Mixmasters that bounce

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based on work done in collaboration with Marco Bruni and John D. Barrow

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Cosmological inflation

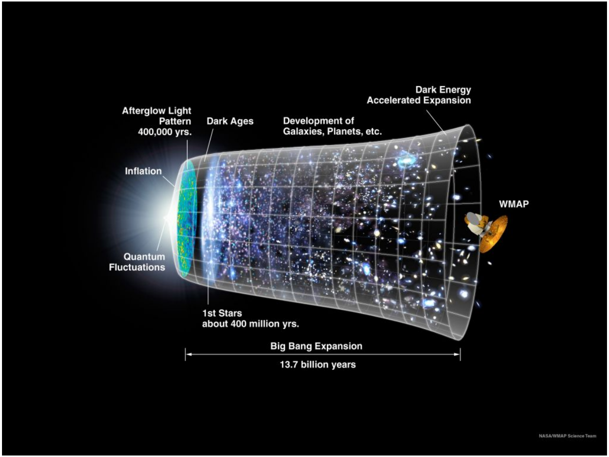

Figure: Standard cosmology with inflation as a model for the early Universe

Inflation solves problems in standard Big Bang cosmology

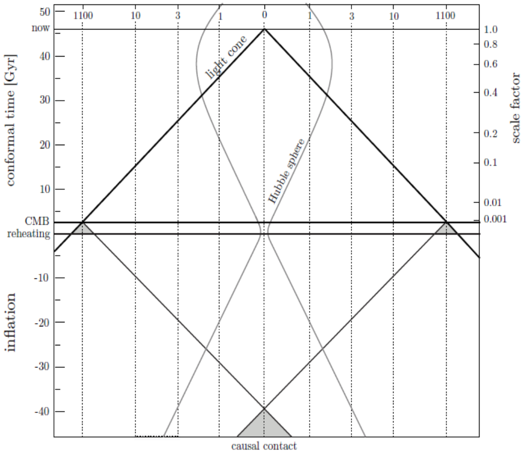

Figure: Inflation as a solution to the horizon problem

Inflation and scale invariance

Starting from an initial quantum vacuum, the curvature perturbations have a scale invariant primordial power spectrum
Using the facts that

- The background is expanding
- Can be said to be dominated by an approximately ideal EOS fluid

we find,

$$P_\zeta(k,\eta) = \frac{1}{2}\left(\frac{a\left[t_H\right]}{z\left[t_H\right]}\right)^2 H^2$$

SCALE-INVARIANT

What can Inflation do for us?

- Causal mechanism of the formation of structure

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- Causal mechanism of the formation of structure
- Horizon problem
- Isotropy problem
- Flatness problem
- Scale-invariance in scalar and tensor spectra
- Small non-Gaussianities

THERE IS ALWAYS A BUT

It has some issues...

- An inevitable Big Bang singularity
- Trans-Planckian problem
- Exit from inflation
- Multiverse and eternal inflation
- The η problem
- The initial conditions problem
- Various fine-tuning problems
- Lack of falsifiability and predictivity
- And so on...

THIS PROMPTED THE SEARCH FOR ALTERNATIVES

A bouncing cosmology

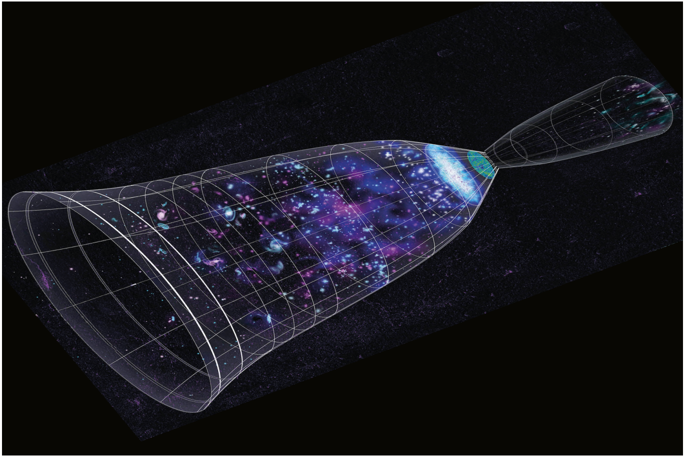

Figure: A bounce as a theory of the early Universe

How do we get a bounce?

- Coming out of the contracting phase the Hubble rate H is negative.
- $H > 0$ in the expanding phase
- So in the transition or 'bounce' phase, $H = 0$ and

$$\dot{H} = \frac{k}{a^2} - \frac{1}{2}(\rho + P)$$

- If the spatial curvature k is 0, then for $\dot{H} > 0$ *and* $H = 0$, we must have $\rho + P < 0$ (NEC violation)
- If we have positive spatial curvature, we can have a bounce, In the closed radiation FRW universe, exact solutions show this but need a NEC violating field to have the bounce occur at non-zero volume.

J.D.Barrow and Christos G.Tsagas, CQG Vol. 26, No. 19 (2009)

How does a bouncing model perform in solving problems of Standard Cosmology?

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4. Isotropy? THIS TALK

Scale-invariant spectra from bouncing cosmologies

Using the facts that ..

- The background is contracting
- Can be said to be dominated by a dust i.e. $p = 0$ fluid

we find,

SCALE-INVARIANT SPECTRUM ON SUPER-HUBBLE SCALES

Theories of the contracting phase

1. Ekpyrosis as a theory of the contracting phase: slow contraction mediated by a fast rolling scalar field

$$V = -|V_0| \exp\left(\sqrt{\frac{2}{p}}\frac{\phi}{M_{Pl}}\right)$$

   This implies that EOS parameter $w \gg 1$.

2. Matter bounce : the contraction is dominated by a matter-like fluid to ensure a scale-invariant scalar spectrum.

But how do we get a bounce then?

1. Modified theories of gravity - galileon bounces, massive gravity (ghost-free bounce)
2. $f(R)$ mediated bounces, Horava-Lifshitz gravity, modified Gauss Bonet terms gave some stable bouncing solutions
3. Introducing a $-\rho^2$ term phenomenologically. Also in the context of LQC models
4. Ghost-condensate bounces - implemented in New Ekpyrotic Cosmology with specific higher-derivative self-interactions of the scalar field
5. String gas mediated bounces for example S-brane mediated bounces

All of these violate the null-energy condition in the case where positive curvature does not dominate the bounce.

Diane Battefeld, Patrick Peter, Physics Reports, 571, 1 − 66 (2015)

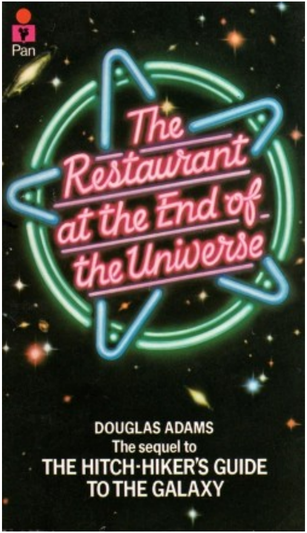
Non-singular cosmology from a quadratic equation of state

- The equation of state used is

$$P = P_0 + \alpha\rho + \frac{\beta}{\rho_C}\rho^2$$

- ρ_C is the energy scale at which non-linearities become relevant
- For this work we choose $P0 = 0$, $\alpha = 1/3$ and $\beta = -1$
- Resembles perfect fluid at $\rho \ll \rho_C$ which in our case is radiation
- Oscillating solutions have been found in the closed FRW case which are always non-singular.

Kishore N. Ananda, Marco Bruni, PRD, 74, 023524 (2006)



"The story so far:
In the beginning the Universe was created.
This has made a lot of people very angry and been widely regarded as a bad move."

-Douglas Adams

Anisotropies grow in the contracting phase.

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Do anisotropic bouncing cosmologies still bounce and isotropise?

How ekpyrosis solves the anisotropy problem

- The metric

$$ds^2 = dt^2 - a^2(t)dx^2 - b^2(t)dy^2 - c^2(t)dz^2$$

- Friedmann equation: $3H^2 = \sigma^2 + \rho_{matter}$,
- The shear evolves as,

$$\dot{\sigma}_{\alpha\beta} + 3H\sigma_{\alpha\beta} = 0$$

- ρ_{matter} should evolve as V^{-n}, $n \gg 2$

J. Khoury, B.A. Ovrut, P. J. Steinhardt and N.Turok, 2001, J. High Energy Phys. 11(2001)041

Bianchi Class A: A generalised study of anisotropies

- The generalised metric

$$ds^2 = dt^2 - h_{ab} d\omega^a d\omega^b$$

- Having an isotropic ultra stiff field of density ρ with equation of state $p = (\gamma - 1)\rho$, such that $\gamma > 2$

Phase plane analysis and expansion normalised variables

- We introduce

$$\sigma_+ \equiv \frac{1}{2}(\sigma_{22} + \sigma_{33}),$$
$$\sigma_- \equiv \frac{1}{2\sqrt{3}}(\sigma_{22} - \sigma_{33}).$$

- Write EFE in terms of expansion normalised variables

$$\Omega \equiv \frac{\rho}{3H^2}, \quad \Sigma^2 \equiv \frac{\sigma^2}{3H^2}, \quad K \equiv -\frac{{}^{(3)}R}{6H^2}.$$

The phase plane system looks like...

- Einstein equations of the form $\mathbf{x}' = \mathbf{f}(\mathbf{x})$
- subject to the Friedmann constraint $\mathbf{g}(\mathbf{x}) = 0$
- where the state vector $\mathbf{x} \in \mathbb{R}^6$ is given by

$$\{H, \underbrace{\Sigma_+, \Sigma_-}_{\text{shear components}}, \underbrace{N_1, N_2, N_3}_{\text{spatial curvature variables}}, \Omega\}$$

- The fact that the matter is ultra stiff $\gamma > 2$ is used and
- A no-hair theorem can be proved for all Bianchi types, I-VIII as well as IX(separately)

Cosmic no-hair theorem

All initially contracting, spatially homogeneous, orthogonal Bianchi Type I-VIII cosmologies and all Bianchi type IX universes sourced by an ultra-stiff fluid with an equation of state such that $(\gamma - 2)$ is positive definite, collapse into an isotropic singularity, where the sink is a spatially flat and isotropic FRW universe.

J.E.Lidsey, CQG, 23, 3517,(2005)

Generalising to Bianchi Type IX

- It is the most general closed homogeneous universe, describable by ODEs
- It has the closed FRW universe as its isotropic sub-case
- It has expansion anisotropy and anisotropic 3-curvature(which has no Newtonian analogue)
- On approach to $t \to 0$, in an open interval $0 < t < T$, exhibits chaotic Mixmaster oscillations, however oscillations become finite in number even if $t \to t_{Pl}$ on the finite interval $t_{Pl} < t < T$ excluding $t \to 0$.

In a Bianchi IX universe, the quadratic equation of state with $p = 1/3\rho - \rho^2$ produces bounces which are anisotropic

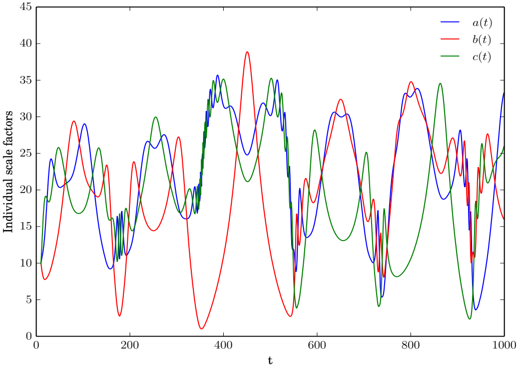

Figure: Scale factors in a diagonal anisotropic closed universe in the presence of the quadratic equation of state fluid

CG and Marco Bruni, PRL, 123, no.202, 201301,(2019)

Anisotropic stresses in a Bianchi I universe

We go back to our simple flat anisotropic universe and add anisotropic pressures in.

- Friedmann equation

$$3H^2 = \sigma^2 + \rho_{matter},$$

- The shear evolves as,

$$\dot{\sigma}_{\alpha\beta} + 3H\sigma_{\alpha\beta} = \underbrace{\boxed{\mu\mathcal{P}_{\alpha\beta}}}_{\text{anisotropic stress}}$$

The equation for the shear isn't homogeneous and we can't say straight away that an ultra stiff field will be able to dominate over it.

Anisotropic stresses in Bianchi Class A

- Resort to the expansion normalised variables and introduce $Z \equiv \frac{\mu}{3H^2}$ where μ is the anisotropic pressure field energy density with EOS, $p_i = (\gamma_i - 1)\mu$ and $\gamma_\star = (\gamma_1 + \gamma_2 + \gamma_3)/3 > \gamma$

- try to perform stability analysis on the state vector $\mathbf{x} = \{H, \Sigma_+, \Sigma_-, N_1, N_2, N_3, \Omega, Z\}$

- Linearise expansion normalised EFE around the FL point

$$\Sigma_+ = 0,\ \Sigma_- = 0,\ N_1 = 0,\ N_2 = 0,\ N_3 = 0,\ \Omega = 1,\ Z = 0$$

Stability analysis with anisotropic pressures:the results

We find the following eigenvalues

- $\frac{3}{2}(2-\gamma)$ of multiplicity 2
- $\frac{3\gamma-2}{2}$ of multiplicity 3
- $3(\gamma-\gamma_{\star})$ of multiplicity 1
- Using the condition $\gamma_{\star} > \gamma > 2$, FL equilibrium point stability cannot be determined

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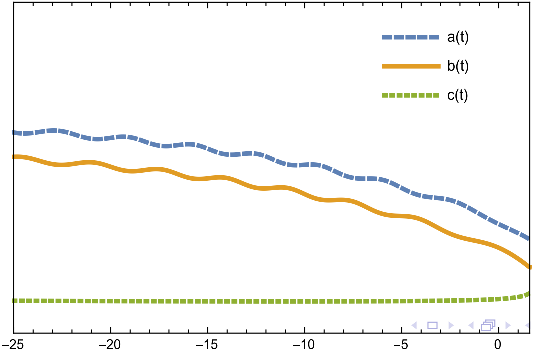
We can no longer determine the stability of the FL point and can't prove a no hair theorem like before.

Growing anisotropies cause Bianchi IX to collapse

Even if the anisotropic pressures are ultra-stiff on average, isotropisation doesn't occur and the Bianchi IX Universe does not re-expand

Figure: Scale factors in a diagonal anisotropic closed universe in the presence of positive anisotropic stress



The inclusion of shear viscosity

$$\dot{\sigma}_{ab} + 3H\sigma_{ab} = \pi_{ab} = \kappa\rho^{1/2}\sigma_{ab}, \quad \kappa < 0 \text{ and } \kappa \text{ is a constant}$$

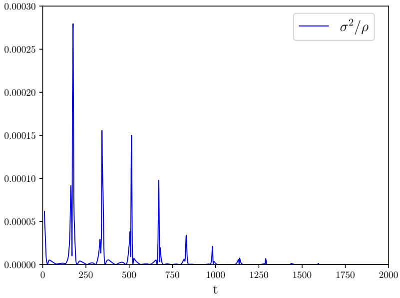
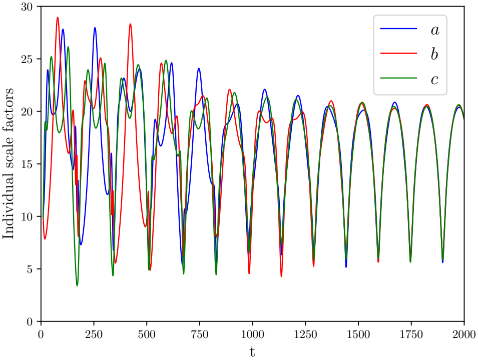


Figure: Normalised dimensionless shear in a diagonal anisotropic closed universe

CG and Marco Bruni, PRL, 123, no.202, 201301,(2019)

Scale factors in the Bianchi IX universe with shear viscosity



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Mixmaster chaotic behaviour mitigated as the Lyapunov index is negative.

The take-home!

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1. A Big Bang singularity isn't the end of the story: non-singular cosmologies may describe our very early Universe.
2. Non -linear fluids sourcing a cosmology is another way to have a bounce.
3. Anisotropies are suppressed in the contracting phase if you include dissipative shear viscous effects.

Outlook and Future work

- Exploring the construction of a field theory example of this kind of quadratic equation of state, as well as a model of how shear viscous effects could arise in the early Universe. A possible idea is through a black hole gas?
- The role of inhomogeneities need to be studied.
- Is it possible to have perturbations travel through this bouncing model?
- The question of whether the Mixmaster chaotic behaviour is truly suppressed instead of just mitigated also needs to be explored in more detail.

Thank you

Definition: *Bianchi models are spatially homogeneous cosmologies admitting a three-parameter local group G_3 of isometries that act simply transitively on spacelike hypersurfaces Σ_t.*

$$ds^2 = dt^2 - h_{ab} d\omega^a d\omega^b$$

where $d\omega^a = \frac{1}{2} C^a_{bc} \omega^b \wedge \omega^c$ and C^a_{bc} are the structure constants of the Lie algebra G_3 As $C^a_{(bc})=0$, there are 9 independent components, and

$$C^a_{bc} = n^{cd} \epsilon_{dab} + \delta^c_{[a} A_{b]}$$

where n_{ab} is a symmetric 3×3 matrix, and $A_b = C^a_{ab}$ is a 3×1 vector.

Using the Jacobi identity, $C^e_{d[a} C^d_{bc]}$, we have $n^{ab} A_b = 0$. Choose $A_b = (A, 0, 0)$ and $n_{ab} =$diag$[n_1, n_2, n_3]$, to get,

$$n_1 A = 0$$

If $A = 0$, Bianchi Class A models, and if $A \neq 0 (n_1 = 0)$, Bianchi Class B.

- We define the unit timelike vector field $\mathbf{u}$ perpendicular to the group orbits and the projection tensor h_{ab}
- $u_{a;b} = \sigma_{ab} + \omega_{ab} + \frac{1}{3}\Theta h_{ab} - \dot{u}_a u_b$
- We have specialised to cases where the total stress tensor(isotropic+anisotropic) is diagonal
- We can write EFE as $\mathbf{x}' = \mathbf{f}(\mathbf{x})$. The functions $\mathbf{f}(\mathbf{x})$ are homogeneous of degree 2
- System is invariant under scale transformation $\tilde{\mathbf{x}} = \lambda \mathbf{x}$ and $d\tilde{t}/dt = \lambda$
- so we can introduce dimensionless variables, as well as because the variables in their current form diverge close to the big bang and tend to zero at late times in ever-expanding models
- Things evolve wrt the scale factor, so it seems natural to normalise wrt the Hubble rate

- We have ρ and μ for isotropic and anisotropic pressure fields which follow the equations of state $p = (\gamma - 1)\rho$ and $p_i = (\gamma_i - 1)\mu$ with $\gamma_\star = (\gamma_1 + \gamma_2 + \gamma_3)/3$ and $\gamma_\star > \gamma$
- the 3 scale factors in the 3 directions are expressed as,

$$a(t) \equiv \mathrm{e}^{\alpha(t)},\ b(t) \equiv \mathrm{e}^{\beta(t)},\ c(t) \equiv \mathrm{e}^{\delta(t)}$$

- Define

$$\begin{aligned} x &\equiv \alpha'(t) - \beta'(t), \\ y &\equiv \alpha'(t) - \delta'(t), \\ H &\equiv \frac{1}{3}\left(\alpha'(t) + \beta'(t) + \delta'(t)\right). \end{aligned}$$

- Choose initial conditions satisfying the Friedmann constraint for the variable system

$$\{x, y, H, \alpha, \beta, \delta, \rho, \mu\}$$

The setup

- The generalised metric

$$ds^2 = -dt^2 + h_{ab} d\omega^a d\omega^b$$

- Having isotropic ultra stiff ghost field of density ρ with equation of state $p = (\gamma - 1)\rho$
- and anisotropic pressure ultra stiff field of density μ with equation of state $p_i = (\gamma_i - 1)\mu$
- with $\gamma_\star = (\gamma_1 + \gamma_2 + \gamma_3)/3$ and $\gamma_\star > \gamma$