Jet Propulsion Laboratory
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Consistency relations of the large scale structure

and how to break them

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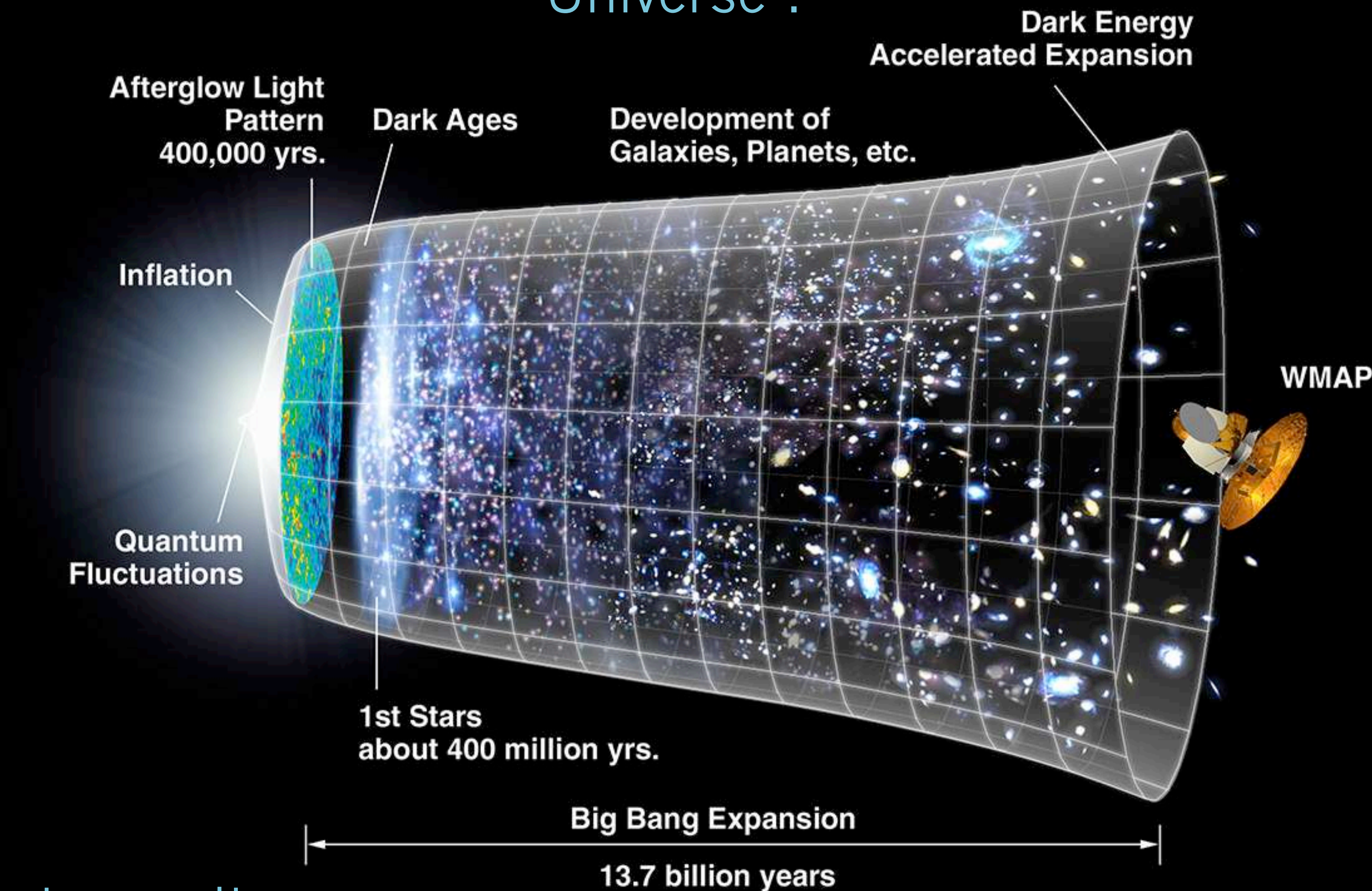
What are the laws ruling the Universe ?



What were its initial conditions ?

What is it made of?

Cosmology and large scale structure

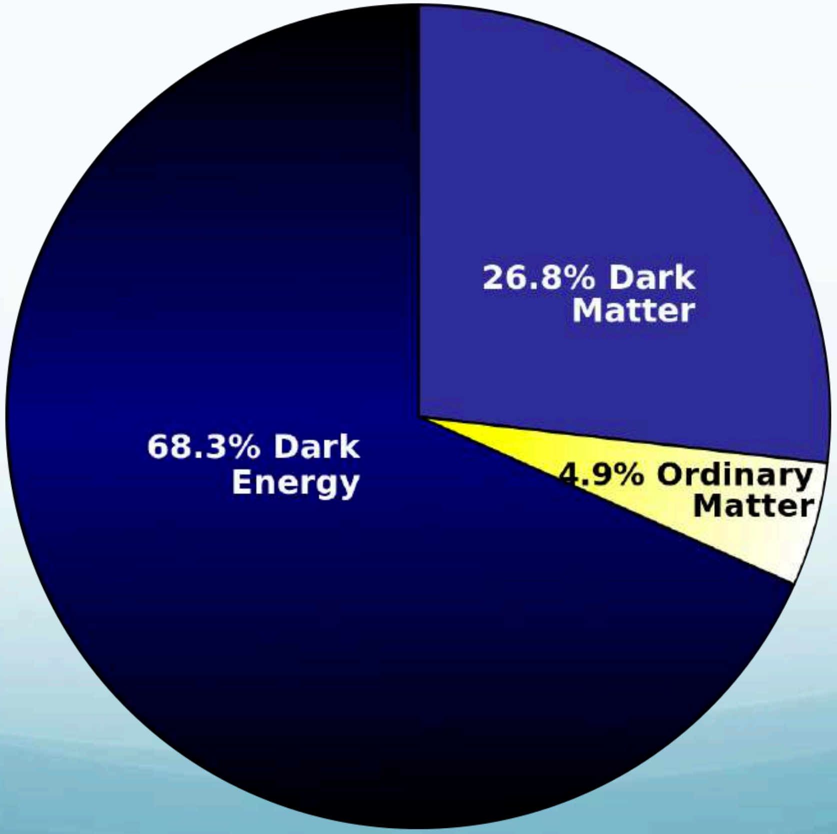
Simplest case and consistency relations

Non Gaussian initial conditions

Cosmology and distances

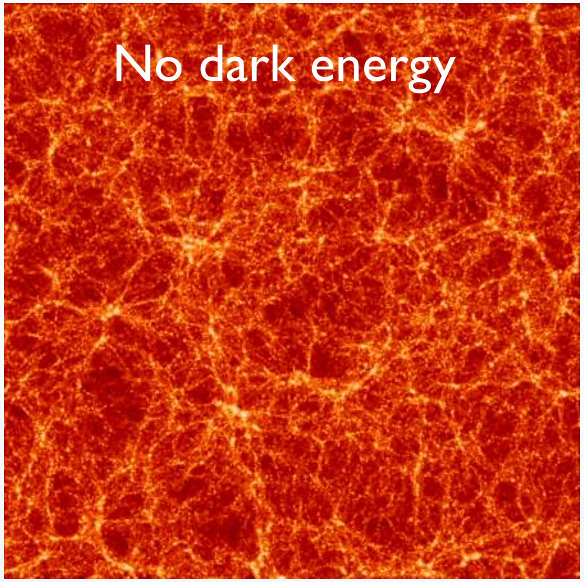
General Relativity: Geometry = Content

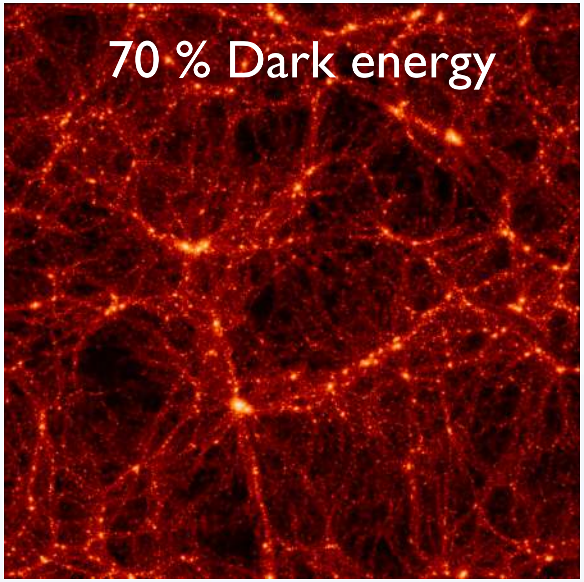
Distances (e.g. Supernovae) ➞ accelerated expansion



Cosmology with structures

Energy content changes the structure





Credit: Joerg Colberg, Virgo simulations, Jenkins et al, 1998 Astrophysical Journal, 499, 20-40

$$\rho(t,\vec{x}) = \bar{\rho}(t)[1 + \delta(t,\vec{x})]$$

Cosmology with structures

Equivalence Principle (EP)

→ Do all objects fall the same way?

Cornerstone of GR

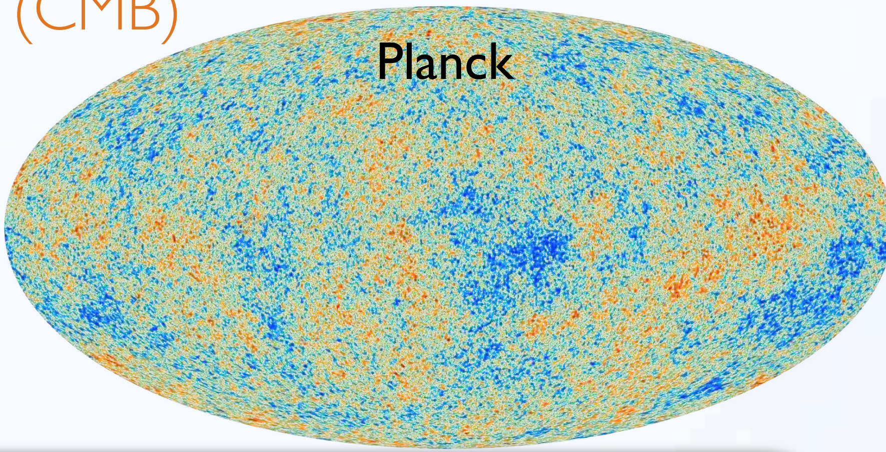
Initial conditions

→ Is the distribution initially Gaussian?

Prediction of simplest inflation models

Probing the structures

✧ Cosmic Microwave Background (CMB)



Snapshot at t = 380'000 years.

$$\delta \sim 10^{-5}$$

2 dimensional

✧ Large scale structure (LSS)

Late Universe (2-10 billion years ago)

$$\delta \geq 1$$

3 dimensional

New surveys to come online: **LSST, WFIRST, EUCLID,…**

Extracting the information

Density field $\delta(\vec{x})$

Extracting the information

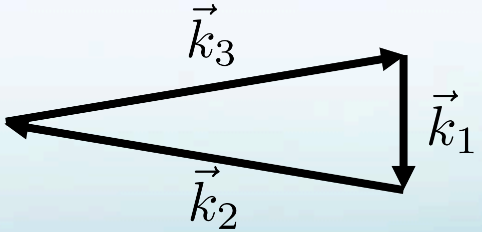
Fourier space $\delta(\vec{k})$

$$\rightarrow \quad \langle \delta(\vec{k}_1) \cdots \delta(\vec{k}_n) \rangle$$

Power Spectrum (n=2)

$$\langle \delta(\vec{k}) \delta(\vec{k}') \rangle = (2\pi)^3 P(k) \delta^{(D)}(\vec{k} + \vec{k}')$$

Bispectrum (n=3)



$$\langle \delta(\vec{k}_1) \delta(\vec{k}_2) \delta(\vec{k}_3) \rangle = (2\pi)^3 \delta^{(D)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) B(k_1, k_2, k_3)$$

Simplest case
and
consistency relations

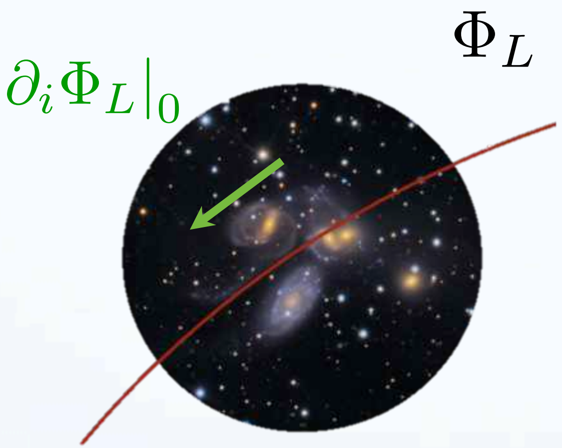
Consistency relations

Assume Equivalence Principle

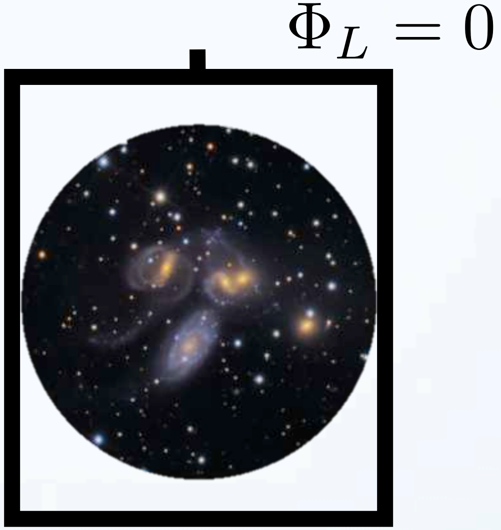
$$\Phi_L(\eta,\vec{x}) = \Phi_L(\eta)\big|_0 + \partial_i\Phi_L(\eta)\big|_0\, x^i + \boxed{\partial_i\partial_j\Phi_L(\eta)\big|_0\, x^i x^j} + \dots$$

Consistency relations

Assume Equivalence Principle



$$\equiv$$



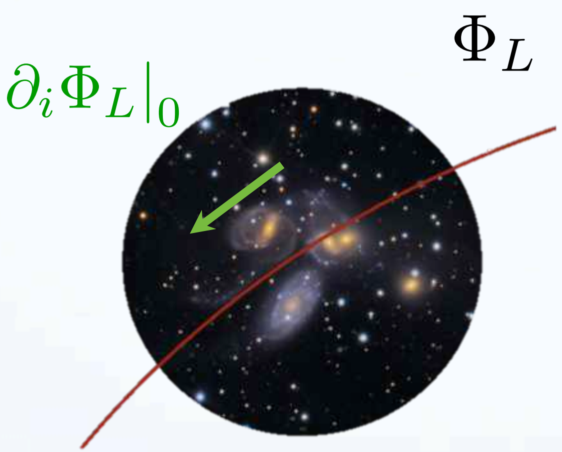
$$\vec{x} \to \vec{\tilde{x}} \equiv \vec{x} + \delta\vec{x}$$

$$\delta\vec{x}(\eta, \vec{p}) = -i\frac{\vec{p}}{p^2}\delta_L(\eta, \vec{p}) \propto \Phi_L(\eta, \vec{p})$$

$$\langle \delta^{(g)}(\eta_1, \vec{x}_1) \cdots \delta^{(g)}(\eta_n, \vec{x}_n) | \Phi_L \rangle = \langle \delta^{(g)}(\eta_1, \vec{\tilde{x}}_1) \cdots \delta^{(g)}(\eta_n, \vec{\tilde{x}}_n) \rangle$$

Consistency relations

Assume Equivalence Principle



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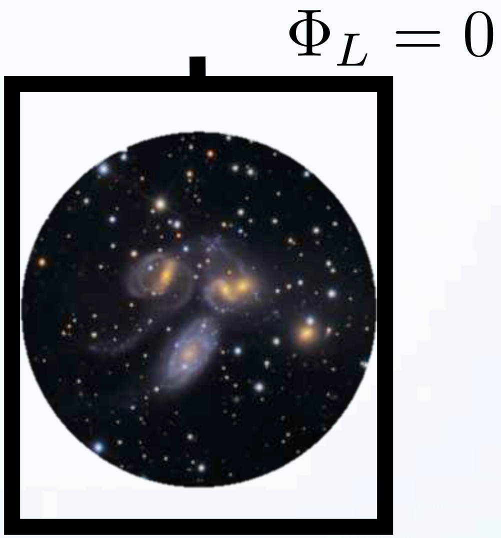
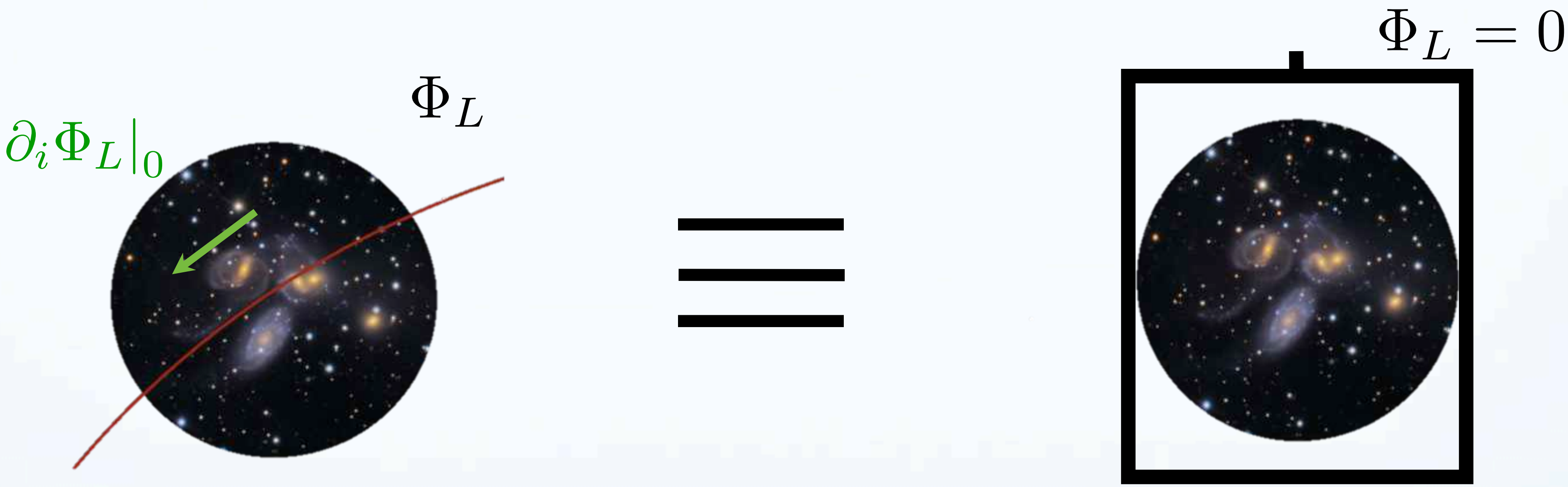


Assume Gaussianity
(no correlations long/short)

$$\langle \delta^{(g)}(\eta_1, \vec{x}_1) \cdots \delta^{(g)}(\eta_n, \vec{x}_n) | \Phi_L \rangle = \langle \delta^{(g)}(\eta_1, \vec{\tilde{x}}_1) \cdots \delta^{(g)}(\eta_n, \vec{\tilde{x}}_n) \rangle$$

Consistency relations

Assume Equivalence Principle



Assume Gaussianity

(no correlations long/short)

$$\langle \delta_{\vec{p}}(\eta)\delta^{(g)}_{\vec{k}_1}(\eta_1)\cdots\delta^{(g)}_{\vec{k}_n}(\eta_n)\rangle'_{\vec{p}\to 0} = -P(p,\eta)\sum_a \frac{D(\eta_a)}{D(\eta)}\frac{\vec{k}_a\cdot\vec{p}}{p^2}\langle\delta^{(g)}_{\vec{k}_1}(\eta_1)\cdots\delta^{(g)}_{\vec{k}_n}(\eta_n)\rangle'$$

Peloso & Pietroni '13, Riotto et al '13 Creminelli et al, '13

Consistency relations

$$\langle \delta_{\vec{p}}(\eta) \delta^{(g)}_{\vec{k}_1}(\eta_1) \cdots \delta^{(g)}_{\vec{k}_n}(\eta_n) \rangle'_{\vec{p}\to 0} = -P(p,\eta) \sum_a \frac{D(\eta_a)}{D(\eta)} \frac{\vec{k}_a \cdot \vec{p}}{p^2} \langle \delta^{(g)}_{\vec{k}_1}(\eta_1) \cdots \delta^{(g)}_{\vec{k}_n}(\eta_n) \rangle'$$

Linear

Non perturbative

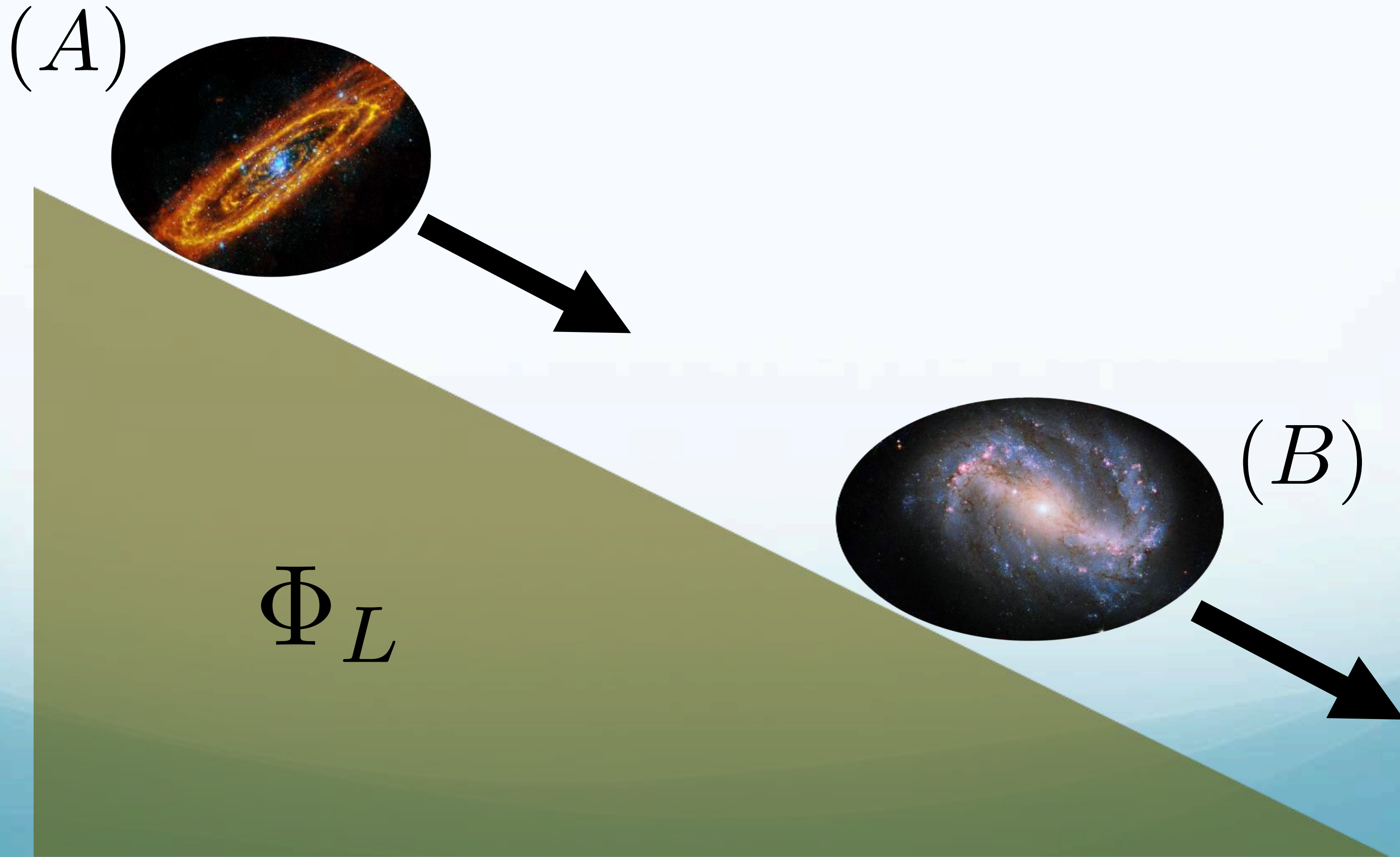
$$\delta(\eta, \vec{k}) = D(\eta) \delta_0(\vec{k})$$

Equal time correlators

$$\langle \delta_{\vec{p}}(\eta) \delta^{(g)}_{\vec{k}_1}(\eta) \cdots \delta^{(g)}_{\vec{k}_n}(\eta) \rangle_{p\to 0} = \mathcal{O}([k/p]^0)$$

Equal time correlators

$$\langle \delta_{\vec{p}}(\eta) \delta^{(A)}_{\vec{k}_1}(\eta) \delta^{(B)}_{\vec{k}_2}(\eta) \rangle = \mathcal{O}[(k/p)^0]$$



Breaking the assumptions

✧ Equivalence principle

$$\langle \delta_{\vec{p}}(\eta)\delta^{(A)}_{\vec{k}_1}(\eta)\delta^{(B)}_{\vec{k}_2}(\eta)\rangle'_{p\to 0} = \left(\epsilon\,\frac{\vec{p}\cdot\vec{k}}{p^2} + \mathcal{O}[(k/p)^0]\right) P(\eta,p)P_{AB}(\eta,k)$$

Model dependent

✧ Correlation short-long modes

→ Local Non Gaussianity $\Phi = \Phi_{\rm G} + f_{\rm NL}^{\rm Loc}(\Phi_{\rm G}^2 - \langle\Phi_{\rm G}\rangle^2)$

$$\langle \delta_{\vec{p}}(\eta)\delta_{\vec{k}_1}(\eta)\delta_{\vec{k}_2}(\eta)\rangle'_{p\to 0} = \left(\frac{6 f_{\rm NL}^{\rm Loc}\Omega_{\rm m,0}H_0^2}{p^2 T(p) D(\eta)} + \mathcal{O}[(k/p)^0]\right) P(\eta,p)P(\eta,k)$$

Peloso & Pietroni '13

Primordial Non-Gaussianity (PNG)

Why study non-Gaussianity?

$$\Phi = \Phi_{\rm G} + f_{\rm NL}^{\rm Loc}(\Phi_{\rm G}^2 - \langle \Phi_{\rm G} \rangle^2)$$

$\rightarrow$ $f_{\rm NL}^{\rm loc}$ $\longrightarrow$ $-\frac{5}{12}(n_s - 1)$ *Maldacena '02*

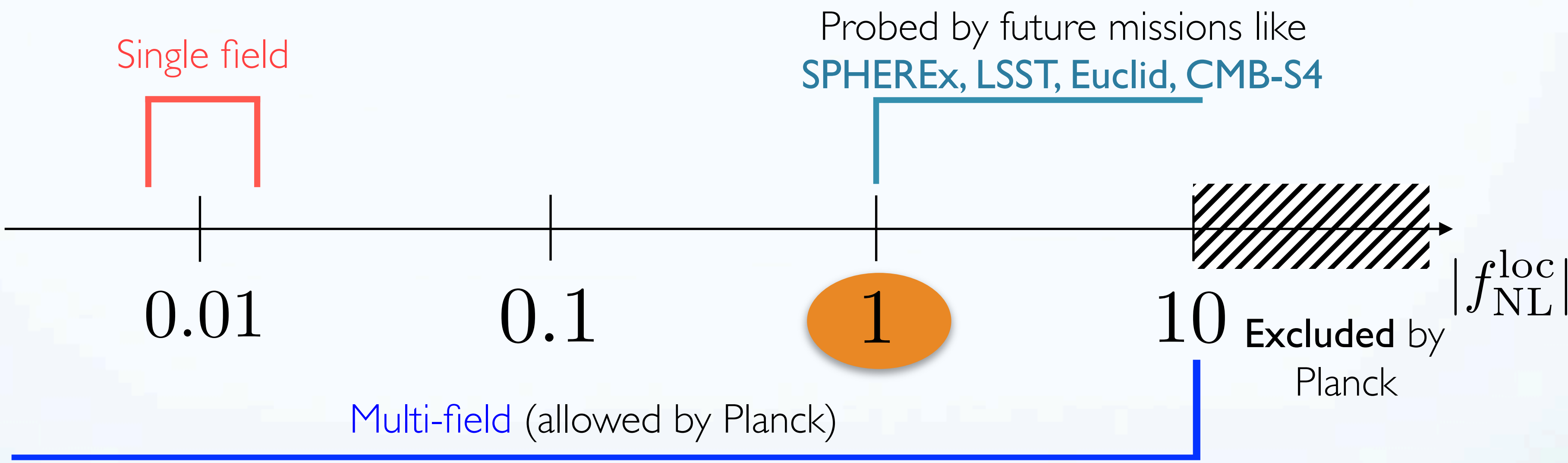
Single field inflation

Creminelli & Zaldarriaga '04

$\longrightarrow$ **?**

Multi-field inflation

Why study non-Gaussianity?



$$\mathrm{Prob}(|f_{\rm NL}^{\rm Loc}| > 1) \gtrsim 50\%^{*}$$ *with de Putter and Doré* **arXiv:1612.05248**

$*$: 2-field models with spectator field

Measuring PNG from surveys

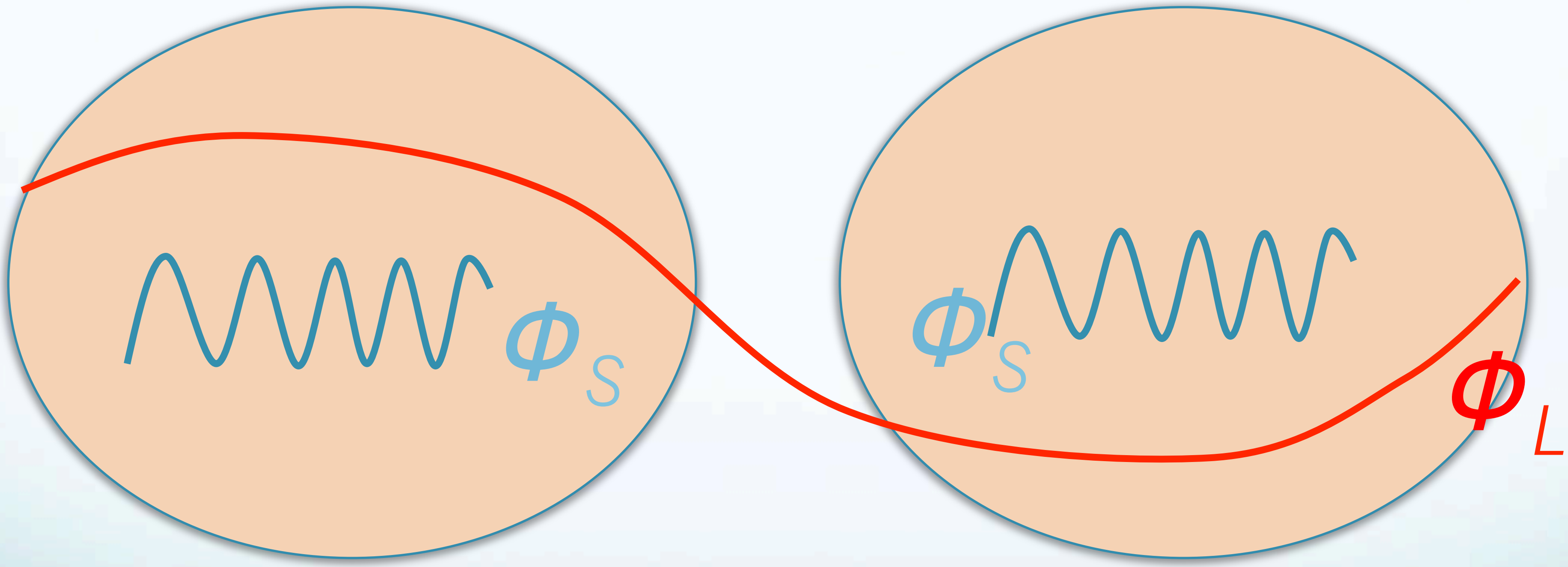
- CMB: Bispectrum

$$\sigma(f_{\rm NL}^{\rm Loc}) \sim 5$$

- Galaxy surveys: scale-dependent bias

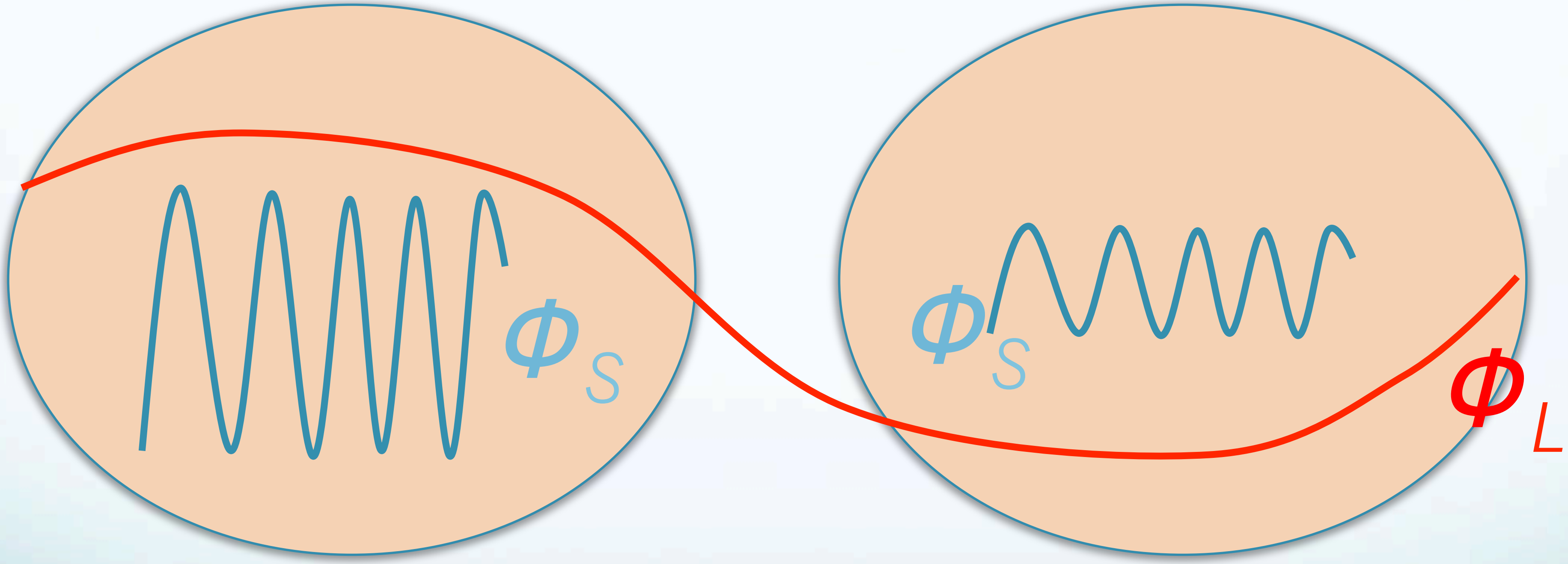
Scale-dependent bias

Single field inflation



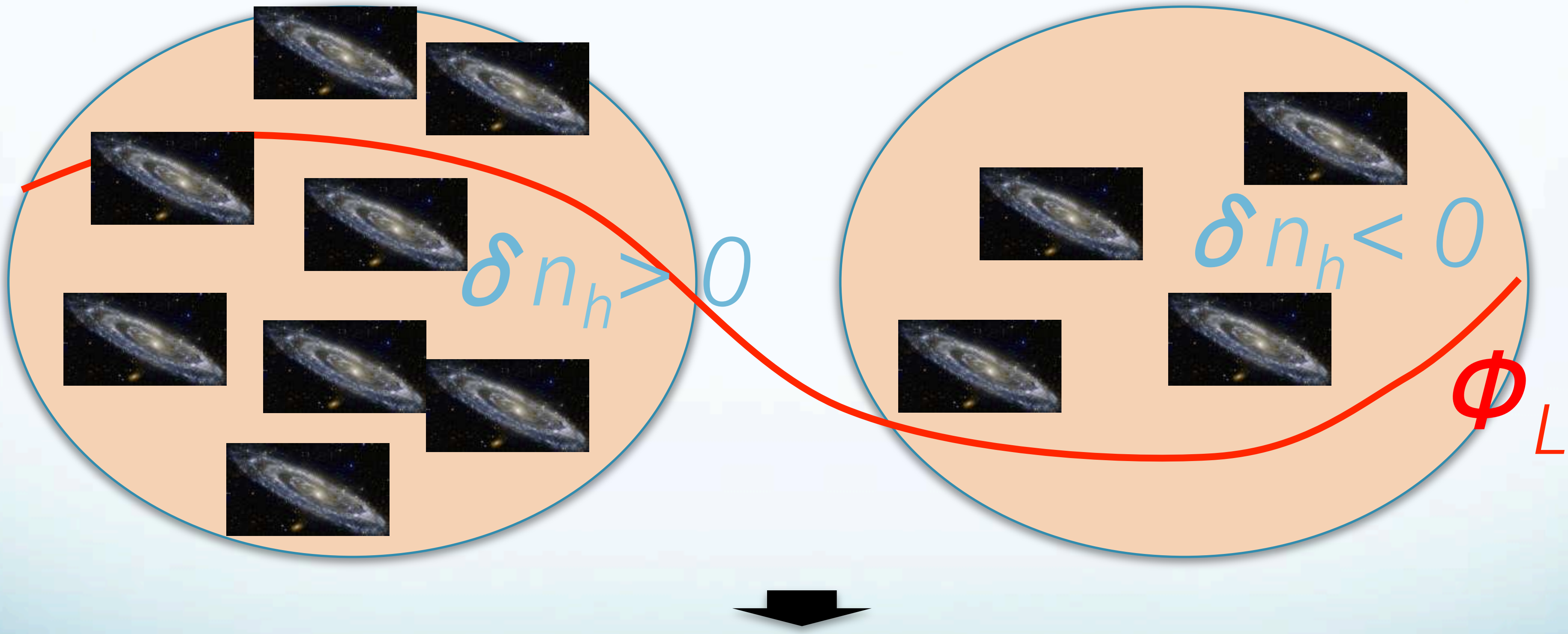
Scale-dependent bias

Multi-field inflation



Scale-dependent bias

Multi-field inflation



Scale-dependent bias

Scale-dependent bias

✧ Local PNG

$$\mathcal{M}(q) \equiv \frac{2q^2 T(q) D(z)}{3\Omega_m H_0^2}$$

$$b_{\rm NG}(q) = 2\, f_{\rm NL}^{\rm Loc}\, (b_\delta - 1)\, \delta_c\, \mathcal{M}^{-1}(q) \sim \frac{1}{q^2 T(q)}$$

⬍

$$\langle \delta_{\vec{p}}(\eta) \delta_{\vec{k}_1}(\eta) \delta_{\vec{k}_2}(\eta) \rangle'_{p \to 0} = \left(\frac{6 f_{\rm NL}^{\rm Loc} \Omega_{\rm m,0} H_0^2}{p^2 T(p) D(\eta)} + \mathcal{O}[(k/p)^0] \right) P(\eta, p) P(\eta, k)$$

Scale-dependent bias

- Local PNG

$$\mathcal{M}(q) \equiv \frac{2q^2 T(q) D(z)}{3\Omega_m H_0^2}$$

$$b_{\rm NG}(q) = 2\, f_{\rm NL}^{\rm Loc}\, (b_\delta - 1)\, \delta_c\, \mathcal{M}^{-1}(q) \sim \frac{1}{q^2 T(q)}$$

- Equilateral PNG

Typical size of halos

$$b_{\rm NG}(q) = 6\, f_{\rm NL}^{\rm Eq}\, (b_\delta - 1)\, \delta_c\, (q\, R_*)^2\, \mathcal{M}^{-1}(q) \sim \frac{1}{T(q)}$$

Consistency relation ***not*** broken

Biasing and PNG

with de Putter, Green and Doré **arXiv:1612.06366**

- Generalized model of bias *McDonald & Roy '09, Assassi et al '15*

$$\delta_h = b_\delta \delta + b_{\rm NG}(q)\delta + F_{\rm nonlocal}[\nabla^2\delta] + F_{\rm nonlinear}[\delta]$$

δ^2

$$\left[b_{q^2}(qR_*)^2 + b_{q^4}(qR_*)^4\right]\delta$$

Seen in simulations *Chan et al '12, Baldauf et al '12*

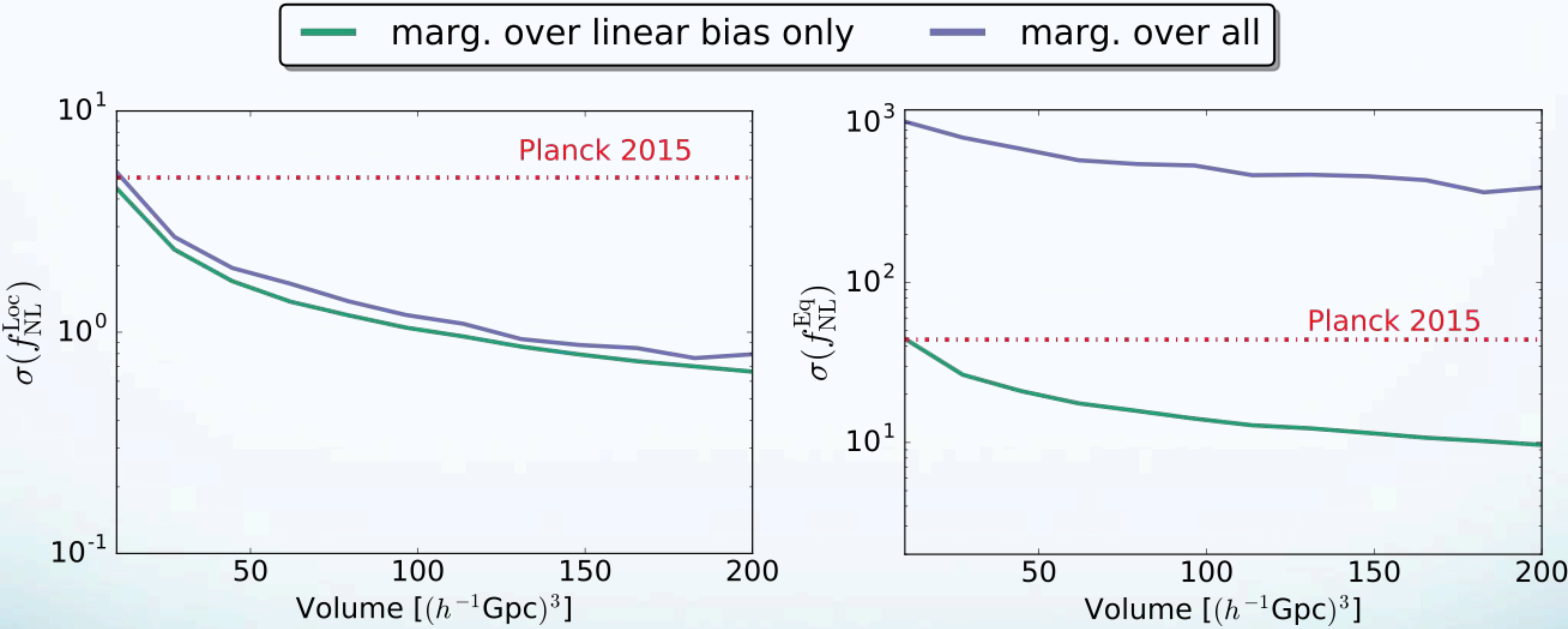
- Evolution or PNG?

$$T(q) \sim 1 + T_1 q^2 + T_2 q^4$$

$b_{\rm NG}^{\rm Loc} \sim q^{-2}$ ✓

$b_{\rm NG}^{\rm Eq} \sim c + c_1 q^2 + \cdots$?

Measuring PNG from surveys



Signal peaks for squeezed

Distinct from evolution

Signal peaks for equilateral

Degenerate with evolution

Conclusions

Consistency relations are robust consequences of $\Lambda\mathrm{CDM}$

Not satisfied if Equivalence principle is broken or local PNG

Unbroken for equilateral PNG $\longrightarrow$ degenerate with evolution

Bispectrum more appropriate than bias for equilateral PNG