Holography in Classical and Quantum Gravity

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Outline

A. Motivation for holography

B. Gauge/gravity duality

C. “Real world” applications

Why holography?

Early motivation came from the Bekenstein-Hawking entropy of black holes:

$$S_{BH} = A / 4$$

Diffeomorphism invariance implies that the Hamiltonian is defined by a surface integral at infinity. If the spectrum of black holes is discrete, one can (in principle) determine the state by measuring the energy at infinity.

(Balasubramanian, Marolf, Rozali)

Holography is not just a property of black holes, but should be a general property of quantum gravity:

Everything that happens in a region of space can be described by degrees of freedom living on the boundary.

('tHooft and Susskind)

Advantages of anti-de Sitter (AdS) for holography

- A static slice in AdS has constant negative curvature, so the spheres at large radius are much bigger than usual
- The conformal boundary at infinity is timelike
- Black hole thermodynamics is better behaved in AdS since the negative curvature acts like a confining box

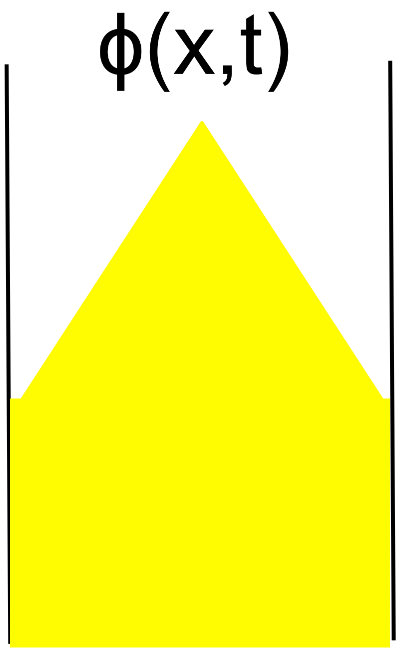
(Hawking and Page)

Another general argument for holography in AdS (Marolf)

In perturbation theory, any field at a point in the interior can be evolved back and expressed as an integral over boundary operators at infinity.

Now use $$\frac{d}{dt}\mathcal{O}(t) = i[H, \mathcal{O}(t)].$$

to evolve all boundary operators to one time.



Gauge/gravity duality

Gauge/gravity duality

(Maldacena; Gubser, Klebanov, Polyakov; Witten)

With anti-de Sitter boundary conditions, string theory (which includes gravity) is completely equivalent to a (nongravitational) gauge theory living on the boundary at infinity.

When string theory is weakly coupled, gauge theory is strongly coupled, and vice versa.

AdS can be written

$$ds^2 = r^2[-dt^2 + dx_i dx^i] + \frac{dr^2}{r^2}$$

The gauge theory lives on the Minkowski spacetime at r = ∞.

Scaling symmetry: $r \rightarrow ar$, $(t, x_i) \rightarrow (t/a, x_i/a)$ so small r corresponds to large distances or low energy in the gauge theory.

Attempt to disprove this conjecture:

A 4D theory should have many more degrees of freedom than a 3D theory. So let's compare the entropy at high temperature:

3D thermal gas: $E \sim T^3 V, \quad S \sim T^2 V$

4D thermal gas: $E \sim T^4 V, \quad S \sim T^3 V$

BUT in a theory with gravity, at high T this gas will collapse to form a large black hole.

The planar black hole has metric

$$ds^2 = r^2[-f(r)dt^2 + dx_i dx^i] + \frac{dr^2}{r^2 f(r)}$$

where $$f(r) = \left(1 - \frac{r_0^3}{r^3}\right)$$

The Hawking temperature is $T \sim r_0$.

The total energy is $E \sim r_0^3 V \sim T^3 V$.

The entropy is $S \sim A \sim r_0^2 V \sim T^2 V$.

So the 3+1 BH energy and entropy are exactly like a thermal gas in 2+1 dimensions.

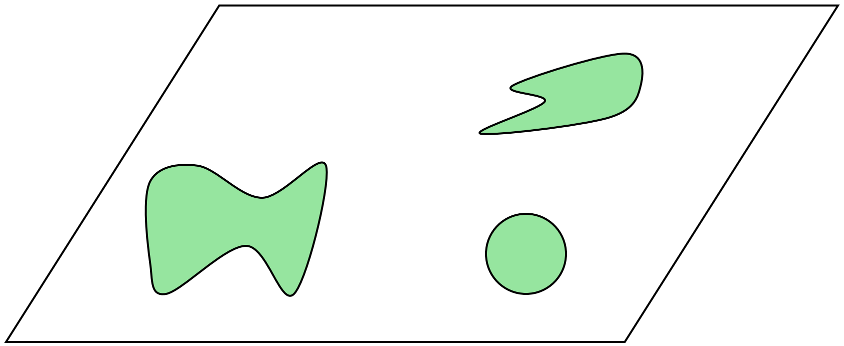
Early Evidence

- Symmetries agree
- Gauge theory analogs of all massless string modes are known
- Gauge theory analogs of many massive string modes have been found (strings arise as long chains of operators)
- Many interactions have been shown to agree

Comparing states

Explicit correspondence has been found for all states preserving half the SUSY (Lin, Lunin, Maldacena)

Gauge theory: All states created by a single homogeneous field - N x N matrix. This matrix model can be quantized exactly and states are labeled by closed curves in a plane.



String theory: For each state there is a corresponding gravity solution. It is stationary and nonsingular, but can have complicated topology. It is characterized by a solution to a 3D linear equation. The boundary condition for the linear equation is again closed curves in a plane.

The curvature is everywhere below the Planck scale if the area enclosed by each loop is large enough.

The full quantum description of this sector of the theory is given by the matrix model.

Many calculations agree

- Microscopic derivation of black hole entropy
- Partition functions
- Expectation value of Wilson loops
- Renormalization group flow
- …

Renormalization group (RG) flow in a QFT corresponds to obtaining an effective low energy action by integrating out high energy modes.

This corresponds to radial dependence on the gravity side:

Gauge theory: add mass terms and follow RG flow to low energies to obtain a new field theory

Gravity theory: modify the boundary conditions for certain matter fields and solve Einstein's equation

One finds detailed numerical agreement between the small r behavior of the gravity solution and the endpoint of the RG flow.

Gauge/gravity duality provides a background independent formulation of quantum gravity.

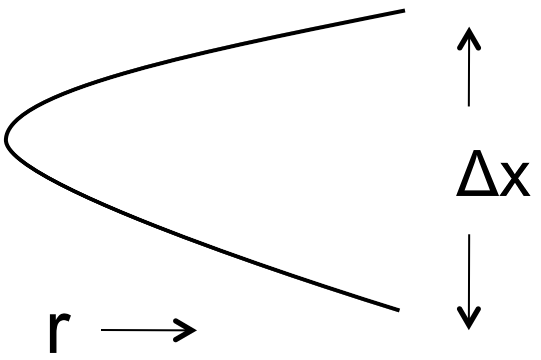
Only the asymptotic metric is fixed. The metric in the interior is free to fluctuate.

Traditional applications of gauge/gravity duality

Gain new insight into strongly coupled gauge theories, e.g., geometric picture of confinement.

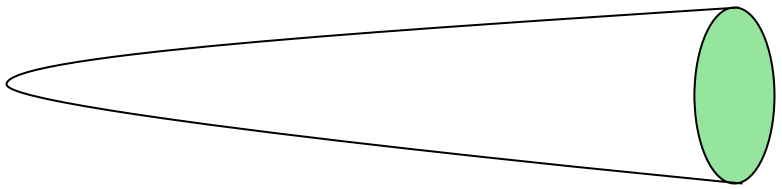
Gain new insight into quantum gravity, e.g., quantum properties of black holes

The potential between two quarks is obtained from the length of a string in the bulk:

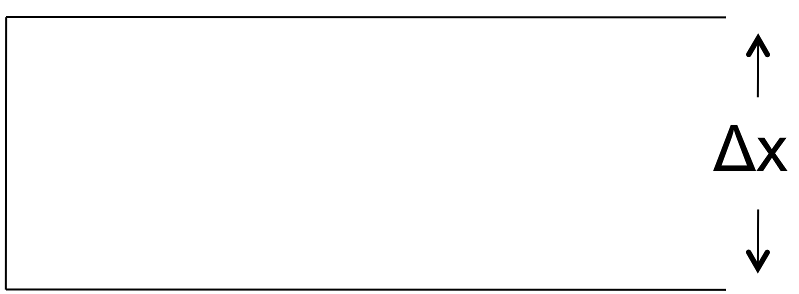


Due to the scaling symmetry of AdS, the length of this curve is independent of Δx. AdS describes the vacuum of a scale invariant theory.

The gravity dual of a confining vacuum differs from AdS in that the spacetime ends at nonzero radius e.g.



In this case, the length of the string grows linearly with Δx when it becomes large:



Quantum Black Holes

The gauge theory has enough microstates to reproduce the entropy of black holes.

The formation and evaporation of small black holes can be described by ordinary Hamiltonian evolution in the gauge theory. It does not lose information.

Unfortunately, this does (yet) not tell us how the information comes out. Still very mysterious.

“Real world” applications of gauge/gravity duality

In a certain limit, all stringy and quantum effects are suppressed and gravity theory is just general relativity

(in higher dimensions, with asymptotically anti-de Sitter boundary conditions).

This duality allows us to compute dynamical transport properties of strongly coupled systems at nonzero temperature.

Theoretical physicists have very few other tools to do this.

It reduces to perturbing an AdS black hole, e.g., dissipation is modeled by energy falling into the black hole.

Basic Ingredients of the Duality

A state of thermal equilibrium at temperature T is dual to a black hole with temperature T.

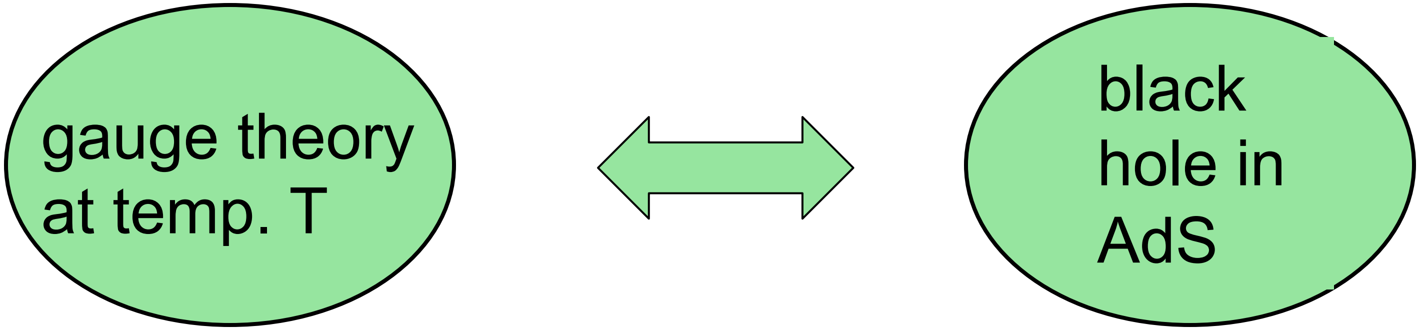
Fields in spacetime are dual to operators in the boundary theory.

Local properties of the gauge theory are related to the asymptotic behavior of the gravity solution.

Hydrodynamics from gravity

One expects that the long distance dynamics of any strongly interacting field theory is described by (relativistic) hydrodynamics.

Gauge/gravity duality predicts that hydrodynamics can be recovered from general relativity. Start with:



Boost black hole to give it four-velocity u^α. Consider metrics where the horizon radius and u^α vary slowly compared to the temperature T.

Find that Einstein's equation implies

$$\partial_\mu T^{\mu\nu} = 0$$

where

$$T^{\mu\nu} = (\pi T)^4(\eta^{\mu\nu} + 4u^\mu u^\nu) - 2(\pi T)^3\sigma^{\mu\nu} + \cdots$$

with $\sigma \sim PP\partial u$. Recover equations for fluid with viscosity.

(Bhattacharyya, Hubeny, Minwalla, Rangamani)

This has led to two corrections to standard treatments of hydrodynamics:

By going to second order, one obtains a form of relativistic hydrodynamics. The standard treatment by Israel and Stewart was shown to be incomplete. Several other second order terms were missing (Baier et al 2007.).

This approach has even led to an addition to the standard Landau-Lifshiftz treatment of magnetohydrodynamics.

When the fluid is charged, there can be an extra (parity violating) term in $T_{\mu\nu}$.

This term was first found by a gravity computation (J. Erdmenger, et. al 2008) and only later was it justified by a purely field theory argument.

For fluids with a gravity dual, the viscosity is very low. The ratio of the (shear) viscosity η to entropy density s is

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

This is much smaller than most materials.

(Kovtun, Policastro, Son, Starinets)

Something close to this is seen in experiment!

Alice: A Large Ion Collider Experiment

Lead nuclei collide at 3TeV per nucleon pair

Gauge/gravity duality and heavy ion colliders

The quark/gluon plasma produced at RHIC and the LHC is strongly coupled and thermalizes quickly. Surprisingly, it is well described by fluid dynamics with a very low viscosity - close to value calculated from gravity.

Gauge/gravity duality currently offers the best explanation of this fact.

Simple model of a conductor

Suppose electrons in a metal satisfy

$$m\frac{dv}{dt} = eE - m\frac{v}{\tau}$$

If there are n electrons per unit volume, the current density is J = nev. Letting $E(t) = Ee^{-i\omega t}$, find J = σ E, with

$$\sigma(\omega) = \frac{K\tau}{1 - i\omega\tau}$$

where $K=ne^2/m$. This is the Drude model.

Conductivity from gravity

Consider gravity coupled to a Maxwell field. Start by perturbing the black hole solution.

Assume time dependence $e^{-i\omega t}$ and impose ingoing wave boundary conditions at the horizon.

The asymptotic behavior is

$$A_x = A_x^{(0)} + \frac{A_x^{(1)}}{r} + \cdots$$

The gauge/gravity duality dictionary says

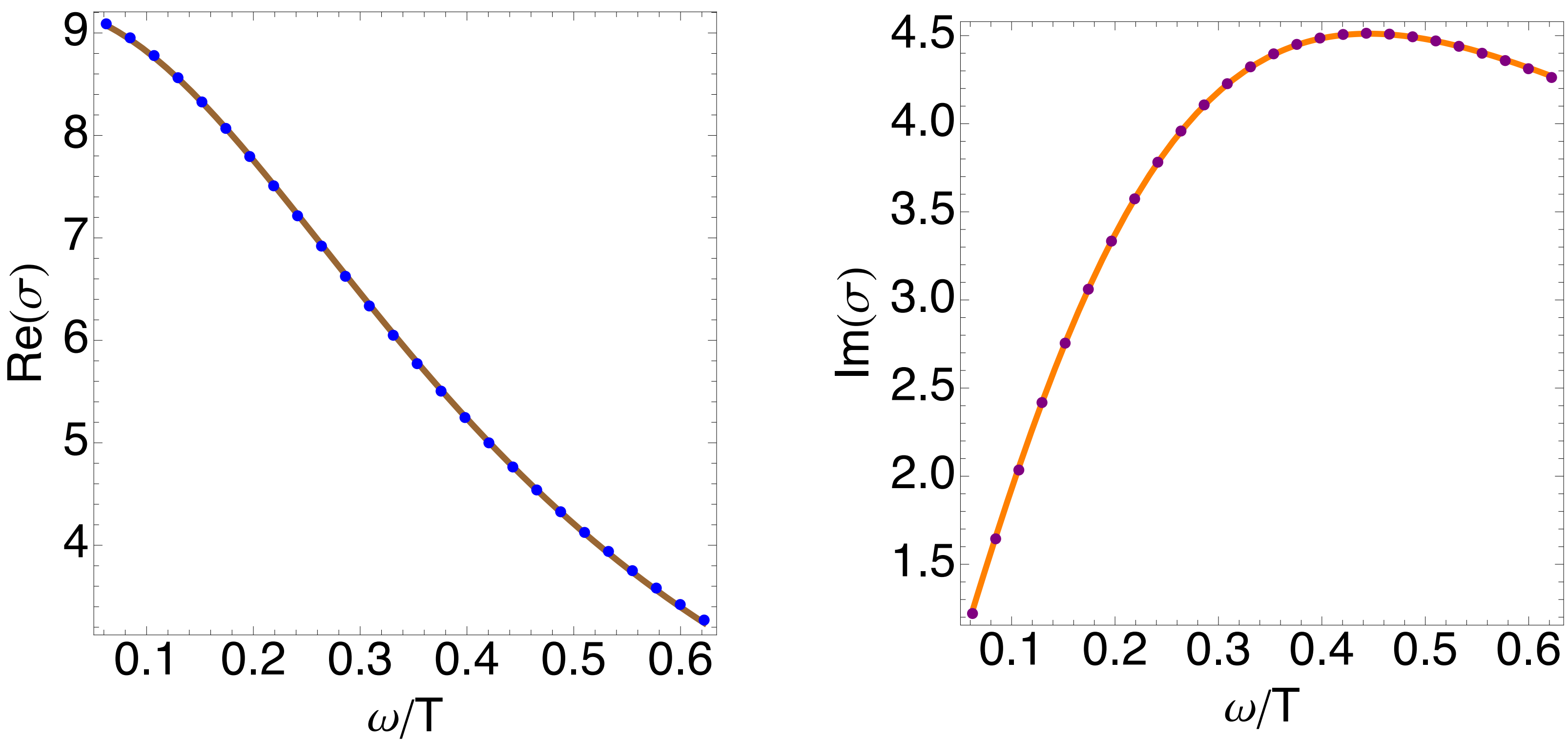
$$E_x = i\omega A_x^{(0)}, \quad J_x = A_x^{(1)}$$

We obtain the conductivity from Ohm's law

$$\sigma(\omega) = \frac{J_x}{E_x} = \frac{A_x^{(1)}}{i\omega A_x^{(0)}}$$

The low frequency conductivity takes the simple Drude form:

$$\sigma(\omega) = \frac{K\tau}{1 - i\omega\tau}$$



(Santos, Tong, G.H., 2012)

Superconductivity 101

In conventional superconductors (Al, Nb, Pb, …) pairs of electrons with opposite spin can bind to form a charged boson called a Cooper pair.

Below a critical temperature T_c, there is a second order phase transition and these bosons condense.

The DC conductivity becomes infinite.

This is well described by BCS theory.

Gravity dual of a superconductor

(Hartnoll, Herzog, and G.H.)

Gravity	Superconductor
Black hole	Temperature
Charged scalar field	Condensate

Need to find a black hole that has scalar hair at low temperatures, but no hair at high temperatures.

This is not an easy task.

Gubser (2008) argued that a charged scalar field around a charged black hole would have the desired property. Consider

$$\mathcal{L} = R + \frac{6}{L^2} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - |\partial\Psi - iqA\Psi|^2 - m^2|\Psi|^2$$

For an electrically charged black hole, the effective mass of Ψ is

$$m_{eff}^2 = m^2 + q^2 g^{tt} A_t^2$$

But the last term is negative. This produces scalar hair at low temperature.

At large radius, the vector potential and charged scalar behave as

$$A_t = \mu - \frac{\rho}{r}, \qquad \psi = \frac{\psi^{(2)}}{r^2}$$

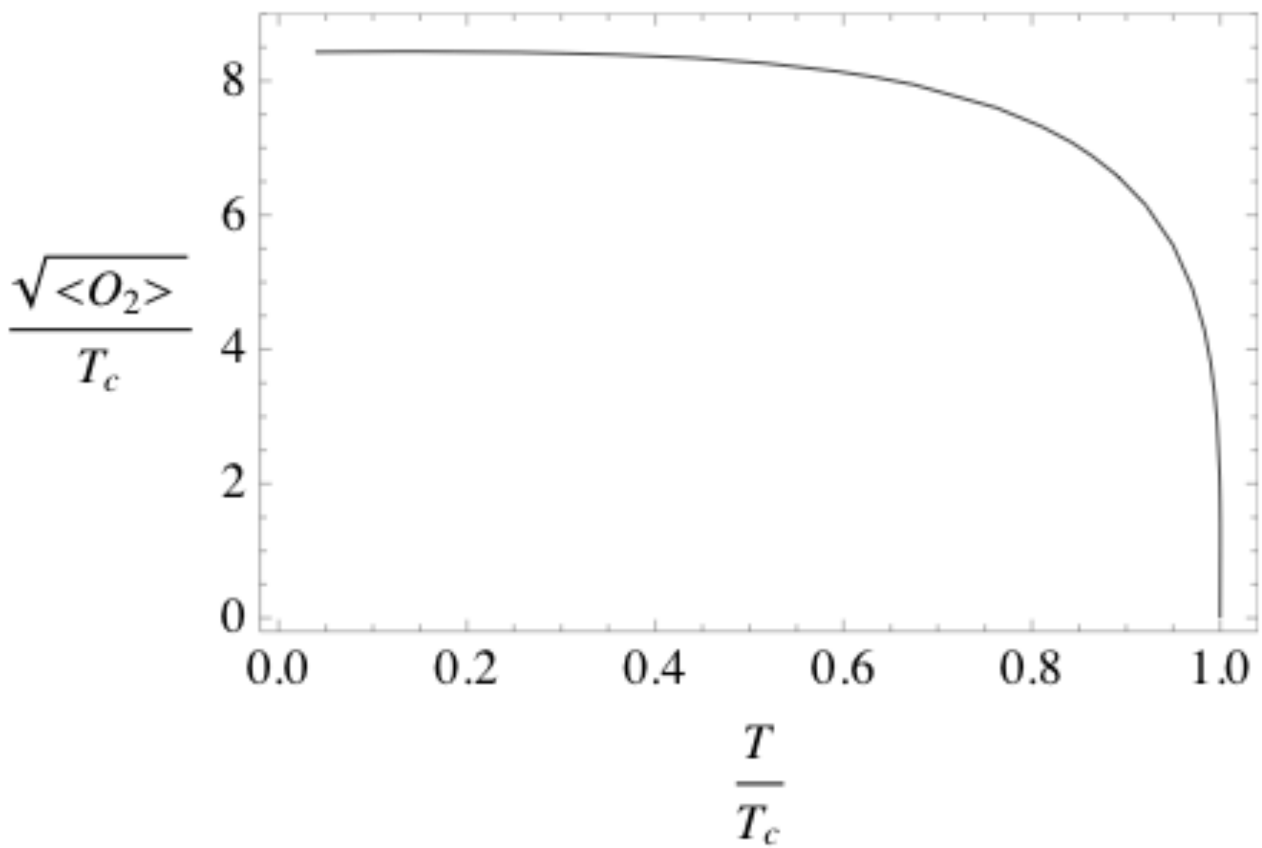
Gauge/gravity duality relates these constants to properties of the dual field theory:

μ = chemical potential, ρ = charge density

There is an operator O_2 dual to ψ, and

$$\langle \mathcal{O}_2 \rangle = \sqrt{2}\psi^{(2)}$$

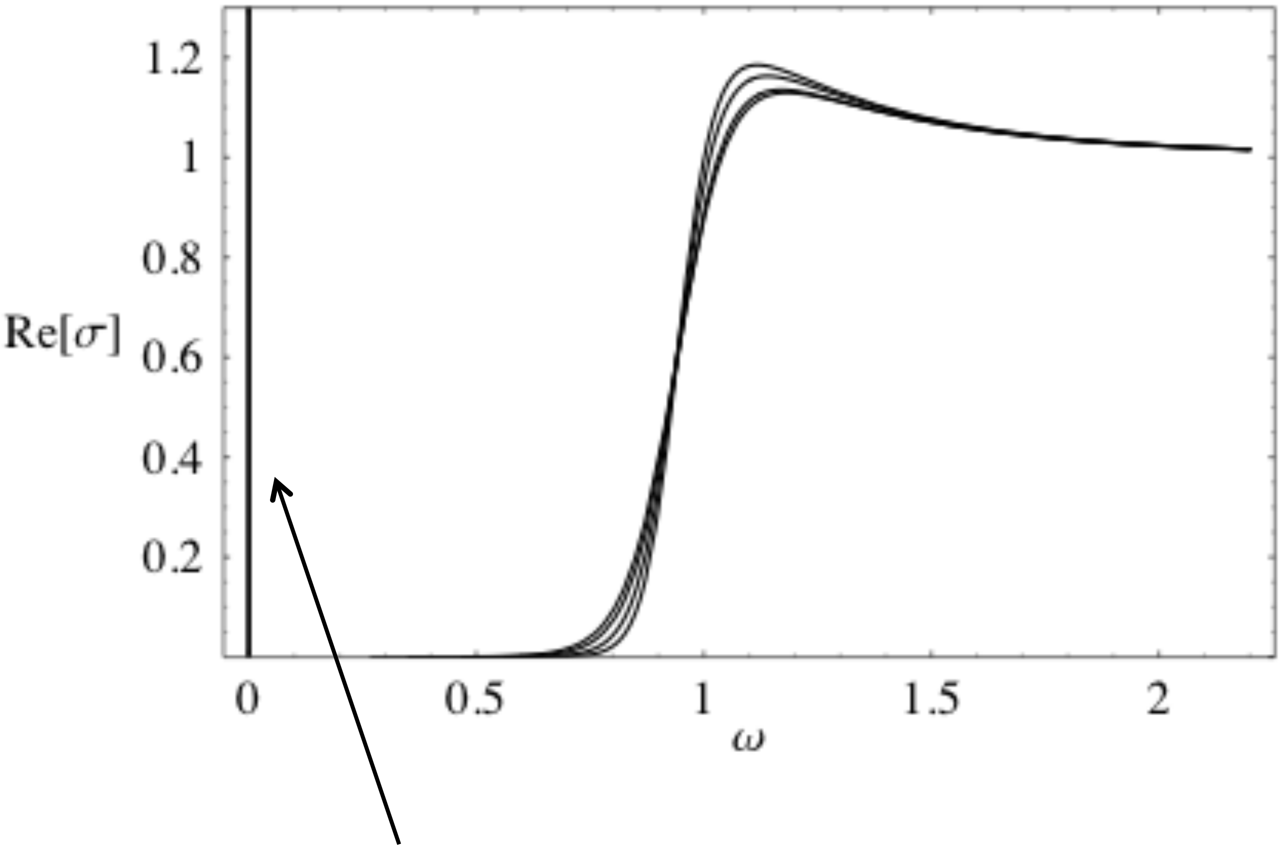
Condensate (hair) as a function of T



$$T_c \propto \mu$$

The conductivity at low T

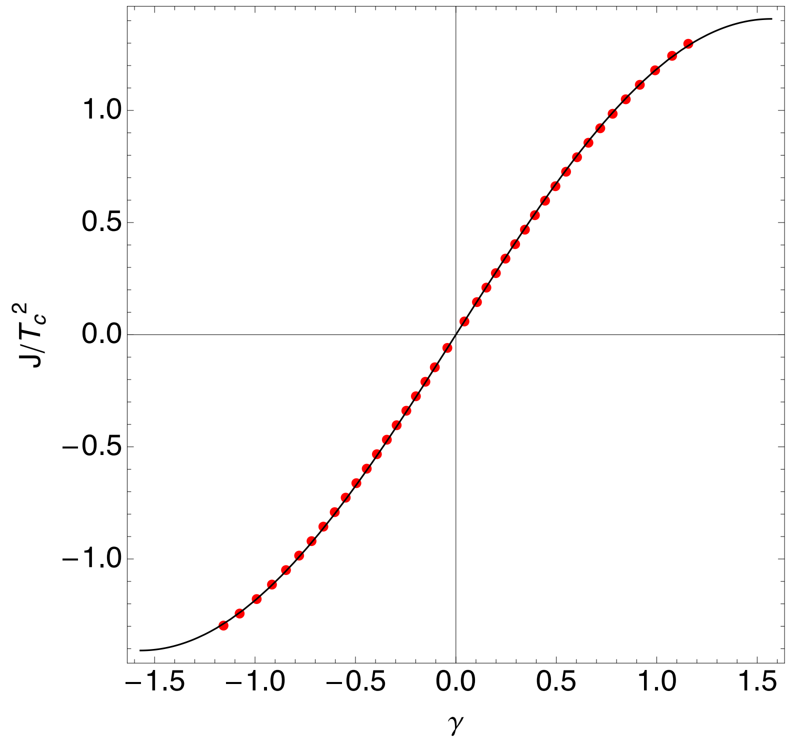
(Hartnoll, Herzog, and G.H.)



Delta function at ω = 0

A Josephson junction consists of two superconductors separated by a weak link.

Predict: $J = J_{max} \sin \gamma$



(Santos, Way, G.H.)

We have only used GR in the bulk

In addition to describing gravitational phenomena (black holes, gravitational waves, etc.) general relativity can also describe aspects of other fields of physics including hydrodynamics, QCD, and superconductivity.