Dark Matter and Hawking radiation:
Using BlackHawk to constrain Primordial Black Holes

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In collaboration with A. Arbey, I. Masina, G. Orlando, M. Geiller, E. R. Livine and F. Sartini

IAP seminar, Paris – January 11th, 2020

Introduction

BlackHawk

HR constraints on PBHs

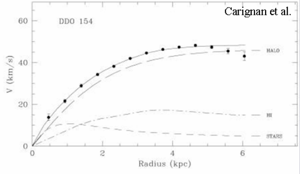
WDM constraints on PBHs

Alternative BH solutions and HR

Conclusion

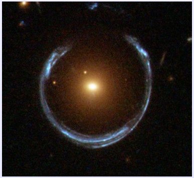
Introduction: The missing Dark Matter

- Galaxy rotation curves

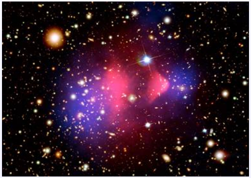


$$\rho_{\text{deduced}} \propto r^{-2} \gg \rho_{\text{stars}} \propto e^{-r/r_0}$$

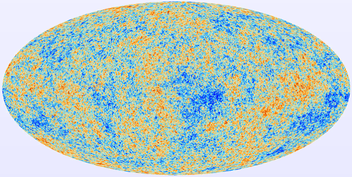
- Gravitational lensing



- Bullet cluster



- Planck data



$\implies \sim 26.8\%$ of Dark Matter

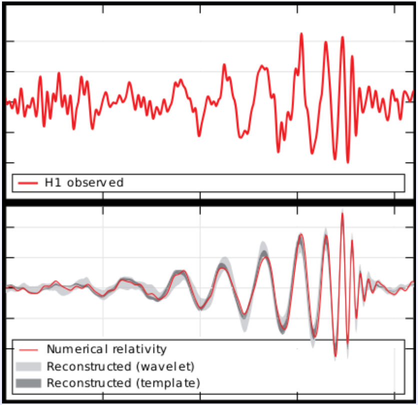
Introduction: The missing Dark Matter

Dark matter candidates

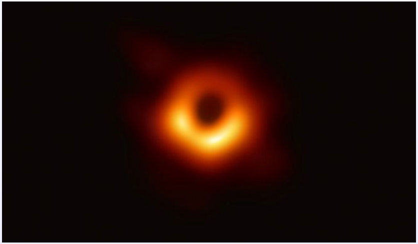
- **Massive neutrinos**
- **Weakly Interacting Massive Particles (WIMPs)**
  In particular, many particle physics models provide WIMP candidates!
- **Other particles/fields:** warm dark matter (WDM), axions, dark fluids, ...
  Exotic and non-baryonic particles
- **Black Holes**
  Not possible with stellar and supermassive black holes
- **Modified Gravitation Laws**
  MOND, TeVeS, Scalar-tensor theories, ...

Introduction: Primordial Black Holes

Renewed interest in Primordial Black Holes



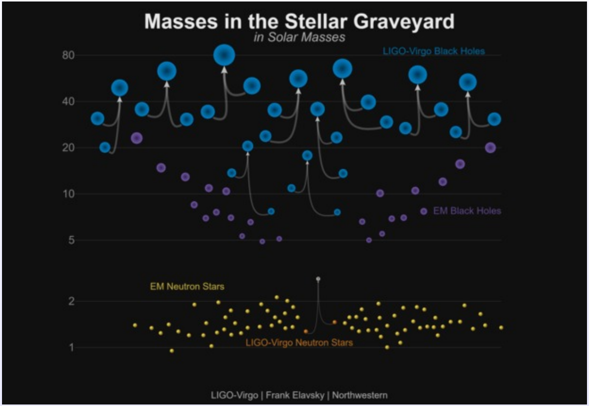
GW150914 - LIGO (2015)



M87 - EHT (2020)

Introduction: Primordial Black Holes

LIGO/VIRGO mergers



[status in Dec. 2018 after run O2]

LIGO/VIRGO BHs are surprisingly too light/heavy and their spin is close to 0

Introduction: Primordial Black Holes

Motivations

PBHs:

- don't require new physics/particles
- existence of BHs confirmed by X-ray and GW signals and shadow reconstruction
- hints of BHs too light/heavy for stellar origin (see in particular GW190521)
- unknown origin of the (seeds of the) supermassive BHs

Formation

Formation at the end of inflation when overdensities re-enter the Hubble horizon

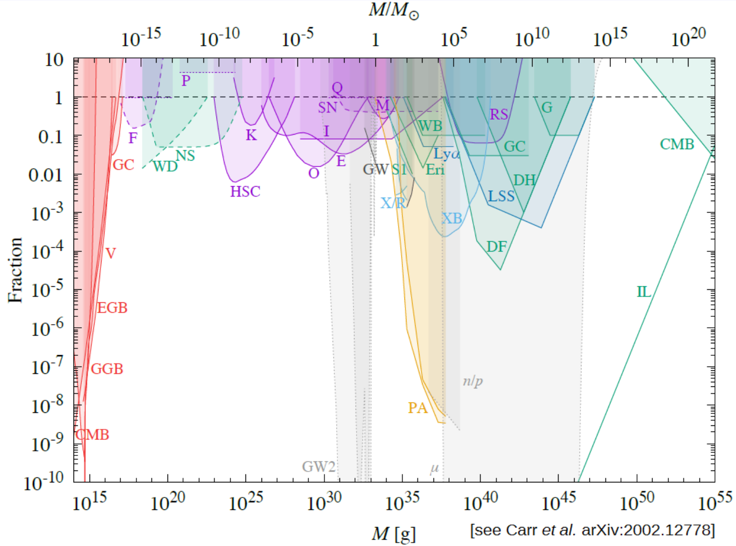
$$M_{\mathrm{PBH}}(t_0) \sim M_{\mathrm{P}} \times \frac{t_0}{t_{\mathrm{P}}} \sim 10^{38}\,\mathrm{g} \times \left(\frac{t_0}{1\,\mathrm{s}}\right) \tag{1}$$

Possible formation of BHs with smaller masses due to incomplete collapse or to other formation channels (1st-order phase transitions, cosmic strings/domain walls collapse, ...)

Spin distribution

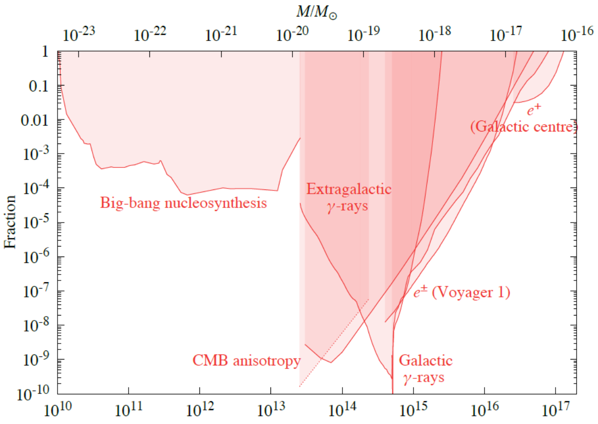
Low or high initial spin depending on radiation/matter domination at time of formation

Introduction: Primordial Black Holes



[see Carr *et al.* arXiv:2002.12778]

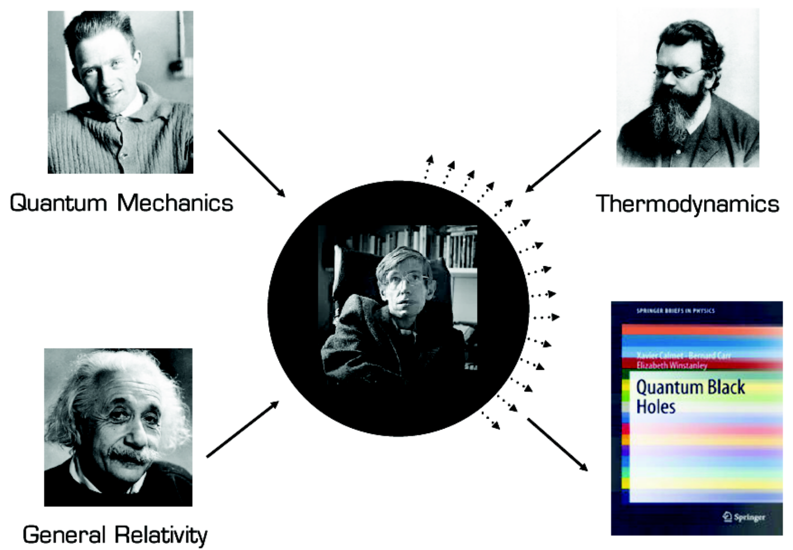
Introduction: Primordial Black Holes



[see Carr *et al.* arXiv:2002.12778]

- no constraints for $M_{\rm PBH} < 10^9\,$g
- these are all monochromatic constraints
- they deal with Schwarzschild (uncharged non-rotating) PBHs

Introduction: Hawking radiation

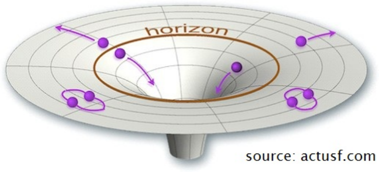


Quantum Mechanics

Thermodynamics

General Relativity

Introduction: Hawking radiation



source: actusf.com

"Thermal" emission of particles with temperature

$$T(M,a^*) = \frac{\kappa}{2\pi} \xrightarrow[a^*=0]{\text{Schwarzschild}} \frac{1}{8\pi M} \xrightarrow[a^*=1]{\text{Kerr extremal}} 0 \tag{2}$$

where κ is the surface gravity (metric influence) and rate for particle i

$$Q_i = \frac{\mathrm{d}^2 N_i}{\mathrm{d}t\mathrm{d}E} = \frac{1}{2\pi}\sum_{\text{dof.}} \frac{\Gamma_i(M,E,a^*)}{e^{E'/T(M,a^*)} \pm 1} \tag{3}$$

Γ_i is the greybody factor (see after)
E' is the energy corrected for horizon rotation
$\pm$ stands for fermions/bosons

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Why creating the public code BlackHawk?

What about Black Hole nature?

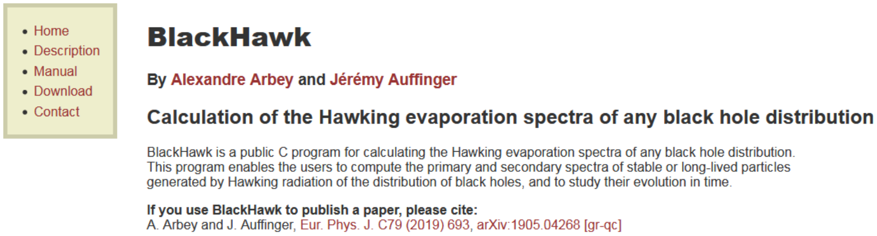
- nature of the horizon?
- nature of the singularity?
- (extended-)GR metric around a BH?
- thermodynamics and information?

Hawking radiation gives access to all of those aspects, and can be used to constrain the abundance of BHs in the remaining open windows for them to represent all of DM.

BlackHawk: General information

`BlackHawk v1.2`

- is open-source
- is written in `C`
- can be run on Linux, MacOS and Windows (using Cygwin)
- can be downloaded at `https://blackhawk.hepforge.org`



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BlackHawk

By Alexandre Arbey and Jérémy Auffinger

Calculation of the Hawking evaporation spectra of any black hole distribution

BlackHawk is a public C program for calculating the Hawking evaporation spectra of any black hole distribution. This program enables the users to compute the primary and secondary spectra of stable or long-lived particles generated by Hawking radiation of the distribution of black holes, and to study their evolution in time.

If you use BlackHawk to publish a paper, please cite:
A. Arbey and J. Auffinger, Eur. Phys. J. C79 (2019) 693, arXiv:1905.04268 [gr-qc]

BlackHawk content: BH distributions

Test distribution

A Dirac distribution for both mass and spin allows one to compute the spectrum of a single BH.

Realistic distributions

Several realistic extended distributions $n(M, a^*)$ for both mass and spin are implemented:

- log-normal
- power-law
- uniform
- ...

Users can provide their own BH distribution.

BlackHawk content: Greybody factors

Computation of the greybody factors

Outside `BlackHawk`, `Mathematica` scripts have been used to solve the Teukolsky equations of the propagation of particles for all spins and for Schwarzschild and Kerr BHs. We solve Schrödinger-like wave equations of the form

$$\frac{\mathrm{d}^2 Z}{\mathrm{d}r^{*2}} + \left(\omega^2 - V(r^*)\right) Z = 0 \tag{4}$$

where $V(r^*)$ is the potential barrier for Hawking radiation (determined by the BH metric) and ω is the particle energy. The greybody factors are defined as the ratio between on-horizon and infinity wave amplitudes

$$\Gamma \equiv \frac{|Z_{\mathrm{inf.}}|^2}{|Z_{\mathrm{hor.}}|^2} \tag{5}$$

Scripts & tables

The scripts and greybody factors tables are publicly available and are interesting *per se* (*e.g.* comparison between different metrics).

BlackHawk content: Evaporation Page coefficients

Evaporation equations

Evaporation of BHs results from the emission of particles with mass and angular momentum, reducing the mass and angular momentum of the BH as a reaction. The equations are

$$\frac{\mathrm{d}M}{\mathrm{d}t} = -\frac{f(M,a^*)}{M^2} \tag{6}$$

$$\frac{\mathrm{d}a^*}{\mathrm{d}t} = \frac{a^*(2f(M,a^*) - g(M,a^*))}{M^3} \tag{7}$$

where the Page coefficients f and g are obtained by integrating the emission rates

$$f(M,a^*) \equiv -M^2\frac{\mathrm{d}M}{\mathrm{d}t} = M^2\int_0^{+\infty}\sum_{\mathrm{dof.}}\frac{E}{2\pi}\frac{\Gamma(E,M,a^*)}{e^{E'/T}\pm 1}\mathrm{d}E \tag{8}$$

$$g(M,a^*) \equiv -\frac{M}{a^*}\frac{\mathrm{d}J}{\mathrm{d}t} = \frac{M}{a^*}\int_0^{+\infty}\sum_{\mathrm{dof.}}\frac{m}{2\pi}\frac{\Gamma(E,M,a^*)}{e^{E'/T}\pm 1}\mathrm{d}E \tag{9}$$

Scripts & tables

The scripts and Page factors are publicly available and interesting *per se* (*e.g.* BH lifetime computation).

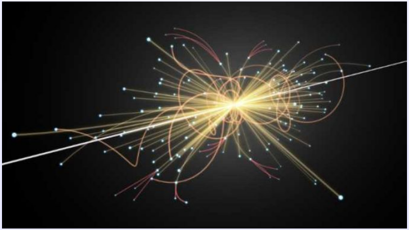
BlackHawk content: Hadronization

Secondary spectra

SM particles emitted by Hawking radiation must be hadronized/decayed following SM interactions. PYTHIA and HERWIG have been used to compute hadronization tables giving the energy distribution of secondary stable particles from primary SM particles.
There is a choice between "BBN stable" particles ($\tau \gtrsim 10^{-8}\,$s) and "cosmologically stable" particles ($\tau \to +\infty$).

Scripts & tables

The tables are computed for c.o.m. energies of $\gtrsim$ GeV and extrapolated at lower energies. The tables and scripts are publicly available and interesting *per se* (*e.g.* dark matter searches).



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Hawking radiation constraints: Principle

4 types of constraints

- local constraints: local PBH(DM) density, local final blow-up rate
- galactic constraints: galactic PBH(DM) density, bulge-integrated signal

$$\frac{\mathrm{d}F_i}{\mathrm{d}E} = \int \frac{\mathrm{d}\Omega}{4\pi} \int \left(\int_{M_{\mathrm{min}}}^{M_{\mathrm{max}}} \frac{\mathrm{d}n(l)}{\mathrm{d}M} \frac{\mathrm{d}^2 N_i}{\mathrm{d}t\mathrm{d}E} \mathrm{d}M \right) \mathrm{d}l \tag{10}$$

- extragalactic constraints: cosmological PBH(DM) density, redshift-integrated signal/background

$$I_i = E\frac{\mathrm{d}F_i}{\mathrm{d}E} = \frac{1}{4\pi} E \int_{t_{\mathrm{min}}}^{t_{\mathrm{max}}} (1+z) \left(\int_{M_{\mathrm{min}}}^{M_{\mathrm{max}}} \frac{\mathrm{d}n}{\mathrm{d}M} \frac{\mathrm{d}^2 N_i}{\mathrm{d}t\mathrm{d}E}((1+z)E)\mathrm{d}M \right) \mathrm{d}t \tag{11}$$

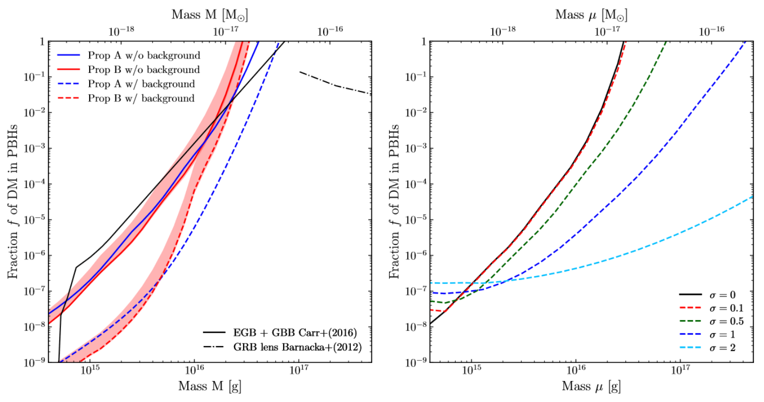
- cosmological constraints: PBH impact on cosmological observables (BBN, CMB), no direct link with DM

These constraints are sizeable only for *light* PBHs of $M_{\mathrm{PBH}} \lesssim 10^{18}\,$g, leaving an open window at $M_{\mathrm{PBH}} \gtrsim 10^{18}\,$g.

Hawking radiation constraints: Local constraints

Voyager 1 constraint

Voyager 1 has entered the interstellar medium; the spacecraft is impinged by $e^{\pm}$ cosmic rays. This sets a limit on the local evaporation rate of PBHs, thus on DM fraction.

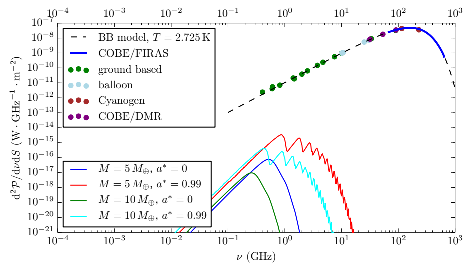


M. Boudaud and M. Cirelli [arXiv:1807.03075]

Hawking radiation constraints: Local constraints

Planet 9 constraint

Planet 9 (if it exists) might be a captured PBH of planetary mass. Its Hawking radiation could be measured by an *in situ* mission.



adapted from A. Arbey, JA [arXiv:2006.02944]

Dense DM cores

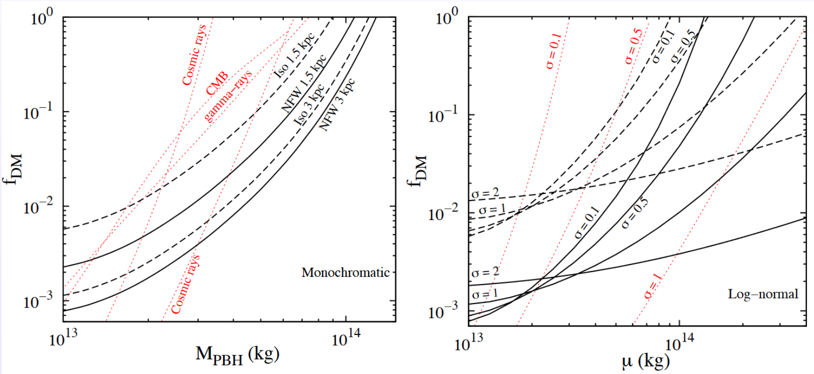
Dense DM cores could accumulate inside astrophysical objects (Sun, Earth, ...), collapse into BHs and then emit Hawking radiation.

J. F. Acevedo, J. Bramante, A. Goodman, J. Kopp and T. Opferkuch [arXiv:2012.09176]

Hawking radiation constraints: Galactic constraints

$e^{\pm}$ constraints

$e^{\pm}$ are emitted by Hawking radiation in the galactic bulge and annihilate in 511 keV photons (INTEGRAL).

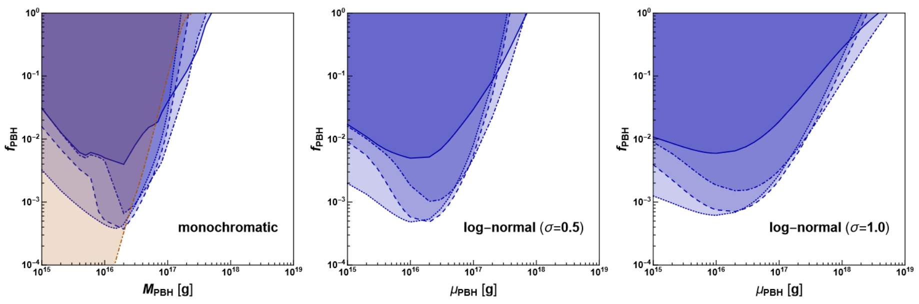


R. Laha [arXiv:1906.09994]

Hawking radiation constraints: Galactic constraints

$e^{\pm}$ constraints

$e^{\pm}$ are emitted by Hawking radiation in the galactic center and annihilate in 511 keV photons (INTEGRAL) [with the effect of PBH spin].

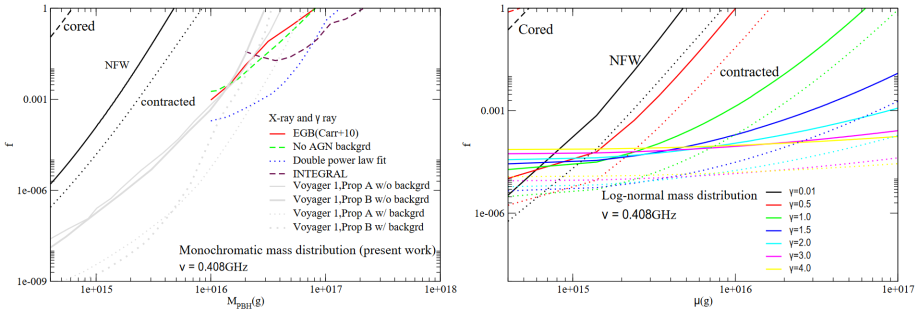


B. Dasgupta, R. Laha and A. Ray [arXiv:1912.01014]

Hawking radiation constraints: Galactic constraints

$e^{\pm}$ constraints

Charged particles produce radio synchrotron radiation due to the strong magnetic field at galactic center (archive radio data from the 70s).

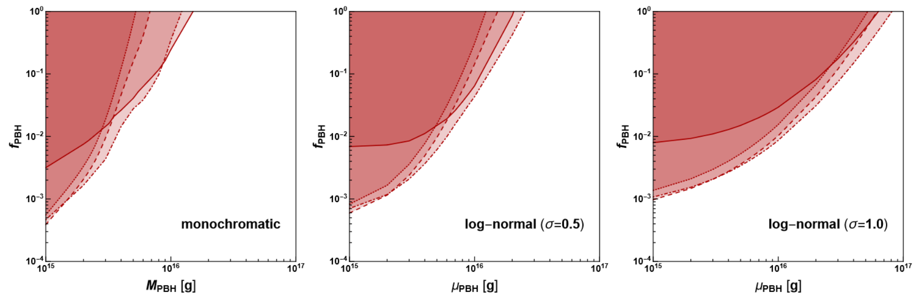


M. H. Chan and C. H. Lee [arXiv:2007.05677]

Hawking radiation constraints: Galactic constraints

Neutrino constraints

$\nu/\bar{\nu}$ are emitted by Hawking radiation in the galactic bulge and propagate in straight line to Earth where they can be detected by Super-Kamiokande [with the effect of PBH spin].

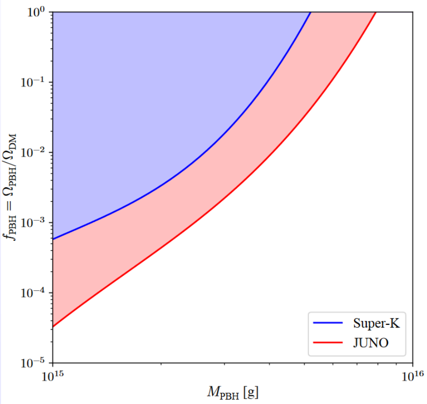


B. Dasgupta, R. Laha and A. Ray [arXiv:1912.01014]

Hawking radiation constraints: Galactic constraints

Neutrino constraints

$\nu/\bar{\nu}$ are emitted by Hawking radiation in the galactic bulge and propagate in straight line to Earth where they can be detected by (future) JUNO.

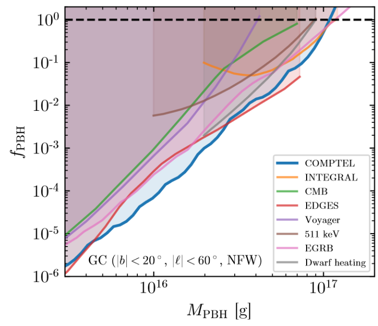


S. Wang, D.-M. Xia, X. Zhang, S. Zhou, and Z. Chang [arXiv:2010.16053]

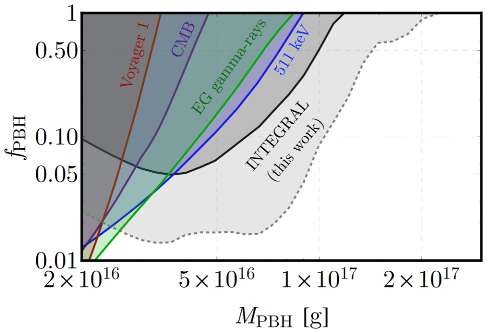
Hawking radiation constraints: Galactic constraints

Gamma-ray constraints

PBHs evaporate to soft gamma-rays in the galaxy, measured by (archival) COMPTEL or INTEGRAL.



A. Coogan, L. Morrison and S. Profumo [arXiv:2010.04797]

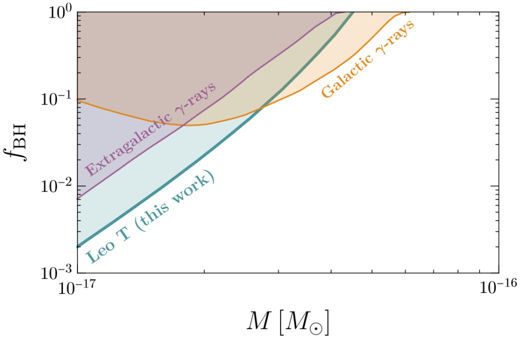


R. Laha, J. B. Muñoz and T. R. Slatyer [arXiv:2004.00627]

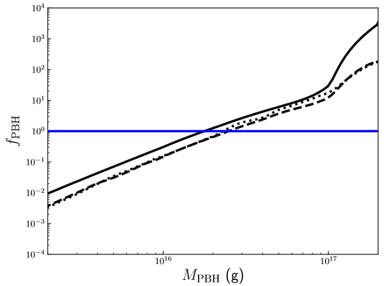
Hawking radiation constraints: Extragalactic constraints

Temperature constraints

PBHs evaporating in dense media such as Leo T dwarf galaxy heat and ionize the interstellar medium [with the effect of PBH spin].



H. Kim [arXiv:2007.07739]

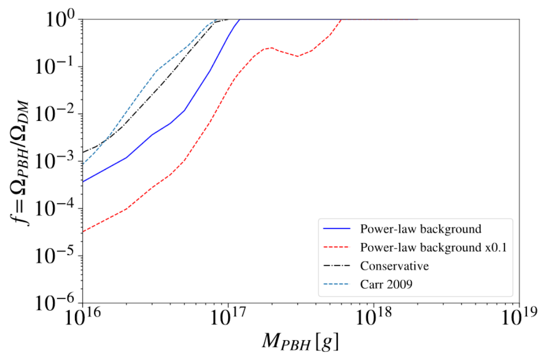


R. Laha, P. Lu, and
V. Takhistov [arXiv:2009.11837]

Hawking radiation constraints: Extragalactic constraints

X-ray constraints

$M_{\rm PBH} \geq 10^{16}$ g PBHs evaporate to soft gamma-rays and X-rays, leading to a isotropic background, which adds up to the AGN (and other sources) background that could be probed soon.

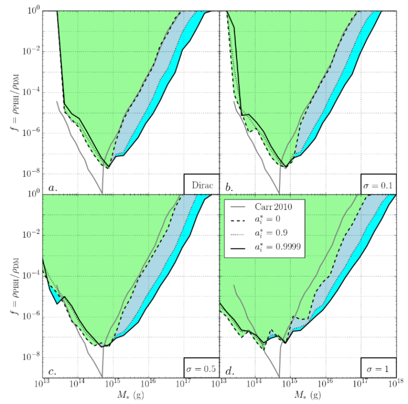


G. Ballestros, J. Coronado-Blázquez and D. Gaggero [arXiv:1906.10113]

Hawking radiation constraints: Extragalactic constraints

Gamma-ray constraints

$M_{\rm PBH} \geq 10^{13}$ g PBHs evaporate to gamma-rays, leading to a isotropic background, which adds up to the AGN (and other sources) background measured *e.g.* by FERMI-LAT.

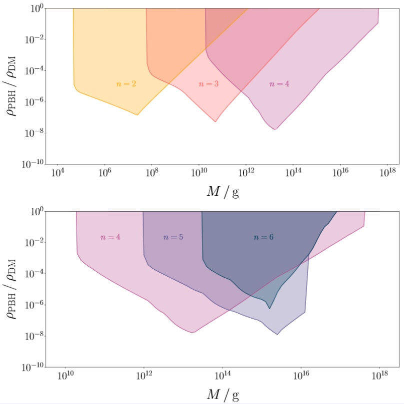


A. Arbey, JA and J. Silk [arXiv:1906.04750]

Hawking radiation constraints: Extragalactic constraints

Gamma-ray constraints

PBHs in higher-dimensional $(4+n)$D space-time are cooler, and thus live longer than 4D PBHs. The gamma-ray constraints are modified depending on n.

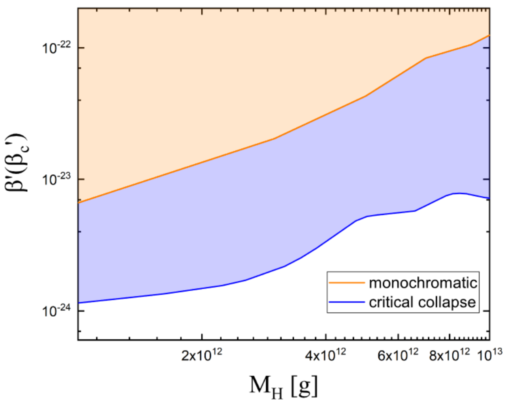


J. Johnson [arXiv:2005.07467]

Hawking radiation constraints: Cosmological constraints

BBN constraints

$M_{\rm PBH} = 10^9 - 10^{13}\,$g PBHs evaporate to hard gamma-rays and energetic particles that can photo-dissociate light elements, altering BBN abundances.



Y. Luo, C. Chen, M. Kusakabe, and T. Kajino [arXiv:2011.10937]

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Warm Dark Matter constraints on Primordial Black Holes

In collaboration with I. Masina and G. Orlando [arXiv:2012.09867]

Context

- No direct constraint on $M_{\rm PBH} \lesssim 10^9\,$g PBHs (although gravitational astronomy)
- Ultra-light PBHs evaporating before BBN could generate all DM through HR
- Different channel of production than the thermal one → WDM
- Depending on fraction of Universe β collapsed into PBHs, they can come to dominate the energy density

$$\rho_{\rm rad.} \propto a^{-4} \qquad \text{whereas} \qquad \rho_{\rm PBH} \propto a^{-3} \tag{12}$$

Early matter domination if

$$\beta \geq \bar{\beta} = \left(\frac{3f(M_{\rm PBH})}{\gamma}\right)^{1/2} \frac{M_{\rm Pl}}{M_{\rm PBH}} \tag{13}$$

where $f(M_{\rm PBH})$ determines the life-time of PBHs in the evolution formula and γ is the ratio of the Hubble horizon content collapsing to PBHs.

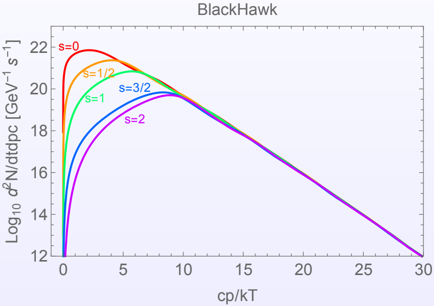
Warm Dark Matter constraints on Primordial Black Holes

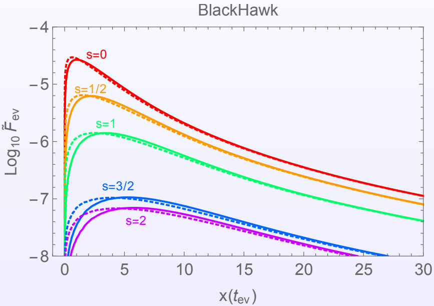
Strategy

Choose a fraction β of PBHs
Choose a DM candidate (spin)
$\downarrow$
Compute the DM yield
[`BlackHawk`]
$\downarrow$
Integrate to obtain momentum distribution at evaporation
Take redshift into account (PBH/R domination)
$\downarrow$
Determine DM mass (fixed DM density = today)
Determine transfer function for structure formation
[`CLASS`]
$\downarrow$
Obtain constraints on β

Warm Dark Matter constraints on Primordial Black Holes

Results





Instantaneous distribution

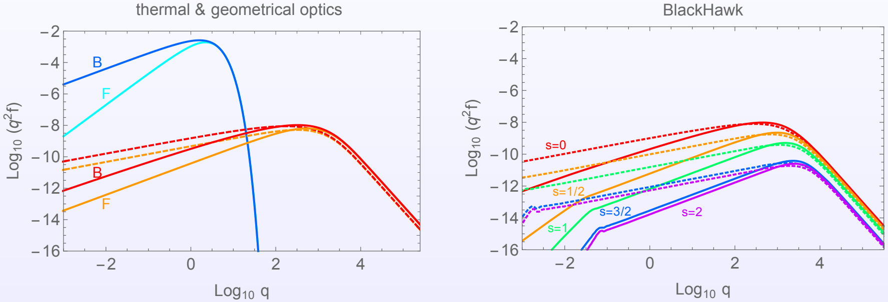
Integrated distribution at evaporation
Solid (dashed): Radiation (PBH) domination

JA, I. Masina and G. Orlando [arXiv:2012.09867]

Warm Dark Matter constraints on Primordial Black Holes

Results

JA, I. Masina and G. Orlando [arXiv:2012.09867]



Dimensionless DM momentum distributions (input to `CLASS`) at PBH evaporation, compared to thermal decoupling.

	Boson	Fermion	$s = 0$	$s = 1/2$	$s = 1$	$s = 3/2$	$s = 2$
$\bar{m}_{\mathrm{BH}} c^2/\mathrm{MeV}$	0.114	0.076	0.112	0.155	0.344	2.28	2.59
$\bar{m}_{\mathrm{R}} c^2/\mathrm{MeV}$	0.086	0.057	0.084	0.116	0.259	1.71	1.94

WDM mass for fixed DM density today, for $\beta_{\mathrm{BH}} = 1$ and $\beta_{\mathrm{R}} = \bar{\beta}$.

Warm Dark Matter constraints on Primordial Black Holes

Conclusions

- necessity to go beyond geometrical optics approximation and thus use `BlackHawk`
- necessity to take the DM spin into account
- WDM emitted during PBH-domination era is excluded for all spins
- WDM could still be a possibility if emitted by PBHs during radiation-dominated era with β small enough
- necessity to use precise structure formation constraints with `CLASS`

Hawking radiation by spherically-symmetric and static black holes for all spins

In collaboration with A. Arbey, M. Geiller, E. R. Livine and F. Sartini [arXiv:2101.02951]

Context

- No clear theory of the nature of BH horizon (information paradox, ...)
- Precise measure of ringdown signals gives access to quasi-normal modes (QNMs)
- QNMs and HR are both sensitive to horizon structure/BH metric
- Prediction of modified signals/constraints due to alternative BH solutions

We restrict ourselves to spherically symmetric and static BHs

$$\mathrm{d}s^2 = -G(r)\mathrm{d}t^2 + \frac{1}{F(r)}\mathrm{d}r^2 + H(r)\mathrm{d}\Omega^2 \tag{14}$$

which are asymptotically flat

$$F(r) \underset{r\to+\infty}{\longrightarrow} 1 \qquad G(r) \underset{r\to+\infty}{\longrightarrow} 1 \qquad H(r) \underset{r\to+\infty}{\sim} r^2 \tag{15}$$

Hawking radiation by spherically-symmetric and static black holes for all spins

Strategy

Choose a metric
(charged BHs, (4+n)D BHs, polimerized BHs)
↓
Determine the equations of motion of particles
Compute the effective potentials for Schrödinger wave equations
↓
Compute the greybody factors
[`Mathematica`]
↓
Compute the Hawking radiation
Determine modified constraints/signals
[`BlackHawk`]
↓
Conclude on the distinguishability of alternative BH metrics

Hawking radiation by spherically-symmetric and static black holes for all spins

Results

We develop the Newman-Penrose equations of motion of massless spin 0, 1, 2, 1/2 and 3/2 particles to obtain a generalized decoupled radial Teukolsky equation

$$A_s\left(B_s\Phi_s'\right)' + \left(\omega^2 + i\omega s\sqrt{\frac{F}{G}}\left(\frac{GH'}{H} - G'\right) + C_s\right)\Phi_s = 0 \tag{16}$$

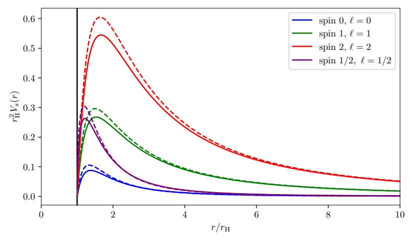
Then we Chandrasekhar transform this equation to obtain a Schrödinger-like wave equation with short-ranged potentials suited for numerical integration

$$\frac{\mathrm{d}^2 Z_s}{\mathrm{d}r^{*2}} + \left(\omega^2 - V_s\left(r(r^*)\right)\right)Z_s = 0 \tag{17}$$

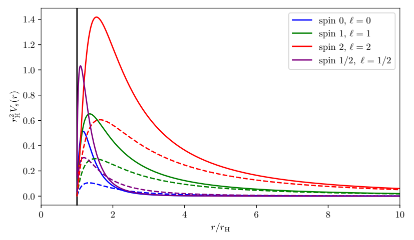
We obtain the general potentials

$$\begin{aligned} V_0 &= \frac{\nu_0 G}{H} + \frac{1}{\sqrt{H}}\frac{\mathrm{d}^2\sqrt{H}}{\mathrm{d}r^{*2}} & V_1 &= \frac{\nu_1 G}{H} \\ V_2 &= \frac{\nu_2 G}{H} + \frac{1}{2H^2}\left(\frac{\mathrm{d}H}{\mathrm{d}r^*}\right)^2 - \frac{1}{\sqrt{H}}\frac{\mathrm{d}^2\sqrt{H}}{\mathrm{d}r^{*2}} & V_{1/2} &= \frac{\nu_{1/2} G}{H} \pm \sqrt{\nu_{1/2}}\frac{\mathrm{d}}{\mathrm{d}r^*}\left(\sqrt{\frac{G}{H}}\right) \end{aligned} \tag{18}$$

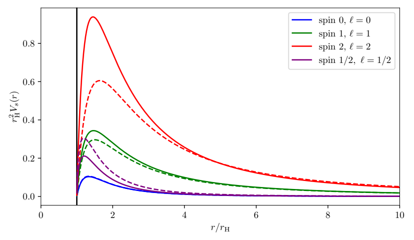
Hawking radiation by spherically-symmetric and static black holes for all spins



Potentials for charged BH ($Q = 2M/3$)



Potentials for $(4 + n)$D BH ($n = 2$)



Potentials for polymerized BH ($\epsilon = 0.8$)

Conclusion and perspectives

- `BlackHawk` is a versatile tool to compute signals and constraints linked to Hawking radiation

`https://blackhawk.hepforge.org`

- constraints come from local, galactic, extragalactic and cosmological observations
- mixed models of PBHs and DM can be probed
- alternative GR can be tested

Thank you for your attention!

Backup

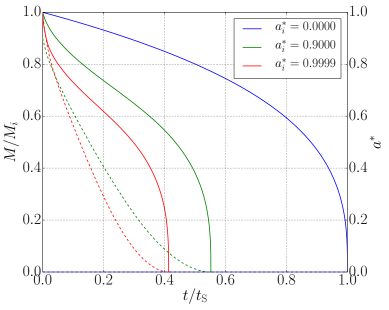
Backup

Backup: BH evolution

Evolution equations

$$\frac{\mathrm{d}M}{\mathrm{d}t} = -\frac{f(M, a^*)}{M^2} \tag{19}$$

$$\frac{\mathrm{d}a^*}{\mathrm{d}t} = \frac{a^*(2f(M, a^*) - g(M, a^*))}{M^3} \tag{20}$$

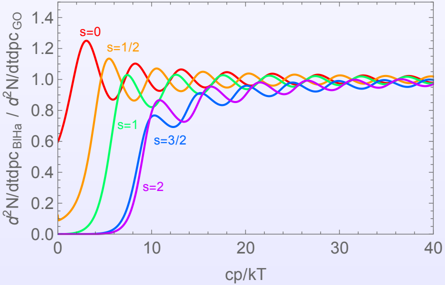


A. Arbey, JA and J. Silk [arXiv:1906.04196]

Backup: HR cross-sections

Hawking radiation cross-section for all spins

$$\sigma_s \underset{E\to 0}{\propto} E^{\alpha_s} \qquad \sigma_s \xrightarrow[E\to+\infty]{} \text{const.} \qquad \text{[GO]} \tag{21}$$

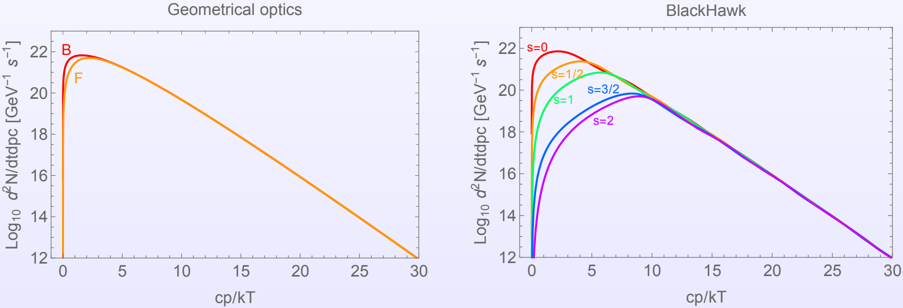


JA, I. Masina and G. Orlando [arXiv:2012.09867]

Backup: HR emission rates

Emission rates for all spins

$$\frac{\mathrm{d}^2 N_s}{\mathrm{d}t\mathrm{d}E} = \frac{1}{2\pi}\frac{\Gamma_s(M,E,a^*)}{e^{E'/T(M,a^*)} - (-1)^{2s}} \tag{22}$$

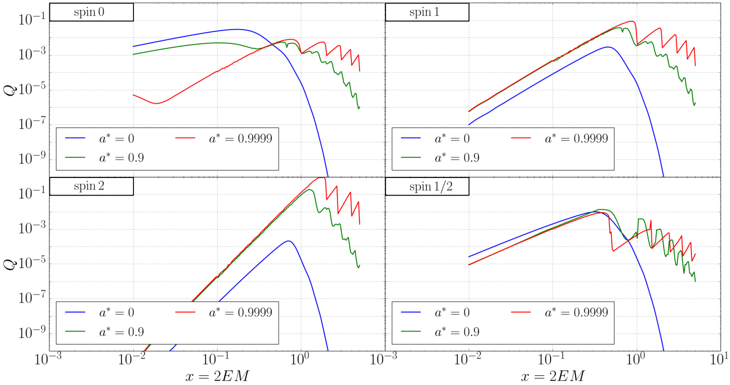


JA, I. Masina and G. Orlando [arXiv:2012.09867]

Backup: HR emission rates (Kerr)

Emission rates for all spins (rotating BHs)

$$a^* \equiv a/M \equiv J/M^2 \qquad (23)$$



Backup: Kerr metric and potentials

Field equations + Kerr metric

$$\begin{aligned}\text{Dirac}:\ & (i\partial\!\!\!/ - \mu)\psi = 0 \ \text{(fermions)}\\ \text{Proca}:\ & (\Box + \mu^2)\phi = 0 \ \text{(bosons)}\end{aligned}$$

$$\begin{aligned} d\tau^2 =& (dt - a\sin^2\theta d\phi)^2\,\frac{\Delta}{\Sigma} - \left(\frac{dr^2}{\Delta} + d\theta^2\right)\Sigma \\ & - ((r^2 + a^2)d\phi - adt)^2\,\frac{\sin^2\theta}{\Sigma}\end{aligned}$$

Teukolsky radial equation

$$\frac{1}{\Delta^s}\frac{\mathrm{d}}{\mathrm{d}r}\left(\Delta^{s+1}\frac{\mathrm{d}R}{\mathrm{d}r}\right) + \left(\frac{K^2 + 2i\,s(r-M)K}{\Delta} - 4i\,sEr - \lambda_{slm} - \mu^2 r^2\right)R = 0$$

Change of variables

$R \to Z$ and $r \to r^*$ defined by

$$\frac{\mathrm{d}r^*}{\mathrm{d}r} = \frac{\rho^2}{\Delta} \implies r^*(r) = r + \frac{r_{\mathrm{H}}r_+ + am/E}{r_+ - r_-}\ln\left(\frac{r}{r_+} - 1\right) - \frac{r_{\mathrm{H}}r_- + am/E}{r_+ - r_-}\ln\left(\frac{r}{r_-} - 1\right)$$

Schrödinger-like wave equation

$$\frac{\mathrm{d}^2 Z}{\mathrm{d}r^{*2}} + \left(E^2 - V(r^*)\right)Z = 0$$

$V(r^*)$ spin-dependent Chandrasekhar-Detweiler potentials (1970's)

Backup: Kerr metric and potentials

Chandrasekhar-Detweiler potentials

$$V_0(r) = \frac{\Delta}{\rho^4}\left(\lambda_{0\,lm} + \frac{\Delta + 2r(r-M)}{\rho^2} - \frac{3r^2\Delta}{\rho^4}\right)$$

$$V_{1/2,\pm}(r) = (\lambda_{1/2\,lm}+1)\frac{\Delta}{\rho^4} \mp \frac{\sqrt{(\lambda_{1/2,l,m}+1)\Delta}}{\rho^4}\left((r-M) - \frac{2r\Delta}{\rho^2}\right)$$

$$V_{1,\pm}(r) = \frac{\Delta}{\rho^4}\left((\lambda_{1\,lm}+2) - \alpha^2\frac{\Delta}{\rho^4} \mp i\alpha\rho^2\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{\Delta}{\rho^4}\right)\right)$$

$$\begin{aligned} V_2(r) = \frac{\Delta}{\rho^8}\Bigg(q - \frac{\rho^2}{(q-\beta\Delta)^2}\bigg(&(q-\beta\Delta)\left(\rho^2\Delta q'' - 2\rho^2 q - 2r(q'\Delta - q\Delta')\right) \\ &+\rho^2(\kappa\rho^2 - q' + \beta\Delta')(q'\Delta - q\Delta')\bigg)\Bigg) \end{aligned}$$

where $\rho^2 \equiv r^2 + \alpha^2$ and $\alpha^2 \equiv a^2 + am/E$ and

$$q(r) = \nu\rho^4 + 3\rho^2(r^2 - a^2) - 3r^2\Delta$$

$$q'(r) = r\left((4\nu+6)\rho^2 - 6(r^2 - 3Mr + 2a^2)\right)$$

$$q''(r) = (4\nu+6)\rho^2 + 8\nu r^2 - 6r^2 + 36Mr - 12a^2$$

$$\beta_\pm = \pm 3\alpha^2$$

$$\kappa_\pm = \pm\sqrt{36M^2 - 2\nu(\alpha^2(5\nu+6) - 12a^2) + 2\beta\nu(\nu+2)}$$

Backup: Spherically symmetric and static metrics

$$\mathrm{d}s^2 = -G(r)\mathrm{d}t^2 + \frac{1}{F(r)}\mathrm{d}r^2 + H(r)\mathrm{d}\Omega^2 \tag{24}$$

Charged BHs (Reissner-Nordström)

$$F = G = 1 - \frac{r_\mathrm{S}}{r} + \frac{r_\mathrm{Q}^2}{r^2}\,, \quad H = r^2 \tag{25}$$

$$r_\mathrm{H} = r_\mathrm{S}\,\frac{1 + \sqrt{1 - 4r_\mathrm{Q}^2/r_\mathrm{S}^2}}{2} \tag{26}$$

where $r_\mathrm{Q}^2 \equiv Q^2$ is the charge of the BH

Higher-dimension BHs $(4+n)$D

$$F = G = 1 - \left(\frac{r_\mathrm{H}}{r}\right)^{n+1}\,, \quad H = r^2 \tag{27}$$

$$r_\mathrm{H} = \frac{1}{\sqrt{\pi}M_*}\left(\frac{M}{M_*}\right)^{1/(n+1)}\left(\frac{8\Gamma\left((n+3)/2\right)}{n+2}\right)^{1/(n+1)} \tag{28}$$

where $M_\mathrm{P}^2 \sim M_*^{n+2}R^n$ defines the fundamental mass scale of the theory

Polimerized (LQG) BHs – triad-connection method

$$F = (r - r_+)(r - r_-)r^4/(r + r_*)^2(r^4 + a_0^2) \tag{29}$$

$$G = (r - r_+)(r - r_-)(r + r_*)^2/r^4 + a_0^2 \tag{30}$$

$$H = r^2 + a_0^2/r^2 \tag{31}$$

$$r_+ = 2m \equiv r_\mathrm{H} \tag{32}$$

$$r_- = 2mP^2 \tag{33}$$

$$r_* = \sqrt{r_+ r_-} \tag{34}$$

where a_0 is the area gap of loop quantum gravity, $P = (\sqrt{1+\epsilon^2} - 1)/(\sqrt{1+\epsilon^2} + 1)$ is the so-called polymeric function, and the parameter m is related to the so-called ADM mass M by $M = m(1+P)^2$