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Compact binary dynamics in the Effective Field Theory approach

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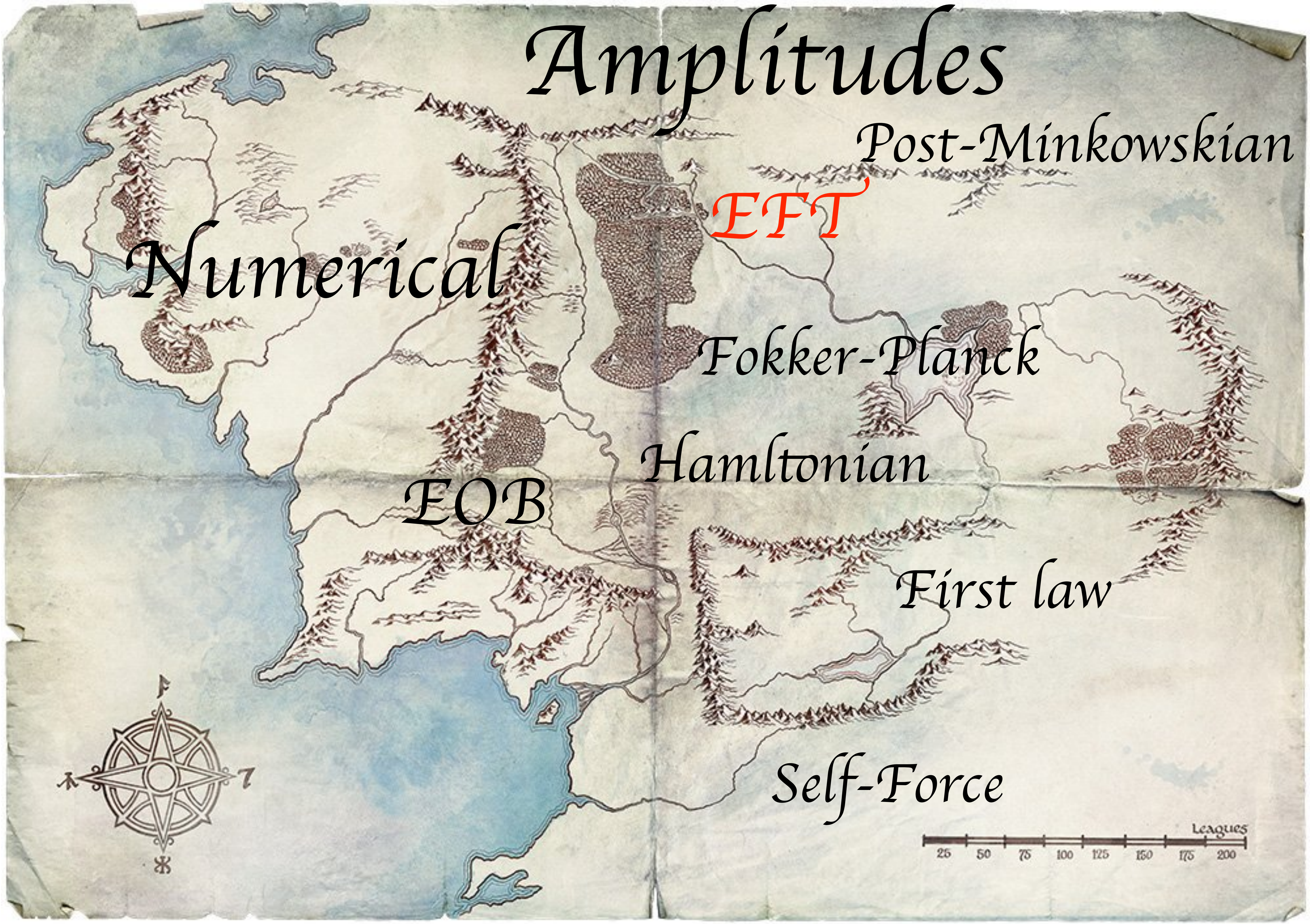
Post-Minkowskian
Post-Newtonian
Leagues
25 50 75 100 125 150 175 200


Amplitudes
Post-Minkowskian
EFT
Numerical
Fokker-Planck
Hamltonian
EOB
First law
Self-Force
Leagues
25 50 75 100 125 150 175 200

Outline

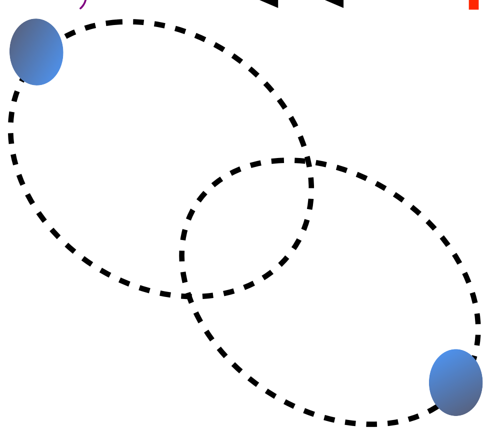
- EFT principles and results
  - diagrammatic expansion, method of regions
  - near zone
  - far zone, tails and memory

- High PN logs
  - relation between logs and power emission
  - leading logs from renormalisation group

Length scales in a binary system: a double hierarchy

r <<* $\lambda_{GW} \simeq \frac{r}{v}$

$l_{BH,NS}$ <<** r



$$S_{m_a} \simeq S^a_{pp} = -m_a \int \sqrt{-g_{\mu\nu} \mathrm{d}x^\mu_a \mathrm{d}x^\nu_a} = -m_a \int \mathrm{d}t \sqrt{-g_{\mu\nu} \dot{x}^\mu_a \dot{x}^\nu_a}$$

$a = 1, 2$

up to 5PN
no spin

*during the inspiral phase

**during the inspiral phase and for compact objects

Integrating out the gravitational field

$$S_g = \frac{1}{32\pi G}\int \mathrm{d}^{(d+1)}x\sqrt{-g}\left[R - g_{\mu\nu}g^{\alpha\beta}g^{\gamma\delta}\Gamma^{\mu}_{\alpha\beta}\Gamma^{\nu}_{\gamma\delta}\right]$$

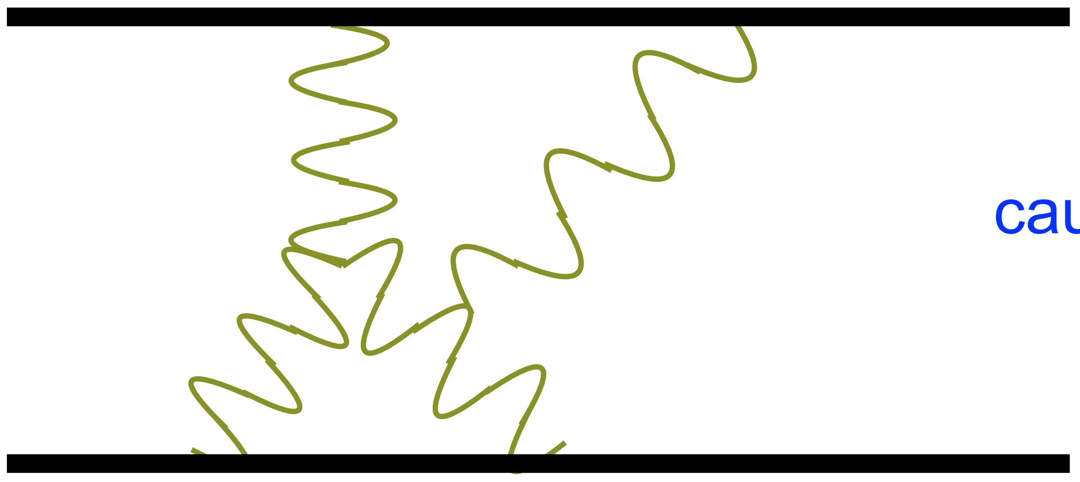
gauge fixing term

$$S_{tot}[x,h] = S_g[h] + S_m[x,h]$$

$$\mathrm{e}^{iS_{eff}[x]} = \int \mathcal{D}[h]\mathrm{e}^{iS_{tot}[x,h]}$$

perturbative expansion in

$$\frac{GM}{r}\ (\simeq v^2)$$



with causal boundary conditions

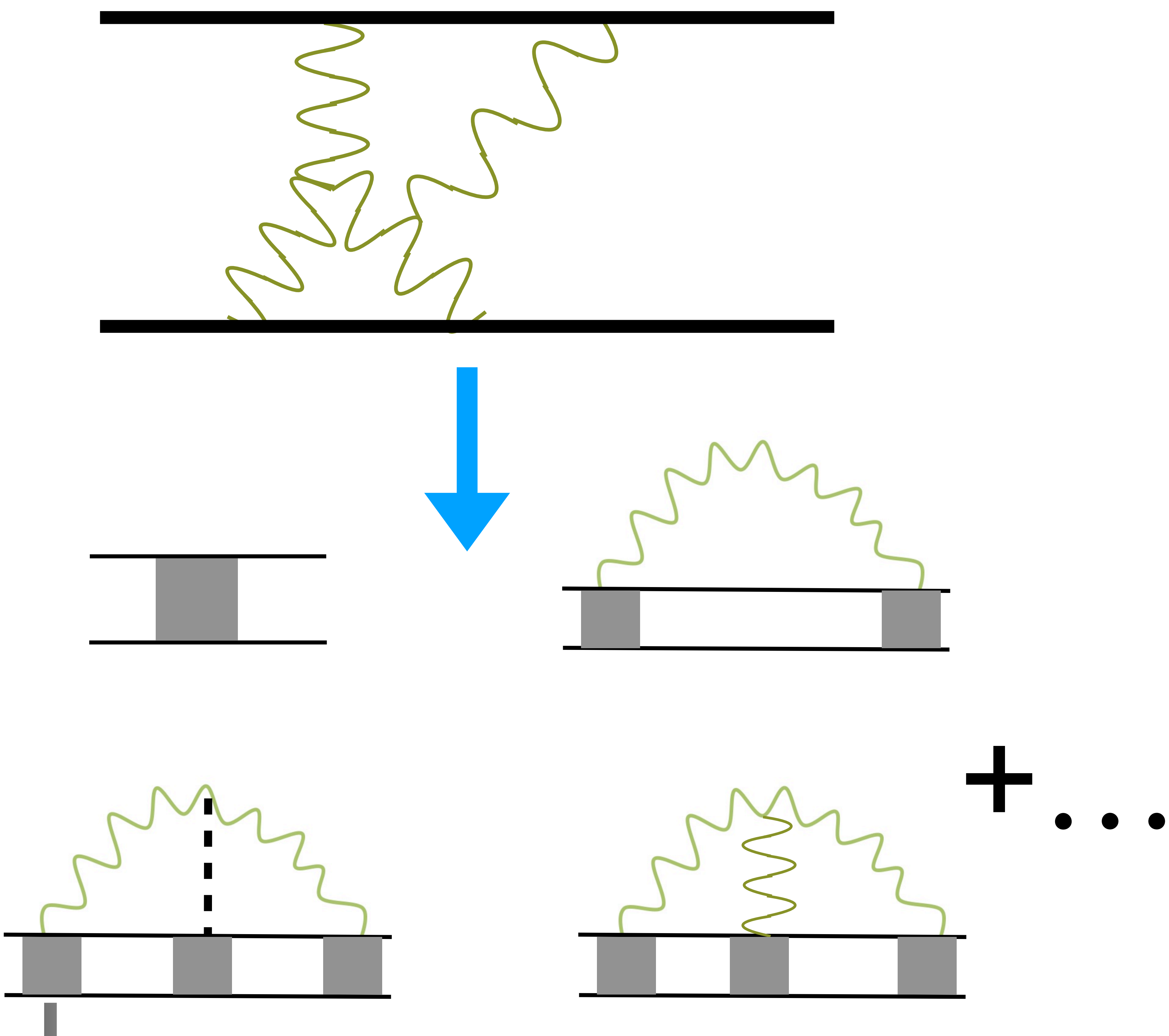
r $\ll$ $\lambda_{GW} \simeq \dfrac{r}{v}$

potential (quasi-static) modes $k \simeq \dfrac{1}{r}$ $\gg$ $\omega_{gw} \simeq \dfrac{v}{r} \simeq k_{gw}$ radiation modes

Method of regions: near zone

$$\frac{i}{\mathbf{k}^2 - k_0^2} \longrightarrow \frac{i}{\mathbf{k}^2} \sum_{n \geq 0} \left(\frac{k_0^2}{\mathbf{k}^2} \right)^n$$

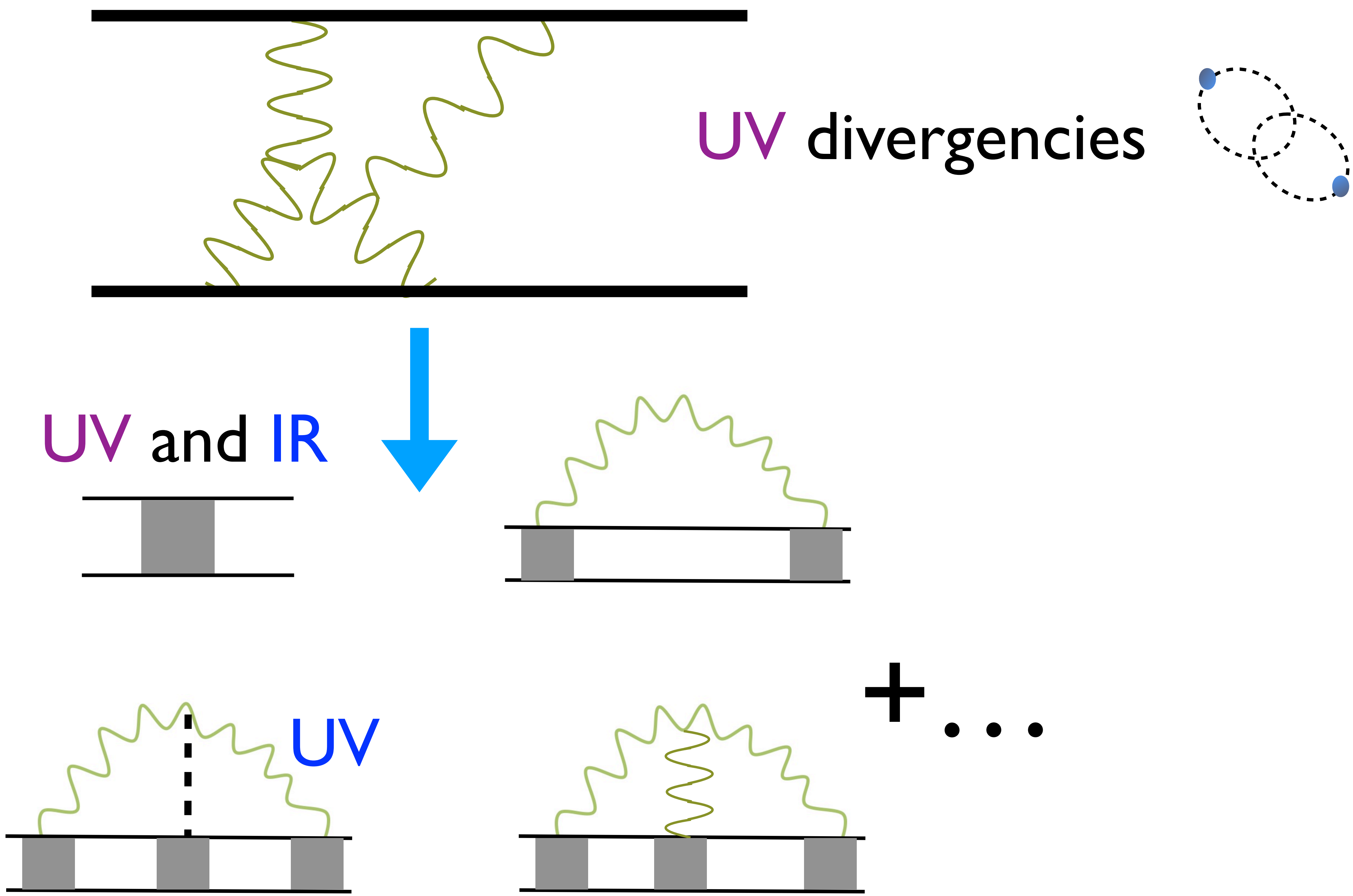
Method of regions



$$S_m[x,h] = -\int \mathrm{d}\tau \left[m + \frac{1}{2}\mathcal{S}_{\mu\nu}\Omega^{\mu\nu} - \frac{1}{2} I_{ij}E^{ij} + \frac{2}{3} J_{ij}B^{ij} + c_E E^{ij}E_{ij} + \ldots \right.$$

multipole expansion and matching

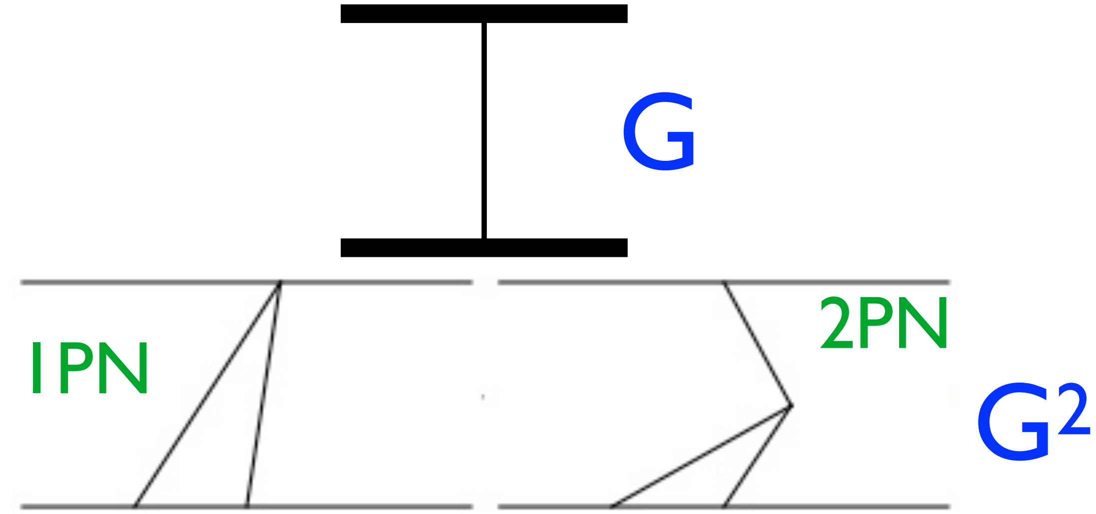
Method of regions

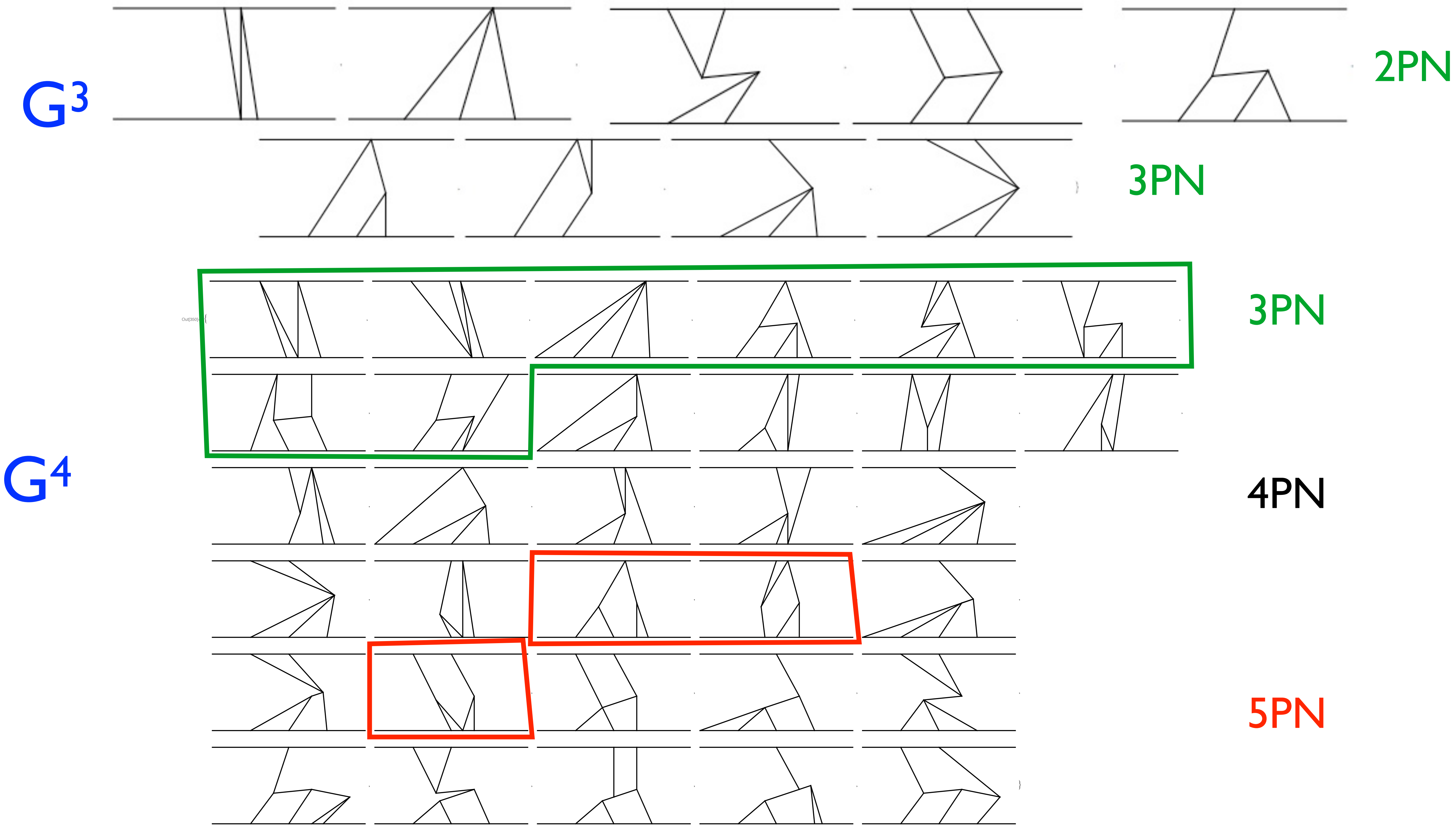


$$S_m[x,h] = -\int \mathrm{d}\tau \Big[m + \frac{1}{2}\mathcal{S}_{\mu\nu}\Omega^{\mu\nu} - \frac{1}{2} I_{ij}E^{ij} + \frac{2}{3} J_{ij}B^{ij} + c_E E^{ij}E_{ij} + \ldots$$

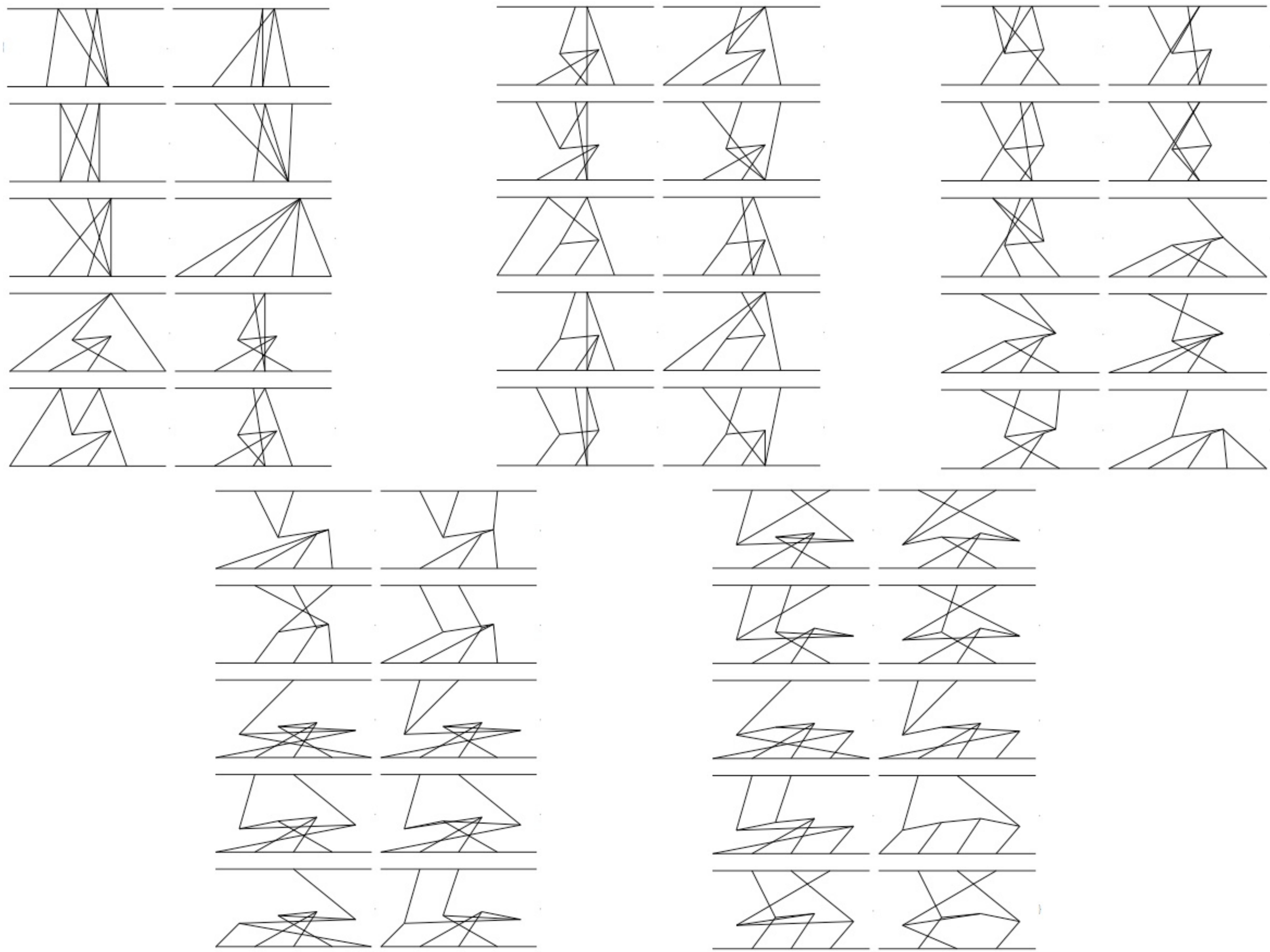
multipole expansion and matching

605 diagrams to evaluate at 4PN





G^5 (50 relevant at 4PN, out of 164)



KK variables

$$c_d = \frac{2(d-1)}{d-2}$$

$$m_p \equiv \frac{1}{\sqrt{32\pi G}}$$

$$g_{\mu\nu} = e^{2\phi}\begin{pmatrix} -1 & A_j \\ A_i & e^{-c_d\phi}(\delta_{ij}+\sigma_{ij}) - A_iA_j \end{pmatrix}$$

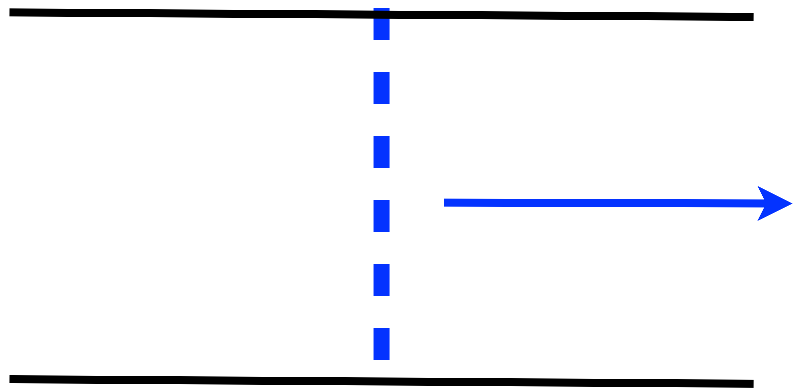
$$\phi \;\; A_i \;\; \sigma_{ij} \longrightarrow \frac{\phi \;\; A_i \;\; \sigma_{ij}}{m_p}$$

$$S_{pp}^{a} = -m_a\int d\tau_a = -m_a\int dt\; e^{\phi/m_{Pl}}\sqrt{\left(1-\frac{A_i}{m_{Pl}}v_a^i\right)^2 - e^{-c_d\phi/m_{Pl}}\left(v^2+\frac{\sigma_{ij}}{m_{Pl}}v_a^i v_a^j\right)}$$

$$\begin{aligned} S_g \simeq \int \mathrm{d}^{d+1}x\sqrt{-\gamma}\Bigg\{&\frac{1}{4}\left[(\vec\nabla\sigma)^2 - 2(\vec\nabla\sigma_{ij})^2 - (\dot\sigma^2 - 2(\dot\sigma_{ij})^2)\,\mathrm{e}^{\frac{-c_d\phi}{m_{Pl}}}\right] - c_d\left[(\vec\nabla\phi)^2 - \dot\phi^2\mathrm{e}^{-\frac{c_d\phi}{m_{Pl}}}\right] \\ &+\left[\frac{F_{ij}^2}{2} + \left(\vec\nabla\cdot\vec A\right)^2 - \dot{\vec A}^2\mathrm{e}^{-\frac{c_d\phi}{m_{Pl}}}\right]\mathrm{e}^{\frac{c_d\phi}{m_{Pl}}} + 2\frac{\left[F_{ij}A^i\dot A^j + \vec A\dot{\vec A}(\vec\nabla\cdot\vec A)\right]\mathrm{e}^{\frac{c_d\phi}{m_{Pl}}} - c_d\dot\phi\vec A\vec\nabla\phi}{m_{Pl}} \\ &+2c_d\left(\dot\phi\vec\nabla\cdot\vec A - \dot{\vec A}\vec\nabla\phi\right) + \frac{1}{m_{Pl}}\left[-\dot\sigma A_i\hat\Gamma^i_{jj} + 2\dot\sigma_{ij}\left(A_k\hat\Gamma^k_{ij} - A_i\hat\Gamma^j_{kk}\right)\right] - c_d\frac{\dot\phi^2\vec A^2}{m_{Pl}^2} \\ &+\frac{1}{m_{Pl}}\sigma^{ij}\left(\frac{1}{2}\sigma_{kl,i}\sigma^{kl}_{,j} + \sigma_{ik,l}\sigma_j^{k,l} + \sigma_{ik,l}\sigma_j^{l,k} - \sigma^k_{i,k}\sigma^l_{j,l} + \sigma_{,i}\sigma^k_{j,k} - \frac{1}{2}\sigma_{ij,k}\sigma^{,k} - \sigma_{ik,j}\sigma^{,k} - \frac{1}{4}\sigma_{,i}\sigma_{,j}\right) \\ &+\frac{1}{2m_{Pl}}\sigma\left(\frac{1}{4}\sigma_k\sigma k + \sigma^{ki}_{,i}\sigma^{,j}_{kj} - \sigma_{ki,j}\sigma^{kj,i} - \frac{1}{2}\sigma_{ki,j}\sigma^{ki,j}\right)\Bigg\} + \mathcal{O}(5PN) \end{aligned}$$

Newton's potential derived in momentum space, and to use dimensional regularization

$$-i\sqrt{32\pi G}m_2 + \mathcal{O}(v_2^2)$$



$$\frac{i}{8(\partial_t^2 - p^2)} = \frac{-i}{8p^2} + \mathcal{O}\left(\frac{\partial_t^2}{p^4}\right)$$

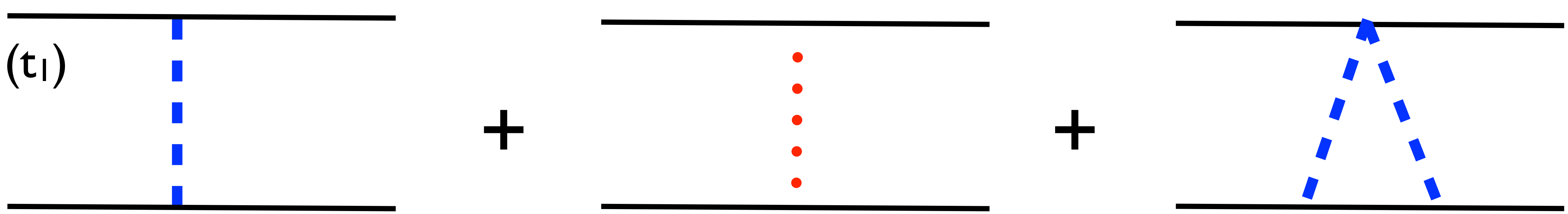
$$-i\sqrt{32\pi G}m_1 + \mathcal{O}(v_1^2)$$

$$V = i\int \frac{\mathrm{d}^3 p}{(2\pi)^3}(-i\sqrt{32\pi G}m_1)(-i\sqrt{32\pi G}m_2)\frac{-i}{8p^2}\mathrm{e}^{ip.(x_1 - x_2)}$$

1PN:

$x_1(t_1)$

$x_2(t_2)$

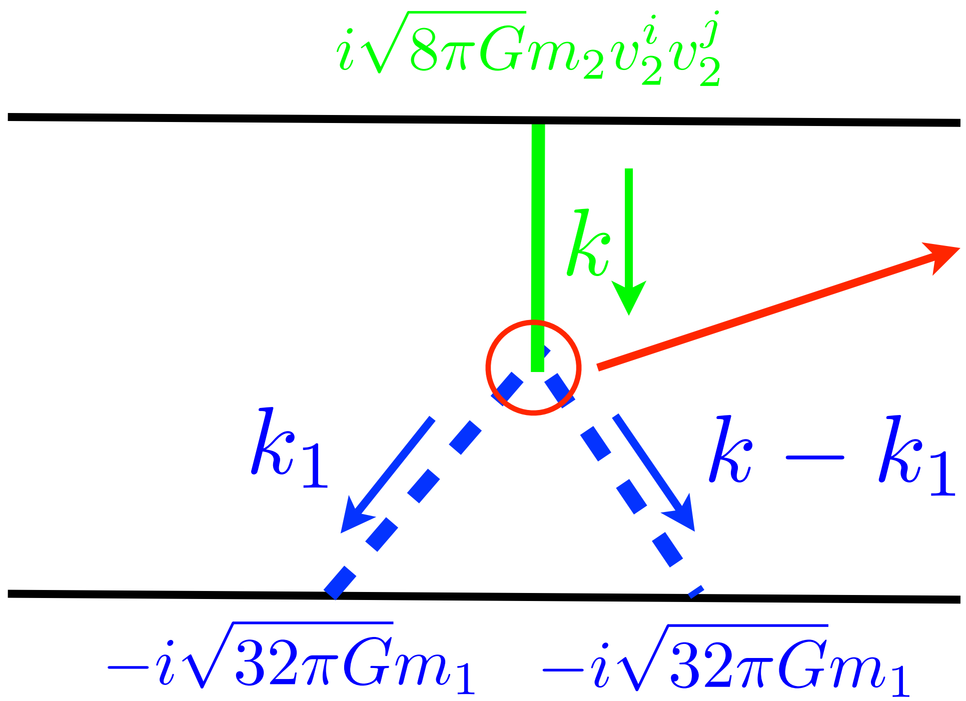


$\mathcal{O}(v^2)$ + $\mathcal{O}(v^2)$ + $\mathcal{O}(1)$

Total 1PN:

$$\mathcal{L}_{EIH} = -\frac{G^2 M m_1 m_2}{r^2} + \frac{G m_1 m_2}{2r}\left[-v_{1r}v_{2r} + 3\left(v_1^2 + v_2^2\right) - 7\vec{v_1}\cdot\vec{v_2}\right]$$

One example at $\mathcal{O}(G^2)$



$$-c_d\sqrt{\gamma}\left[\left(\vec{\nabla}\phi\right)^2 - \dot{\phi}^2\right] \simeq -c_d\left(1-\frac{\sigma}{2m_{Pl}}\right)\left(\delta_{ij}+\frac{\sigma_{ij}}{m_{Pl}}\right)\phi^{,i}\phi^{,j}$$

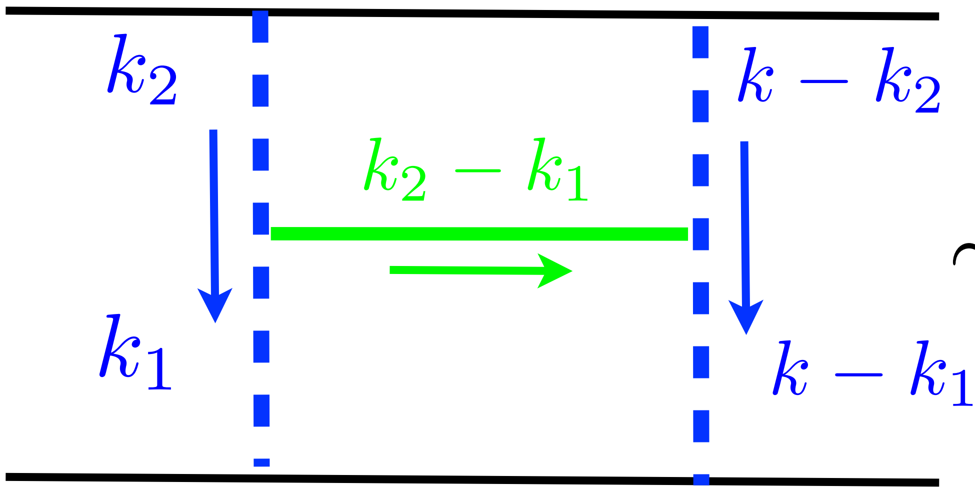
$$S_{eff}[x_1,x_2] \supset -i\int_x\int_{t,t_{1a},t_{1b},t_2}\left(\frac{-im_1 V_\phi(t_{1a})}{m_p}\right)\left(\frac{-im_2 V_\phi(t_{1b})}{m_p}\right)\left(\frac{-im_2 V_\sigma^{ij}(t_2)}{m_p}\right)V_{ij}^{\phi^2\sigma}\times\{propagators\}$$

$$= 64\pi^2 G^2 m_1^2 m_2 \int_{t,k,k_1}\frac{(k_1.v_2)^2-(k.v_2)(k_1.v_2)}{k^2(k-k_1)^2 k_1^2}\mathrm{e}^{ik.r(t)}$$

$$I_0 = \int\frac{\mathrm{d}^d k_1}{(2\pi)^d}\frac{1}{k_1^{2\alpha}(k-k_1)^{2\beta}} = \frac{\Gamma(\alpha+\beta-d/2)\Gamma(d/2-\alpha)\Gamma(d/2-\beta)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(d-\alpha-\beta)(4\pi)^{d/2}}(k^2)^{d/2-\alpha-\beta}$$

One-loop master integral

$\mathcal{O}(G^3)$: two-loop integrals



$$\sim \frac{\mathrm{d}^d k \mathrm{d}^d k_1 \mathrm{d}^d k_2 \{k^4\}\, \mathrm{e}^{ik.r}}{k_1^2 (k-k_1)^2 k_2^2 (k-k_2)^2 (k_2-k_1)^2}$$

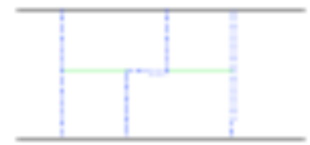
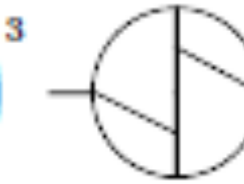
Integration by parts:

$$\begin{aligned} I(\alpha,\beta,\gamma,\delta,\epsilon) &\equiv \int_{\mathbf{p}_1,\mathbf{p}_2} \frac{1}{p_1^{2\alpha}(\mathbf{p}-\mathbf{p}_1)^{2\beta} p_2^{2\gamma} (\mathbf{p}-\mathbf{p}_2)^{2\delta} (\mathbf{p}_1-\mathbf{p}_2)^{2\epsilon}} \\ &= [\gamma\,(I(\alpha-1,\beta,\gamma+1,\delta,\epsilon) - I(\alpha,\beta,\gamma+1,\delta,\epsilon-1)) + \\ &\quad \delta\,(I(\alpha,\beta-1,\gamma,\delta+1,\epsilon) - I(\alpha,\beta,\gamma,\delta+1,\epsilon-1))]\,/\,(2\epsilon+\gamma+\delta-d) \end{aligned}$$

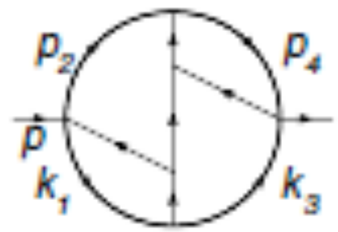
⇩

$$\int_{k_1,k_2} \frac{1}{k_1^2 (k-k_1)^2 k_2^2 (k-k_2)^2 (k_2-k_1)^2} = \int_{k_1,k_2} \frac{\frac{k_1^2}{k_2^2} - \frac{(k_2-k_1)^2}{k_2^2} + \frac{(k-k_1)^2}{(k-k_2)^2} - \frac{(k_2-k_1)^2}{(k-k_2)^2}}{k_1^2 (k-k_1)^2 k_2 (k-k_2)^2 (k_2-k_1)^2}$$

$\mathcal{O}(G^4, G^5)$: a systematic reduction pipeline

$\mathcal{A}_{49} =$  $= -2\,\mathrm{i}\,(8\pi G_N)^5 \left(\frac{(d-2)}{(d-1)}\, m_1 m_2\right)^3$  $[N_{49}]$,

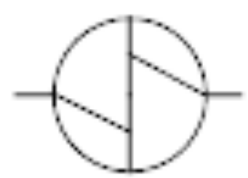
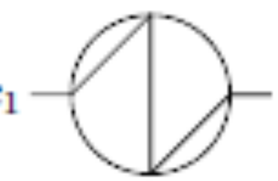
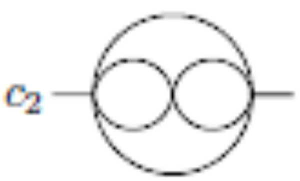
write amplitude
using Feynman rules

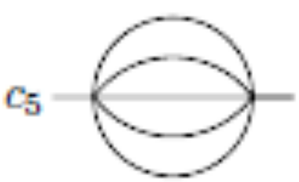


$$[N_{49}] \equiv \int_{k_1,k_2,k_3,k_4} \frac{N_{49}}{k_1^2\; p_2^2\; k_3^2\; p_4^2\; k_{12}^2\; k_{13}^2\; k_{23}^2\; k_{24}^2\; k_{34}^2} ,$$

$$N_{49} \equiv (k_1 \cdot k_3\; k_{12} \cdot k_{23} - k_1 \cdot k_{12}\; k_3 \cdot k_{23} - k_1 \cdot k_{23}\; k_3 \cdot k_{12}) \times$$
$$(p_2 \cdot k_{23}\; p_4 \cdot k_{34} + p_4 \cdot k_{23}\; p_2 \cdot k_{34} - p_2 \cdot p_4\; k_{23} \cdot k_{34}) ,$$

express amplitude in terms of
Master Integrals

 $[N_{49}] = c_1$  $+\, c_2$  $+\, c_3$  $+$

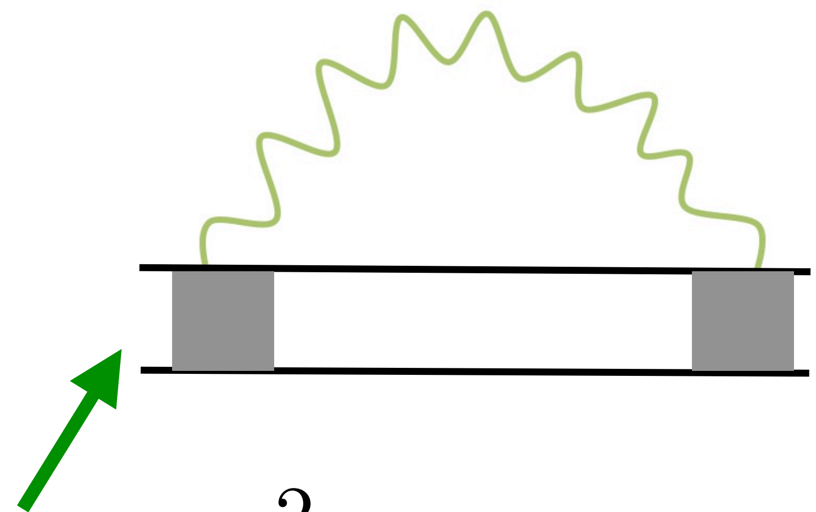
$+\, c_4$  $+\, c_5$  ,

get the result:

$$\mathcal{L}_{49} = -\mathrm{i} \lim_{d\to 3} \int_p e^{\mathrm{i}p\cdot r} \mathcal{A}_{49} = (32 - 3\pi^2) \frac{G_N^5 m_1^3 m_2^3}{r^5}$$

Procedure automatised at 5PN and
(partially) up to 6PN by Blumlein et al.

Simple self-energy diagram (emission)



$$S_m[x,h] = -\int \mathrm{d}\tau \left[m + \frac{1}{2}\mathcal{S}_{\mu\nu}\Omega^{\mu\nu} - \frac{1}{2}I_{ij}E^{ij} + \frac{2}{3}J_{ij}B^{ij} + c_E E^{ij}E_{ij} + \ldots \right.$$

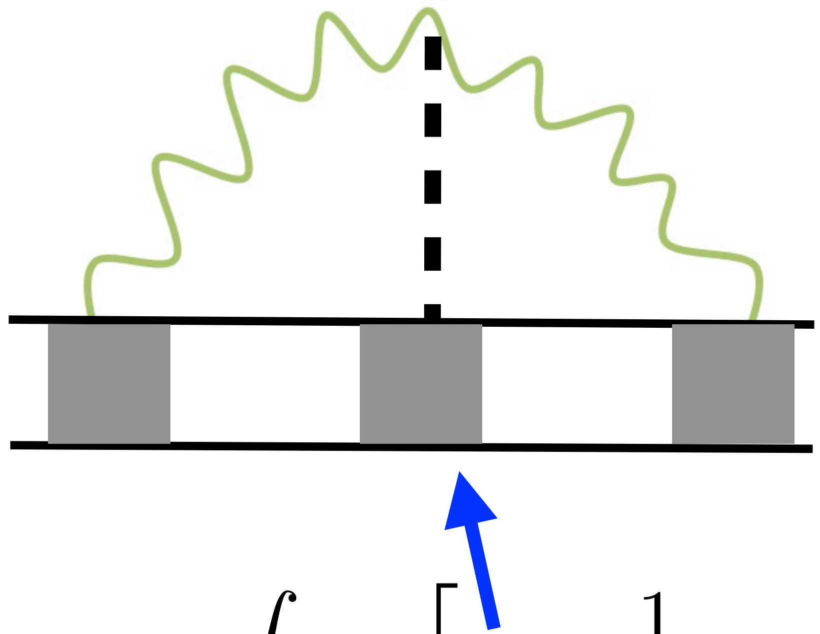
does not contribute to the conservative dynamics, gives only dissipative effects

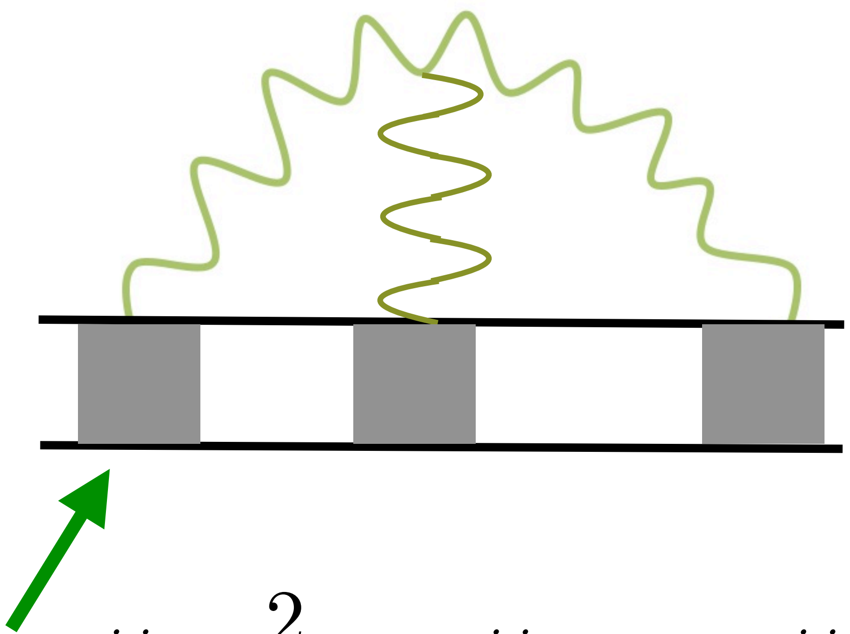
$$-\frac{2\pi}{5}G\int_{\mathbf{k},k_0} I_{ij}(k_0)I_{ij}(-k_0)\frac{k_0^4}{\mathbf{k}^2-k_0^2}$$

Power emission from imaginary part with Feynman boundary conditions (optical theorem)

Radiation-reaction in Keldysh formalism

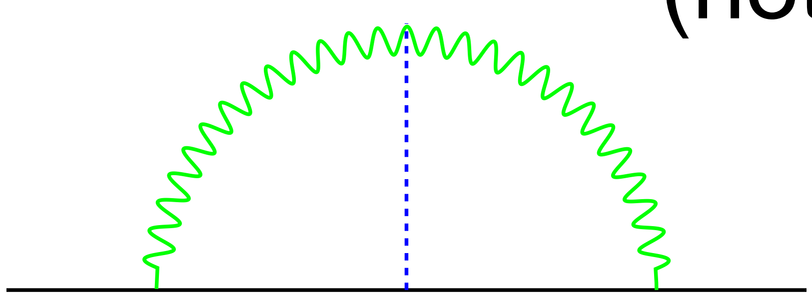
Hereditary terms: tail and memory





$$S_m[x,h] = -\int \mathrm{d}\tau \left[m + \frac{1}{2}\mathcal{S}_{\mu\nu}\Omega^{\mu\nu} - \frac{1}{2}I_{ij}E^{ij} + \frac{2}{3}J_{ij}B^{ij} + c_E E^{ij}E_{ij} + \ldots\right.$$

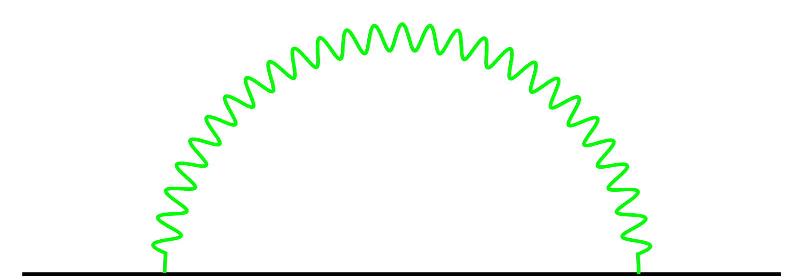
UV poles and nonlocal terms associated with tails:
(not with memory)



$\mathcal{A}_{tail}(k_0)$

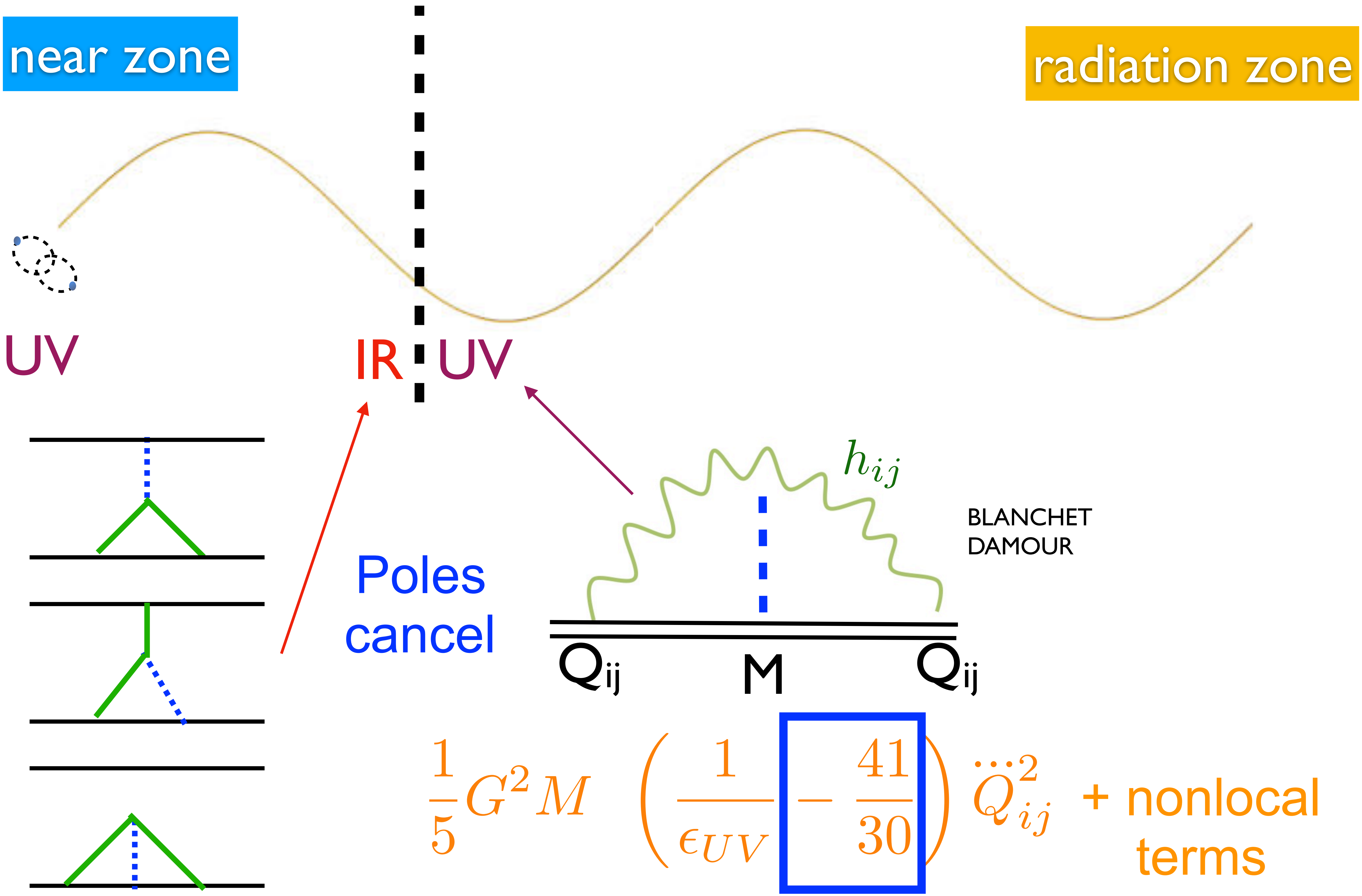
$$\simeq \quad -\frac{GMk_0}{c^2}\left[\frac{1}{(d-3)} + 2\log\left(\frac{|k_0|}{\mu}\right) - i\pi\right]$$

$$\times \ \mathrm{Im}:$$



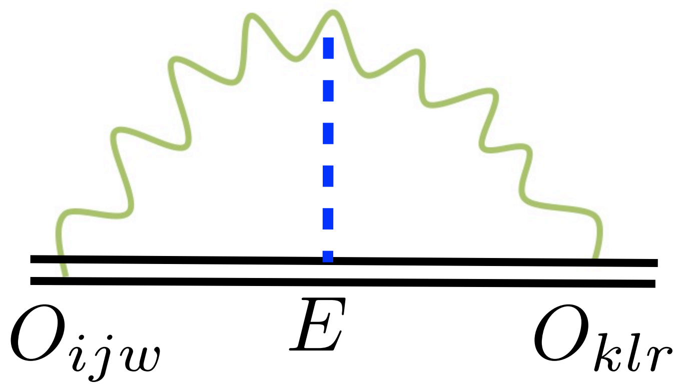
$\mathcal{A}_{rad}(k_0)$

IR and UV divergencies



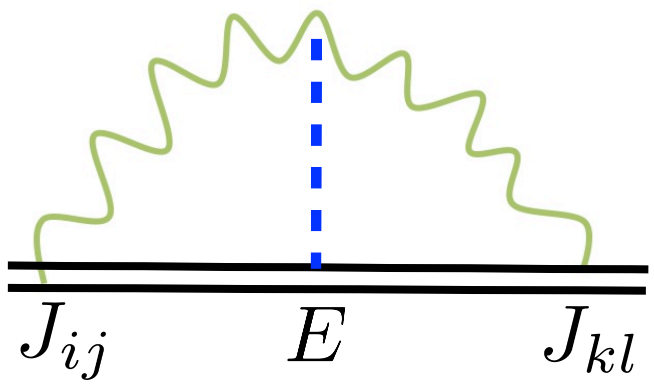
$$\frac{1}{5}G^2M\left(\frac{1}{\epsilon_{UV}}-\frac{41}{30}\right)\dddot{Q}_{ij}^{2}$$ + nonlocal terms

A glimpse at 5PN



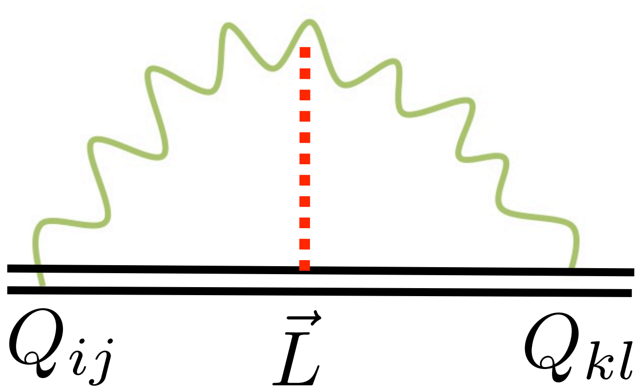
$$-\frac{1}{189}G^2M\left(\frac{1}{\epsilon_{UV}}-\frac{82}{35}\right)\ddddot{O}^{\,2}_{ijk}$$

+ nonlocal terms

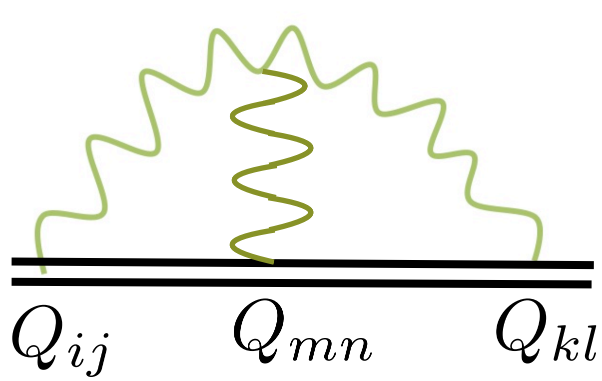


$$-\frac{16}{45}G^2M\left(\frac{1}{\epsilon_{UV}}-\frac{127}{60}\right)\dot{J}^2_{ij}$$

+ nonlocal terms



$$\frac{8}{15}G^2\epsilon_{ijk}L_k\ddddot{Q}_{il}\dddot{Q}_{jl}$$



$$-\frac{G^2}{15}\left(\ddddot{Q}_{il}\ddddot{Q}_{jl}Q_{ij}+\frac{4}{7}\dddot{Q}_{il}\dddot{Q}_{jl}\ddot{Q}_{ij}\right)$$

Gauge invariant energy

$$E = -\frac{\mu x}{2}\left\{1 - \left[\frac{3}{4} + \frac{\nu}{12}\right]x + \left[-\frac{27}{8} + \frac{19}{8}\nu - \frac{\nu^2}{24}\right]x^2\right.$$ 012PN

$$\nu \equiv \frac{m_1 m_2}{M}$$

$$x \equiv \frac{(GM\omega)^{2/3}}{c}$$

$$+\left[-\frac{675}{64} + \left(\frac{34445}{576} - \frac{205}{96}\pi^2\right)\nu - \frac{155}{96}\nu^2 - \frac{35\nu^3}{5184}\right]x^3$$ 3PN

$$+\left[-\frac{3969}{128} + \left(\frac{9037}{1536}\pi^2 - \frac{123671}{5760} + \frac{448}{15}\left[2\gamma_E + \log(16x)\right]\right)\nu + \right.$$

Tail

4PN

$$\left. + \left(\frac{3137}{576}\pi^2 - \frac{498449}{3456}\right)\nu^2 + \frac{301}{1728}\nu^3 + \frac{77}{31104}\nu^4\right]x^4$$

Gauge invariant energy up to 4PN

$$E = -\frac{\mu x}{2}\left\{1 - \left[\frac{3}{4} + \frac{\nu}{12}\right] x + \left[-\frac{27}{8} + \frac{19}{8}\nu - \frac{\nu^2}{24}\right] x^2\right.$$

012PN

$$\nu \equiv \frac{m_1 m_2}{M}$$

$$x \equiv \frac{(GM\omega)^{2/3}}{c}$$

$$+ \left[-\frac{675}{64} + \left(\frac{34445}{576} - \frac{205}{96}\pi^2\right)\nu - \frac{155}{96}\nu^2 - \frac{35\nu^3}{5184}\right] x^3$$

3PN

$$+ \left[-\frac{3969}{128} + \left(\frac{9037}{1536}\pi^2 - \frac{123671}{5760} + \frac{448}{15}\left[2\gamma_E + \log(16x)\right]\right)\nu + \right.$$

Tail

$$\left. + \left(\frac{3137}{576}\pi^2 - \frac{498449}{3456}\right)\nu^2 + \frac{301}{1728}\nu^3 + \frac{77}{31104}\nu^4\right] x^4$$

4PN

BH perturbation

post-Minkoskian

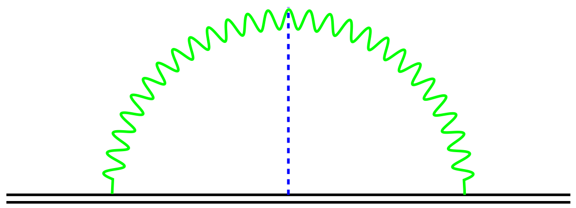
$$\frac{1-2x}{\sqrt{1-3x}} - 1$$

test particle in Schwarzschild

$$+ \, \nu P(x^5, \ldots, x^{22})$$

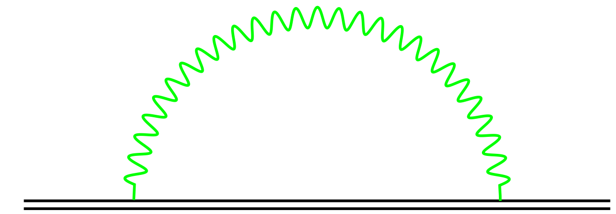
BH perturbation

Logarithmic terms up to 7PN



$\mathcal{A}_{tail}(k_0)$

$$\simeq \; -\frac{GMk_0}{c^2}\left[\frac{1}{(d-3)} + 2\log\left(\frac{|k_0|}{\mu}\right) - i\pi\right] \times \mathrm{Im} :$$



$\mathcal{A}_{rad}(k_0)$

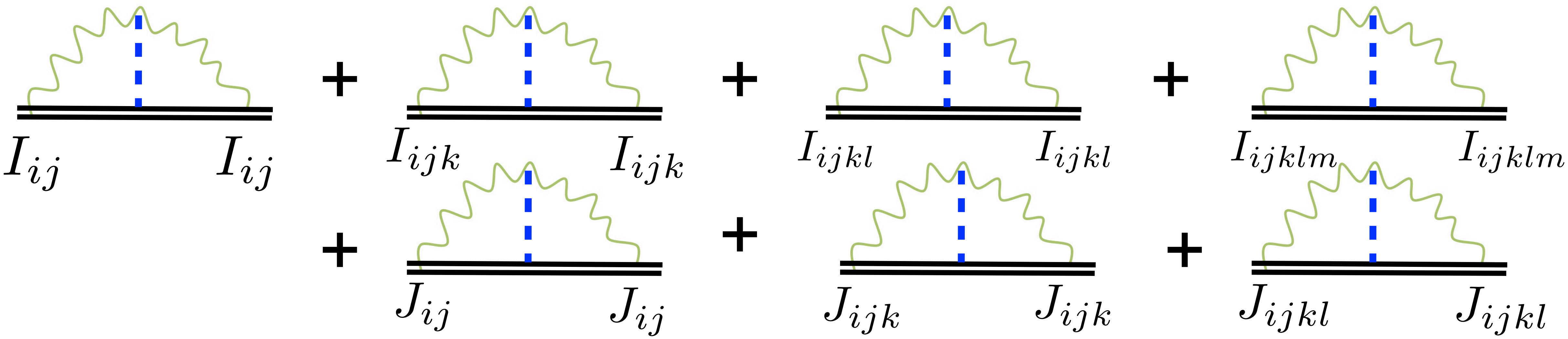
$$S^{\mathrm{nl}}_{\mathrm{tail}} = \sum_{\ell=2}^{+\infty} \frac{G^2 M}{c^{2\ell+4}} \underset{\tau_0}{\mathrm{Pf}} \iint \frac{\mathrm{d}t\mathrm{d}t'}{|t-t'|}\left[a_\ell\, I_L^{(\ell+1)}(t) I_L^{(\ell+1)}(t') + \frac{b_\ell}{c^2} J_L^{(\ell+1)}(t) J_L^{(\ell+1)}(t')\right]$$

$$a_\ell = \frac{1}{5}\,,\,\frac{1}{189}\,\ldots \qquad b_\ell = \frac{16}{45}\,\ldots$$

Thorne emission coefficients

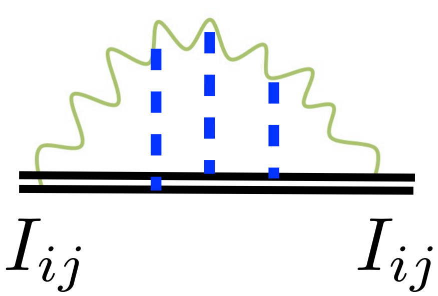
Circular orbit: nonlocal → local: $\log\left(\frac{v}{c}\right)$

$$S^{\mathrm{circ}}_{\mathrm{tail}} = -\sum_{\ell=2}^{+\infty} \frac{2G^2 M}{c^{2\ell+4}} \int \mathrm{d}t \log\left(\frac{v}{c}\right)\left[a_\ell\,\left(I_L^{(\ell+1)}(t)\right)^2 + \frac{b_\ell}{c^2}\,\left(J_L^{(\ell+1)}(t)\right)^2\right]$$



$$E^{\log}_{\text{simple-tail}} = -\frac{m\nu^2}{2}x^5\log x\left[\frac{448}{15} + \left(-\frac{4988}{35} - \frac{656}{5}\nu\right)x + \left(-\frac{1967284}{8505} + \frac{914782}{945}\nu + \frac{32384}{135}\nu^2\right)x^2 + \left(\frac{16785520373}{2338875} - \frac{1424384}{1575}\log\left(\frac{r}{r_0}\right) + \left(\frac{2132}{45}\pi^2 - \frac{41161601}{51030}\right)\nu - \frac{13476541}{5670}\nu^2 - \frac{289666}{1215}\nu^3\right)x^3 + \mathcal{O}\left(x^4\right)\right]$$

(tail)^3



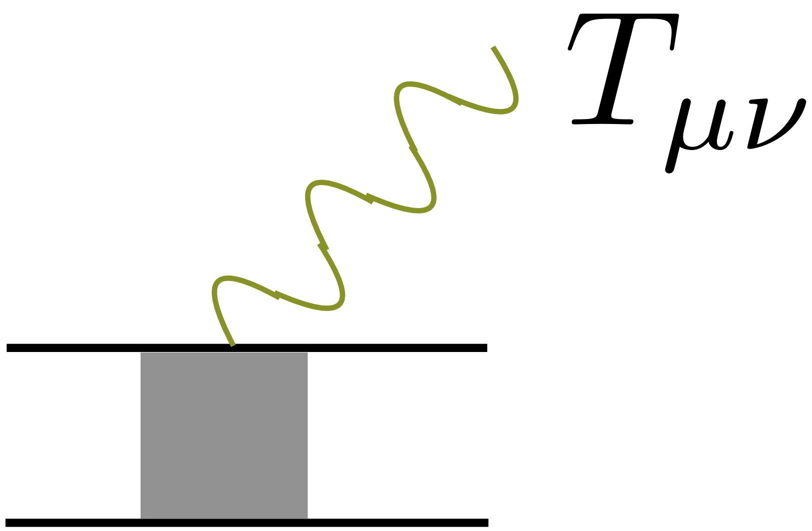
+ ...

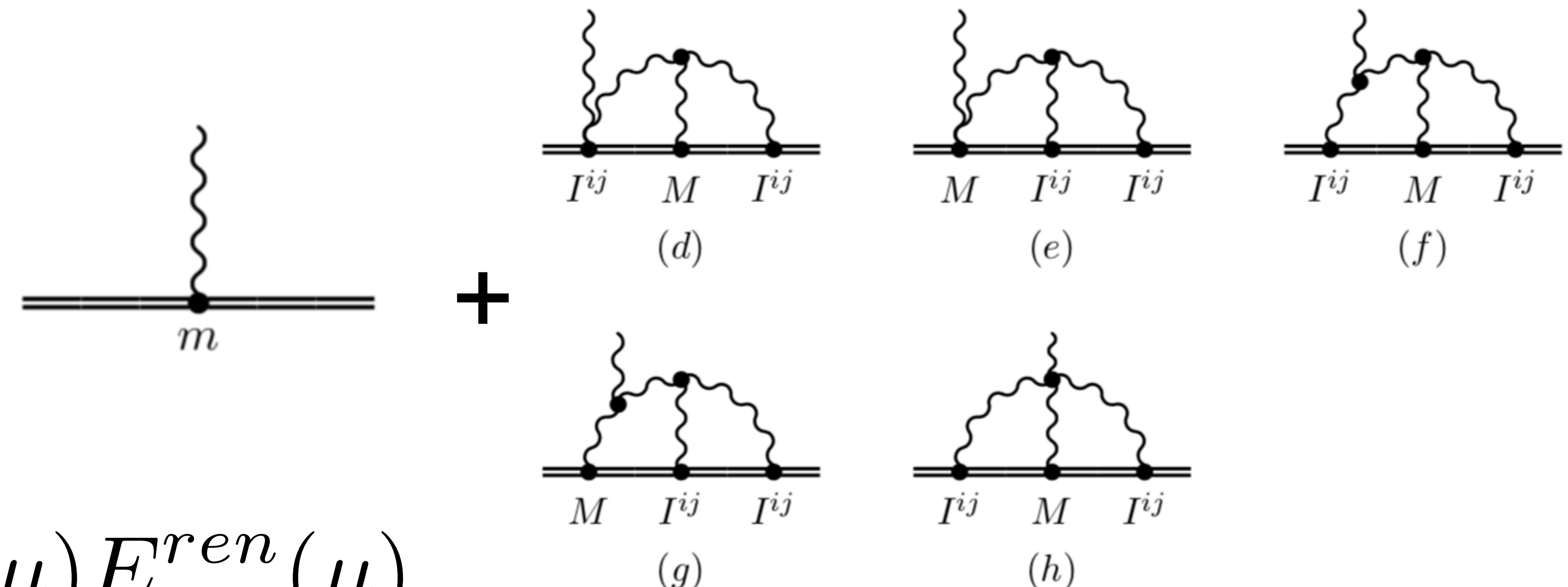
difficult, but

can deduce it from self-force:

$$E^{\log}_{(\text{tail})^3}(x) = -\frac{m\nu^2}{2}x^8\log x\left\{-\frac{356096}{1575}\log x - \frac{108649792}{55125} + \frac{1424384}{1575}\left[\log\left(\frac{r}{r_0}\right) - \gamma_E - \log 4\right] + \mathcal{O}(x)\right\}$$

Leading logs from renormalisation group analysis at arbitrary high PN order

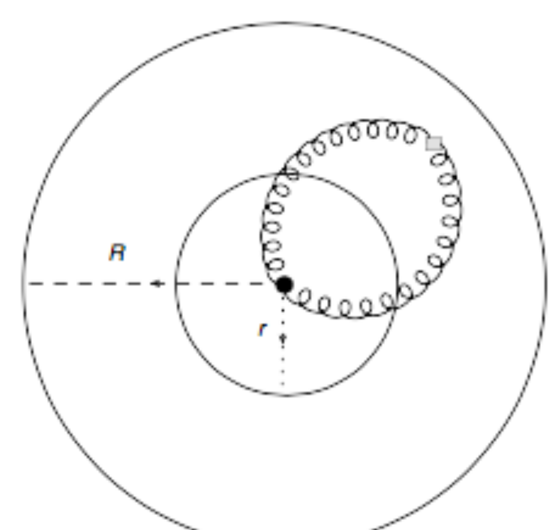




$$E^{bare} = Z(\mu) E^{ren}(\mu)$$

$$\frac{\mathrm{d}\log E(\mu)}{\mathrm{d}\log\mu} = -\frac{2G^2}{5}\left[2Q_{ij}^{(1)}Q_{ij}^{(5)} - 2Q_{ij}^{(2)}Q_{ij}^{(4)} + Q_{ij}^{(3)}Q_{ij}^{(3)}\right]$$

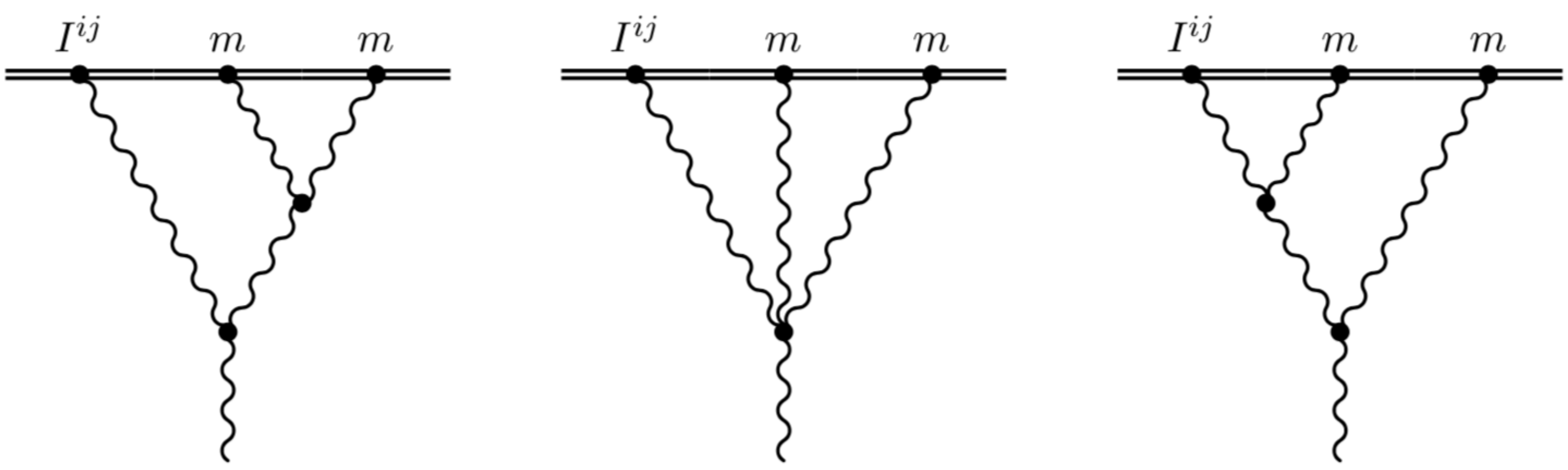
The same result can be derived from the energy-balance equation in presence of tail interactions



Similarly, for the angular momentum:

$$\frac{\mathrm{dJ^i}(\mu)}{\mathrm{d}\log\mu} = -\frac{8G^2M}{5}\varepsilon^{ijk}\left[Q_{jl}Q^{(5)}_{kl} - Q^{(1)}_{jl}Q^{(4)}_{kl} + Q^{(2)}_{jl}Q^{(3)}_{kl}\right]$$

and for the quadrupole:



(a) (b) (c)

$$\frac{\mathrm{d}Q_{ij}(\omega,\mu)}{\mathrm{d}\log\mu} = \beta_Q\,(GM\omega)^2\,Q_{ij}(\omega,\mu) \qquad \beta_Q = -\frac{214}{105}$$

$$Q_{ij}(t,\mu) = \sum_{n\geq 0}\frac{1}{n!}\left(-\beta_Q G^2M^2\log\bar{\mu}\right)Q^{(2n)}_{ij}(t,\mu_0)$$

$$\left\langle \frac{E(\mu)}{M} \right\rangle = 1 - G^2 \sum_{n=1}^{+\infty} \frac{(2\log\bar{\mu})^n}{n!} \left(\beta_Q G^2 M^2\right)^{n-1} \langle Q_{ij}^{(n+2)}(\mu_0) Q_{ij}^{(n+2)}(\mu_0)\rangle$$

$$\langle J^i(\mu)\rangle = \langle J^i(\mu_0)\rangle - \frac{12 G^2 M}{5} \varepsilon^{ijk} \sum_{n=1}^{+\infty} \frac{(2\log\bar{\mu})^n}{n!} \left(\beta_Q G^2 M^2\right)^{n-1} \langle Q_{jl}^{(n+1)}(\mu_0) Q_{kl}^{(n+2)}(\mu_0)\rangle$$

First law of binary dynamics: $$\frac{\mathrm{dE}}{\mathrm{d}\omega} = \omega \frac{\mathrm{dJ}}{\mathrm{d}\omega}$$

$$E_{\text{leading-(log)}^n} = -\frac{m\nu\, x}{2}\left[\frac{64\nu}{15} \sum_{n=1}^{+\infty} \frac{6n+1}{n!} (4\beta_Q)^{n-1} x^{3n+1} (\log x)^n\right]$$

$$c_{log}^{4PN} = \frac{448}{15} \qquad c_{log^2}^{7PN} = -\frac{356096}{1575}$$

Perfect agreement between

self-force and PN EFT + RG flow + 1st law

$$c^{10PN}_{\log^3} = \frac{445503488}{496125}$$

$$c^{13PN}_{\log^4} = -\frac{5017776128}{2083725}$$

$$c^{16PN}_{\log^5} = \frac{133151707332608}{27348890625}$$

$$c^{19PN}_{\log^6} = -\frac{68019046365134848}{8614900546875}$$

$$c^{22PN}_{log^7} = \frac{6766608266507793203 2}{633195190195312 5}$$

$$\frac{64}{15}\frac{6n+1}{n!}(4\beta_Q)^{n-1}$$

$$n = 3\ldots 7$$

Gauge invariant energy up to 4PN

$$\nu \equiv \frac{m_1 m_2}{M} \qquad x \equiv \frac{(GM\omega)^{2/3}}{c}$$

$$E = -\frac{\mu x}{2}\left\{1 - \left[\frac{3}{4}+\frac{\nu}{12}\right]x + \left[-\frac{27}{8}+\frac{19}{8}\nu-\frac{\nu^2}{24}\right]x^2\right.$$

012PN

$$+\left[-\frac{675}{64}+\left(\frac{34445}{576}-\frac{205}{96}\pi^2\right)\nu-\frac{155}{96}\nu^2-\frac{35\nu^3}{5184}\right]x^3$$

3PN

$$+\left[-\frac{3969}{128}+\left(\frac{9037}{1536}\pi^2-\frac{123671}{5760}+\frac{448}{15}\left[2\gamma_E+\log(16x)\right]\right)\nu+\right.$$

Tail

4PN

BH perturbation

$$\left.+\left(\frac{3137}{576}\pi^2-\frac{498449}{3456}\right)\nu^2+\frac{301}{1728}\nu^3+\frac{77}{31104}\nu^4\right]x^4$$

post-Minkoskian

$$= -\frac{m\nu^2}{2}x^5\log x\left\{\left(-\frac{4988}{35}-\frac{656}{5}\nu\right)x+\left(-\frac{1967284}{8505}+\frac{914782}{945}\nu+\frac{32384}{135}\nu^2\right)x^2\right.$$

$$+\left[\frac{85229654387}{16372125}-\frac{1424384}{1575}(\gamma_{\rm E}+\log 4)+\left(\frac{2132}{45}\pi^2-\frac{41161601}{51030}\right)\nu-\frac{13476541}{5670}\nu^2\right.$$

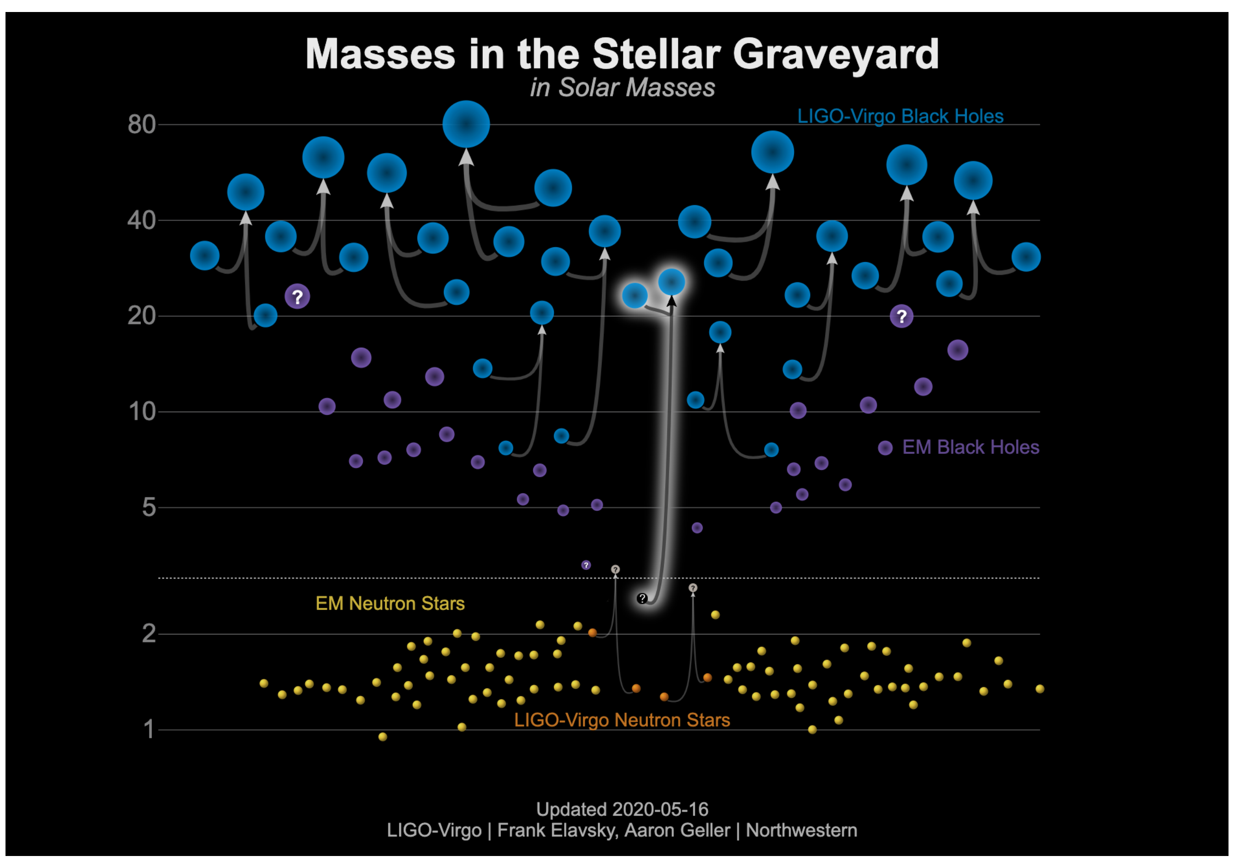
$$\left.\left.-\frac{289666}{1215}\nu^3-\frac{356096}{1575}\log x\right]x^3+\frac{64}{15}\sum_{n=3}^{+\infty}\frac{(6n+1)\,(4\beta_I)^{n-1}}{n!}x^{3(n-1)}(\log x)^{n-1}+\cdots\right\}$$

$$\frac{1-2x}{\sqrt{1-3x}}-1+\frac{16\nu x^2}{15\beta_Q}\left[\left(1+24\beta_Q x^3\log x\right)x^{4\beta_Q x^3}-1\right] + \nu P(x^5,\ldots,x^{22})$$

Conclusions

Compact binary systems
as hydrogen atom for GR

phenomenology



theory

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