the potential are saddles of the conditional mean of δ. Outflowing direction (filament) has higher mean density.

Saddle point of the potential



- Halo A (filament): large $\langle\delta|S\rangle$, more likely, smaller σ, larger M
- Halo B (void): low $\langle\delta|S\rangle$, less likely, larger σ, smaller M
- Halo C (filament): same σ as B, shallow slope, high accr., late forming

Saddle point of the potential

- Upcrossing probability at σ ($\rightarrow$mass function) given saddle $\mathcal{S}$:

$$f_{\rm up}(\sigma;\mathbf{r}) = p(\delta_c - \langle\delta(\mathbf{r})|\mathcal{S}\rangle)\mu_{\mathbf{r}}F(X_{\mathbf{r}}) \qquad F(x) = \frac{1+\mathrm{erf}(x/\sqrt{2})}{2} + \frac{e^{-x/2}}{x\sqrt{2\pi}}$$

$$\mu_{\mathbf{r}} \equiv \langle\delta'(\mathbf{r})|\delta_c,\mathcal{S}\rangle \qquad X_{\mathbf{r}} \equiv \mu_{\mathbf{r}}/\sqrt{\mathrm{Var}(\delta'(\mathbf{r})|\delta_c,\mathcal{S})}$$

- Conditional probability of α ($\rightarrow$accretion rate) given σ and $\mathcal{S}$:

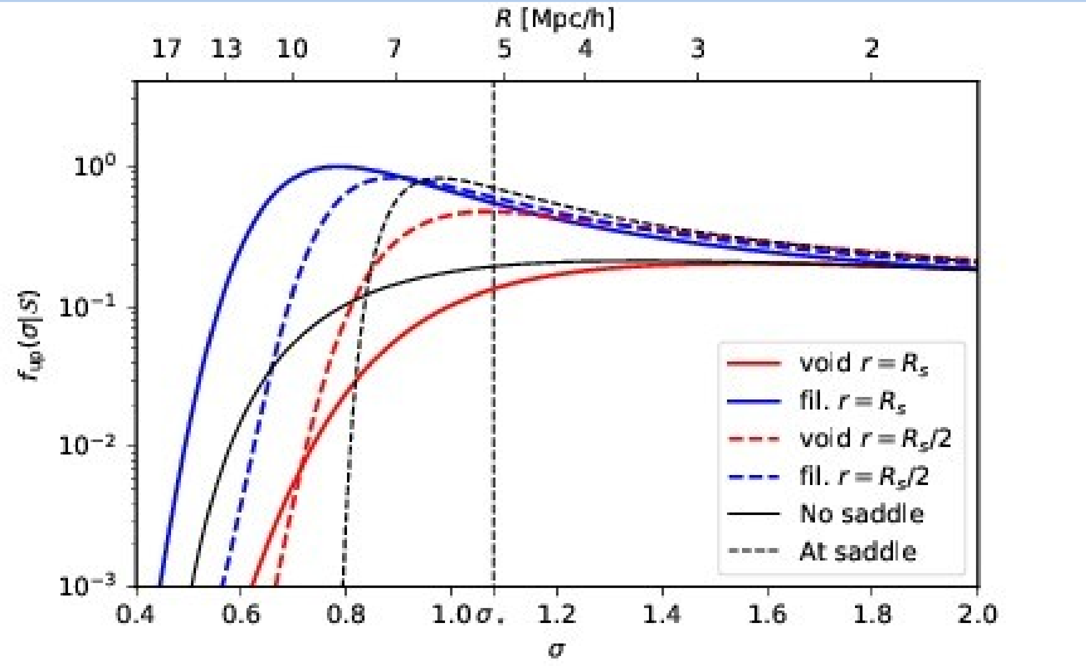
$$f_{\rm up}(\alpha|\sigma;\mathbf{r}) = \frac{\delta_c^2}{\sigma^2\alpha^3}\frac{p\left(\delta_c/\sigma\alpha - \mu_{\mathbf{r}}\right)}{\mu_{\mathbf{r}}F(X_{\mathbf{r}})}$$

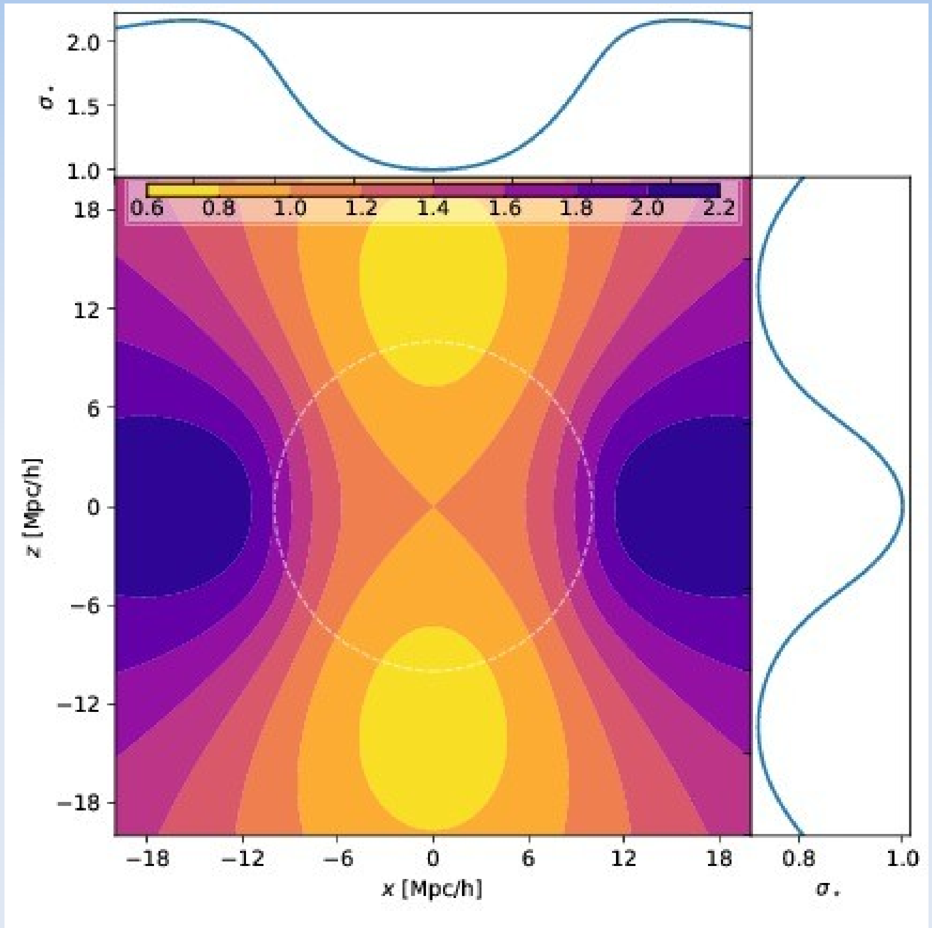
- Conditional probability of $D_{\rm f}$ ($\rightarrow$formation time) given σ and $\mathcal{S}$:

$$f_{\rm up}(D_{\rm f}|\sigma;\mathbf{r}) = \frac{\delta_c}{D_{\rm f}^2}p(\delta_c/D_{\rm f}|\delta_c,\mathcal{S})\,\frac{\mu_{\rm f,\mathbf{r}}F(X_{\rm f,\mathbf{r}})}{\mu_{\mathbf{r}}F(X_{\mathbf{r}})}$$

$$\mu_{\rm f,\mathbf{r}} \equiv \langle\delta'(\mathbf{r})|\delta_c,\delta_c/D_{\rm f},\mathcal{S}\rangle \qquad X_{\rm f,\mathbf{r}} \equiv \mu_{\rm f,\mathbf{r}}/\sqrt{\mathrm{Var}(\delta'(\mathbf{r})|\delta_c,\delta_c/D_{\rm f},\mathcal{S})}$$

Assembly bias



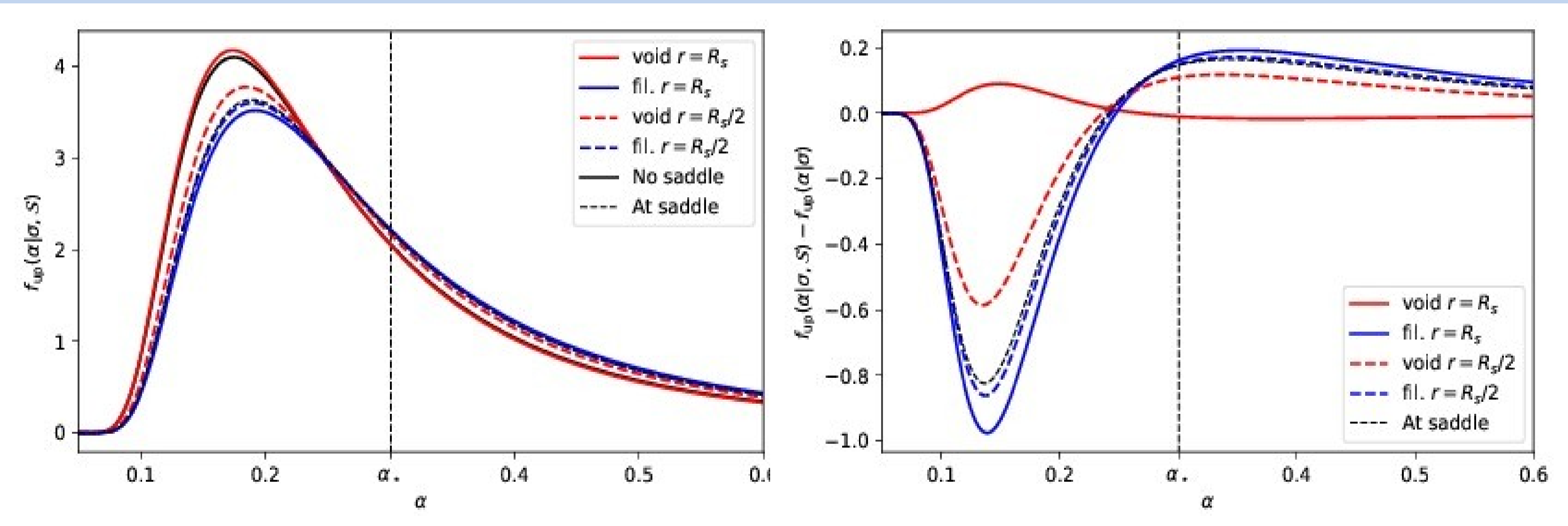


- Saddle point of typical mass too.
- Max of $\sigma_\star$ and min of $M_\star$ along the filament. Receding from nodes, halos are less massive

MM++ (2017)

Assembly bias

$$\frac{\mathrm{d}(\delta-\delta_c)}{\mathrm{d}\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}D}=-\frac{\delta_c}{D^2} \qquad \alpha\equiv-\frac{D}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}D}=\frac{\delta_c}{\sigma(\delta'-\delta_c')D}$$

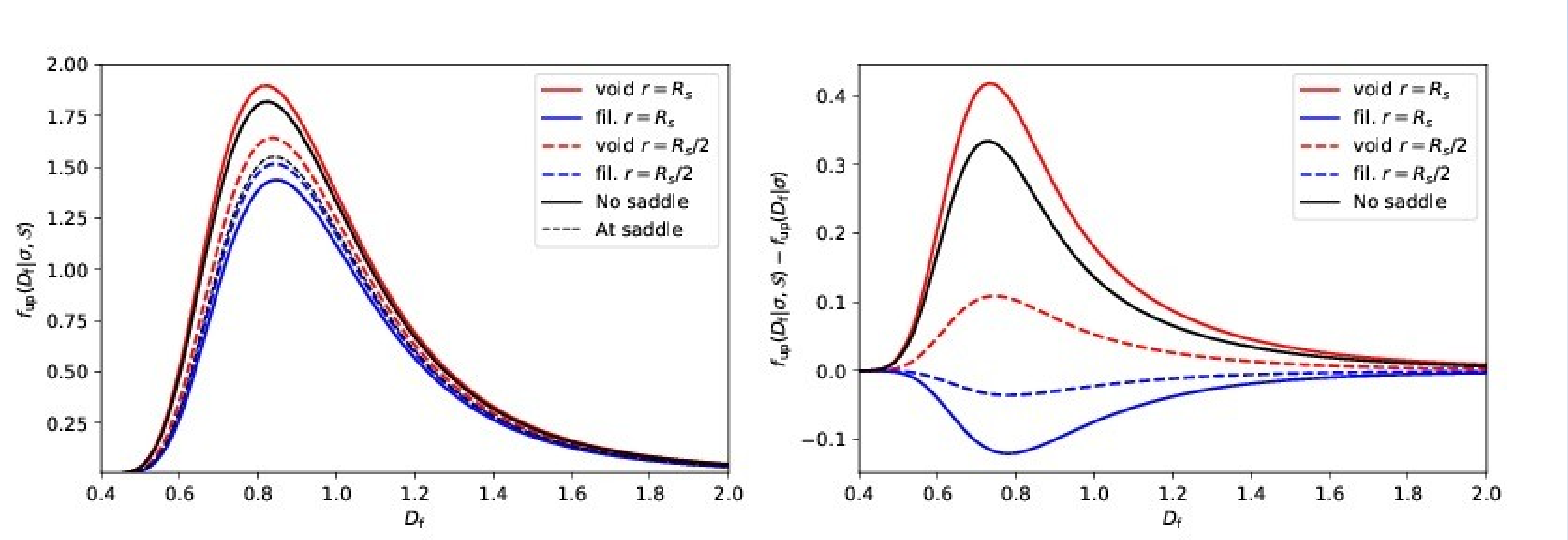


- Slowly accreting halos (small α) are more likely in voids
- Most likely α grows moving to saddle point and then to nodes

MM++ (2017)

Assembly bias

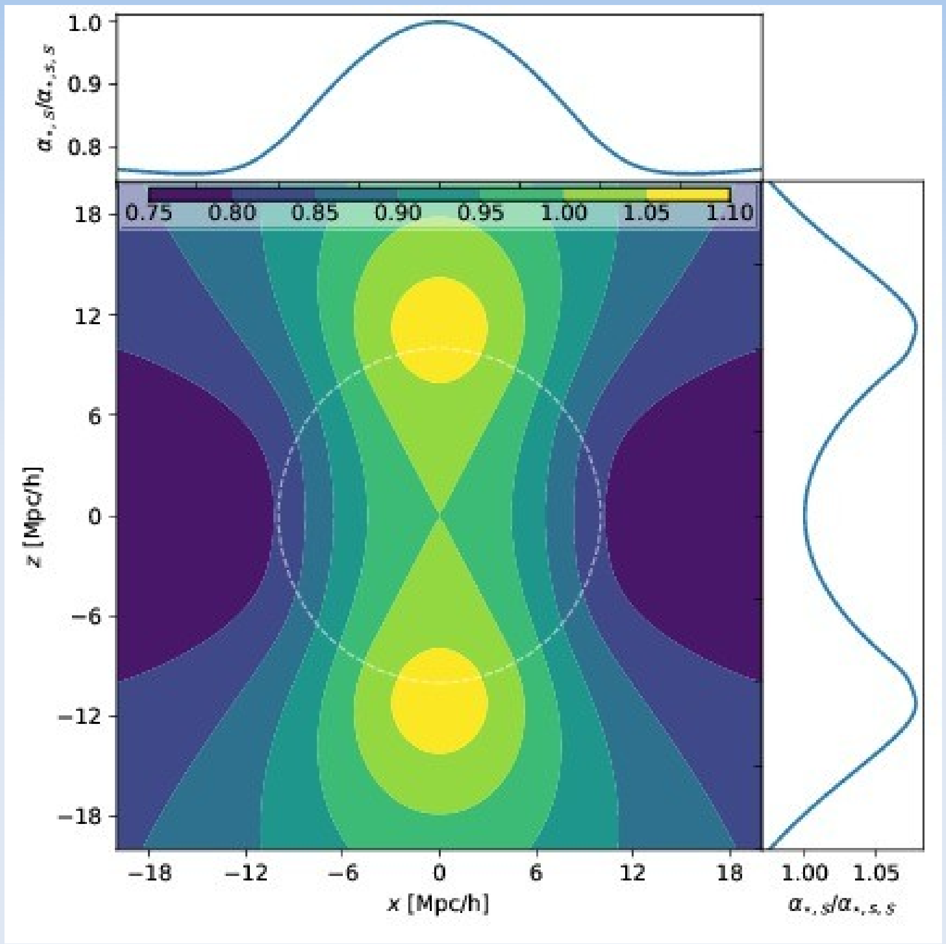
$$\frac{\delta_c}{D_{\rm form}} \equiv \delta(\sigma(M/2))$$



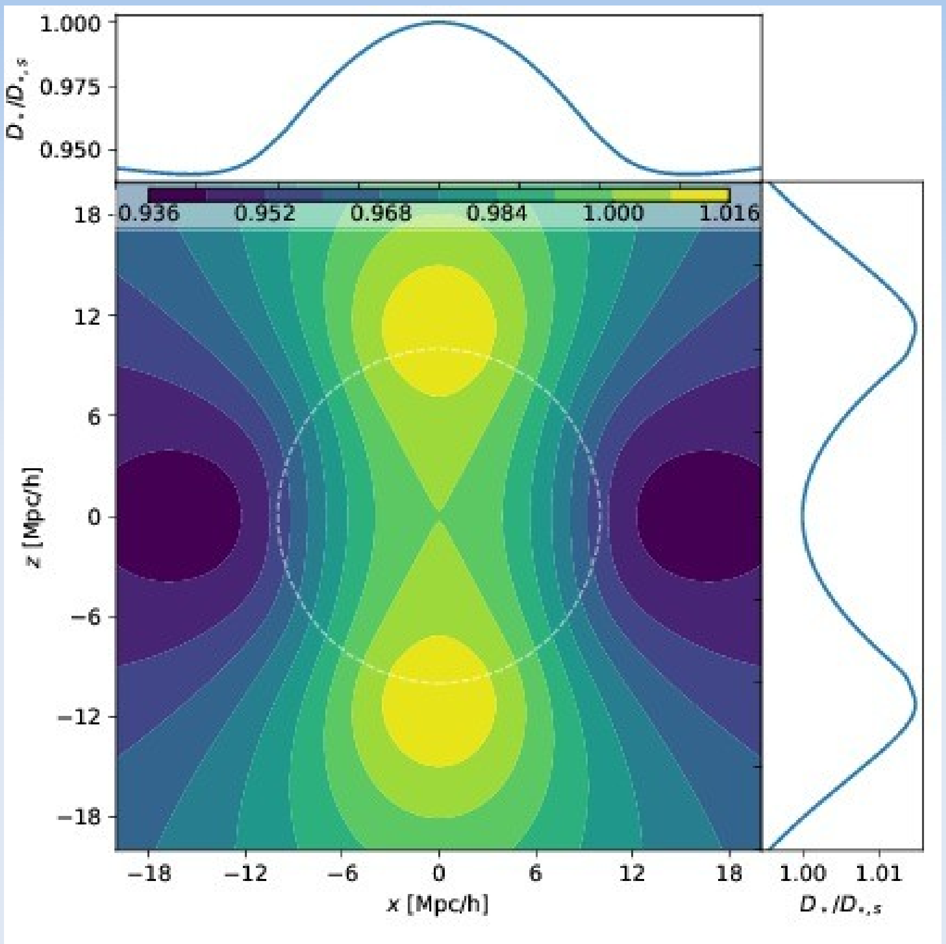
- Early forming halos (small D_f) are more likely in voids
- Most likely D_f grows moving to saddle point and then to nodes

MM++ (2017)

Assembly bias



Typical accretion rate

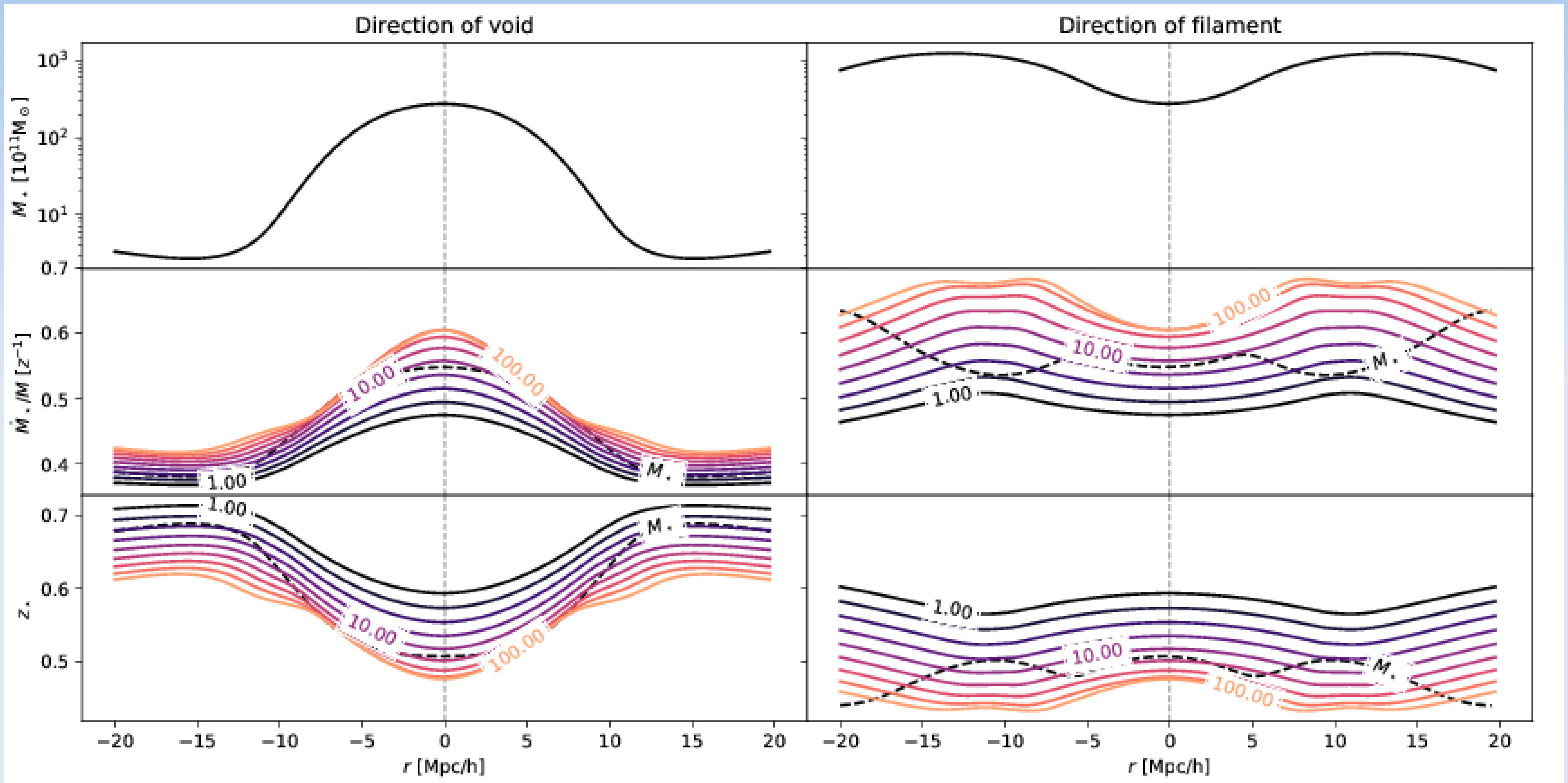


Typical formation time

- Saddle point of $\alpha_\star$ and of $D_\star$. Receding from nodes, halos form earlier and accrete less today. Different level surfaces (and ≠ from mass)

MM++ (2017)

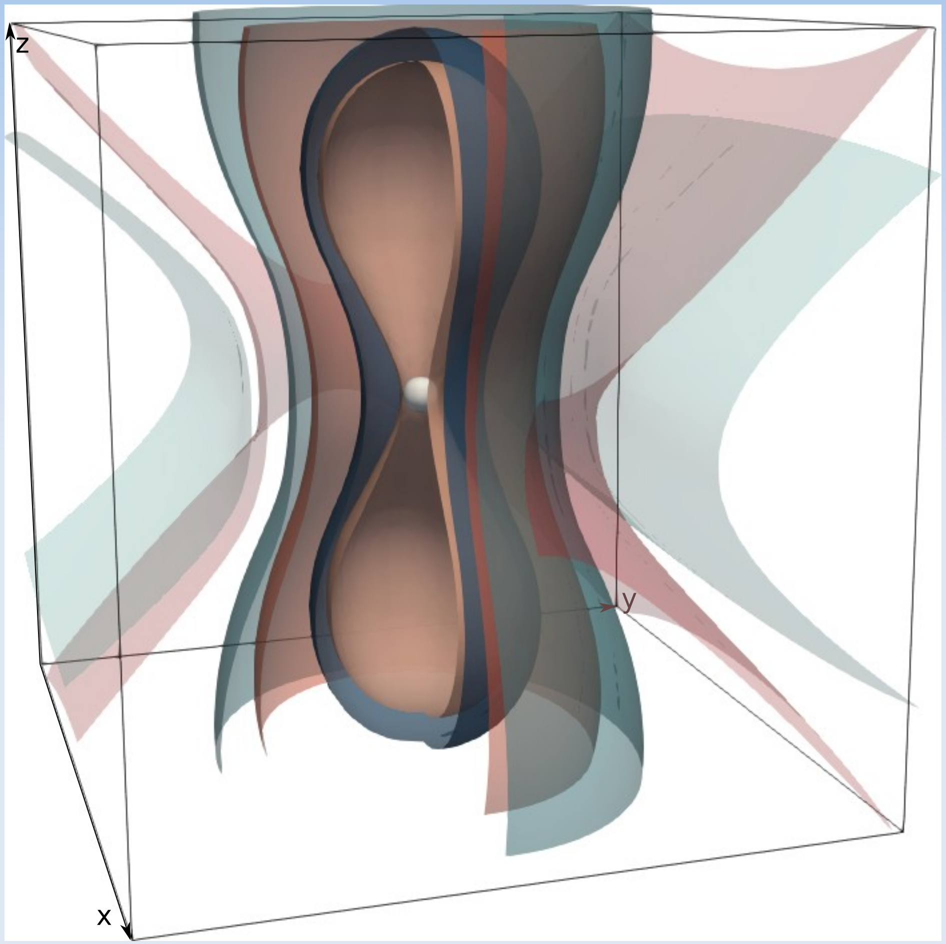
Assembly bias

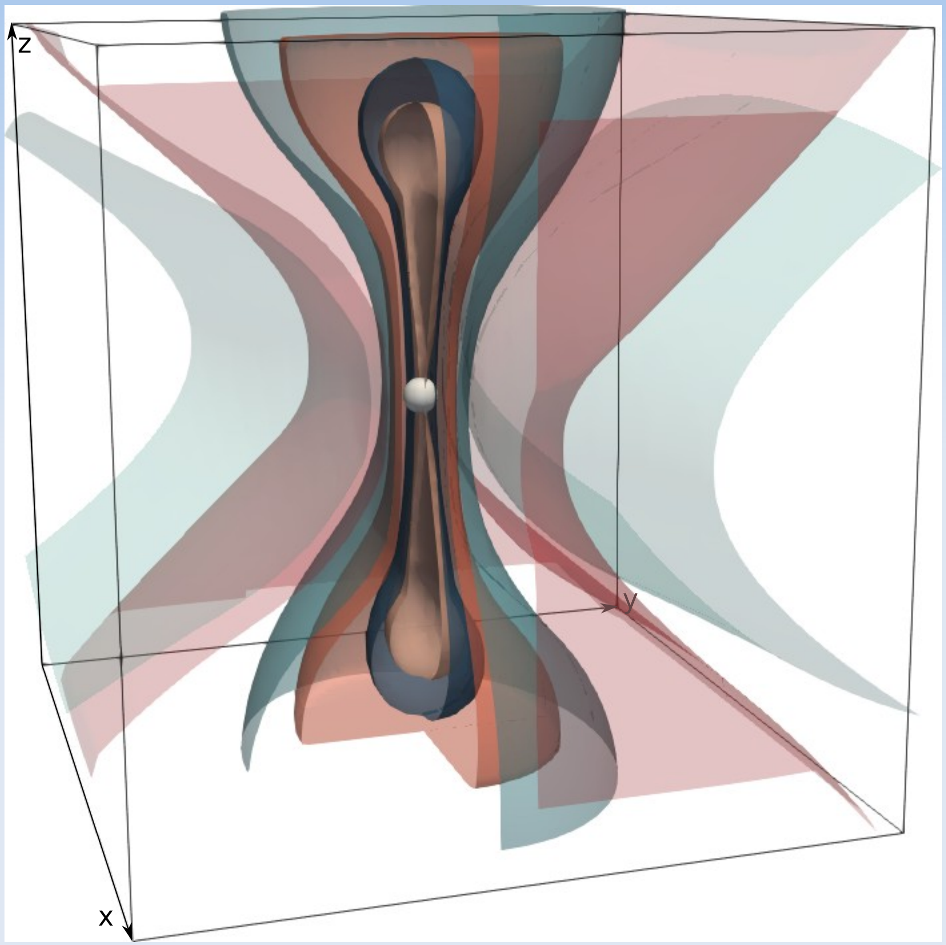


- Typical mass $M_\star$, fixed mass accretion rate $\alpha_\star(M)$ and formation time $D_\star(M)$, along the filament and perpendicularly. Masses in units of $10^{11} M_\odot/h$

MM++ (2017)

Assembly bias

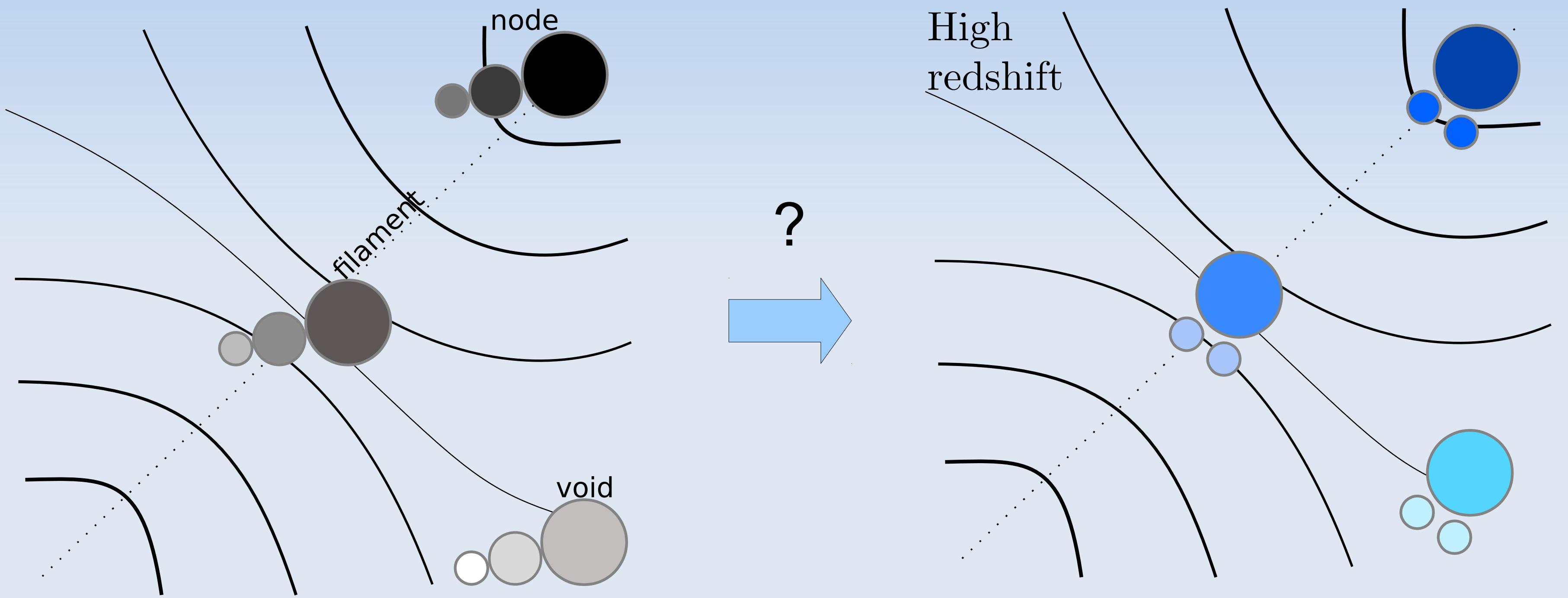




- Level surfaces are initially not aligned, but they get stretched
- More non-linear scales are more aligned (cfr. GAMA, Kralijc et al 2017)

MM++ (2017)

Assembly bias



- Accretion may be related to star formation rate and color. At low redshift, the picture may be reversed by AGN feedback.

MM++ (2017)

Large-scale bias near saddle

- Bias is response to changes in large-scale mean density as usual
- Bias coefficients are derivatives wrt all decorrelated conditional means

$$b_{1,I}(M;\mathbf{r}) = \frac{\partial}{\partial\mu_I}\log\left[f_{\rm up}(\sigma;\mathbf{r})p(\mathrm{saddle})\right]$$

$$\{\mu_I\} = \{\delta_S, \hat{r}_i g_i, \hat{r}_i\bar{q}_{ij}\hat{r}_j, \langle\delta|\mathrm{saddle}\rangle, \langle\delta'|\delta_c, \mathrm{saddle}\rangle\}$$

- Correlation with the large-scale environment δ_0 is

$$\langle\delta_0 n_h(M,\mathbf{r})\rangle = \sum b_I(M;\mathbf{r})C_I$$

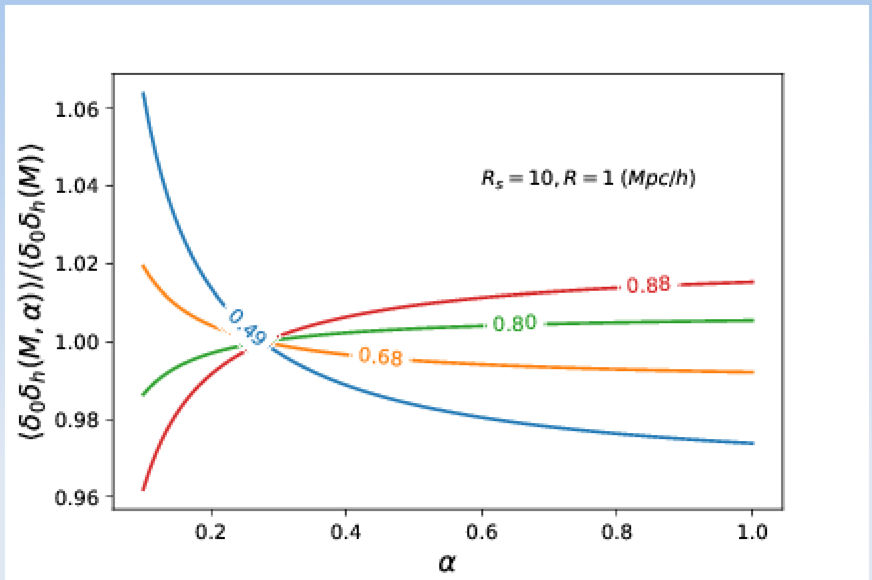
$$\{C_I\} = \{\langle\delta_0\delta_S\rangle, 0, 0, \mathrm{Cov}(\delta_0\delta|\mathrm{saddle}), \mathrm{Cov}(\delta'|\delta_c, \mathrm{saddle})\}$$

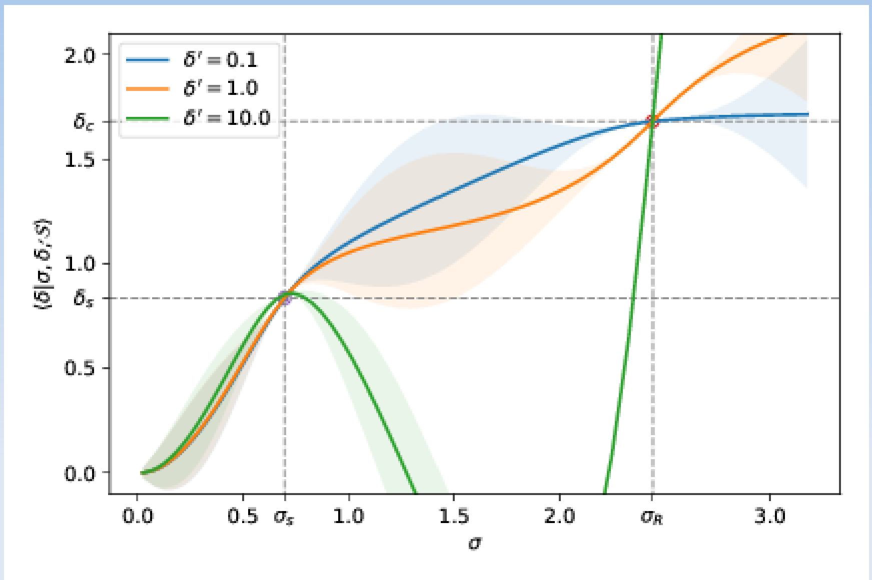
- Same at fixed accretion rate:

$$b_{1,I}(M,\alpha,\mathbf{r}) = \frac{\partial}{\partial\mu_I}\log\left[f_{\rm up}(\sigma,\alpha;\mathbf{r})p(\mathrm{saddle})\right]$$

MM++ (2017)

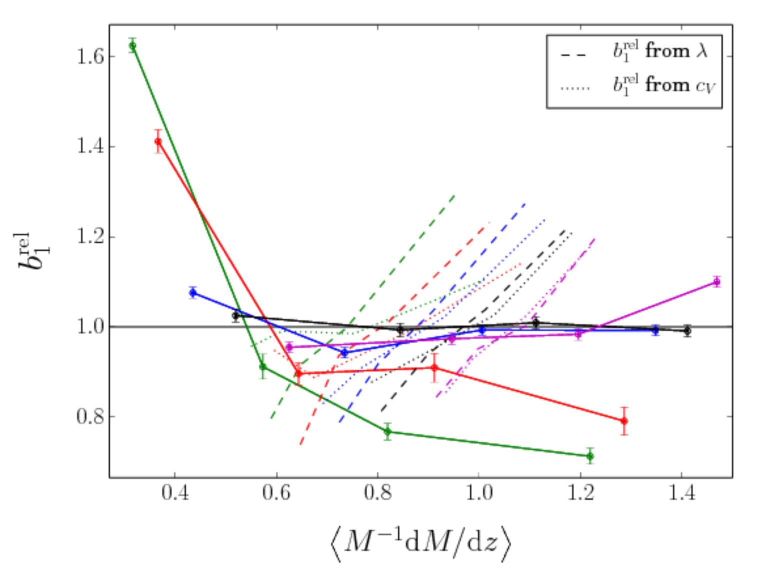
Large-scale bias near saddle





MM++ (2017)

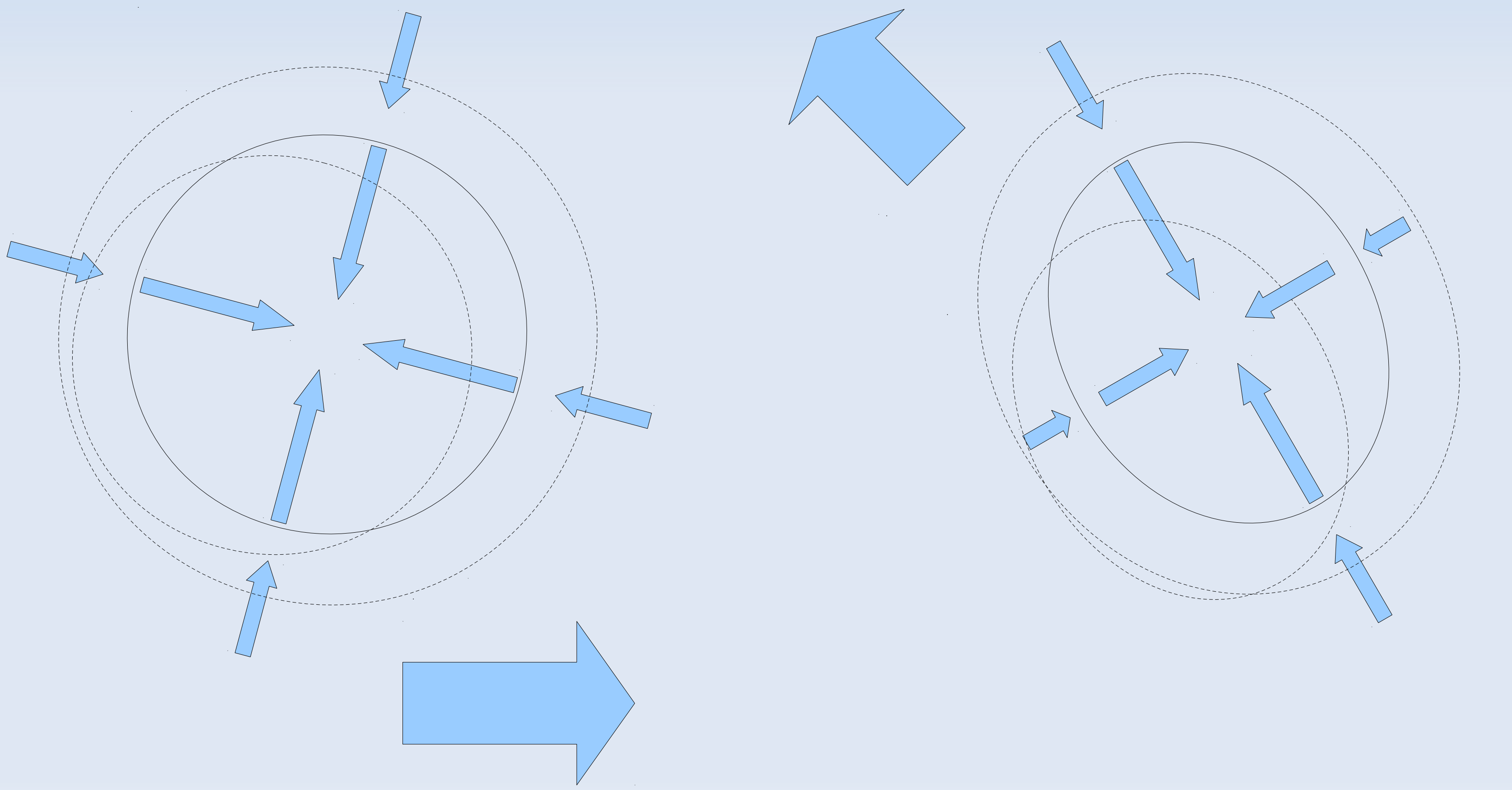
- Near the filament center halos with small accretion rate are more biased, opposite near the nodes
- Consequence of inversion in the constrained excursion set walks
- Same qualitative trend measured in N-body as a function of mass



Lazeyras, MM, Schmidt (2016)

What's next?

Halos as centers of convergence of the velocity field



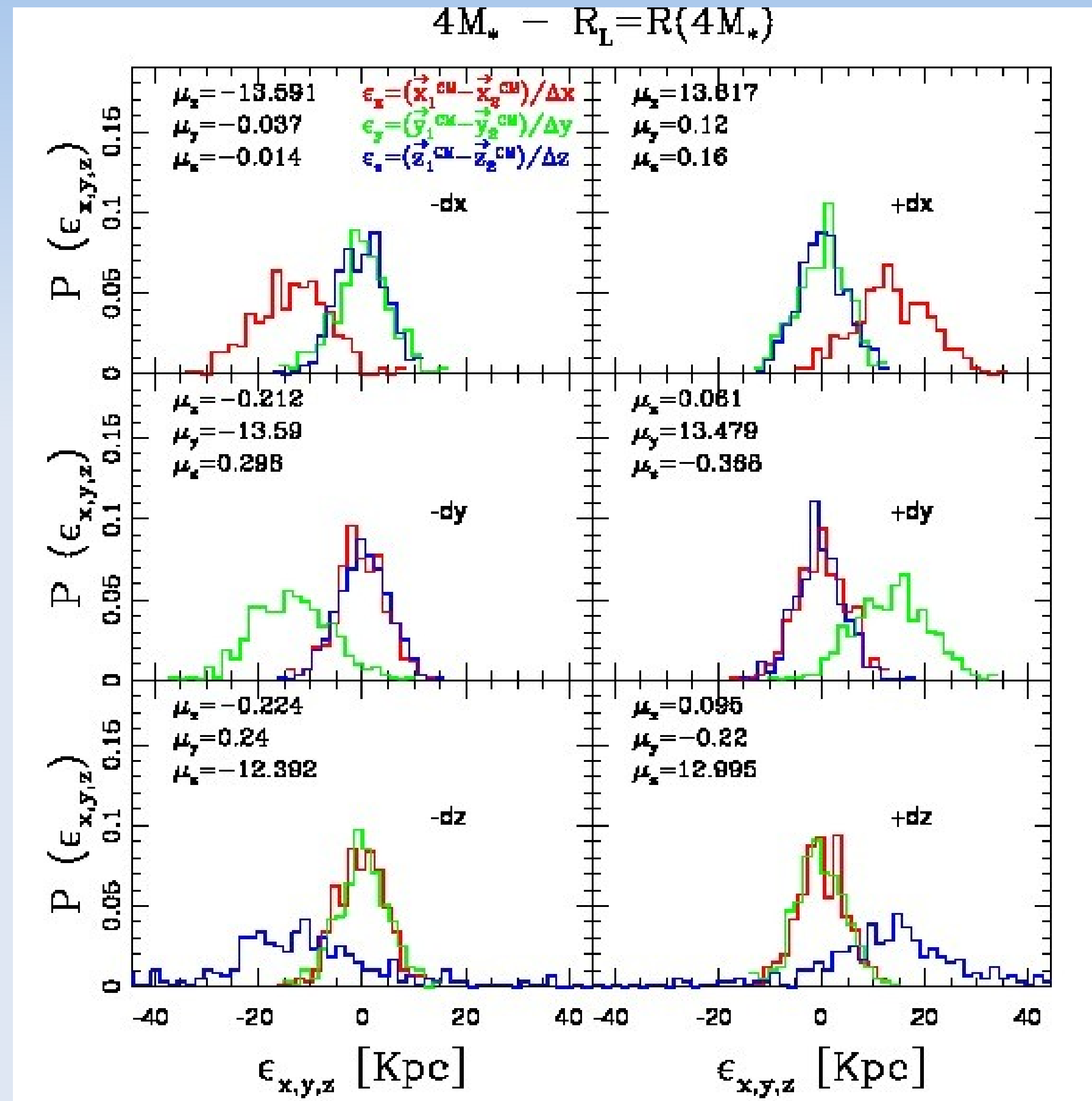
What's next?

- Halos as convergence points of the acceleration field
- Identified by spheres with null dipole moment D_i. That is, set the origin of the coordinates on the center of mass.
- Replace $\nabla_i\delta = 0$ with $D_i = 0$, $\zeta_{ij} = -\nabla_i\nabla_j\delta$ with $-\nabla_i D_j$
- For TH filter: $$\zeta_{ij} \equiv -\nabla_i D_j = \int \frac{\mathrm{d}^3k}{(2\pi)^3} \frac{k_i k_j}{k^2} \delta(\mathbf{k}) \frac{\partial W_{\mathrm{TH}}(kR)}{\partial R}$$
- Describes change of δ as any axis shrinks. Triaxial excursion sets!
- Infall from any direction must decrease with distance: pos def ζ_{ij} , like for peaks
- For a sphere, D_i is the gradient of the binding energy. Halos are minima of the energy!

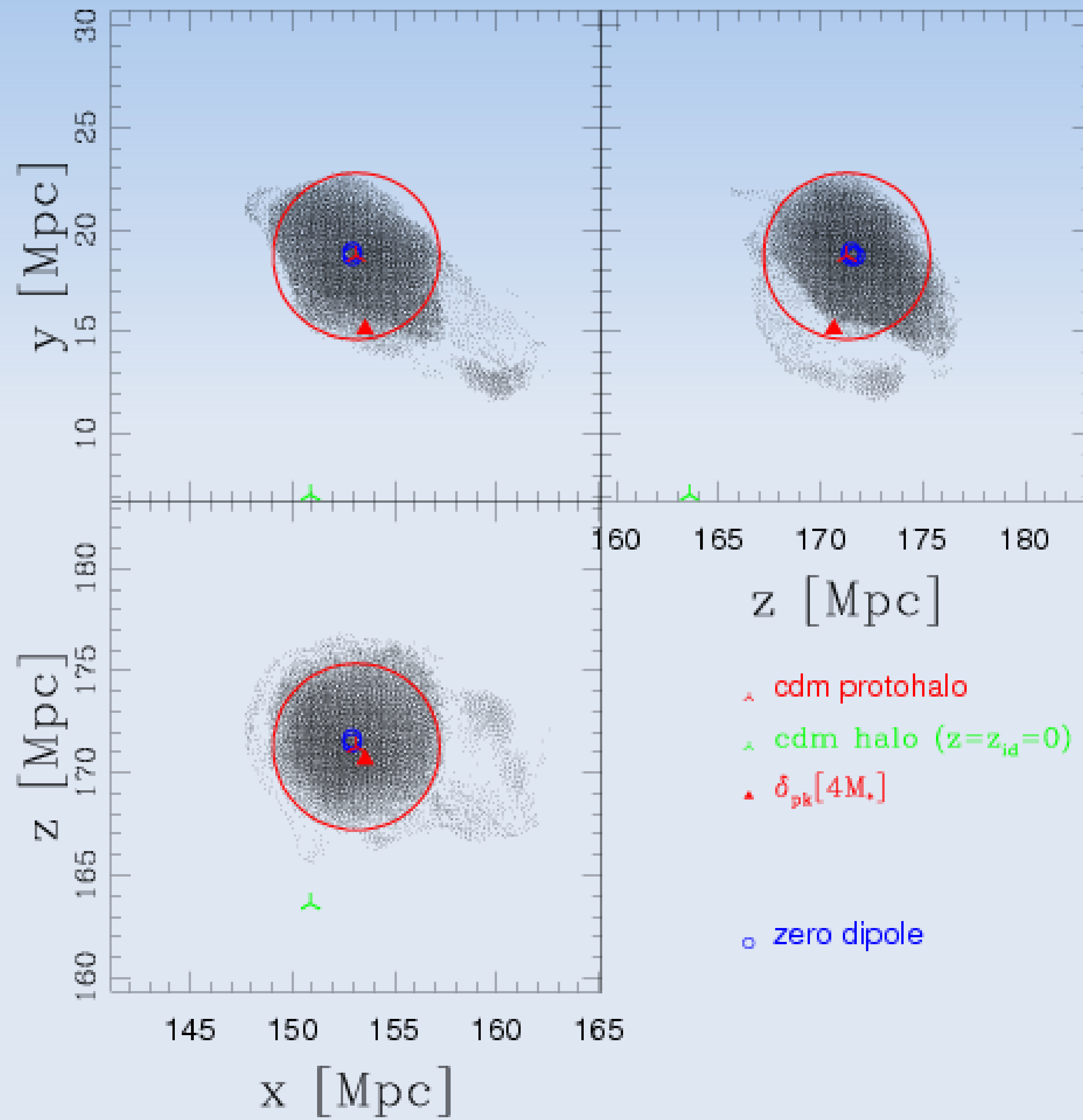
M.Musso (in prep)

What's next?

- The center of mass of a sphere of Lagrangian radius near the center of mass of the protohalo moves in the direction opposite to the displacement
- $D_i = 0$ at the center of mass of the protohalo
- $\nabla_i D_j$ is indeed neg. definite



What's next?



y [Mpc]
z [Mpc]
x [Mpc]
cdm protohalo
cdm halo (z=z$_{id}$=0)
δ_{pk}[4M$_*$]
zero dipole

Conclusions

- Excursion sets can now be done in a mathematically sound way
- They allow to model accretion rates and formation times
- Qualitatively correct prediction of the distribution of secondary halo properties in the cosmic web after conditioning on the proximity to stationary points of the potential
- Saddles define a local metric for the various halo properties. The position in the cosmic web is part and parcel of assembly bias
- Accretion (plus AGN feedback) is a key ingredient to understand galaxy colors. Correlation with angular momentum induced by tidal torques may be used to mitigate the problem of intrinsic alignments
- Need better models with clear dynamical content to improve accuracy and control the errors. Halos as minima of the potential are a very promising candidate

Thanks!!

Large scale bias

- Realistic models involve additional variables, which need not be scalars. For instance, ESP has $\eta_i \equiv \nabla_i\delta$ and $\zeta_{ij} \equiv -\nabla_i\nabla_j\delta$
- Expansion in derivatives of the field, inducing scale dependent bias
- Only rotational invariant combinations η^2, $\mathrm{tr}(\zeta)$, $\mathrm{tr}(\zeta^2)$, $\det(\zeta)$ are relevant. But they are no longer Gaussian variables
- Need to find the appropriate orthogonal basis to expand n_h. This is a suitable, non-trivial combination of Hermite, Laguerre and Legendre

$$H_{ij}(\nu, (\zeta))L_k^{(1/2)}(3\eta^2/2)F_{lm}(\mathrm{tr}(\bar{\zeta}^2), \mathrm{tr}(\bar{\zeta}^3))$$

Lazeyras, MM & Desjacques (2015)

- Can now compute all the scale dependent coefficients, and measure them by cross-correlating halos with these orthogonal polynomials!