The exoplanet HD 80606b as a new laboratory for gravity

François Larrouturou
working with: L. Blanchet & G. Hébrard

based on: arXiv:1905.06630

Institut d'Astrophysique de Paris

$\mathcal{G}\mathbb{R}\epsilon\mathbb{C}\mathcal{O}$ seminar
30. September 2019

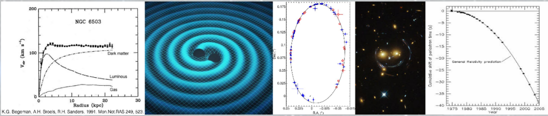
Motivation: Testing our theory of gravitation

- The current tests can be roughly decomposed in two classes

↪ *local* tests: *ie.* Earth-based and Solar System tests.

They are limited in range but have an extremely good accuracy ($eg.$ $|\gamma^{\text{PPN}} - 1| \lesssim 2 \cdot 10^{-5}$).

↪ *distant* tests: *ie.* Hulse-Taylor pulsar, gravitational radiation of binary Black Holes, motion of *S2*, motion of the stars in distant galaxies,...



For a good accuracy, only extremal regimes.

Motivation: Testing our theory of gravitation

⇒ The aim of this work is to address the usual Solar-System tests in a distant stellar system.

- The periastron precession in GR is given by

$$\Delta_{\mathrm{GR}} = \frac{6\pi GM}{ac^2(1-e^2)} .$$

- Detecting the periastron advance in star binaries was proposed by A. Gimenez in 1985[1]. But in all currently investigated systems, the tidal effects dominate[2] ⇒ no possible clean detection...

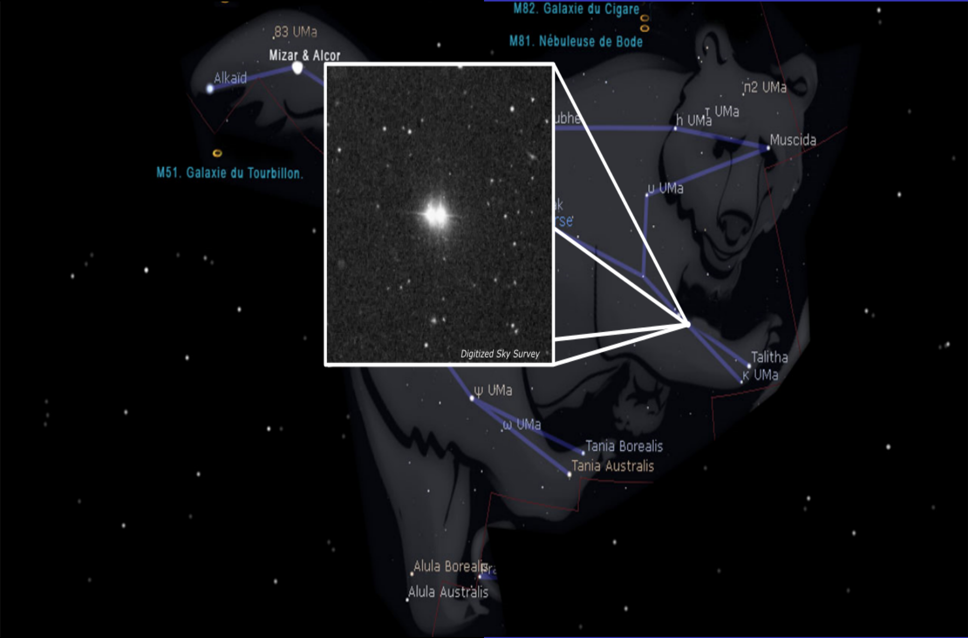
↪ So we need a (transiting) exoplanet

  - with high eccentricity,
  - and high compactness.

[1]Gimenez, 1985, Astrophys. J., 405, 167.
[2]Wolf *et al.*, 2010, Astron. and Astrophys., 509, A18.

Contents


M82. Galaxie du Cigare
M81. Nébuleuse de Bode
83 UMa
Mizar & Alcor
Alkaïd
Alioth
Megrez
Dubhe
h UMa
τ UMa
π2 UMa
Muscida
M51. Galaxie du Tourbillon.
υ UMa
Phad
Merak
Grande Ourse
M 97.
M106
Alkaphrah
Talitha
κ UMa
ψ UMa
ω UMa
Tania Borealis
Tania Australis
Alula Borealis
Alula Australis


M82. Galaxie du Cigare
M81. Nébuleuse de Bode
83 UMa
Mizar & Alcor
Alkaïd
M51. Galaxie du Tourbillon.
η2 UMa
h UMa
τ UMa
Muscida
υ UMa
Talitha
κ UMa
ψ UMa
ω UMa
Tania Borealis
Tania Australis
Alula Borealis
Alula Australis
Digitized Sky Survey

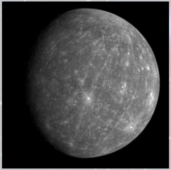
A Midsummer Night's Star

- The Solar-type star HD 80606 is located at 58pc from Earth,
- it has a companion star, distant from 1200 AU, HD 80607.

- Around it is orbiting a Jupiter-like planet ($M_p \simeq 4M_J$), HD 80606b, quite peculiar:

↪ it has a high eccentricity $e = 0.933$,

↪ and a strong spin-orbit misalignment $\lambda \simeq 42°$.



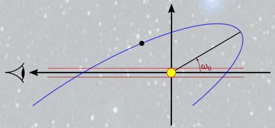
Mercury		HD 80606b
0.206	Eccentricity	0.933
0.31 AU	Periastron	0.03 AU
0.47 AU	Apastron	0.88 AU
116 days	Period	111 days
42 "/ct	Δ_{GR}	215 "/ct

A lucky detection

- HD 80606b was discovered through radial velocity in 2001[3]
  ⇒ due to its eccentricity, the odds for observing a transit were estimated to be 1/100,

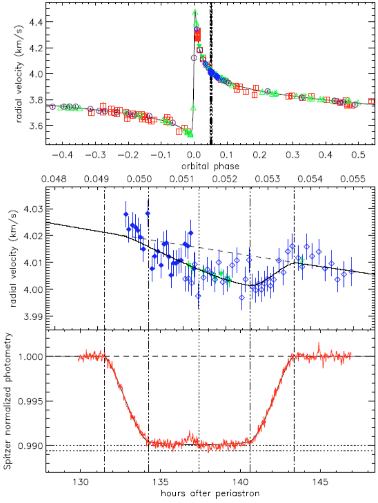
↪ an eclipse (anti-transit) was fortunately detected in 2009[4]
  ⇒ the odds for observing the transit increased to 1/10,

↪ during the night of January 13, 2010, the full 12h-long transit was recorded[5] by the *Spitzer* satellite and the *SOPHIE* spectrograph of the Haute-Provence Observatory.



[3] Naef *et al.*, 2001, Astron. and Astrophys., 375, L27.
[4] Laughlin *et al.*, 2009, Nature, 457, 562.
[5] Hébrard *et al.*, 2011, Astron. and Astrophys., 516, A95.

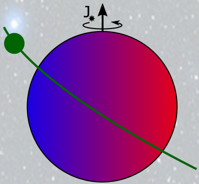
A lucky detection



From Hébrard *et al.*, A&A, A95(2011)516.

- The simultaneous observation of the transit in photometry and radial velocities allowed to fully characterize the system

↪ notably the spin-orbit misalignment is constrained *via.* the Rossiter-McLaughlin effect.



Formation mechanism

The currently favoured formation mechanism, is a "Kozaï migration"[6]

- The planet is formed
  - ↪ in a plane inclined *wrt.* the companion's plane,
  - ↪ further away ($\gtrsim 5$ AU),
  - ↪ with a small eccentricity $e \lesssim 0.1$.

⇝ Kozaï-Lidov oscillations increase e, keeping a and $\sqrt{1-e^2}\cos I$ constant,

⇝ tidal dissipative effects of the companion shrink the orbit,

⇒ today the Kozaï mechanism is negligible: the orbit is quite stable.

Other disfavoured possibilities are

- dynamical friction between the planet and the gas disks, but seems impossible to produce $e \gtrsim 0.60$,
- planet-planet scattering, but seems not strong enough to produce $e \gtrsim 0.90$.

[6] Wu & Murray, 2003, Astr. Journal, 589, 1.

Atmospheric properties

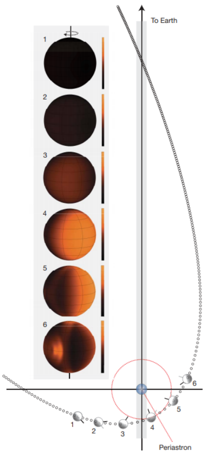


Figure from

Laughlin *et al.*, Nature,457(2009)562.

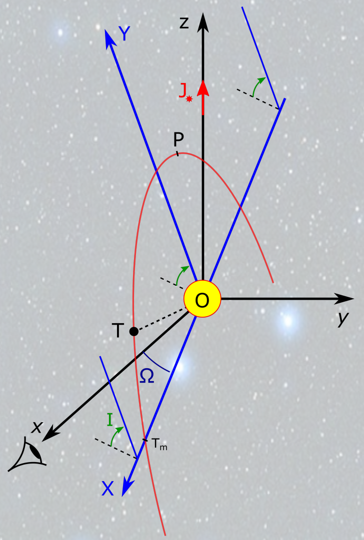
- At the apastron, HD 80606b grazes the limit of its parent's habitable zone,
- at the periastron, it is only a few $R_\star$ away from the star.

$\Rightarrow$ Extreme atmospheric conditions:

$\hookrightarrow$ within 6 hours, the temperature increases from $\sim 800K$ to $\sim 1500K$, to be compared with $T_\star^{\rm eff} \simeq 5800K$,

$\hookrightarrow$ heat shock waves that induce violent storms.

General conventions: Geometry



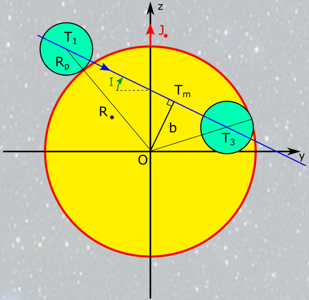
- There are two types of contributions:
  - ↪ the periodic ones (that average to 0 over one cycle),
  - ↪ the secular ones (that induce long-term contributions).
- *Spoiler:* the secular relativistic corrections affect the motion by:
  - ↪ shifting the trajectory (periastron shift),
  - ↪ shifting the period.
- *NB:* for practical purposes, we have taken $\vec{z} \propto \vec{J}_\star$, but of course there is a rotational invariance in the plane of the sky.

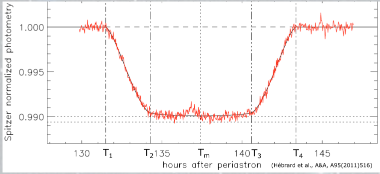
General conventions: Transit times

Let's define the transit times as

- T_1 and T_2: the beginning and end of entrance,
- T_3 and T_4: the beginning and end of exit,
- T_m: the time of passage at closest point from the center of the star.

The eclipse times are defined similarly.





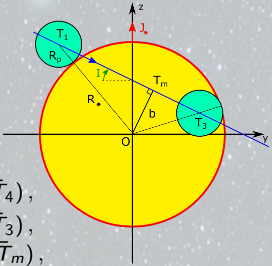
General conventions: Transit times

Mathematically, those points are defined as

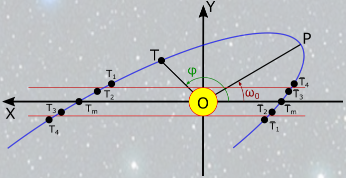
$$r(\varphi)\sin\varphi = Y(b),$$

with

$$Y(b) = \begin{cases} b\cot I + \sqrt{(R_\star + R_p)^2 - b^2} & (T_1 \text{ and } \bar{T}_4)\,, \\ b\cot I + \sqrt{(R_\star - R_p)^2 - b^2} & (T_2 \text{ and } \bar{T}_3)\,, \\ b\cot I & (T_m \text{ and } \bar{T}_m)\,, \\ b\cot I - \sqrt{(R_\star - R_p)^2 - b^2} & (T_3 \text{ and } \bar{T}_2)\,, \\ b\cot I - \sqrt{(R_\star + R_p)^2 - b^2} & (T_4 \text{ and } \bar{T}_1)\,. \end{cases}$$



The impact parameter is simply
$b \simeq r(\pi)\sin\Omega\sin I$
$\bar{b} \simeq r(0)\sin\Omega\sin I$.



NB: we neglect the effects of the local ellipticity, they are $\lesssim 1\%$.

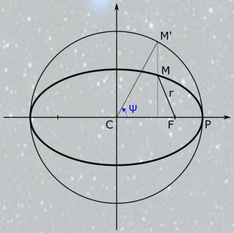
Reminder: Keplerian parametrisation of the motion

We need $r(\varphi)$ and $t(\varphi)$ to solve

$$r(\varphi)\sin\varphi = Y(b), \qquad \text{with} \qquad b \simeq r(\pi)\sin\Omega\sin I.$$

The Keplerian parametrisation uses

- $\hookrightarrow$ the mean anomaly, $\ell = n_0(t - t_{0,P})$, with $n_0 = 2\pi/P = \sqrt{GM/a^3}$ the usual Kepler's third law,
- $\hookrightarrow$ the eccentric anomaly $\psi = 2\arctan\left(\sqrt{\frac{1-e}{1+e}}\tan\frac{\varphi-\omega_0}{2}\right)$,



$\Rightarrow$ so that $\ell = \psi - e\sin\psi$ and

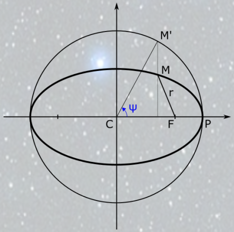
$$r = \frac{a(1-e^2)}{1+e\cos(\varphi-\omega_0)} = a(1-e\cos\psi).$$

Post-Keplerian parametrisation of the motion

The first relativistic correction[7] can be put in an elegant form, called *quasi-Keplerian representation*[8]

It uses

- ↪ the mean anomaly, $\ell = n(t - t_P)$, with $n = 2\pi/P$,
- ↪ the eccentric anomaly $\psi = 2\arctan\left(\sqrt{\frac{1-e_\varphi}{1+e_\varphi}}\tan\frac{\varphi-\omega_0}{2K}\right)$,



⇒ so that $\ell = \psi - e_t \sin\psi$ and

$$r = \frac{a_r(1-e_r^2)}{1 + e_r \cos\left[\frac{\varphi-\omega_0}{K} - \frac{1}{6} k\, e_r\, \nu \sin\left(\frac{\varphi-\omega_0}{K}\right)\right]} = a_r\left(1 - e_r\cos\psi\right).$$

[7] Wagoner & Will, 1976, Astr. J., 210, 764.
[8] Damour & Deruelle, 1985, Annales Inst. H. Poincaré Phys. Théor., 43, 107.

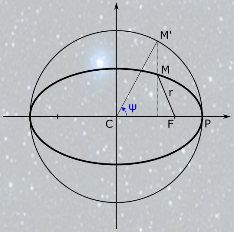
Post-Keplerian parametrisation of the motion

The first relativistic correction[9] can be put in an elegant form, called *quasi-Keplerian representation*[10]

It uses

↪ the mean anomaly, $\ell = n(t - t_P)$, with $n = 2\pi/P$,

↪ the eccentric anomaly $\psi = 2\arctan\left(\sqrt{\frac{1-e_\varphi}{1+e_\varphi}}\tan\frac{\varphi-\omega_0}{2K}\right)$,



⇒ so that $\ell = \psi - e_t \sin\psi$ and

$$r = \frac{a_r(1-{e_r}^2)}{1 + e_r\cos\left[\frac{\varphi-\omega_0}{K} - \frac{1}{6}k\, e_r\, \nu \sin\left(\frac{\varphi-\omega_0}{K}\right)\right]} = a_r\left(1 - e_r\cos\psi\right).$$

[9] Wagoner & Will, 1976, Astr. J., 210, 764.
[10] Damour & Deruelle, 1985, Annales Inst. H. Poincaré Phys. Théor., 43, 107.

Post-Keplerian parametrisation of the motion

- The quasi-Keplerian representation is roughly a Keplerian one, where some parameters receive corrections (with $\nu = M_\star M_p / M^2$)

$$K = 1 + k = 1 + \frac{3GM}{ac^2(1-e^2)} ,$$

$$n = n_0 \left(1 + \zeta\right) = n_0 + \frac{GMn_0}{8ac^2}\left(-15 + \nu\right) ,$$

$$a_r = a\left(1 + \xi\right) = a + \frac{GM}{4c^2}\left(-7 + \nu\right) ,$$

$$e_r = e + \varepsilon_r = e + \frac{GM}{8ac^2}\left[\frac{9+\nu}{e} + \left(15 - 5\nu\right)e\right] ,$$

$$e_t = e + \varepsilon_t = e + \frac{GM}{8ac^2}\left[\frac{9+\nu}{e} + \left(-17 + 7\nu\right)e\right] ,$$

$$e_\varphi = e + \varepsilon_\varphi = e + \frac{GM}{8ac^2}\left[\frac{9+\nu}{e} + \left(15 - \nu\right)e\right] .$$

Relativistic effects on the motion

- The "real" motion is the fully relativistic one.

↪ But it is well approximated by taking a Keplerian one, together with the first relativistic corrections.

⇒ We will thus decompose any quantity as $q = q_0 + \delta q$,

↪ solve the Newtonian problem for q_0,

↪ add the perturbation δq on top of it, and solve for δq.

- In order to do so we have to express the perturbation in terms of the conserved quantities E and J.
  For convenience, we will use the Newtonian formulae

$$E = -\frac{GM\mu}{2a}, \qquad \text{and} \qquad J = \mu\sqrt{GMa(1-e^2)}.$$

Relativistic effects on the motion

- Applying the method to the time of passage at a point i, it comes after N orbits

$$t = t_{0,i} + \frac{2\pi N}{n_0} + \delta t,$$

where δt can be split as

$$\delta t_{\rm sec} = \frac{1}{n_0}\left[(1 - e\cos\psi_0)\delta\psi_{\rm sec} - (\psi_0 - e\sin\psi_0)\zeta\right],$$

$$\delta t_{\rm per} = \frac{1}{n_0}\left[(1 - e\cos\psi_0)\delta\psi_{\rm per} - \varepsilon_t\sin\psi_0\right],$$

with, for a transit,

$$\delta\psi_{\rm sec} = \frac{2k\left[(\cos\psi_0 - e)\cos\omega_0 - \sqrt{1-e^2}\sin\psi_0\sin\omega_0\right]\arctan\left(\sqrt{\frac{1+e}{1-e}}\tan\frac{\psi_0}{2}\right) - a^{-1}\delta Y_{\rm sec}}{\sin\psi_0\sin\omega_0 - \sqrt{1-e^2}\cos\psi_0\cos\omega_0},$$

$$\delta Y_{\rm sec} = -\left.\frac{\partial Y}{\partial b}\right|_{b_0}\frac{k\,b_0\,e\sin\omega_0}{1 - e\cos\omega_0}\left[(2N+1)\pi - \omega_0 + \frac{e\nu\,\sin\omega_0}{6}\right].$$

Relativistic effects on the transit

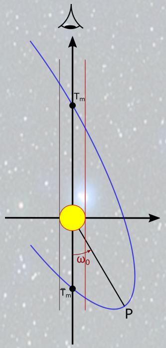
In the case of HD 80606b, taking the reference time $\delta t = 0$ at the periastron passage of January 9, 2010, it comes (all times are in seconds)

N	$\delta t_1(N)$	$\delta t_m(N)$	$\delta t_{14}(N)$	$\delta \bar{t}_1(N)$	$\delta \bar{t}_m(N)$	$\delta \bar{t}_{14}(N)$
0	-2.65	-2.73	0.04	$7.3 \cdot 10^{-3}$	$8.6 \cdot 10^{-3}$	$1.0 \cdot 10^{-3}$
1	-7.70	-7.94	0.09	0.33	0.34	$6.8 \cdot 10^{-3}$
2	-12.76	-13.16	0.15	0.65	0.66	$1.3 \cdot 10^{-2}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
32	-164.3	-169.5	1.93	10.35	10.46	0.19
33	-169.3	-174.7	1.99	10.67	10.78	0.19
⋮	⋮	⋮	⋮	⋮	⋮	⋮
48	-245.1	-252.9	2.88	15.51	15.68	0.28
49	-250.2	-258.1	2.94	15.84	16.01	0.29

Seeking for an observable quantity

For the N^{th} orbit after the reference point, let's define $t_{\text{tr}-\text{ec}}(N) = t_m(N) - \bar{t}_m(N)$, which yields the observable quantity

$$\Delta t_{\text{tr}-\text{ec}}(N) = t_{\text{tr}-\text{ec}}(N) - t_{\text{tr}-\text{ec}}(0) = \delta t_m(N) - \delta \bar{t}_m(N) - \delta t_m(0) + \delta \bar{t}_m(0)\,.$$



N (transit date)	$\Delta t_{\text{tr}-\text{ec}}$ [s]
0 (January 2010)	0
1 (April 2010)	−5.54
2 (August 2010)	−11.08
⋮	⋮
32 (October 2019)	−177.3
33 (February 2020)	−182.8
⋮	⋮
48 (September 2024)	−265.9
49 (December 2024)	−271.4

The 1PN Hamiltonian

- A second way to compute the relativistic effects is by using a Hamiltonian integration of the equations of motion.

$\Rightarrow$ At the first post-Newtonian correction, the reduced Hamiltonian of the relative motion is constructed of

- $\hookrightarrow$ $\vec{X} = \vec{y}_\star + \vec{y}_p$ (with $R = |\vec{X}|$),
- $\hookrightarrow$ its reduced conjugate momentum $\vec{P} = (\vec{p}_\star + \vec{p}_p)/\mu$ (with $P^2 = \vec{P}^2$),
- $\hookrightarrow$ $P_R = \vec{P} \cdot \vec{X}/R$,

and reads

$$\frac{H}{\mu} = \frac{1}{2}P^2 - \frac{GM}{R} + \frac{1}{c^2}\left[\frac{3\nu - 1}{8}P^4 - \frac{GM}{2R}\left(\nu P_R^2 + (3+\nu)P^2\right) + \frac{G^2M^2}{2R^2}\right].$$

The 1PN Hamiltonian

$$\frac{H}{\mu} = \frac{1}{2} P^2 - \frac{GM}{R} + \frac{1}{c^2} \left[\frac{3\nu - 1}{8} P^4 - \frac{GM}{2R} \left(\nu P_R^2 + (3+\nu) P^2 \right) + \frac{G^2 M^2}{2R^2} \right] .$$

- Neglecting the spins, the angular momentum is conserved $\Rightarrow$ the motion takes place in a plane $\Rightarrow$ I and Ω are fixed.
- One can use the Newtonian-looking parametrisation

$$R = a\,(1 - e\cos\psi) \ , \quad P^2 = \frac{GM}{a} \frac{1 + e\cos\psi}{1 - e\cos\psi} , \quad P_R^2 = \frac{GM}{a} \frac{e^2 \sin^2\psi}{(1 - e\cos\psi)^2} ,$$

with $\ell = \psi - e\sin\psi$, to deduce

$$\frac{H}{\mu} = -\frac{GM}{2a} + \frac{1}{2} \left(\frac{GM}{ac} \right)^2 \left[\frac{3\nu - 1}{4} + \frac{4 - \nu}{\mathcal{X}} - \frac{6 + \nu}{\mathcal{X}^2} + \nu \frac{1 - e^2}{\mathcal{X}^3} \right] ,$$

where we introduced $\mathcal{X} = 1 - e\cos\psi$ for convenience.

The Delaunay-Poincaré canonical variables

- Let's introduce the Delaunay-Poincaré canonical variables

$$\lambda = \ell + \omega\,, \qquad \Lambda = \mu\sqrt{GMa}\,,$$
$$h = -\omega\,, \qquad \mathcal{H} = \mu\sqrt{GMa}\left(1 - \sqrt{1-e^2}\right)\,,$$

↪ We have the rough correspondences:

$$\lambda \sim (k, \zeta), \quad h \sim k, \quad \Lambda \sim E, \quad \mathcal{H} \sim (E, J).$$

- Those variables naturally satisfy the Hamilton-Jacobi equations

$$\frac{\mathrm{d}\lambda}{\mathrm{d}t} = \frac{\partial H}{\partial \Lambda}\,, \qquad \frac{\mathrm{d}\Lambda}{\mathrm{d}t} = -\frac{\partial H}{\partial \lambda}\,,$$
$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\partial H}{\partial \mathcal{H}}\,, \qquad \frac{\mathrm{d}\mathcal{H}}{\mathrm{d}t} = -\frac{\partial H}{\partial h}\,.$$

Variation of the parameters

- By integrating the Hamilton-Jacobi equations, it comes

$$\delta a = \frac{GM}{c^2}\left(\frac{1-3\nu}{4} + \frac{\nu-4}{\mathcal{X}_0} + \frac{6+\nu}{\mathcal{X}_0^2} - \nu\,\frac{1-e_0^2}{\mathcal{X}_0^3}\right),$$

$$\delta e = \frac{1-e_0^2}{2a_0\, e_0}\,\delta a\,,$$

$$\delta\omega = \frac{6\,GM}{a_0 c^2(1-e_0^2)} \arctan\left[\sqrt{\frac{1+e_0}{1-e_0}}\tan\frac{\psi}{2}\right] + \frac{GM}{2a_0 e_0\sqrt{1-e_0^2}\,c^2}\left(\frac{(\nu+2)(1-e_0^2)+6e_0^2}{\mathcal{X}_0} + 6\,\frac{1-e_0^2}{\mathcal{X}_0^2} - \nu\,\frac{(1-e_0^2)^2}{\mathcal{X}_0^3}\right)\sin\psi\,,$$

$$\delta\ell = \frac{GM(\nu-15)}{8a_0 c^2}\,\ell_0 - \frac{GM}{2a_0 e_0 c^2}\left((4-\nu)e_0^2 + \frac{2+\nu+4e_0^2}{\mathcal{X}_0} + 6\,\frac{1-e_0^2}{\mathcal{X}_0^2} - \nu\,\frac{(1-e_0^2)^2}{\mathcal{X}_0^3}\right)\sin\psi\,,$$

Relativistic effects on the motion

- Finally the modification of the instants of transits are

$$\delta t = \frac{1}{n_0}\left[(1 - e\cos\psi_0)\delta\psi - \delta\ell - \sin\psi_0\,\delta e\right],$$

where $\delta\psi$ is computed taking in account $\delta\omega$ and

$$\delta b = b_0\left[\frac{\delta a}{a_0} - \frac{2e_0\,\delta e}{1-e_0^2} + \frac{\cos\omega_0\,\delta e}{1-e_0\cos\omega_0} - \frac{\sin\omega_0\,\delta\omega}{1-e_0\cos\omega_0}\right].$$

⇒ This methods agrees with the quasi-Keplerian one within 0.5% after 33 cycles,

↪ the difference is due to different approaches for computing δb.

- *Reminder:* in the quasi-Keplerian derivation,

$$\delta t = \frac{1}{n_0}\left[(1 - e_0\cos\psi_0)\delta\psi - (\psi_0 - e_0\sin\psi_0)\zeta - \varepsilon_t\sin\psi_0\right].$$

Lagrangian perturbation theory

- A nice way to control our derivation is to compute the effect of the secular effects, *via.* celestial perturbation theory.

⇒ At the first post-Newtonian correction, the Lagragian can be expressed as

$$\frac{L}{\mu} = \frac{v^2}{2} - \frac{GM}{r} + \mathcal{R}(\vec{x}, \vec{v}),$$

with the perturbation function

$$\mathcal{R} = \frac{1}{2c^2}\left[\frac{1-3\nu}{4}v^4 + \frac{GM}{r}\left(\nu\,\dot{r}^2 + (3+\nu)\,v^2\right) - \frac{G^2M^2}{r^2}\right].$$

Lagrangian perturbation theory

- When dealing with secular effects, one can apply directly the usual perturbation equations of celestial mechanics to $\mathcal{R}$

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{2}{an}\frac{\partial \mathcal{R}}{\partial \ell} \qquad \frac{\mathrm{d}e}{\mathrm{d}t} = \frac{1-e^2}{ea^2 n}\frac{\partial \mathcal{R}}{\partial \ell},$$

$$\frac{\mathrm{d}\ell}{\mathrm{d}t} = n - \frac{1}{a^2 n}\left[2a\frac{\partial \mathcal{R}}{\partial a} + \frac{1-e^2}{e}\frac{\partial \mathcal{R}}{\partial e}\right], \qquad \frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{\sqrt{1-e^2}}{ea^2 n}\frac{\partial \mathcal{R}}{\partial e}.$$

$\hookrightarrow$ When averaged over one orbit, it comes

$$\left\langle \frac{\mathrm{d}a}{\mathrm{d}t} \right\rangle = \left\langle \frac{\mathrm{d}e}{\mathrm{d}t} \right\rangle = 0, \qquad \left\langle \frac{\mathrm{d}\ell}{\mathrm{d}t} \right\rangle = n_0 + \frac{GMn_0\,(\nu - 15)}{8\,a_0 c^2} = n_0\,(1+\zeta)\,,$$

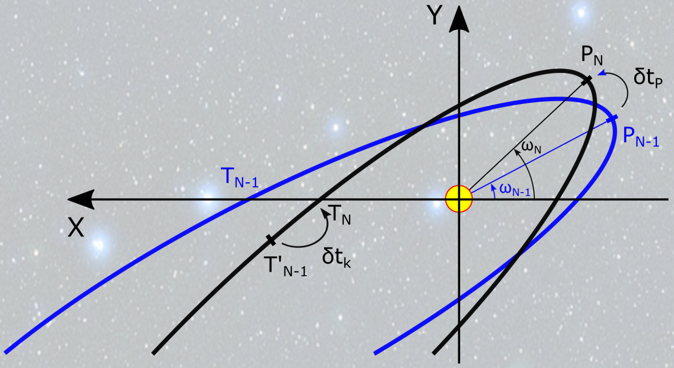
$$\left\langle \frac{\mathrm{d}\omega}{\mathrm{d}t} \right\rangle = \frac{3GMn_0}{a_0 c^2 (1-e_0^2)} = n_0 k\,.$$

In a nutshell : a rough but reliable estimate

- Taking only the two secular effects:

↪ considering a sequence of successive Keplerian orbits,

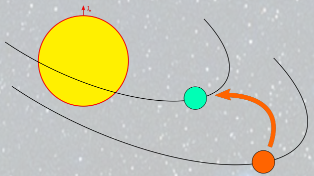
↪ and correcting at each step for the change in period.



Previsions agree with the full effect up to 9%, after 10 years.

Cleanliness of the measure: Theoretical point of view

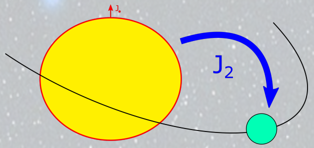
- Other effects may induce a periastron shift:



↪ Presence of a companion (star or planet),

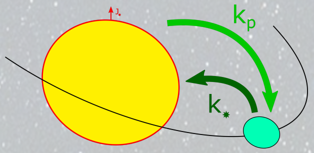
↪ Oblateness of the star,

$$\Delta_{J_2} = \frac{3\pi J_2 R_\star^2}{a^2(1-e^2)^2}\,.$$



Cleanliness of the measure: Theoretical point of view

- Other effects may induce a periastron shift:



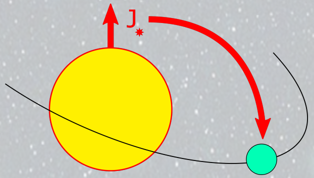
↪ Tidal interactions,

$$\Delta_{\mathrm{T}}^{\star\to p} = 30\pi\, k_p \frac{M_\star R_p^5}{M_p a^5} \frac{1+\frac{3}{2}e^2+\frac{1}{8}e^4}{(1-e^2)^5},$$

$$\Delta_{\mathrm{T}}^{p\to\star} = 30\pi\, k_\star \frac{M_p R_\star^5}{M_\star a^5} \frac{1+\frac{3}{2}e^2+\frac{1}{8}e^4}{(1-e^2)^5},$$

↪ Lense-Thirring effect,

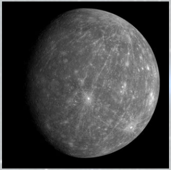
$$\frac{\mathrm{d}\Omega}{\mathrm{d}t} = \frac{2G\,J_\star}{c^2 a^3 (1-e^2)^{3/2}},$$



Cleanliness of the measure: Theoretical point of view

- HD 80607 is too far, and no hints for a planetary companion *via.* radial velocity measurements.
- The effects of the mass loss due to stellar winds are also negligible.[11]
- To estimate other effects, typical values for the Sun and hot Jupiters have been used.
- Let's recall

$$\Delta_{GR}^{Merc} \simeq 42 \text{ arcsec/century} \quad \text{and} \quad \Delta_{GR}^{HD} \simeq 215 \text{ arcsec/century}.$$



Mercury		HD 80606b
12.4	$\Delta_{comp}/\Delta_{GR}$	/
$5 \cdot 10^{-3}$	Δ_{obl}/Δ_{GR}	$2 \cdot 10^{-3}$
$5 \cdot 10^{-6}$	Δ_{tid}/Δ_{GR}	0.16
$5 \cdot 10^{-5}$	Δ_{LT}/Δ_{GR}	$3 \cdot 10^{-4}$

⇒ HD 80606b is much cleaner than the Solar System !

[11] Lecavelier des Étangs, 2007, A&A, 461, 1185 and Boué *et al.*,2012, A&A, 537, L3.

Cleanliness of the measure: In real life

- The final method would be to relativistically fit the trajectory and infer its parameters (*cf.* Hulse-Taylor pulsar).
- But we lack precision: we determine the effect from the parameters.

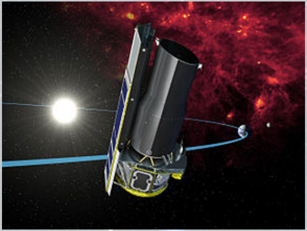
↪ The precision of the measured values[12] will affect our prediction of $\mathcal{O} = \Delta t_{\text{tr}-\text{ec}}(49)$.

Parameter	$\delta p/p$	$\delta\mathcal{O}/\mathcal{O}$
a	1.7 %	1.7 %
e	$5 \cdot 10^{-4}$	$5 \cdot 10^{-4}$
ω	$5 \cdot 10^{-3}$	$5 \cdot 10^{-4}$
b	$9 \cdot 10^{-3}$	$3 \cdot 10^{-4}$
$M_\star$	5.0 %	5 %
M_p	3.4 %	$1.3 \cdot 10^{-4}$
$R_\star$	2.4 %	$\lesssim 10^{-7}$
R_p	2.3 %	$\lesssim 10^{-7}$

[12] Taken in: Hébrard *et al.*, 2011, Astron. and Astrophys., 516, A95.

Cleanliness of the measure: In space

Spitzer



- In operation since 2003, but "hot" since 2009.
- Fortunately, the 3.6 and 4.5 μm bands of the IRAC camera can operate even without liquid He.

$\hookrightarrow$ Measured $t_{\text{tr}-\text{ec}}$ (Jan. 2009) $\sim$ 5.9 days with precision $\pm$275 sec.

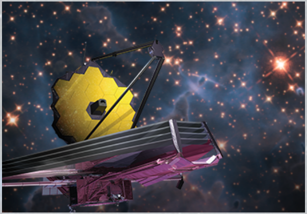
$\Rightarrow$ At the edge of precision to detect

$$\Delta t_{\text{tr}-\text{ec}}\,(\text{Dec. } 2024) = -271\,\text{s}.$$

Cleanliness of the measure: In space

James-Webb Space Telescope

- Shall be launched in 2021 (??).
- Two potentially interesting cameras:
  - ↪ NIRCam: $0.6 - 5\,\mu m$,
  - ↪ MIRI: $5 - 28\,\mu m$,



⇒ Should be able to detect

$$\Delta t_{\mathrm{tr-ec}}\,(\mathrm{Dec.}\ 2024) = -271\,\mathrm{s}.$$

Summary

- We proposed a new way to test our current theory of gravitation: detecting the relativistic effects on the motion of exoplanets.

↪ This will be the first "Solar-System"-like test in a distant stellar system.

- We have deduced an observable quantity: the shift in time elapsed between successive eclipses and transits.

- By focusing on the remarkable case of HD 80606b, we computed

$$t_{\mathrm{tr-ec}}\,(\mathrm{Dec.\,2024}) = t_{\mathrm{tr-ec}}\,(\mathrm{Jan.\,2010}) - 271\,\mathrm{s}.$$

⇒ This effect should be detectable around 2025, with *Spitzer* or *JWST*.

Summary

I would like to thank A. Lecavelier des Étangs and S. Dalal for their patience in answering my questions on exoplanets.


TOUTES LES PYRÉNÉES
Thank you for your attention
A. Villatte, Editeur, Tarbes
Montreurs d'Ours . . .