Primordial Black Hole, Dark Matter, and LIGO's Gravitational Wave Event

Yuichiro Tada (IAP)

w/ K. Inomata, M. Kawasaki, A. Kusenko,
K. Mukaida, T. T. Yanagida

Refs. 1512.03515, 1606.07631,
1611.06130, 1701.02544

Topic

Primordial Black Hole

- **Dark Matter**
- **binary PBH**
  → LIGO's Gravitational Wave

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Hybrid Inflation

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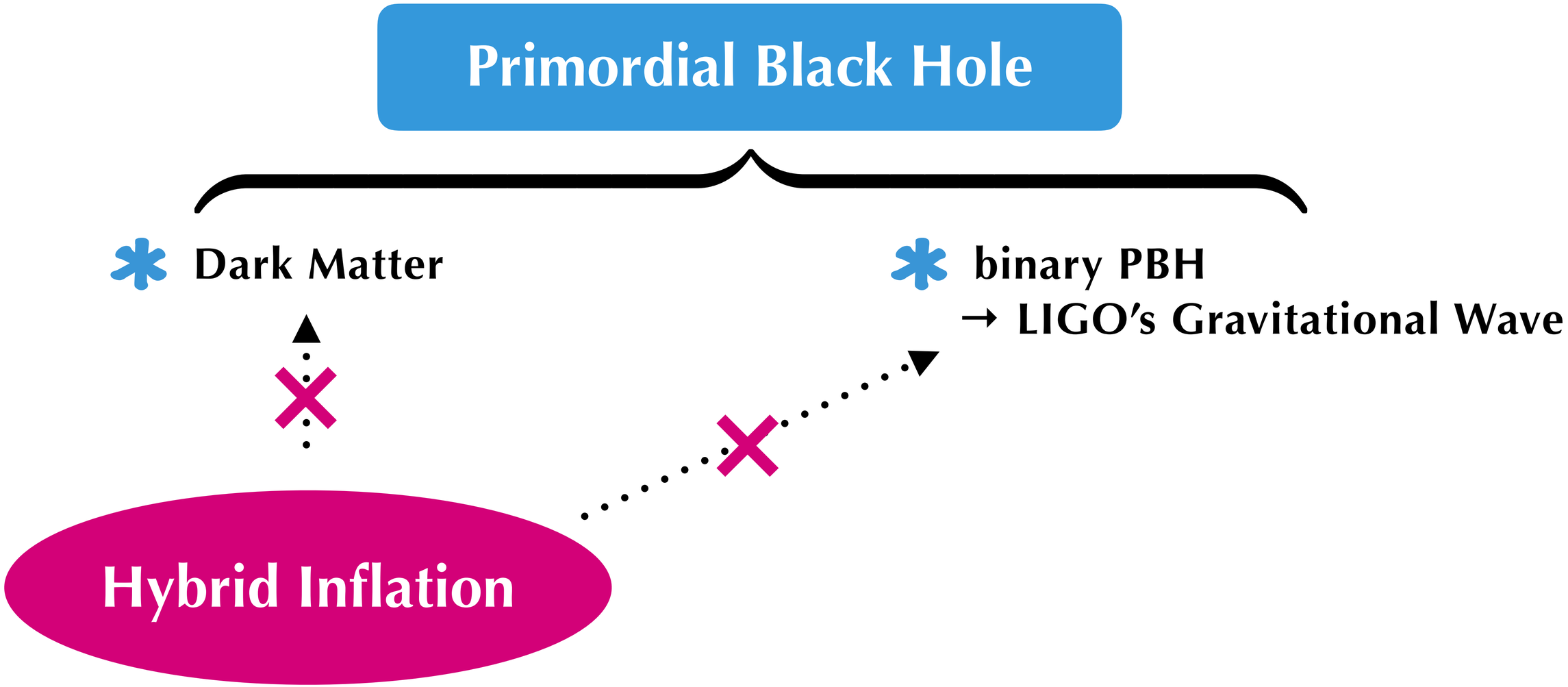
Topic


Primordial Black Hole
Dark Matter
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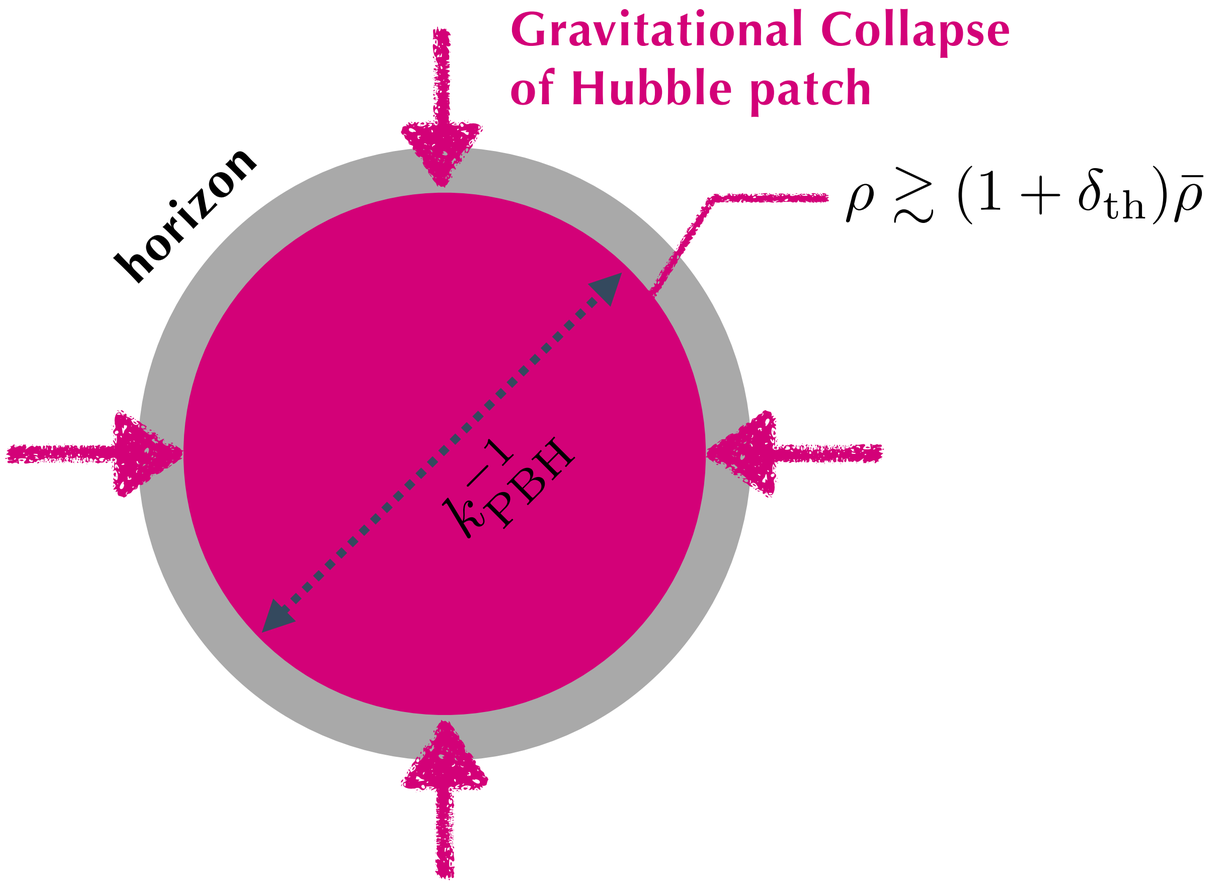
Hybrid Inflation

Double Inflation

Primordial Black Hole

Primordial Black Hole Carr & Hawking 1970s

Radiation Dominated



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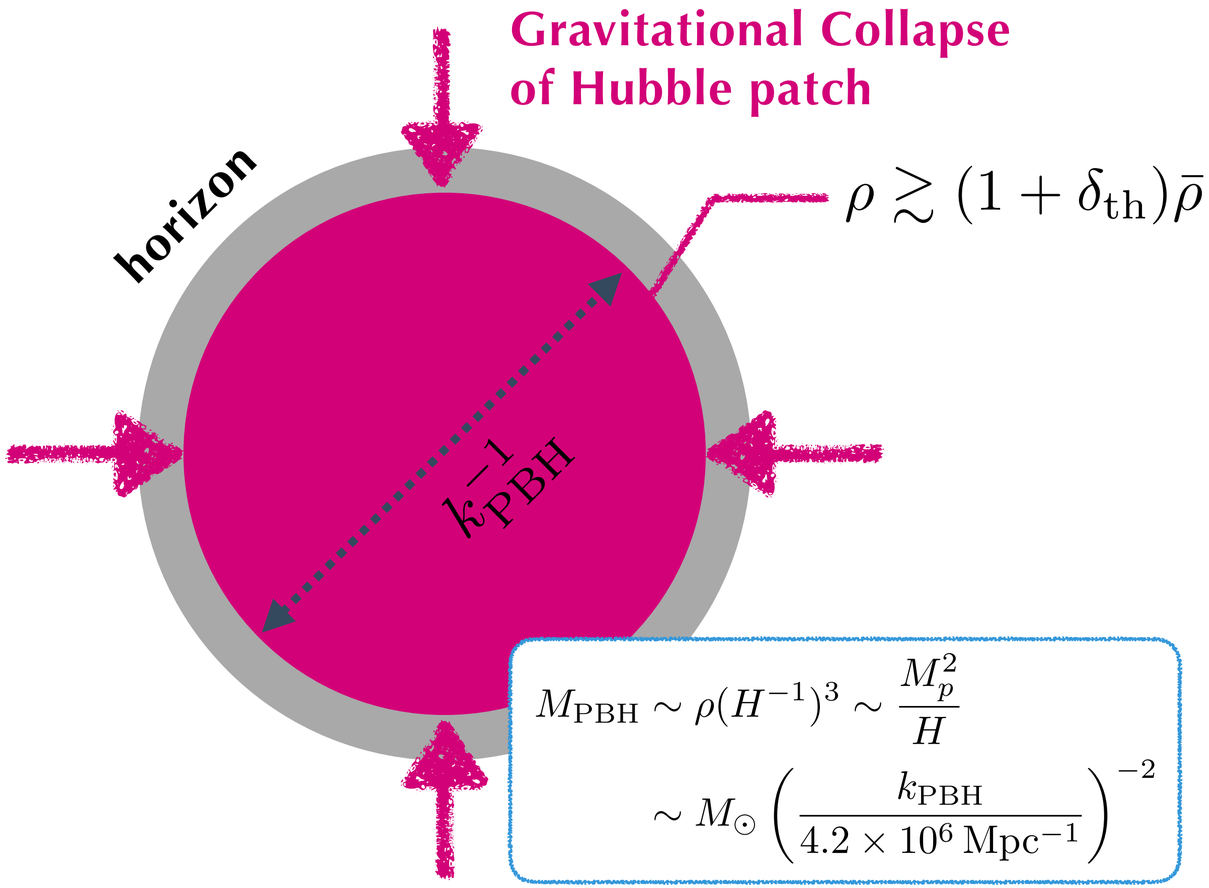

$$M_{\rm PBH} \sim \rho (H^{-1})^3 \sim \frac{M_p^2}{H}$$
$$\sim M_\odot \left(\frac{k_{\rm PBH}}{4.2 \times 10^6\,{\rm Mpc}^{-1}} \right)^{-2}$$

Primordial Black Hole Carr & Hawking 1970s

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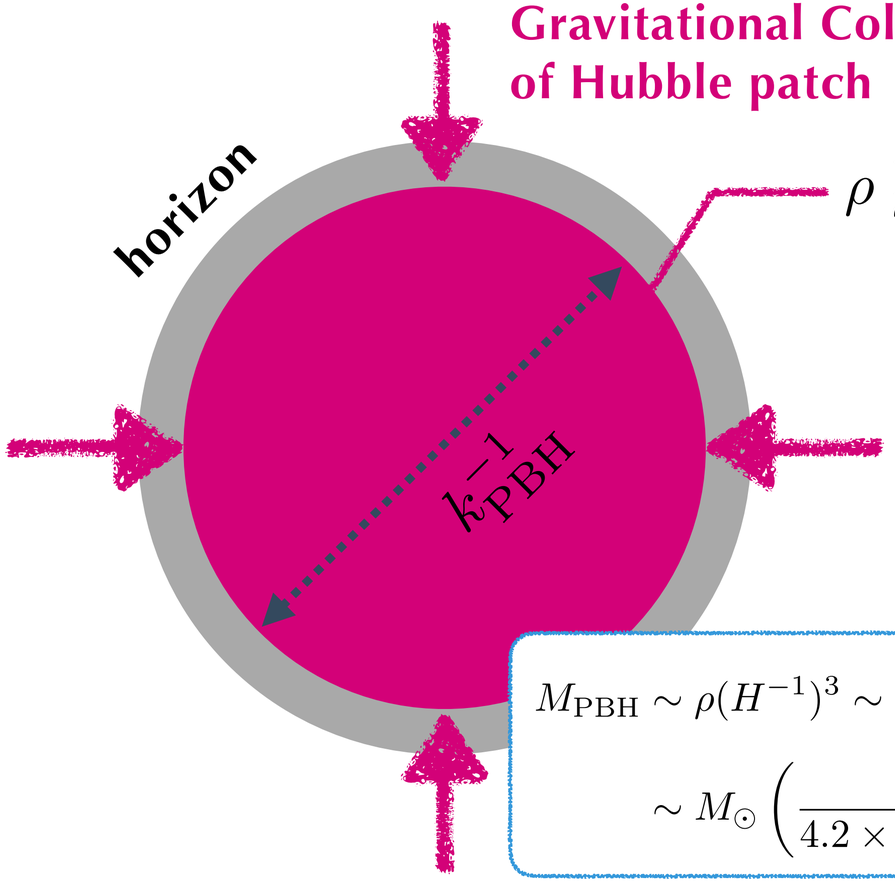

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* comoving gauge density P.

$$\delta_{\rm th} = \begin{cases} w = 1/3 & \text{Carr 1975} \\ 0.7 & \text{Niemeyer \& Jedamzik 1999, Musco et al. 2005} \\ 0.4 & \text{Hawke \& Stewart 2002} \\ \sin^2[\pi\sqrt{w}/(1+3w)] \simeq 0.62 & \text{Harada, Yoo, Kohri 2013} \end{cases}$$

Primordial Black Hole Carr & Hawking 1970s

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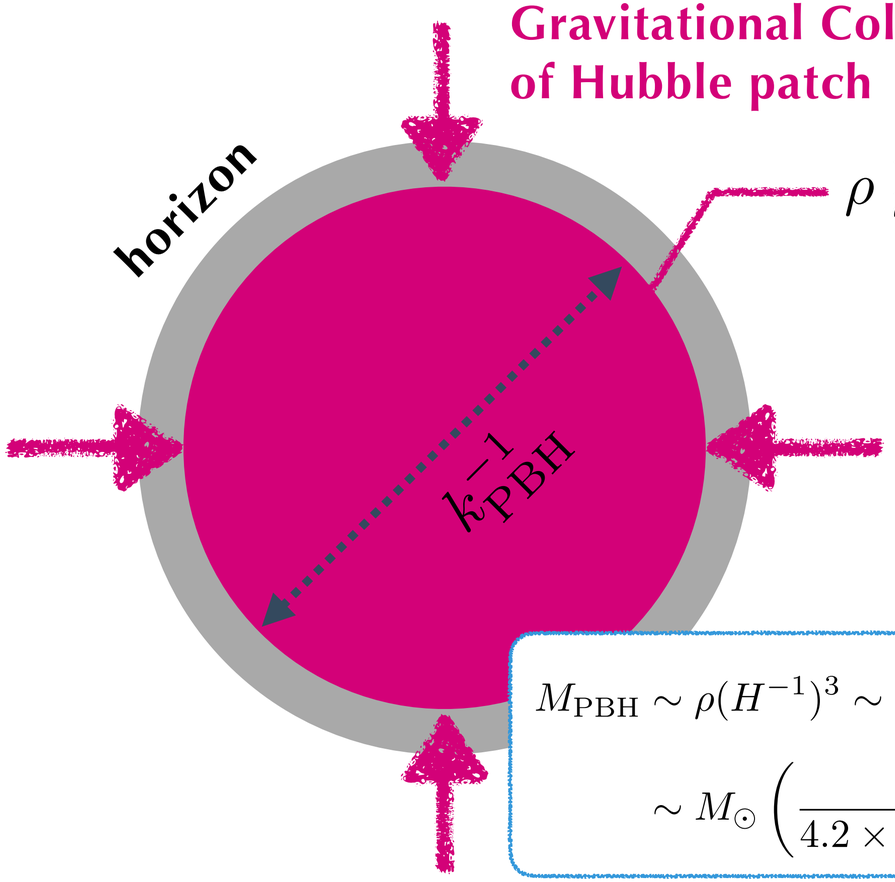

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Harada, Yoo, Kohri 2013

or

* gauge-inv. curvature P.

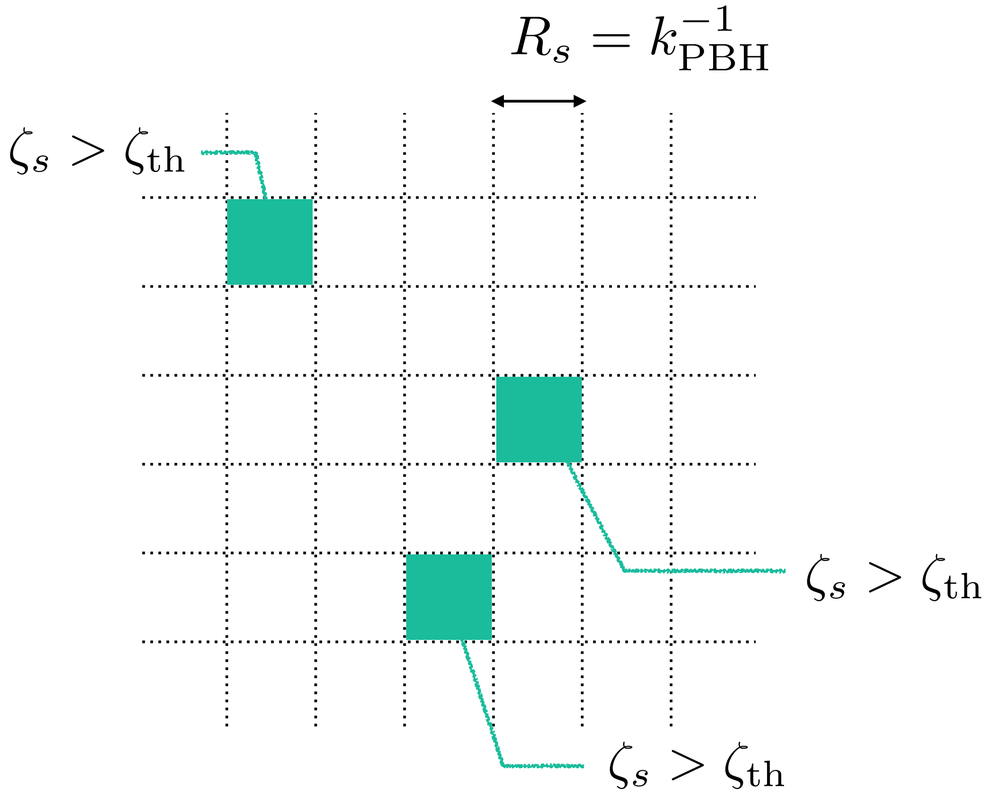
$$\delta = \frac{2(1+w)}{5+3w}\left(\frac{k}{aH}\right)^2 \zeta = \frac{4}{9}\left(\frac{k}{aH}\right)^2 \zeta$$

$$\rightarrow \quad \zeta_{\rm th} \sim \frac{9}{4}\delta_{\rm th} \sim 1$$

Green, Liddle, Malik, Sasaki 2004

PBH abundance

- Press-Schechter 1974



$$\zeta_s(\mathbf{x}) = \int {\rm d}^3 y W_{R_s}(|\mathbf{y}-\mathbf{x}|)\zeta(\mathbf{x})$$

PBH abundance

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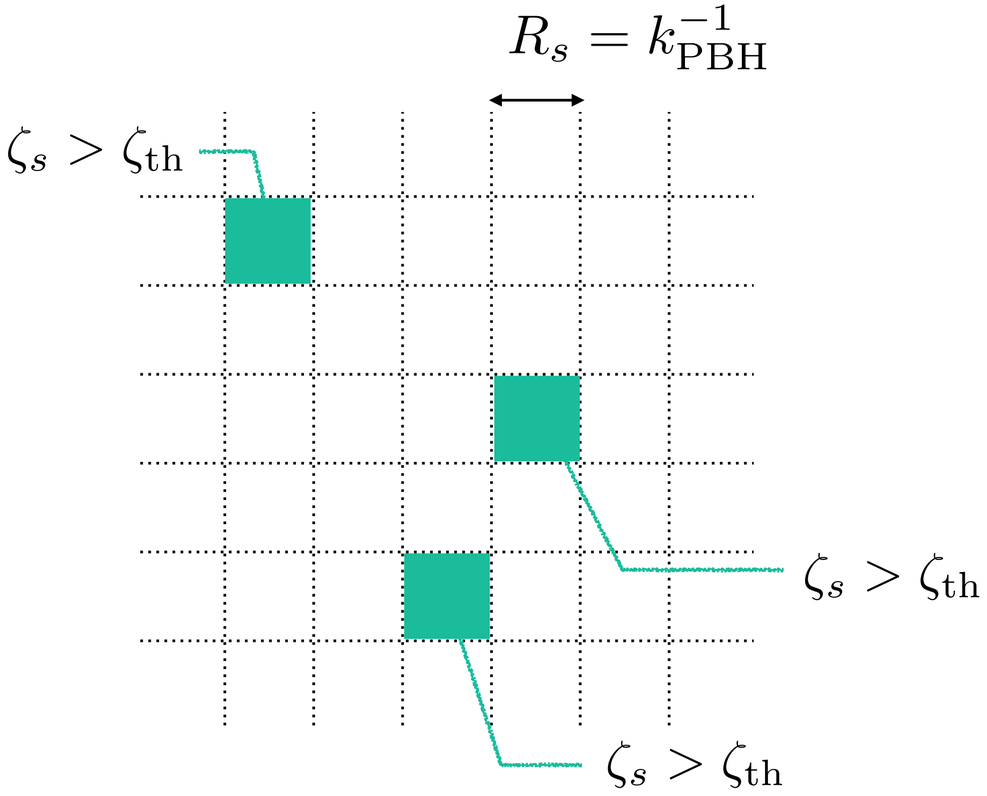

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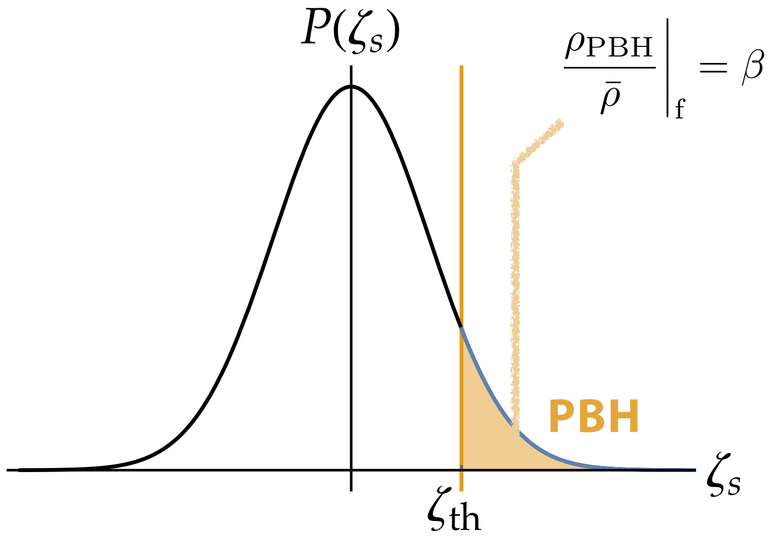
Formation rate ?



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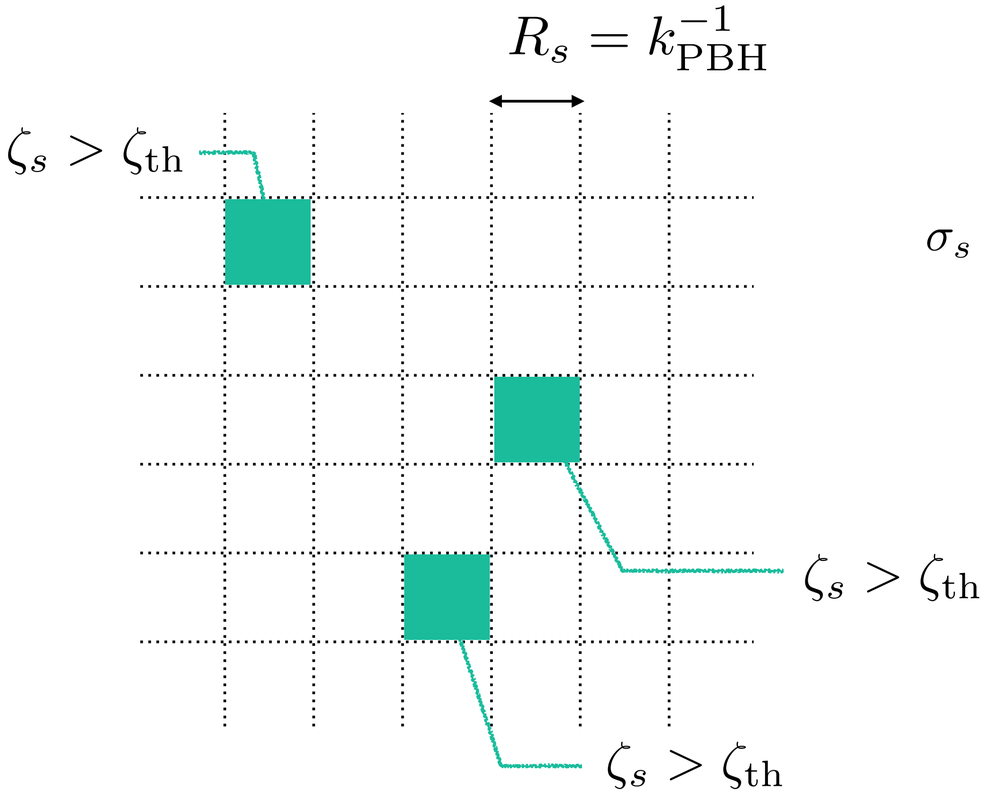

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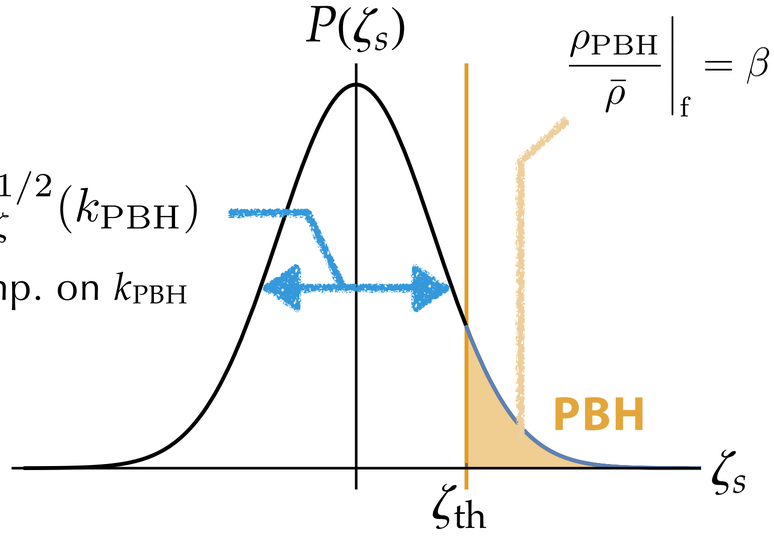

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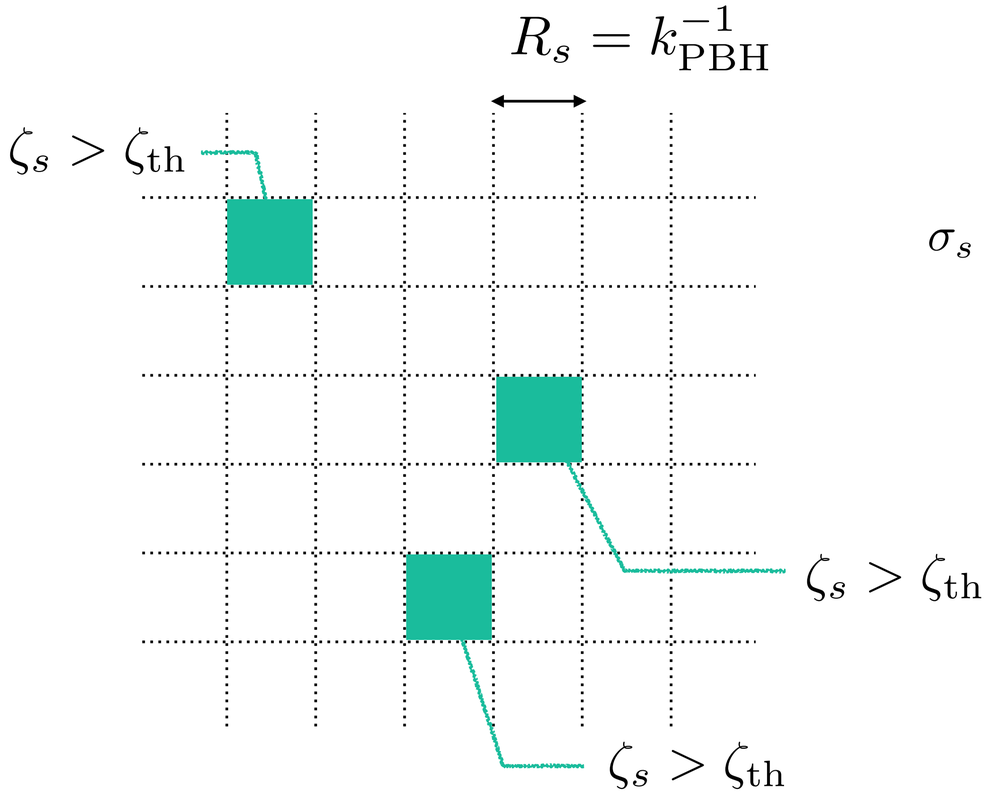

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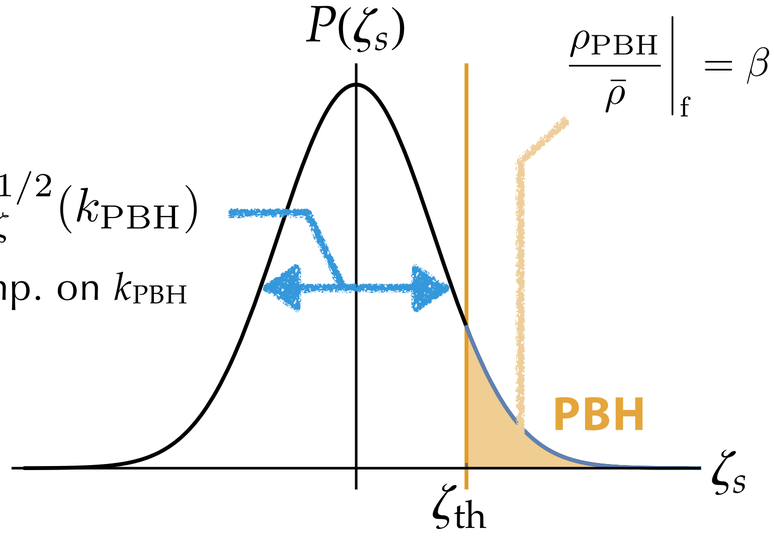

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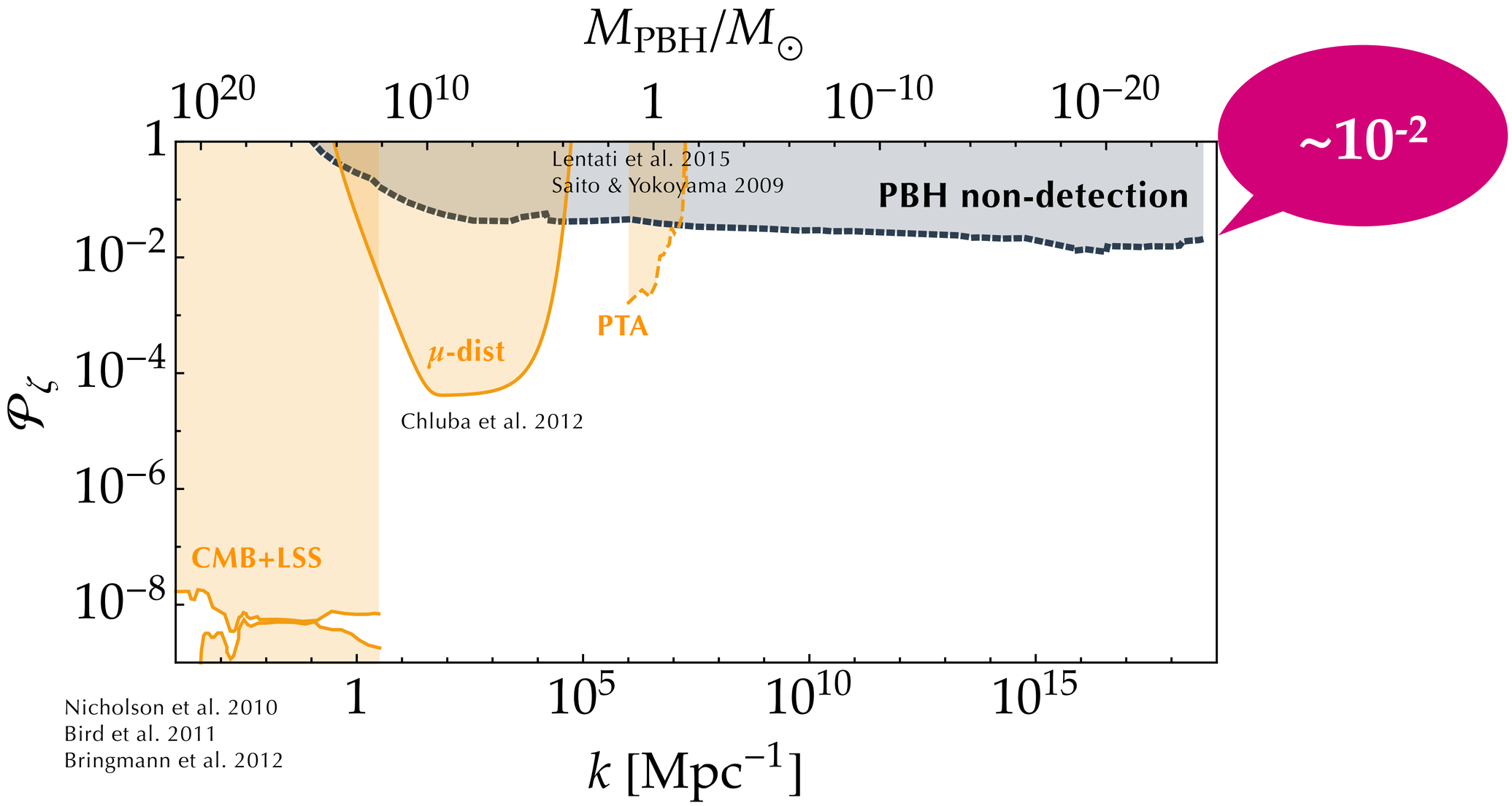
- PBHs should be rarer than about $10\sigma \leftrightarrow \beta \lesssim 10^{-18}$ - 10^{-23}

Carr, Kohri, Sendouda, Yokoyama 2010

$$\mathcal{P}_\zeta(k_{\rm PBH}) \lesssim \left(\frac{1}{10}\zeta_{\rm th}\right)^2 \sim 0.01$$

PBH constraints

PBH as probe for primordial perturbations



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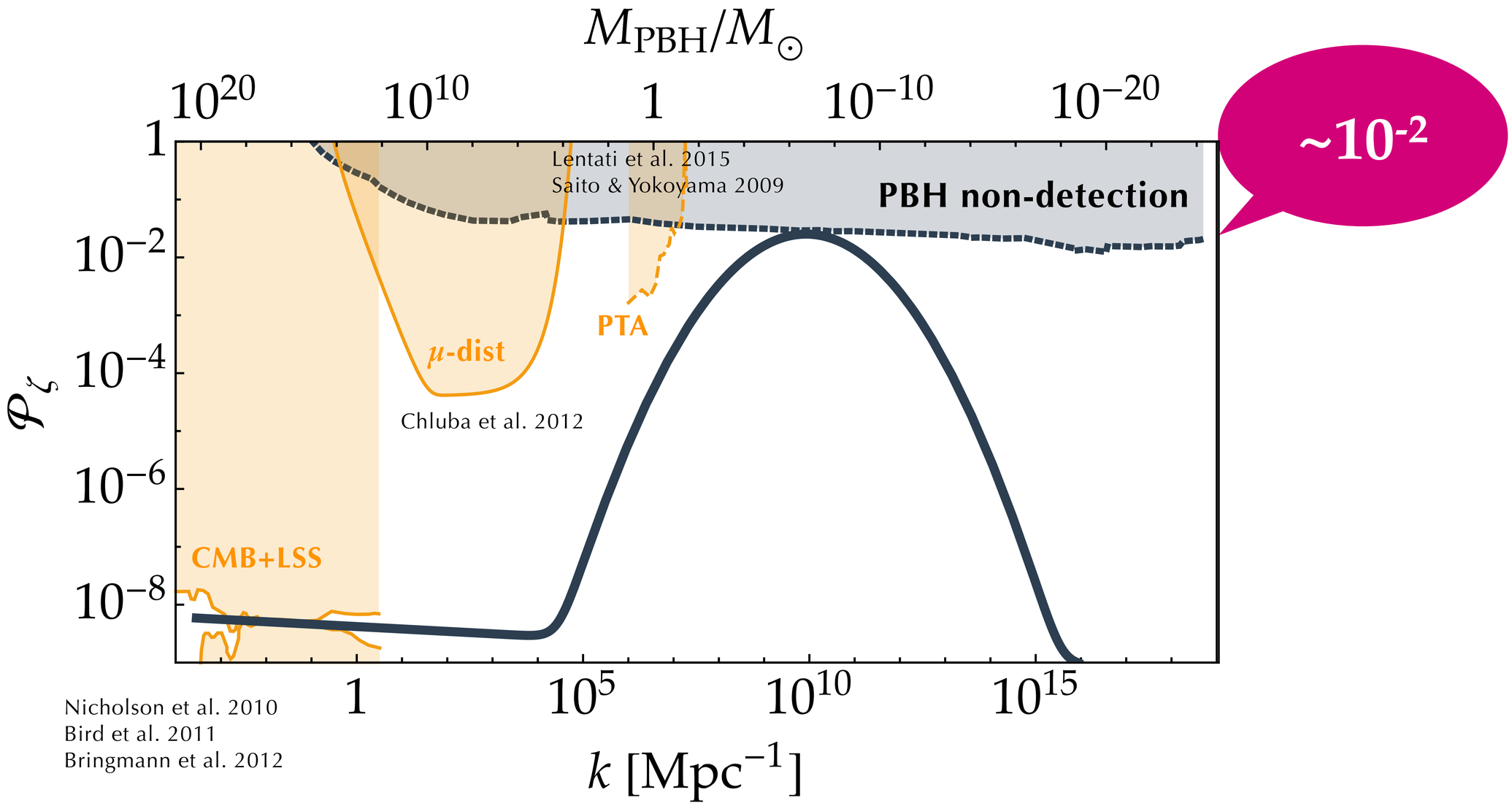
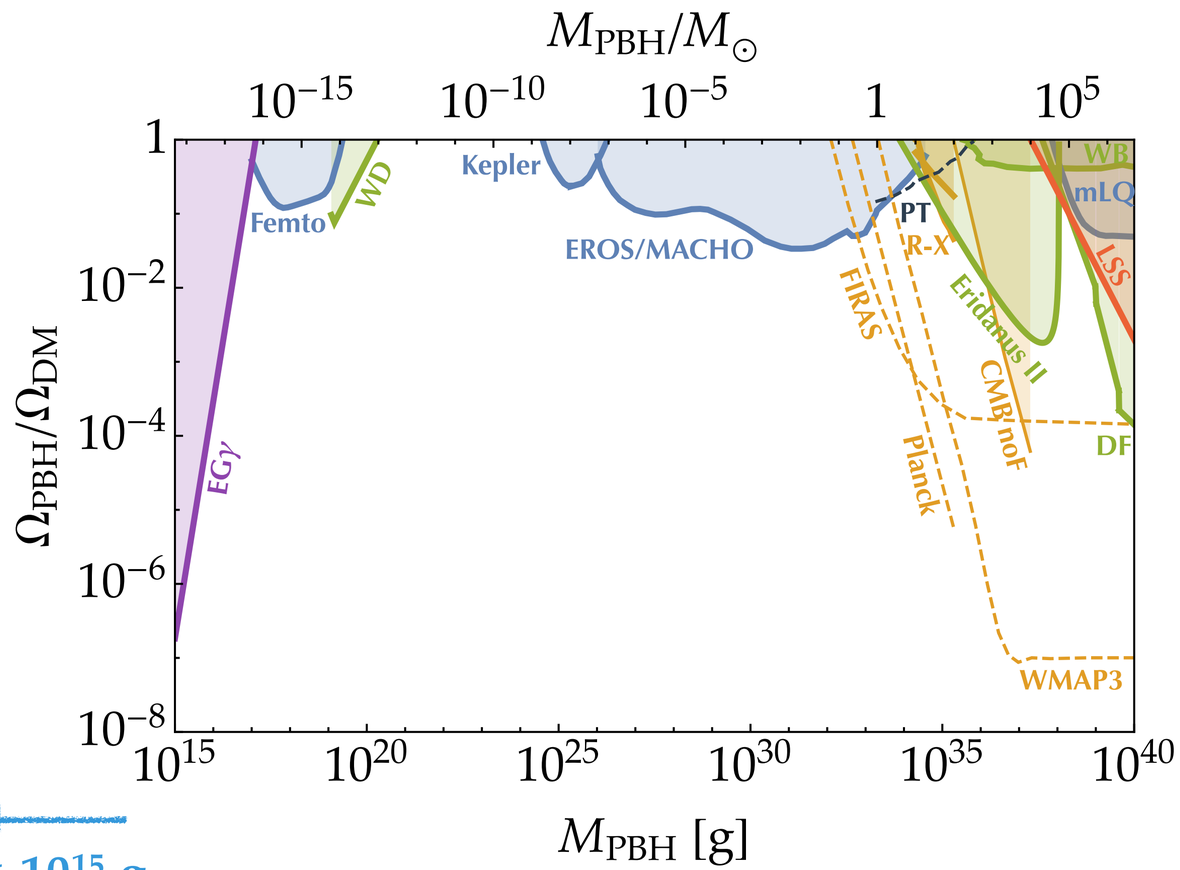


PBH as astrophysical object

Dark Matters



$\lesssim 10^{15}$ g

Hawking evaporated

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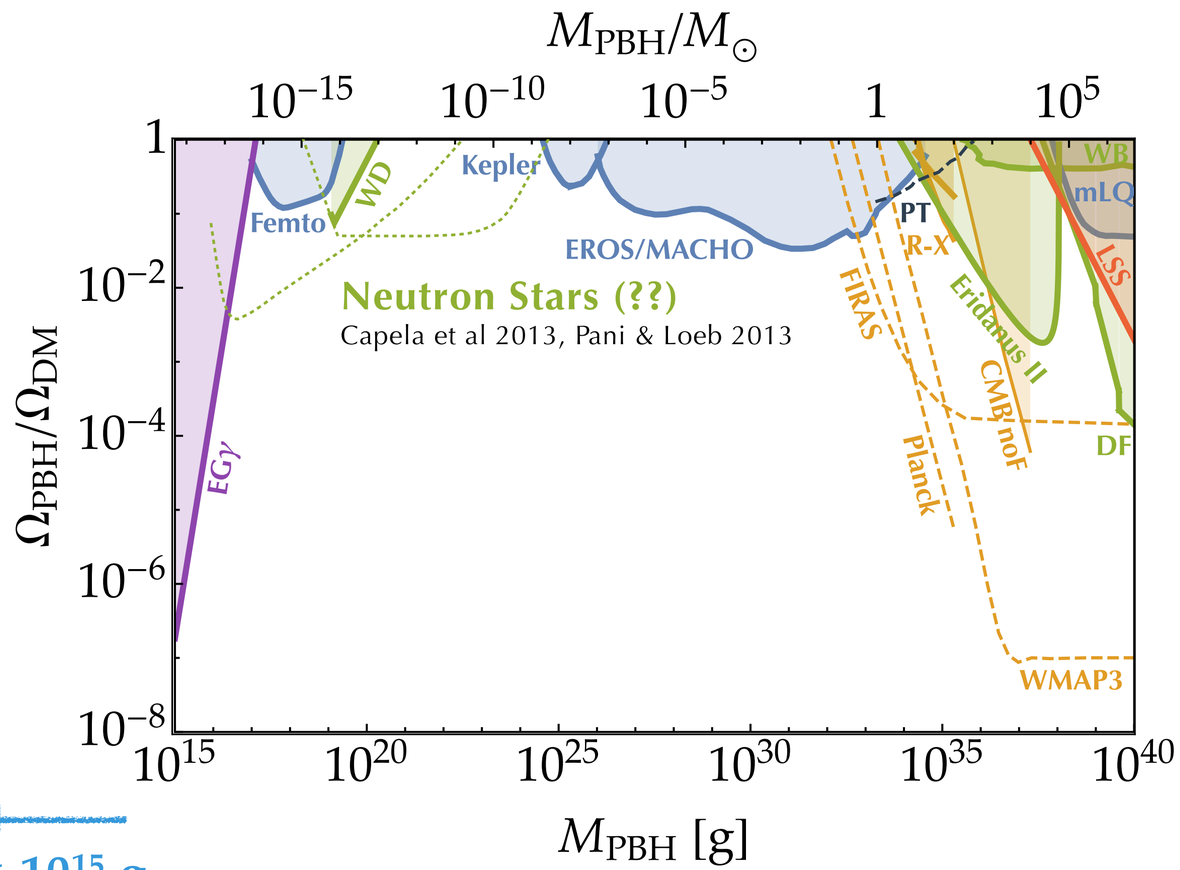

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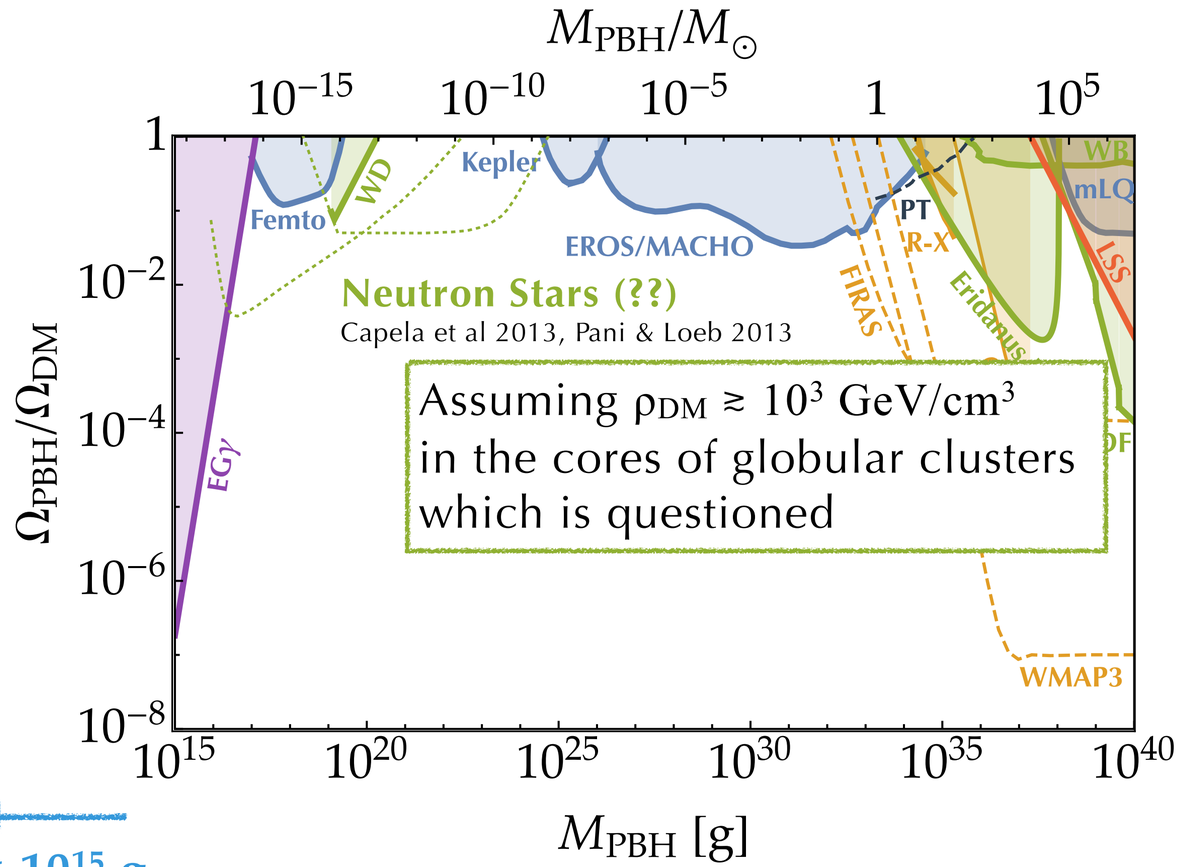

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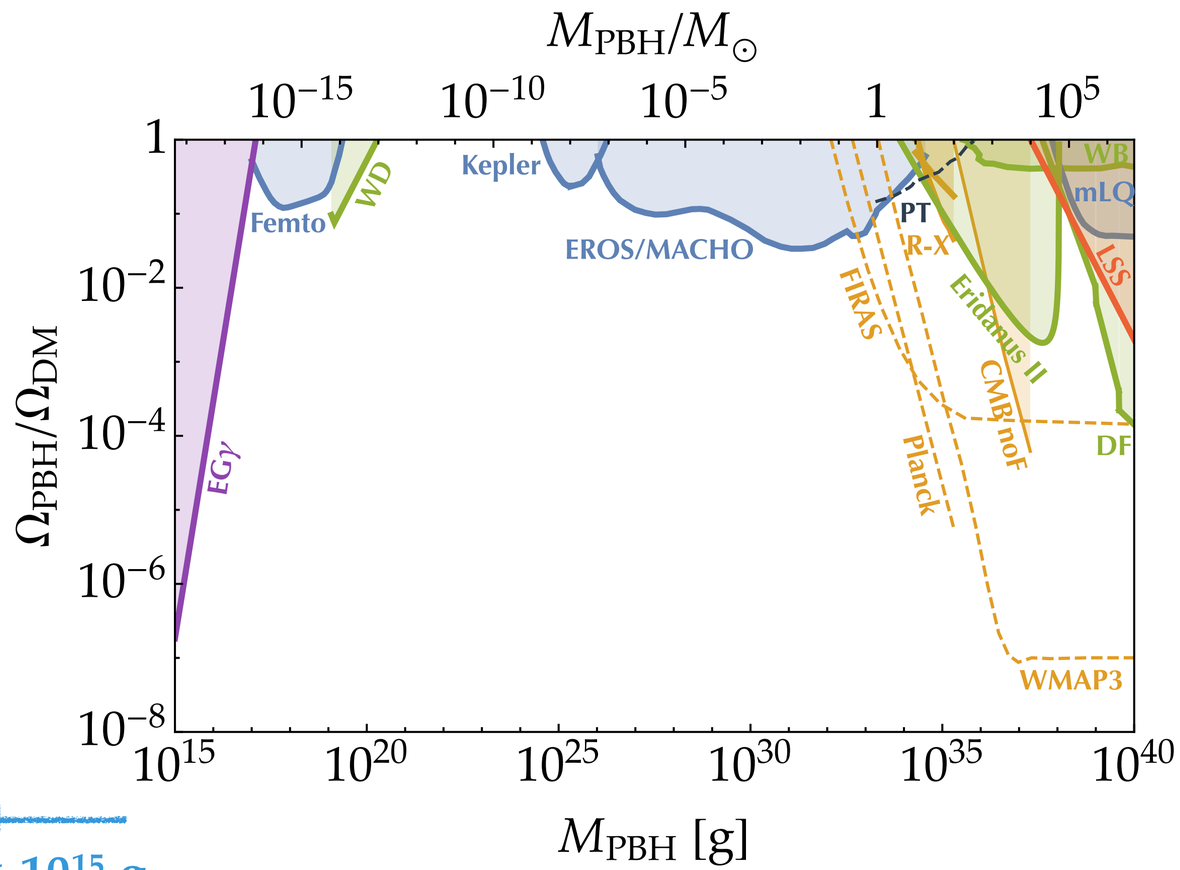

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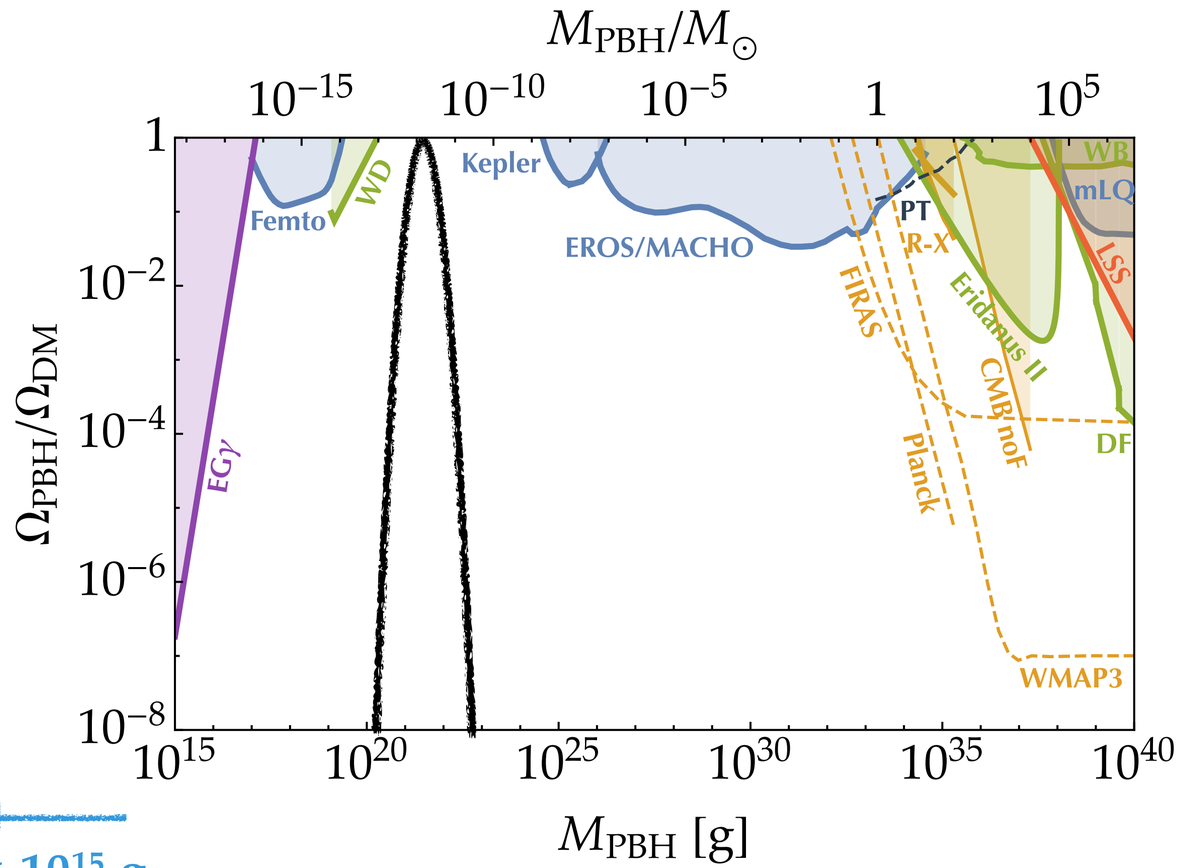

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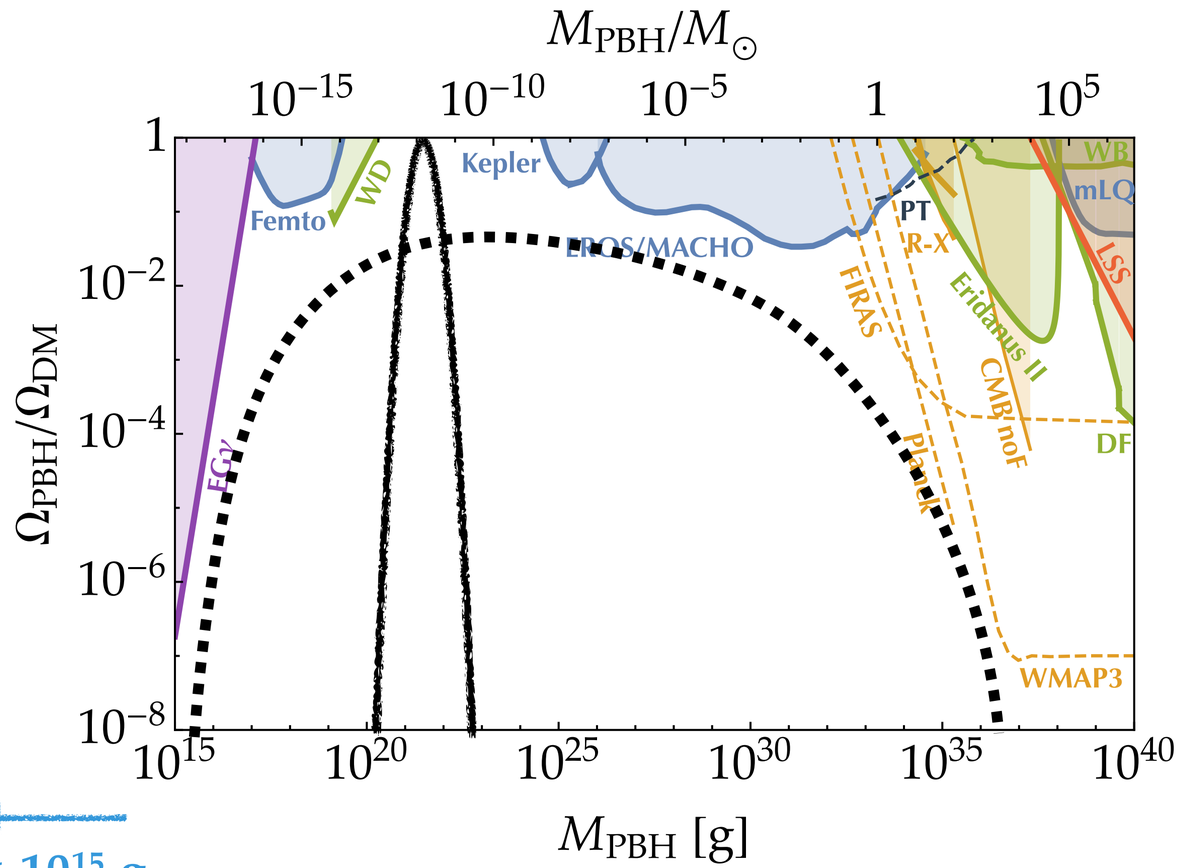

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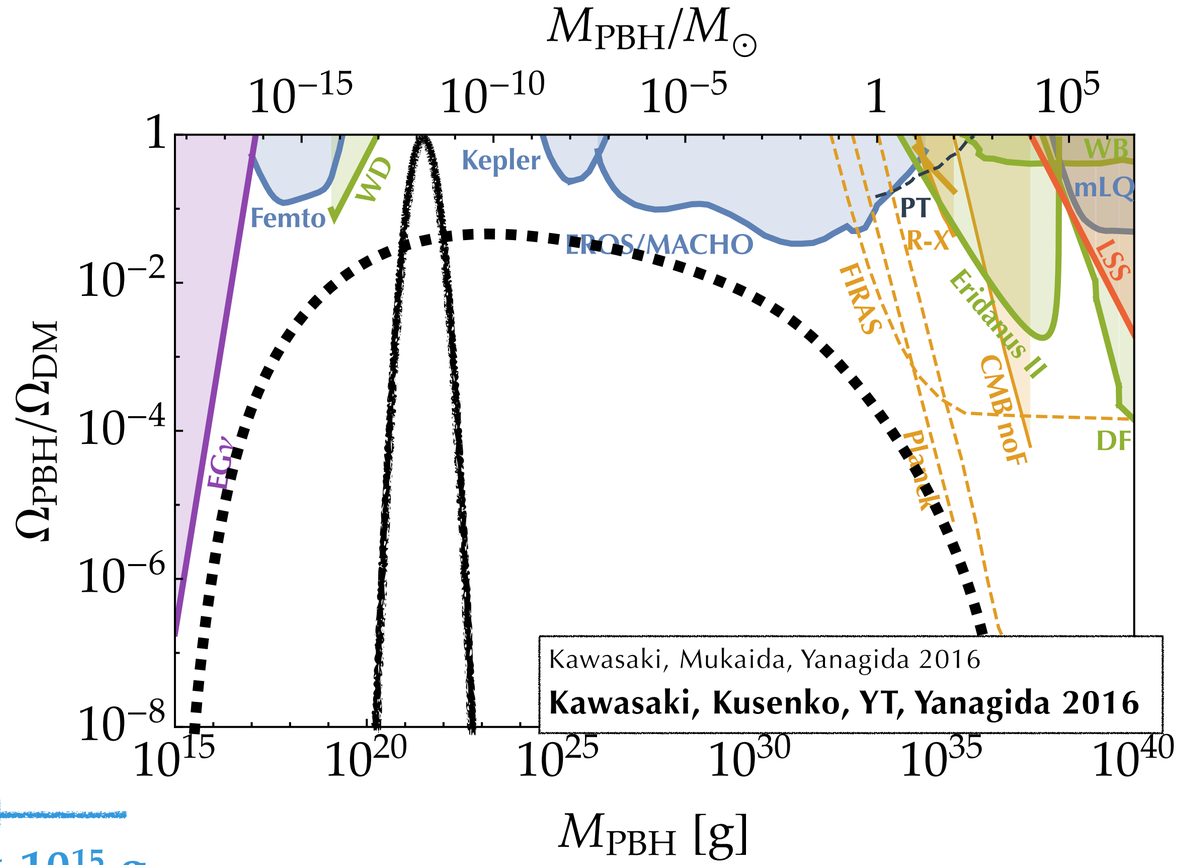

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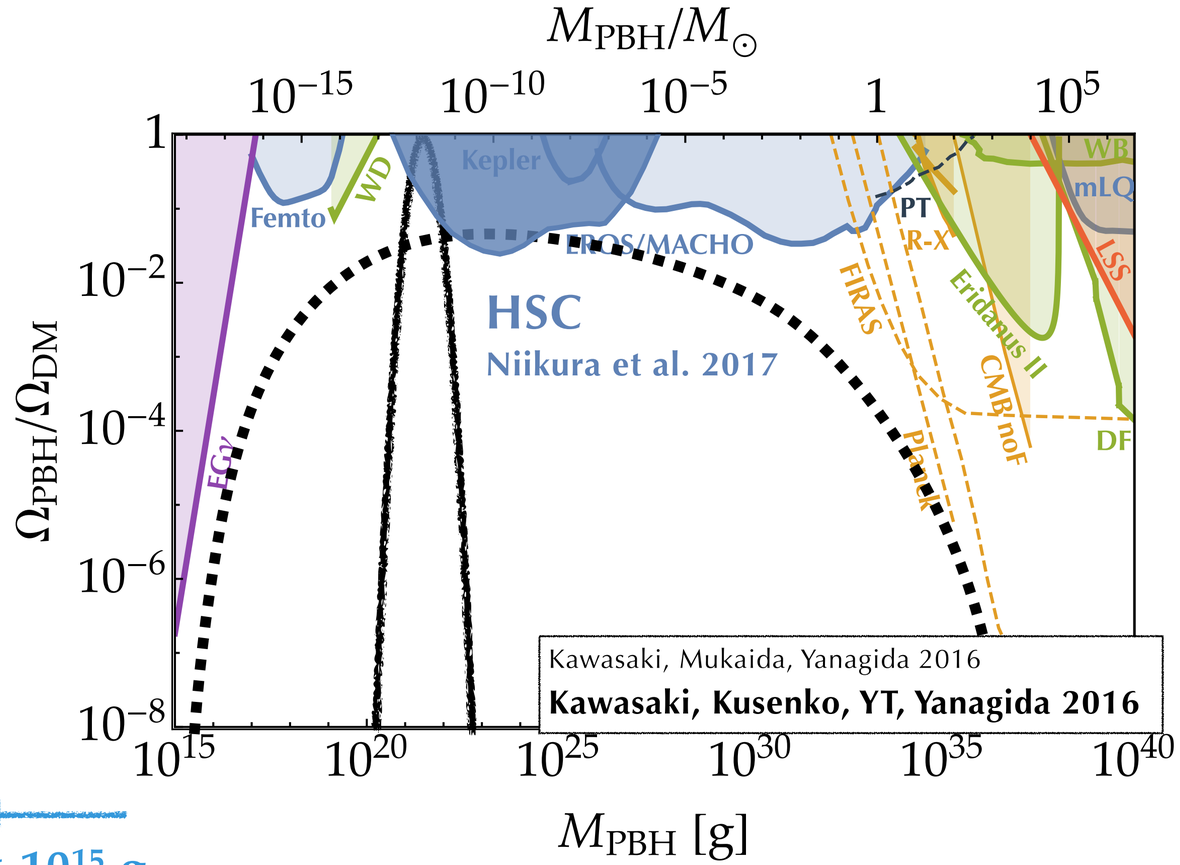

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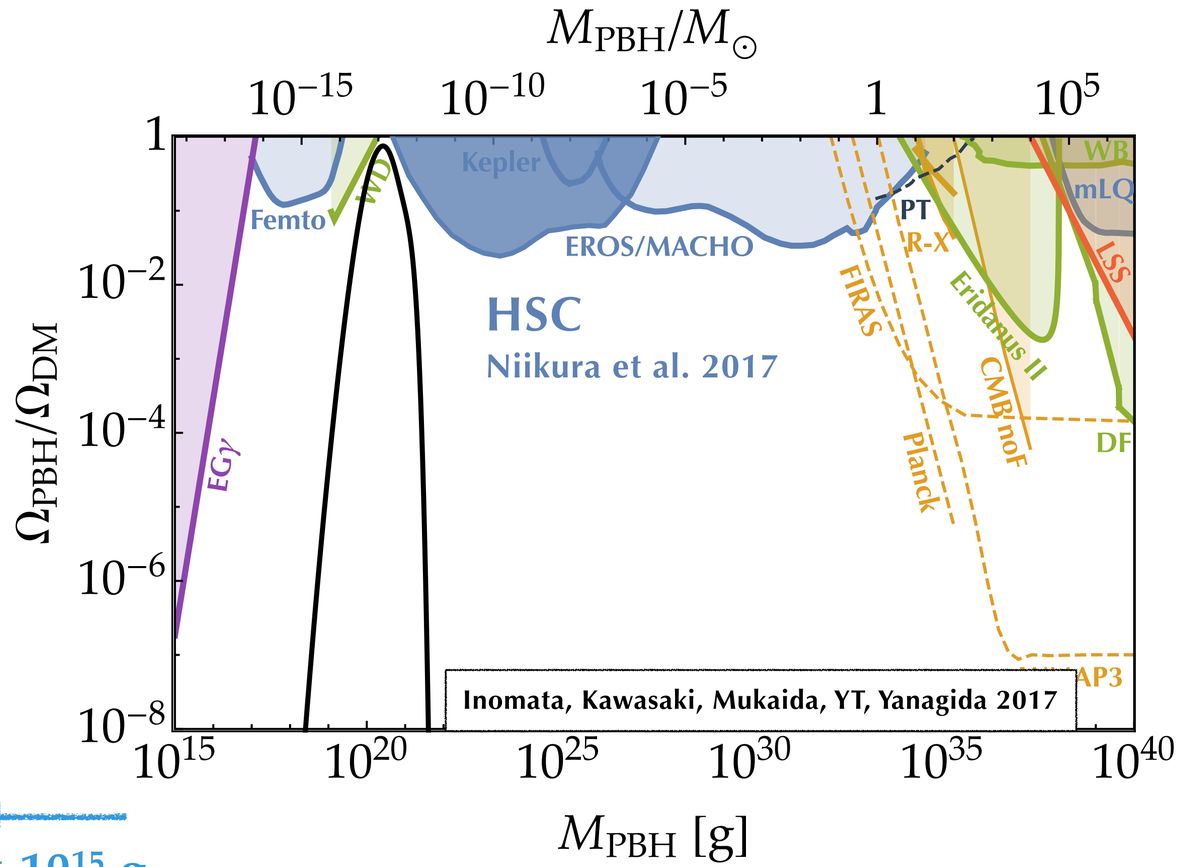

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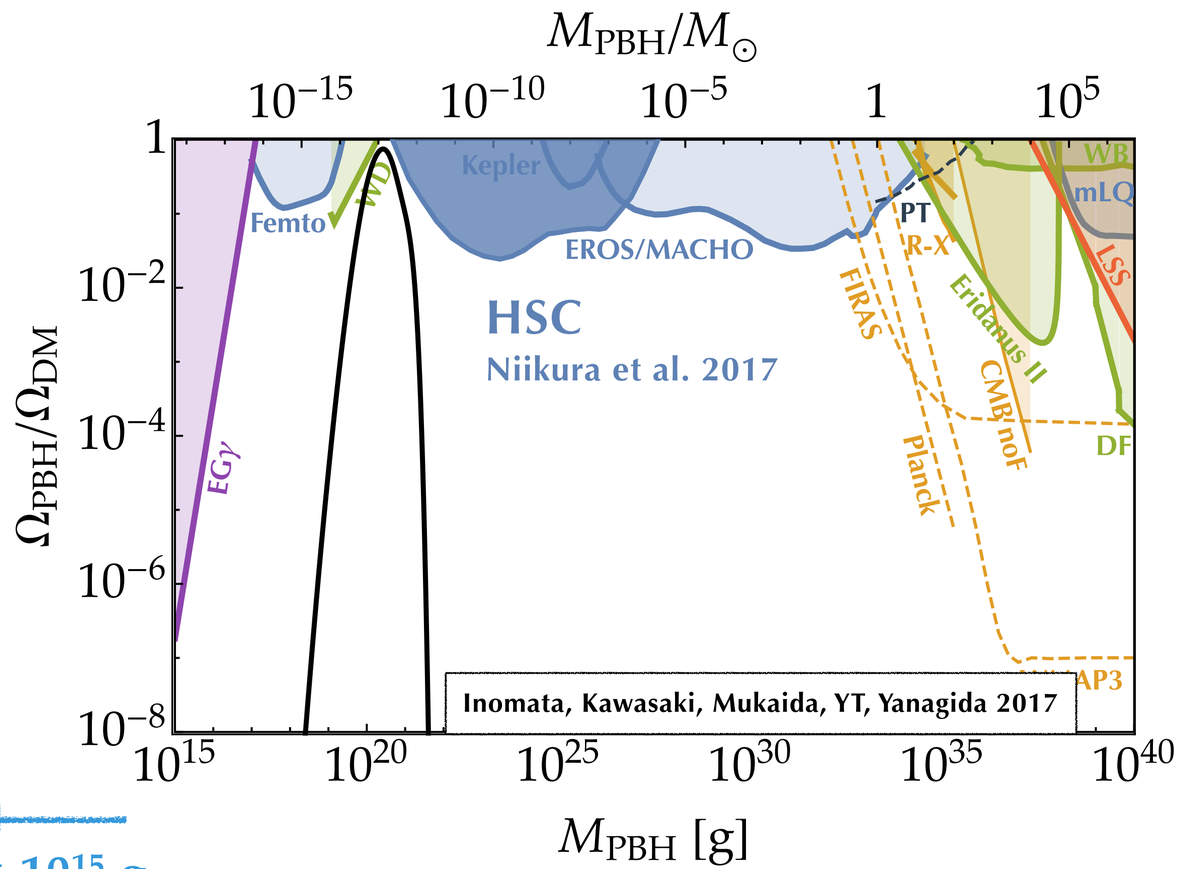

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Hawking evaporated

Gravitational Waves

Selected for a Viewpoint in *Physics*

PRL **116,** 061102 (2016) PHYSICAL REVIEW LETTERS week ending 12 FEBRUARY 2016

Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.*[*]
(LIGO Scientific Collaboration and Virgo Collaboration)
(Received 21 January 2016; published 11 February 2016)

- GW150914 by LIGO & Virgo
  Merger of BH binary: $36M_\odot + 29M_\odot$

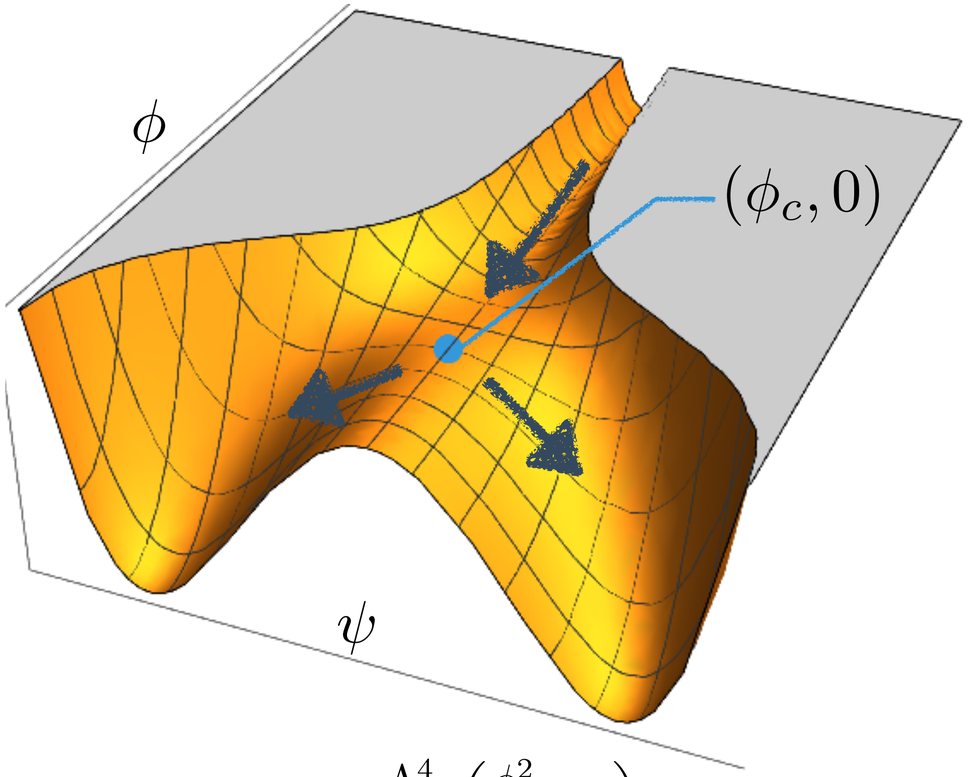
They might be PBHs!

- Bird et al., Clesse & Garcia-Bellido. } in DM halo, $\Omega_{\rm PBH} \sim \Omega_{\rm DM}$
- Sasaki et al. in RD, $\Omega_{\rm PBH} \sim (10^{-3}$–$10^{-4})\,\Omega_{\rm DM}$
- Eroshenko including ad. pert., $\Omega_{\rm PBH} \sim (10^{-2}$–$10^{-3})\,\Omega_{\rm DM}$

Hybrid Inflation

Hybrid Inflation Linde 1994

$$V(\phi,\psi) = V(\phi) + \Lambda^4\left(1-\frac{\psi^2}{M^2}\right)^2 + 2\Lambda^4\frac{\phi^2\psi^2}{\phi_c^2 M^2}$$



$$m_\psi^2|_{\psi\sim 0} = V_{\psi\psi}|_{\psi\sim 0} = 2\frac{\Lambda^4}{M^2}\left(\frac{\phi^2}{\phi_c^2}-1\right)$$

✱ instant waterfall

Chaotic IC + Small Field

$$V(\phi) = \begin{cases} \dfrac{1}{2}m^2\phi^2 & \text{original, blue-tilted} \\ \log|\phi/\phi_c| & \text{SUSY-flat + CW, } n_s \sim 0.98 \\ & \text{Dvali, Shafi, Schaefer 1994} \\ \log|\phi/\phi_c| - \mathrm{Re}(\phi) & \text{SUSY breaking, } n_s \text{ can be 0.96} \\ & \text{Buchmuller et. al. 2000-} \end{cases}$$

✱ long waterfall ($N_{\rm water} > 60$) Clesse 2011

Relax the IC problem of Hilltop Inflation

Mild Waterfall Case

Half & Half case and Massive PBH

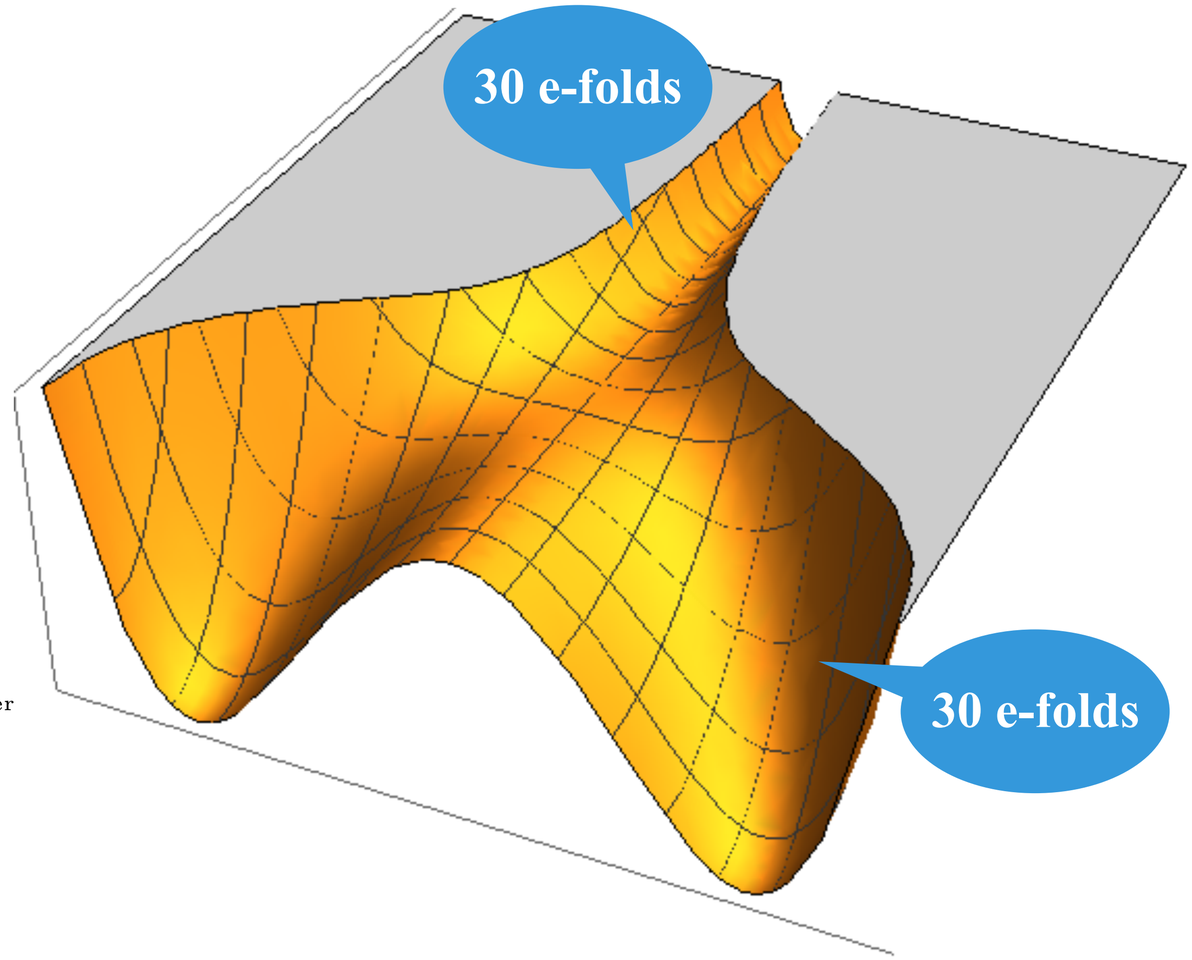
Garcia-Bellido, Linde, Wands 1996
Lyth 2010, 2012
Bugaev, Klimai 2011, 2012
Clesse, Garcia-Bellido 2015
⋮

Perturbations become large around ϕ_c
because of the flatness of the potential.

+

Following inflation enlarges the
perturbation scale to make PBH massive.

$$M_{\rm PBH} \simeq \frac{M_{\rm Pl}^2}{H_{\rm inf}} e^{2N_{\rm water}} = 10^4\,{\rm g}\left(\frac{10^9\,{\rm GeV}}{H_{\rm inf}}\right) e^{2N_{\rm water}}$$



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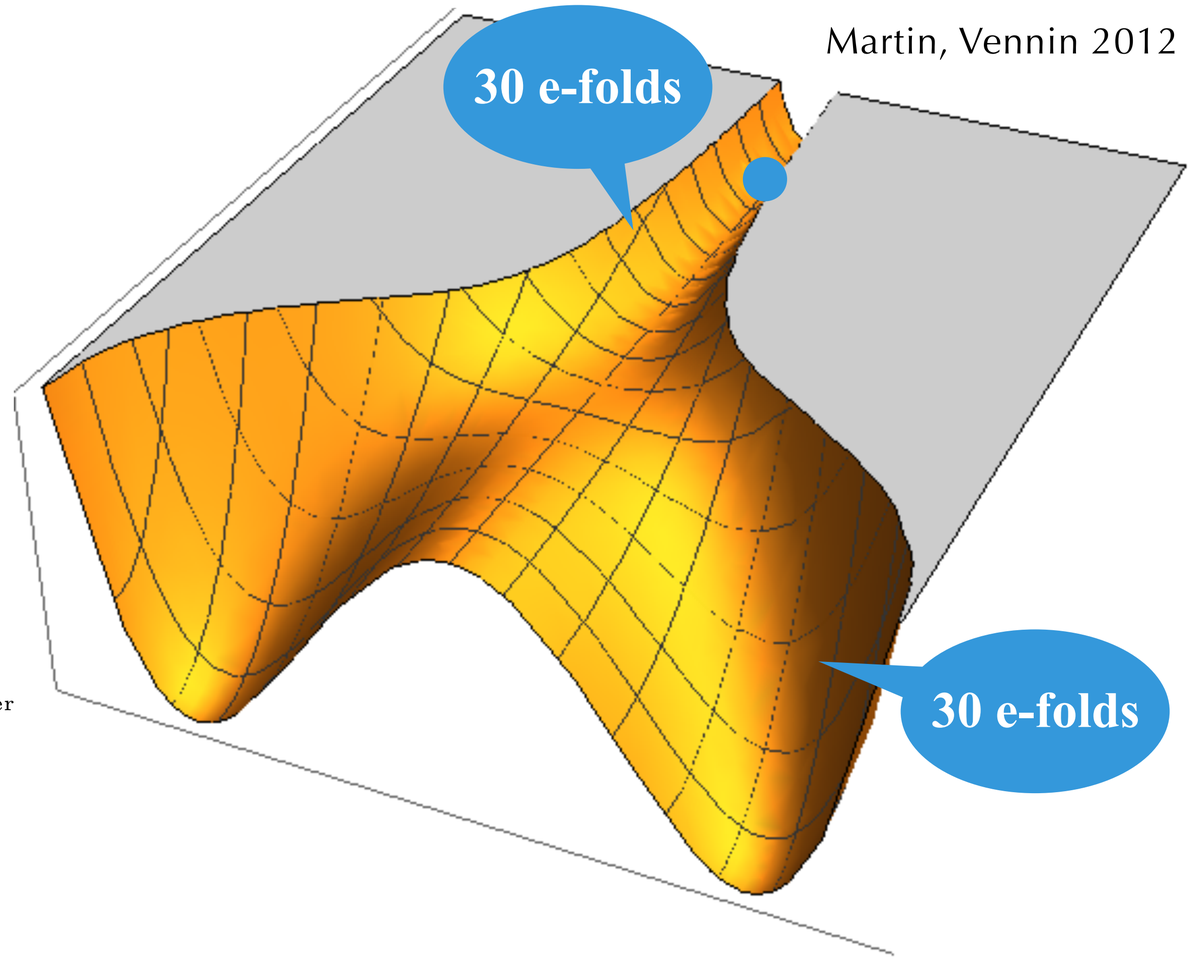

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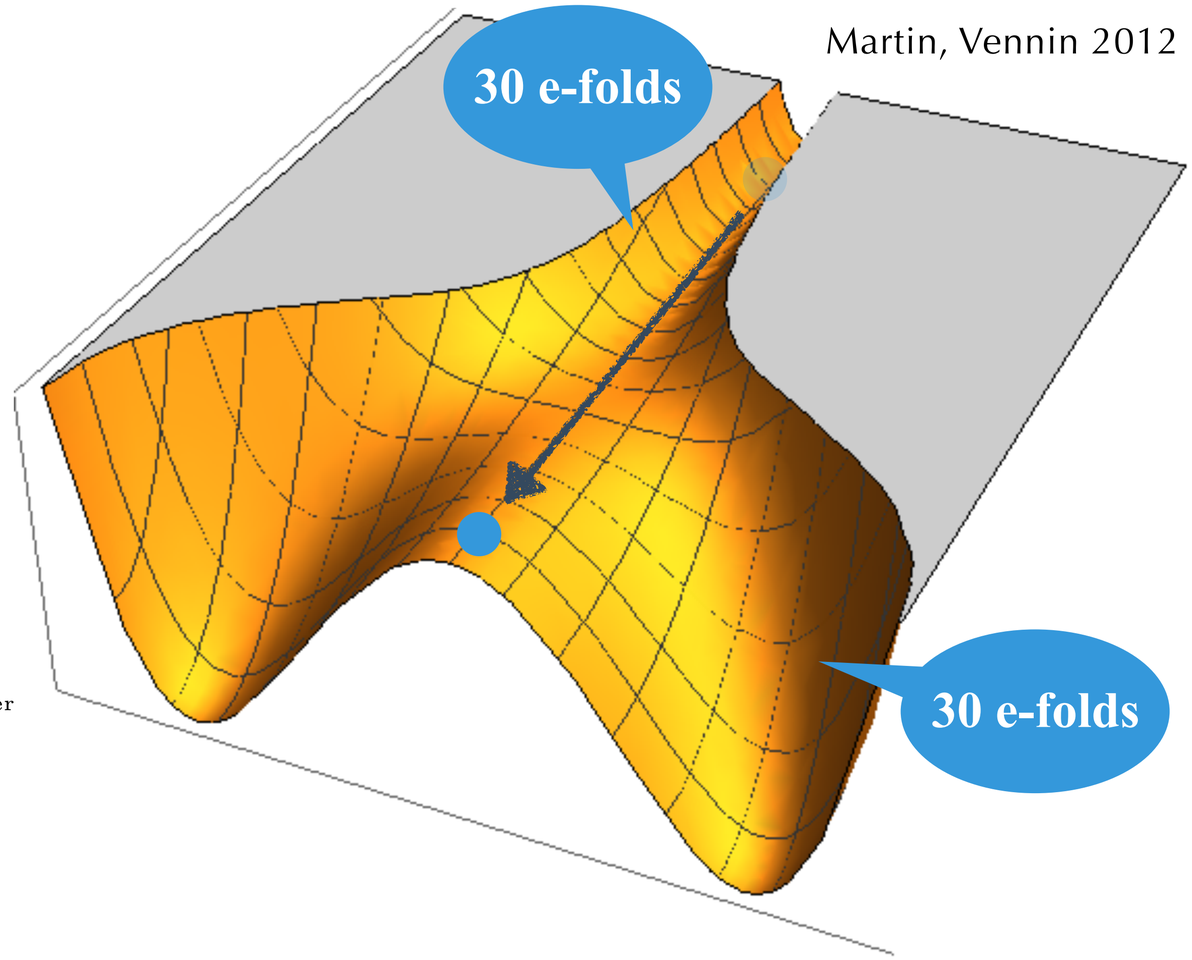

Martin, Vennin 2012

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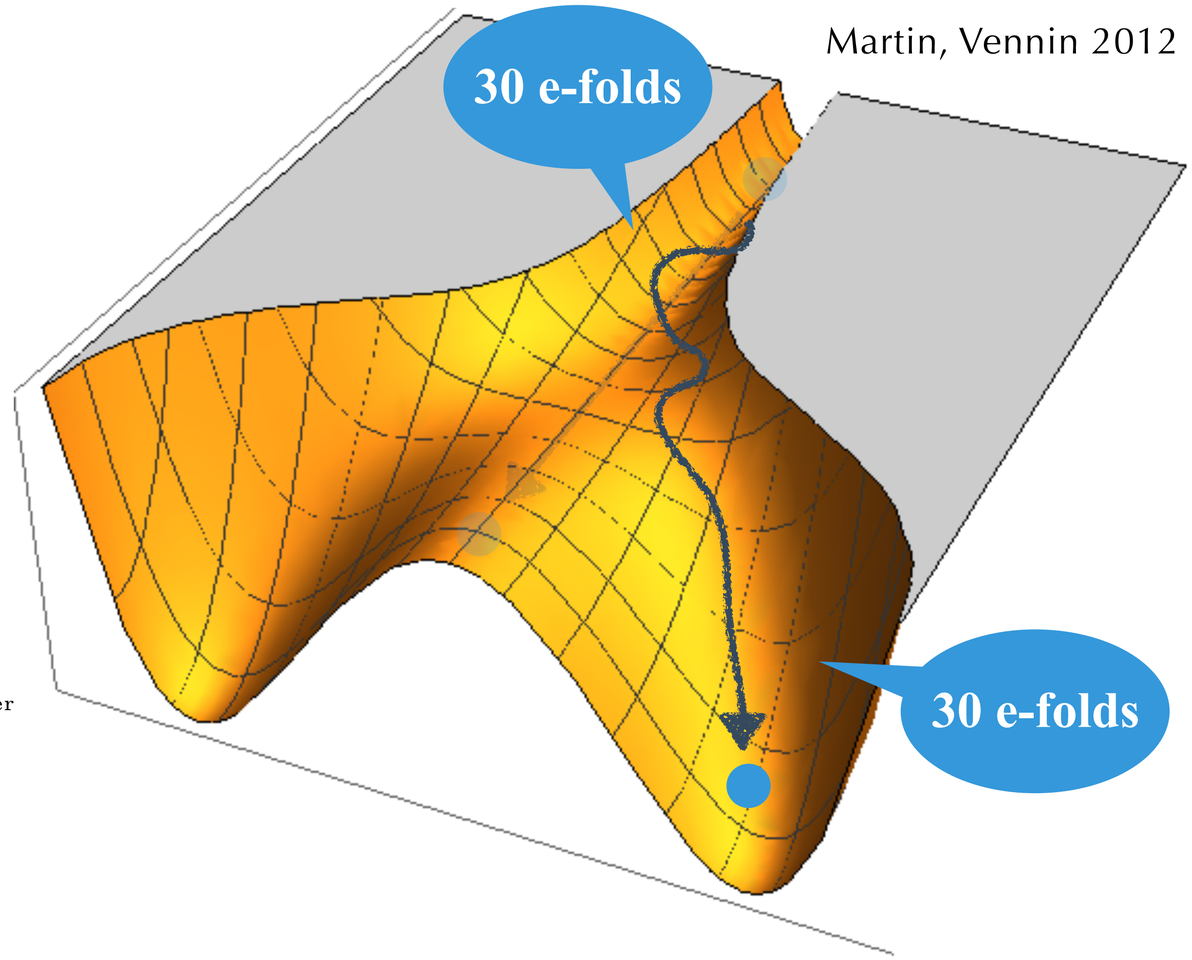

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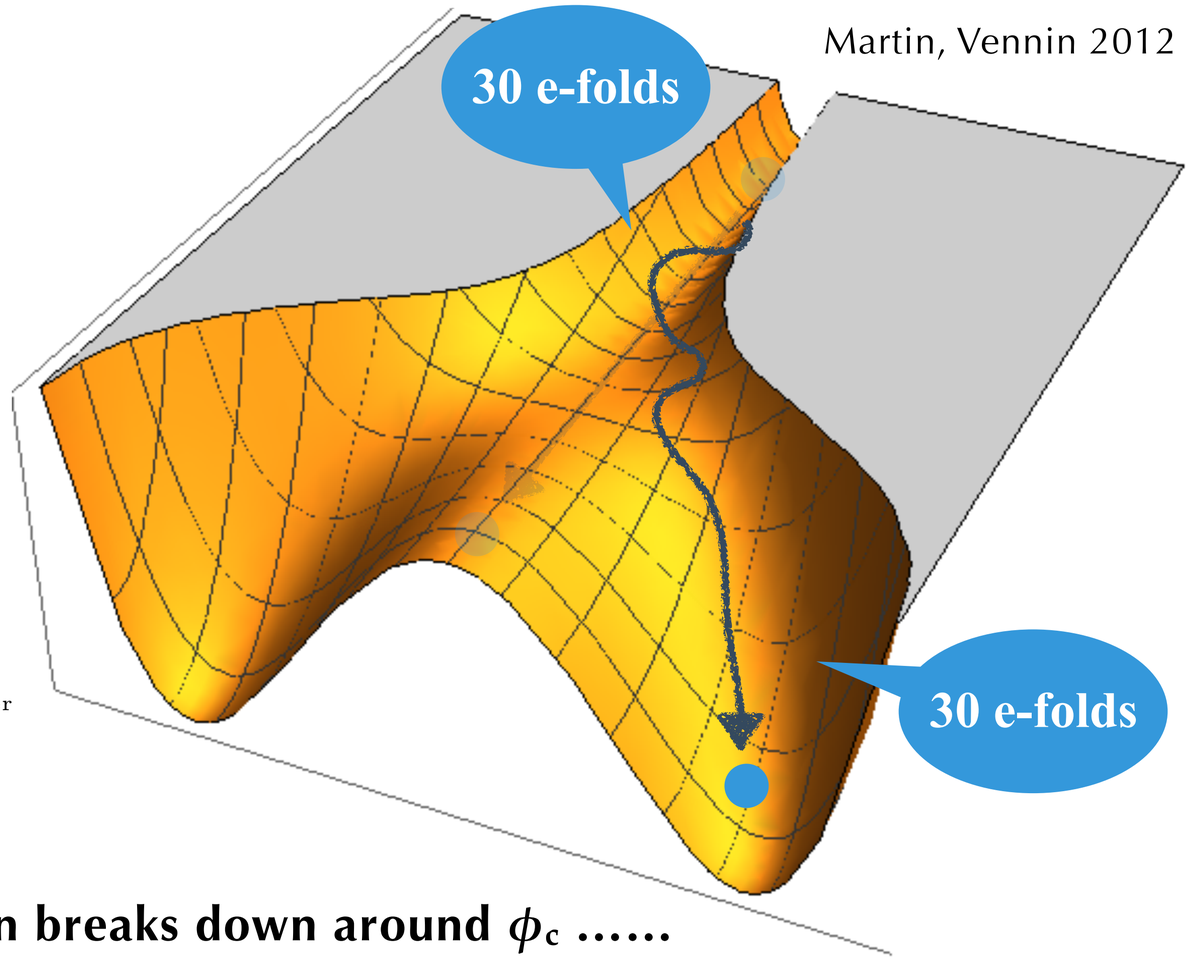
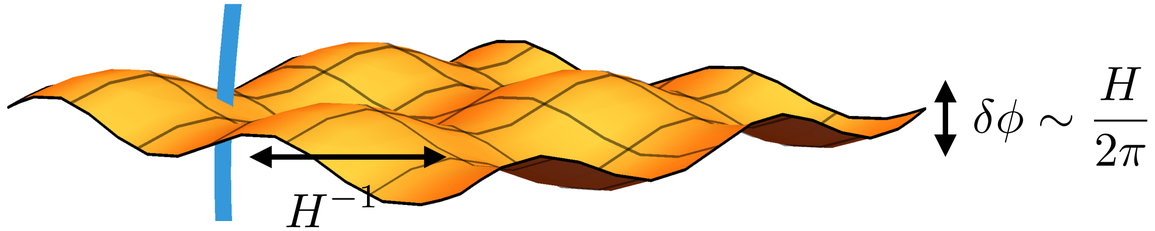


Martin, Vennin 2012

However the perturbative expansion breaks down around ϕ_c ……

→ **Numerical calculation in non-perturbative way with Stochastic formalism!**

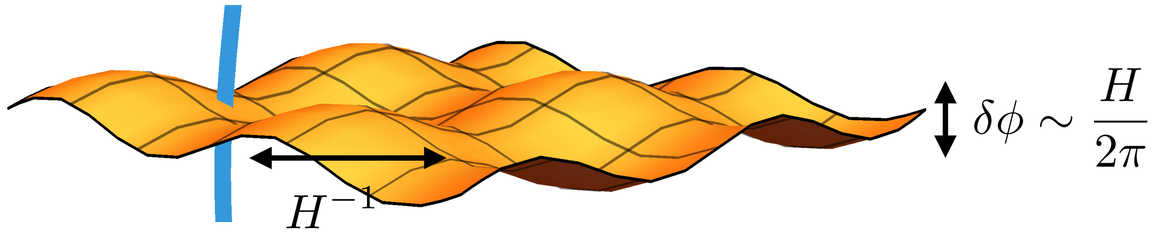
Stochastic formalism Starobinsky 1986



classical b.g.

$$\phi_{\rm IR}(t,\mathbf{x}) = \int \frac{{\rm d}^3k}{(2\pi)^3}\theta(\epsilon aH-k)\phi_{\mathbf{k}}(t){\rm e}^{i\mathbf{k}\cdot\mathbf{x}}, \quad \epsilon \ll 1$$

Stochastic formalism Starobinsky 1986



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$$\phi_{\rm IR}(t,\mathbf{x}) = \int \frac{{\rm d}^3k}{(2\pi)^3}\theta(\epsilon aH-k)\phi_{\mathbf{k}}(t){\rm e}^{i\mathbf{k}\cdot\mathbf{x}}, \quad \epsilon \ll 1$$

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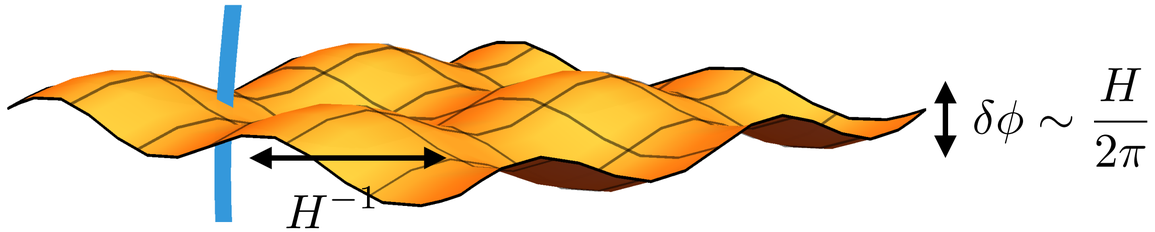
ξ can be interpreted as white and Hubble-patch-independent Gaussian noise

Stochastic formalism Starobinsky 1986



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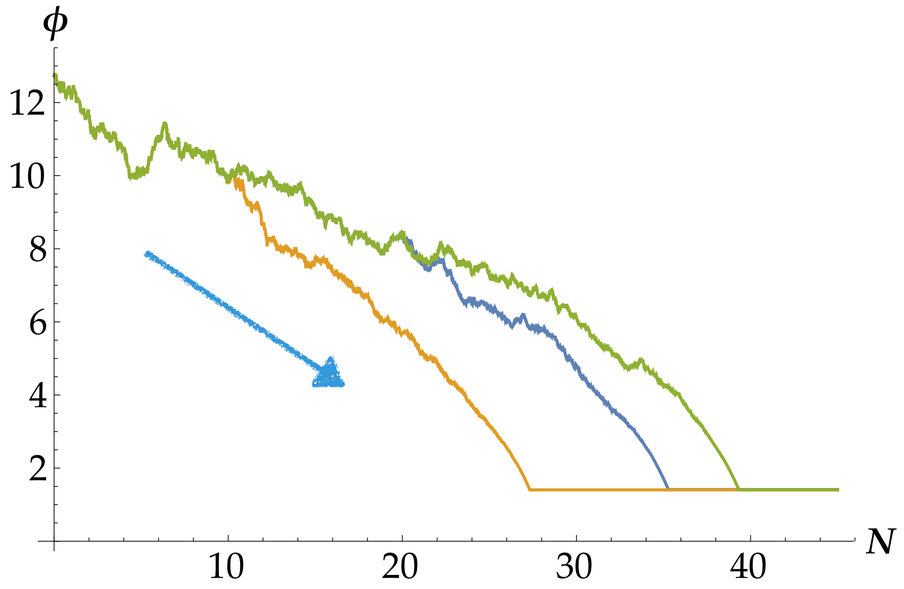

Stochastic formalism

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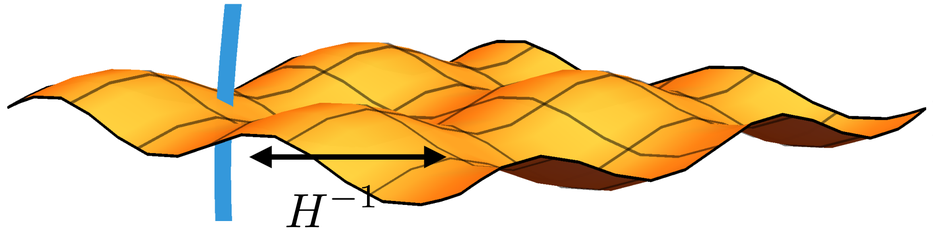

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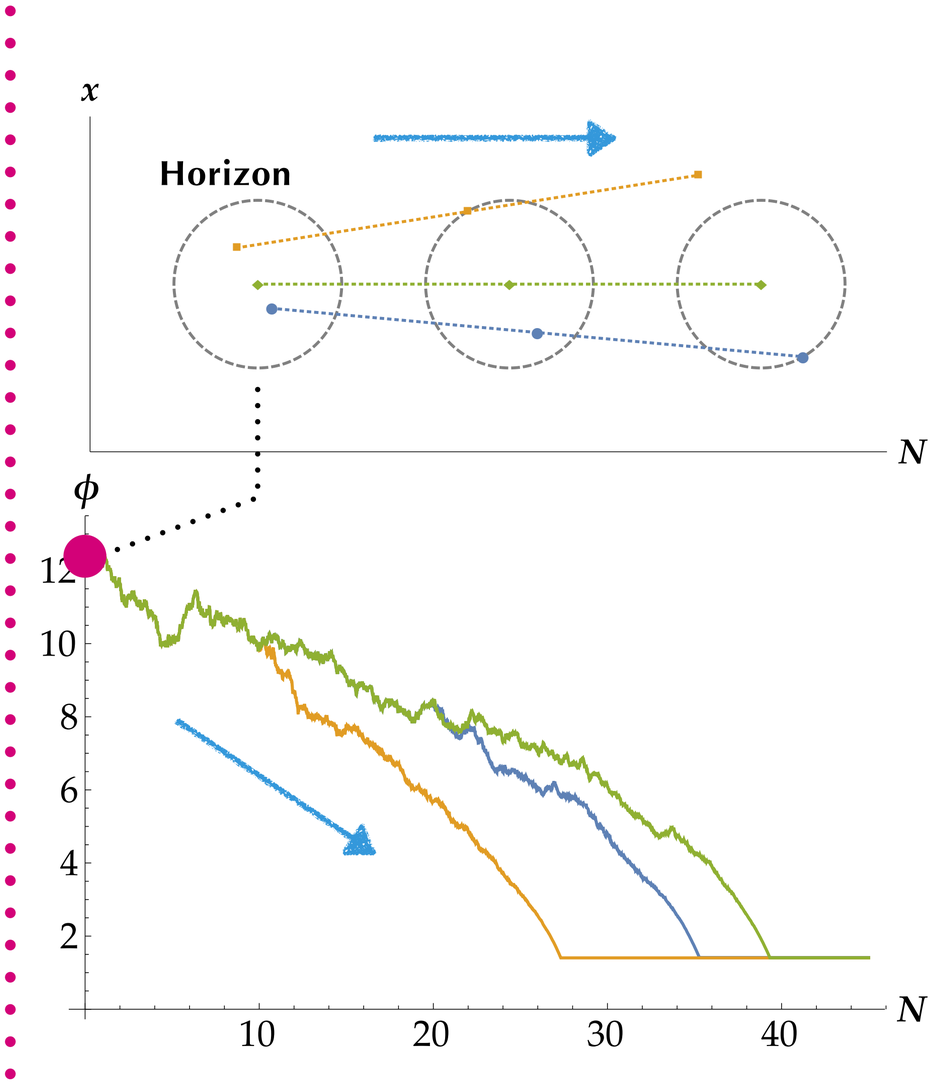

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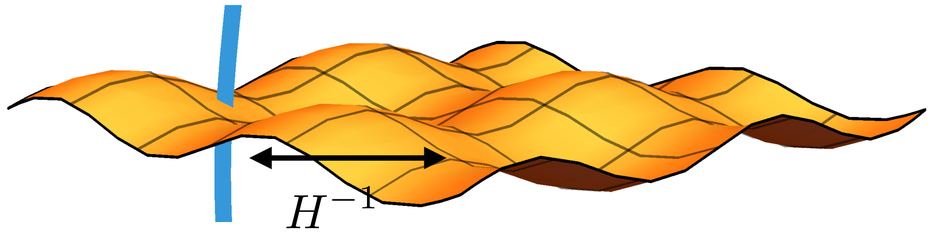

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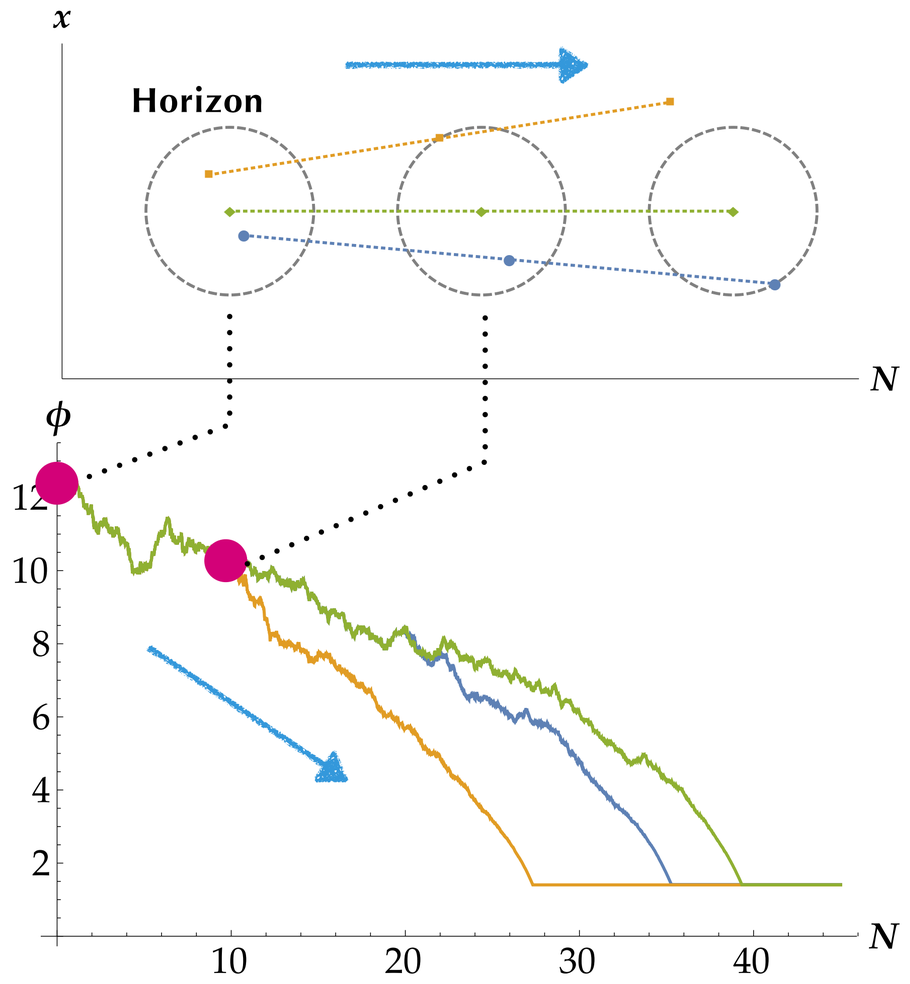

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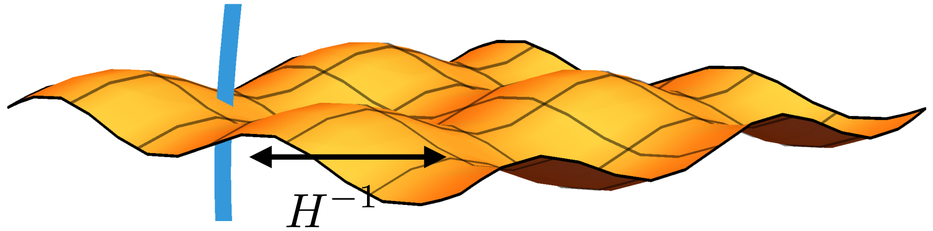

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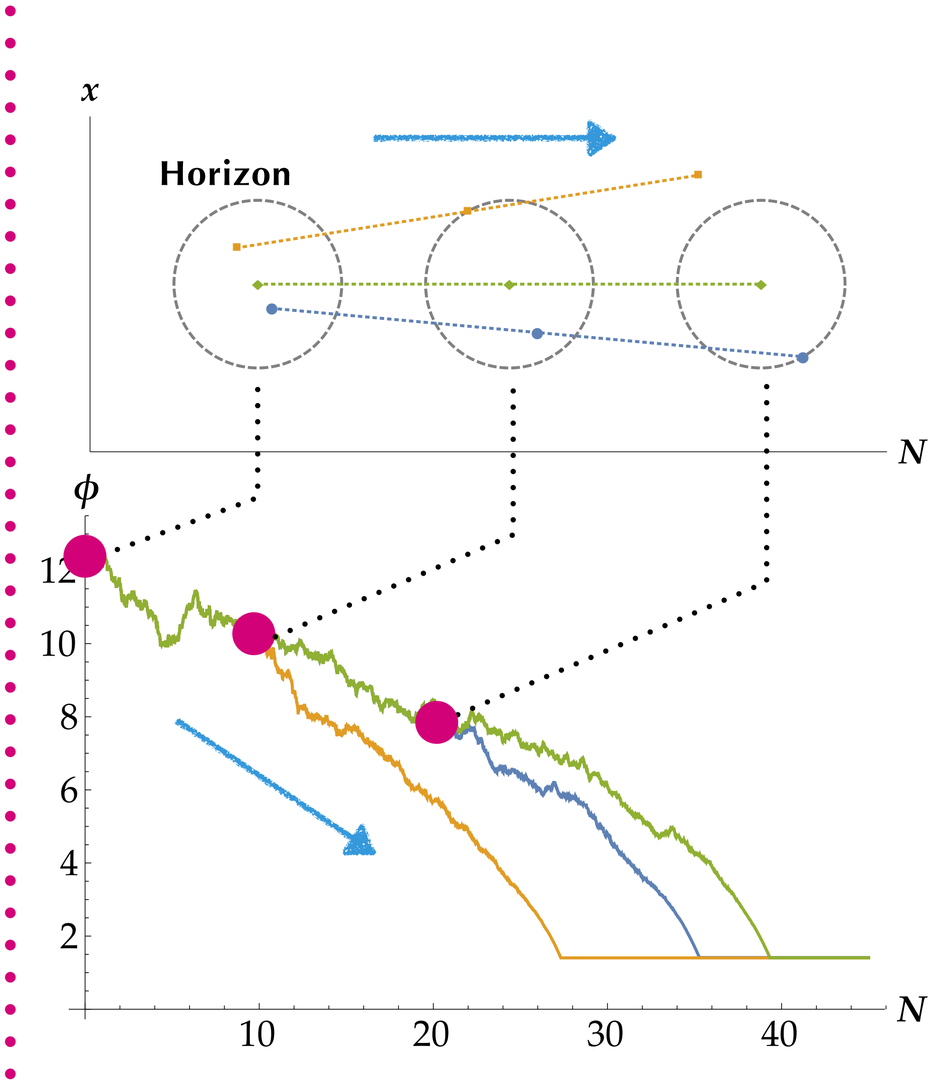

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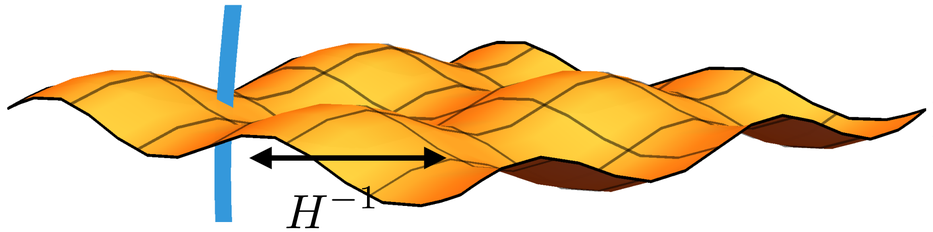

$$\delta\phi \sim \frac{H}{2\pi}$$

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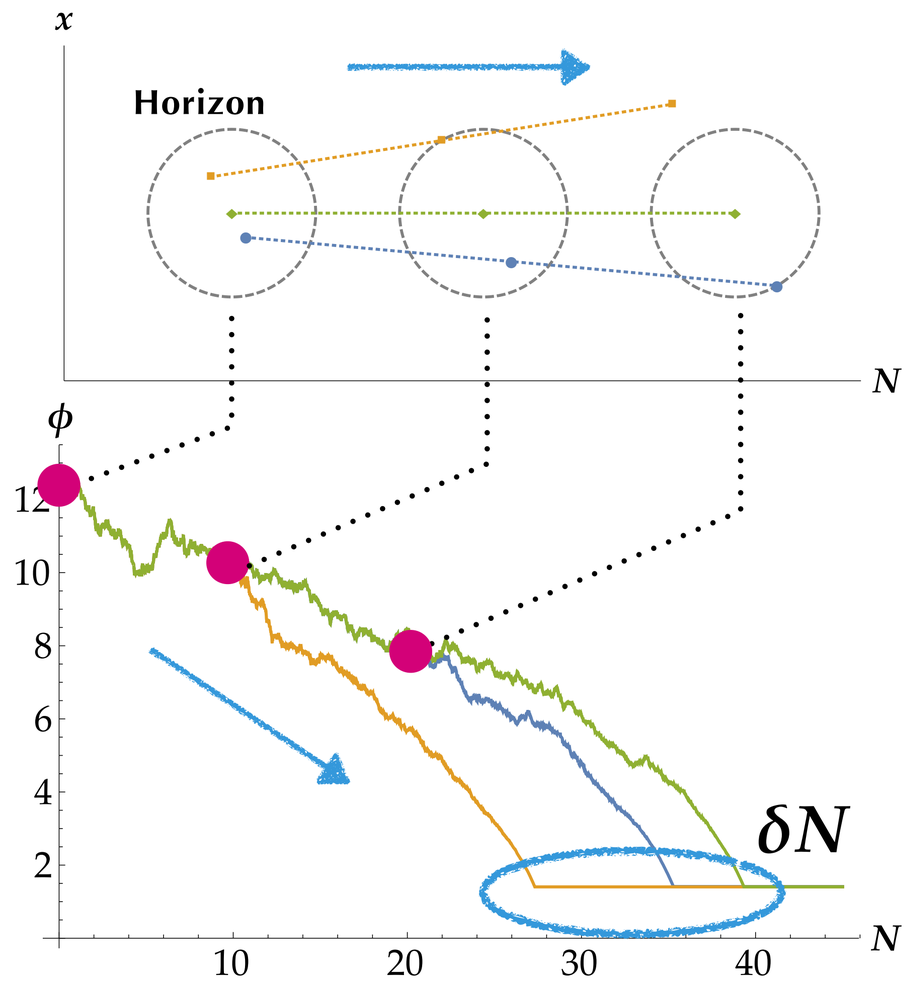

before Parameter Search

$$V(\phi,\psi) = \Lambda^4 \left[\left(1 - \frac{\psi^2}{M^2} \right)^2 + 2\frac{\phi^2\psi^2}{\phi_c^2 M^2} \right] + V(\phi) \simeq \Lambda^4 \frac{\phi - \phi_c}{\mu_1}$$

Taylor expansion in mild case

There are still 4 parameters (Λ, M, ϕ_c, μ_1) ...

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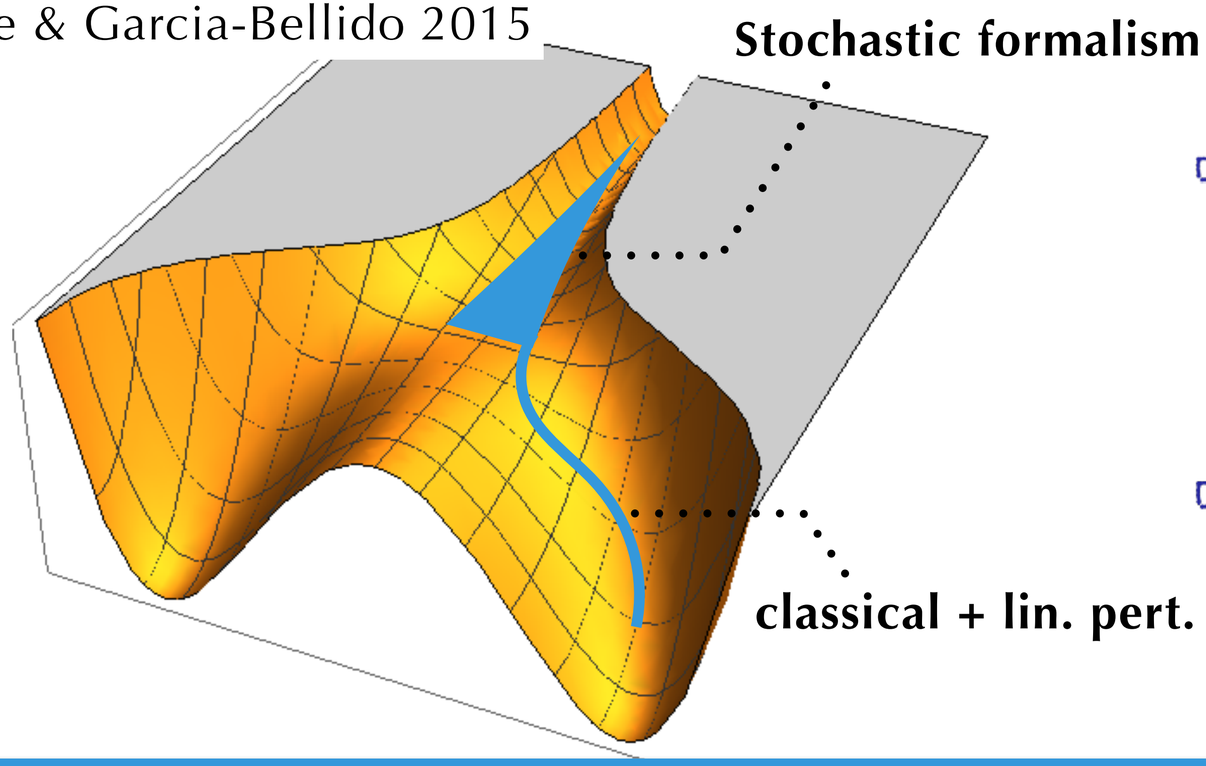
Clesse & Garcia-Bellido 2015



- Both $N_{\rm water}$ and $\mathcal{P}_\zeta$ are determined almost only by the combination $\Pi^2 := M^2\phi_c\,\mu_1/M_{\rm Pl}{}^4$
- The peak of $\mathcal{P}_\zeta$ is @ ϕ_c

Parameter Search

Kawasaki, YT 2015

$$V(\phi,\psi)=\Lambda^4\left[\left(1-\frac{\psi^2}{M^2}\right)^2+2\frac{\phi^2\psi^2}{\phi_c^2M^2}+\frac{\phi-\phi_c}{\mu_1}-\frac{(\phi-\phi_c)^2}{\mu_2^2}\right]$$

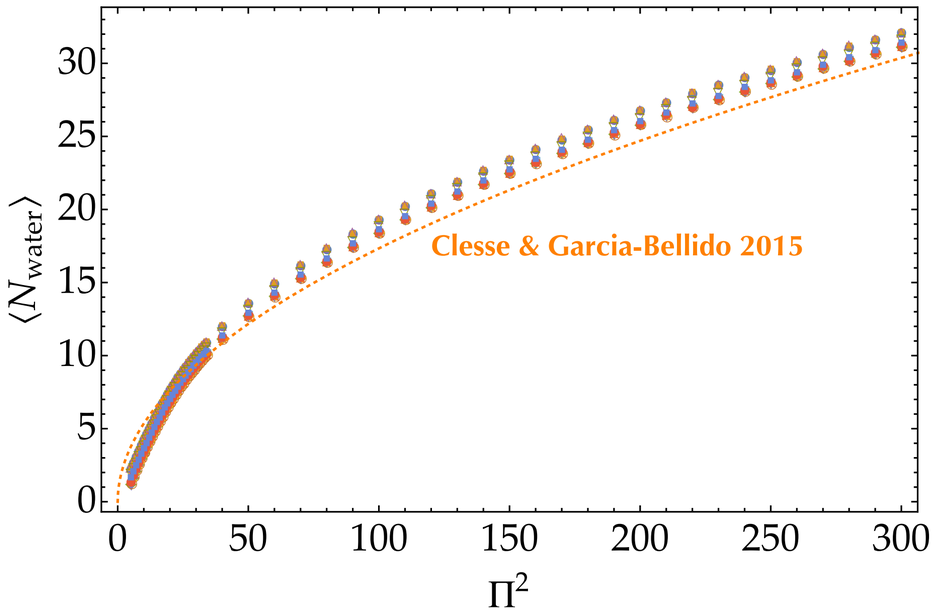
$$\begin{cases}\dfrac{\mu_2}{M_p}=11, & n_s=0.9655\\ \left(\dfrac{\Lambda}{M_p}\right)^4=2.198\times10^{-9}\times12\pi^2\left(\dfrac{\mu_1}{M_p}\right)^{-2}, & A_s=2.198\times10^{-9}\end{cases}$$

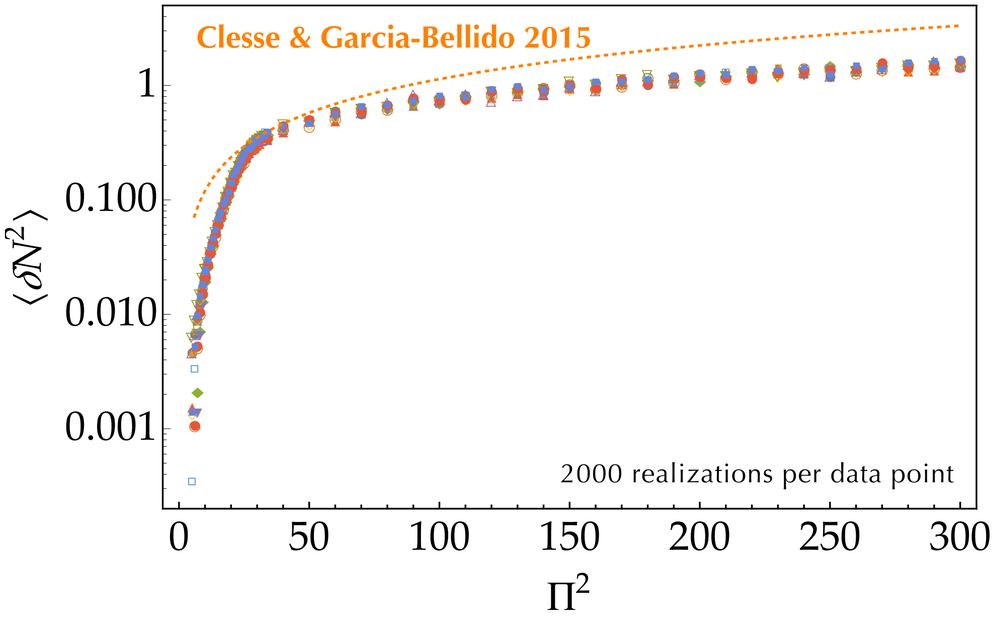
Planck 2015

Searching region

$$\begin{cases}\bullet\ \phi_c=\sqrt{2}M\ (\text{SUSY like}), & 10^{-4}M_p\le M\le10^{-1}M_p\\ \bullet\ \phi_c=0.1M_p\ (\text{fixed}), & 10^{-4}M_p\le M\le10^{-1}M_p\\ \bullet\ M=0.1M_p\ (\text{fixed}), & 10^{-4}M_p\le\phi_c\le10^{-1}M_p\end{cases}$$

μ_1 is also varied among $0\lesssim\Pi^2=\dfrac{M^2\phi_c\mu_1}{M_p^4}\lesssim300$



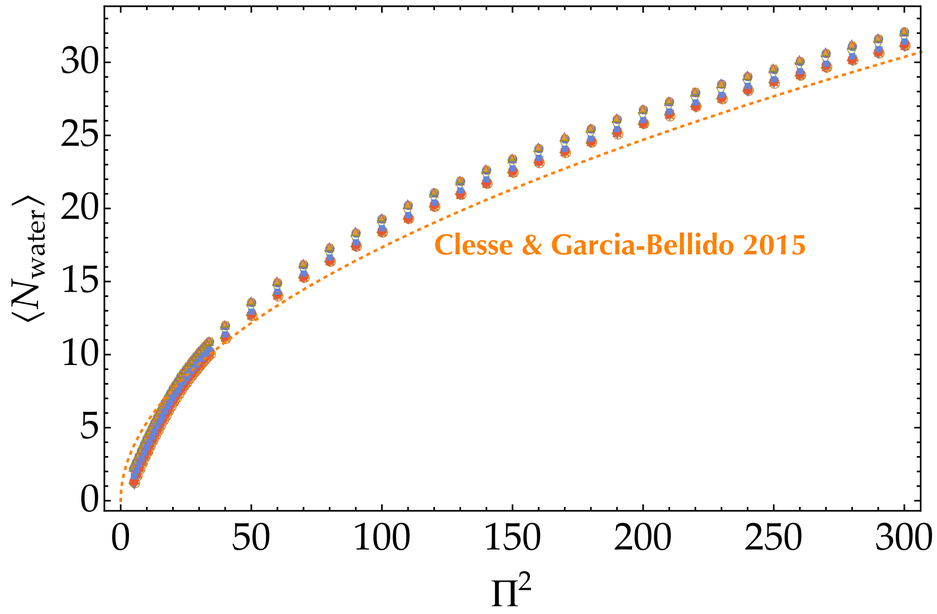


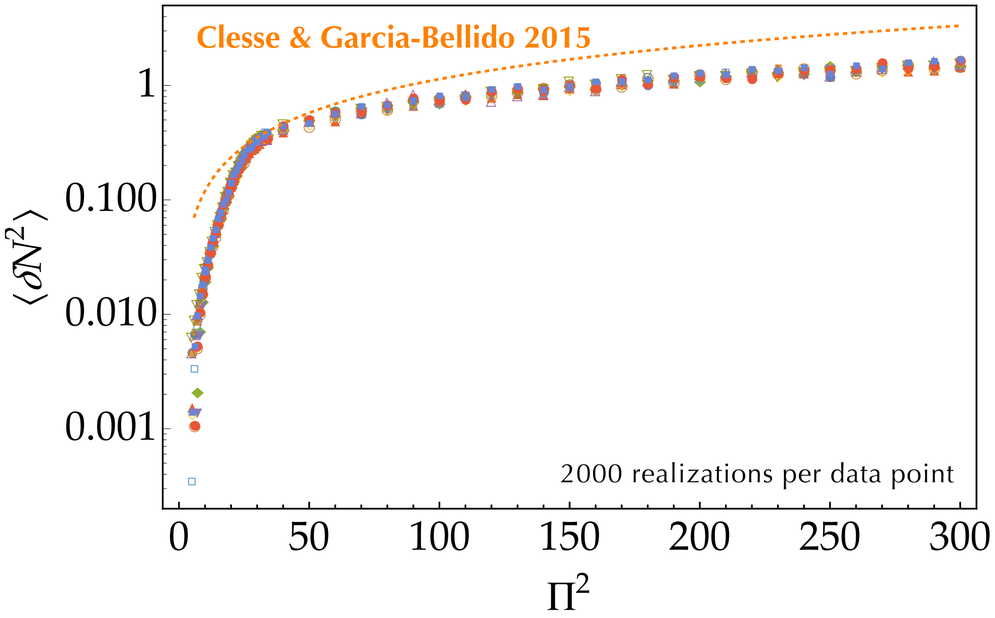
Parameter Search

- Indeed Π^2 plays key role beyond the pert. th.
- There are factor differences @ $\langle \delta N^2 \rangle$
- $\langle \delta N^2 \rangle \lesssim 0.01$ corresponds with $\Pi^2 \lesssim 10$.

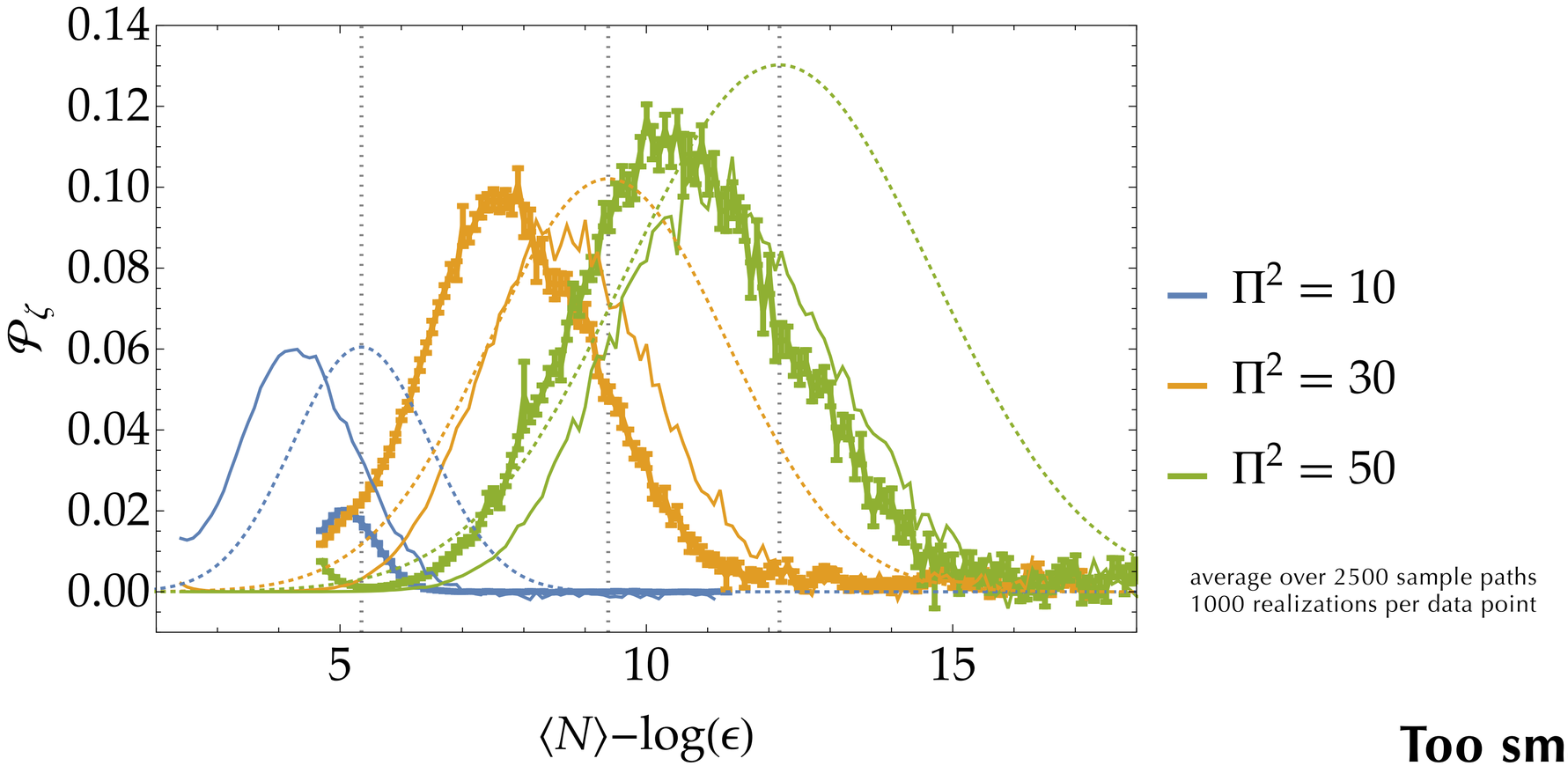
  → Waterfall phase $\lesssim$ 5 e-folds

Inversely, if waterfall phase $\gtrsim$ 5 e-folds, PBHs will be overproduced!!





corresponding PBH scale



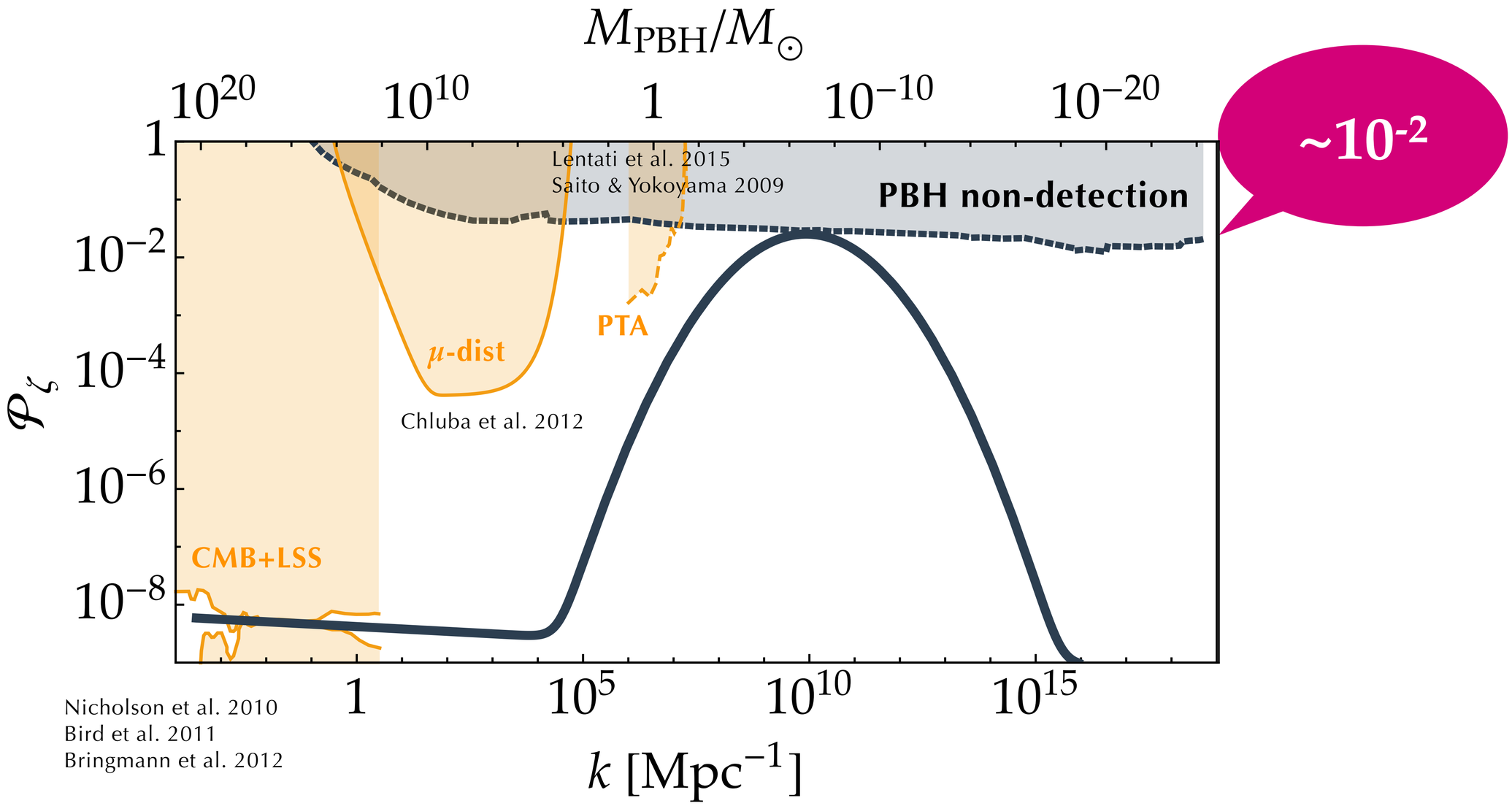
average over 2500 sample paths
1000 realizations per data point

Too small!

$$\Pi^2 \sim 10 \quad \leftrightarrow \quad N_{\rm peak} \sim 4 \quad \leftrightarrow \quad M_{\rm PBH} \sim \frac{M_p^2}{H_{\rm inf}} \mathrm{e}^{2\times 4} = 1.5 \times 10^{-26} M_\odot \left(\frac{10^9\,\mathrm{GeV}}{H_{\rm inf}} \right)$$

Double Inflation

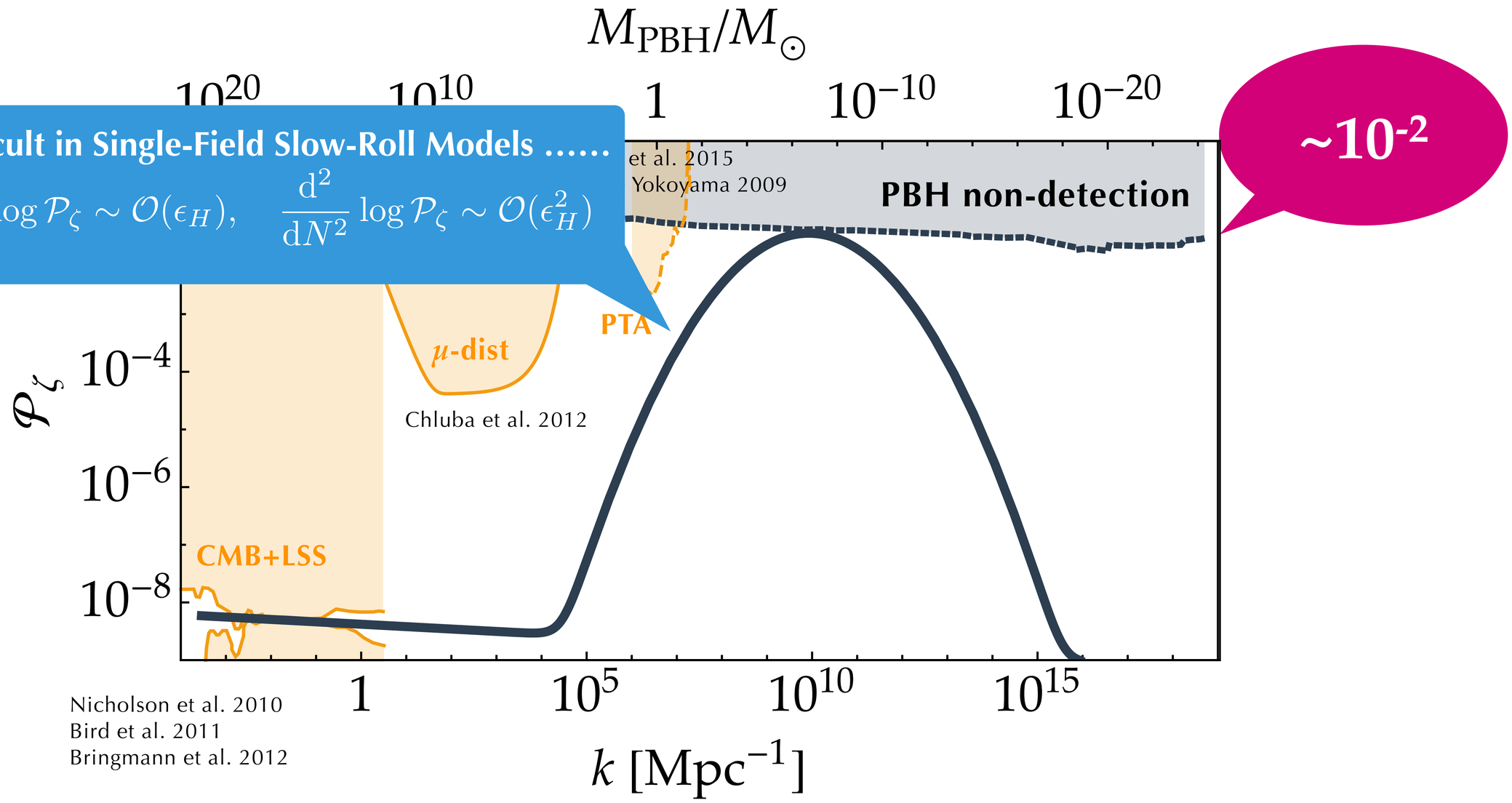
Separated Double Inflation



Separated Double Inflation

Difficult in Single-Field Slow-Roll Models ……

$$\frac{\mathrm{d}}{\mathrm{d}N}\log\mathcal{P}_\zeta \sim \mathcal{O}(\epsilon_H), \qquad \frac{\mathrm{d}^2}{\mathrm{d}N^2}\log\mathcal{P}_\zeta \sim \mathcal{O}(\epsilon_H^2)$$



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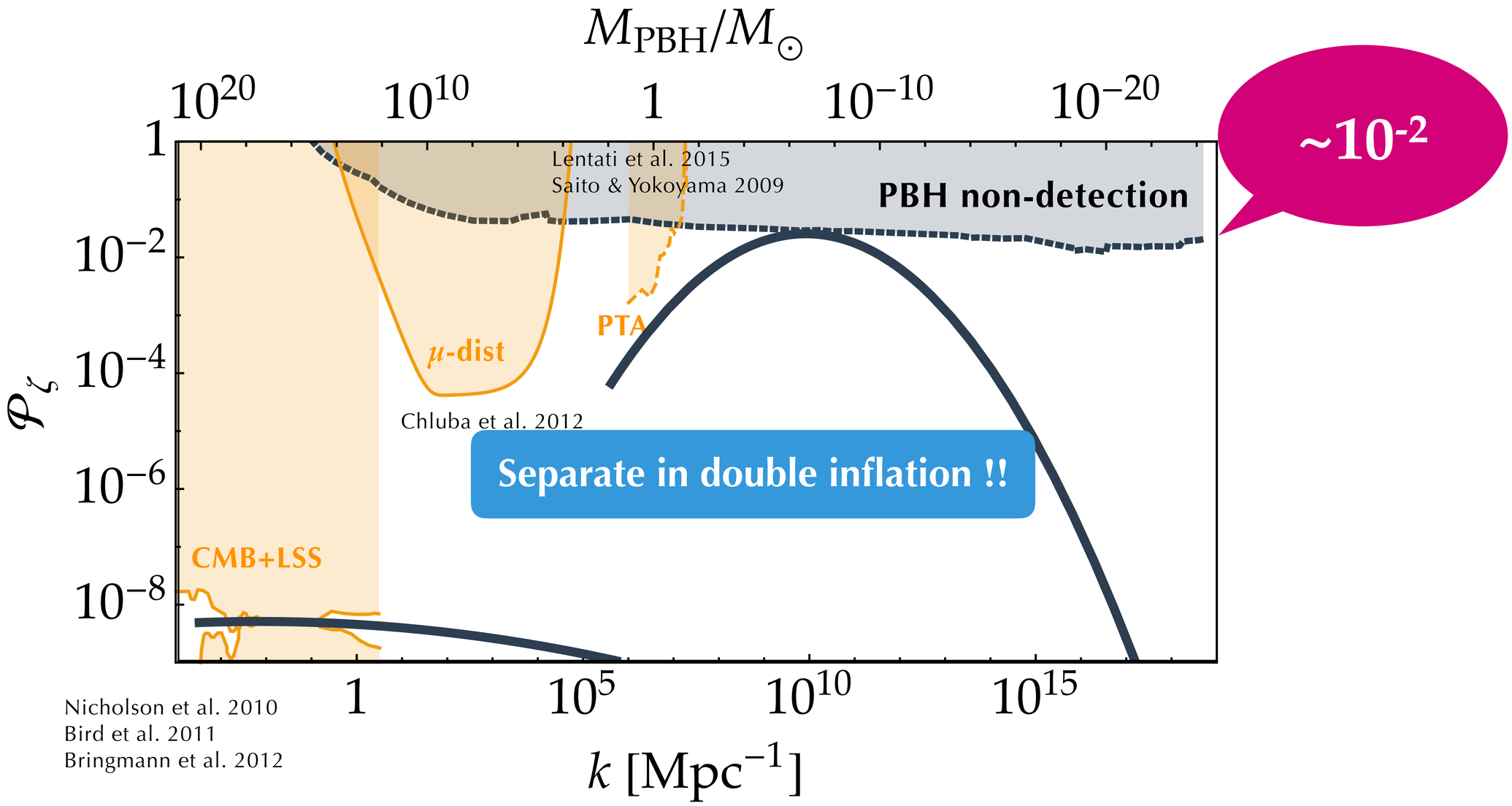

Model

- pre-inflation + new inflation Kawasaki, Sugiyama, Yanagida 1998

$$V(\chi,\phi)=V_{\rm pre}(\chi)+\left(v^2-g\frac{\varphi^n}{M_{\rm Pl}^{n-2}}\right)^2-\varepsilon v^4\frac{\varphi}{M_{\rm Pl}}-\frac{1}{2}\kappa v^4\frac{\varphi^2}{M_{\rm Pl}^2}+\frac{c_{\rm pot}}{2}V_{\rm pre}(\chi)\frac{\varphi^2}{M_{\rm Pl}^2}$$

$$\mathcal{L}_{\rm kin}=-\frac{1}{2}\partial_\mu\chi\partial^\mu\chi-\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi+\frac{c_{\rm kin}}{4}\frac{\varphi^2}{M_{\rm Pl}^2}\partial_\mu\chi\partial^\mu\chi$$

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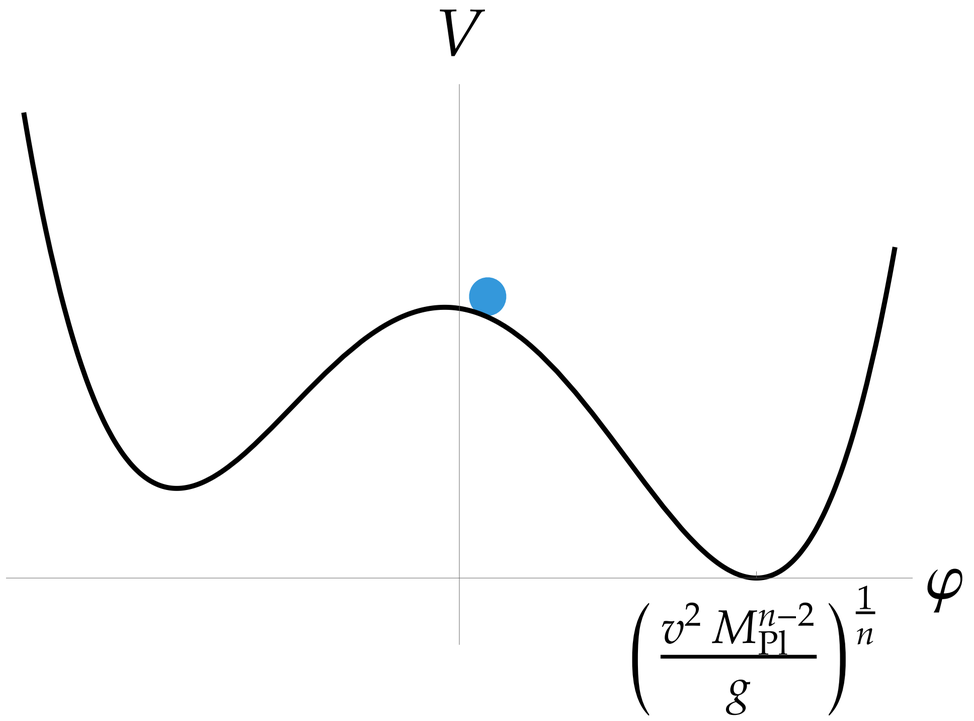

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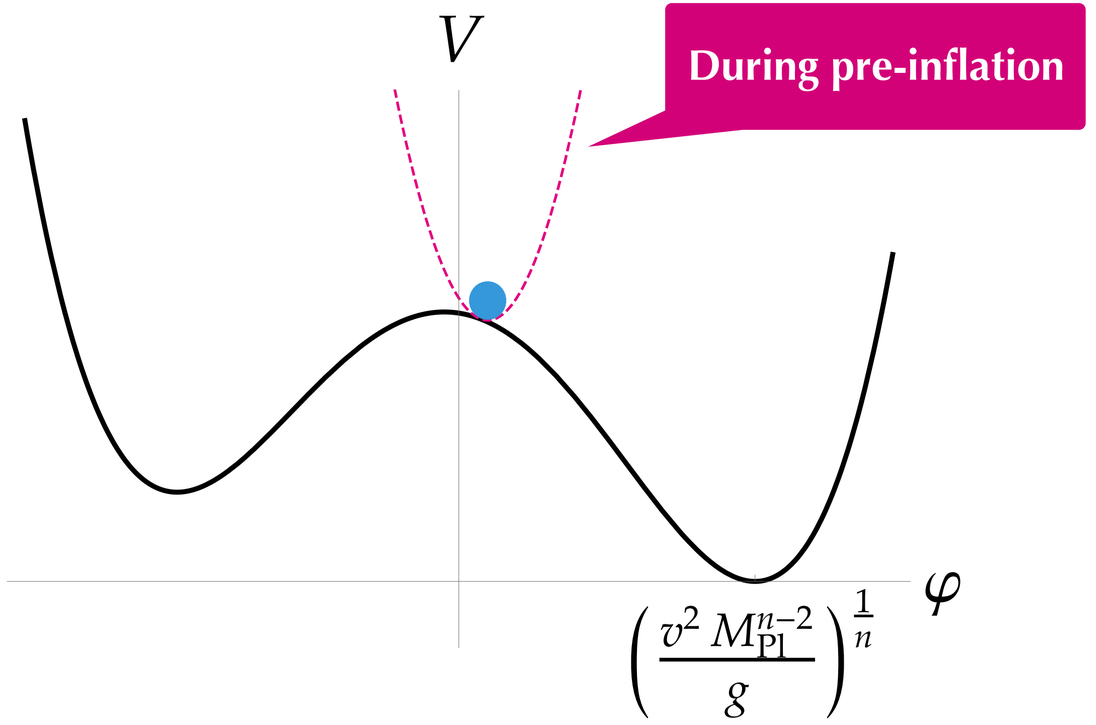

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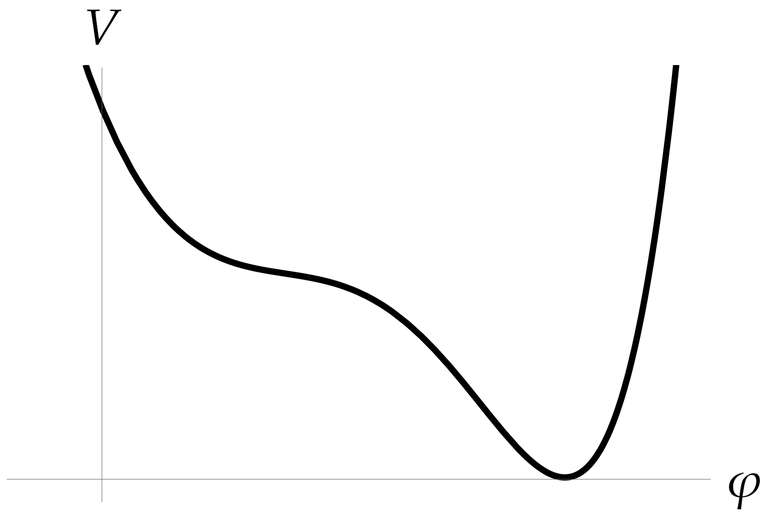
  - $\kappa < 0$
    The potential can have a flat inflection point



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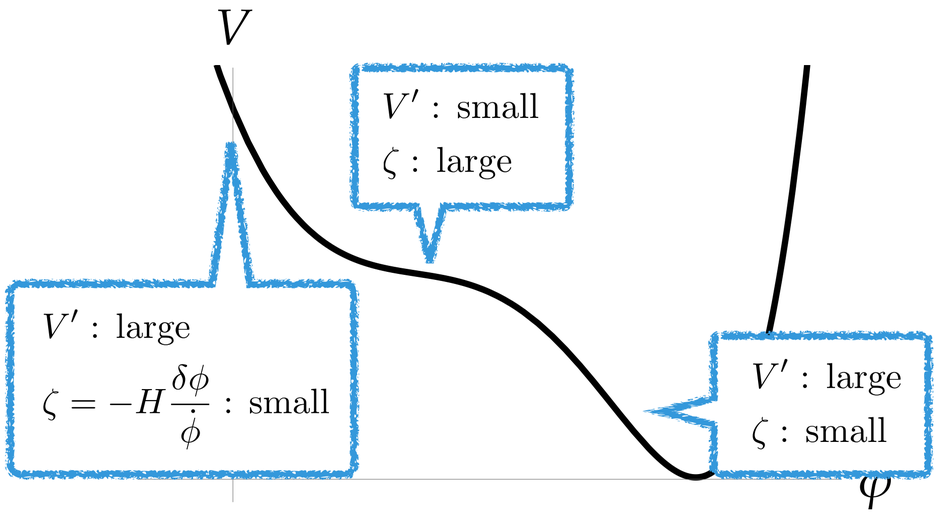

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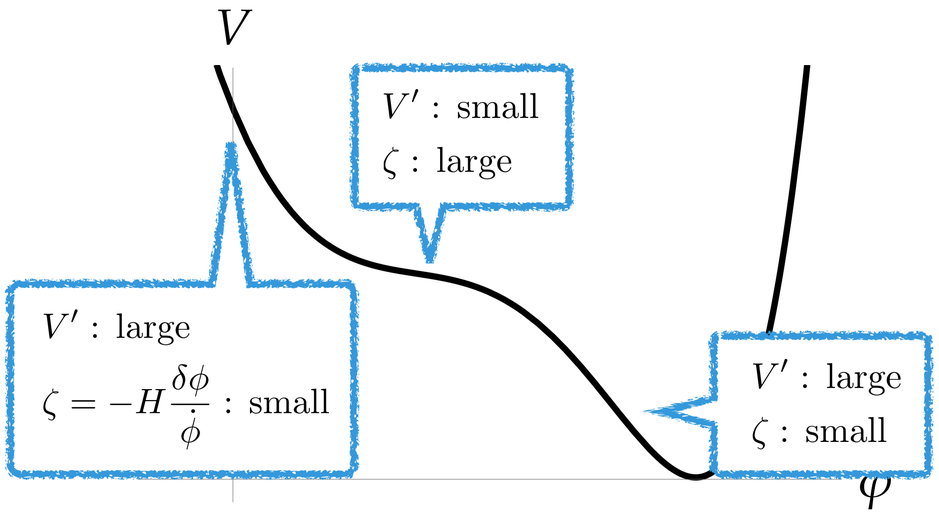

ζ becomes maximal
in the middle of new inflation
irrespectively of pre-inflation

Model

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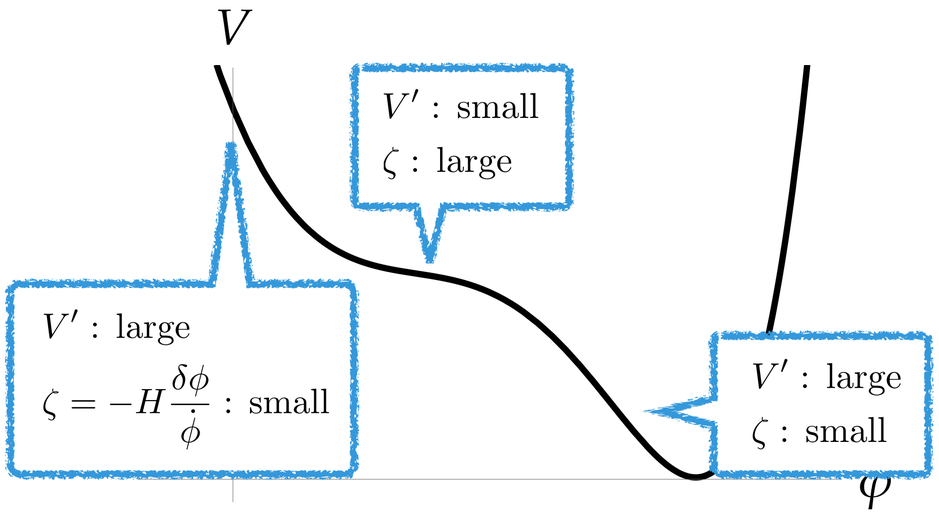

Model

- pre-inflation + new inflation Kawasaki, Sugiyama, Yanagida 1998

$$V(\chi,\phi) = V_{\rm pre}(\chi) + \left(v^2 - g\frac{\varphi^n}{M_{\rm Pl}^{n-2}}\right)^2 - \varepsilon v^4 \frac{\varphi}{M_{\rm Pl}} - \frac{1}{2}\kappa v^4 \frac{\varphi^2}{M_{\rm Pl}^2} + \frac{c_{\rm pot}}{2} V_{\rm pre}(\chi)\frac{\varphi^2}{M_{\rm Pl}^2}$$

$$\mathcal{L}_{\rm kin} = -\frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi$$

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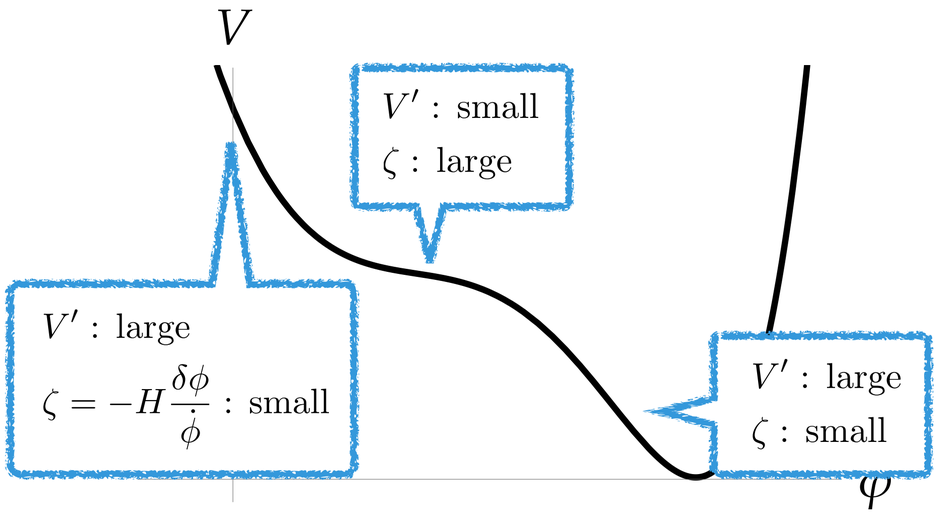


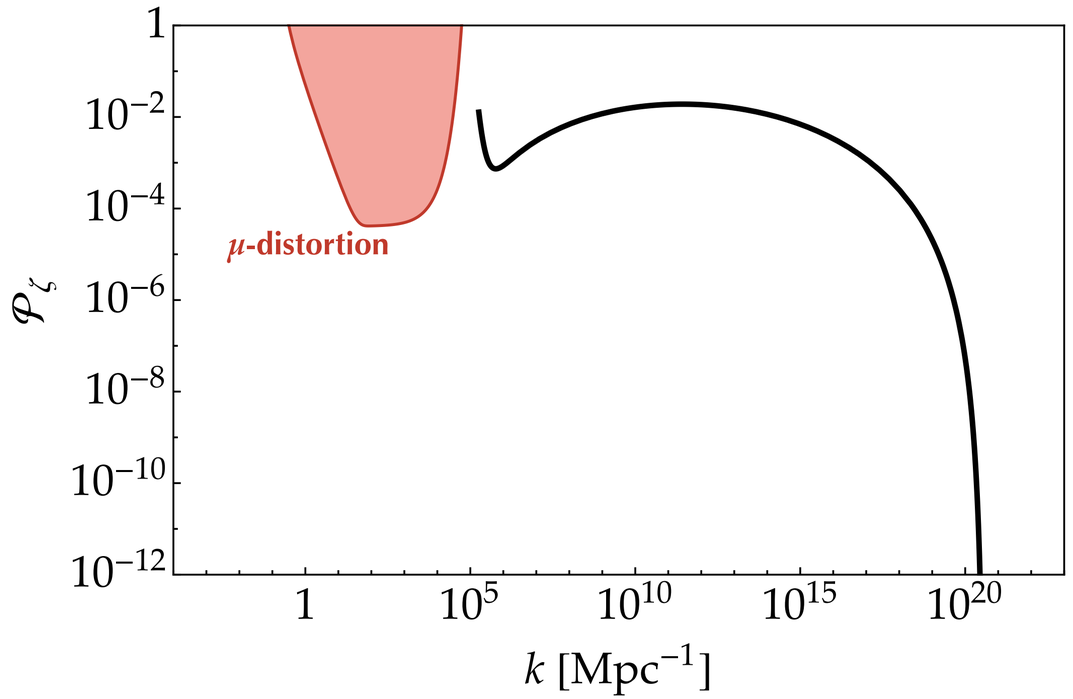
- $\kappa > 0$

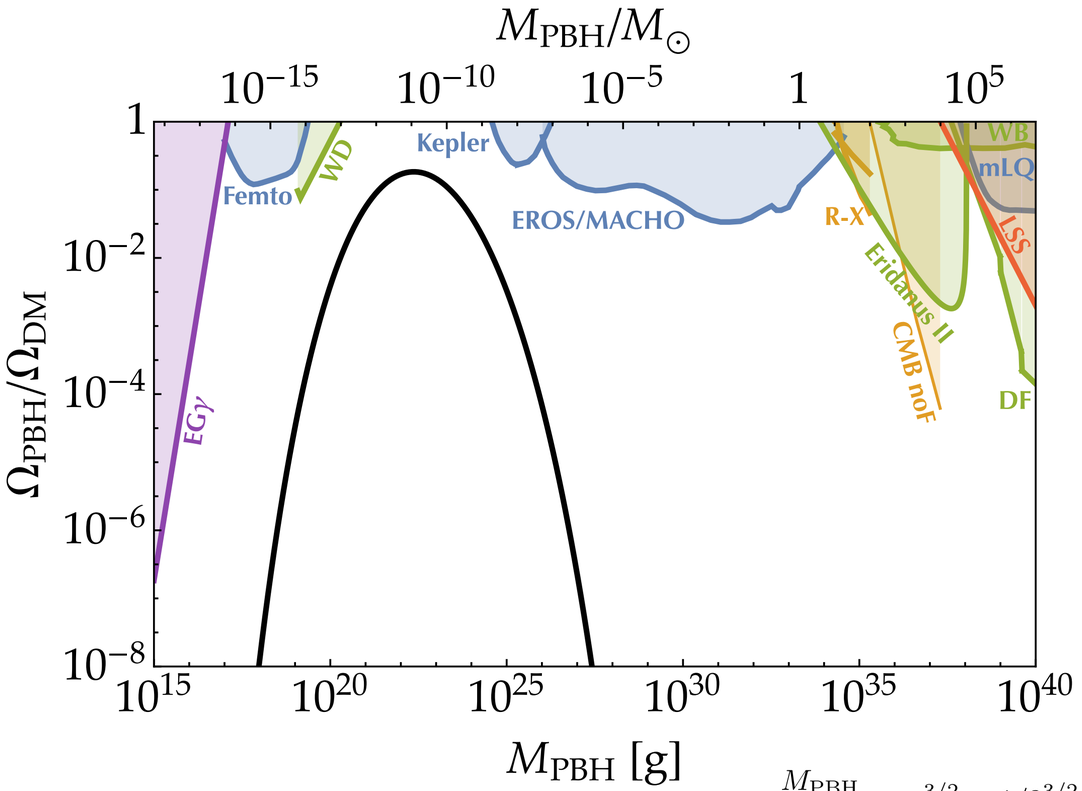
  - ζ is maximal at the beginning of new inflation
  - The precise value depends on pre-inflation

$\kappa < 0$ - slow-roll approximation -

Kawasaki, Kusenko, YT, Yanagida 2016

$$n = 3, \quad v = 10^{-4} M_{\rm Pl}, \quad \kappa = -0.44, \quad \varepsilon = 5.2 \times 10^{-8}, \quad g = 1.78 \times 10^{-3}$$





$$\frac{M_{\rm PBH}}{M_{\rm H}} = w^{3/2} = 1/3^{3/2}$$

$$\delta_{\rm th} = w = 1/3$$

Carr 1975

Gaussian window

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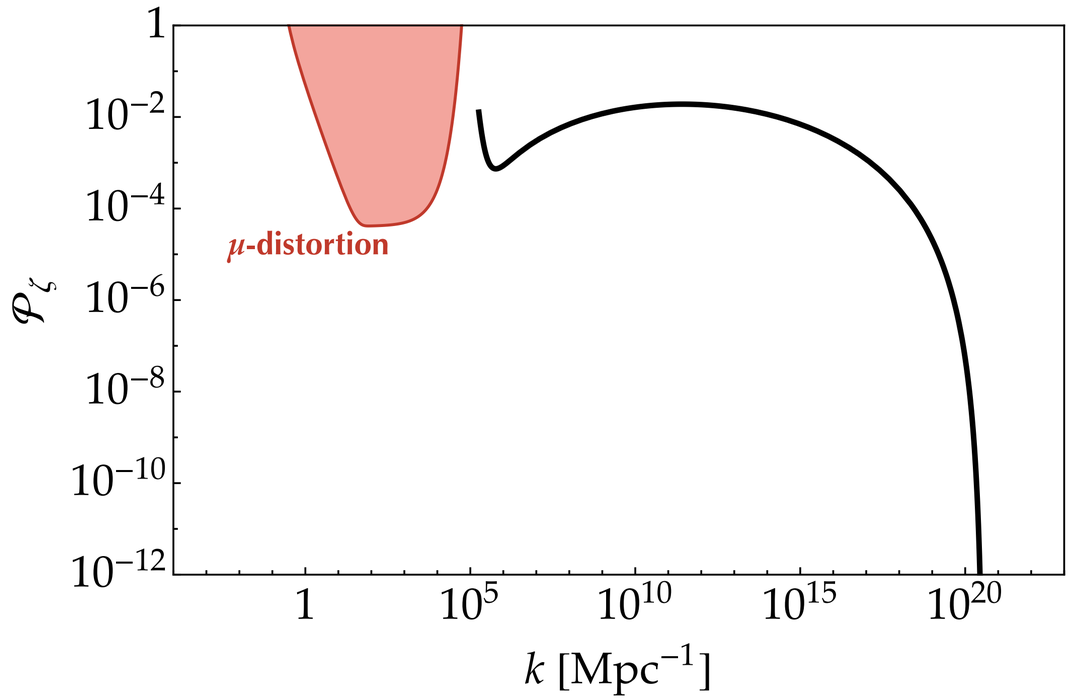

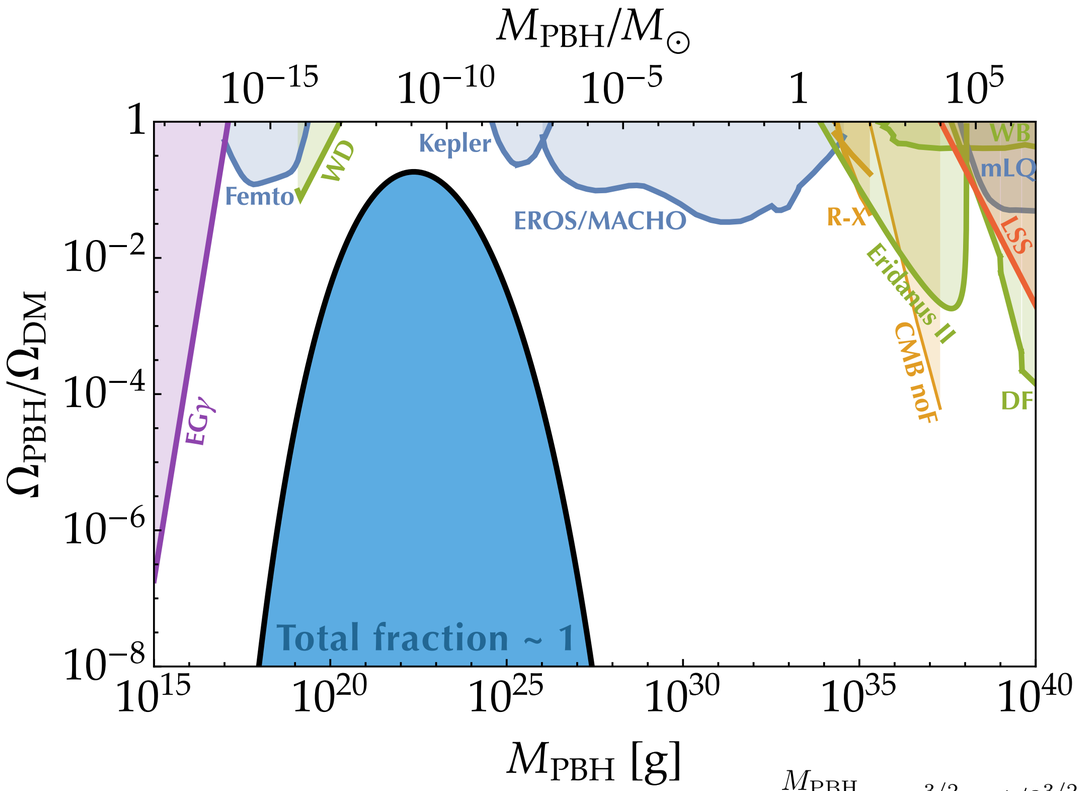

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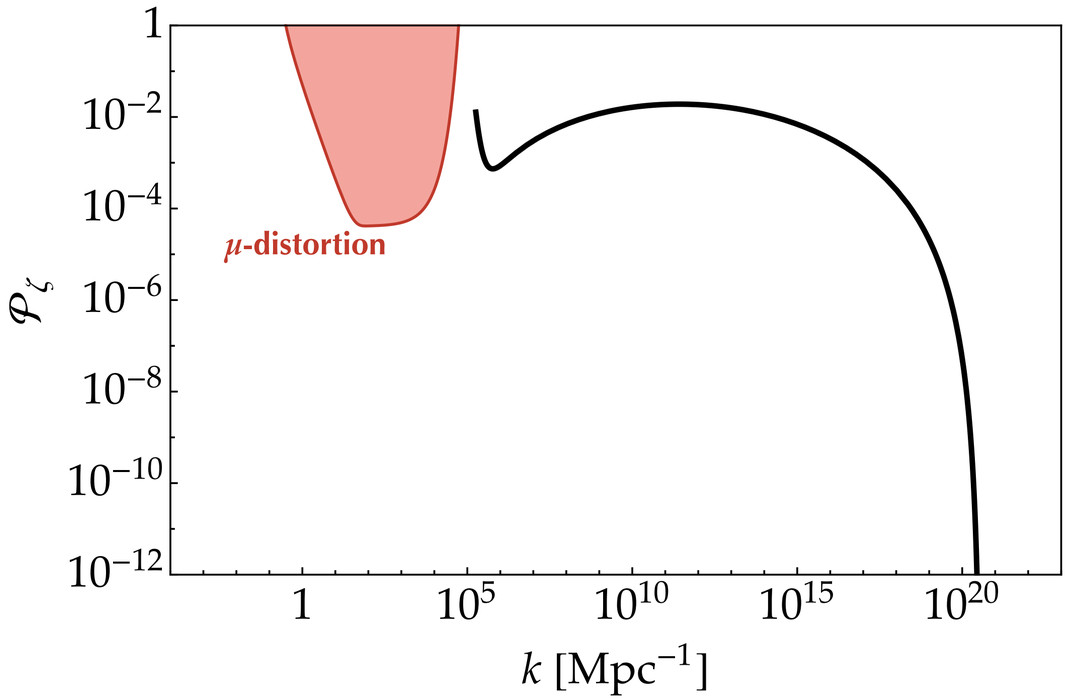

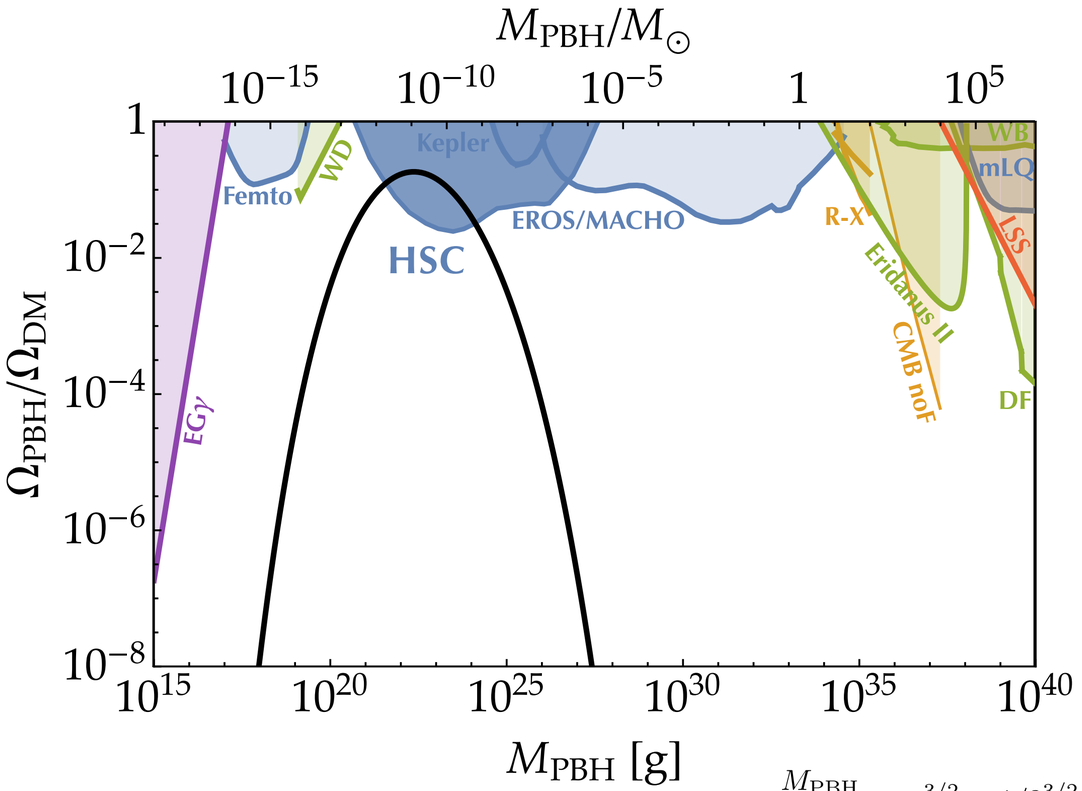

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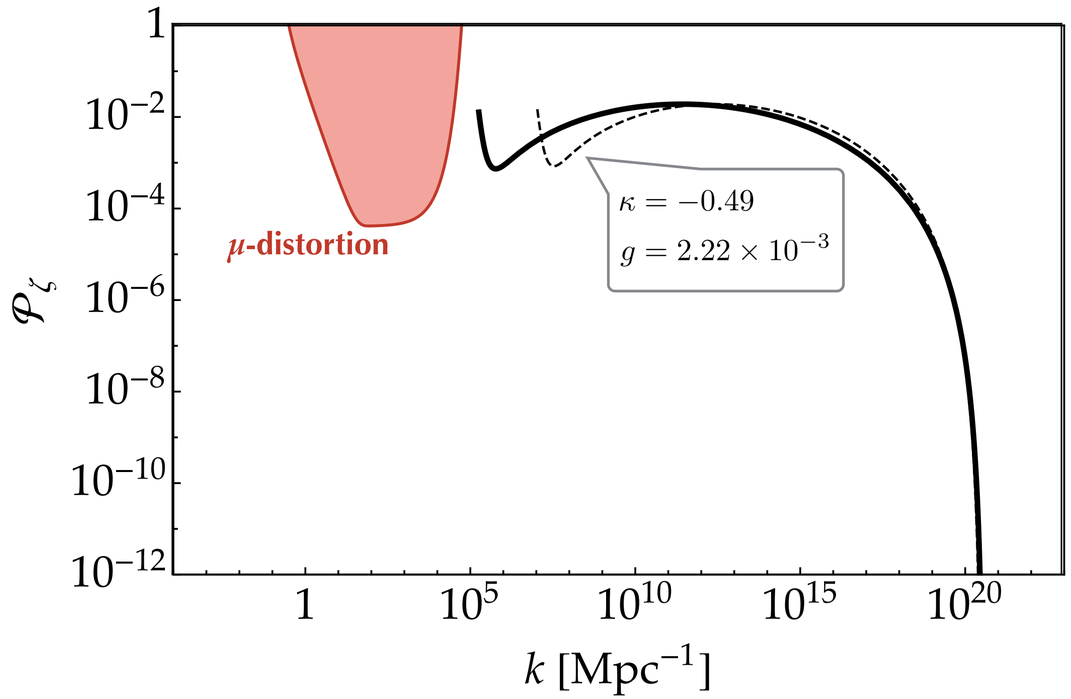

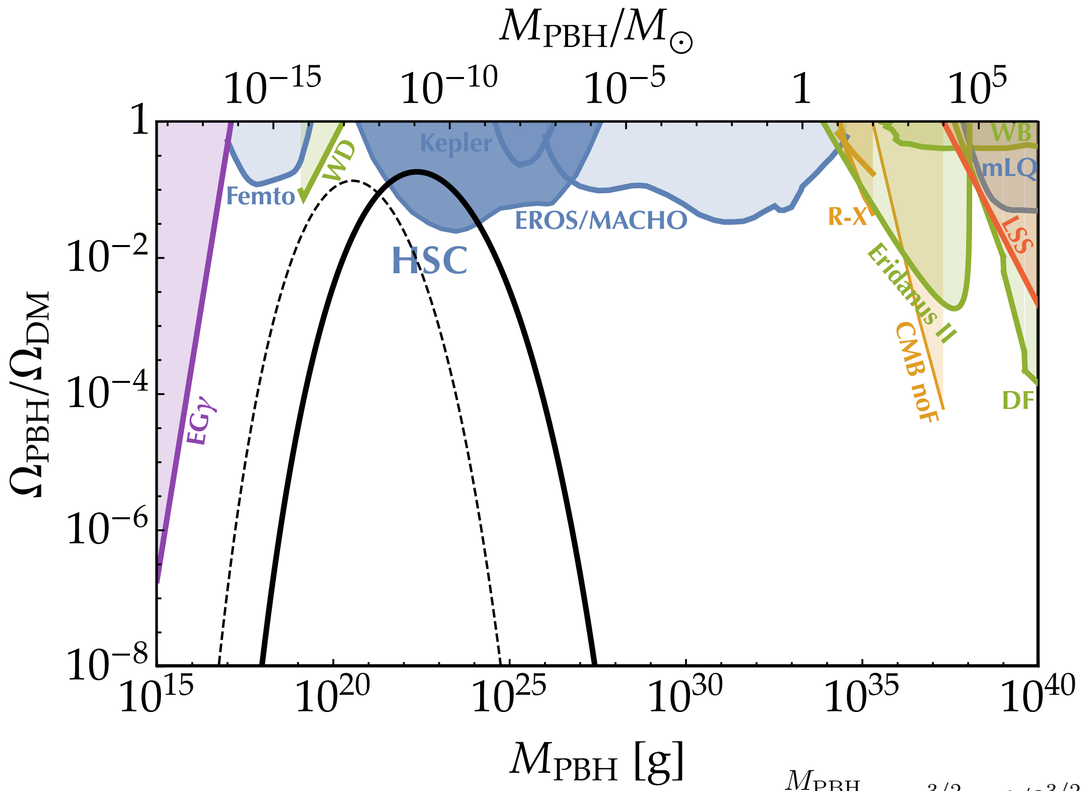

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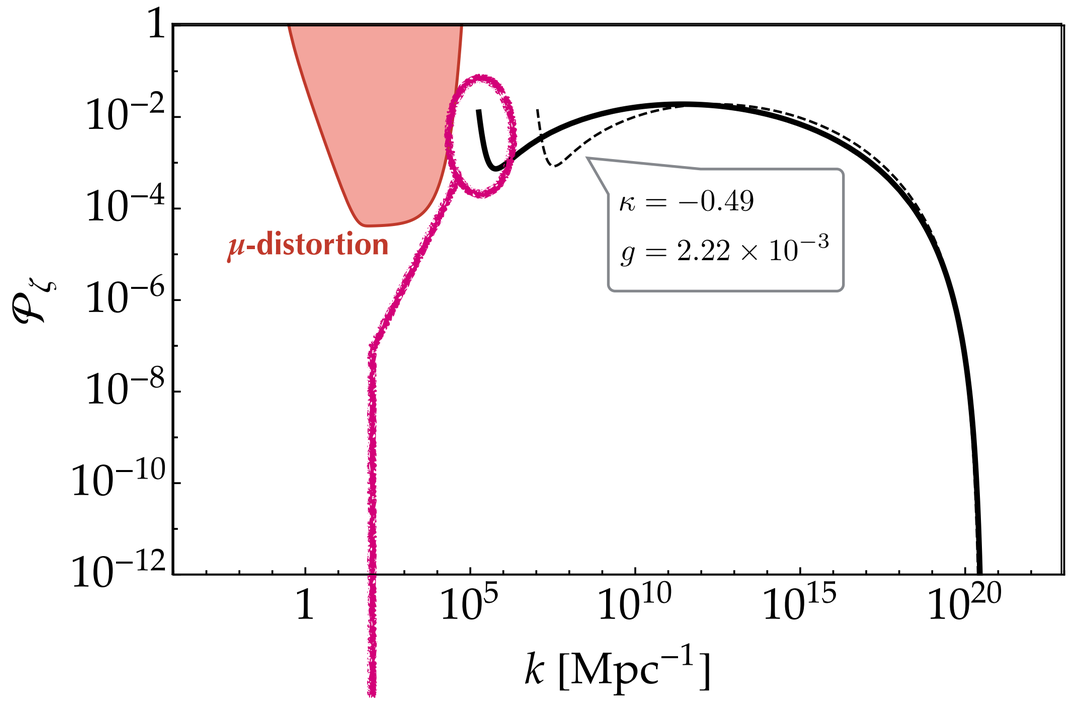

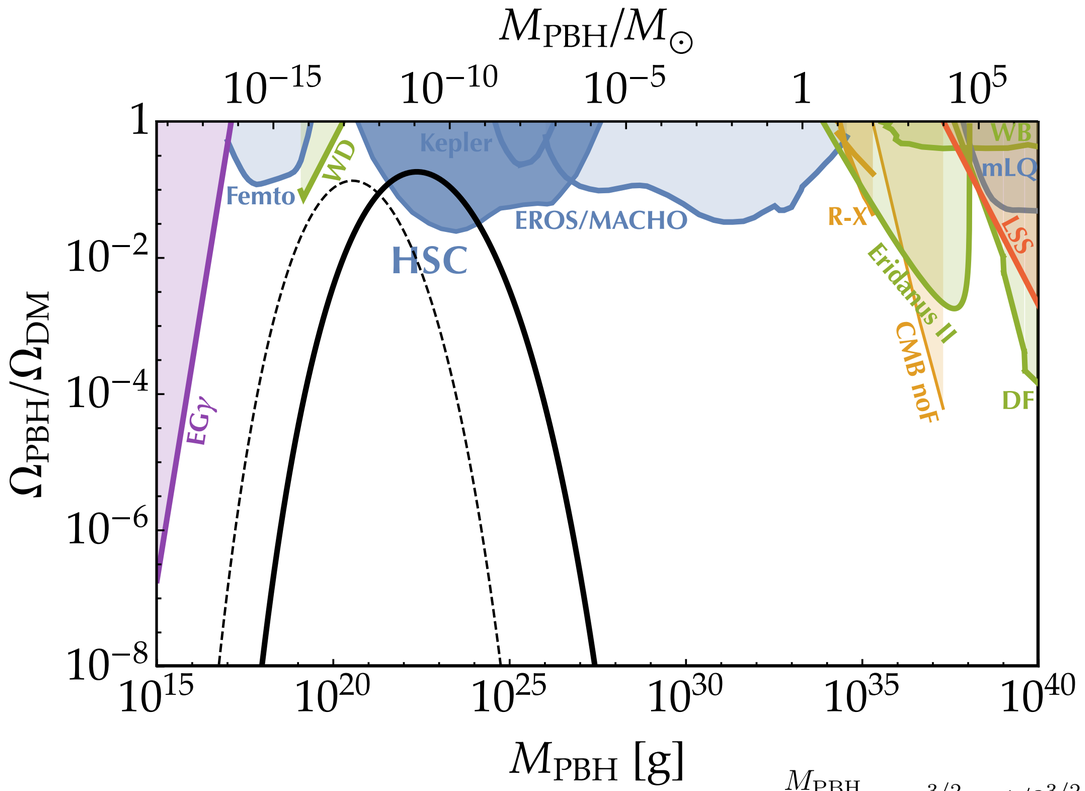

- multi-horizon-cross mode
- invalid slow-roll approx.

$$\frac{M_{\rm PBH}}{M_{\rm H}} = w^{3/2} = 1/3^{3/2}$$
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Carr 1975

Gaussian window

$\kappa > 0$ - full numerical simulation -

Inomata, Kawasaki, Mukaida, YT, Yanagida 2016, 2017

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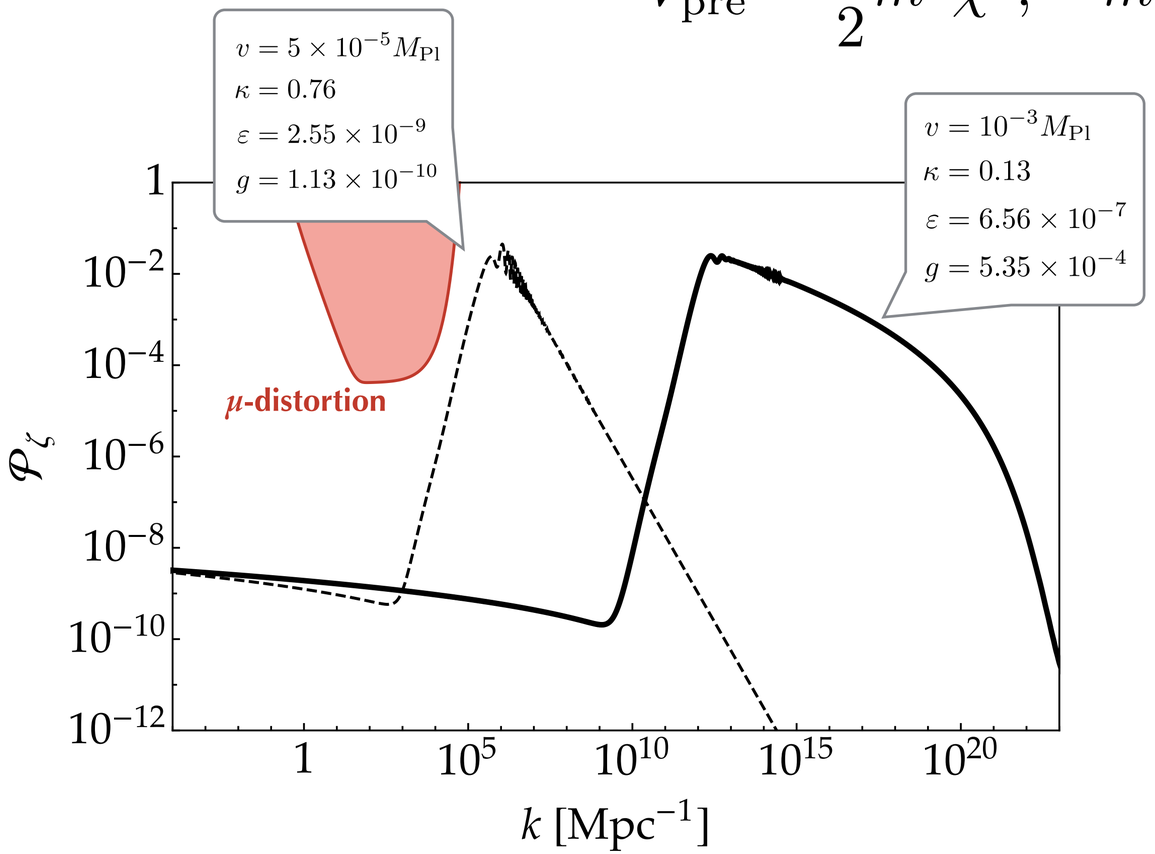
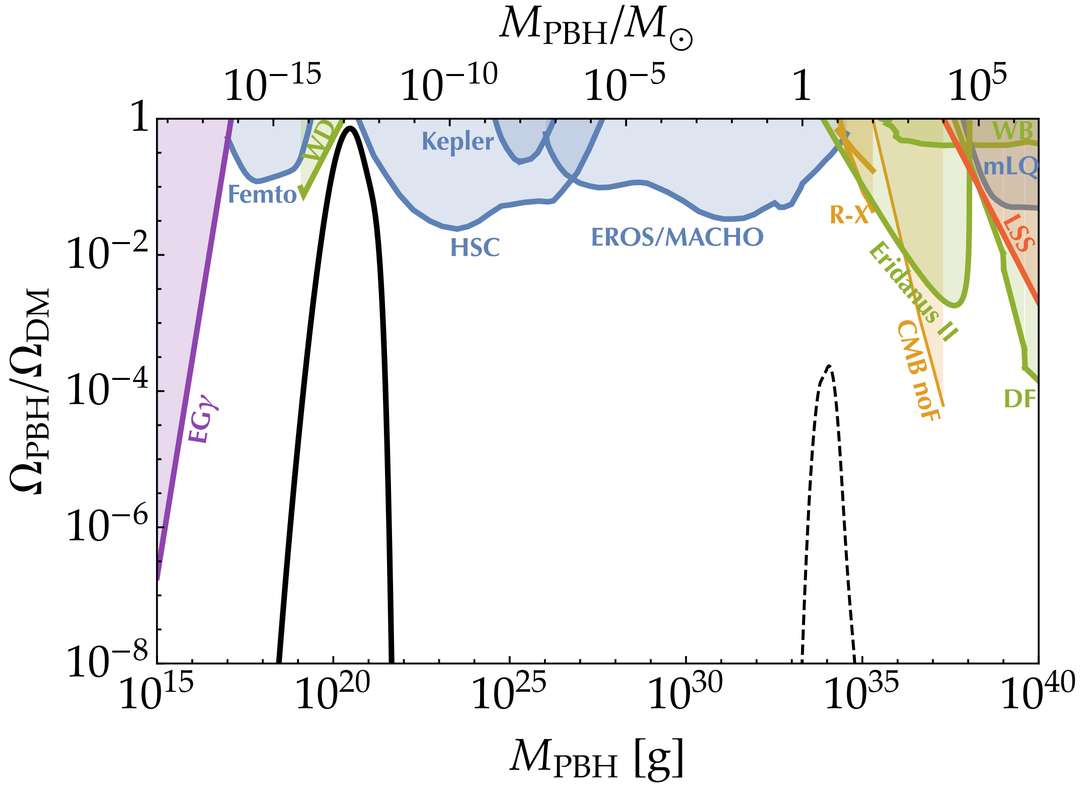
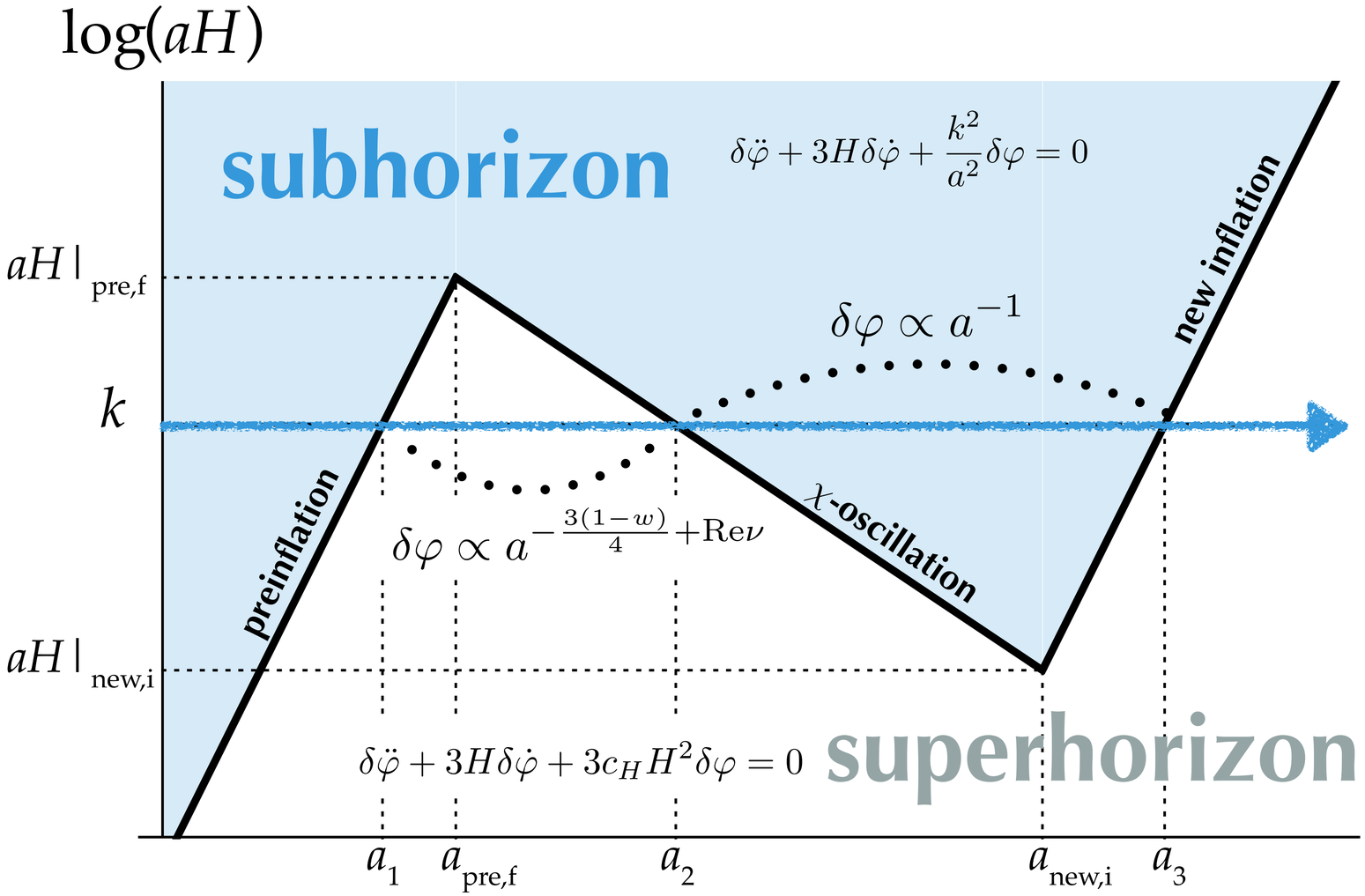




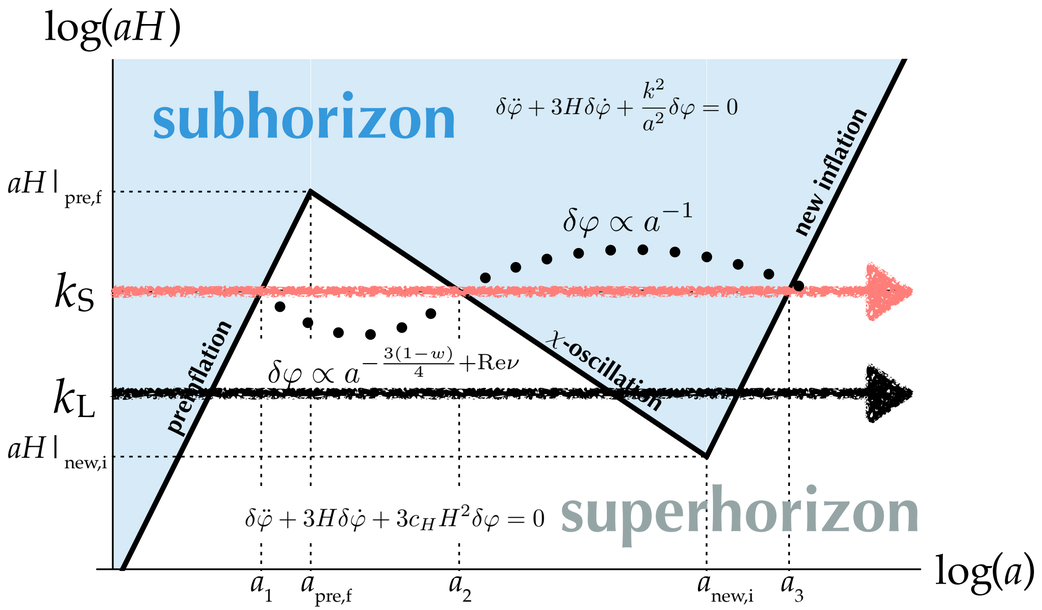
Multi-horizon-cross mode



$$\nu = \sqrt{\left(\frac{3(1-w)}{4}\right)^2 - 3c_H}$$

$$w = p/\rho$$

Multi-horizon-cross mode



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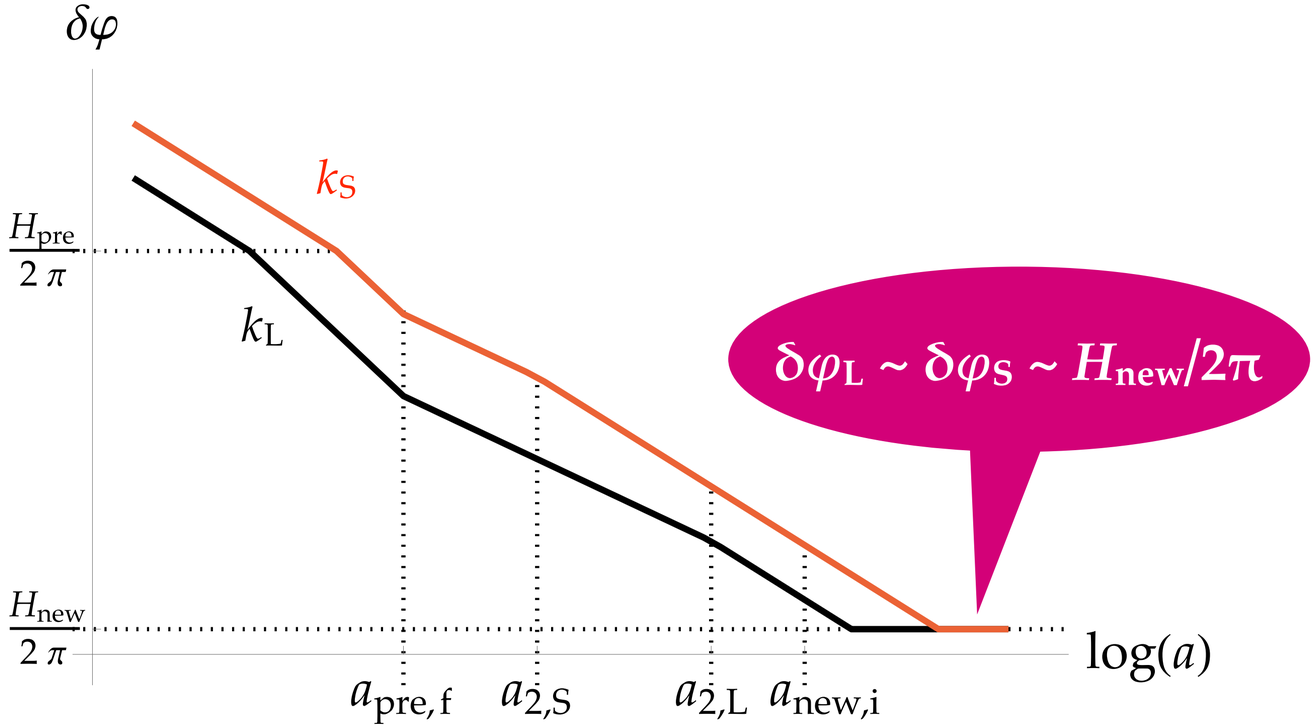
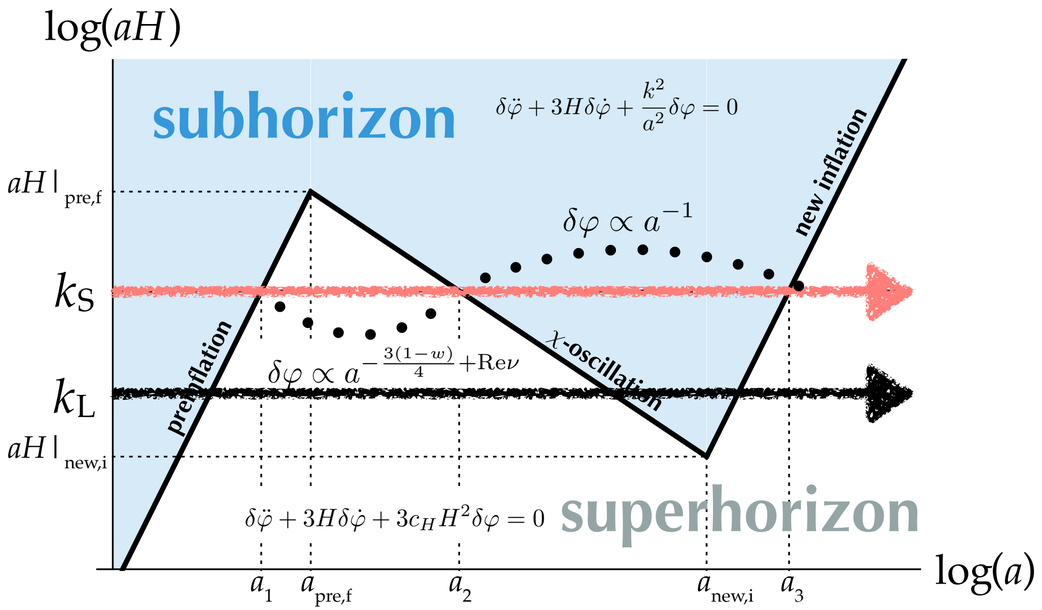
✱ massive (Re$\nu=0$)



Multi-horizon-cross mode



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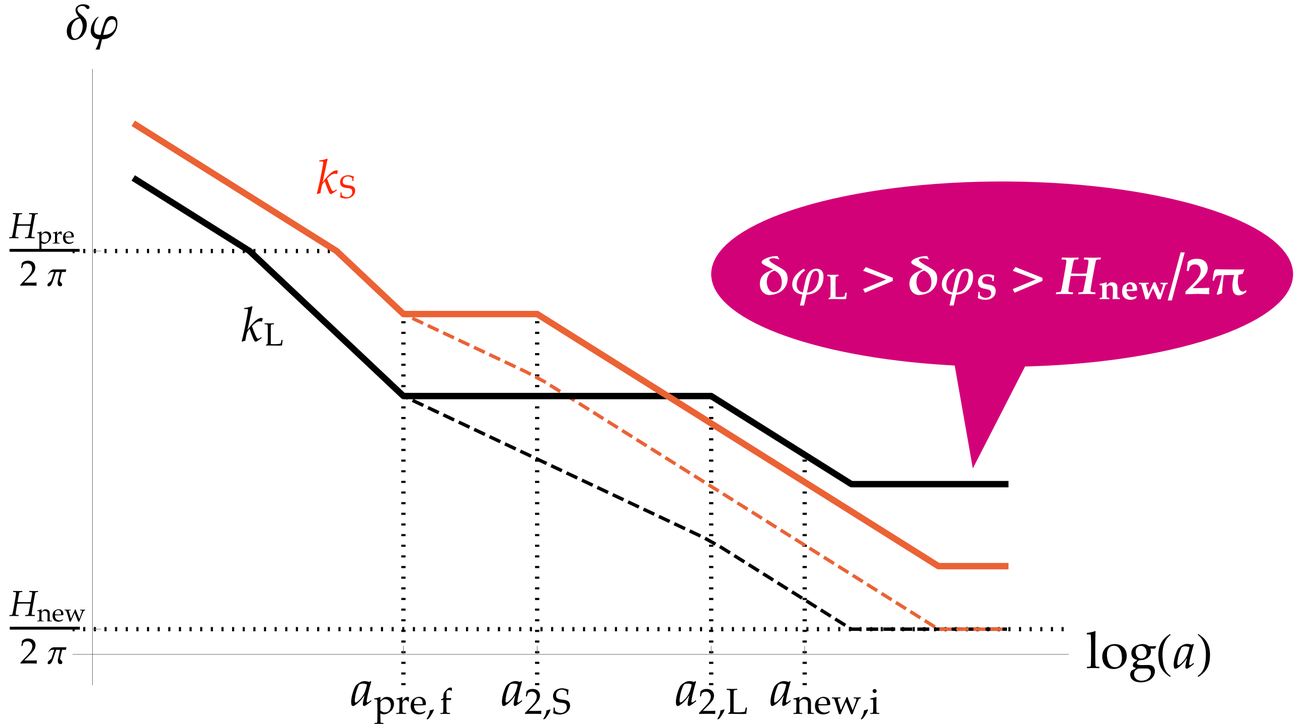
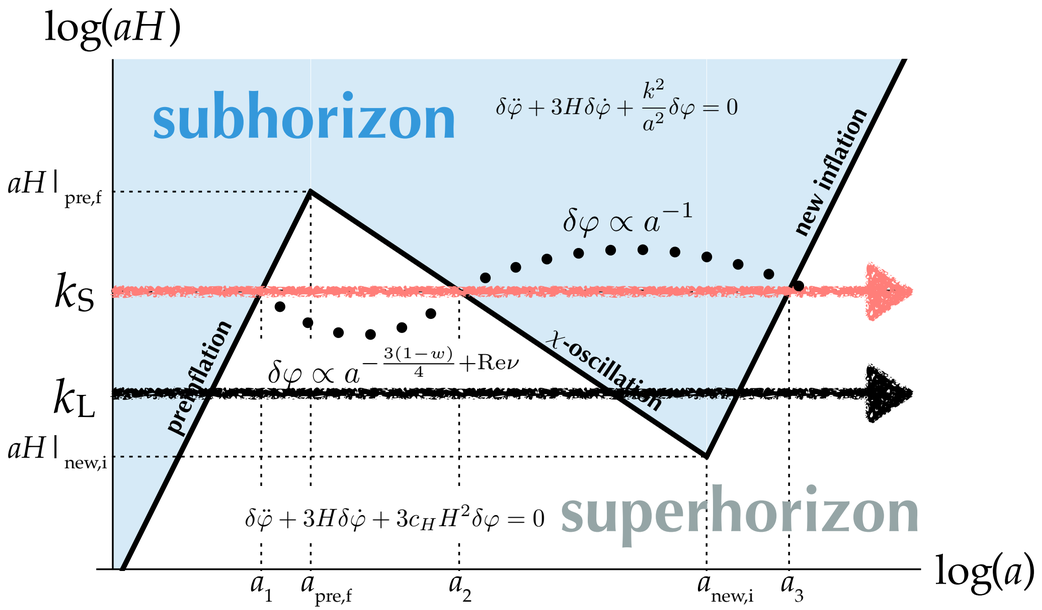


Multi-horizon-cross mode



$$\nu=\sqrt{\left(\frac{3(1-w)}{4}\right)^2-3c_H}\qquad w=p/\rho$$

$$\mathcal{L}_{\rm kin}\supset\frac{c_{\rm kin}}{4}\frac{\varphi^2}{M_{\rm Pl}^2}\partial_\mu\chi\partial^\mu\chi\quad\rightarrow\quad c_{H,\rm osc}\simeq\frac{c_{\rm pot}+c_{\rm kin}}{2}$$

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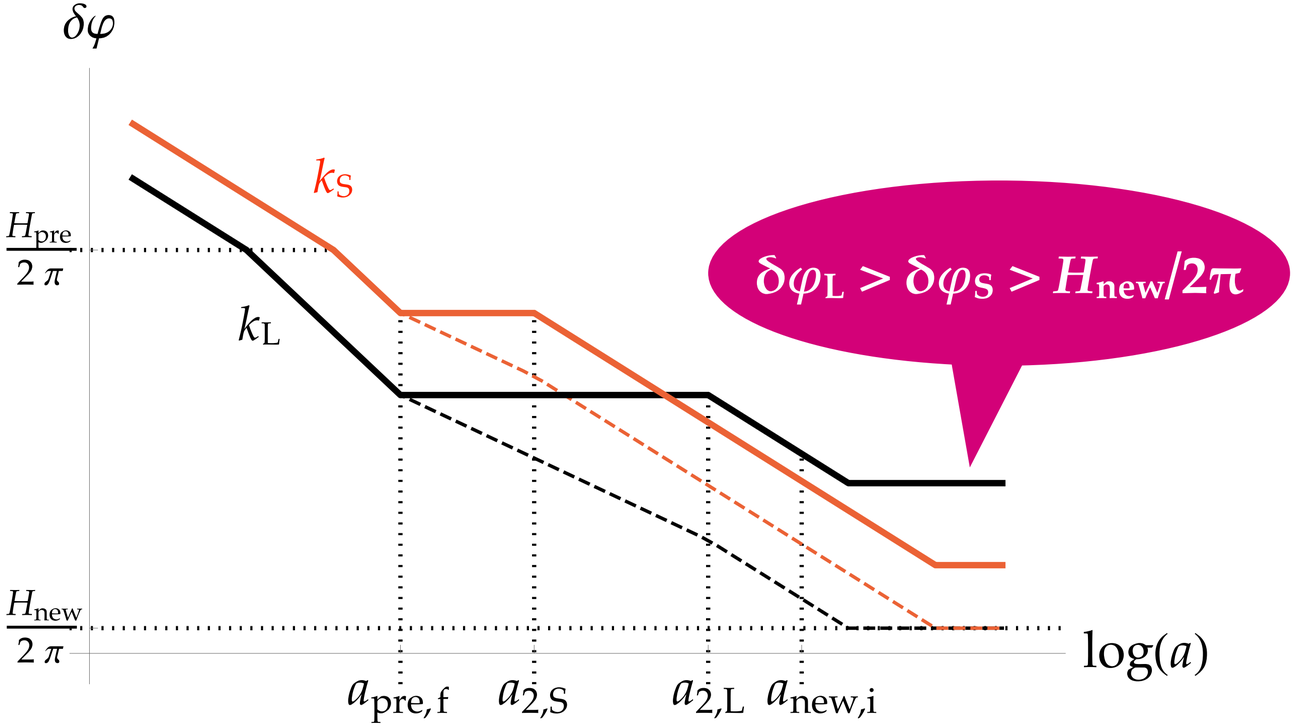
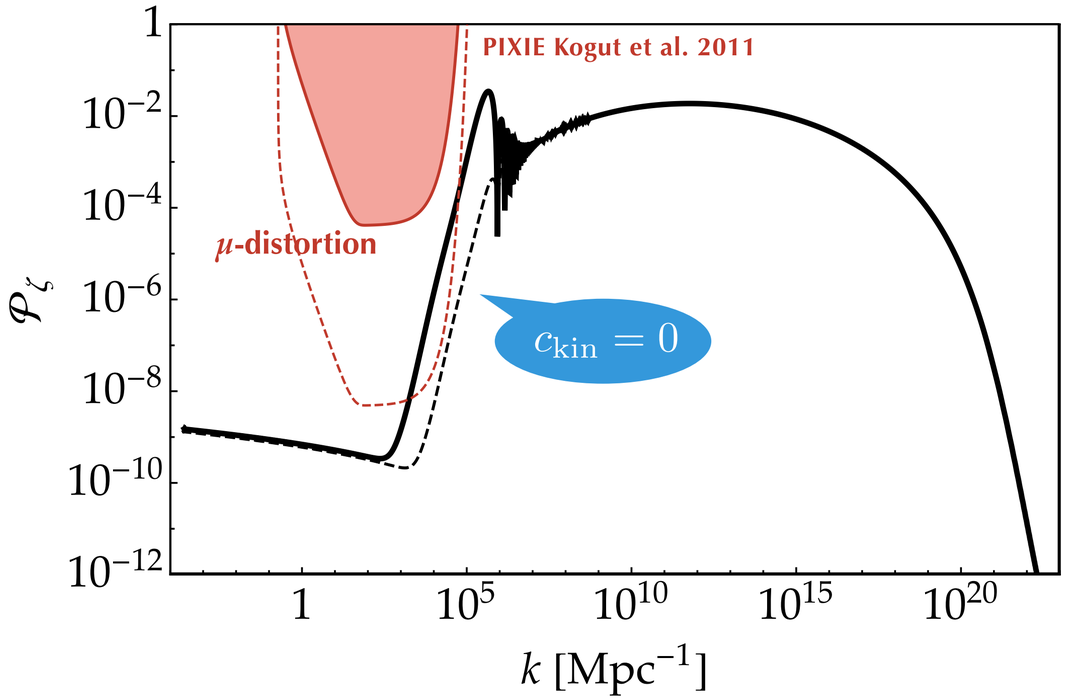
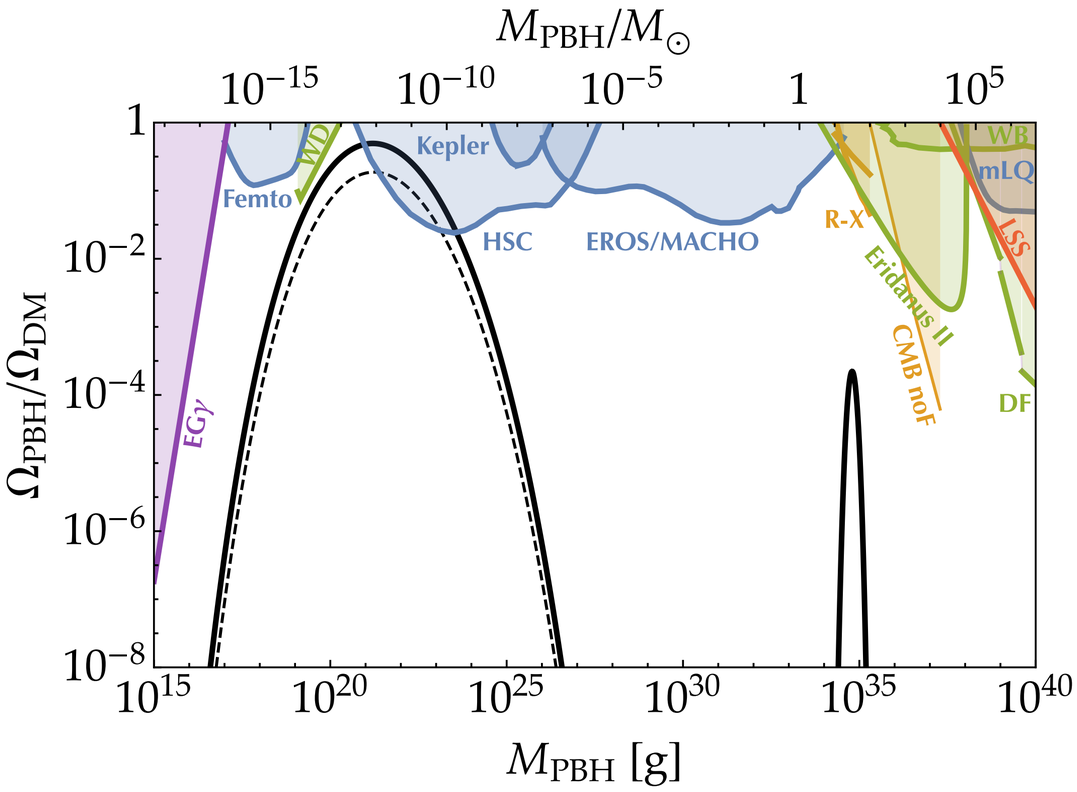


$30M_\odot$ Primordial Black Hole

Inomata, Kawasaki, Mukaida, YT, Yanagida 2016

$$V_{\rm pre} = \frac{1}{2}m^2\chi^2, \quad m = 10^{-5}M_{\rm Pl},$$
$$n = 3, \quad v = 10^{-4}M_{\rm Pl}, \quad \varepsilon = 3.68 \times 10^{-8}, \quad \kappa = -0.335$$
$$g = 1.587 \times 10^{-3}, \quad c_{\rm pot} = 1, \quad c_{\rm kin} = -0.917$$





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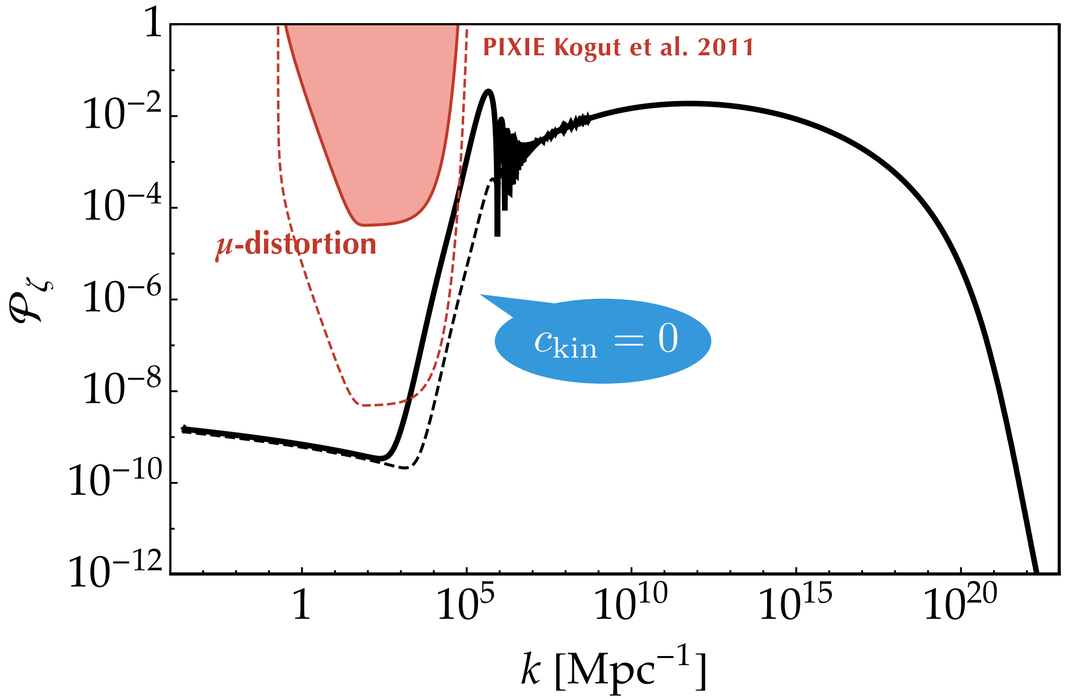

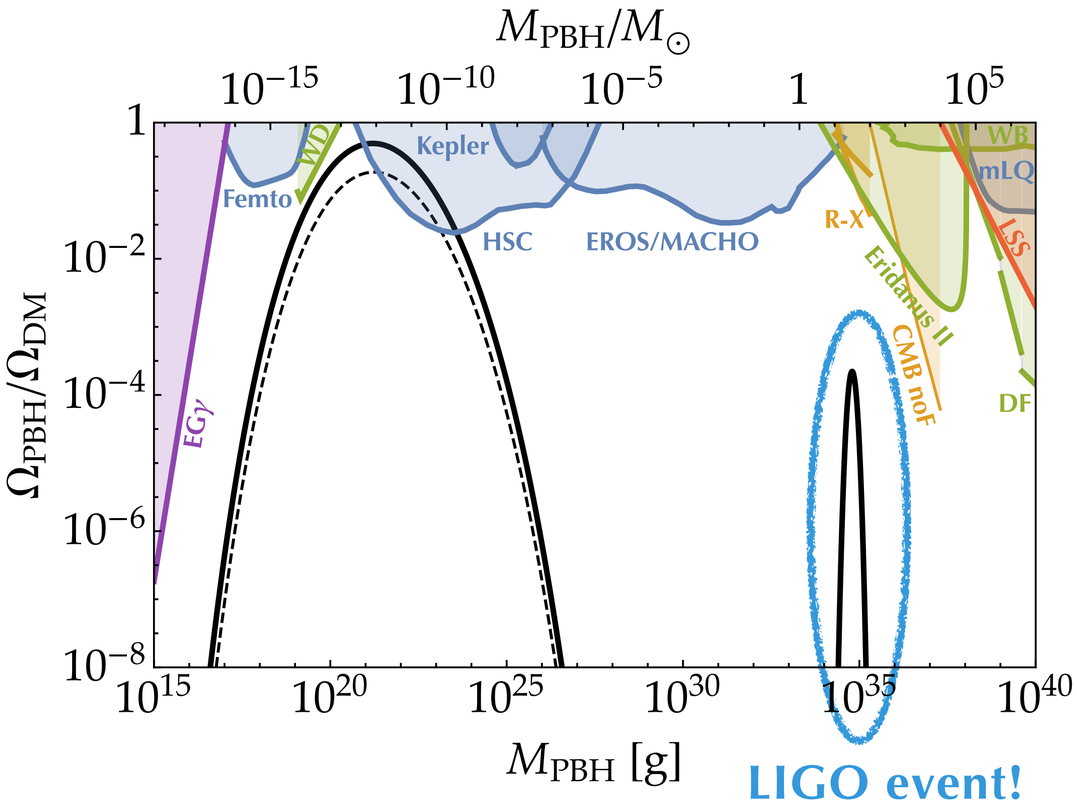

Secondary GW

- Saito & Yokoyama 2009, 2011

large density P. → **PBH**

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2nd order eff. @ HC

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wavelength　Amp. of tensor　Amp. of scalar

$$\rho_{\rm GW}|_{\rm RD} \sim H^2 \underline{(M_{\rm Pl}h)^2} \sim H^2 M_{\rm Pl}^2 \Psi^4$$

canonical mode

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$$\left.\frac{\rho_{\rm GW}}{\rho_{\rm r}}\right|_{\rm RD} \sim \frac{H^2 M_{\rm Pl}^2 \Psi^4}{H^2 M_{\rm Pl}^2} \sim \Psi^4 = \left(\frac{2}{3}\zeta\right)^4$$

Newtonian gauge

$$\Omega_{\rm GW} h^2 \sim \frac{16}{81} \Omega_{\rm r} h^2 \mathcal{P}_\zeta^2 \sim 10^{-9} \left(\frac{\mathcal{P}_\zeta}{0.01}\right)^2$$

4×10^{-5}

PTA const. on ~pc

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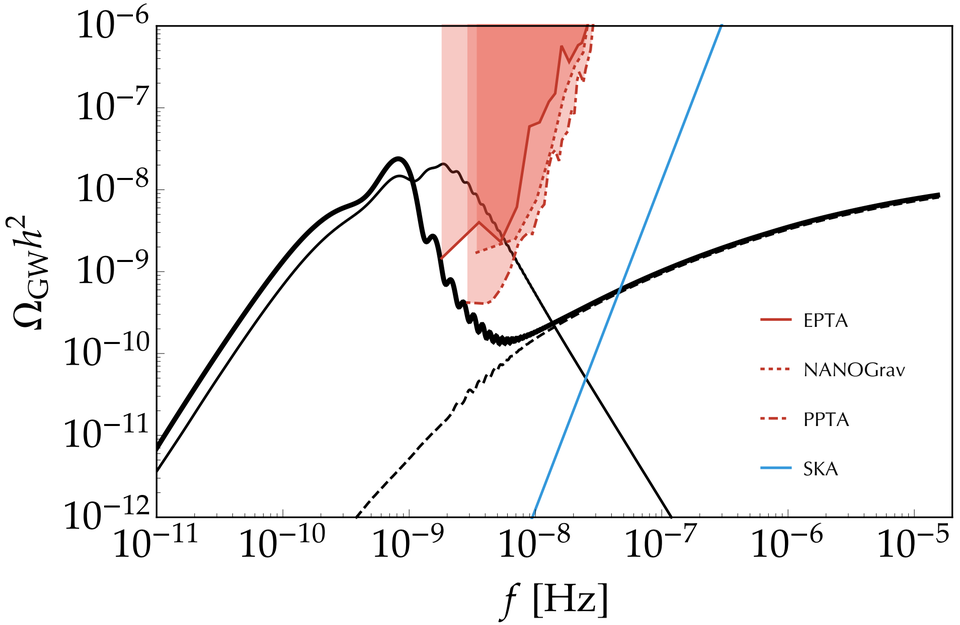
numerical result



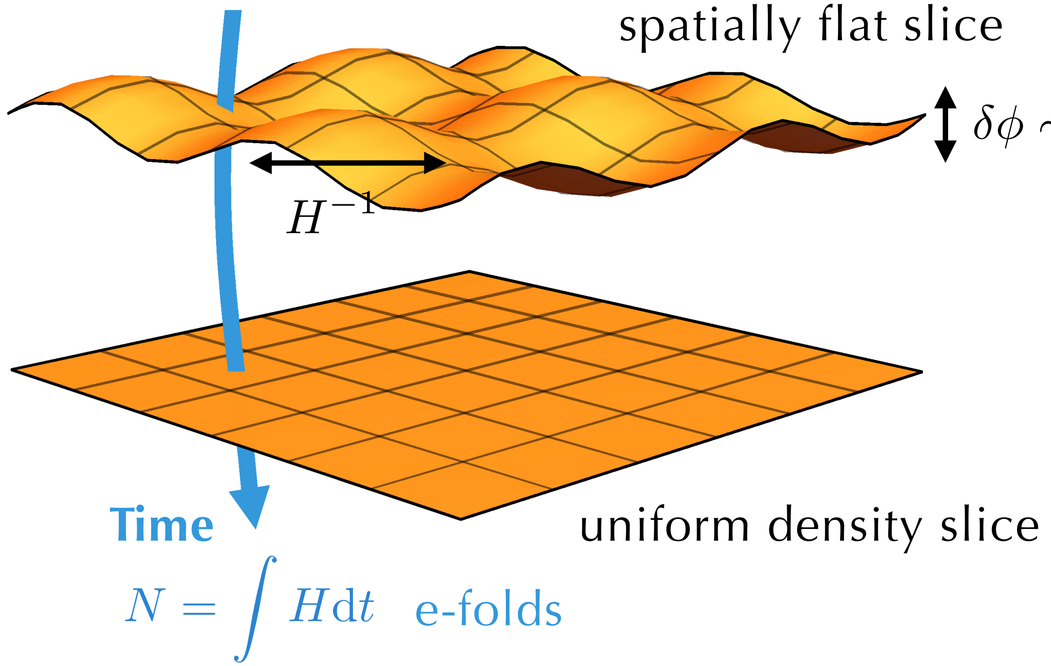
Inomata, Kawasaki, Mukaida, YT, Yanagida 2016

Conclusions

- In hybrid inflation, detectably massive PBHs are inevitably overproduced.
- In double inflation, PBH can be produced in any mass and in any abundance.
  Furthermore, if Hubble induced mass is canceled during oscillation phase,
  another peak can be made on $30M_\odot$.
  These scenario would be tested by CMB spectral dist. or secondary GW
  with future PIXIE/PRISM or SKA collaborations.

Appendix

δN formalism Starobinsky 1985



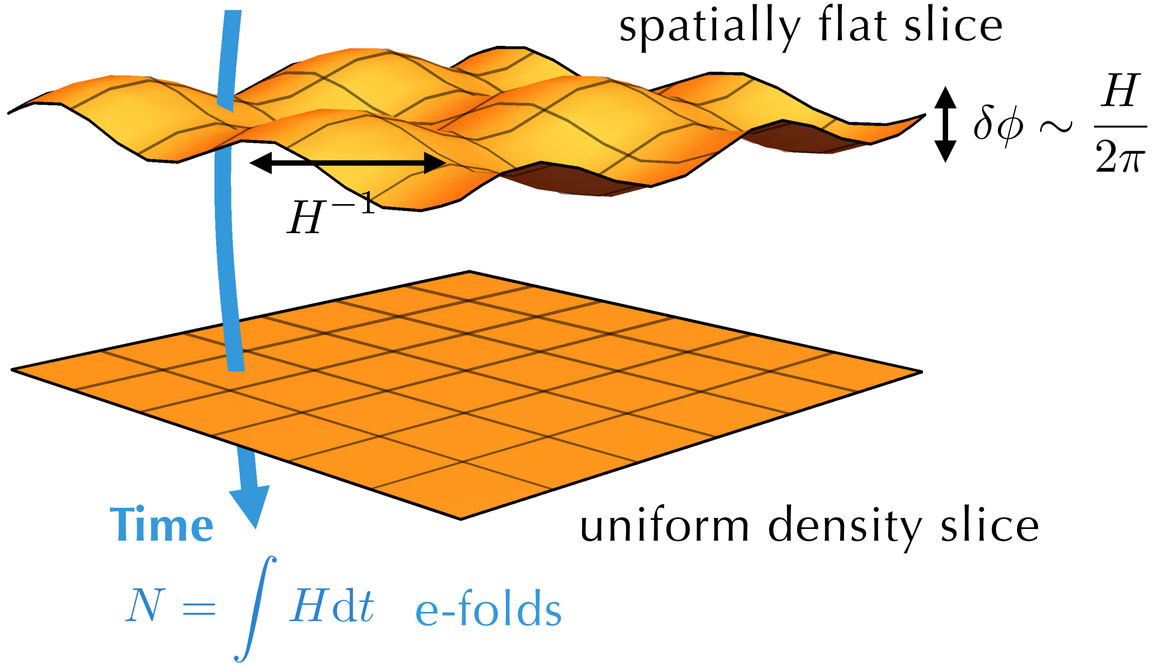
$N = \int H \mathrm{d}t$ e-folds

Fluctuation of elapsed time is the curvature perturbation conserved on superhorizon scale:

$$\delta N = \zeta = \psi + \frac{\delta\rho}{3(\rho + p)}$$

non-perturbative proof Lyth, Malik, Sasaki 2004

δN formalism Starobinsky 1985



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- Simple Example
  - single-field linear perturbation

$$\zeta(\mathbf{x}) = \delta N(\mathbf{x}) \simeq \frac{\mathrm{d}N}{\mathrm{d}\phi}\delta\phi(\mathbf{x})$$

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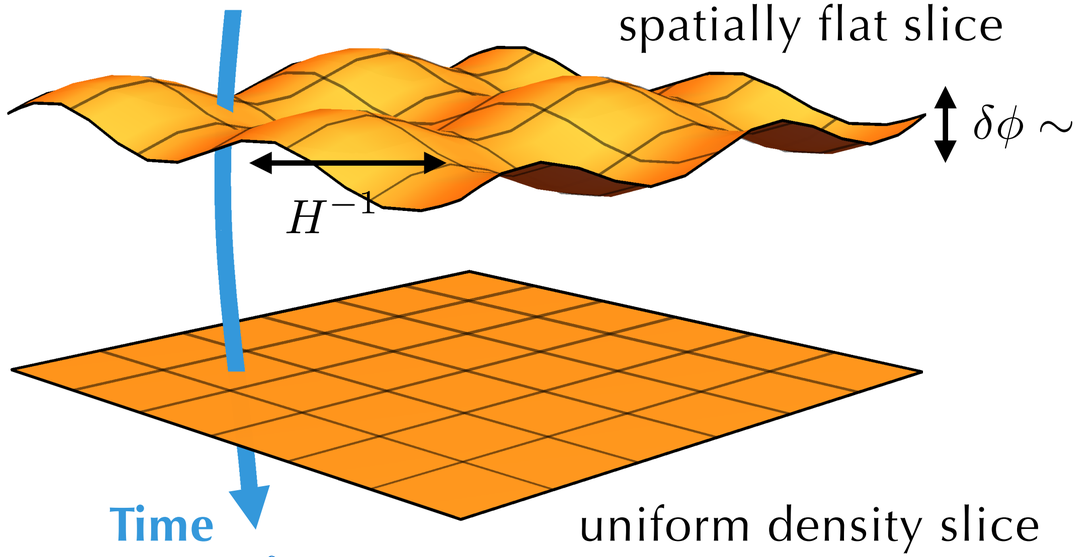

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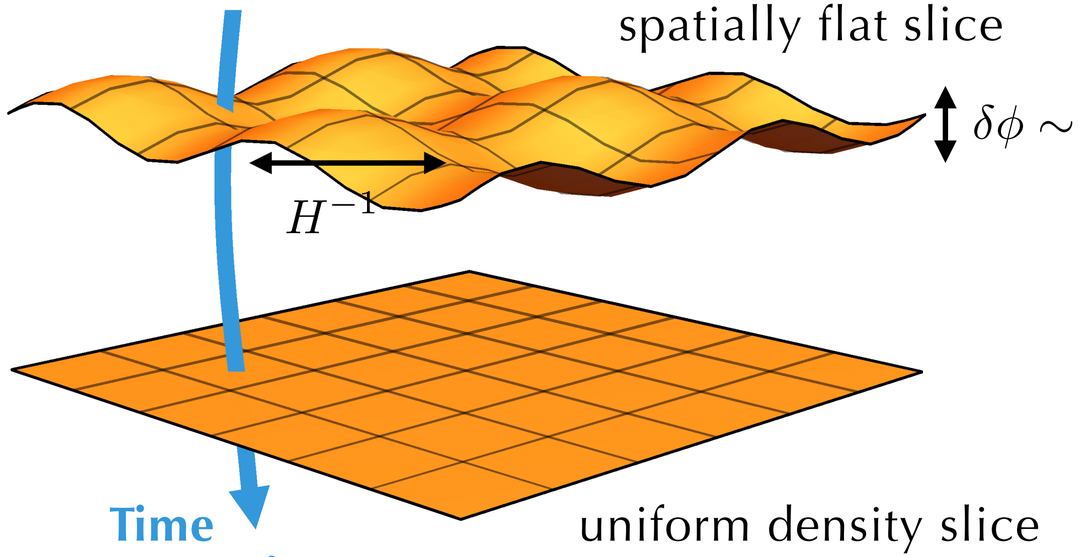
$$\rightarrow \quad \mathcal{P}_\zeta(k) \simeq \left(\frac{\mathrm{d}N}{\mathrm{d}\phi}\right)^2 \mathcal{P}_\phi(k) = \left(\frac{\mathrm{d}N}{\mathrm{d}\phi}\right)^2 \left(\frac{H}{2\pi}\right)^2$$

+

- slow-roll EoM $\quad \frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{V'}{V}$

δN formalism Starobinsky 1985



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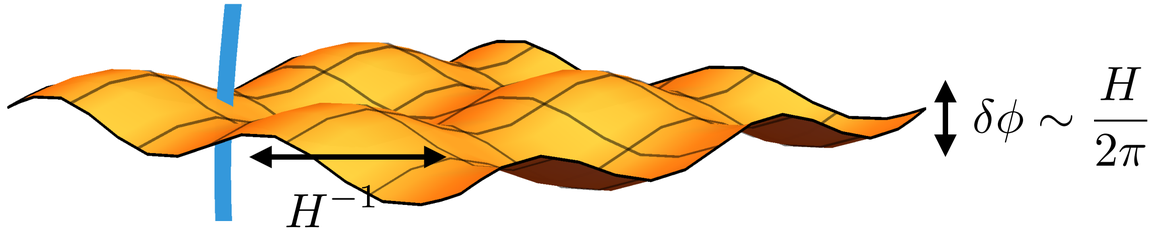
$$\mathcal{P}_\zeta(k) \simeq \left(\frac{V}{V'}\right)^2 \left(\frac{H}{2\pi}\right)^2 \Bigg|_{k=aH}$$

flat (small V') or high (large V) ↔ large ζ

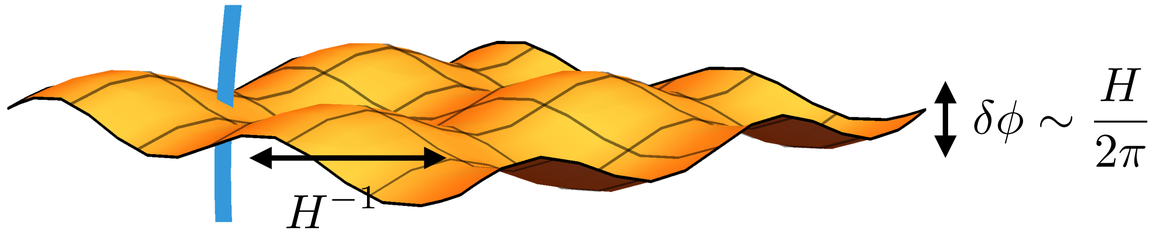
Stochastic formalism Starobinsky 1986



classical b.g.

$$\phi_{\rm IR}(t,\mathbf{x}) = \int \frac{{\rm d}^3k}{(2\pi)^3}\theta(\epsilon aH-k)\phi_{\mathbf{k}}(t){\rm e}^{i\mathbf{k}\cdot\mathbf{x}}, \quad \epsilon \ll 1$$

Stochastic formalism Starobinsky 1986



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- EoM for $\phi_{\rm IR}$

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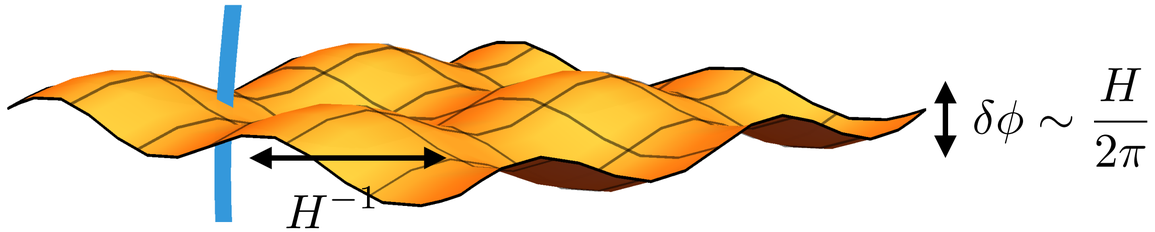
ξ can be interpreted as white and Hubble-patch-independent Gaussian noise

Stochastic formalism Starobinsky 1986



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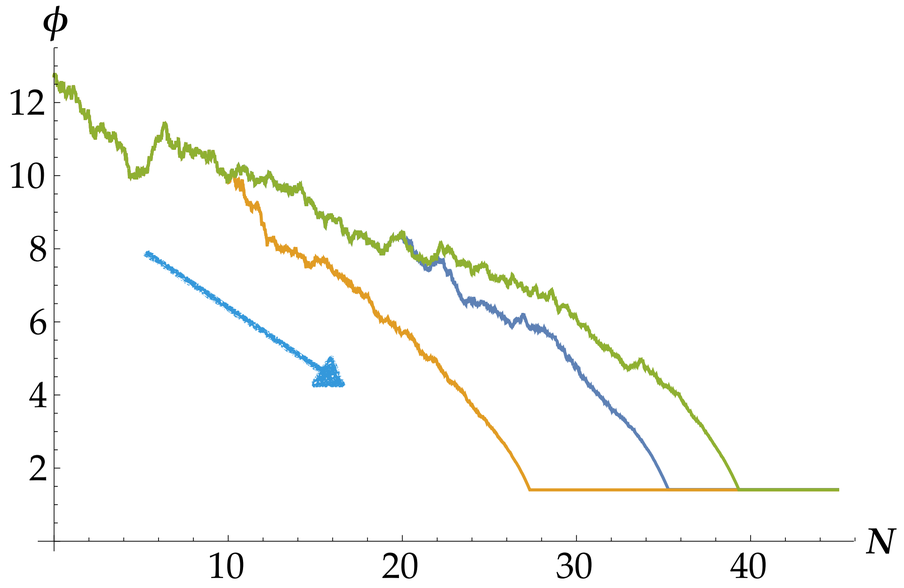
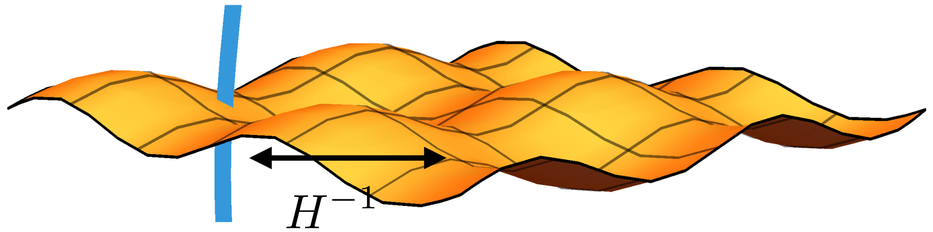


Stochastic formalism Starobinsky 1986



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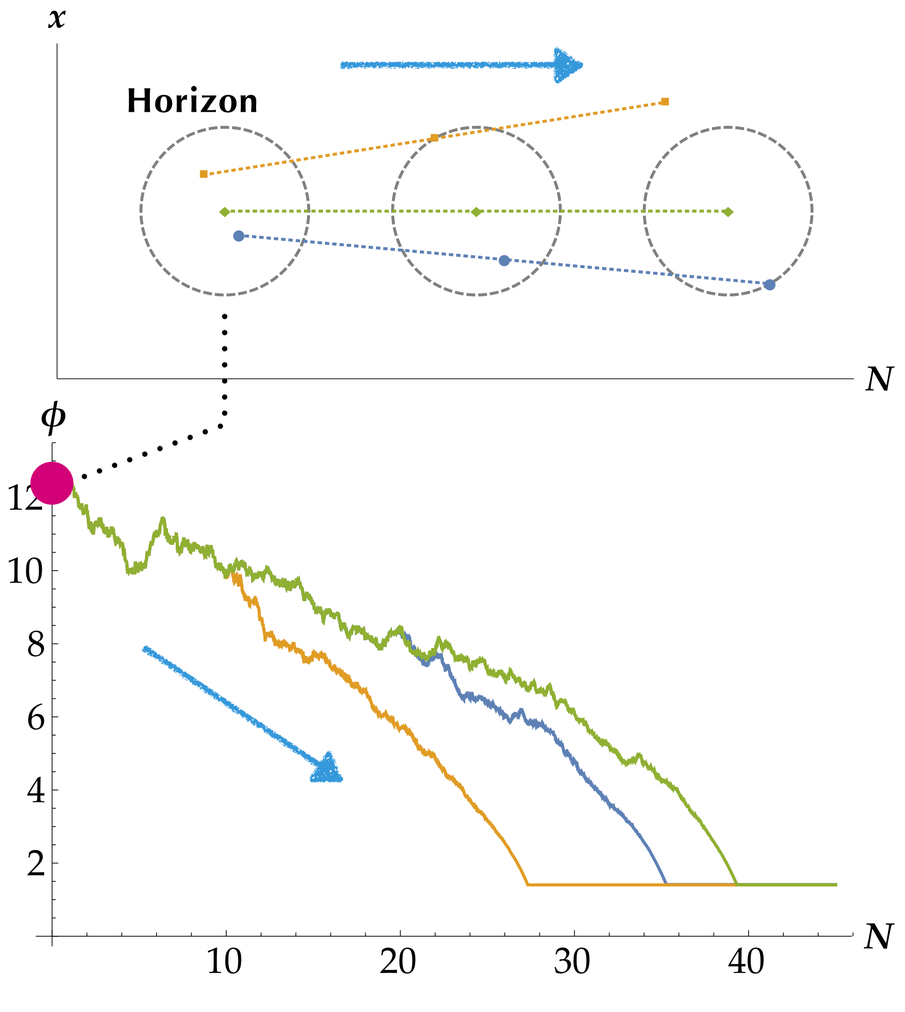
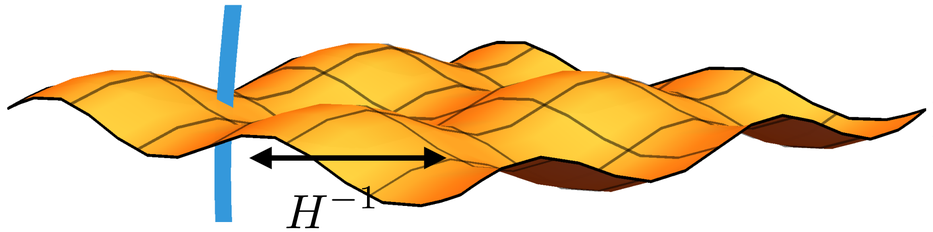


Stochastic formalism Starobinsky 1986



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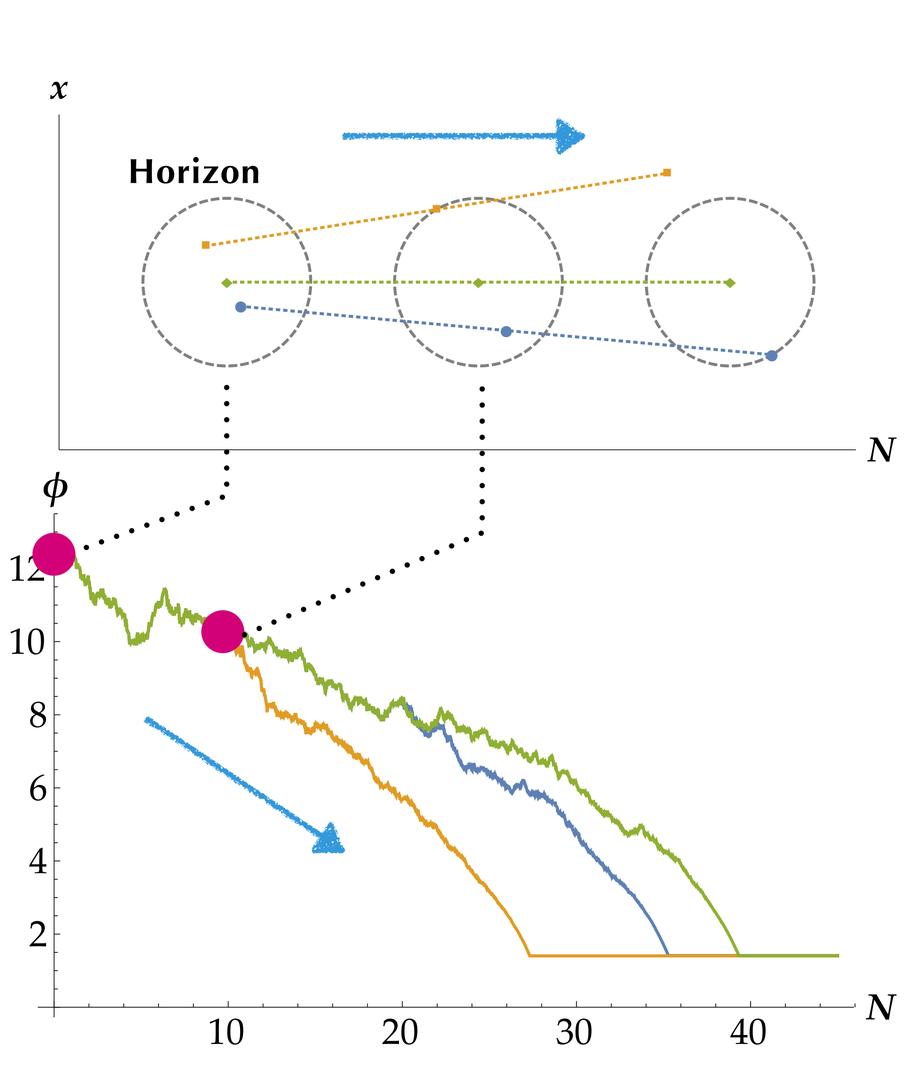
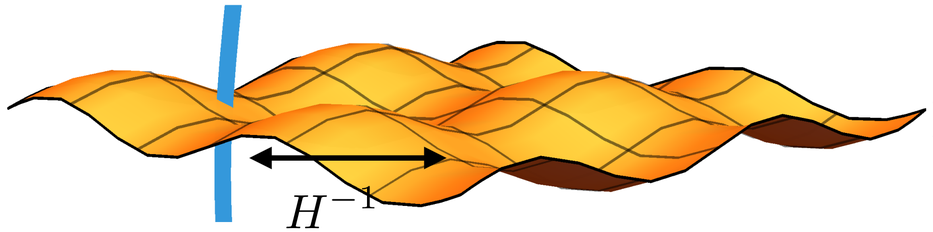


Stochastic formalism Starobinsky 1986



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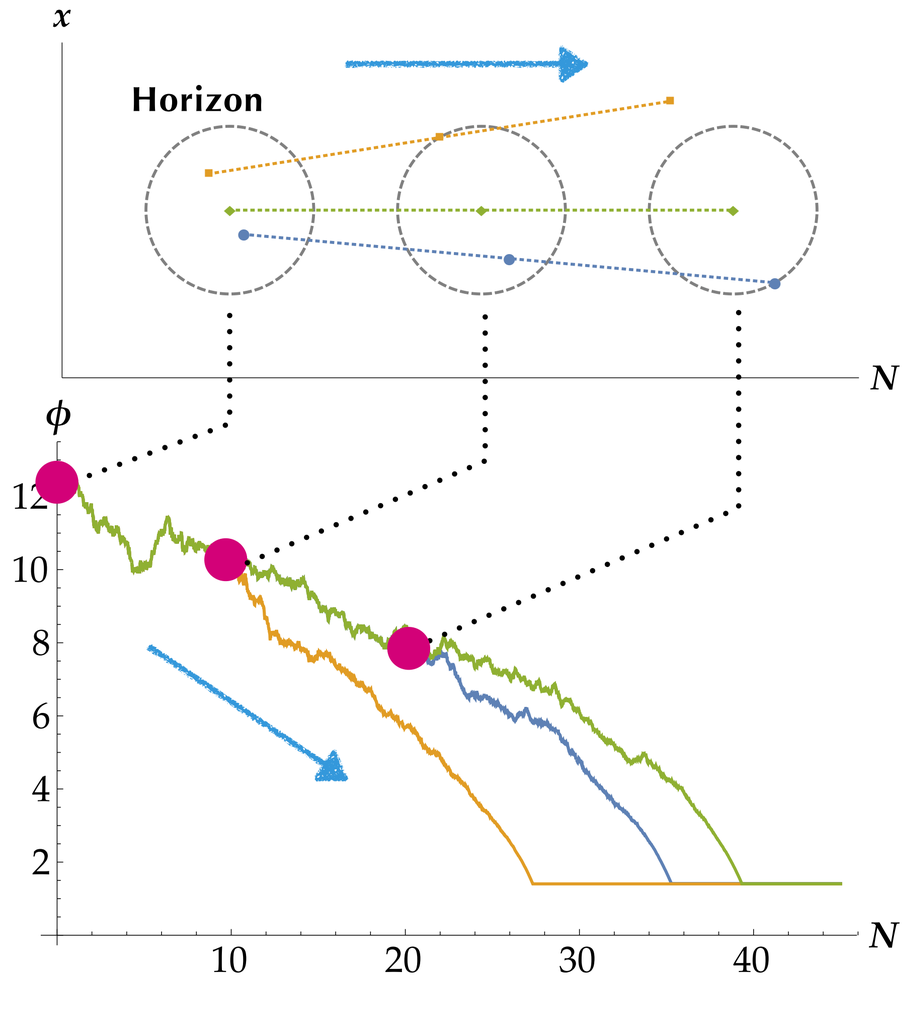
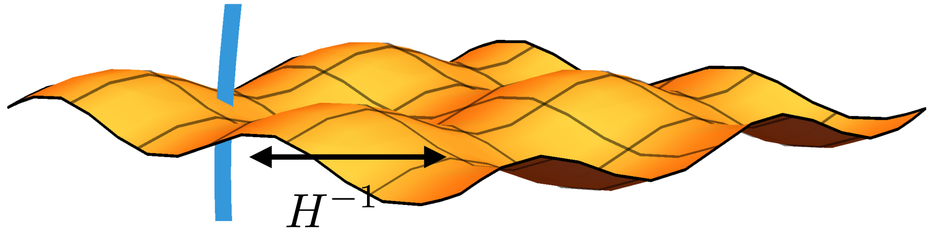


Stochastic formalism Starobinsky 1986



$$\delta\phi \sim \frac{H}{2\pi}$$

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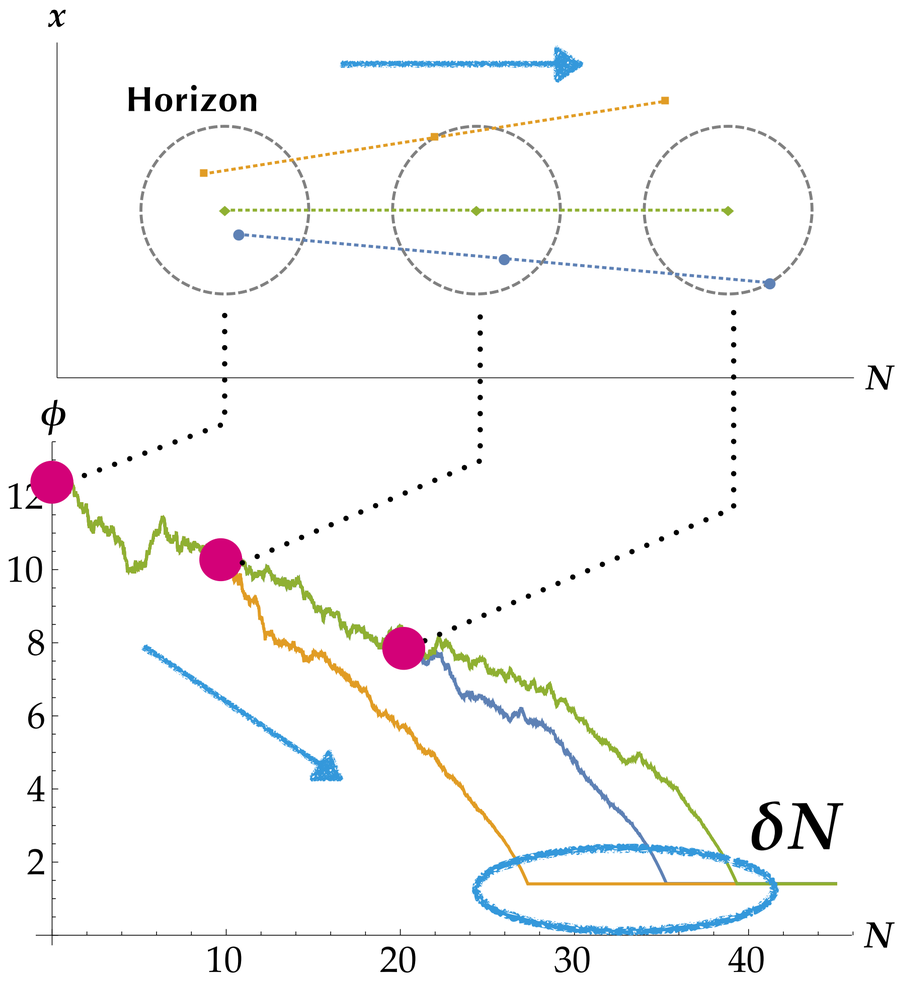


Stochastic-δN formalism

Fujita, Kawasaki, YT ,Takesako 2013

Solve one-point-dynamics from some point (ϕ_*) many times

→ mean e-folds to end slice is $\langle N \rangle$

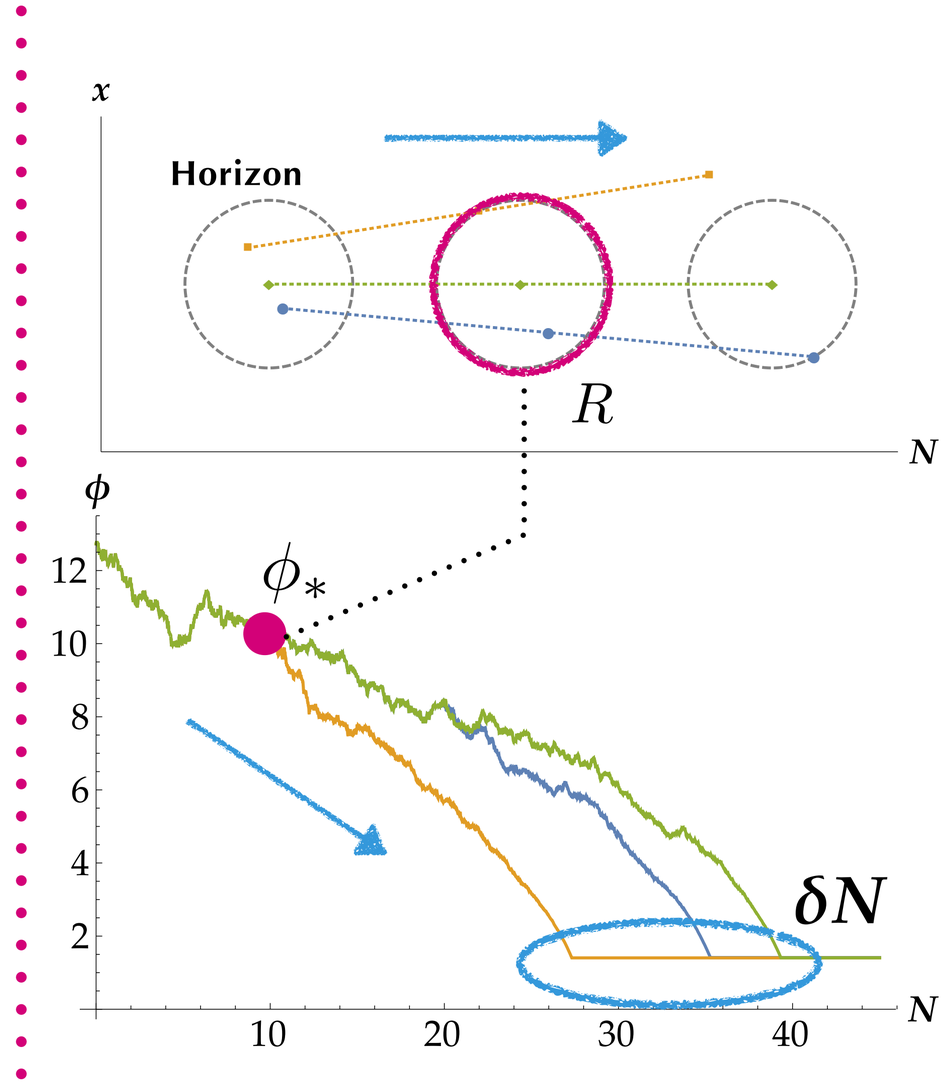
$$(\text{size of } R) = H_f^{-1} \mathrm{e}^{\langle N \rangle}$$

all branches are in this range!

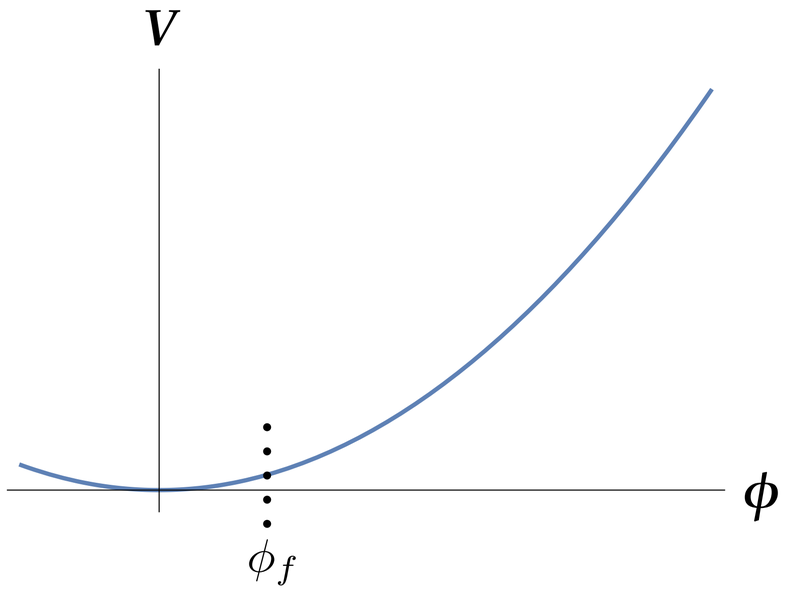
$$\langle \delta N^2 \rangle = \int_{\log(aH|_f \mathrm{e}^{-\langle N \rangle}) = \log aH|_f - \langle N \rangle}^{\log aH|_f} \mathcal{P}_\zeta \, \mathrm{d} \log k$$

→

$$\mathcal{P}_\zeta(k) = \left. \frac{\mathrm{d} \langle \delta N^2 \rangle}{\mathrm{d} \langle N \rangle} \right|_{\langle N \rangle = \log(aH|_f / k)}$$



Algorithm



Algorithm

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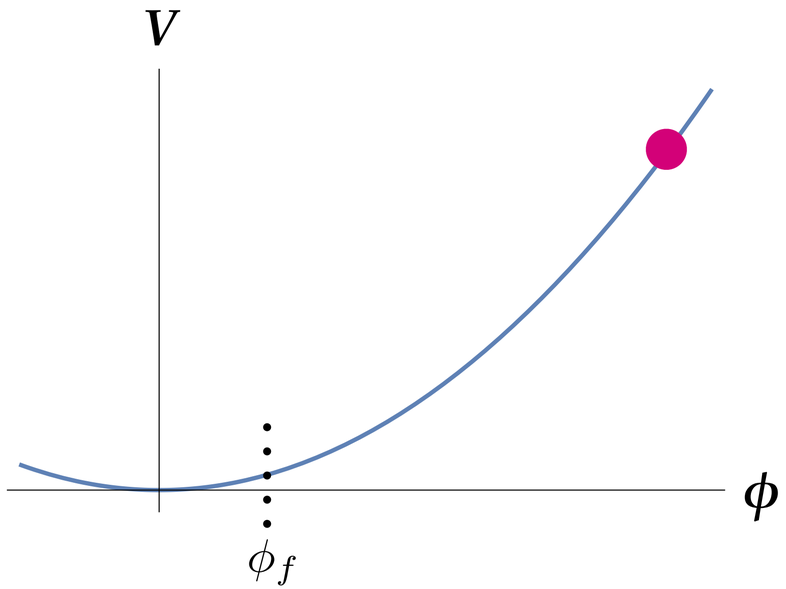

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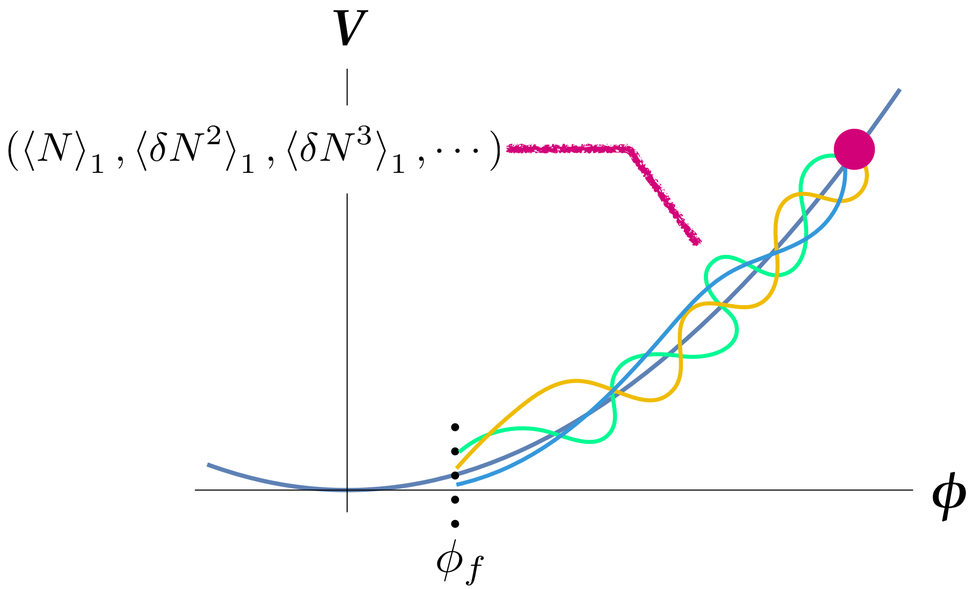

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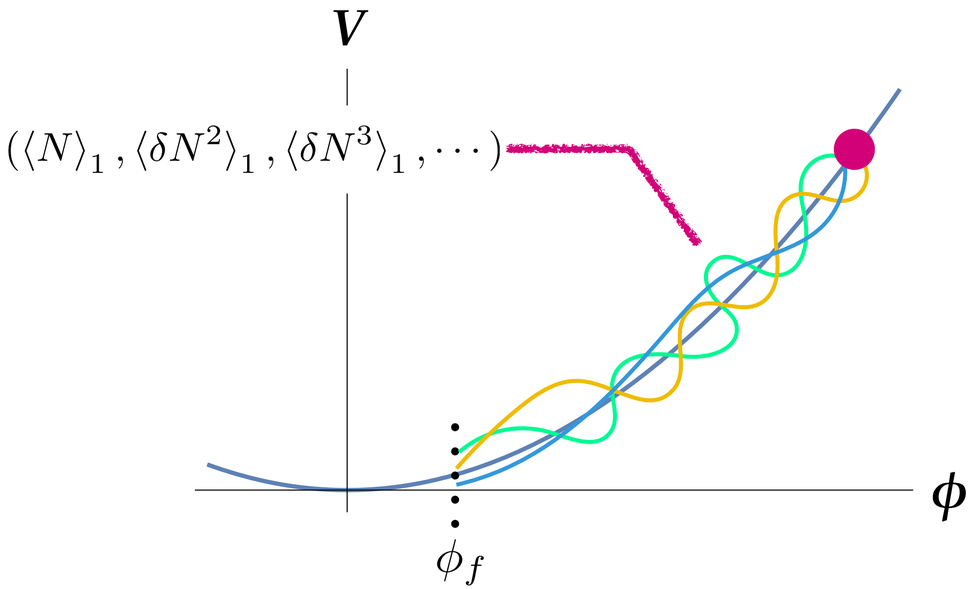

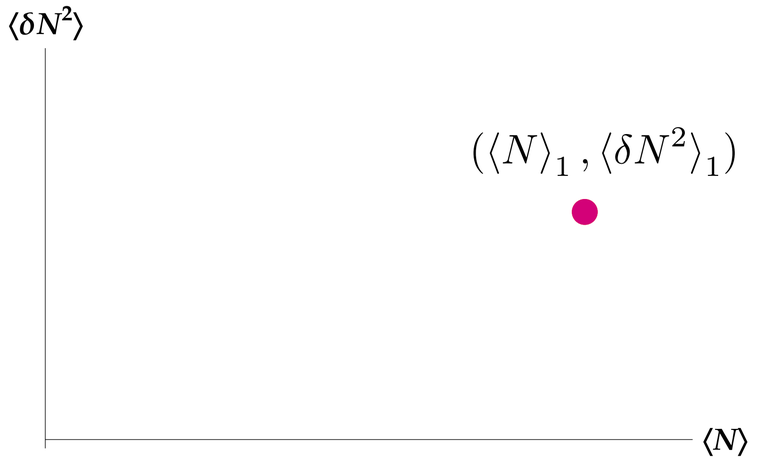

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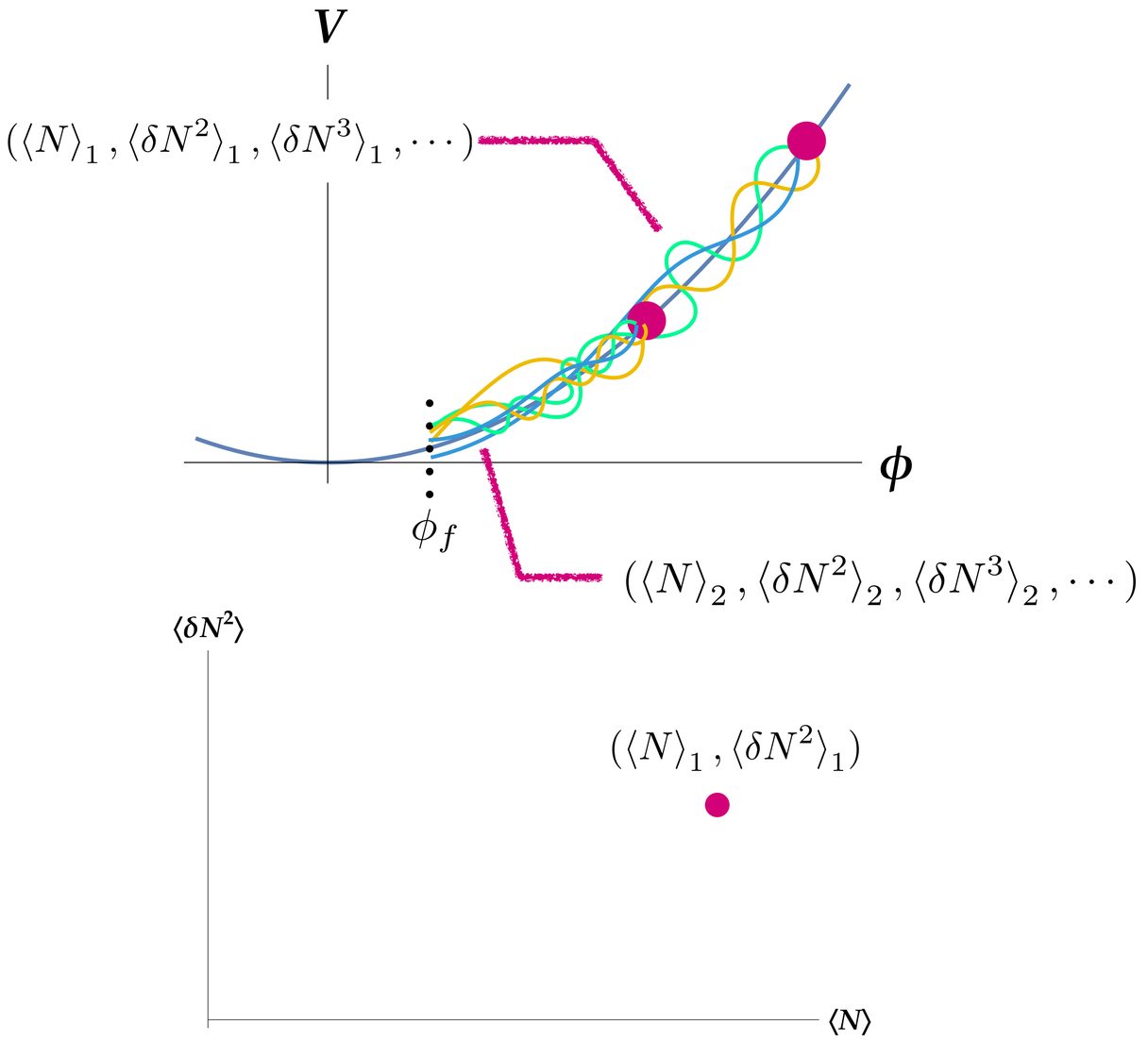

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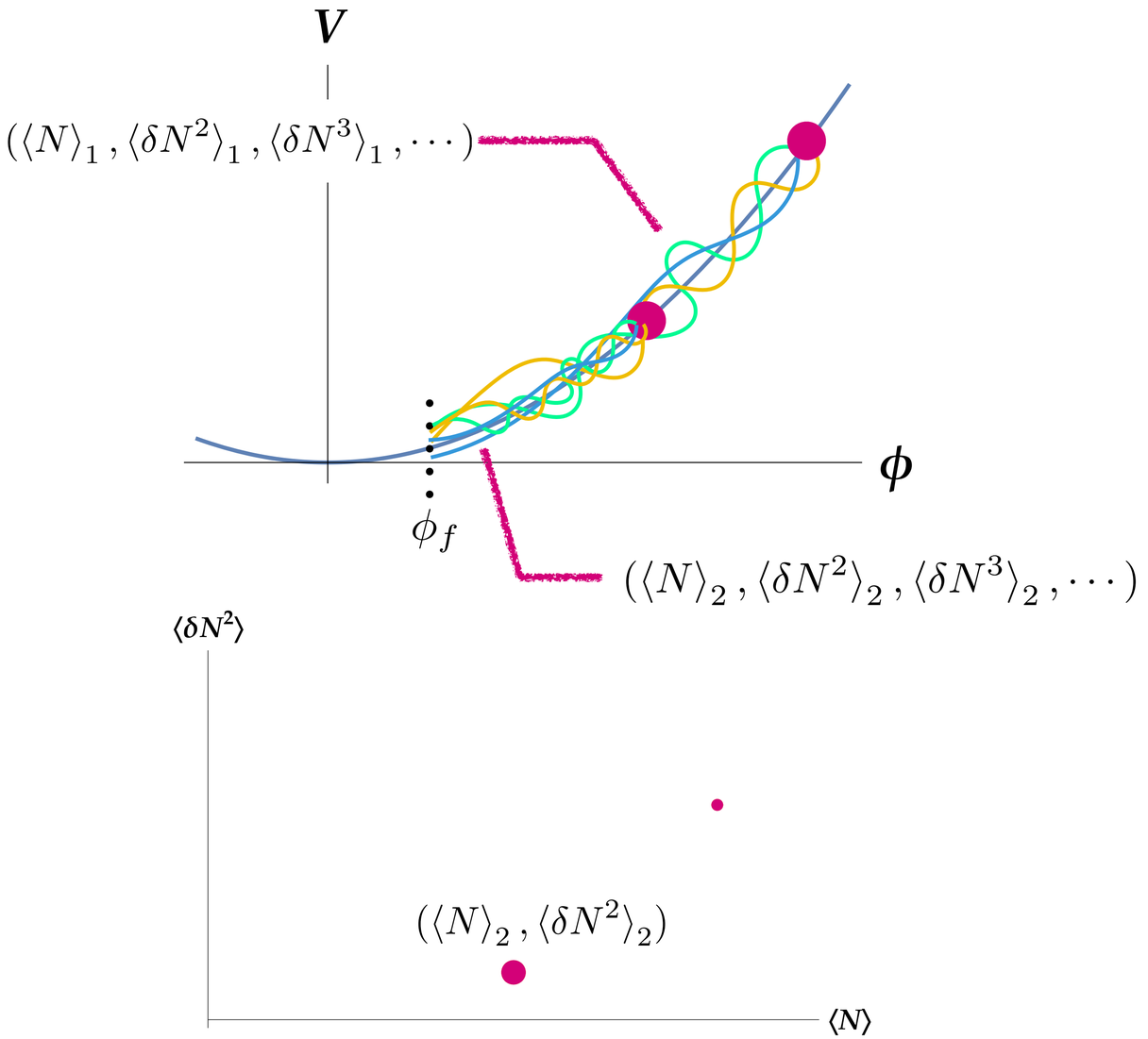

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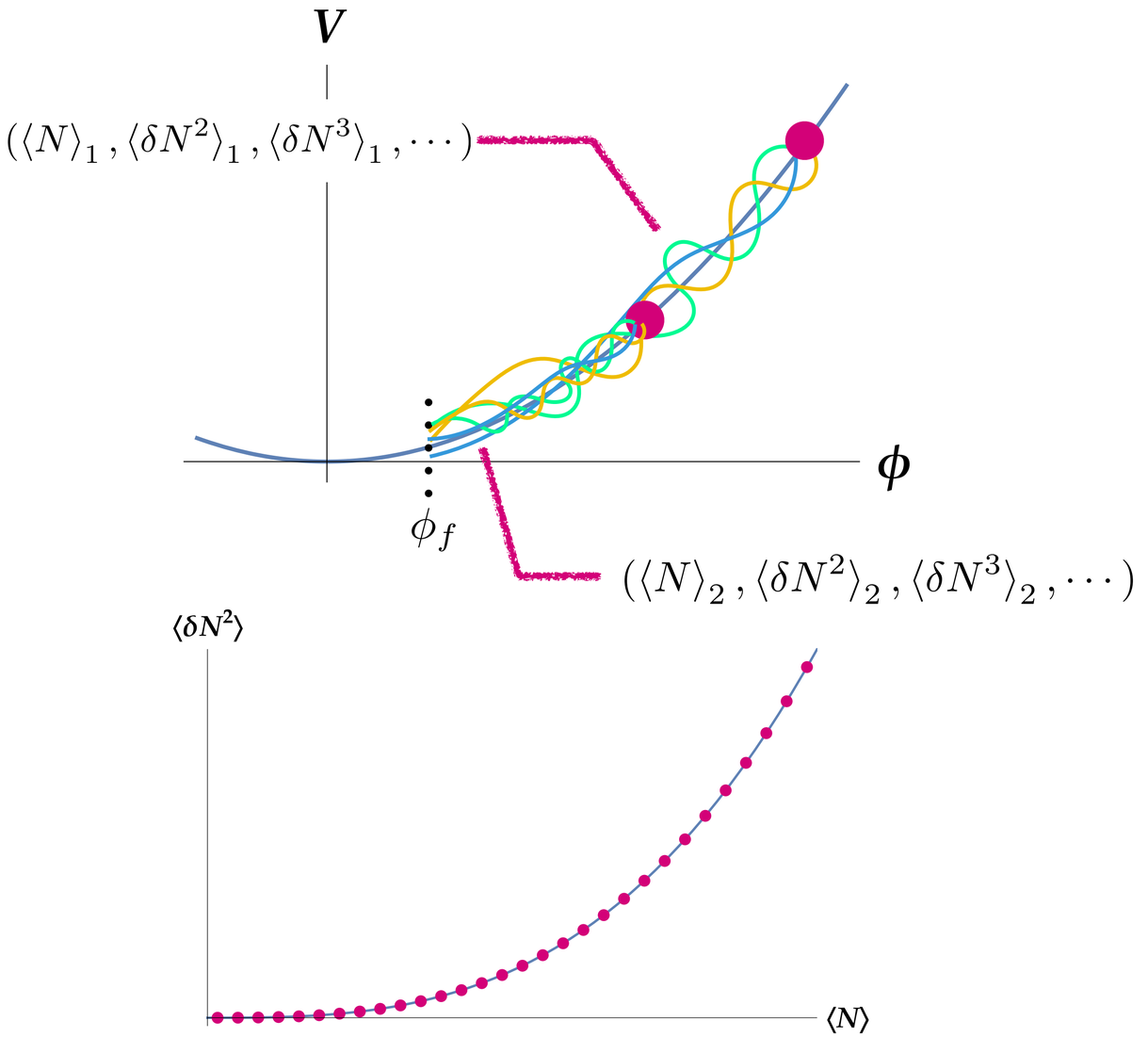

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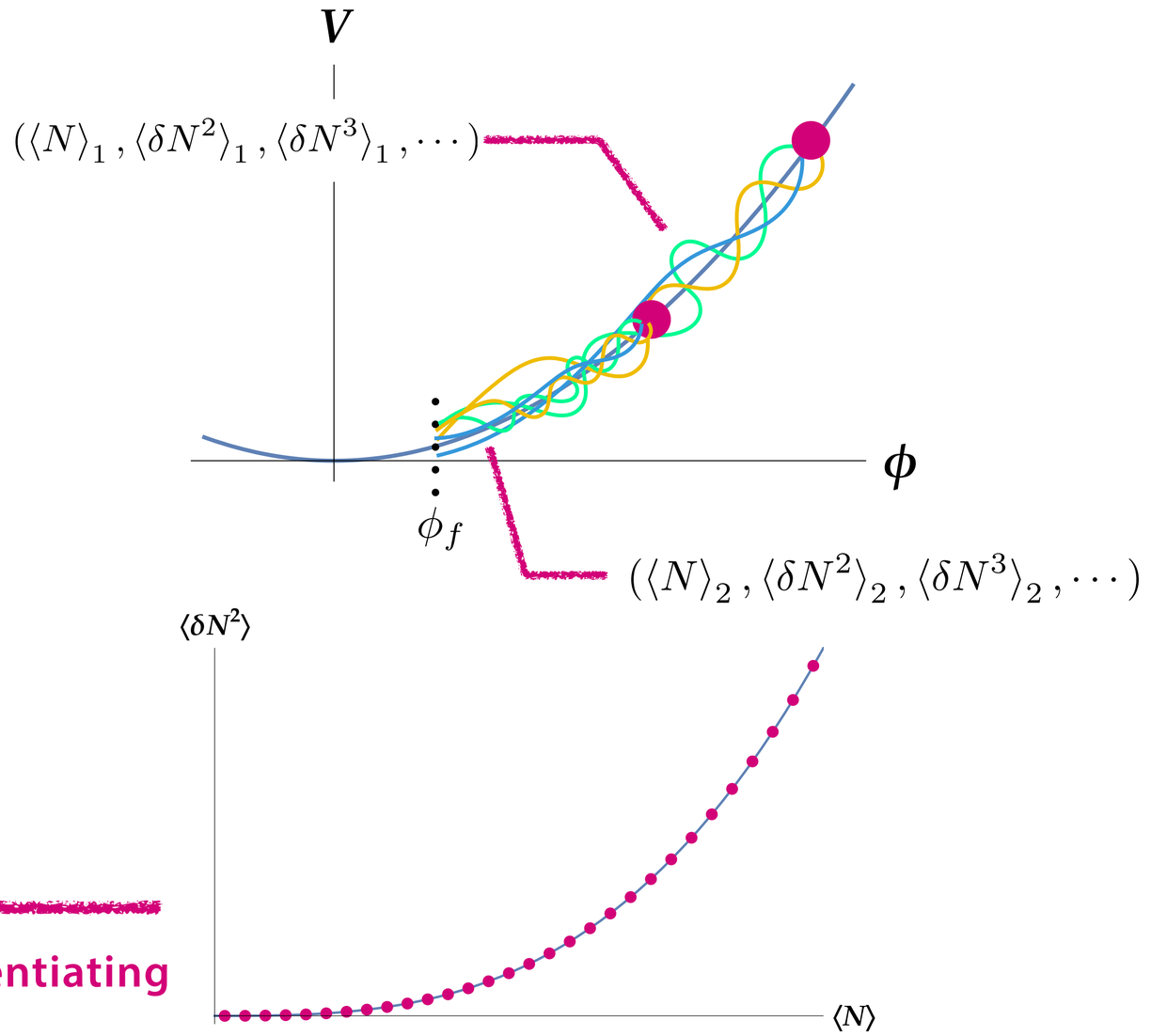

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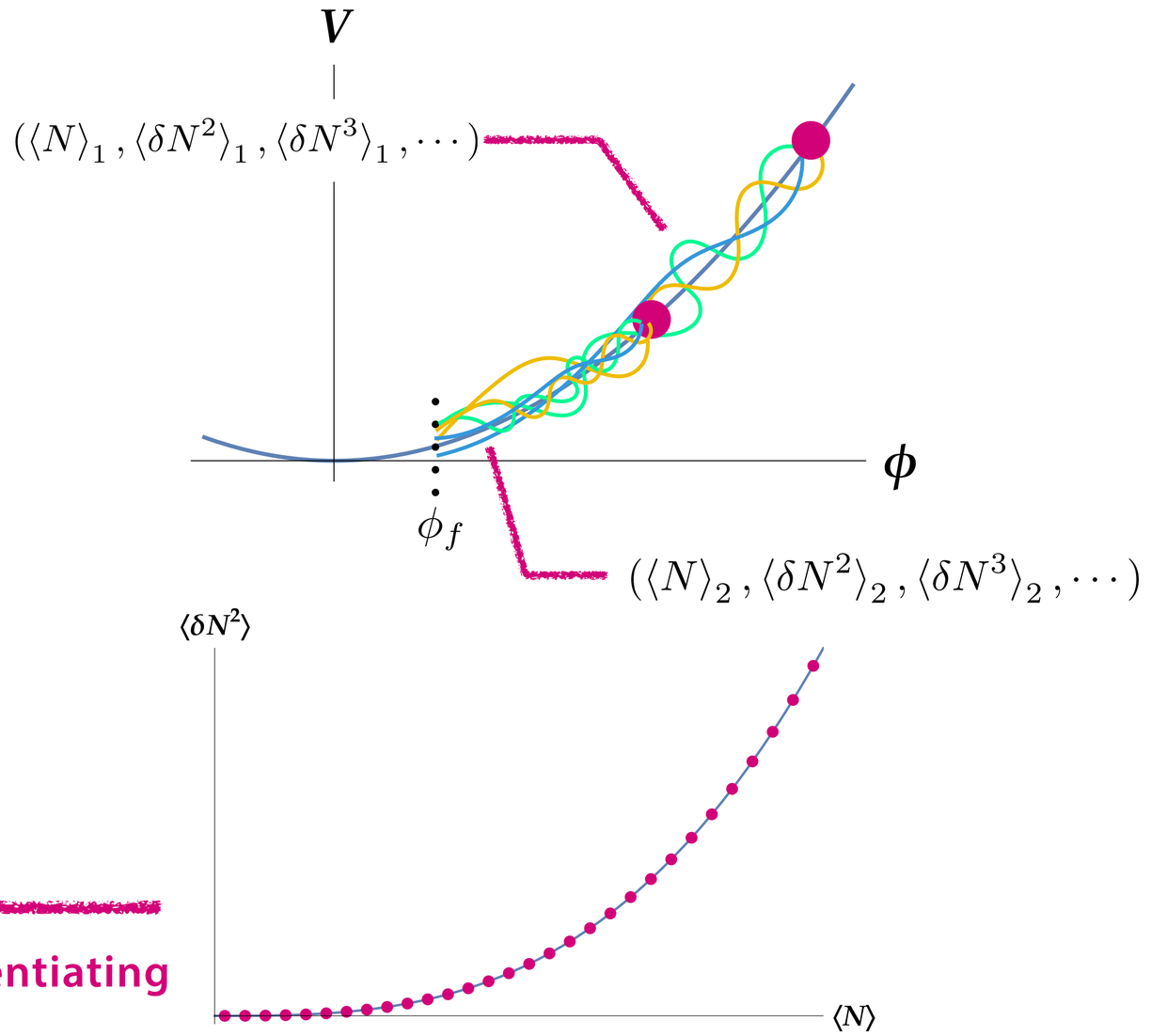

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differentiating

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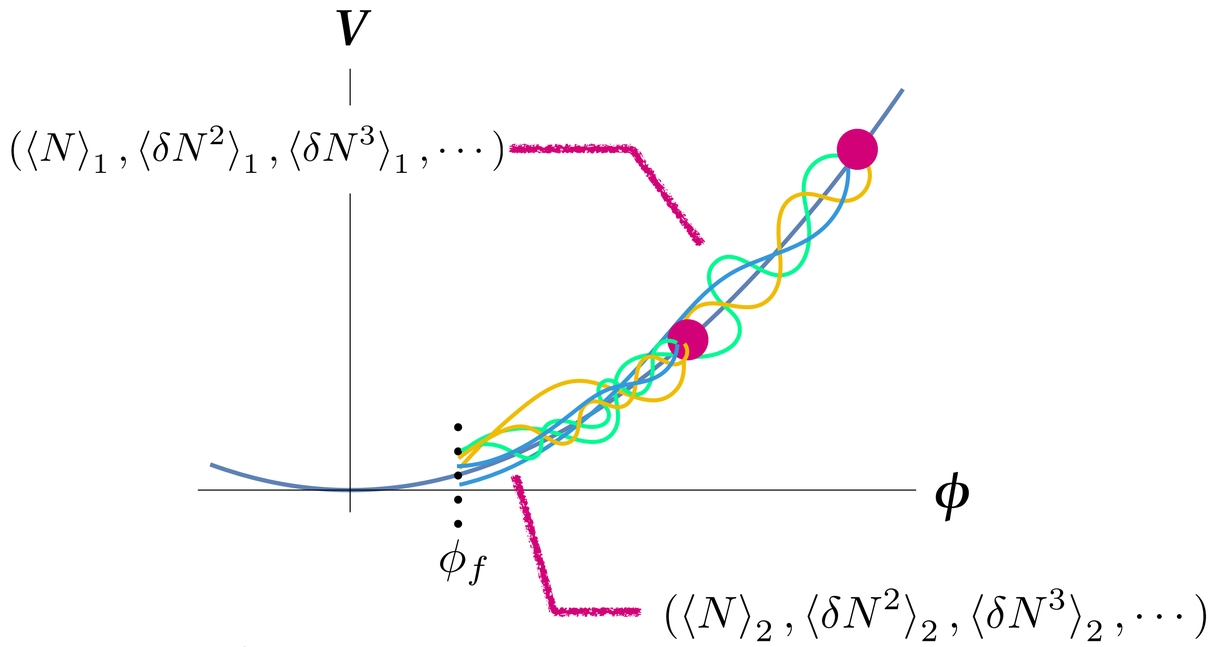

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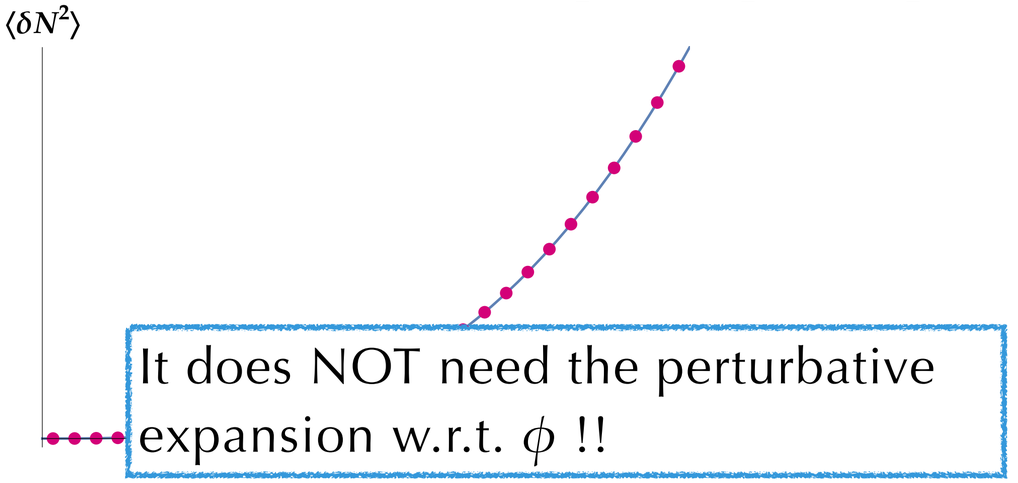
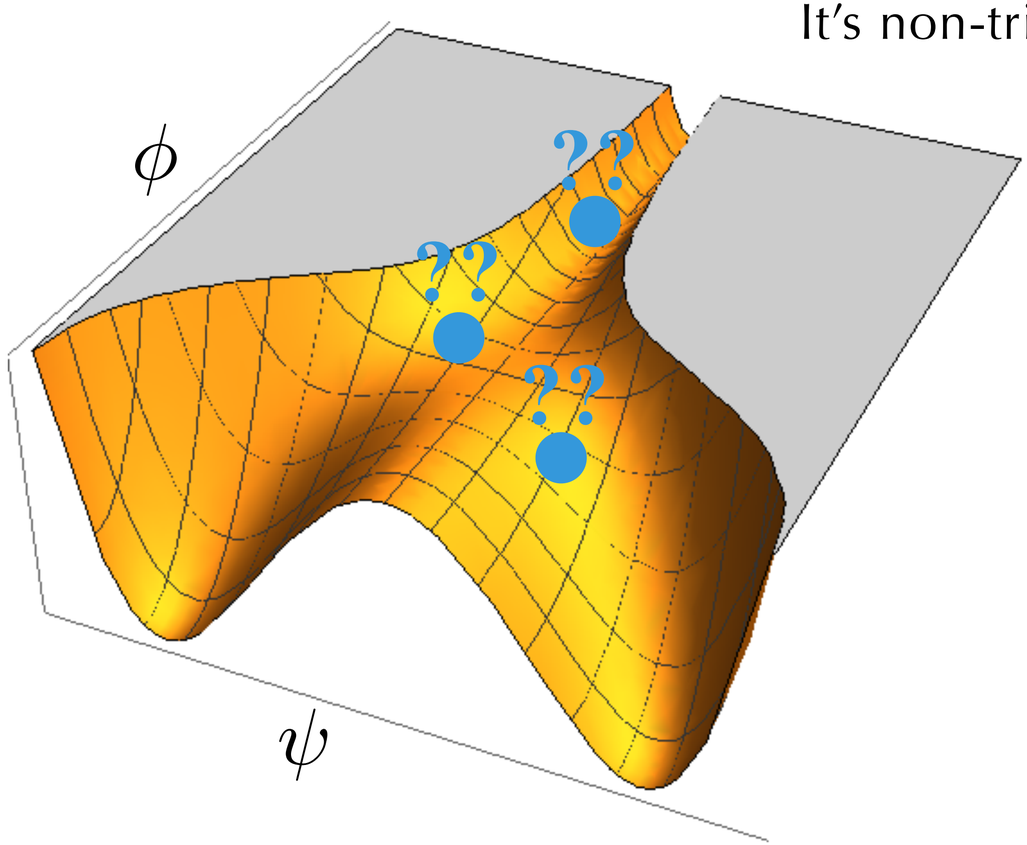


It does NOT need the perturbative expansion w.r.t. ϕ !!

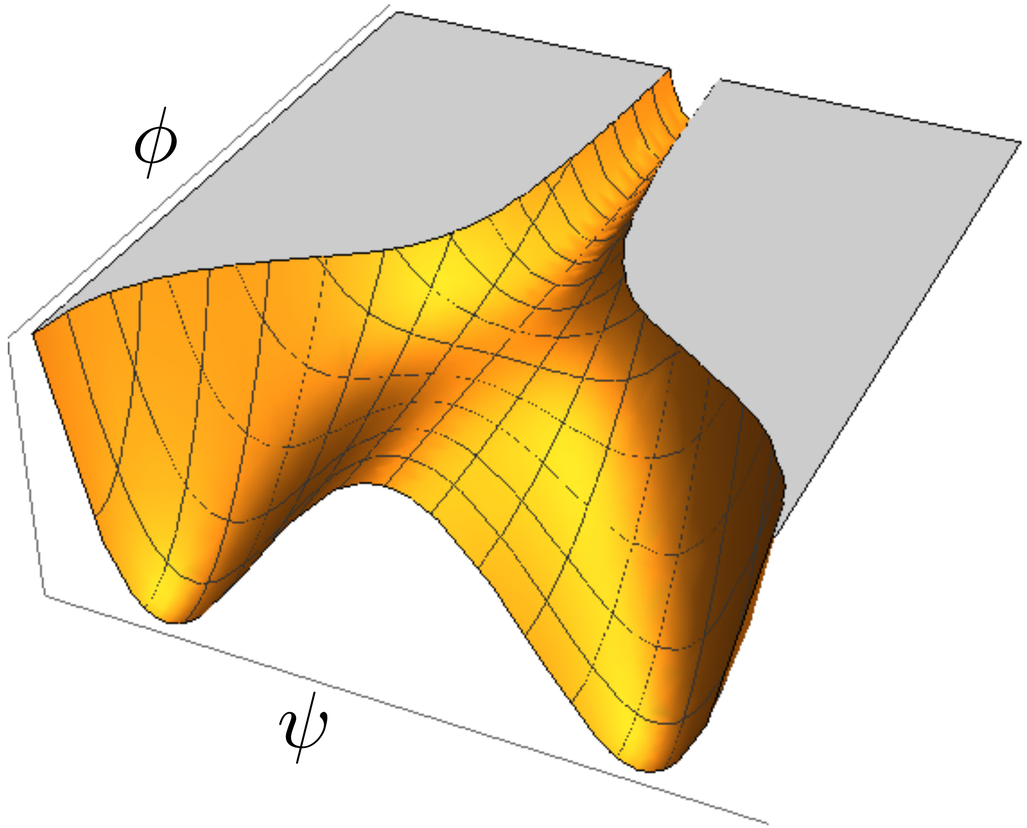
Extension to multi-field case



It's non-trivial where "initial value" should be…

Extension to multi-field case

Take “initial value” on various sample path!!



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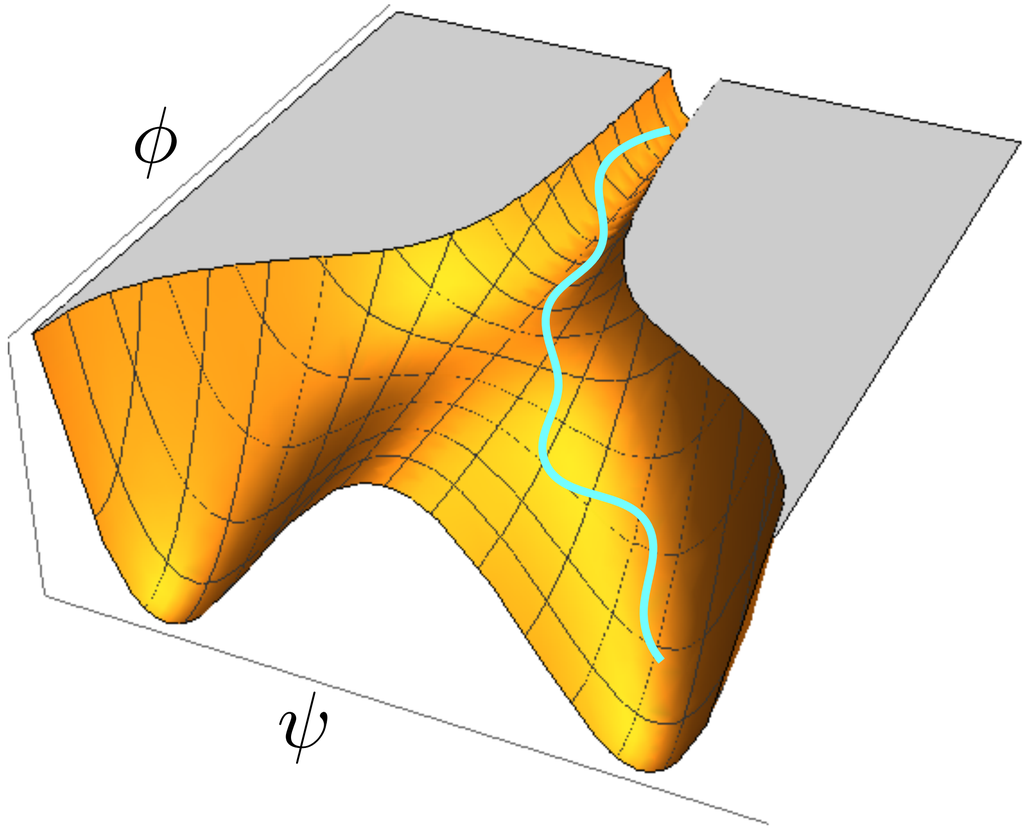

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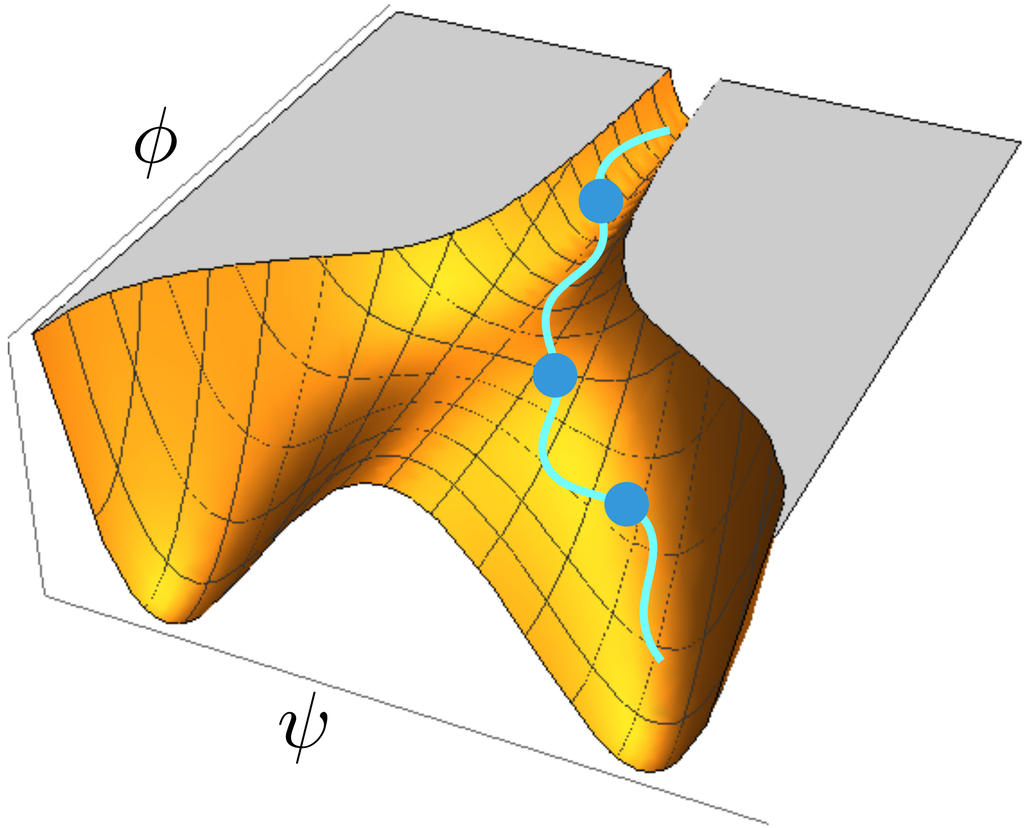

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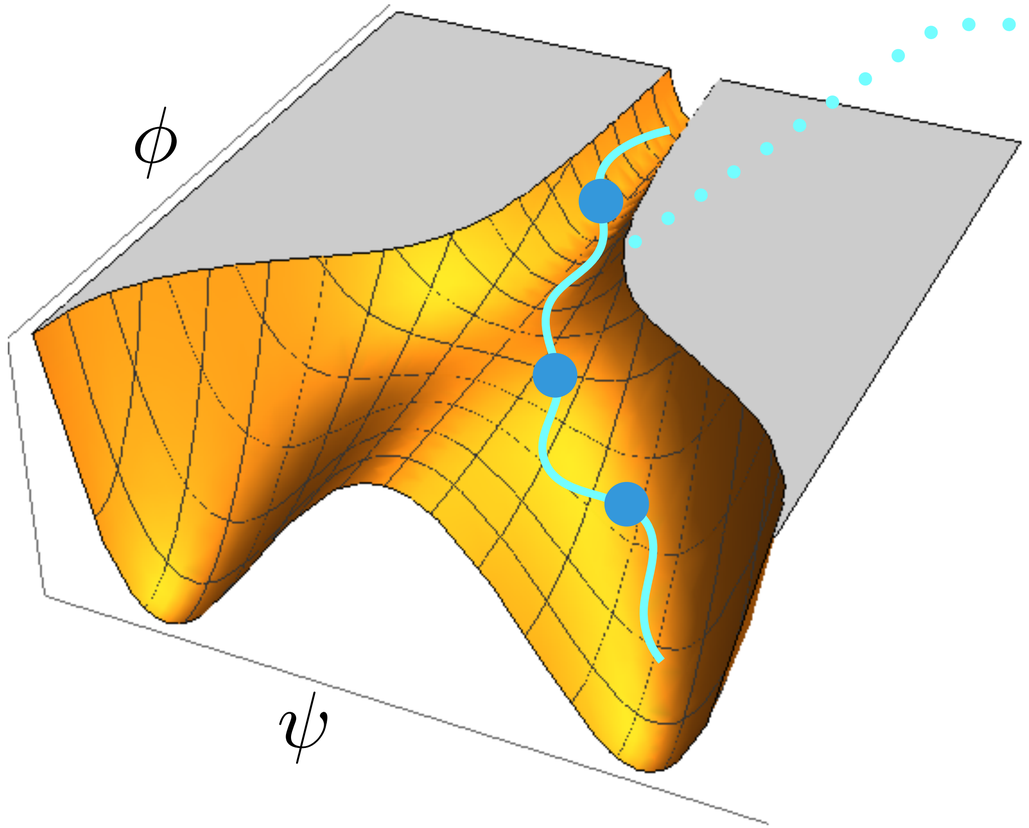

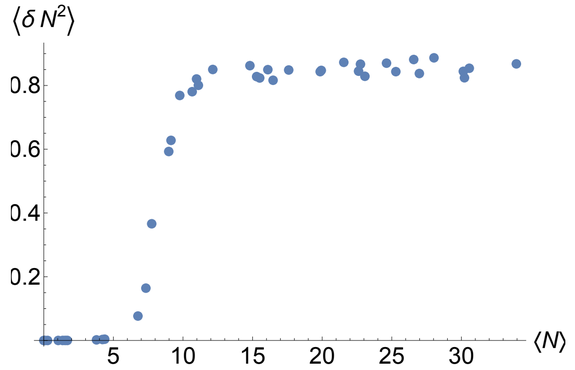

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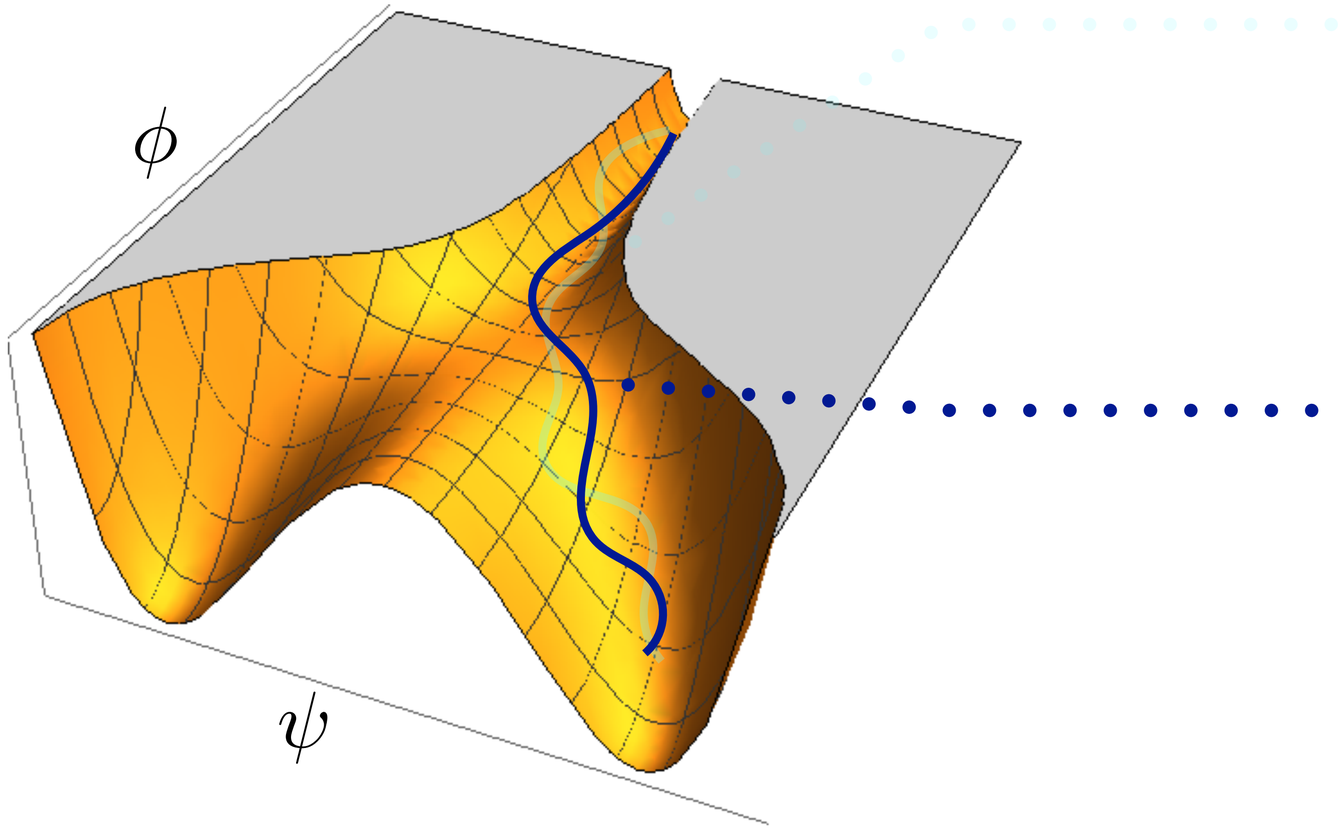

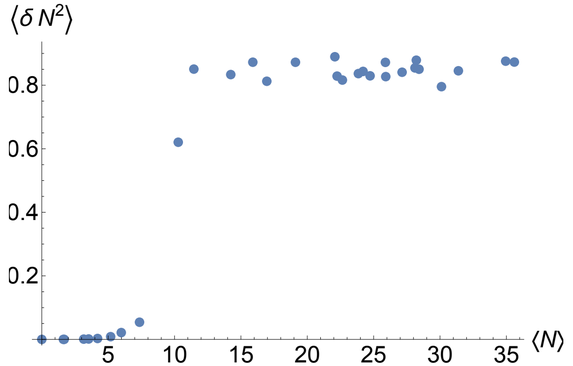

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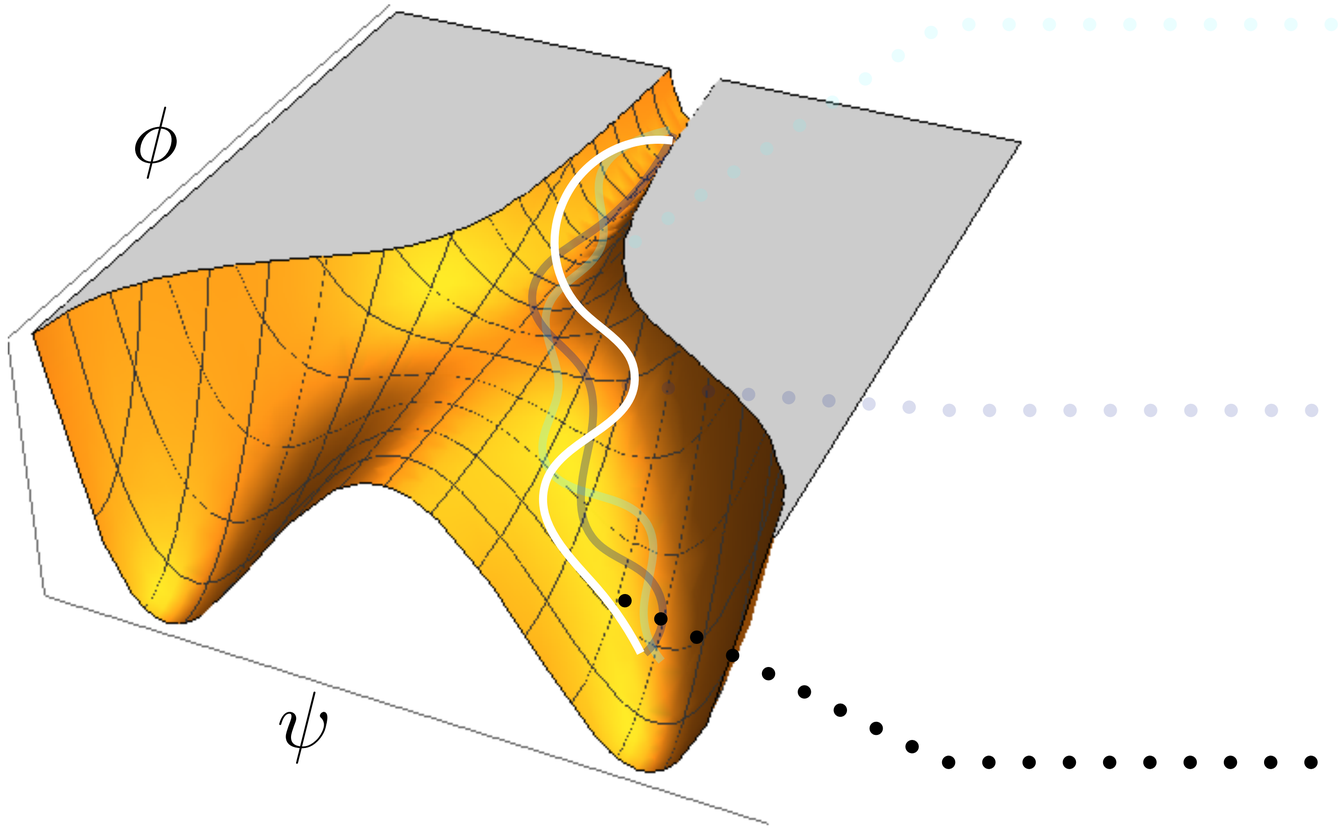

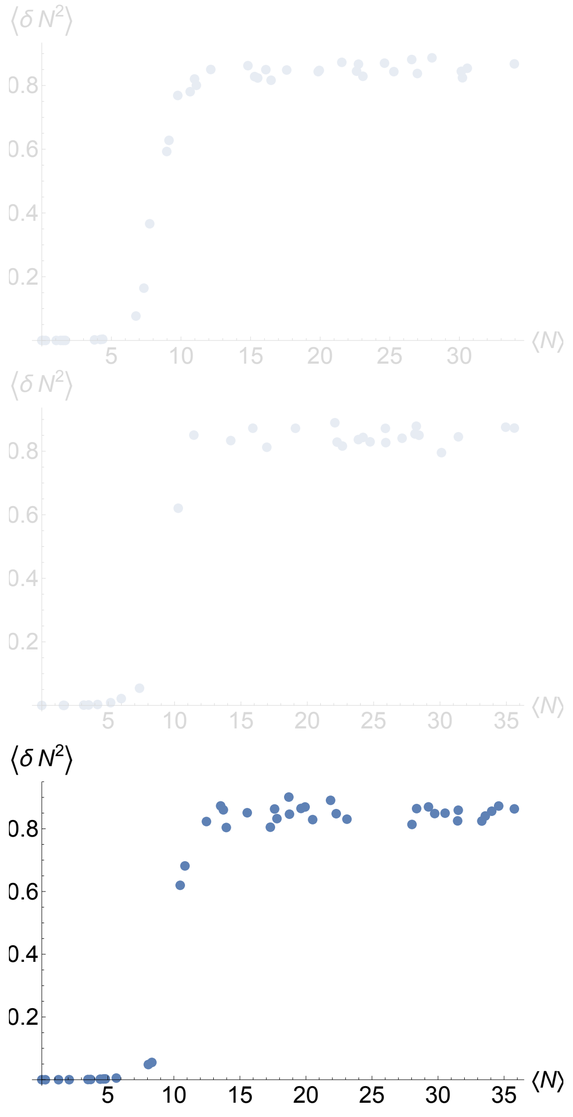

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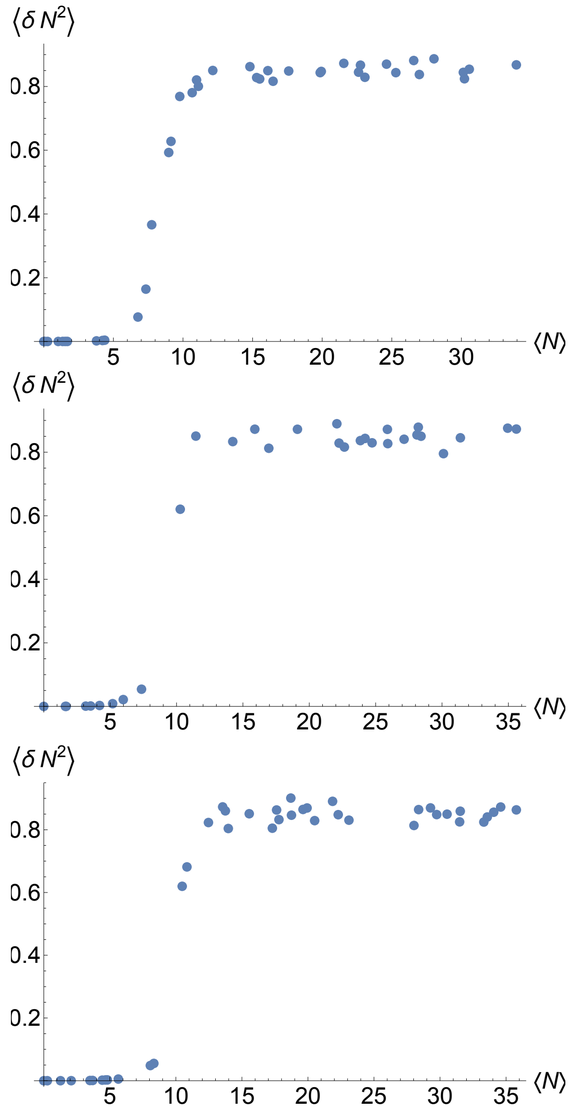

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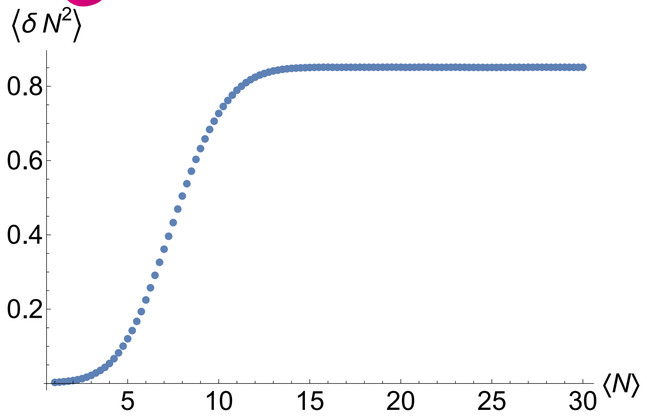
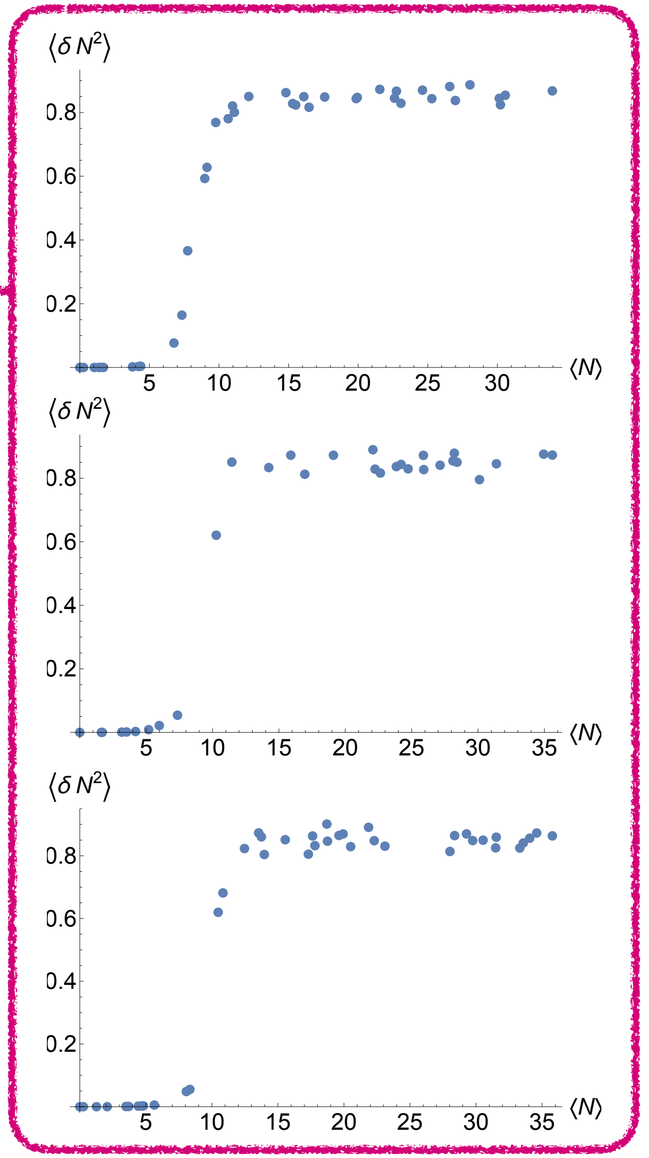
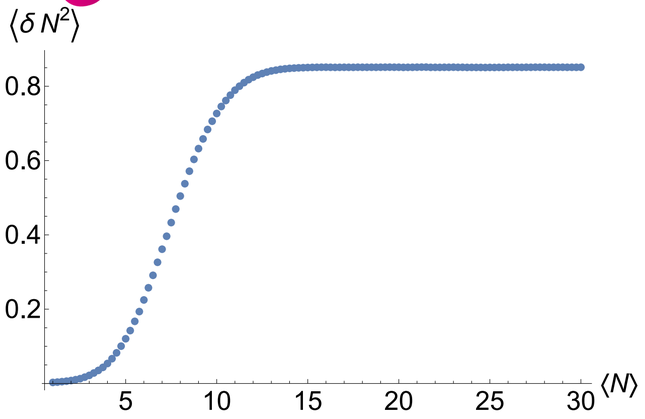


average them



Extension to multi-field case



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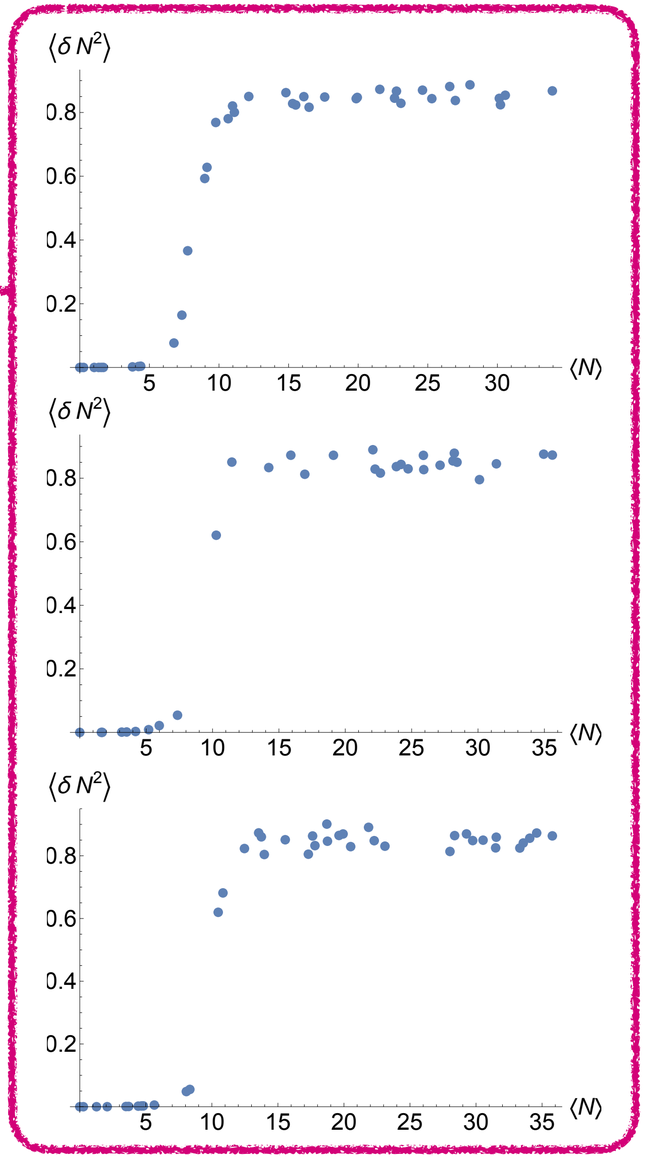
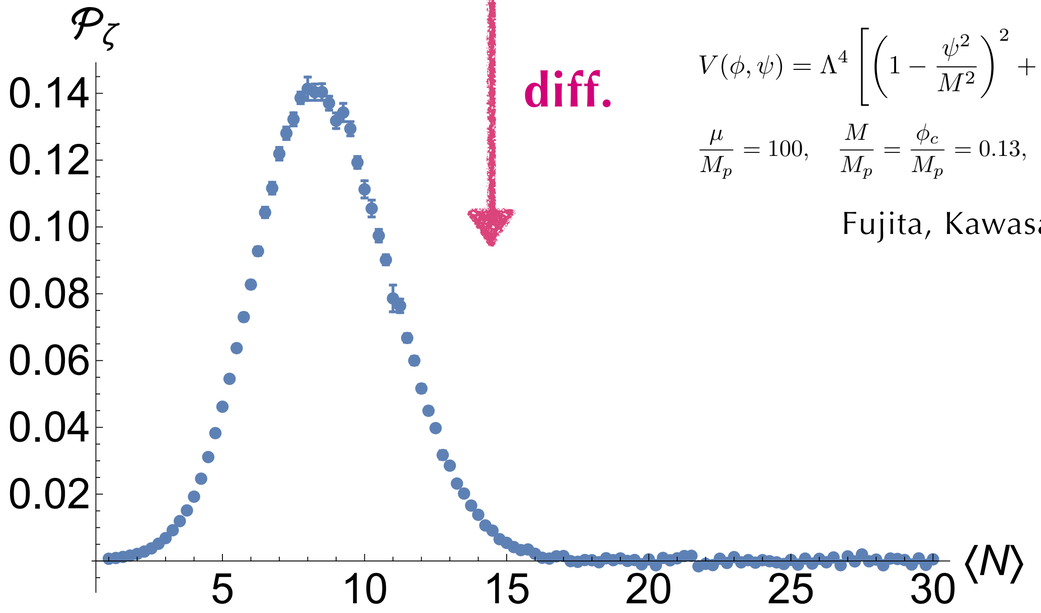


diff.



$$V(\phi,\psi) = \Lambda^4\left[\left(1-\frac{\psi^2}{M^2}\right)^2 + 2\frac{\phi^2\psi^2}{\phi_c^2 M^2} + \frac{\phi^2}{\mu^2}\right]$$

$$\frac{\mu}{M_p} = 100, \quad \frac{M}{M_p} = \frac{\phi_c}{M_p} = 0.13, \quad \frac{\Lambda}{M_p} = 1.2\times 10^{-4}$$

Fujita, Kawasaki, YT 2014

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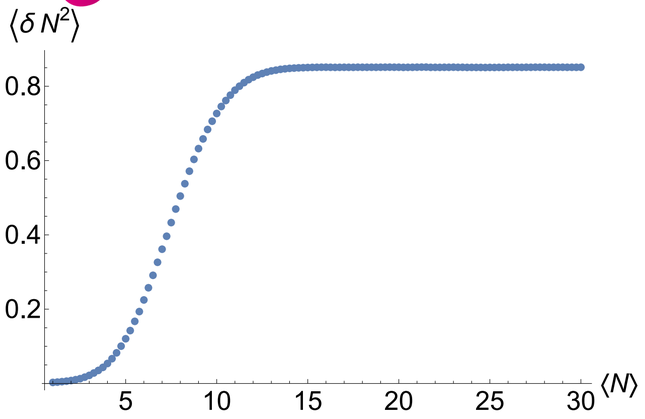

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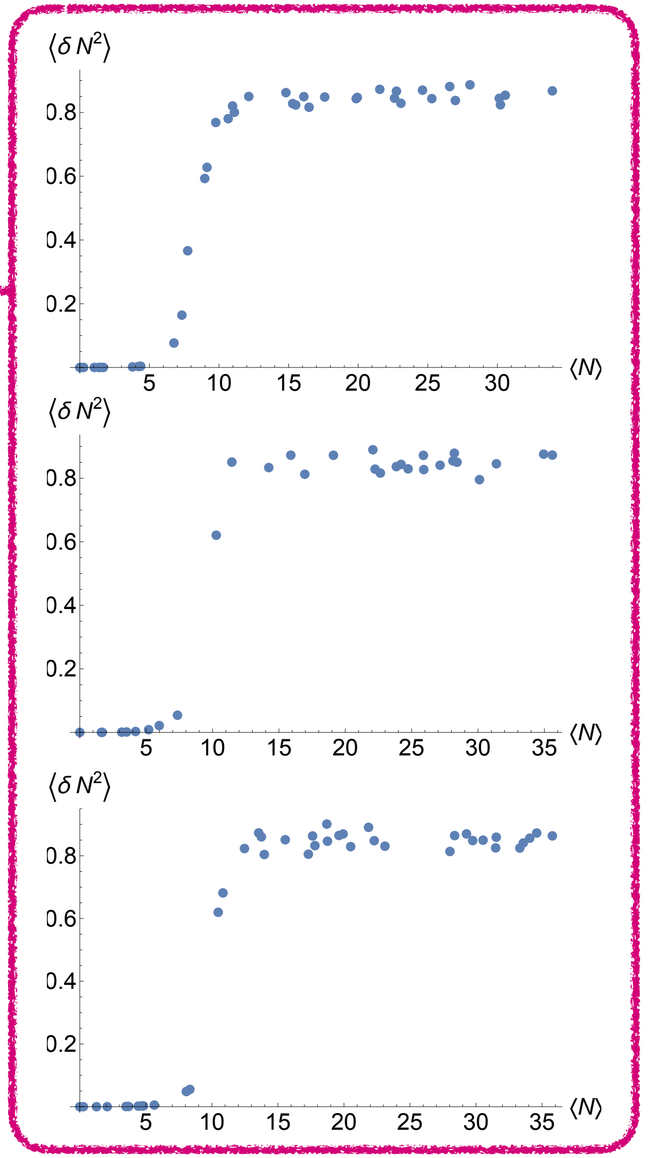
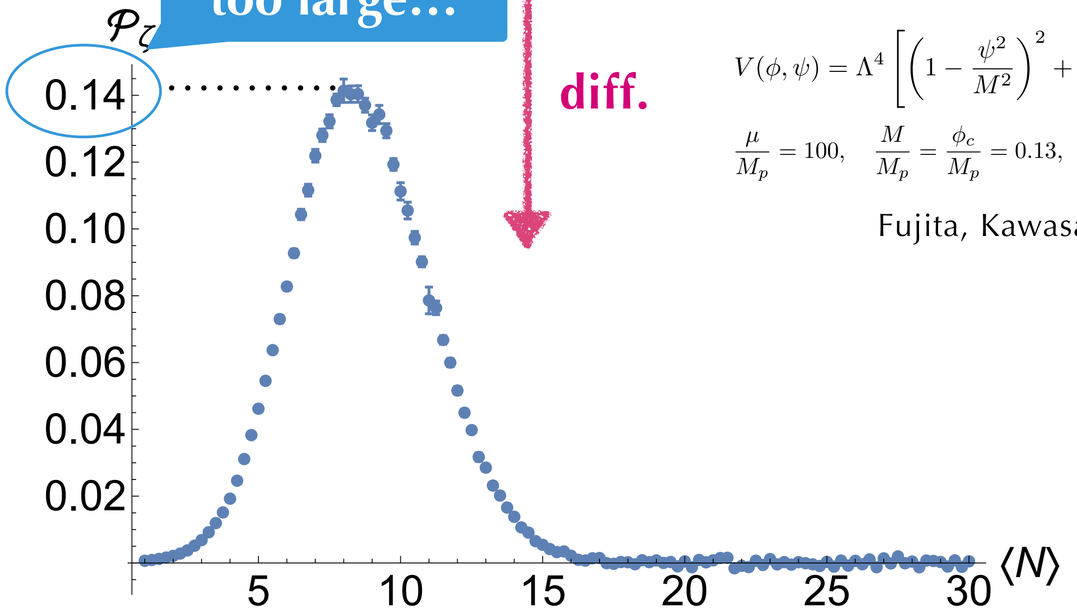


too large…

diff.



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Fujita, Kawasaki, YT 2014

Analytic solution -consistency check-

In the single-field slow-roll case where ζ is almost Gaussian, our algorithm should be consistent with std. pert. th.

Simple assumptions

$$H(\phi) \simeq H_0, \quad H'(\phi) \simeq \sqrt{6\epsilon_H \frac{V}{M_p^4}} : \text{const.}$$

Slow-roll EoM

$$X(N) = \mu N + \int_0^N \xi(N) \mathrm{d}N$$

uniform linear — Brownian

$$\begin{cases} X(N) = \dfrac{2\pi}{H_0}\left(\phi(N) - \phi_i\right) \\ \mu = 12\pi\sqrt{\dfrac{\epsilon_H}{6}\dfrac{M_p^4}{V}} \end{cases}$$

For a drifted Brownian, the generating function of the first passage time N_f for $X_f = 2\pi(\phi_f - \phi_i)/H_0$ is given by,

$$\left\langle \mathrm{e}^{-JN_f} \right\rangle = \mathrm{e}^{X_f(\mu - \sqrt{2J+\mu^2})}, \quad \text{for } {}^\forall J > 0$$

J derivative $J \to 0$

$$\langle N_f \rangle = \frac{X_f}{\mu}, \quad \langle N_f^2 \rangle = \frac{X_f^2}{\mu^2} + \frac{X_f}{\mu^3}, \quad \cdots$$

consistent w/ SPT

$$\left\langle \delta N_f^2 \right\rangle = \left\langle N_f^2 \right\rangle - \left\langle N_f \right\rangle^2 = \frac{X_f}{\mu^3} = \frac{1}{\mu^2}\left\langle N_f \right\rangle$$

$$\mathcal{P}_\zeta = \frac{\mathrm{d}\left\langle \delta N_f^2 \right\rangle}{\mathrm{d}\left\langle N_f \right\rangle} = \frac{1}{\mu^2} = \frac{1}{24\pi^2 M_p^4}\frac{V}{\epsilon_H}$$

Further analysis

- Vennin & Starobinsky 2015

  PDE for $f_n(\phi_i) := \langle N^n \rangle (\phi_i^{\rm in} = \phi_i)$

$$\sum_i \left(v \frac{\partial^2}{\partial \phi_i^2} - \frac{v_i}{v} \frac{\partial}{\partial \phi_i} \right) f_n = -n \frac{f_{n-1}}{M_{\rm Pl}^2}, \quad f_0 = 1, \quad \left(v = \frac{V}{24\pi^2 M_{\rm Pl}^4} \right)$$

  → in single-field inflation

  classicality criterion

$$\mathcal{P}_{\zeta,\rm full} \simeq \mathcal{P}_{\zeta,\rm lin} \left(1 + 5v - 4 \frac{v^2 v''}{v'^2} \right), \quad \text{if } \eta_{\rm cl} := \left| 2v - \frac{v^2 v''}{v'^2} \right| \ll 1$$

- Assadullahi et al. 2016

  if $g := \Delta v / |\nabla v|^2$ is a funtion only of v (harmonic),

  the problem boils down to solving ODE w.r.t. v

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classicality criterion

$$\frac{\eta_V}{2\epsilon_V} v = \frac{\eta_V}{2} \mathcal{P}_{\zeta,\rm lin}$$

- Assadullahi et al. 2016

if $g := \Delta v / |\nabla v|^2$ is a funtion only of v (harmonic),

the problem boils down to solving ODE w.r.t. v

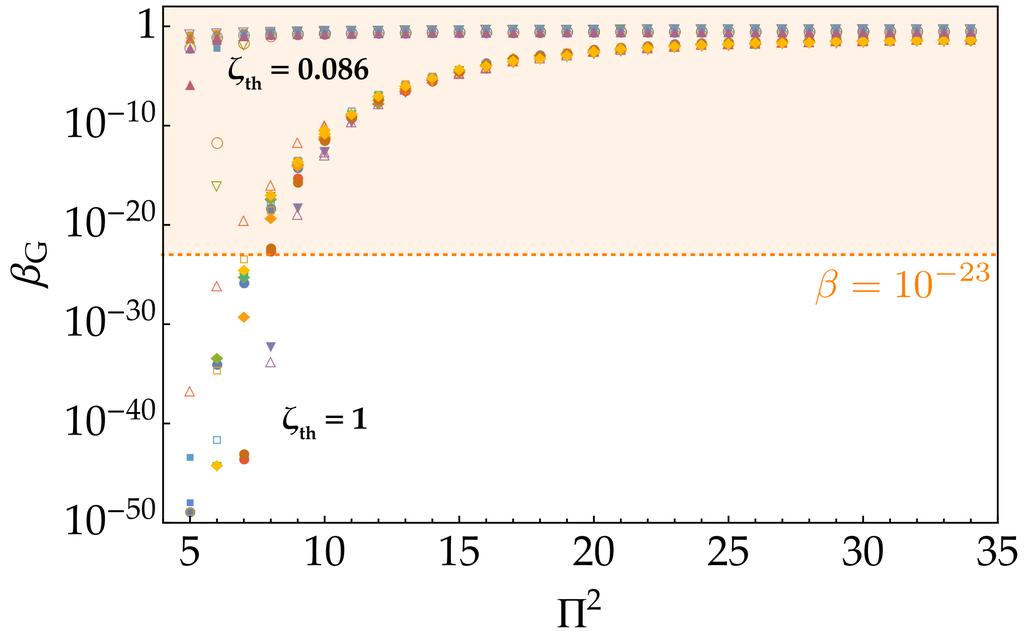
Precise PBH abundance

PBH formation rate

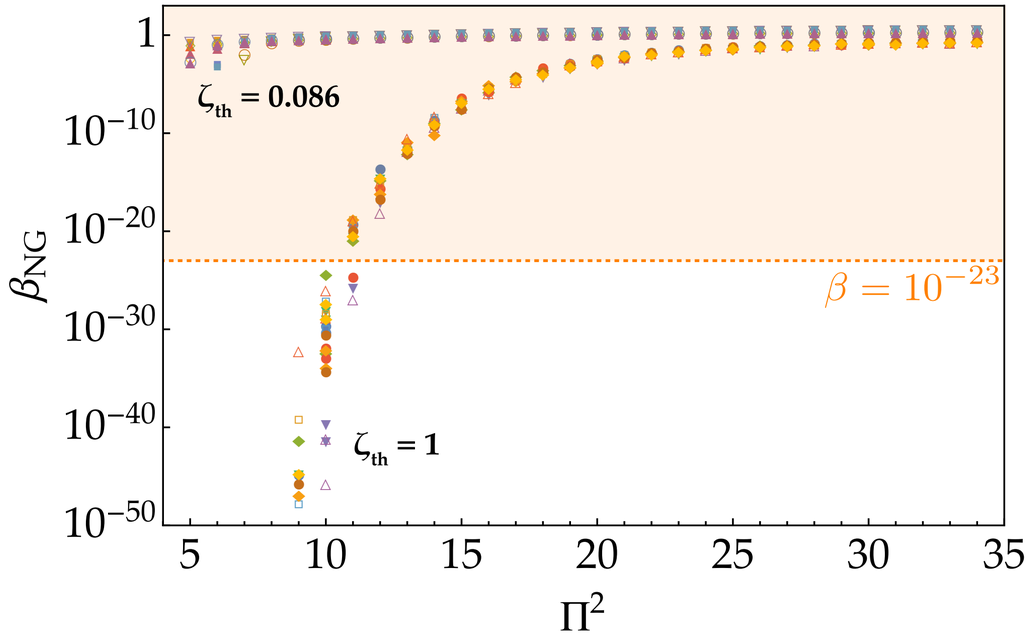
$$\beta = \int_{\zeta_{\rm th}} P(\zeta)\mathrm{d}\zeta \qquad \zeta_{\rm th} = \begin{cases} 1, \\ \dfrac{1}{3}\log\dfrac{3(\chi_a - \sin\chi_a\cos\chi_a)}{2\sin^3\chi_a}\Big|_{\chi_a=\pi\sqrt{w}/(1+3w)} \end{cases} \simeq 0.086.$$

Harada, Yoo, Kohri 2013

- $P(\zeta) = \dfrac{1}{\sqrt{2\pi\sigma^2}}\mathrm{e}^{-\frac{\zeta^2}{2\sigma^2}}, \quad \sigma^2 = \langle\delta N^2\rangle$



- $P(\zeta) = \exp\left[\sum_{n=3}\dfrac{(-1)^n}{n!}\langle\delta N^n\rangle_c\dfrac{\partial^n}{\partial\zeta^n}\right]\exp\left[-\dfrac{\zeta^2}{2\sigma^2}\right], \quad n \le 4$



Double Inflation in Supergravity ($M_{\rm Pl} = 1$)

	Z_{2nR}	Z_2	shift sym.
Φ	2	0	
X	0	1	$\mathrm{Im}X \to \mathrm{Im}X + \alpha$
S	2	1	

Kumekawa, Moroi, Yanagida 1997
Izawa, Kawasaki, Yanagida 2000

Kawasaki, Yamaguchi, Yanagida 2000

$$W = v^2\Phi - \frac{g}{n+1}\Phi^{n+1} + mSX + C$$

$$K = |\Phi|^2 + \frac{1}{2}(X+X^*)^2 + |S|^2 + \frac{\kappa}{4}|\Phi|^4 + \lambda|\Phi|^2|S|^2 + \frac{\xi}{2}|\Phi|^2(X+X^*)^2$$

$$V = e^K(K^{\bar{J}I}D_I W D_{\bar{J}}W^* - 3|W|^2)$$
$$\simeq v^4 - 2\sqrt{2}Cv^2\varphi - \frac{\kappa}{2}v^4\varphi^2 - \frac{g}{2^{\frac{n}{2}-1}}v^2\varphi^n + \frac{g}{2^n}\varphi^{2n} + \frac{1}{2}m^2\chi^2\left(1 + \frac{c_{\rm pot}}{2}\varphi^2\right)$$

$$\mathcal{L}_{\rm kin} = -K_{I\bar{J}}\partial_\mu\bar{\phi}^{\bar{J}}\partial^\mu\phi^I$$
$$\supset -\frac{1}{2}\left(1 - \frac{c_{\rm kin}}{2}\varphi^2\right)\partial_\mu\chi\partial^\mu\chi - \frac{1}{2}\left(1 + \frac{\kappa}{2}\varphi^2\right)\partial_\mu\varphi\partial^\mu\varphi$$

$$\chi = \sqrt{2}\mathrm{Im}X, \quad \varphi = \sqrt{2}\mathrm{Re}\Phi$$
$$c_{\rm pot} = 1 - \lambda, \quad c_{\rm kin} = -\xi$$

Secondary GW

$$\Omega_{\rm GW}h^2 = \frac{8}{243}\Omega_{r,0}h^2 \int_0^\infty \mathrm{d}\nu \int_{|1-\nu|}^{1+\nu} \mathrm{d}u \left[\frac{4\nu^2-(1-u^2+\nu^2)^2}{4\nu u}\right]^2 \mathcal{P}_\zeta(k\nu)\mathcal{P}_\zeta(ku)\overline{I^2(\nu,u,k\eta_c)}$$

$$I(\nu,u,x) = \int_0^x \mathrm{d}\bar{x}\,\bar{x}\sin(x-\bar{x})\left[3\Psi(\nu\bar{x})\Psi(u\bar{x}) + \bar{x}\left\{\Psi(\nu\bar{x})u\Psi'(u\bar{x}) + \nu\Psi'(\nu\bar{x})\Psi(u\bar{x})\right\} + \bar{x}^2 u\nu\Psi'(u\bar{x})\Psi'(\nu\bar{x})\right]$$

$$\Psi(x) = \frac{9}{x^2}\left[\frac{\sin(x/\sqrt{3})}{x/\sqrt{3}} - \cos(x/\sqrt{3})\right]$$