Long-Distance Dynamics of Quantum Fields & Cosmology

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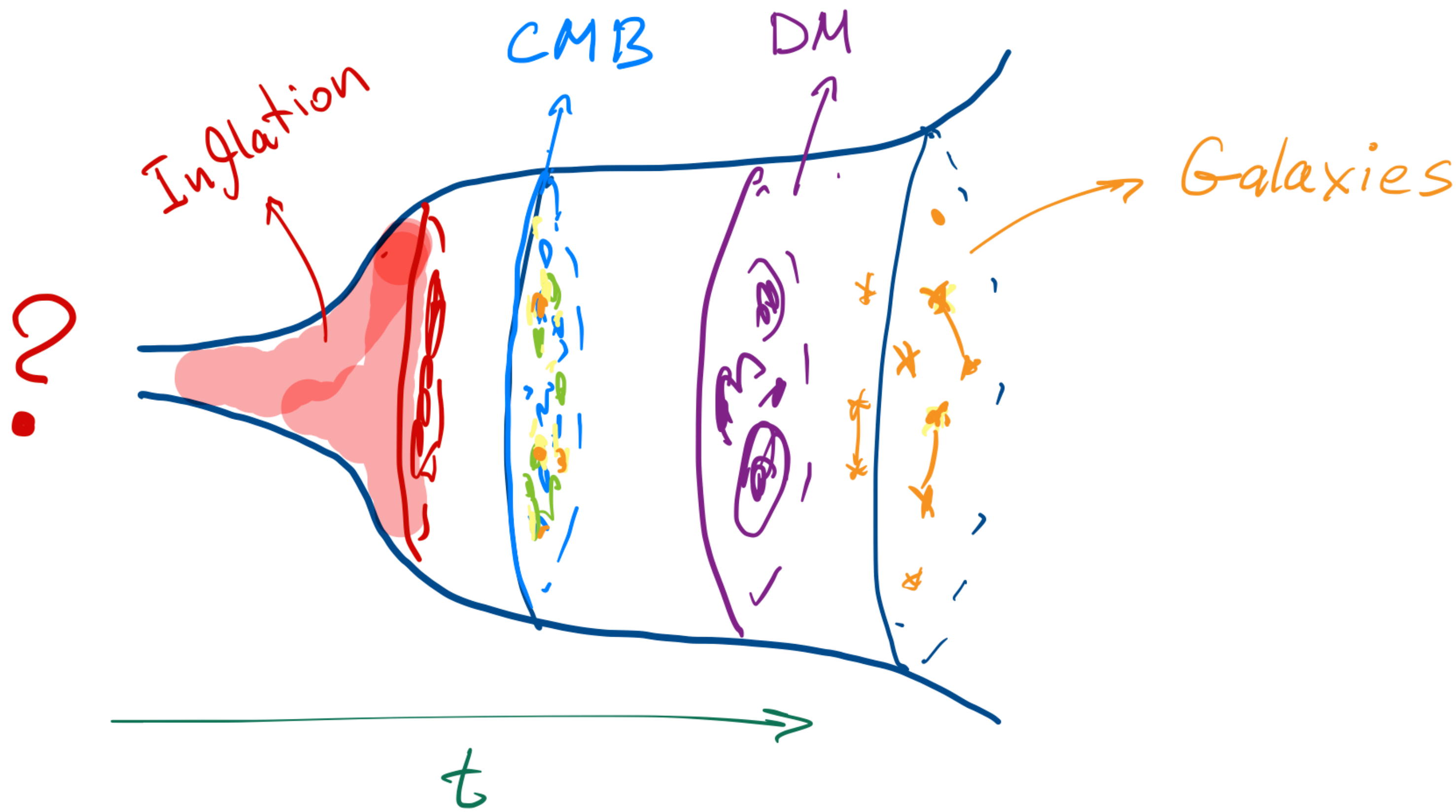
Outline

- Motivation: early Universe cosmology
- Review the problem of IR divergences
- Develop new systematic formalism for QFT in dS-like spacetimes which resolves the problem
- Applications, generalizations and future developments

Early Universe:



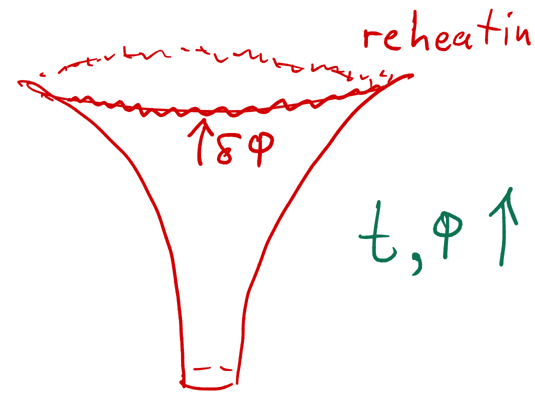
Inflation is the earliest period in the history of the universe that we have access to.
It is a period of exponentially fast accelerating expansion.

$$\text{Inflation} \simeq GR + \Lambda + \varphi$$

positive c.c. "clock" field

$$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2, \quad H^2 \simeq \frac{\Lambda}{M_{pl}^2}$$

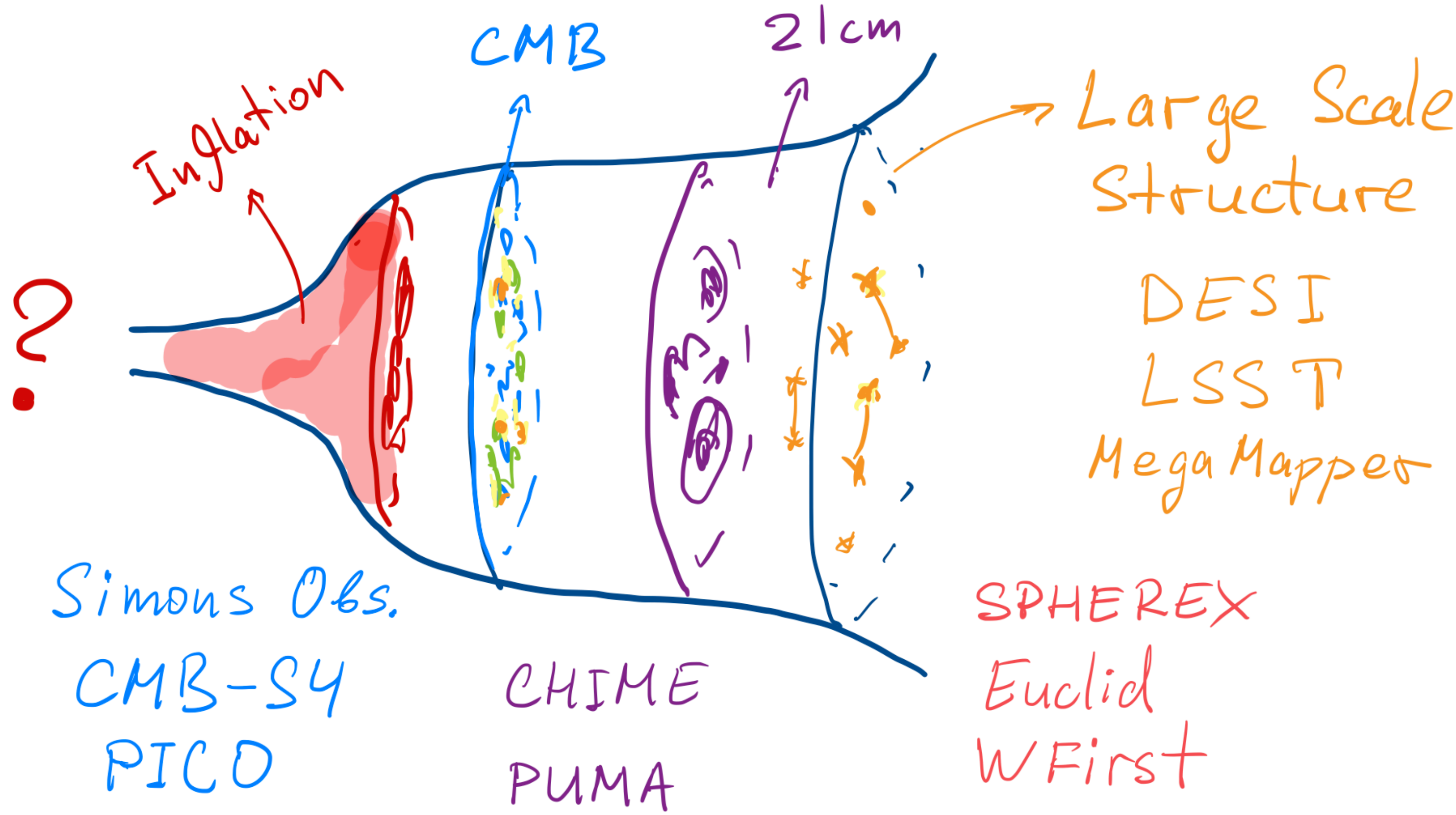
de Sitter: $\dot{H} = 0$ Inflation: $\dot{H} \simeq 0$



$$\langle \delta\varphi \, \delta\varphi \rangle \simeq H^2$$

- All structure in the Universe originates from quantum fluctuations of the "clock" field (inflaton), $\delta\varphi$.
- By expansion, and later by gravity, they get amplified to macroscopic scales.

Observational Signatures of Inflation



Upcoming experiments will provide an enormous amount of new data, e.g. $\langle \delta\varphi^3 \rangle \sim g_{NL}$ forecast:

	Planck	Puma	SPHEREX	MegaMapper	
f_{NL}^{loc}	5	~ 0.5	~ 0.2	~ 0.07	...

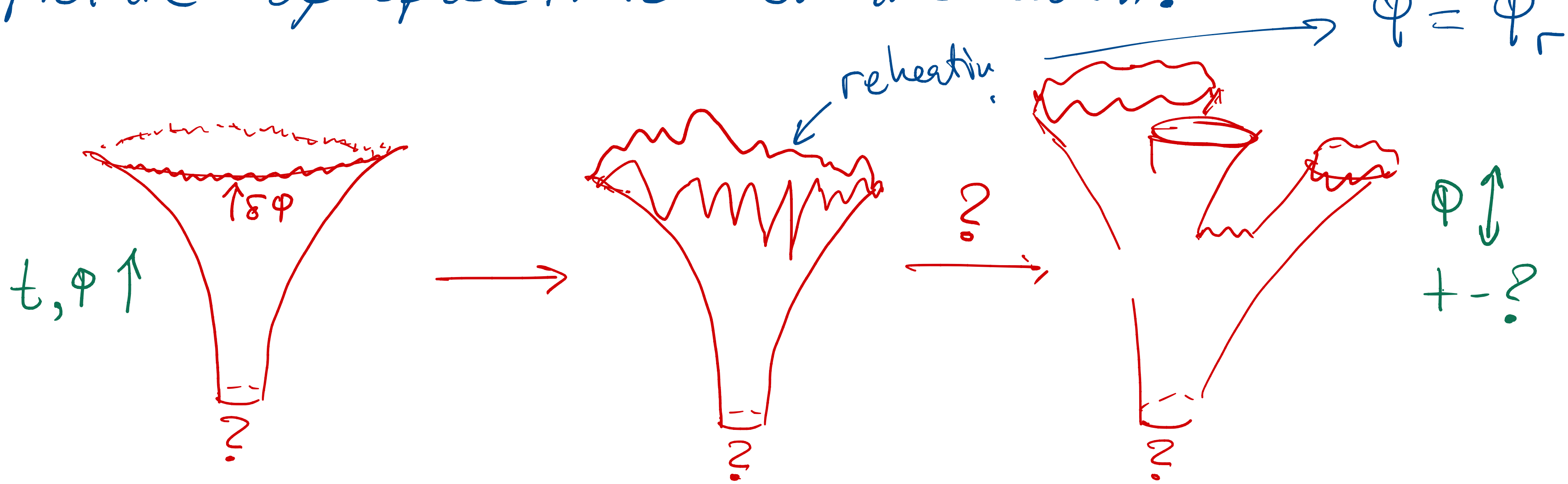
Properties of Inflationary Perturbations

- Presently, we lack techniques to do reliable calculations, at least in some inflationary models. At the same time, many of the searches are "template-based".

- Part of the problem is the infrared divergences present in some Quantum Field Theories (and Gravity) in quasi-dS spacetime.

Foundational Problems in Cosmology

- In some models of inflation semiclassical picture of spacetime breaks down:

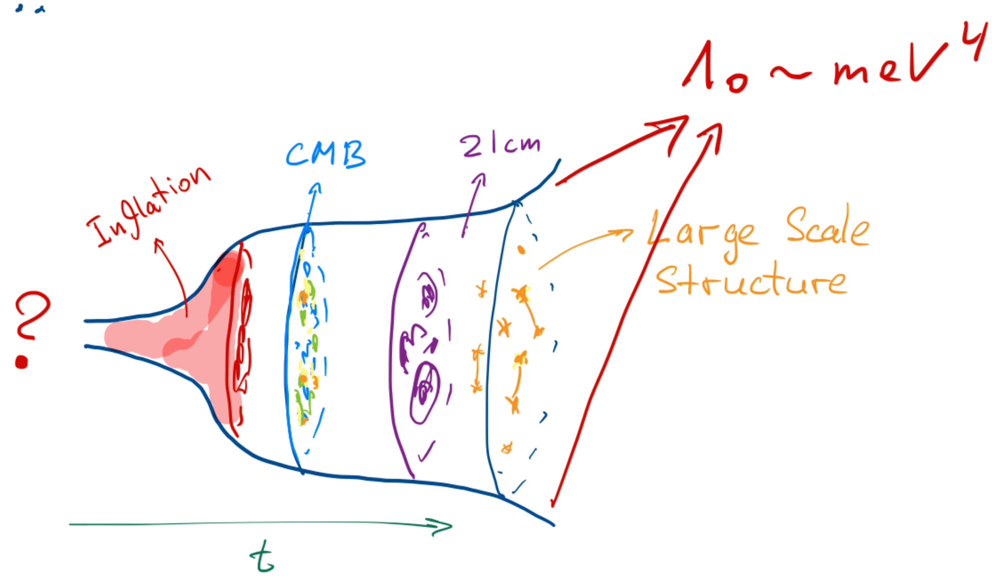


- The "clock" breaks and inflation becomes "eternal".
- QM probabilities $\longrightarrow$ measure problem
- It is an IR phenomena, which happens on long distances and timescales.

- Related IR issues lead some to question even perturbative stability of dS space:

Polyakov `07, `09, `12...
Giddings and Sloth `11
Burgess et al `10
...

- Initial conditions? Microscopic description, string theory?
- We should not forget that currently the universe is accelerating again...



IR Dynamics of Light Fields.

- Let us focus on the issue of IR divergences
- Consider a light scalar field on rigid dS

$$\mathcal{L} = (\partial\varphi)^2 - V(\varphi) \qquad ds^2 = -dt^2 + e^{2Ht} d\vec{x}^{\,2}$$

$M_{pl} \to \infty$, $H = \text{const}$

e.g. $V(\varphi) \simeq m^2\varphi^2 + \lambda\varphi^4$

focus on $m^2 \ll H^2$, $\lambda \ll 1$ $\qquad (\varphi \neq \phi)$

- Our goal is to compute correlation functions of φ:

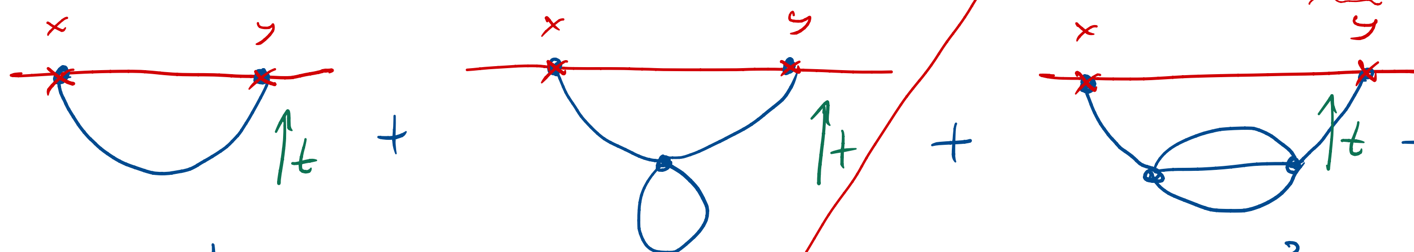
$$\langle \varphi(\vec{x}_1, t) \ldots \varphi(\vec{x}_n, t) \rangle \qquad \text{(equal } t \text{ first)}$$

at long distances, $a(t)\, x_{ij} \to \infty$

- Let us try to compute correlators perturbatively, as we would do in flat space.
- Of course, there are very similar diagrammatic techniques (Schwinger – Keldysh formalism):

$$\mathcal{L} = \partial\varphi^2 - \lambda\varphi^4 - m^2\varphi^2 :$$

$$m^2 \approx \sqrt{\lambda}\, H^2 \quad \Downarrow \quad m^2 \sim \lambda \Lambda_w^2$$

$\langle \varphi(x)\varphi(y)\rangle \simeq$  $+ \dots$

~ 1 $\qquad \sim \underbrace{\frac{\lambda}{m_P^4} \gg 1}$ $\qquad \sim \frac{\lambda^2}{m_P^8} \gg 1$

$$\left(\overset{k}{\bullet\!\!\longrightarrow\!\!\bullet} \sim \frac{a^{-m^2}}{k^{3-m^2}} \right)$$

- If mass is small enough, perturbation theory is badly divergent!

Baumgart, Sundrum `19

- This simple-looking problem did not have a systematic solution

- This regime is relevant for
  - primordial fluctuations (if φ is a spectator field), e.g. (Panagopoulos, Silverstein '19) for primordial BH's
  - PQ symetry breaking during inflation (if φ is an axion)
  - Stability of dS space
  - Slow roll eternal inflation (if φ is an inflaton)

Arkani-Hamed, Dubovsky et. al. '07, '08

- We developed a constructive "EFT-like" formalism to treat QFT in this regime.

VG, Senatore 1911.00022
(inspired by Starobinsky '84)

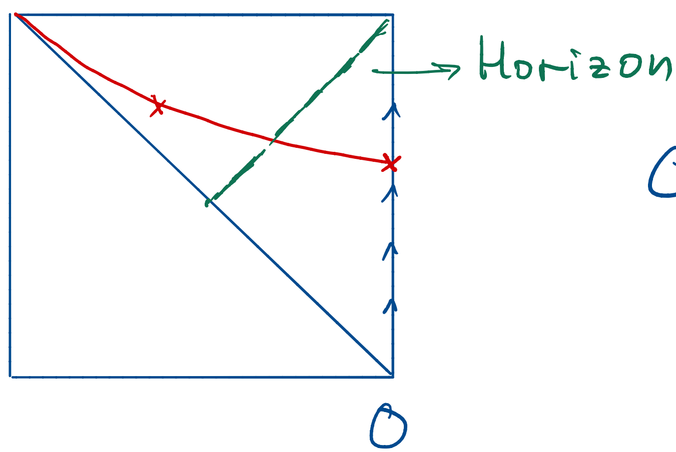
- The construction is a bit complex and it proceeds in several steps:

  1. Calculation of the wave function
  2. Separation of "long" and "short" modes
  3. Derivation of equations for probability distributions
  4. Solution of these equations allows to calculate correlation functions

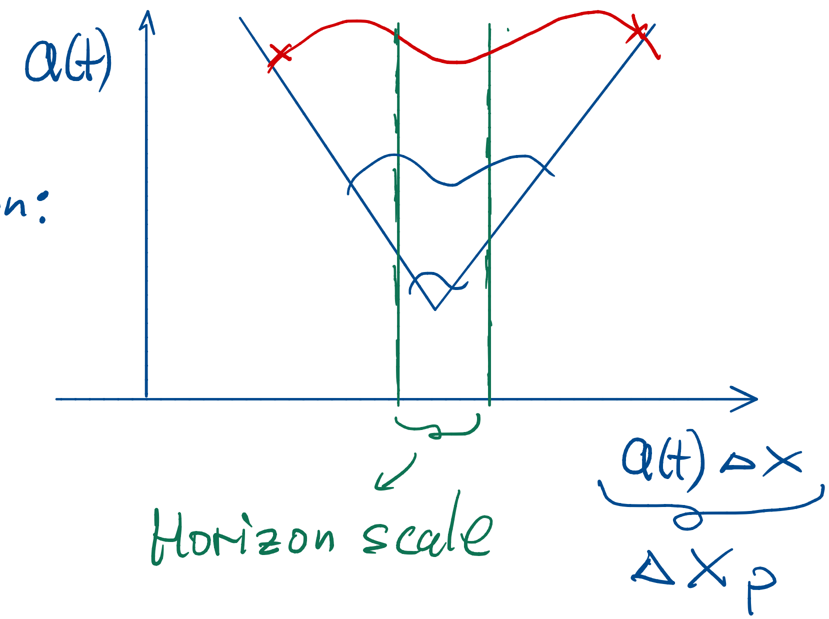
Basic facts about dS space (expanding part)

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 = \frac{-d\eta^2 + d\vec{x}^2}{H^2\eta^2}, \quad a(t) = e^{Ht} = \frac{-1}{H\eta}$$

Penrose:



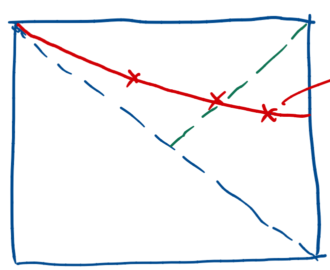
Cartoon:



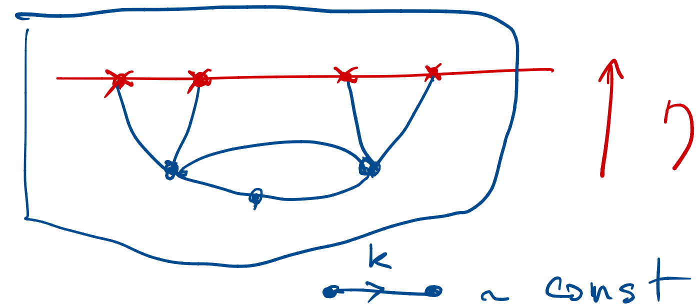
- Long modes: $a(t)\Delta X \gg H^{-1} \Leftrightarrow k \ll a(t) H$
- For $\Delta X \ll (aH)^{-1}$ looks like Minkowski space

The Wave Function of QFT in dS

- We will first compute correlators in a particular state, an analog of the Bunch-Davies state, and later show that at late times it is an attractor.



$\Psi_{BD}[\varphi, \eta]$



$\sim \text{const}_{k \to 0}$ vs $\frac{1}{k^3}$ $\Downarrow$ for corr.

- Ψ_{BD} does **not** suffer from IR divergences. For those familiar with AdS/CFT, it may appear natural due to the relation

$$\Psi_{BD}[\varphi, \eta] = Z_{EAdS}[\varphi, z] \Big|_{\substack{z = i\eta \\ L_{AdS} = i L_{dS}}}$$

- It can also be seen in a direct dS calculation.

$$\log \Psi_{BD}[\varphi,\eta] \underset{\eta\to 0}{\sim} \frac{i}{\eta^3}\int dx \left(V(\varphi) + V'(\varphi)^2 + \dots\right) + \frac{i}{\eta}\int \varphi\Delta\varphi + \dots$$

$$\underbrace{+\int dx\,dy\,\varphi(x)\varphi(y)\langle O_x O_y\rangle + \lambda\int dx_i\,\varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)\langle \prod_i^4 O_{x_i}\rangle + \dots}_{\log Z_{CFT}}$$

- We can obtain a meaningful perturbative expansion for Ψ_{BD}, assuming $\varphi \ll \lambda^{-\frac{1}{2}} H$
- Initial conditions are fixed by demanding

$$\Psi_{BD}[\varphi_k] \xrightarrow[\eta\to-\infty]{} \Psi_{Mink}[\varphi_k]$$

e.g.

Guth and Pi \`85
Anninos, Anous, Freedman, Konstantinidis \`14

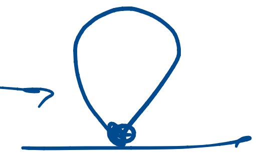
- Of course, we cannot just compute correlators from Ψ:

$$\langle \varphi(x_1)\dots\varphi(x_n)\rangle = \int D\varphi\, \Psi\Psi^*\varphi(x_1)\dots\varphi(x_n)$$

is still IR divergent

$$\langle \varphi \varphi \rangle = \int D\varphi\, \varphi \varphi^* = \int D\varphi\, e^{\varphi k^3 \varphi + k^3 \lambda \varphi^4}$$

$$\int \frac{d^3 k}{k^3}$$



- Instead, let us split $\varphi = \varphi_\ell + \varphi_s$

$$\varphi_\ell = \int_0^{\Lambda(t)} d^3k\, e^{ikx} \varphi_{\vec{k}} \,, \qquad \Lambda(t) = \varepsilon a(t) H \,, \qquad \varepsilon \ll 1 \,.$$

- $\Lambda(t)$ grows with time $\rightsquigarrow$ more modes become long
- Not surprisingly, long modes will give dominant contribution:

$$\varphi_\ell \sim \frac{H}{\sqrt{m}} + \frac{H}{\lambda^{1/4}} \gg \varphi_s \sim H \qquad \text{(to be checked later)}$$

- ε is similar to RG scale (e.g. Polchinski '84), it will cancel from all physical observables!
- We will chose $e^{-\frac{1}{\sqrt{\lambda}}} \ll \varepsilon \ll \sqrt{\lambda}$
- ε and λ will be our main expansion parameters.

- Next, define n-point distributions of long modes:

$$P_n(\varphi_1 \ldots \varphi_n; \vec{x}_{ij}, t) = \int D\varphi(\vec{x}) \prod_{i=1}^{n} \delta(\varphi_i - \varphi_\ell(\vec{x}_i))\, \Psi[\varphi, t]\, \Psi^*[\varphi, t]$$

fixed coordinate distance

only long modes

- P_n's generate correlators of φ_ℓ:

$$\langle \varphi_\ell(x_1) \ldots \varphi_\ell(x_n) \rangle = \int d\varphi_1 \ldots d\varphi_n\, \varphi_1 \ldots \varphi_n\, P_n(\varphi_1 \ldots \varphi_n, \vec{x}_{ij}, t)$$

- We still cannot compute them directly, but we can derive an equation which guides their time evolution:

$$\partial_t P_n(\varphi_1 \ldots \varphi_n; \vec{x}_{ij}, t) = \text{"Drift"} + \text{"Diffusion"}$$

$$\text{"Drift"} \to \partial_t \Psi\Psi^* \qquad \text{"Diffusion"} \to \delta(\varphi_i - \partial_t \varphi_\ell(\vec{x}_i))$$

$$\varphi_\ell = \int_0^{\Lambda(t)} d^3k\, e^{ikx} \varphi_{\vec{k}}$$

- Let us study in some detail the "Drift" term for the one-point distribution:

$$\partial_t \Psi\Psi^* = i a^{-3} \frac{\delta}{\delta\varphi}\left(\Psi^* \frac{\delta}{\delta\varphi}\Psi\right) + c.c. \qquad \text{(continuity eqn.)}$$

$$i a^{-3} \frac{\delta}{\delta\varphi_\ell}\Psi_{BD} \equiv \Gamma_\ell(\varphi)\Psi_{BD}, \qquad \Gamma_\ell(\varphi, x) = V'(\varphi(x)) + V'V'' + O(\lambda, \varepsilon)$$

The long part contributes

We use knowledge of the W.F.

gradients suppressed by ε^2

$$\log \Psi_{BD} \underset{\eta\to 0}{\sim} \frac{i}{\eta^3}\int dx \left(V(\varphi) + V'(\varphi)^2 + \ldots + \eta^2 \varphi \Delta \varphi\right)$$

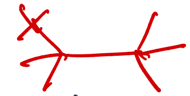
- We get:

$$P_1 \approx \int D\varphi\, \delta(\varphi_1 - \varphi_\ell(x_1)) \frac{\delta}{\delta\varphi_\ell}\left[\Gamma_\ell(\varphi)\Psi\Psi^*\right] \equiv \frac{\partial}{\partial\varphi_1}\left(\langle \Gamma_\ell(\varphi, x_1)\rangle_{\varphi_1} \cdot P_1(\varphi_1)\right)$$

exp. value, with fixed φ_1

$\Psi = \Psi_\ell + \Psi_s$

$$\langle \Pi_\ell(\varphi), x \rangle_{\varphi_1} \simeq \langle \lambda \varphi^3(x_1) \rangle_{\varphi_1} + \langle \lambda^2 \varphi^5(x_1) \rangle_{\varphi_1} =$$



$$= \underbrace{\lambda\varphi_1^3}_{\sim\lambda^{1/4}} + \underbrace{\lambda\varphi_1 \langle \varphi_s^2 \rangle_{\varphi_1} + \lambda^2 \varphi_1^5}_{\lambda^{3/4}\log\varepsilon} + \underbrace{\langle \int dx_2\, \lambda \varphi_\ell^3(x_2) \cdot \Omega_\Lambda(x_2 - x_1) \rangle_{\varphi_1} + \dots}_{\lambda^{5/4}\log\varepsilon}$$

$+ m^2 \varphi_1$

- $\langle \varphi_s^2 \rangle_{\varphi_1} \simeq \log\varepsilon \cdot H^2 + \log\varepsilon\, \lambda \varphi_1^2 + \dots \longrightarrow \lambda^{1/2} \log\varepsilon \ll 1 \longrightarrow \varepsilon \gg e^{-1/\sqrt{\lambda}}$

  $\varphi_\ell \sim \lambda^{-1/4} H \ll \lambda^{-1/2} H$ ✓

- Three long momenta can make short, we need to project.

$$\langle \int dx_2\, \lambda \varphi_\ell^3(x_2) \cdot \Omega_\Lambda(x_2 - x_1) \rangle_{\varphi_1} P_1(\varphi_1, t) =$$

"projector" on $k \le \Lambda(t)$

$$= \int d\varphi_2\, dx_2\, \Omega_\Lambda(x_{12}) \cdot P_2(\varphi_1, \varphi_2; x_{12}, t)$$

- Crucially, we never need to take non-trivial path integrals over long modes.

$\psi(x,t) \quad \langle x(t) \rangle = \int dx\, \psi^* \psi\, x$

$x \sim \varphi$

- To summarize, we get

$$\partial_t P_1(\varphi_1, t) = \frac{\partial}{\partial \varphi_1}\Big[\left(\lambda \varphi_1^3 + \lambda^2 \varphi_1^5 + \lambda \varphi_1 \log \varepsilon\right) P_1(\varphi_1, t) +$$

$$+ \int d\varphi_2\, dx\, \Omega_\Lambda(x) \cdot P_2(\varphi_1, \varphi_2; x, t)\Big] + \mathcal{O}(\lambda, \varepsilon) + \text{"Diffusion".}$$

- Derivation of the "Diffusion" term proceeds similarly, by carefully treating

$$\delta'\left(\varphi_i - \int^{\Lambda(t)} \varphi_k e^{ikx} dk\right)$$

- Also analagous steps lead us to the equation for all P_n's.

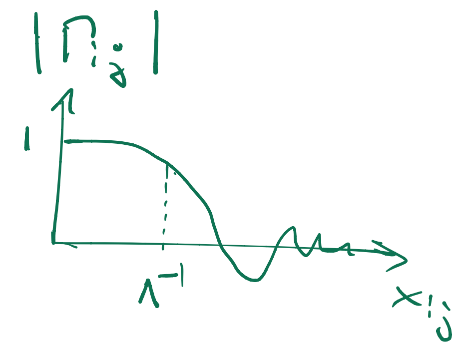
- Instead of going into further details, let us present the final equations, which determine all equal-time long-modes correlators.

Crucial building blocks are one- and two-field diff. operators Γ_i and Γ_{ij}:

$$\Gamma_i = \frac{\partial^2}{\partial \varphi_i^2} + \frac{\partial}{\partial \varphi_i} V'(\varphi_i) + \mathcal{O}(\lambda, \varepsilon)$$

"Diffusion" "Drift"

$$\Gamma_{ij} = \frac{\sin \varepsilon a x_{ij}}{\varepsilon a x_{ij}} \frac{\partial^2}{\partial \varphi_i \partial \varphi_j} + \mathcal{O}(\lambda, \varepsilon)$$



$$\partial_t P_1(\varphi_1, t) = \underline{\Gamma_1 P_1} + D_{12} P_2 + \dots \qquad D_{n\,n+1} \sim \int d\varphi_{n+1}\, \Omega\, P_{n+1} \sim \lambda$$

$$\partial_t P_2(\varphi_1, \varphi_2; x_{12}, t) = \underline{(\Gamma_1 + \Gamma_2 + \Gamma_{12}) P_2} + D_{23} P_3 + \dots$$

...

$$\partial_t P_n(\{\varphi_i\}; \{x_{ij}\}, t) = \underline{\left(\sum_i^n \Gamma_i + \sum_{i \neq j}^n \Gamma_{ij}\right) P_n} + D_{n\,n+1} P_{n+1} + \dots$$

- We also need initial conditions:

$$P_2(\varphi_1, \varphi_2, t)\Big| = P_1(\varphi_1) \cdot \delta(\varphi_1 - \varphi_2) \,,$$ and similarly for all P_n's

- At the leading order P_n only depends on P_k, $n<k$ $\rightsquigarrow$ we only need finite number of PDE's.

- Let us discuss how one can solve the above equations. First, we need to find Eigenvalues and Eigenfunctions of Γ:

$$\Gamma \Phi_n = \frac{\partial^2}{\partial \varphi^2} \Phi_n + \frac{\partial}{\partial \varphi}\left(V' \Phi_n\right) = -\lambda_n \Phi_n$$

e.g. $V' \simeq \lambda \varphi^3 + m^2 \varphi$. Unless the mass term dominates, it has to be done numerically, but this is just a 1d problem. For bounded potentials

$\lambda_0 = 0$, $\lambda_{n \geq 1} > 0$, e.g. $\lambda\varphi^4$: $\lambda_n \sim \sqrt{\lambda}$

$m^2\varphi^2$: $\lambda_n \sim m^2/H^2$

- One point distribution is time-independent:

$$P_1(\varphi_1) = \Phi_0 = e^{-\frac{V(\varphi_1)}{H^4}}$$

- To find two-point distribution we need to solve

$$\partial_t P_2(\varphi_1, \varphi_2; x_{12}, t) = \left(\Gamma_1 + \Gamma_2 + \Gamma_{12}\right) P_2(\varphi_1, \varphi_2; x_{12}, t)$$

This can be done by using "sudden" perturbation theory for Γ_{12}. At long distance one finds:

$$\langle \varphi(x_1)\varphi(x_2)\rangle \sim (a x_{12})^{-\lambda_1} \rightarrow \text{decays at large distances}$$

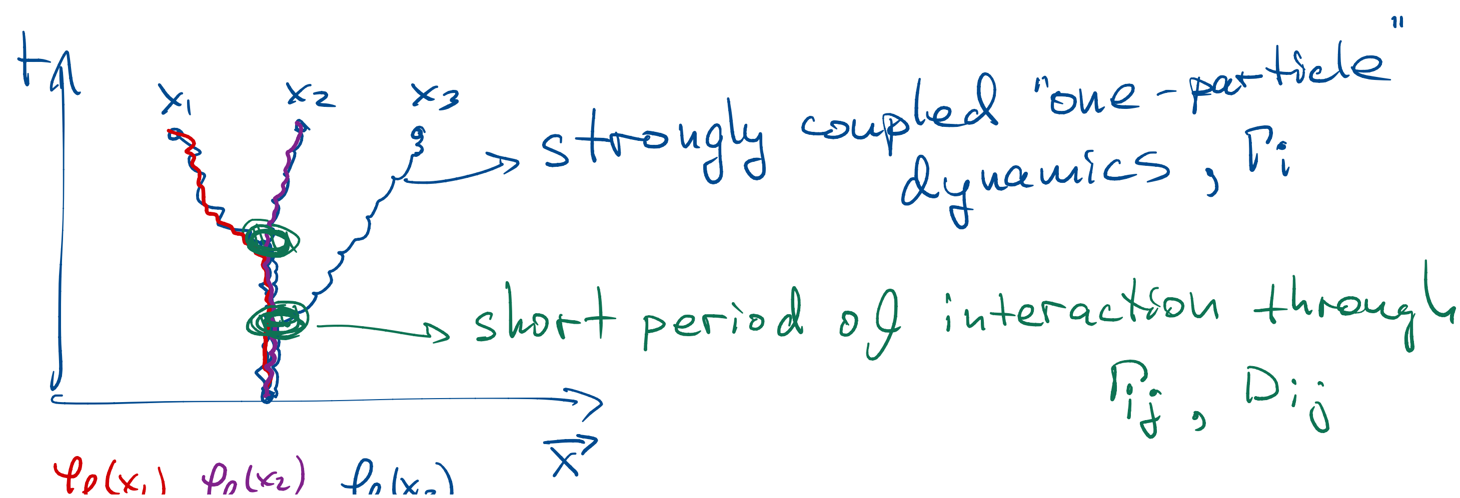
$\lambda_1 \sim \sqrt{\lambda}$ for λe^4

- Higher point functions have "conformal" form:

$$\langle \varphi(x_1)\varphi(x_2)\varphi^2(x_3)\rangle \sim \frac{C_{112}}{a x_{12}^{2\lambda_1-\lambda_2}\, a x_{13}^{\lambda_2}\, a x_{23}^{\lambda_2}},$$

$$C_{112} = \int d\varphi\, \Phi_1^2\, \Phi_2$$

- Let us summarize the "technical" part:
  - We derived an "EFT-like" description for long modes
  - It is given in terms of a hierarchically structured system of PDE's
  - Expansion is organized in the number of space-time points in which we fix the field
  - Each term in the PDE's can be derived from perturbation theory, using the wave function, and in principle, to any order in λ, ε.



$$\partial_t P_1 = \Gamma_1 P_1 + \lambda D_{12} P_2 + \dots \qquad \Gamma_{ij} \sim \frac{\sin \varepsilon a x_{ij}}{\varepsilon a x_{ij}} \xrightarrow[t\to\infty]{} 0$$

$$\partial_t P_2 = (\Gamma_1 + \Gamma_2 + \Gamma_{12}) P_2 + \lambda D_{23} P_3 + \dots$$

Several more comments:

- Non-equal time correlators can be computed in a similar way.
- All correlators are dS-invariant and decay at large separations.
- This shows that at late times correlators are state-independent (for states created by insertions of local operators): $|\Psi_0\rangle = O(t_0)|\Psi_{BD}\rangle$

$$\langle \Psi_0 | \varphi(x_1) \dots \varphi(x_n) | \Psi_0 \rangle = \langle \varphi(x_1) \dots \varphi(x_n) O(t_0) O^\dagger(t_0) \rangle \xrightarrow[t \gg t_0]{}$$

$$\to \langle \varphi(x_1) \dots \varphi(x_n) \rangle$$

- Leading eqn. agrees w. Starobinsky "stochastic" approach.
- Nothing is really "stochastic".
- There is also no classical saddle that dominates.

Explicit form of subleading corrections:

$\Gamma \tilde{\varphi}_n =$

$$\boxed{\left[-\frac{H^3}{8\pi^2}\frac{\partial^2}{\partial\phi'^2} + W_0(\phi') + W_1(\phi')\right]\tilde{\Phi}_n(\phi') = (\lambda_n + \delta\lambda_n)\,\tilde{\Phi}_n(\phi')\,,}$$

$O(\sqrt{\lambda}) \sim$

$$W_0(\phi') \equiv \frac{2\pi^2\lambda^2\phi'^6}{9H^5} - \frac{\lambda\phi'^2}{2H}\,,$$

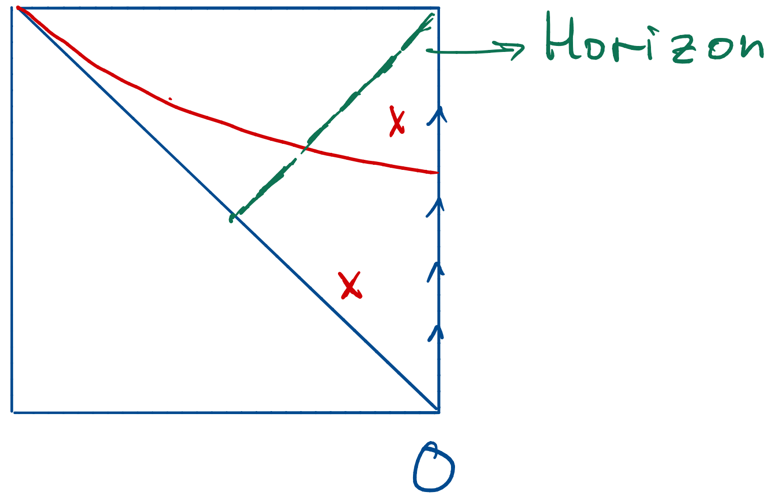
$O(\lambda) \sim$

$$W_1(\phi') \equiv \frac{4\pi^2\lambda^3\phi'^8}{27H^7} + \frac{4\pi^2\lambda\bar{m}^2\phi'^4}{9H^5} - \frac{5\lambda^2\phi'^4}{18H^3} - \frac{\bar{m}^2}{6H}\,.$$

physical mass

- Corrections to Eigenvalues can be computed as in time-independent QM perturbation theory.

Thermal properties of dS correlators:



Restricted to a single static patch correlators satisfy the KMS condition:

$$\boxed{\left\langle \hat{\phi}(\vec{x}_1, t_1)\hat{\phi}(\vec{x}_2, t_2 + i\beta)\right\rangle = \left\langle \hat{\phi}(\vec{x}_1, t_1)\hat{\phi}(\vec{x}_2, t_2)\right\rangle^\dagger}$$

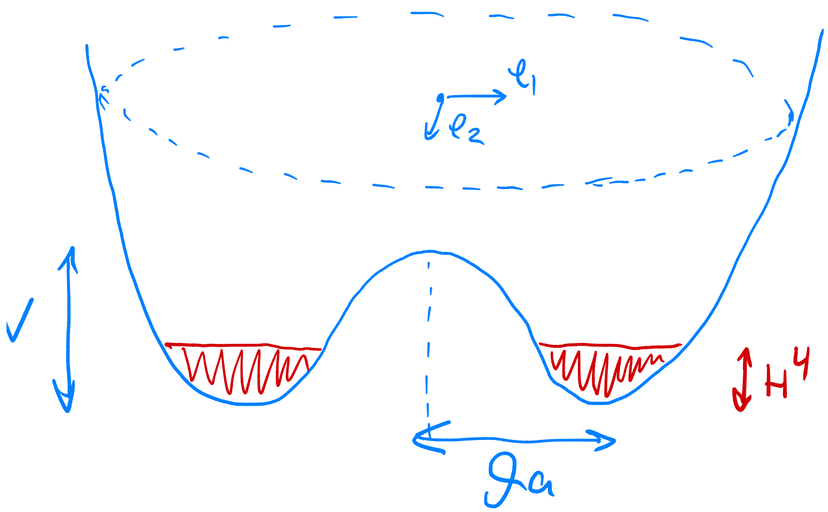
$\beta = 2\pi H^{-1}$

Applications: Spontaneous Symmetry Breaking

- There is no SSB in dS, and in particular no Goldstone bosons.

- What happens if we put an axion in dS?

$$V = \lambda(\varphi_i\varphi_i)^2 - m^2\varphi_i\varphi_i$$



Even if we take $f_a \gg H$, symmetry gets dynamically restored.

$$\Gamma \approx \left(\frac{\partial}{\partial\varphi_i}\right)^2 + \frac{\partial}{\partial\varphi_i}\left(\frac{\partial}{\partial\varphi_i} V(\varphi_1,\varphi_2)\right)$$

$$a x_{ij} \to \infty : \quad \langle \varphi_i(x_1)\,\varphi_j(x_2)\rangle \approx (a x_{12})^{-\frac{H^2}{f_a^2}} \cdot \delta_{ij}$$

Generalization: Gravitational Backreaction.

- How much does the story change when we turn on gravity?
- Our formalism still applies. Long-wavelength metric perturbations can have large amplitude but carry little energy.
- This is exactly the situation when our formalism works (and perturbation theory breaks down).
- We expect large observable effects for very shallow potential: $m^2 \ll \frac{H^4}{M_{pl}^2}$, $\sqrt{\lambda} \ll \frac{H^2}{M_{pl}^2}$

(Work in progress w. Senatore

Conclusion

- Problem of IR divergences is instrumental for our understanding of both inflationary perturbations and the global fate of cosmological spacetimes.

- We constructed a systematic framework suited to adress this problem for QFT on de Sitter space.

- The formalism is being applied to include gravity and the inflaton field.