Queen Mary
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Dr Chris Clarkson
Dr Sophia Goldberg
Dr Karim Malik

Multi-Scale Perturbation Theory

A theoretical tool for late universe cosmology with ***nonlinear structures***

Kit Gallagher - PhD Student - QMUL

MOTIVATION

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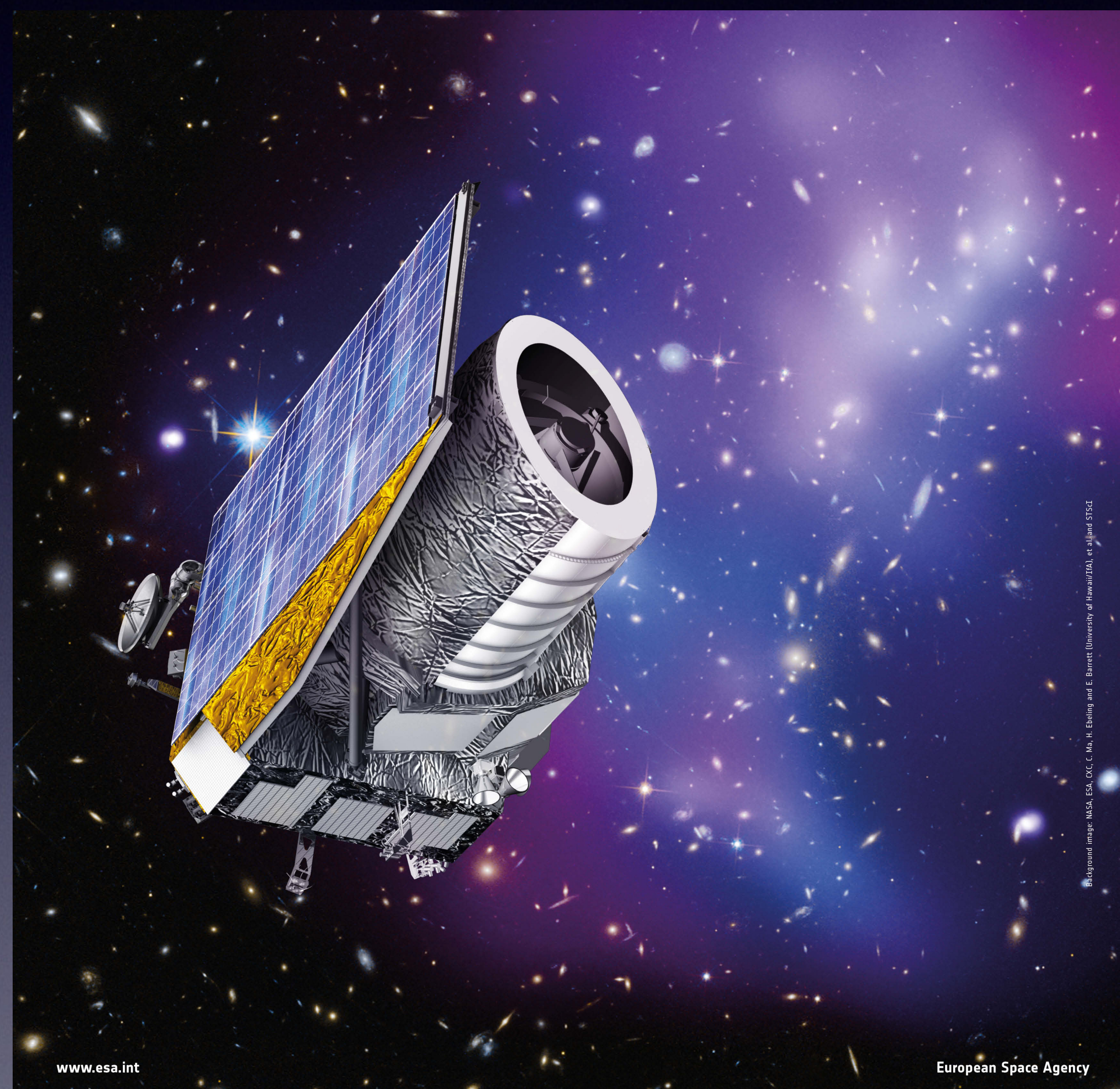

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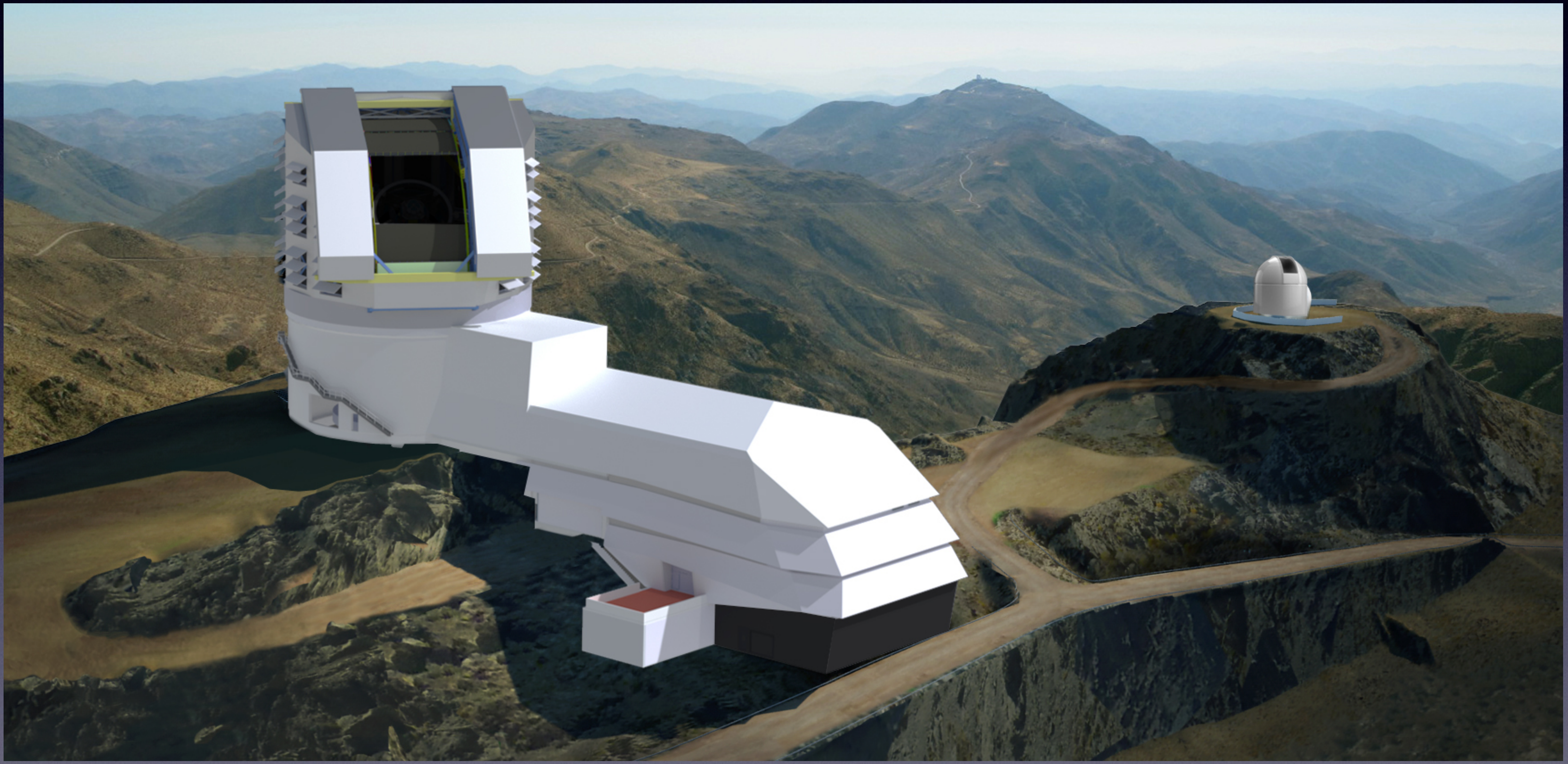

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- So our notion of **"realism"** becomes a question of approximation…
- What are the ***most physical*** simplifying approximations we can make?

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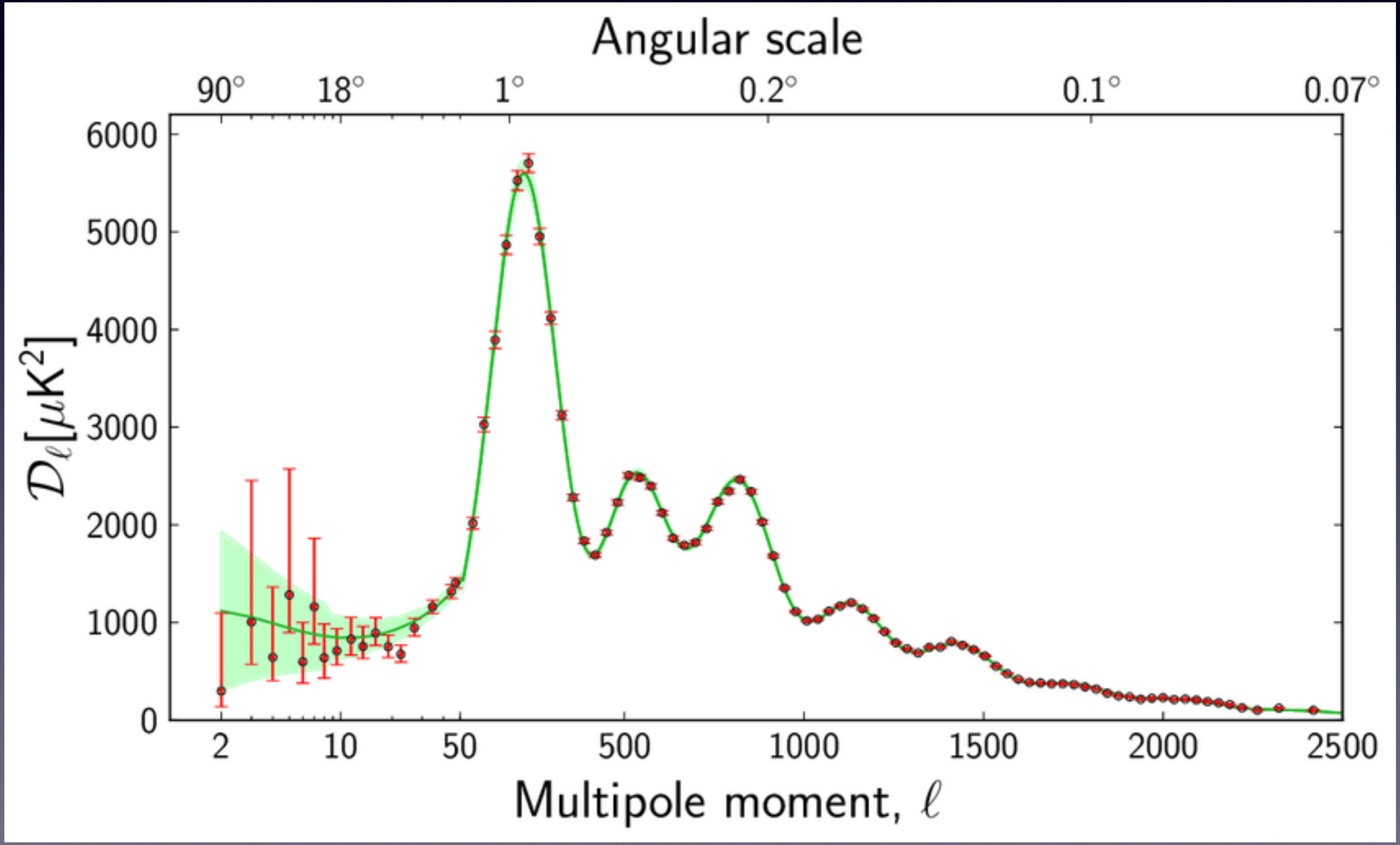
- Equations linearise at **ALL ORDERS** in ϵ
- Smaller corrections can be calculated by considering inhomogeneous linear problem with ***higher order ($\epsilon^{n>1}$) source terms***

SUCCESS!

Image taken from - Durrer. R. (2015) - The Cosmic Microwave Background - *Class. Quant. Grav. 32*

SUCCESS!

THIS APPROACH IS **EXTREMELY** SUCCESSFUL



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BUT…

Small fluctuation assumption **breaks down** in the late universe due to **GRAVITATIONAL COLLAPSE** on small scales!

This →

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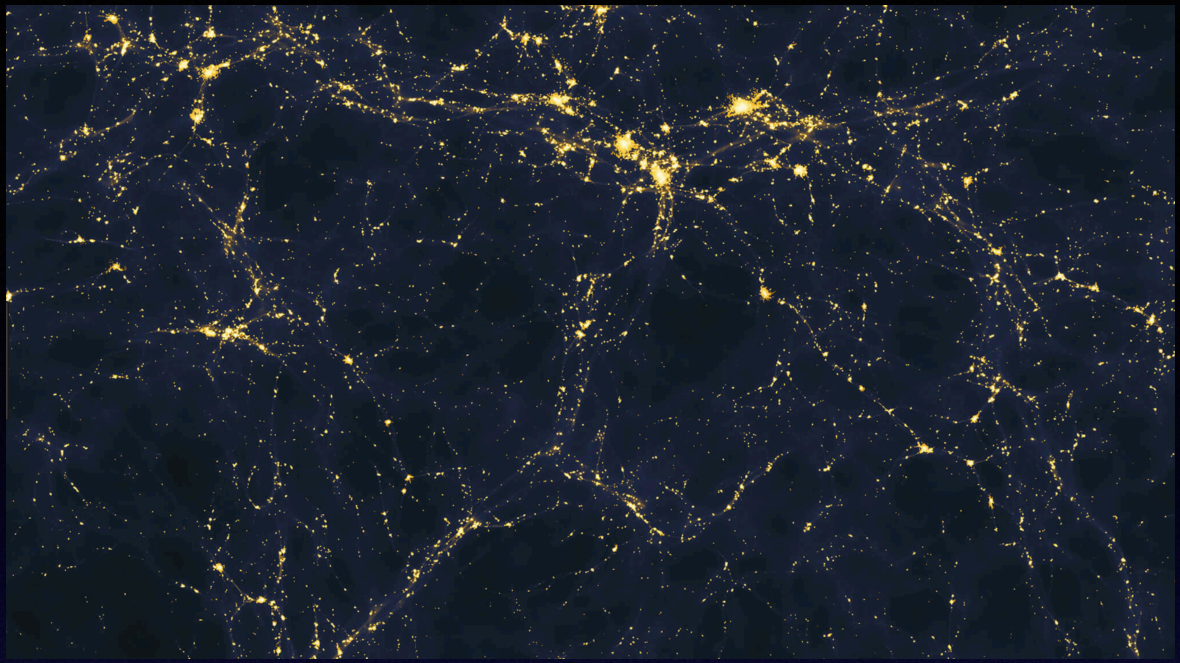

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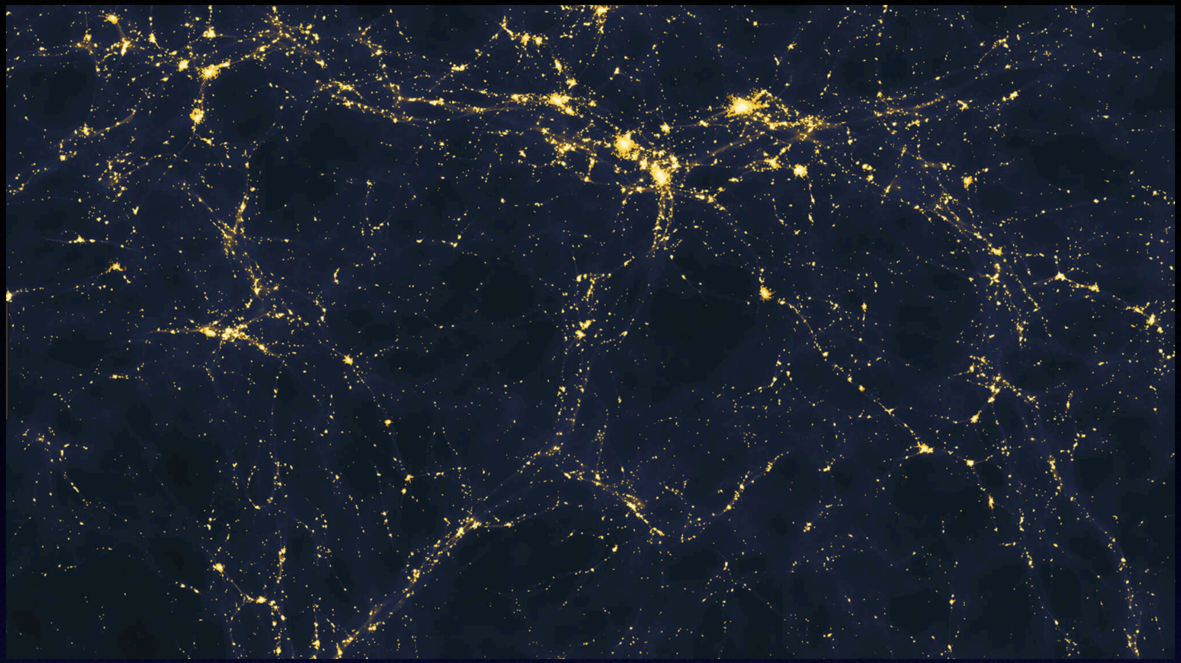

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$\bar{\rho}$ $+$ $\delta\rho \sim \epsilon\bar{\rho}$

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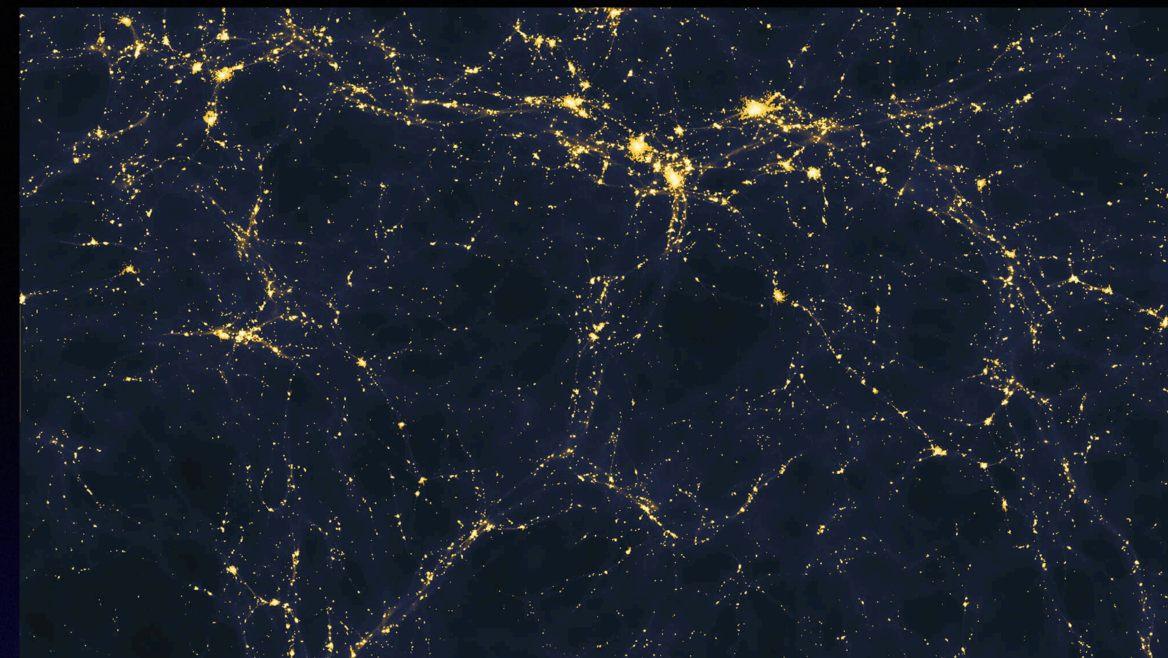

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…more like…

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BUT NONLINEARITY COUPLES SCALES!

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$$\phi_2(k) \sim \int dq\, \phi_1(q-k)\, \delta_1(q)$$

Conceivable that large δ at small scales could have an effect on quantities at larger scales!

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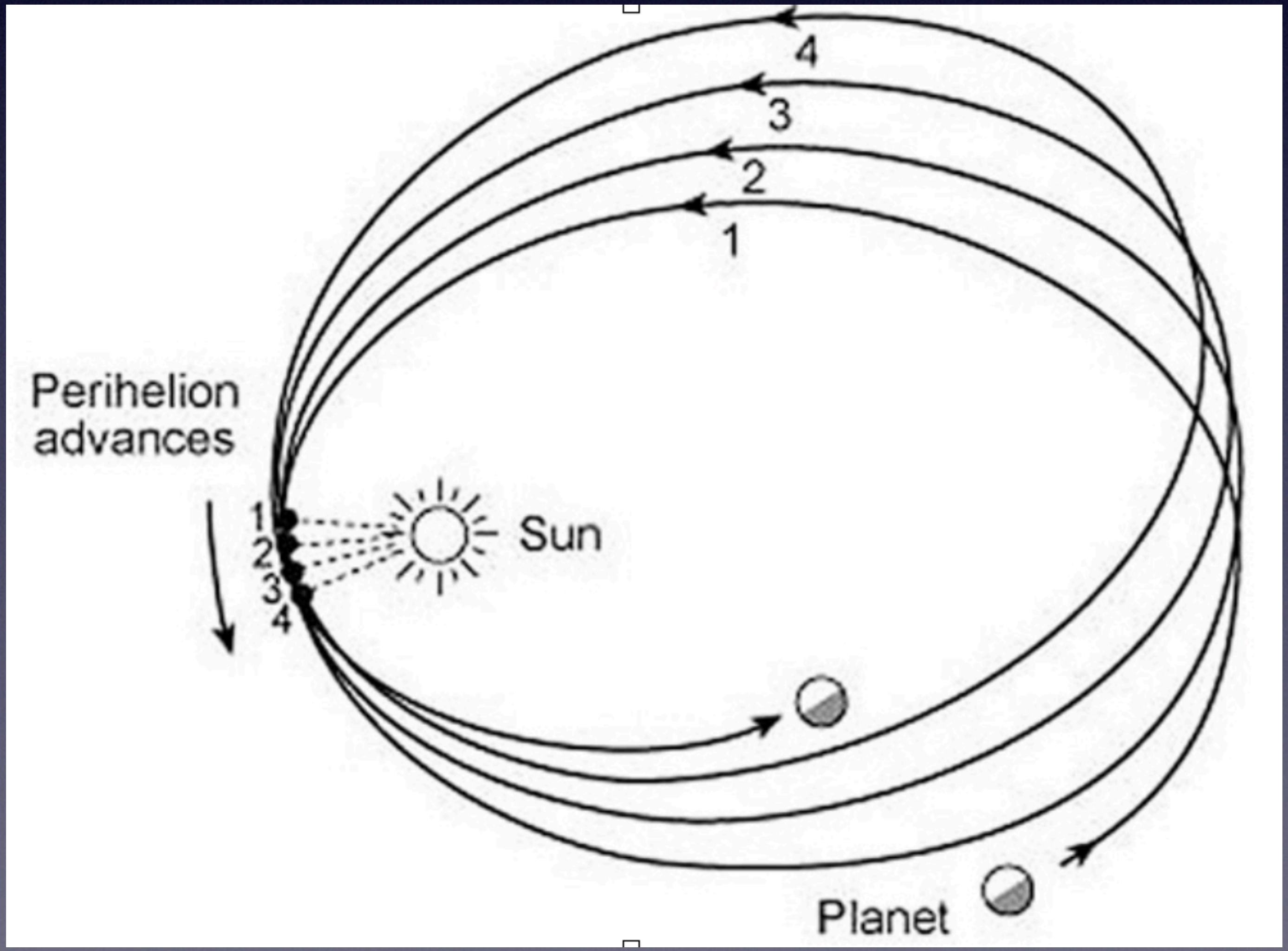

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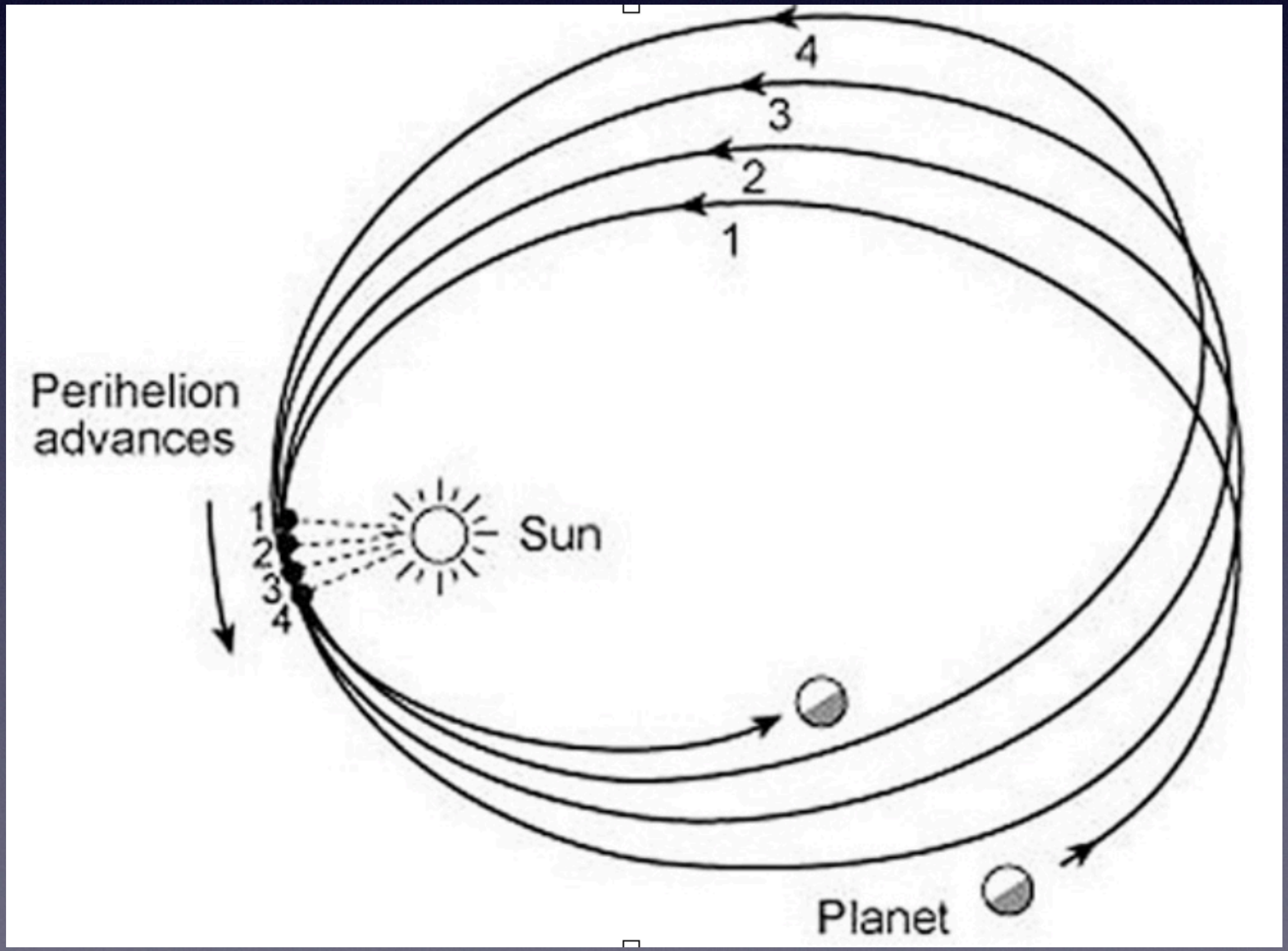

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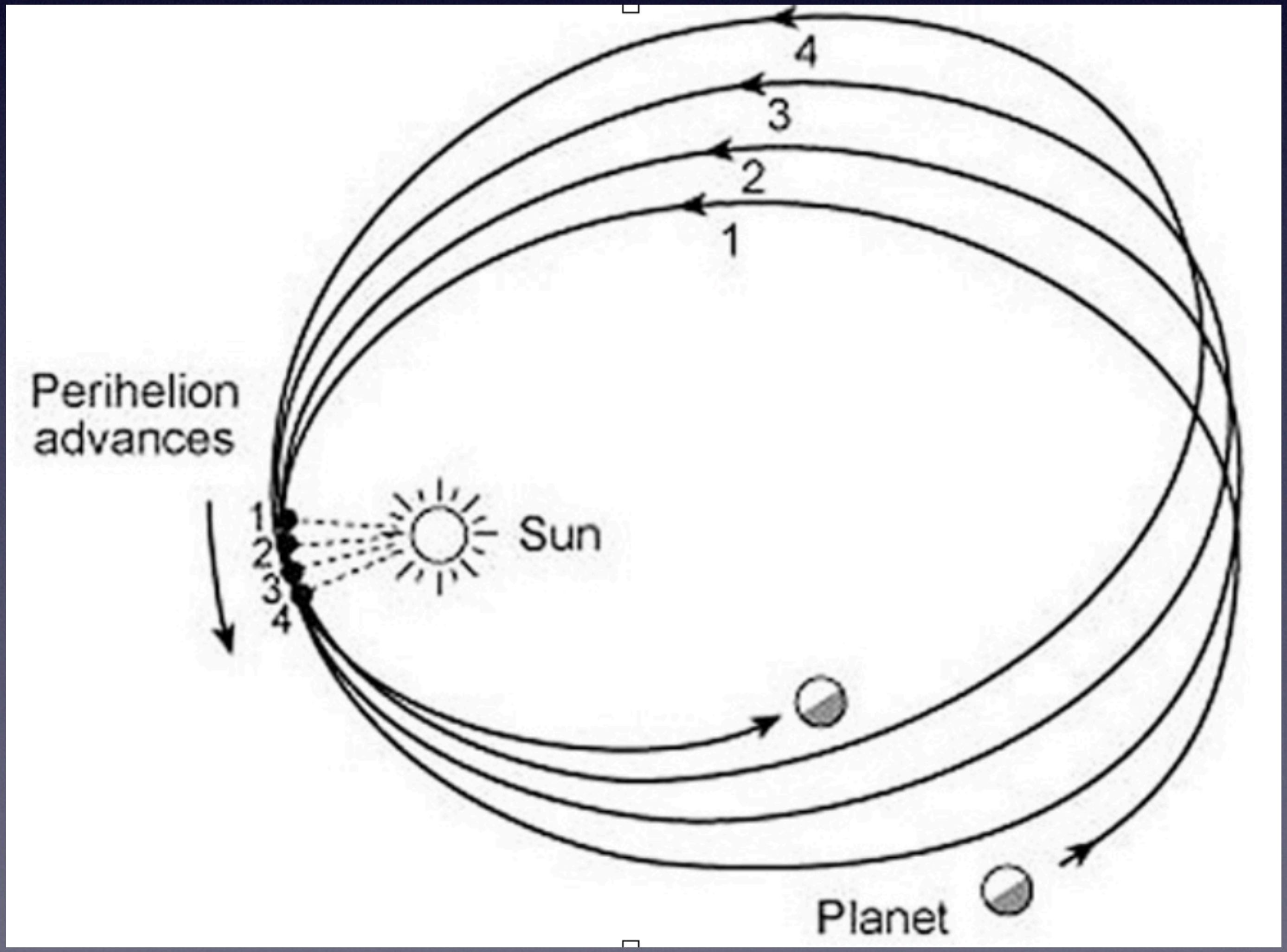

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3. The gravitational field is weak everywhere:

$$U \sim \eta^2$$

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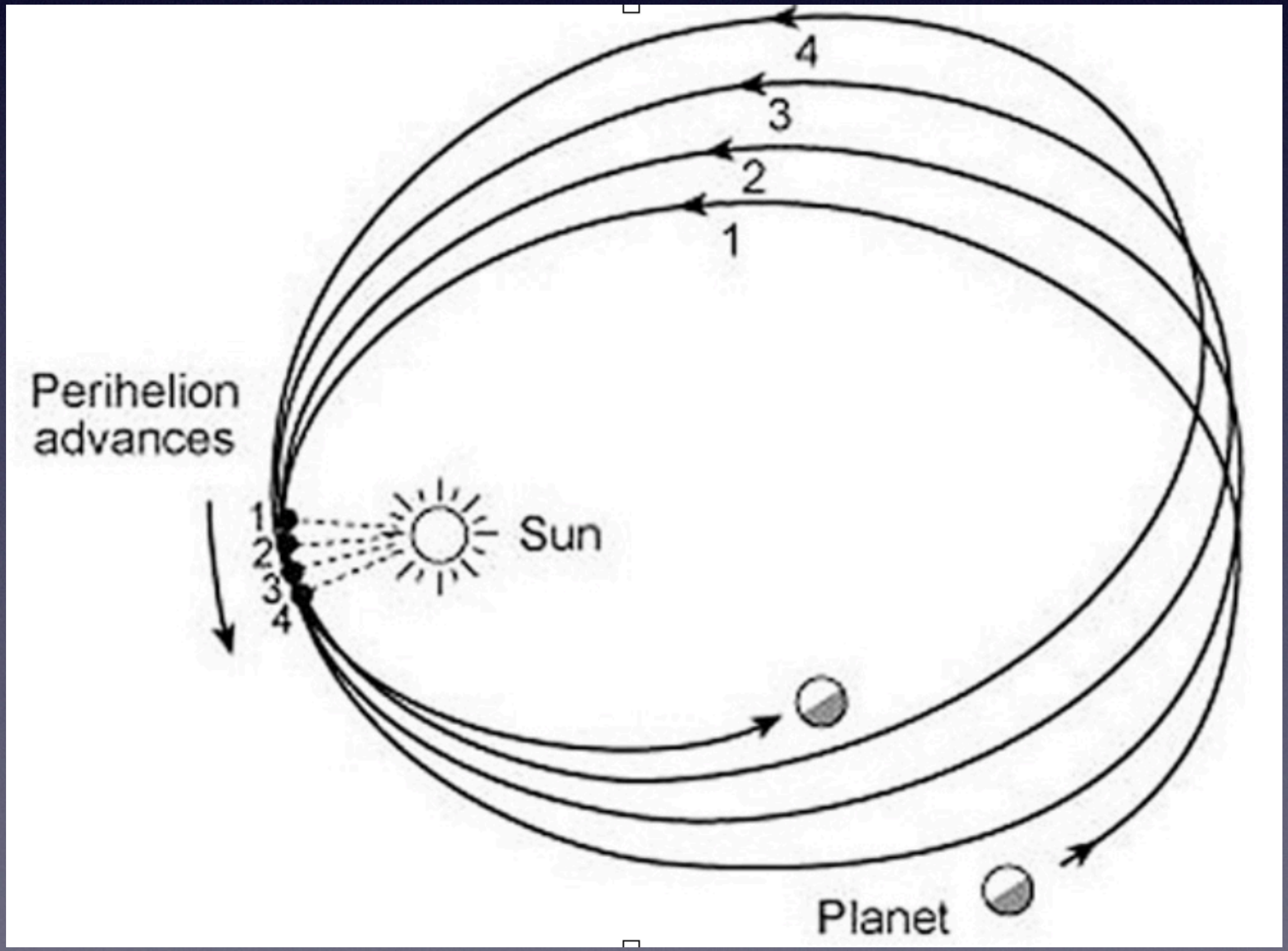


Small velocities and weak fields lead to…

$$\frac{v}{c} = \frac{1}{c}\frac{dx}{dt} \sim \eta \quad \Longrightarrow \quad \frac{d}{dt} \sim \eta \frac{d}{dx}$$

$$v \sim \eta$$

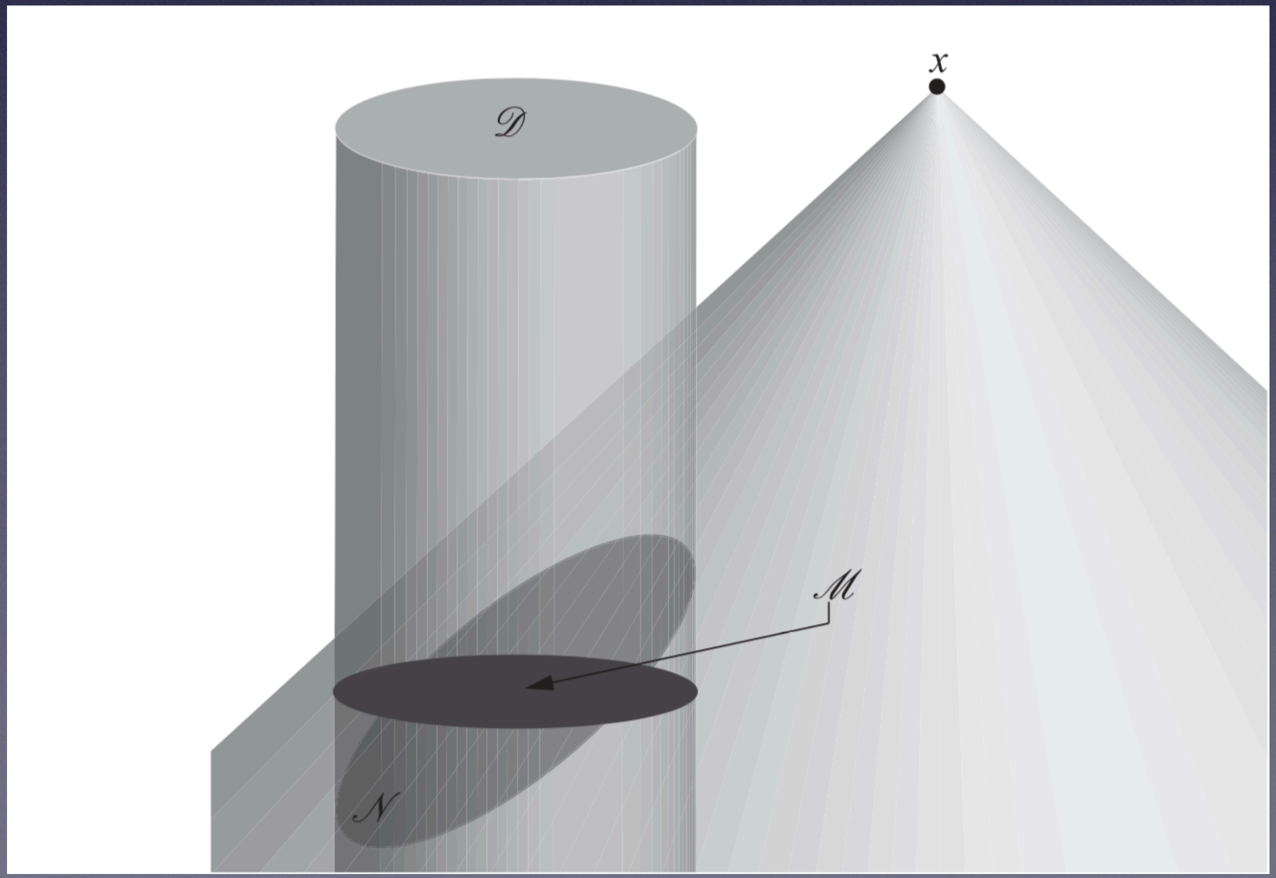
$$\rho \sim U \sim \eta^2$$

$$B_i \sim \eta^3$$

$$h_{ij} \sim \eta^4$$

SMALL TIME DERIVATIVES:

Equations change structure from wave equations to Poisson equations



Can think of approximating $\mathcal{N}(x,t)$ by $\mathcal{M}(x)$

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…and see how they might **interact**?

ANSWER…YES!

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Leading order Newtonian perturbation (→ $F^{(0,2)}$)

First order cosmological perturbation (→ $F^{(1,0)}$)

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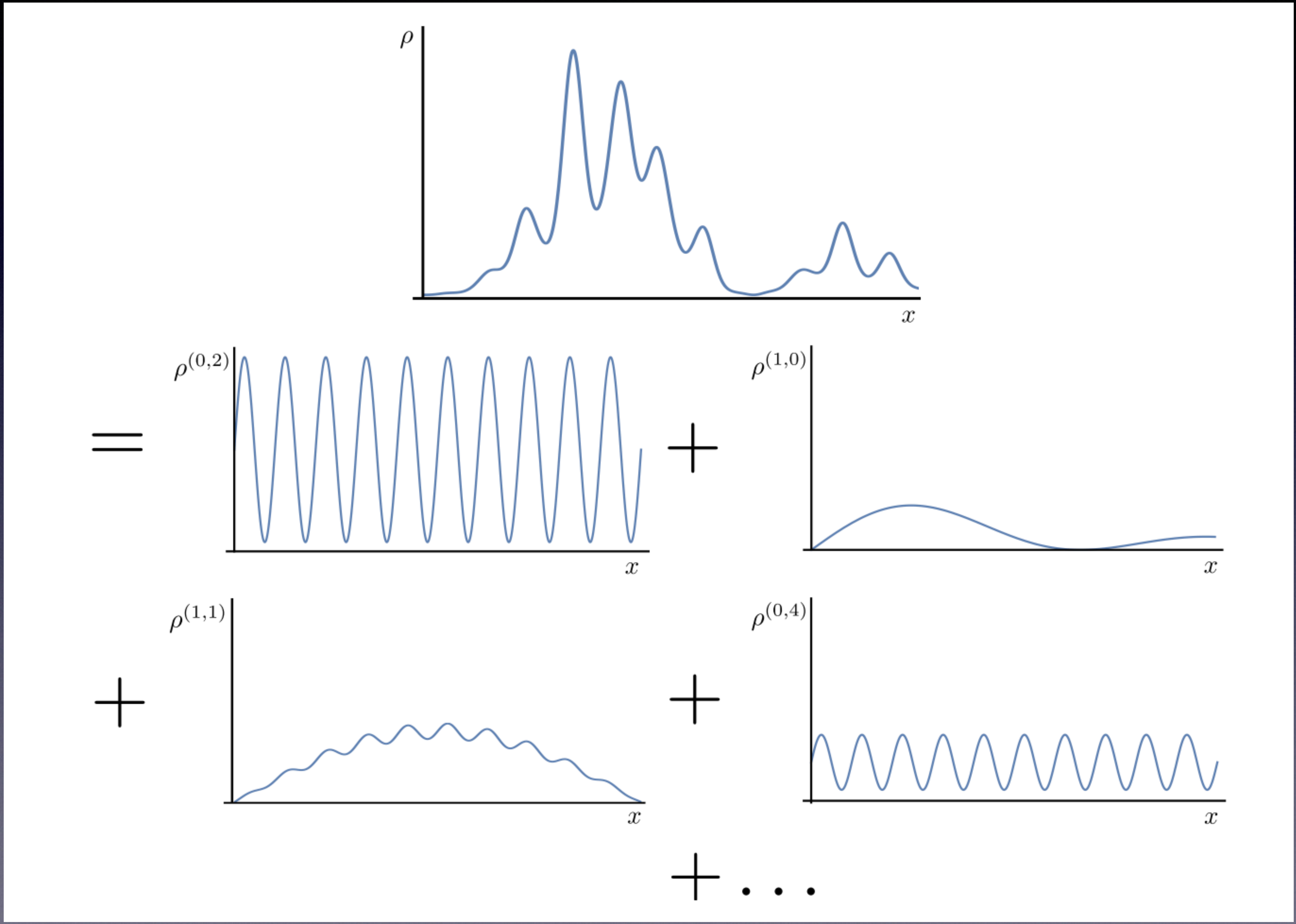

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This choice is **well motivated by physical observations**!

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…where $\{U, \rho_N, v_{Ni}\}$ are treated like the leading order parts of a **post-Newtonian expansion**…

…and $\{\phi, \psi, \rho, v_i\}$ are treated like **standard 1st order cosmological perturbations**.

At **LEADING ORDER:**

Obtain **homogeneous Friedmann-like behaviour…**

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$$\rho^{(0,2)} = \rho_N = \bar{\rho} + \delta\rho_N \sim \frac{\eta^2}{L_N^2} \implies \delta_N = \frac{\delta\rho_N}{\bar{\rho}} \sim 1$$

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- …get **quadratic products** of **nonlinear Newtonian source terms**
- **AND spatially varying coefficients in linear operator**!?!

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- Or…make **further approximations** and try to proceed **analytically**…

We take **approximate solutions from Newtonian perturbation theory** for $\{U, \rho_N, v_{Ni}\}$
- see what happens?
Compare to normal perturbation theory?

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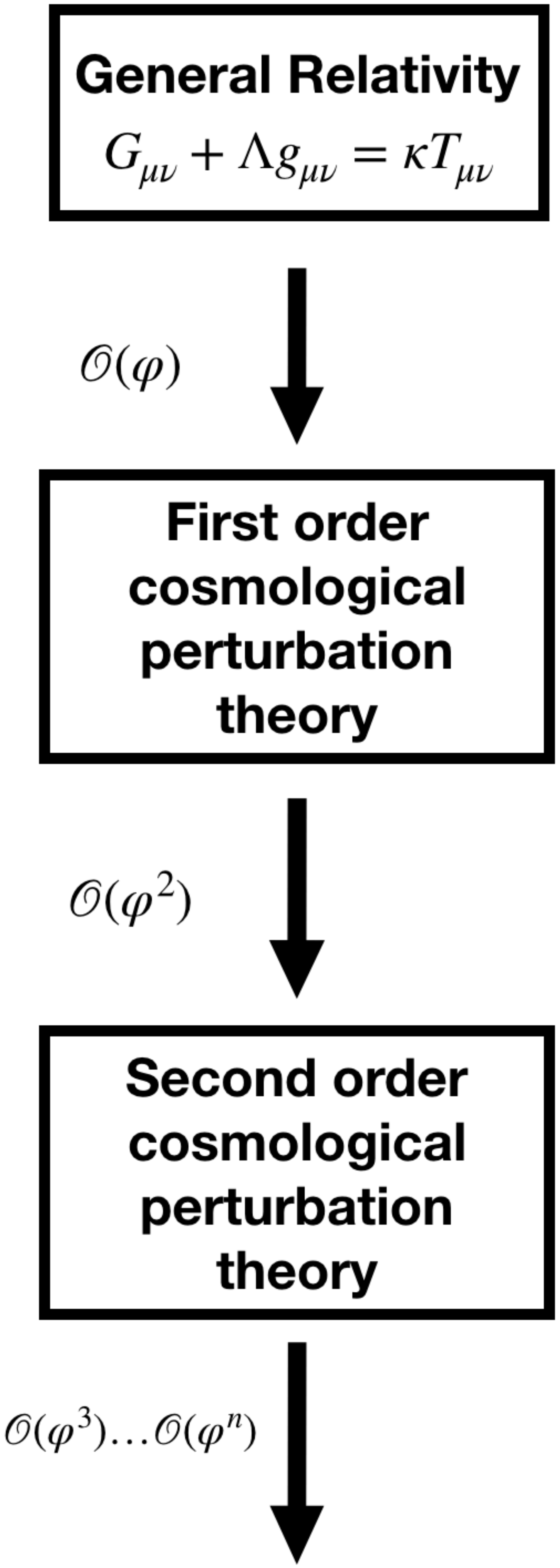
…we can do the same thing for the **cosmological ones**!

$$\psi = \psi^{(1)} + \psi^{(2)} + \dots$$

$$\delta = \delta^{(1)} + \delta^{(2)} + \dots$$

General Relativity

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$$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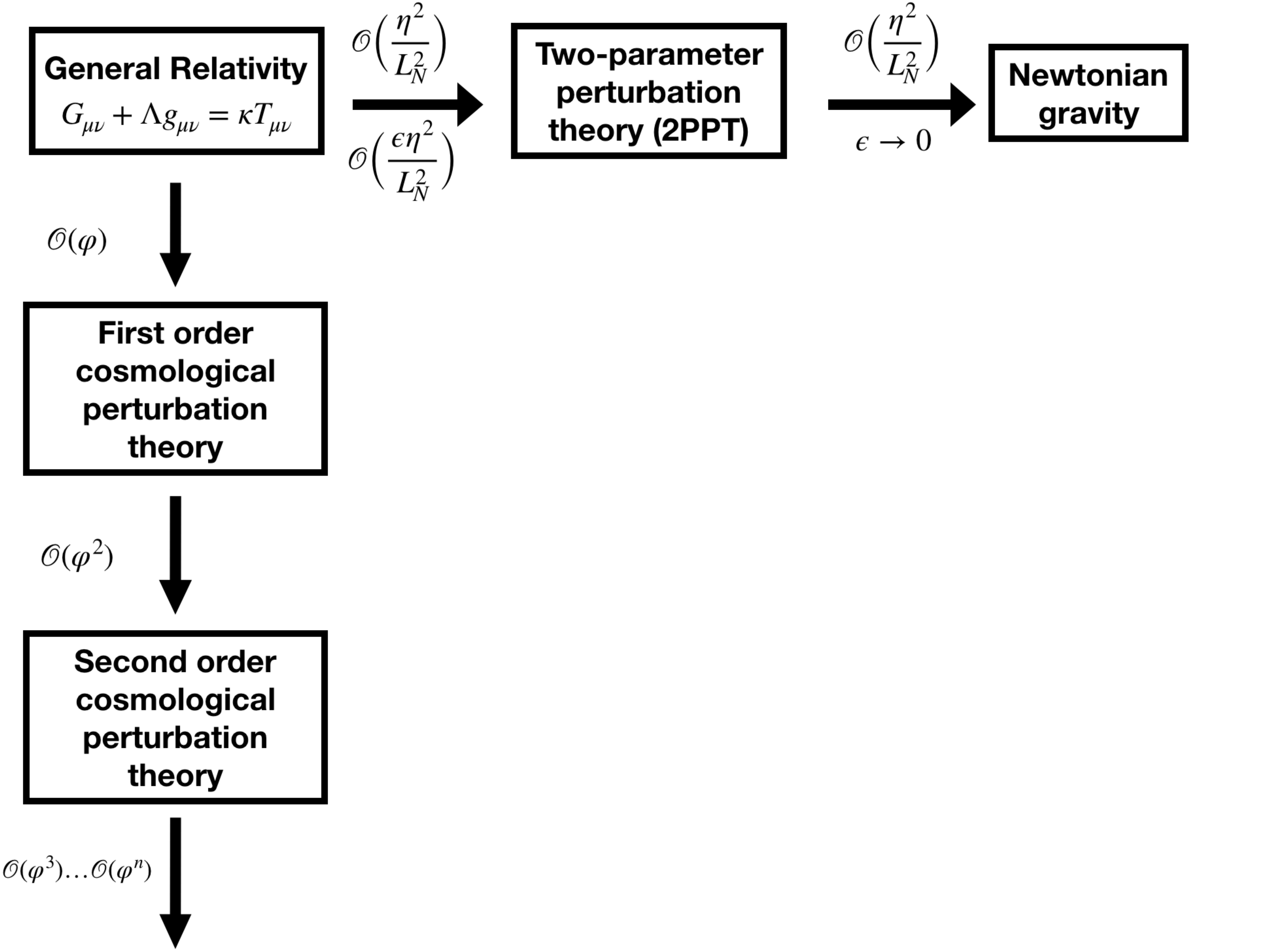
First order cosmological perturbation theory
$\mathcal{O}(\varphi^2)$
Second order cosmological perturbation theory
$\mathcal{O}(\varphi^3)\ldots\mathcal{O}(\varphi^n)$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$\mathcal{O}\left(\frac{\eta^2}{L_N^2}\right)$
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Two-parameter perturbation theory (2PPT)
$\mathcal{O}\left(\frac{\eta^2}{L_N^2}\right)$
$\epsilon \to 0$
Newtonian gravity
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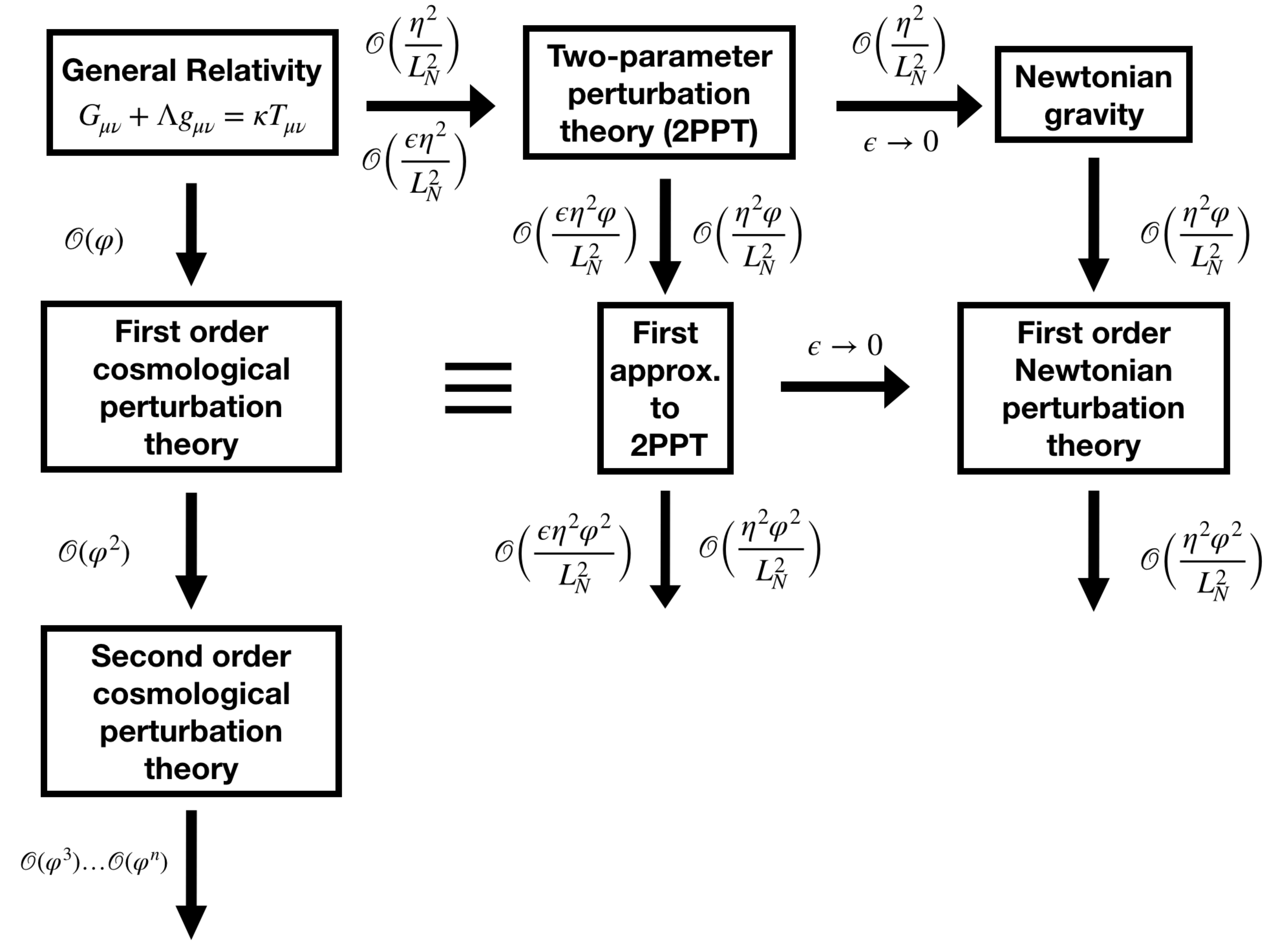
General Relativity
$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$
$\mathcal{O}\left(\frac{\eta^2}{L_N^2}\right)$
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Two-parameter perturbation theory (2PPT)
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Newtonian gravity
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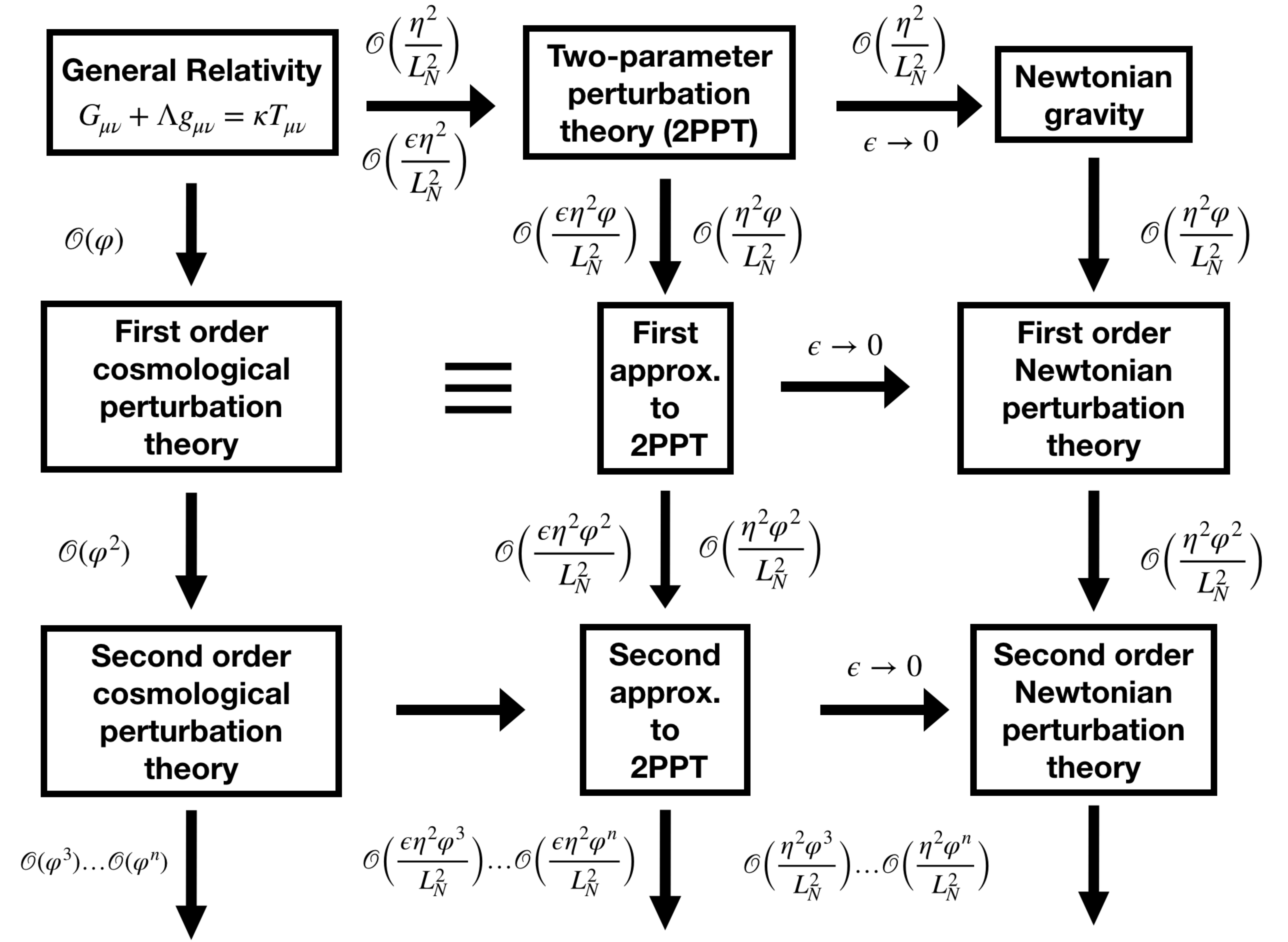

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- At **second approximation** - recover an approximation to 2nd order perturbation theory (**missing some terms**)
- At **third approximation** - recover an approximation to 3rd order perturbation theory (missing **A LOT** of terms)

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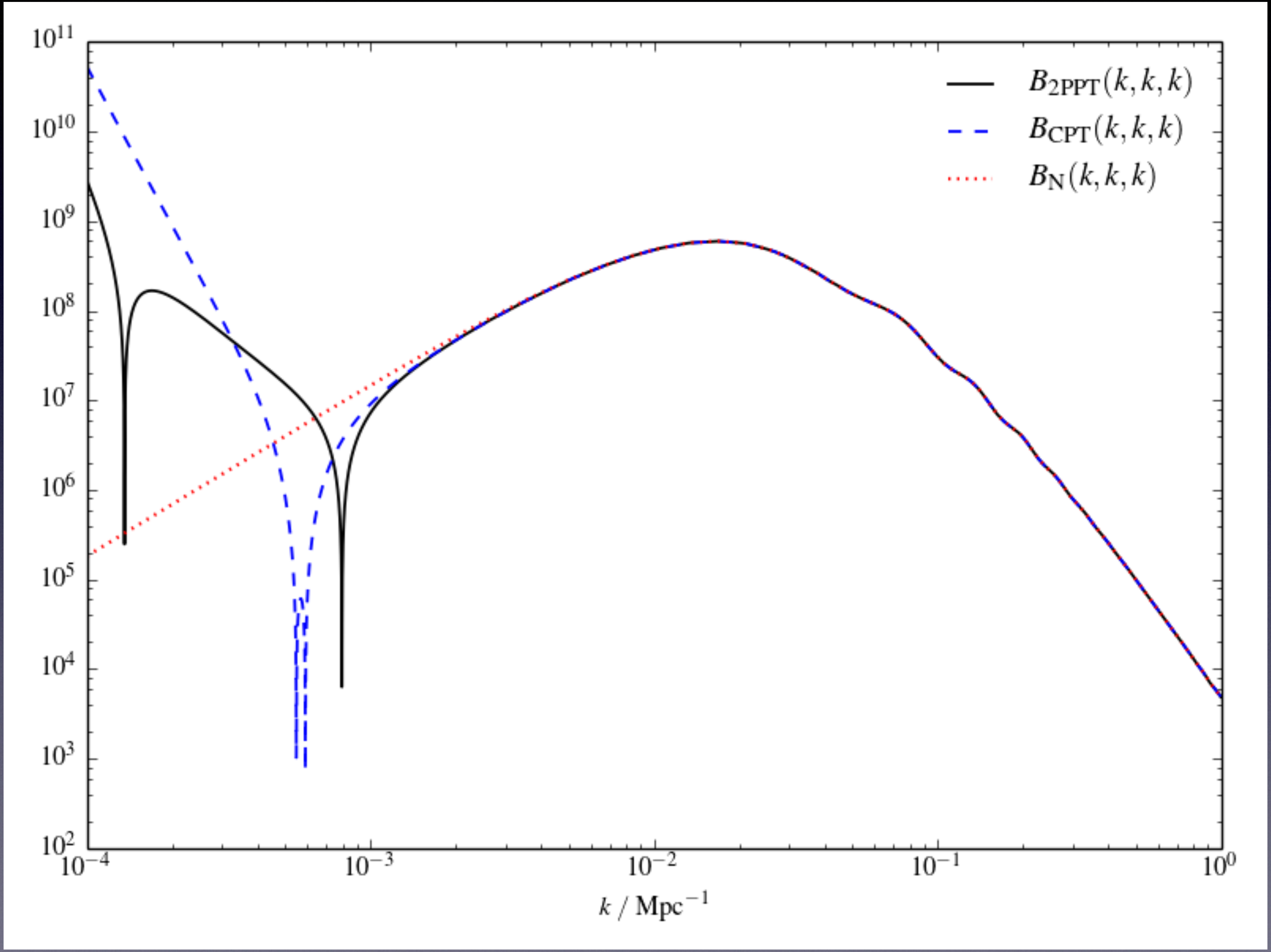

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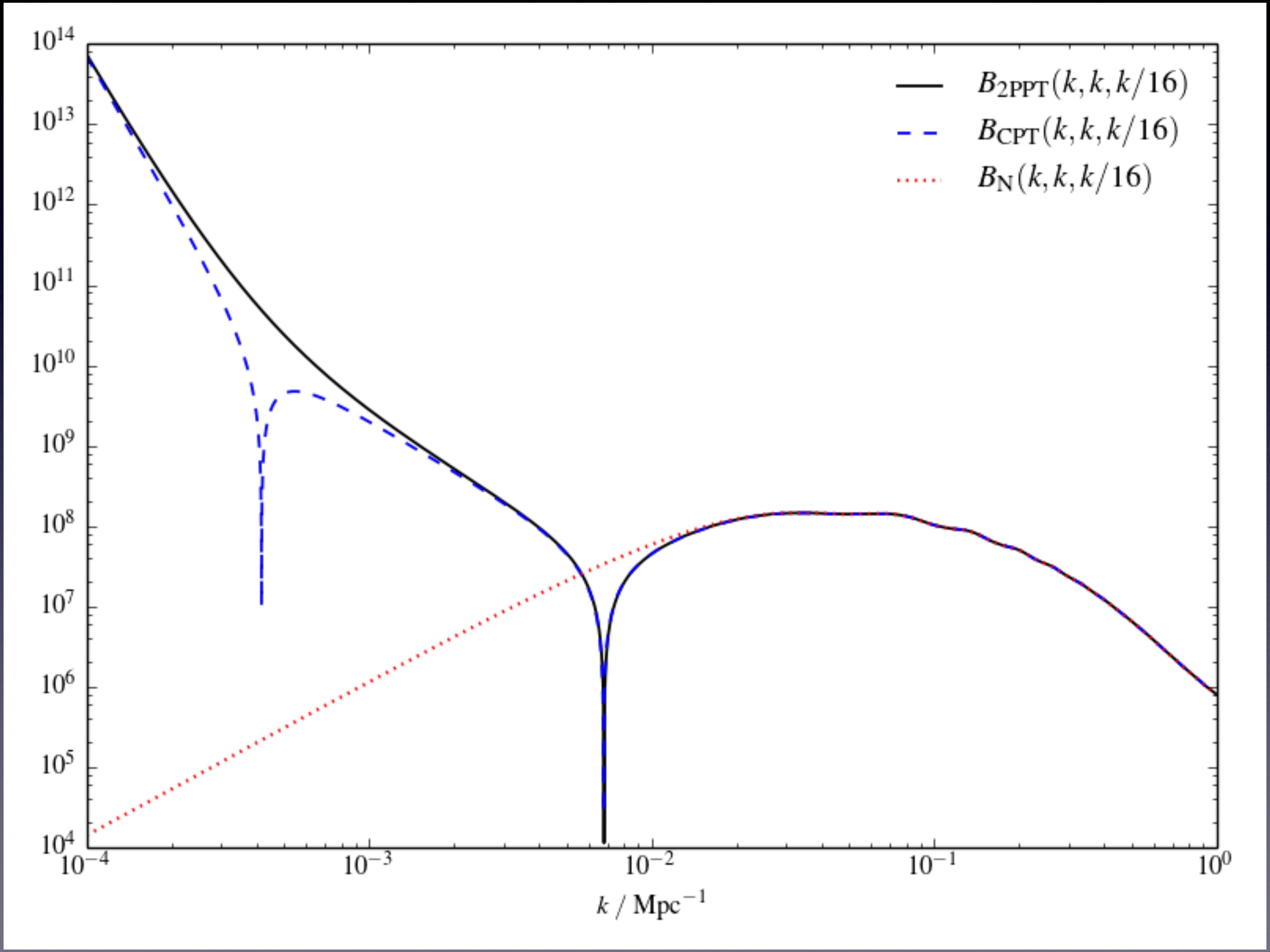

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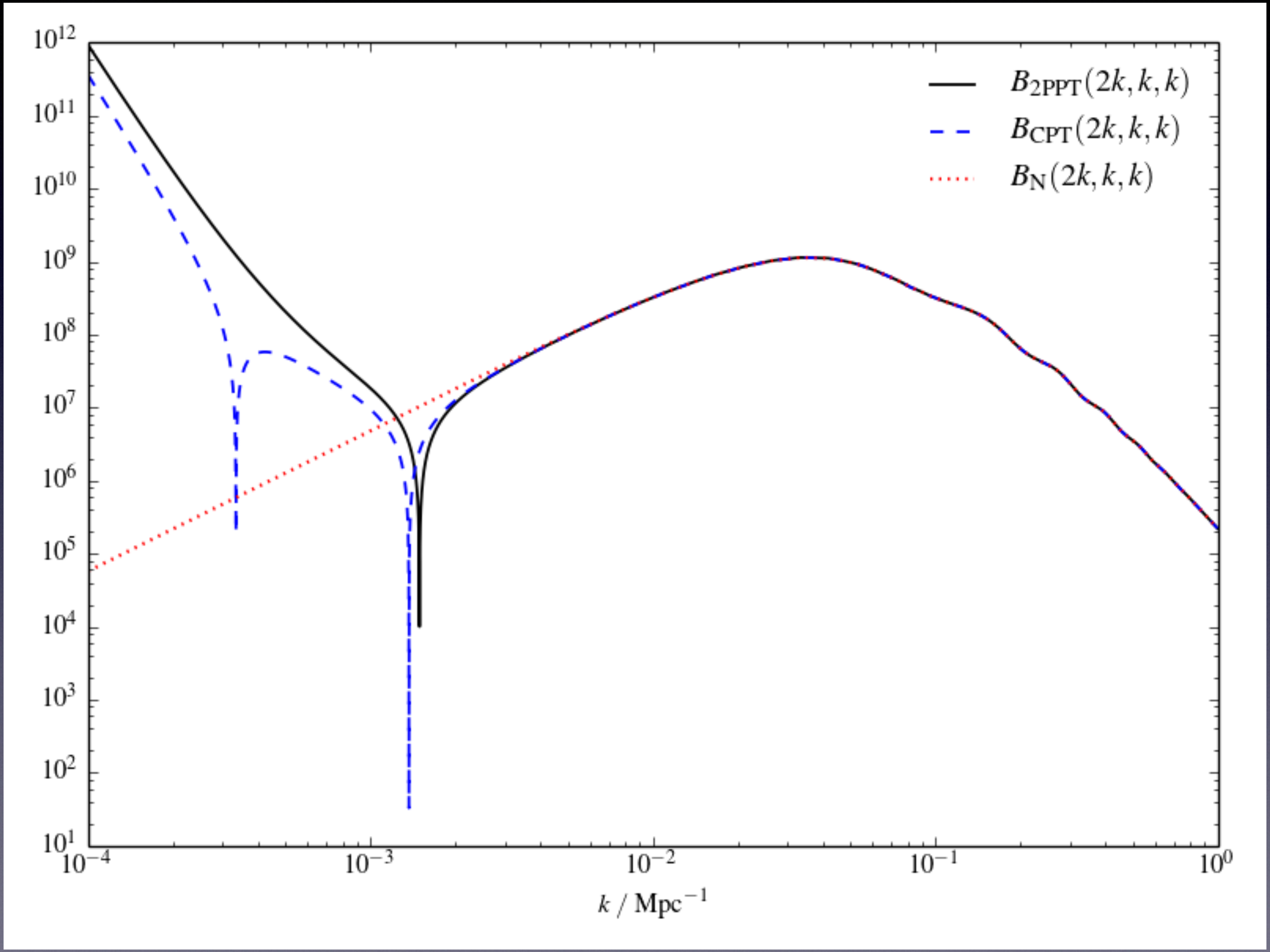

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Seem to have **good agreement** down to $k > 10^{-3}\ \mathrm{Mpc}^{-1}$

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The **2PPT correctly identifies** all the Poisson gauge terms scaling as $\sim \dfrac{\mathcal{H}^2}{k^2}$

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…**BUT** then would have to consider **even higher order products of the nonlinear terms**

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4. "Upgrade" the Newtonian perturbation theory sector (Renormalised PT, EFTofLSS)
5. Check the soft theorems!

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THANK YOU!