Probing cosmic string networks with gravitational waves

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GReCO seminar

Outline

Introduction to cosmic strings

References :
(Nielsen & Olesen, 1973)
(Kibble, 1976)
(Vilenkin & Shellard, 2001)
(Ringeval, Sakellariadou, & Bouchet, 2007)
(Ringeval, 2010)
(Vachaspati, Pogosian, & Steer, 2015)

Cosmic strings (Kibble, 1976)

1D topological defects

- Cosmic strings are 1D topological defects that may appear after a symmetry breaking phase transition
- After the phase transition the field *falls* into the new vacuum manifold $\mathcal{M}$
- Strings arise if $\mathcal{M}$ is not simply connected, i.e. $\mathcal{M}$ contains holes around which loops can be trapped
- We expect strings to be formed in most models of spontaneous symmetry breaking

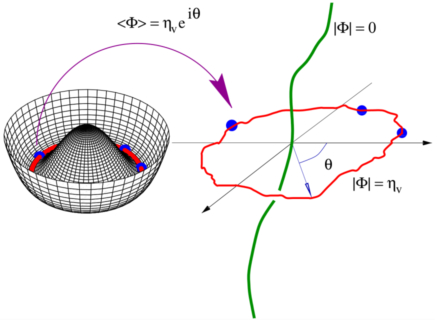

Figure: String formation in the "Mexican hat" potential $V(|\phi|)$. Figure taken from (Ringeval, 2010)

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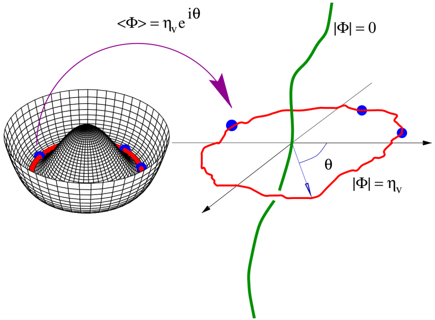


Figure: String formation in the "Mexican hat" potential $V(|\phi|)$. Figure taken from (Ringeval, 2010)

As an example, the Lagrangian for the Nielsen-Olesen string (Nielsen & Olesen, 1973)

$$\mathcal{L} = -\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + (\mathcal{D}_\mu\phi)^*\mathcal{D}^\mu\phi - \frac{\lambda}{4}\left(|\phi|^2 - \eta^2\right)$$

Nambu-Goto strings: the one-dimensional limit

- The width of the string is very small compared to the other length scales in the problem, and the thin string limit is commonly adopted.
- Then the string is simply modeled as a line with mass per unit length $\mu \propto T^2$ using the Nambu-Goto action which minimizes the area swept by the string

$$\mathcal{S} = -\mu \int \sqrt{-\det(\gamma)} \mathrm{d}^2\zeta$$

$\zeta^{\mathrm{a}} = (t, \zeta)$ and γ_{ab} the induced metric on the string

Energy scale	Width	Linear density
GUT : 10^{16} GeV	2×10^{-32} m	$G\mu \approx 10^{-6}$
3×10^{10} GeV	5×10^{-27} m	$G\mu \approx 10^{-17}$
10^8 GeV	2×10^{-24} m	$G\mu \approx 10^{-22}$
EW : 100 GeV	2×10^{-18} m	$G\mu \approx 10^{-34}$

Closed loops of cosmic strings

Oscillation and gravitational wave emissions

The general solution for a Nambu-Goto string in a Minkowski background is

$$\vec{X}(t,\zeta) = \frac{1}{2}\left[\vec{a}(\zeta - t) + \vec{b}(t+\zeta)\right]$$
$$\vec{a}'^2 = \vec{b}'^2 = 1$$

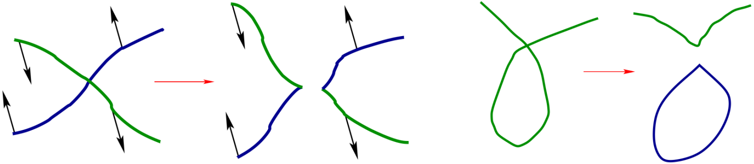
For a closed loop $X^\mu(t,\zeta+\ell) = X^\mu(t,\zeta)$. One can show that the loop oscillates with a period $T = \dfrac{\ell}{2}$.
These oscillations lead to a gravitational radiation. The *quadrupole formula* can give a **rough** estimate of the power emitted (Vilenkin & Shellard, 2001)

$$\dot{E} \approx G\left(\frac{\mathrm{d}^3 D}{\mathrm{d}t^3}\right)^2 \approx GM^2L^4\omega^6 \approx \Gamma G\mu^2$$

in which $D \approx ML^2$ is the quadrupole moment, $M = \mu L$ is the mass and $\omega \approx L^{-1}$ the characteristic frequency.
NOTE : it does not depend on the loop length !

Typical properties of cosmic strings

Loop formation and scaling



- When strings intersect, they change partner
- Analytical arguments and numerical simulations show the existence of an attractor solution independent of initial conditions called scaling
- During scaling, all length-scales are proportional to t cosmic time.
- In particular, it means loop can survive until today

$$\rho_\infty \propto t^{-2} \propto \begin{cases} a^{-4} & \text{during radiation era} \\ a^{-3} & \text{during matter era} \end{cases}$$

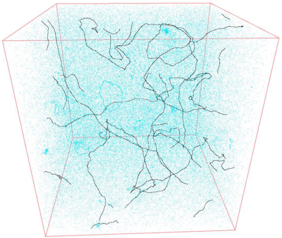


Figure: (Ringeval et al., 2007)

Observational signatures of cosmic strings

Selection of observational signatures

- CMB : line discontinuities in the temperature or polarization patterns, and statistical methods based on calculations of various correlation functions. $G\mu <$ few $\times 10^{-7}$
- 21-cm : brightness fluctuations or spatial correlations between the 21 cm and CMB anisotropies. Future experiments can in principle constrain $G\mu \approx 10^{-10} - 10^{-12}$
- The metric around a cosmic string can result in characteristic lensing patterns of distant light sources.

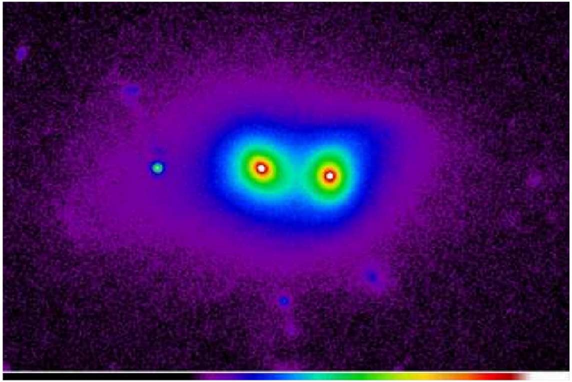


Figure: CLS-1, discovered in 2003, raised a lot of interest from the cosmic strings community but turned out to be two similar galaxies close to each other

Gravitational wave emission from cosmic strings

References in this section :
(Vachaspati & Vilenkin, 1985)
(Damour & Vilenkin, 2001)
(Siemens et al., 2006)
(Blanco-Pillado & Olum, 2017)
(Abbott et al., 2018)
(Collaboration & the Virgo Collaboration, 2019)
(Auclair, Blanco-Pillado, et al., 2019)

Bursts of gravitational waves

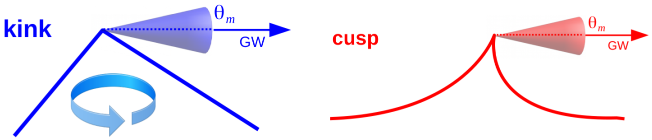


Figure: F. Robinet

A typical loop will have a number of kinks and cusps, and the spectrum of high frequency gravitational radiation emitted from a string depends on these features

- kinks are discontinuities in the tangent vector of the string. Kinks are formed when strings intercommute and travel along the string at the speed of light, $q = 5/3$.
- cusps travel instantly at the speed of light, $q = 4/3$.

The waveform of the gravitational wave arriving at the detector is known (Damour & Vilenkin, 2001)

$$h_q(\ell, z, f) = A_q(\ell, z, f) f^{-q} \quad , \quad A_q = g_{1,q} \frac{G\mu\ell^{2-q}}{(1+z)^{q-1} r(z)} \tag{1}$$

Rate of bursts

For a given loop distribution, you can estimate the *GW burst rate* (Siemens et al., 2006)

$$\frac{\mathrm{d}^2\mathcal{R}_q}{\mathrm{d}V\mathrm{d}\ell} = \frac{1}{1+z} \times \frac{\mathrm{d}^3\nu_q}{\mathrm{d}t\mathrm{d}\ell\mathrm{d}V} \times \Delta_q$$

as a function of

- Δ_q geometrical factor for the fraction of GWs you can access (linked to a beaming angle)
- $\frac{\mathrm{d}^3\nu_q}{\mathrm{d}t\mathrm{d}\ell\mathrm{d}V} = \frac{2}{\ell} N_q \frac{\mathrm{d}^2\mathcal{N}}{\mathrm{d}\ell\mathrm{d}V}$ number of events per space time volume per unit length
- N_q mean number of events per oscillation, which is suposed to be a fixed number.
- z redshift at emission

The effective burst rate in the detector depends on its sensitivity.

$$\mathcal{R}_q = \int \mathrm{d}A_q \; \mathrm{e}_q(A_q) \frac{\mathrm{d}\mathcal{R}_q}{\mathrm{d}A_q}(G\mu, N_q) \tag{2}$$

LIGO/Virgo burst search during O1

The parameter space $(G\mu, N_q)$, is scanned and excluded at a 95% level when $\mathcal{R}_q$ exceeds $2.996/T_{\text{obs}}$ which is the rate expected from a random Poisson process over an observation time T_{obs}.

- No cosmic string burst detected during O1 and O2 runs
- Allows to put upper bounds on the string tension which are not very competitive with respect to the Stochastic Background of GW
- We are currently involved in the LIGO/Virgo collaboration to produce constraints for the O3 run

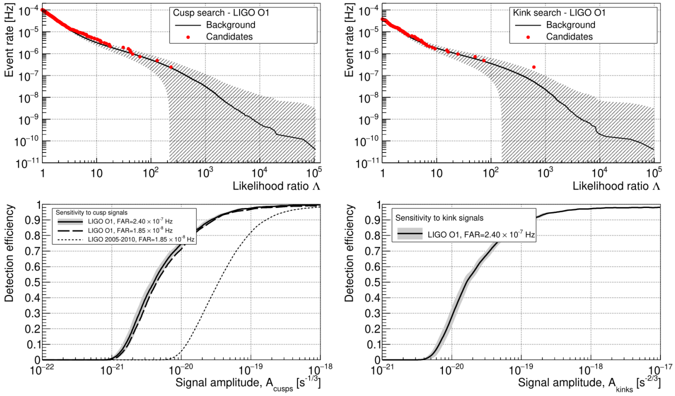


Figure: (Abbott et al., 2018)

Emission of gravitational waves by a cosmic string loop

$$\dot{E} = \Gamma G \mu^2, \quad \Gamma = \sum_m \mathrm{P}_m = \mathcal{O}(50)$$

- All the energy radiated by loops is converted to gravitational waves
- An effective average power P_m emitted in mode m determined by simulations and/or analytical arguments

The high frequency regime is dominated by contributions from burst-like events

$$\mathrm{P}_m \propto \begin{cases} m^{-4/3} & \text{for cusps} \\ m^{-5/3} & \text{for kinks} \end{cases}$$

Low-frequency modes are dominated by the oscillations of the loops

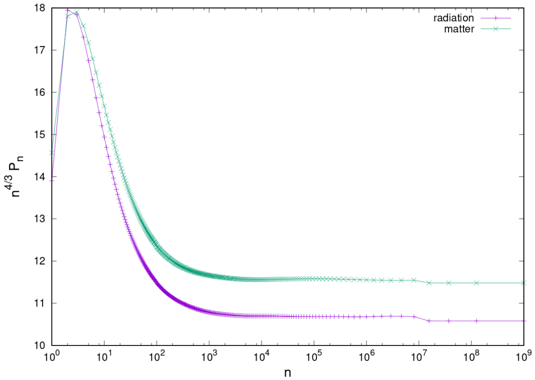


Figure: Averaged power spectrum determined numerically in (Blanco-Pillado & Olum, 2017)

The stochastic background of gravitational waves

The uncorrelated sum of all the GW signals produced by cosmic string loops during the History of the Universe constitutes a Stochastic Background of GW.
We can estimate this background using energetic arguments

$$\Omega_{\rm GW}(\ln f) = \frac{8\pi G}{3\mathrm{H}_0^2} f \rho_{\rm GW}$$

$$\rho_{\rm GW}(f) = \int_0^{t_0} \frac{\mathrm{d}t}{[1+z(t)]^4} \mathrm{P}_{\rm gw}(t, f') \frac{\partial f'}{\partial f}$$

$$\mathrm{P}_{\rm gw}[t, f'] = G\mu^2 \sum_m \frac{2m}{f'^2} \mathrm{P}_m \frac{\mathrm{d}^2\mathcal{N}}{\mathrm{d}\ell \mathrm{d}V} \left[\frac{2m}{f'}, t\right]$$

The loop distribution $\frac{\mathrm{d}^2\mathcal{N}}{\mathrm{d}\ell \mathrm{d}V}$ remains to be specified, more in the next section.

Existing constrained from LIGO/Virgo O1 run

- The constraint from burst is less stringent than the one from stochastic
- The intercommutation probability p is set to 1 in the present seminar
- There is a huge disparity between different models especially on these relatively high-frequency experiments. More on that later

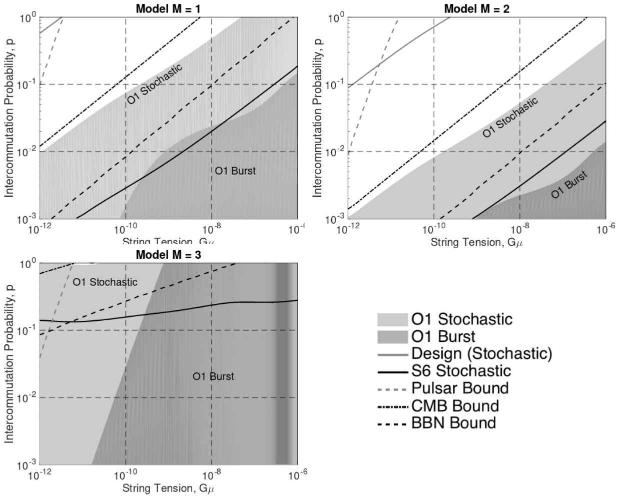
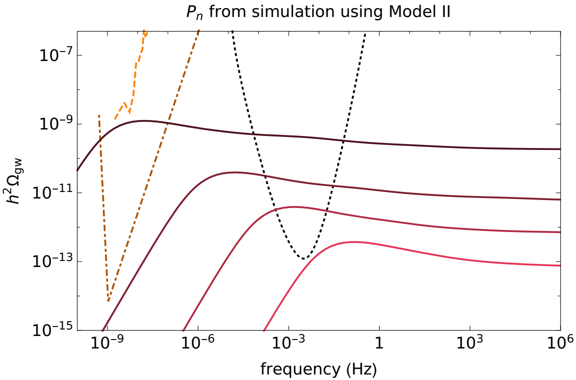


Figure: 95% confidence exclusion regions (Abbott et al., 2018)

Projected constraints for LISA (Auclair, Blanco-Pillado, et al., 2019)

Analysis done within the LISA cosmology working group



$$\Omega_{\rm gw}(f \to \infty) \propto \sqrt{\frac{G\mu}{\Gamma}}$$

Figure: A comparison of the LISA sensitivity curve to the predicted SBGW. LISA will probe strings with tensions higher than $G\mu = 10^{-17}$ with little dependence on the cosmic string model.

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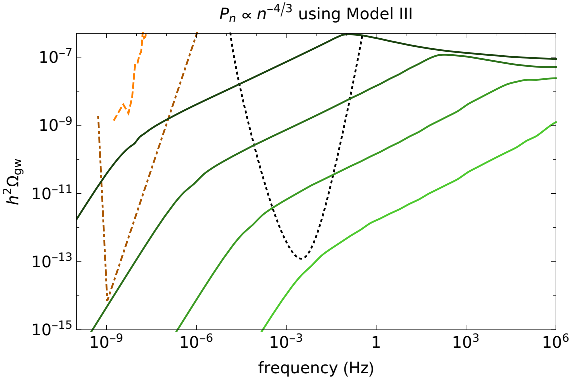

$$\Omega_{\rm gw}(f \to \infty) \propto (G\mu)^{0.16}$$

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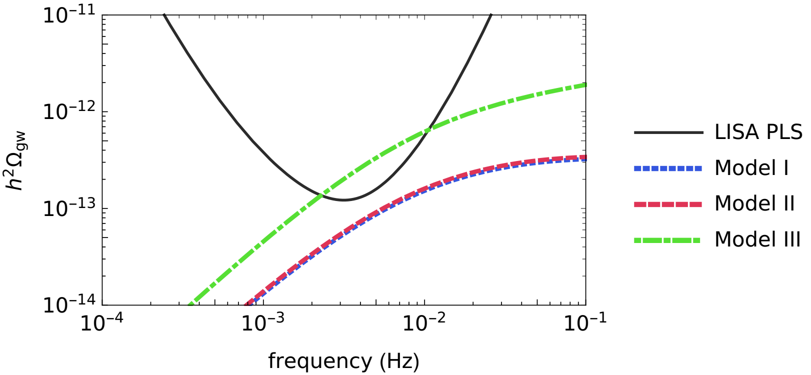

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The loop distribution: beyond the Nambu-Goto approximation

References
(Hindmarsh, Stuckey, & Bevis, 2009)
(Vachaspati, 2010)
(Blanco-Pillado, Olum, & Shlaer, 2011)
(Mota & Hindmarsh, 2015)
(Matsunami, Pogosian, Saurabh, & Vachaspati, 2019)
(Auclair, Steer, & Vachaspati, 2019)

Field-Theory (FT) simulations of individual loops

Formation, evolution and decay

- So far we have studied Nambu-Goto strings, ie. infinitely thin strings
- Large-scale field-theory simulations find that cosmic strings decay rapidly into particles (Hindmarsh et al., 2009)
- High resolution field theory simulation of single loops tend to show that their lifetime is actually longer that previously expected (Matsunami et al., 2019)
- The rate at which strings emitt particles has been measured in high-resolution numerical simulations
- We propose a first step to bridge the gap between Nambu-Goto strings and field-theory strings

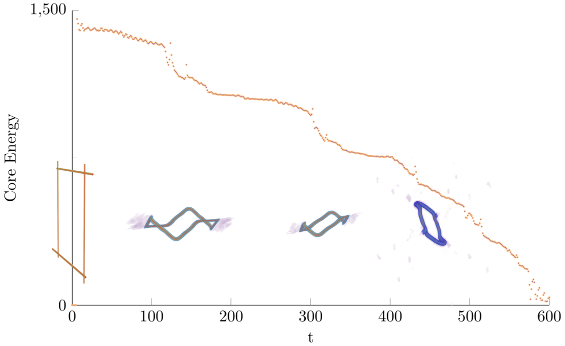


Figure: Energy of a loop with the initial size of 390 lattice spacings plotted vs time. (Matsunami et al., 2019)

Energy budget for a cosmic string loop

We parametrize the energy lost by an average loop with $\mathcal{J}$, remember that for cosmic string loops, $E = \mu\ell$

$$\frac{\mathrm{D}\ell}{\mathrm{D}t} = -\Gamma G\mu\mathcal{J}(\ell)$$

Where

- $\mathcal{J}(\ell) = 1$ if GW emission is the only channel for losing energy
- $\mathcal{J}(\ell) = 1 + \ell_\mathrm{k}/\ell$ if kinks are present on the loop
- $\mathcal{J}(\ell) = 1 + \sqrt{\ell_\mathrm{c}/\ell}$ if cusps are present on the string
- $\mathcal{J}(\ell) = \Theta(\ell - \ell_\mathrm{V})$ in the case of superconducting strings

$$\ell_\mathrm{k} \sim \beta_\mathrm{k}\frac{w}{\Gamma G\mu} \propto (G\mu)^{-3/2}, \quad \ell_\mathrm{c} \sim \beta_\mathrm{c}\frac{w}{(\Gamma G\mu)^2} \propto (G\mu)^{-5/2}, \quad \ell_\mathrm{V} = \frac{N}{\sqrt{\mu}}$$

Modeling the loop distribution with a continuity equation (Auclair, Steer, & Vachaspati, 2019)

Non self-intersecting loops are produced from the network of infinite strings and then lose energy

$$\frac{\partial}{\partial t}\left(a^3\frac{\mathrm{d}^2\mathcal{N}}{\mathrm{d}\ell\mathrm{d}V}\right)+\frac{\partial}{\partial \ell}\left(a^3\frac{\mathrm{D}\ell}{\mathrm{D}t}\frac{\mathrm{d}^2\mathcal{N}}{\mathrm{d}\ell\mathrm{d}V}\right)=a^3\mathcal{P}(\ell,t)$$

which, in terms of our length-dependent energy-loss channel becomes

$$\frac{\partial}{\partial t}\left(a^3\frac{\mathrm{d}^2\mathcal{N}}{\mathrm{d}\ell\mathrm{d}V}\right)-\Gamma G\mu\frac{\partial}{\partial \ell}\left(a^3\mathcal{J}(\ell)\frac{\mathrm{d}^2\mathcal{N}}{\mathrm{d}\ell\mathrm{d}V}\right)=a^3\mathcal{P}(\ell,t)$$

Introducing the new variables

$$\tau\equiv\Gamma G\mu t\ , \qquad \xi\equiv\int\frac{\mathrm{d}\ell}{\mathcal{J}(\ell)}.$$

the continuity equation becomes

$$\left(\left.\frac{\partial}{\partial \tau}\right|_{\xi}-\left.\frac{\partial}{\partial \xi}\right|_{\tau}\right)\left(\Gamma G\mu\mathcal{J}a^3\frac{\mathrm{d}^2\mathcal{N}}{\mathrm{d}\ell\mathrm{d}V}\right)=a^3\mathcal{J}\mathcal{P},$$

Modeling the loop distribution

Solution for a δ-function loop production function

The shape of the loop production function (LPF) has been studied in numerical simulations but it is still a matter of debate. Simplest choice coming from the standard one-scale model is to assume

$$\mathcal{P}(\ell, t) = Ct^{-5}\delta\left(\frac{\ell}{t} - \alpha\right)$$

which seams to reproduce well (Blanco-Pillado et al., 2011) and can be used as a Green's function for more elaborate LPF. The loop formation time $t_\star$ satisfies the following equation

$$\Gamma G\mu t_\star + \xi(\alpha t_\star) = \Gamma G\mu t + \xi(\ell),$$

and the loop distribution is given by

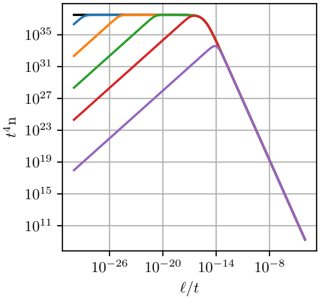
$$t^4 \frac{\mathrm{d}^2\mathcal{N}}{\mathrm{d}\ell \mathrm{d}V} = C\frac{1}{\mathcal{J}(\ell)}\frac{\mathcal{J}(\alpha t_\star)}{\alpha + \Gamma G\mu \mathcal{J}(\alpha t_\star)}\left(\frac{t_\star}{t}\right)^{-4}\left(\frac{a(t_\star)}{a(t)}\right)^3.$$

If $J(\ell) = 1$ then $\xi(\ell) = \ell$ it reduces to the standard scaling Nambu-Goto loop distribution for a delta-function loop production function

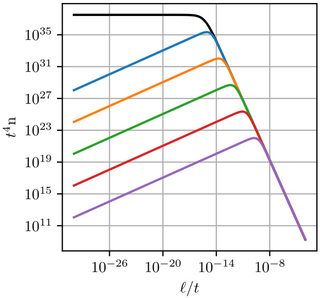
$$t^4 \frac{\mathrm{d}^2\mathcal{N}}{\mathrm{d}\ell \mathrm{d}V} = C\frac{(\alpha + \Gamma G\mu)^{3-3\nu}}{(\gamma + \Gamma G\mu)^{4-3\nu}}$$

Consequences on the number of loops

Modeling the loop number density with both GW and particle emission



(a) Influence of kinks, $G\mu = 10^{-17}$



(b) Influence of cusps, $G\mu = 10^{-17}$

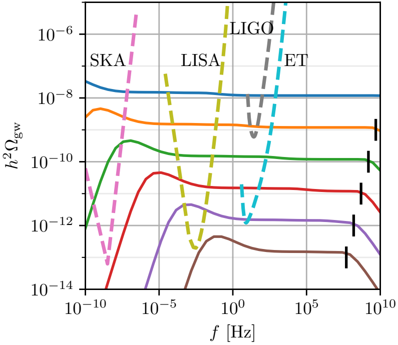
Figure: From bottom to top, the curves show snapshots of the loop distribution at redshifts $z = 10^{13}, 10^{11}, 10^{9}, 10^{7}, 10^{5}$, and the black curve is the scaling NG loop distribution

Impact on the SBGW

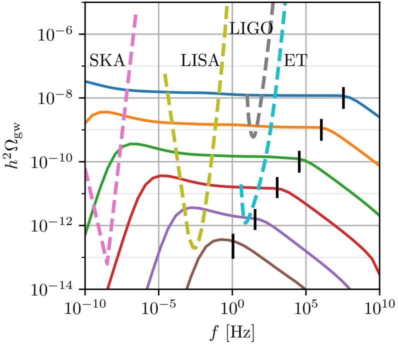
Breaking of the high frequency plateau

A consequence of the introduction of $\ell_{\rm k}, \ell_{\rm c}$ is that the high frequency plateau is cutoff at

$$f = \sqrt{\frac{2H_0\sqrt{\Omega_{\rm rad}}c}{\ell_{\rm c,k}\Gamma G\mu}}$$



(a) SBGW : kinks



(b) SBGW : cusps

Particle emission bounds

Injected energy by cosmic strings (Mota & Hindmarsh, 2015; Vachaspati, 2010)

- The emitted particles are heavy and in the dark particle physics sector corresponding to the fields that make up the string
- We assume that there is some interaction of the dark sector with the standard model sector

The energy density injected by cosmic strings per unit of time

$$\Phi_{\rm H}(t) = \int_0^{\alpha t} {\rm P}_{\rm c,k} \frac{{\rm d}^2\mathcal{N}}{{\rm d}\ell {\rm d}V} {\rm d}\ell'$$

in which

$$ {\rm P}_{\rm k} = \Gamma G\mu \frac{\ell_{\rm k}}{\ell} \qquad {\rm P}_{\rm c} = \Gamma G\mu \sqrt{\frac{\ell_{\rm c}}{\ell}}$$

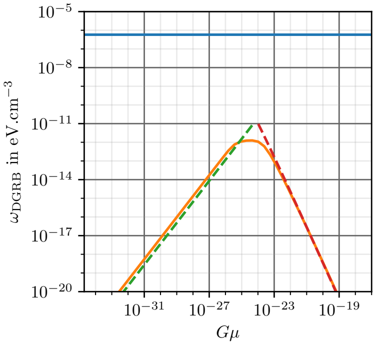
Then the emitted particle radiation will eventually decay, and a significant fraction of the energy $f_{\rm eff} \sim 1$ will cascade down into γ-rays.

$$\omega_{\rm DGRB} = f_{\rm eff} \int_{t_c}^{t_0} \frac{\Phi_{\rm H}(t)}{(1+z)^4} {\rm d}t$$

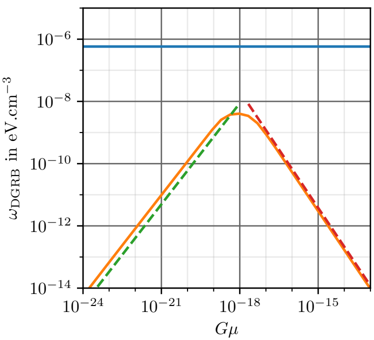
Contribution of cosmic strings to the Diffuse Gamma-Ray Background

Constraints from Fermi-LAT

$$\omega_{\text{DGRB}}^{\text{obs}} \leq 5.8 \times 10^{-7}\ \text{eV.cm}^{-3}$$



(a) Contribution from kinks, for small $G\mu$, $\omega_{\text{DGRB}} \propto \mu^{9/8}$ and $\mu^{-2}\log\mu$ for large $G\mu$



(b) Contribution from cusps, for small $G\mu$, $\omega_{\text{DGRB}} \propto \mu^{13/12}$ and $\mu^{-5/3}$ for large $G\mu$

Conclusion

Summary

- Cosmic strings are a general prediction of most symmetry-breaking models
- **Scaling** means that the network of cosmic strings survives for a very long time
- Gravitational wave astronomy is one of the most promising technique to probe for cosmic strings, especially with the space-based detector LISA which will be able to probe cosmic strings with tension $G\mu \geq 10^{-17}$
- We have tried to go beyond the Nambu-Goto approximation by taking into account the emission of particles which seems to dominate in Field-Theory simulations on small scales
- Our analysis show that this phenomenon has little effect on the Stochastic Background
- We have also checked that this emission of particles does not violate bounds for the diffuse Gamma-Ray Background

Conclusion

Future developments

- It is important to evaluate more carefully the prevalence of kinks versus cusps on cosmological string loops
- It would also be interesting to study other loop production functions, particularly power-law LPF which predict a larger number of small loops; hence one might expect a larger gamma ray background from strings
- We are also applying these tools to the study of vortons together with Danièle Steer, Patrick Peter and Christophe Ringeval.

Thank you

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