Dark Energy Instabilities induced by Gravitational Waves

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Introduction

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- General Relativity (GR) is very well tested on solar system scales. On cosmological scales it is equally successful, but we don't understand the nature of the dark sector
- The accelerated expansion could hint at new physics (e.g. Dark Energy (DE) theories)
- Surprisingly, we still lack powerful tests of GR on cosmological scales

$$-\frac{k^2}{a^2}\Phi = 4\pi\mu\, G_{\rm N}\rho_m\Delta_m \ , \quad \frac{\Phi}{\Psi} = \gamma$$

- Current constraints on μ, γ are $\sim \mathcal{O}(1)$
- Upcoming surveys (LSST, Euclid, SKA..) promise to achieve $\sim \mathcal{O}(10^{-2})$

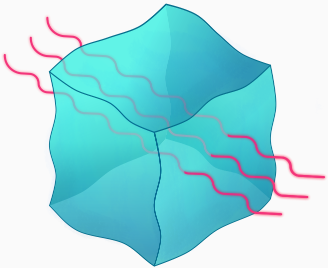
Introduction

- On shorter scales GR is much better constrained: viable modifications of gravity need some form of screening [1]
- Gravitational wave (GW) observations provide new powerful tests of GR
- The emission of GWs from binaries not so useful so far (due to screening and computational difficulties)
- The propagation is promising: linear regime and sensitive to dark components

[1] A. I. Vainshtein, Phys. Lett. B (1972), J. Khoury, A. Weltman Phys. Rev. Lett. (2004), K. Hinterbichler and J. Khoury, Phys. Rev. Lett. (2010)

Models of Dark Energy

Additional scalar field: Lorentz-violating fluid



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Scalar-tensor theories

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$$\mathscr{L} = R - \frac{1}{2}X - V(\phi)$$ Quintessence

$$\mathscr{L} = f(\phi)R - \frac{1}{2}X - V(\phi)$$ Brans-Dicke

$$\mathscr{L} = R - \mathcal{P}(\phi, X)$$ k-essence

$$X \equiv g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

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Scalar fluctuations $\phi = \phi_0(t) + \pi(x)$ in FRW have generic speeds of sound c_s

$$X^2 \supset \dot{\phi}_0^2\,\dot{\pi}^2$$

$$\mathscr{L}_\pi \sim \dot{\pi}^2 - c_s^2(\partial_i\pi)^2$$

- More general approach: consider also (stable) theories with higher-derivatives [2] $\mathscr{L}(\phi, \partial_\mu \phi, \nabla_\mu \nabla_\nu \phi)$

[2] G. W. Horndeski Int. J. Theor. Phys. (1974), C. Deffayet et al. PRD (2011)
M. Zumalacárregui and J. García-Bellido PRD (2014), J. Gleyzes et al. PRL (2014)

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$$\mathscr{L}_2 = G_2(\phi, X)$$

$$\mathscr{L}_3 = G_3(\phi, X)\Box\phi$$

$$\mathscr{L}_4 = G_4(\phi, X)R - 2G_{4,X}(\phi, X)\left[(\Box\phi)^2 - \phi_{\mu\nu}\phi^{\mu\nu}\right]$$

$$-F_4(\phi, X)\epsilon^{\mu\nu\rho}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma}\phi_\mu\phi_{\mu'}\phi_{\nu\nu'}\phi_{\rho\rho'}$$

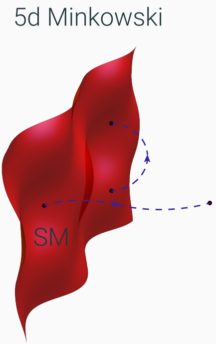
$$\mathscr{L}_5 = G_5(\phi, X)G_{\mu\nu}\phi^{\mu\nu} + \frac{1}{3}G_{5,X}(\phi, X)[(\Box\phi)^3 - 3\Box\phi\phi_{\mu\nu}\phi^{\mu\nu} + 2\phi_{\mu\nu}\phi^{\sigma\mu}\phi^{\nu}_{\sigma}]$$

$$-F_5(\phi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\phi_\mu\phi_{\mu'}\phi_{\nu\nu'}\phi_{\rho\rho'}\phi_{\sigma\sigma'}\,, \qquad \phi_\mu \equiv \nabla_\mu\phi$$

[2] G. W. Horndeski Int. J. Theor. Phys. (1974), C. Deffayet et al. PRD (2011)
M. Zumalacárregui and J. García-Bellido PRD (2014), J. Gleyzes et al. PRL (2014)

Why so complicated theories?

- Massive Gravity: [3] longitudinal mode $g_{\mu\nu} \supset \partial_\mu \partial_\nu \phi$
- DGP: [4] higher dimensional gravity



- Brane-bending mode $g_{5\mu} \sim \partial_\mu \phi$
- Action with many derivatives but 2^{nd} order equations: we can trust the classical non-linear regime

[3] C. de Rham, G. Gabadadze and A. Tolley PRL (2010)
[4] G. Dvali, G. Gabadadze and M. Porrati Phys. Lett. B (2000)

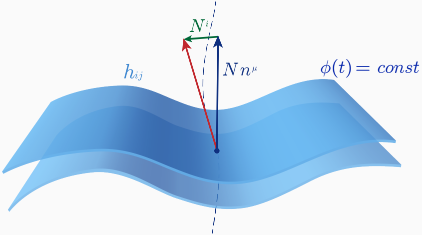
- The cosmological background solution $\phi_0(t)$ spontaneously breaks time-translation invariance
- Second derivatives give interesting phenomenology for tensors

For example:

$$(\nabla_\mu \nabla_\nu \phi)^2 \supset \dot{\phi}_0 (\dot{\gamma}_{ij})^2$$

$$\mathscr{L}_\gamma \sim (\dot{\gamma}_{ij})^2 - c_T^2 (\partial_k \gamma_{ij})^2$$

- Efficient way to study perturbations
- Clear connection with cosmological observables
- In a FRW and in unitary gauge $\delta\phi(t,x) = 0$ the action is geometrical

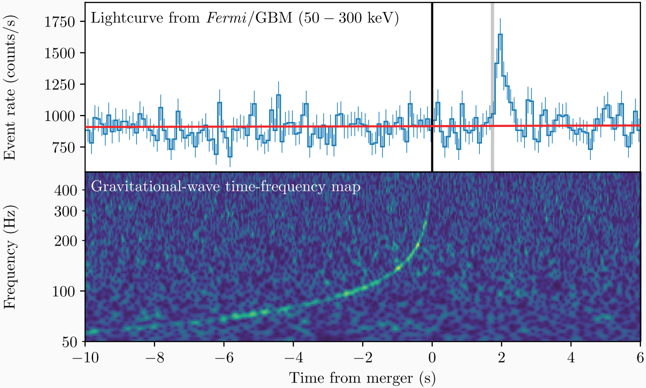


$$ds^2 = -N^2 dt^2 + h_{ij}(N^i dt + dx^i)(N^j dt + dx^j)$$

$$S_{\text{EFT}} = \int d^4x\, N\sqrt{h}\left[\frac{M_\star^2}{2} f(t) R - \Lambda(t) - c(t) N + \frac{m_2(t)^4}{2}\delta N^2 + \ldots\right]$$

[4] C. Cheung et al. JHEP (2008)
G. Gubitosi, F. Piazza and F. Vernizzi JCAP (2013)

Dark Energy after GW170817 and GRB170817A



$$\left|\frac{c_T - c}{c}\right| \lesssim 10^{-15}$$

LIGO Scientific and Virgo and Fermi-GBM and INTEGRAL Collaborations Astrophys. J. (2017)

- The condition $|\Delta c/c| \leq 10^{-15}$ sets tight constraints on DE models [5]

$$\mathscr{L}_2 = G_2(\phi, X)$$

$$\mathscr{L}_3 = G_3(\phi, X)\Box\phi$$

$$\mathscr{L}_4 = G_4(\phi, X)R - 2G_{4,X}(\phi, X)\left[(\Box\phi)^2 - \phi_{\mu\nu}\phi^{\mu\nu}\right] - F_4(\phi, X)\epsilon^{\mu\nu\rho}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma}\phi_{\mu}\phi_{\mu'}\phi_{\nu\nu'}\phi_{\rho\rho'}$$

$$\mathscr{L}_5 = G_5(\phi, X)G_{\mu\nu}\phi^{\mu\nu} + \frac{1}{3}G_{5,X}(\phi, X)[(\Box\phi)^3 - 3\Box\phi\phi_{\mu\nu}\phi^{\mu\nu} + 2\phi_{\mu\nu}\phi^{\sigma\mu}\phi^{\nu}_{\sigma}] - F_5(\phi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\phi_{\mu}\phi_{\mu'}\phi_{\nu\nu'}\phi_{\rho\rho'}\phi_{\sigma\sigma'}$$

$$XF_4 = 2G_{4,X} + G_{5,\phi}$$

[5] P. Creminelli and F. Vernizzi PRL (2017), J. M. Ezquiaga and M. Zumalacárregui PRL (2017)
T. Baker et al. PRL (2017), J. Sakstein and B. Jain PRL (2017)

Dark Energy after GW170817 and GRB170817A

- Only few models survive after this observation

$$\mathscr{L}_{c_T=1} = B_4(\phi, X)R + G_2(\phi, X) + G_3(\phi, X)\Box\phi$$
$$- \frac{4}{X} B_{4,X}(\phi, X) \left[\phi^\mu \phi^\nu \phi_{\mu\nu}\Box\phi - \phi^\mu \phi_{\mu\nu}\phi_\lambda \phi^{\lambda\nu}\right] , \qquad X \equiv -\frac{1}{2}(\nabla_\mu \phi)^2$$

- Only few models survive after this observation

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- In the EFT language in unitary gauge this corresponds to

$$\mathscr{L}^{\text{EFT}}_{c_T=1} = \frac{M_\star^2}{2}f(t)R - \Lambda(t) - c(t)g^{00} + \frac{m_2^2(t)}{2}(\delta g^{00})^2 - \frac{m_3^3(t)}{2}\delta g^{00}\delta K$$
$$+\frac{\tilde{m}_4^2(t)}{2}\delta g^{00}\left({}^{(3)}R - \delta\mathcal{K}_2\right)$$

- Is this enough? Interactions could modify the GW signal

- We will consider $S = S_0 + S_{m_3} + S_{\tilde{m}_4}$

$$\mathscr{L}_0 = \sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - \lambda(t) - c(t)g^{00} + \frac{m_2^4(t)}{2}(\delta g^{00})^2\right]$$ m_2 changes c_s

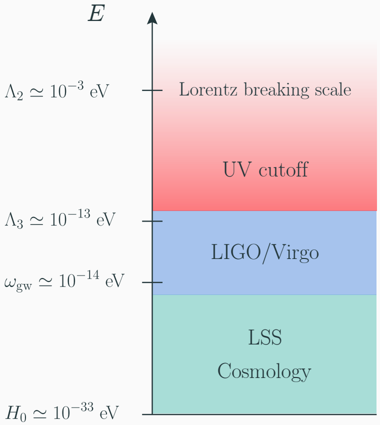
$$\mathscr{L}_{m_3} = -\sqrt{-g}\,\frac{m_3^3(t)}{2}\delta K \delta g^{00}$$ m_3 cubic Horndeski

$$\mathscr{L}_{\tilde{m}_4} = \sqrt{-g}\,\frac{\tilde{m}_4^2(t)}{2}\delta g^{00}\left({}^{(3)}R + \delta K^{\nu}_{\mu}\delta K^{\mu}_{\nu} - \delta K^2\right)$$ $\tilde{m}_4$ quartic beyond Horndeski

Gravitational wave decay

Radiative corrections

- The cosmological background solution $\phi(t)$ spontaneously breaks time-translation invariance, with $\dot{\phi} \sim \Lambda_2^2$



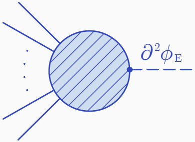
$$\Lambda_2 \equiv (M_{\rm Pl} H_0)^{1/2}$$

$$\Lambda_3 \equiv (M_{\rm Pl} H_0^2)^{1/3}$$

C. de Rham and S. Melville PRL (2018)

Radiative corrections

- Is $c_T = 1$ stable against radiative corrections?
  Yes, for theories with approx. Galilean invariance $\phi \to \phi + c + b_\mu x^\mu$
- Non-renormalization theorem [6] for Galileons. $\mathscr{L} = (\partial\phi)^2 + (\partial\phi)^2\Box\phi/\Lambda_3^3$



$$\partial^\mu\phi_\mathrm{E}\partial_\mu\phi\Box\phi = \partial^\mu\phi_\mathrm{E}\partial_\nu\left[\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}\eta_{\mu\nu}\partial^\rho\phi\partial_\rho\phi\right]$$

- With Gravity the symmetry is weakly-broken [7]. Galilean operators get loop corrections suppressed by $\Lambda_3^4/\Lambda_2^4 \sim 10^{-40}$

[6] M. Luty, M. Porrati and R. Rattazzi JHEP (2003)
[7] D. Pirtskhalava et. al. JCAP (2015), L. Santoni, E. Trincherini and L. G. Trombetta JHEP (2018)

- We look for interactions between gravitons γ_{ij} and scalars $\phi = \phi_0(t) + \pi(x)$
- After imposing $c_T = 1$ we still have $c_s \neq 1$
- The action contains

$$S_{\tilde{m}_4} = \alpha_{\rm H} M_{\rm Pl}^2 \int \mathrm{d}^4x \sqrt{-g}\, \delta g^{00} \left[{}^{(3)}R + \delta K_{ij}\delta K^{ij} - \delta K^2 \right]$$
$$\supset \frac{\alpha_{\rm H}}{\Lambda_3^3} \int \mathrm{d}^4x\, \ddot{\gamma}_{ij}\partial_i\pi\partial_j\pi$$

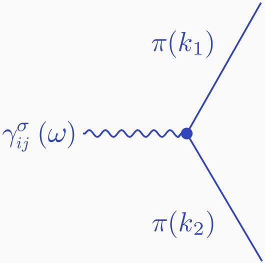
- In covariant language [8] $\alpha_{\rm H} = -2XB_{4,X}/B_4 \sim \tilde{m}_4^2/M_{\rm Pl}^2$

[8] E. Bellini and I. Sawicki JCAP (2014), Gleyzes et al. JCAP (2015)

- For $c_s < 1$
  GW can decay into Dark Energy fluctuations $\gamma \to \pi\pi$
- Similar to photon absorption in a material (e.g. photon-phonon interaction)
- Rate of the process:

$$\Gamma_{\gamma\to\pi\pi} \simeq \left(\frac{\alpha_{\rm H}}{\Lambda_3^3}\right)^2 \frac{\omega^7(1-c_s^2)^2}{480\pi c_s^7}$$

$$\omega = 2\pi f_{\rm gw}\ , \qquad \Lambda_3 \simeq (1000\,{\rm Km})^{-1}$$



Gravitational Wave Decay

- LIGO/Virgo events are at a distance $d_s \sim 40\,\mathrm{Mpc}$ with $f_{\rm gw} \sim 100\,\mathrm{Hz}$
- The coupling $\alpha_{\rm H}$ is compatible with observations if $d_s\,\Gamma_{\gamma\to\pi\pi} \lesssim 1$

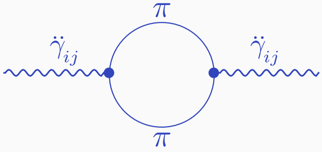
$$\alpha_{\rm H} < 10^{-10}$$

- Appreciable modifications of GR on large-scales require $\alpha_{\rm H} \sim 0.1$. Thus $\alpha_{\rm H}$ cannot affect cosmological observations [9]

[9] P. Creminelli, M. Lewandowski, GT, F. Vernizzi JCAP (2018)

Gravitational Wave Dispersion

- For generic c_s, the coupling affects the dispersion of GWs



- Similar to a (frequency-dependent) refractive index in a material
  - Loops generate quadratic operators with higher derivatives $\Delta\mathscr{L} \sim (\partial^4\gamma_{ij})^2$

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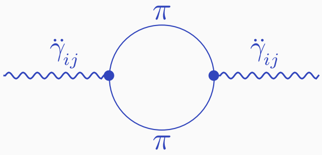


  - Loops generate quadratic operators with higher derivatives $\Delta\mathscr{L} \sim (\partial^4\gamma_{ij})^2$
  - The modified dispersion relation for GWs is

$$\omega^2 = \boldsymbol{k}^2 - \left(\frac{\alpha_{\rm H}}{\Lambda_3^3}\right)^2 \frac{\boldsymbol{k}^8(1-c_s^2)^2}{480\pi^2 c_s^7} \log\left(-(1-c_s^2)\frac{\boldsymbol{k}^2}{\mu_0^2} - i\epsilon\right)$$

Coherent effects

Coherent effects

- However, realistic GWs have large occupation numbers: coherent effects lead to a Bose enhancement of the decay rate
- Cubic Horndeski gives a decay, but suppressed by $(\Lambda_3/\Lambda_2)^4 \sim 10^{-40}$

$$\mathscr{L}_0 + \mathscr{L}_{m_3} = \frac{1}{4}(\dot{\gamma}_{ij})^2 - \frac{1}{4}(\partial_k \gamma_{ij})^2 + \frac{1}{2}\dot{\pi}^2 - \frac{c_s^2}{2}(\partial_i \pi)^2 + \frac{1}{\Lambda^2}\dot{\gamma}_{ij}\partial_i\pi\partial_j\pi + \dots$$

- Also, $\mathscr{L}_{\tilde{m}_4} \supset \frac{1}{\Lambda_\star^3}\ddot{\gamma}_{ij}\partial_i\pi\partial_j\pi$.
  Computations are the same, with the replacement $\Lambda^2 \to \Lambda_\star^3\omega^{-1}$
- The energy scales are defined as

$$\Lambda^2 \simeq \frac{\Lambda_2^2}{\alpha_{\rm B} c_s^2} \qquad \Lambda_\star^3 \simeq \frac{\Lambda_3^3}{\alpha_{\rm H} c_s^2}$$

$$\Lambda_2^2 \equiv M_{\rm Pl} H_0 \;, \quad \Lambda_3^3 \equiv M_{\rm Pl} H_0^2 \;, \quad \alpha_{\rm B} \equiv -\frac{m_3^3}{2M_{\rm Pl}^2 H} \;, \quad \alpha_{\rm H} \equiv \frac{2\tilde{m}_4^2}{M_{\rm Pl}^2}$$

- Non-linear galilean terms $(\partial\pi)^2\partial^2\pi$ and terms $\gamma\gamma\pi$ are also present

Coherent effects

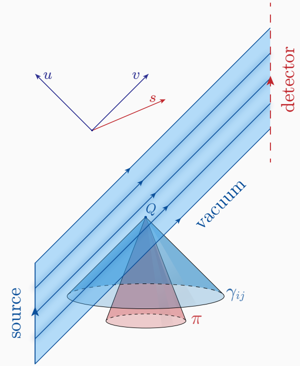
- For cubic Horndeski the quadratic action for π becomes

$$\mathscr{L}_\pi = \frac{\dot{\pi}^2}{2} - c_s^2 \frac{(\partial_k \pi)^2}{2} + \frac{1}{\Lambda^2} \dot{\gamma}_{ij}\, \partial_i \pi \partial_j \pi$$

- Enhancement: large occupation number

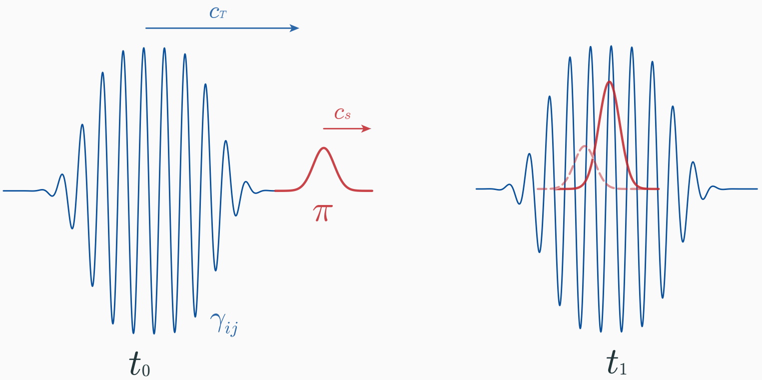
$$\gamma_{ij} = M_{\rm Pl} h_0^+ \sin(\omega u) \epsilon_{ij}^+ \ , \quad \beta = \frac{h_0^+ \omega M_{\rm Pl}}{\Lambda^2 c_s^2}$$

$$\ddot{\pi} - c_s^2 \partial_k^2 \pi + c_s^2 \beta \cos(\omega u)\, \epsilon_{ij}^+ \partial_i \partial_j \pi = 0$$



$$u = t - z \ , \quad s = -t + c_s^{-2} z \ , \quad v = t + z$$

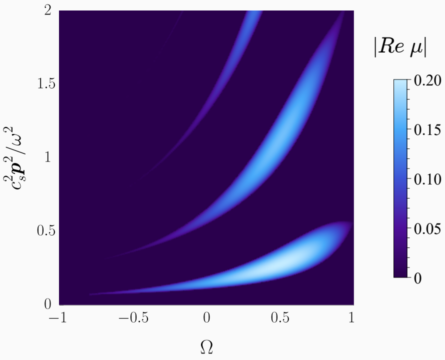
Coherent effects



- In momentum space for $c_s < 1$

$$\frac{\mathrm{d}^2\pi_{\boldsymbol{p}}}{\mathrm{d}\tau^2} + \left(A_{\boldsymbol{p}} - 2q_{\boldsymbol{p}}\cos(2\tau)\right)\pi_{\boldsymbol{p}} = 0$$

- Mathieu equation: instability bands
- Resonant modes $\pi_{\boldsymbol{p}} \sim e^{\mu_{\boldsymbol{p}}\tau}$
- For small coupling $\beta \ll 1$ the characteristic exponent $\mu \sim \beta$
- For $\beta \sim 10^{-1}$ we have $\rho_\pi \sim \rho_\gamma$ after $\mathcal{N}_{cyc} = \tau/\pi \sim 10^3$



$$\tau = \frac{\omega u}{2}\ , \quad \Omega = p_z/|\boldsymbol{p}|$$

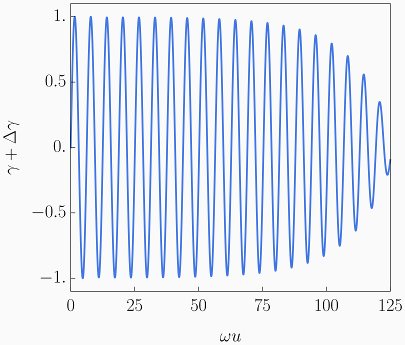
$$A_{\boldsymbol{p}} = \frac{4c_s^2\boldsymbol{p}^2(1-c_s\Omega)^2}{\omega^2(1-c_s^2)^2}$$

$$q_{\boldsymbol{p}} = 2\beta\frac{c_s^2\boldsymbol{p}^2}{\omega^2}\frac{1-\Omega^2}{1-c_s^2}\cos(2\varphi)$$

- DE back-reacts on the metric

$$\Box\gamma_{ij} + \frac{2}{\Lambda^2}\Lambda_{ij,kl}\partial_t \langle \partial_k\pi\partial_l\pi \rangle = 0$$

- $\Lambda_{ij,kl}$ is the projector along the direction of propagation
- Can be evaluated in the saddle-point approximation ($\beta\tau \to \infty$)



$$\Delta\gamma_{ij}(u,v) \simeq -\frac{\omega^4 v}{4c_s^5\sqrt{\beta}\Lambda^2}\frac{(1-c_s^2)^2}{(8\pi\omega u)^{3/2}}\sin(\omega u)\, e^{\beta\omega u/4}\epsilon_{ij}^+$$

- The resonance is altered when non-linearities become comparable with resonant term (rescattering) [10]

- Galilean structure of self-interactions:

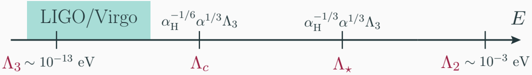
$$\mathscr{L}_3 = \frac{1}{M^3} \Box\pi \, (\partial_\mu \pi)^2 \, , \qquad M^3 \sim \frac{\Lambda_3^3}{\alpha_{\rm B} c_s^3} \qquad vs \qquad \frac{1}{\Lambda^2} \dot{\gamma} (\partial_i \pi)^2$$

- For $\mathscr{L}_3$ this happens when: $(\partial_i \pi)^2 \sim h_0^2 \Lambda_2^4$

$$\frac{\Delta\gamma}{\bar{\gamma}} \sim \frac{v (\partial_i \pi)^2}{\Lambda^2 M_{\rm Pl} h_0^+} \sim (v H_0) h_0 \alpha_{\rm B} c_s^2 \ll 1$$

[10] T. Prokopec and T. G. Roos PRD (1997)
P. Adshead, J. T. Giblin, and Z. J. Weiner PRD (2017)

Non-Linearities: $\tilde{m}_4^2$



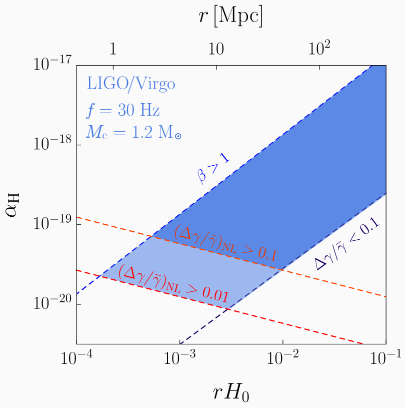
- For quartic galileons the dominant NL is quartic:

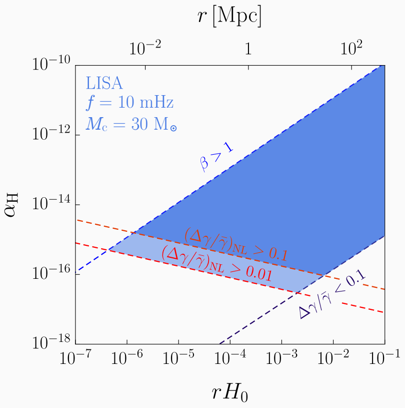
$$\mathscr{L}_4 = \frac{1}{\Lambda_c^6}(\partial_\mu\pi)^2\left[(\Box\pi)^2 - (\partial_\mu\partial_\nu\pi)^2\right] , \qquad \Lambda_c^6 \sim \frac{\Lambda_3^6}{\alpha_{\rm H} c_s^4} \qquad vs \qquad \frac{1}{\Lambda_\star^3}\ddot{\gamma}(\partial_i\pi)^2$$

- The geometry of the system suppresses the interactions (quartic Galileon interactions vanish in $2d$)

$$\frac{\Delta\gamma}{\bar{\gamma}} \sim \frac{v\,\omega(\partial_i\pi)^2}{\Lambda_\star^3 M_{\rm Pl} h_0^+} \lesssim \beta c_s (vH_0)\frac{H_0}{\omega h_0^+}\sqrt{\beta\tau}$$

Observational signatures for $\tilde{m}_4^2$





P. Creminelli, GT, F. Vernizzi, V. Yingcharoenrat JCAP (2019)

Surviving models

- Modified dispersion and decay arise from spontaneous breaking of Lorentz invariance

- The landscape of viable DE models is reduced to

$$\mathscr{L}_{c_T=1}^{\text{no decay}} = G_2(\phi, X) + G_3(\phi, X)\Box\phi + f(\phi)R$$

- The Cubic Galileon gives a decay, but perturbative analysis is inconclusive. We can study the non-linear stability of the model in the regime $\beta > 1$

Classical Stability

Classical Stability: Regime $\beta > 1$

- Let us focus on m_3^3 / cubic Galileon model with $c_s < 1$
- Even though the parametric resonance is not relevant for this operator, the regime $\beta > 1$ looks problematic

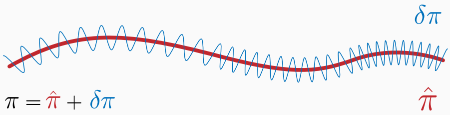
$$\mathscr{L}_\pi = \frac{\dot{\pi}^2}{2} - c_s^2 \frac{(\partial_k \pi)^2}{2} + \frac{1}{\Lambda^2} \dot{\gamma}_{ij}\, \partial_i \pi \partial_j \pi + \dots$$
$$\sim \frac{\dot{\pi}^2}{2} - \frac{c_s^2}{2}(1-\beta)(\partial_k \pi)^2 \qquad \Longrightarrow \text{Gradient instability } \pi \sim e^{\Lambda_{\rm UV} t}$$

- Do non-linearities quench the instability?
  We need to study the full stability of the model at non-linear level: GW can generate a background for π [11]

[11] A. Nicolis and R. Rattazzi JHEP (2004), S. Dubovsky et.al. JHEP (2006)

Classical Stability: conditions for stability

Conditions for stability



- The quadratic Lagrangian and EOM for $\delta\pi$ around a configuration $\hat{\pi}(x)$ (for frequencies $\omega \gg \partial\hat{\pi}(x)/\hat{\pi}(x)$) are

$$\mathscr{L}_{\delta\pi} = Z_{\mu\nu}[\hat{\pi}(x)]\partial^\mu \delta\pi \partial^\nu \delta\pi \ , \qquad Z_{\mu\nu}[\hat{\pi}(x)]\partial^\mu \partial^\nu \delta\pi = 0$$

- The dispersion relation is obtained by the solutions of $Z_{\mu\nu} k^\mu k^\nu = 0$
- Absence of gradient instabilities $\Rightarrow$ all solutions for k^μ are real. Equivalently the matrix $Z_{0i}Z_{0j} - Z_{00}Z_{ij}$ has to be positive-definite
- Absence of ghost instabilities $\Rightarrow Z_{00} > 0$

Classical Stability: stability theorem for the cubic Galileon

- It seems difficult the system becomes unstable because of the stability theorem for the cubic Galileon [12]
- The background EOM and kinetic matrix for perturbations are

$$\tilde{K}^2 - \tilde{K}^2_{\mu\nu} - 3\tilde{K} = -\frac{T}{4\Lambda^3 M_4}\ , \quad Z_{\mu\nu} = -3\eta_{\mu\nu} - 2\left(\tilde{K}_{\mu\nu} - \eta_{\mu\nu}\tilde{K}\right)$$

$$\tilde{K}_{\mu\nu} \equiv -\frac{1}{M^3}\partial_\mu\partial_\nu\hat{\pi}$$

- $Z_{\mu\nu}$ is assumed to be diagonalizable at each point (corresponding to non-relativistic matter sources)
- If the system is stable at one point, then it is stable everywhere: marginally stable configurations (e.g. $Z_{00} = 0$) are not solutions of the EOM
- Other Galileons do not have this property [13]

[12] A. Nicolis and R. Rattazzi JHEP (2004)
[13] S. Endlich and J. Wang JHEP (2011)

Classical Stability: Regime $\beta > 1$

- We need to consider the full cubic action ($\gamma\pi\pi$ and $\gamma\gamma\pi$ vertexes)

$$\mathscr{L} = -\frac{1}{2}\bar{\eta}^{\mu\nu}\partial_\mu\pi\partial_\nu\pi - \underbrace{\boxed{\frac{1}{M^3}\Box\pi(\partial\pi)^2}}_{\text{cubic NL}} + \underbrace{\boxed{\frac{\dot{\gamma}_{\mu\nu}}{\Lambda^2}\partial^\mu\pi\partial^\nu\pi}}_{\text{decay}} - \underbrace{\boxed{\frac{M^3}{2\Lambda^4}\dot{\gamma}_{\mu\nu}\dot{\gamma}^{\mu\nu}\pi}}_{\text{GR non-linear term}}$$

- $\bar{\eta}_{\mu\nu} = \mathrm{diag}(-1, c_s^2, c_s^2, c_s^2)$. The coefficient $\beta \sim \frac{\dot{\gamma}_{\mu\nu}}{\Lambda^2} > 1$
- The kinetic matrix for perturbations is

$$Z_{\mu\nu} = -\frac{1}{2}\bar{\eta}_{\mu\nu} - 2(K_{\mu\nu} - \eta_{\mu\nu}K) + \frac{\dot{\gamma}_{\mu\nu}}{\Lambda^2}\,, \qquad K_{\mu\nu} \equiv \partial_\mu\partial_\nu\hat{\pi}/M^3$$

- The equation of motion for $\hat{\pi}$ is

$$\frac{1}{3}Z^2 - Z^2_{\mu\nu} = \frac{3c_s^2 - 1}{6}$$

The GW cancels out

Classical Stability: Regime $\beta > 1$ and $c_s < 1$

- The system is apparently stable since the equations are as in flat space. If $\gamma_{\mu\nu} = \gamma_{\mu\nu}(u)$ we can solve the equation explicitly
- Translational invariance in (x, y, v) and $c_s < 1$ imply $\hat{\pi}''(u) = -\frac{M^3}{2(1-c_s^2)\Lambda^4}\dot{\gamma}_{\mu\nu}^2$
- The matrix $Z_{\mu\nu}$ has components

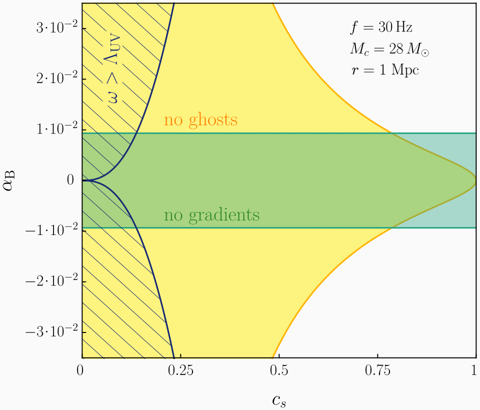
$$Z^{00} = \frac{1}{2} + 2\frac{\hat{\pi}''}{M^3}\,, \quad Z^{03} = Z^{30} = 2\frac{\hat{\pi}''}{M^3}\,, \quad Z^{33} = -\frac{c_s^2}{2} + 2\frac{\hat{\pi}''}{M^3}$$

$$Z^{11} = -\frac{c_s^2}{2} + \frac{\dot{\gamma}_{11}}{\Lambda^2}\,, \quad Z^{22} = -\frac{c_s^2}{2} + \frac{\dot{\gamma}_{22}}{\Lambda^2}$$

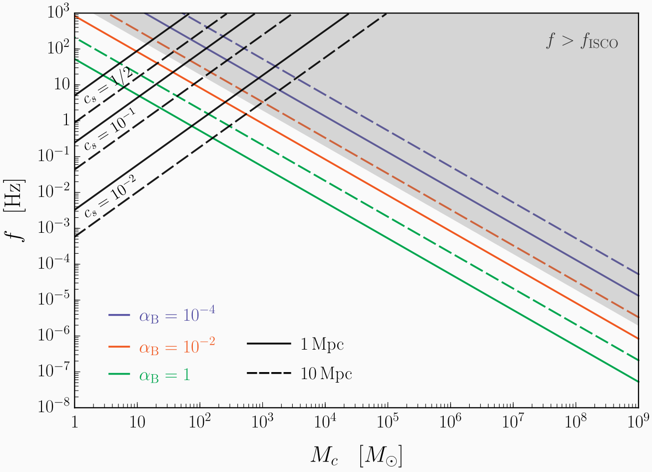
- No contradictions: instabilities appear when $Z_{\mu\nu}$ is not diagonalizable

P. Creminelli, GT, F. Vernizzi, V. Yingcharoenrat, JCAP (2020)

- The entries Z^{11} and Z^{22} can have the wrong sign:
  $\beta > 1 \Rightarrow$ gradient instability
- Ghost instabilities are present when Z^{00} changes sign:
  $\beta^2 > (1 - c_s^2)c_s^{-4} \Rightarrow$ ghost instability

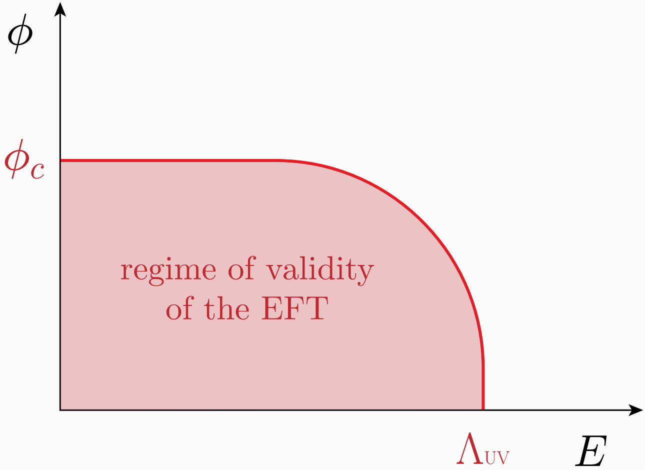


- For massive BHs (MBHs), $\omega \ll \Lambda_{\rm UV}$. In a region of $10\,\mathrm{Mpc}$ we have a large number of halo mergers (and so MBHs mergers) in the last Hubble time [14]



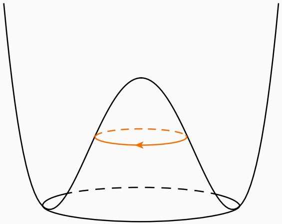
[14] M. Bonetti et. al. Mon. Not. Roy. Astron. Soc. (2018)
M. Bonetti et. al. Mon. Not. Roy. Astron. Soc. (2019)

Fate of the instability



- The theory is unstable: the fastest-growing modes are at the cutoff
- Thus, the fate of the instability depends on the UV completion (not known for Horndeski theories)
- Gradient and ghost instabilities can appear in the low-energy EFT of stable UV complete models

- Example: $\mathcal{P}(X)$ around a constant $\hat{X}$ background can develop gradient and ghost instabilities. Here the UV completion is stable: $\mathrm{U}(1)$ complex scalar ϕ in the broken phase



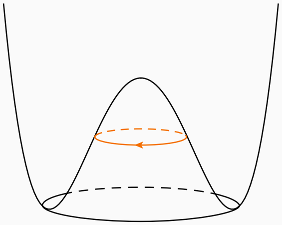
$$\mathscr{L} = -|\partial h|^2 - \lambda(|h|^2 - v^2)^2$$

$$h = \rho\, e^{i\phi}\,, \;\; \langle\rho\rangle = v^2 - \frac{X}{2\lambda}\,, \;\; X \equiv (\partial\phi)^2$$

$$h_0 = -\frac{X}{2\lambda} + v^2\,, \;\; \phi = \hat{\phi} + \pi\,, \;\; h = h_0 + \delta h$$

Fate of the instability: $\mathrm{U}(1)$ example

- At low energies and for small coupling, h can be integrated out
- The IR theory is $\mathscr{L}_{\rm IR} \simeq -\frac{X}{4\lambda}(\mu^2 - X)\ ,\quad \mu \equiv 4\lambda v^2$
- Around a background $\hat{X} < 0$, the fluctuations π are stable For $\hat{X} > 0$ the system becomes gradient unstable for $\mu^2/6 < \hat{X} < \mu^2/2$. For larger values it is also ghost unstable
- In the UV, gradient instabilities correspond to tachyons: rate of growth $\sim \mu$ (saturated by the cutoff of the EFT)
- When we encounter ghosts, the EFT breaks down: δh becomes massless



Conclusions

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- GW propagation provides new stringent constraints on DE theories
- Spontaneous breaking of Lorentz invariance allows for GW decay
- For $\beta < 1$ quantum and classical decay rule out quartic beyond Horndeski. Inconclusive analysis for cubic Horndeski
- For $\beta > 1$ cubic Horndeski becomes gradient and ghost unstable: theory becomes UV sensitive: the EFT is not viable
- The fate of $\gamma_{\mu\nu}$ is undetermined in absence of a concrete UV completion. New constrains on α_{B}

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Thank you for listening

Backup slides

Constrains for Degenerate Theories

- Degenerate Higher Order Theories (DHOST) [15] are compatible with $c_\mathrm{T} = 1$

$$\mathscr{L}_\mathrm{DHOST} = \frac{M^2}{2}\left[-\frac{2}{3}\alpha_\mathrm{L}(t)\delta K^2 + 4\beta_1(t)\delta K\, V + \beta_2(t)V^2 + \beta_3(t)a_i a^i\right]$$

$$V \equiv -\frac{1}{2}(\dot{g}^{00} - N^i\partial_i g^{00})/g^{00}\ ,\ a_i \equiv -\frac{1}{2}\partial_i g^{00}/g^{00}$$

- Degeneracy conditions between β_i's keeps 1 scalar d.o.f.
- In the case of α_B, relevant parameter for $\gamma \to \pi\pi$ is

$$\alpha_\mathrm{B} - \frac{\alpha_\mathrm{M}}{2}(1-\beta_1) + \beta_1 - \dot{\beta}_1 H\ , \qquad \alpha_\mathrm{M} \equiv \frac{(M^2)^{\dot{}}}{M^2 H}$$

- For α_H instead the constrained parameter is

$$\alpha_\mathrm{H} + 2\beta_1$$

[15] D. Langlois and K. Noui JCAP (2016), M. Crisostomi, K. Koyama, and G. Tasinato JCAP (2016)

- Kinetic Braiding: $\alpha_{\rm B}\delta g^{00}\delta K$ gives a mixing $\Phi \sim \alpha_{\rm B} H\pi$.
  It enhances the gravitational attraction on short scales, weakens it on large scales. Observational signatures in e.g. the matter power spectrum [16]

- Kinetic Matter Mixing: $\alpha_{\rm H}\delta g^{00}\left({}^{(3)}R + \delta K^{\nu}_{\mu}\delta K^{\mu}_{\nu} - \delta K^2\right)$
  gives a mixing $\Phi \sim \alpha_{\rm H}\dot{\pi}$. It weakens gravitational attraction: on short scales the matter power spectrum gets suppressed [17]

$$\alpha_{\rm B} \equiv -\frac{m_3^3}{2M_{\rm Pl}^2 H}\ , \quad \alpha_{\rm H} \equiv \frac{2\tilde{m}_4^2}{M_{\rm Pl}^2}$$

[16] M. Zumalacárregui et. al. JCAP (2017)
[17] G. D’Amico et. al. JCAP (2017)

Summary

EFT of DE operator	$\frac{1}{2}\tilde{m}_4^2\,\delta g^{00}\left({}^{(3)}R + \delta\mathcal{K}_2\right)$	$m_3^3\,\delta g^{00}\delta K$
GLPV operator	$-\frac{2Xf_{,X}}{f}$	$\frac{2Xf_{,X}}{f} + \frac{\dot{\phi}XQ_{,X}}{2Hf}$
α_i	$\alpha_{\rm H}$	$\alpha_{\rm B}$
After conf. transformation	$\alpha_{\rm H} + 2\beta_1$	$\alpha_{\rm B} - \frac{\alpha_{\rm M}}{2}(1-\beta_1) + \beta_1 - \frac{\dot{\beta}_1}{H}$
Perturbative decay	$\lvert\alpha_{\rm H}\rvert \gtrsim 10^{-10}$	Irrelevant
Narrow resonance	$10^{-20} \lesssim \lvert\alpha_{\rm H}\rvert \lesssim 10^{-17}$ $10^{-16} \lesssim \lvert\alpha_{\rm H}\rvert \lesssim 10^{-10}$	Not applicable
Instability	$\lvert\alpha_{\rm H}\rvert \gtrsim 10^{-20}$	$\lvert\alpha_{\rm B}\rvert \gtrsim 10^{-2}$

$$\mathscr{L}_{c_T=1} = P(\phi,X) + Q(\phi,X)\Box\phi + f(\phi,X)R - \frac{4f_{,X}}{X}\left(\phi^{;\mu}\phi^{;\nu}\phi_{;\mu\nu}\Box\phi - \phi^{;\mu}\phi_{;\mu\nu}\phi_{;\lambda}\phi^{;\lambda\nu}\right)$$

Viable theory ($c_T = 1$, no decay and no gradients):

$$\mathscr{L} = P(\phi,X) + f(\phi)R \;\text{, plus conf. transformations }\; g_{\mu\nu} \to C(\phi,X)g_{\mu\nu}$$