Gravitational Waves from Axions

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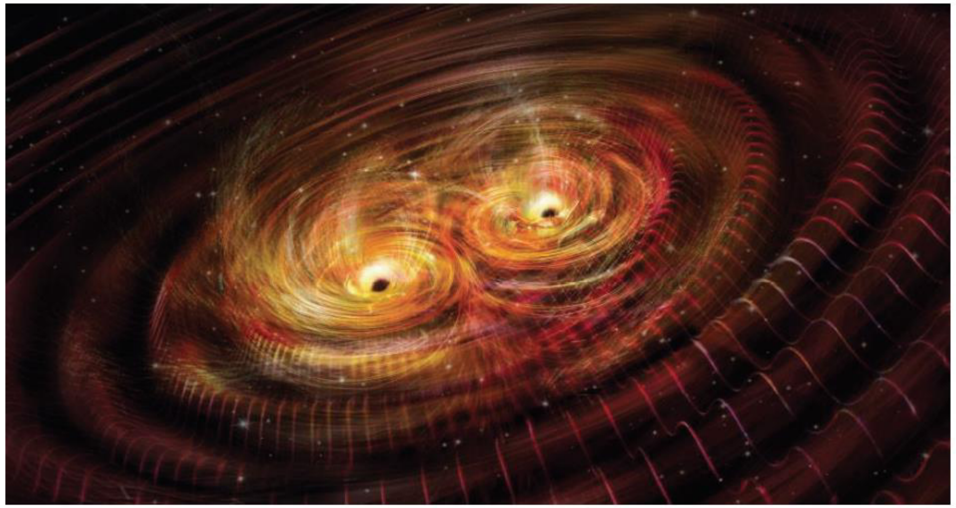
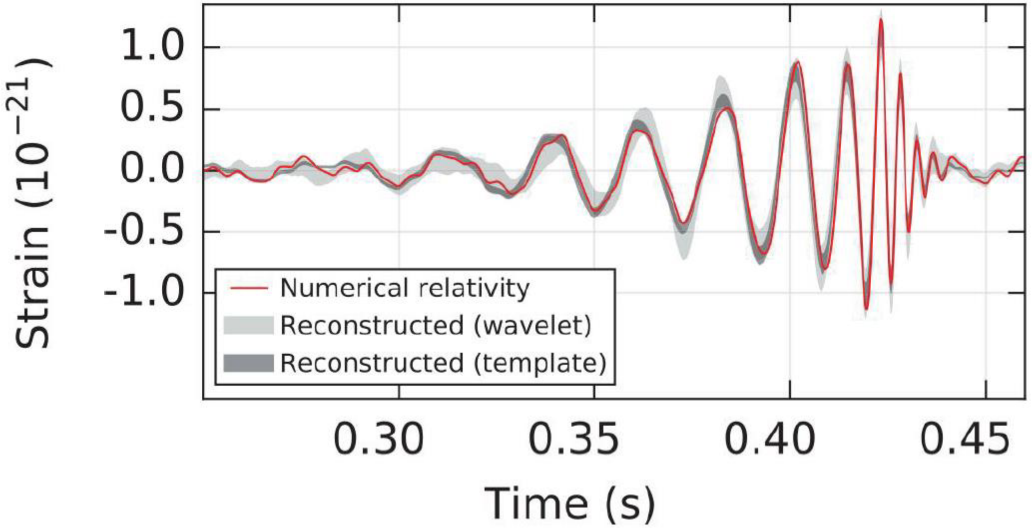
Outline

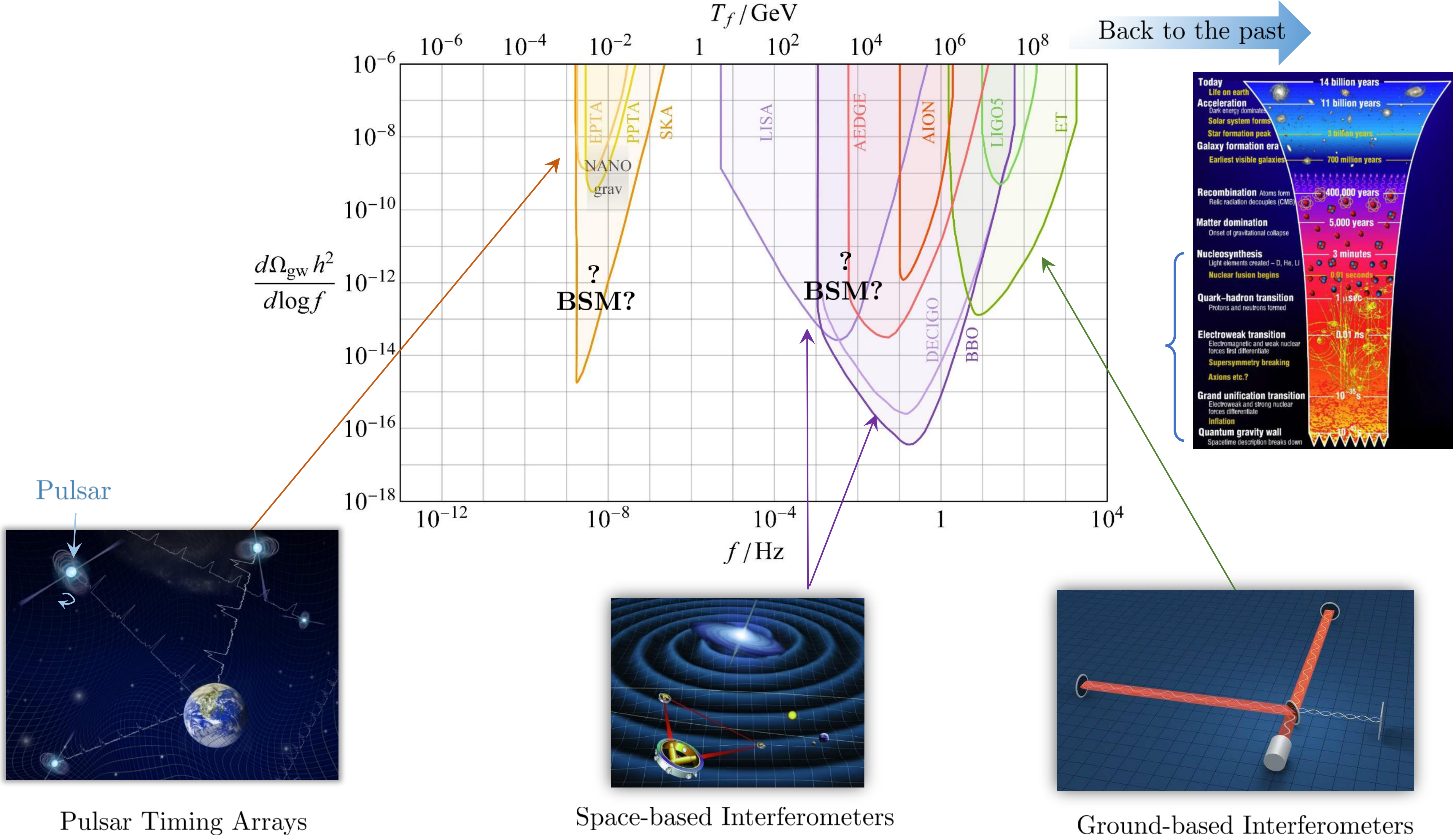
- **Prospects of gravitational wave dectection**
- **Axions and axion strings**
- **String dynamics and generation of gravitational waves**

  → **Bounds on axion mass and decay constant**
- **Conclusion and Outlook**

Gravitational Waves

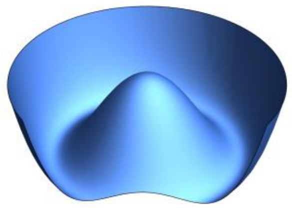






T_f / GeV
10^{-6} 10^{-4} 10^{-2} 1 10^{2} 10^{4} 10^{6} 10^{8}
Back to the past
$\frac{d\Omega_{\rm gw} h^2}{d\log f}$
10^{-6} 10^{-8} 10^{-10} 10^{-12} 10^{-14} 10^{-16} 10^{-18}
EPTA
PPTA
SKA
NANO grav
LISA
AEDGE
AION
LIGO5
ET
DECIGO
BBO
?
BSM?
?
BSM?
10^{-12} 10^{-8} 10^{-4} 1 10^{4}
f / Hz
Today
Life on earth
14 billion years
Acceleration
Dark energy dominates
11 billion years
Solar system forms
Star formation peak
3 billion years
Galaxy formation era
Earliest visible galaxies
700 million years
Recombination Atoms form
Relic radiation decouples (CMB)
400,000 years
Matter domination
Onset of gravitational collapse
5,000 years
Nucleosynthesis
Light elements created – D, He, Li
Nuclear fusion begins
3 minutes
0.01 seconds
Quark–hadron transition
Protons and neutrons formed
1 μsec
Electroweak transition
Electromagnetic and weak nuclear forces first differentiate
Supersymmetry breaking
Axions etc.?
0.01 ns
Grand unification transition
Electroweak and strong nuclear forces differentiate
Inflation
Quantum gravity wall
Spacetime description breaks down
Pulsar
Pulsar Timing Arrays
Space-based Interferometers
Ground-based Interferometers

Axion $\equiv a$



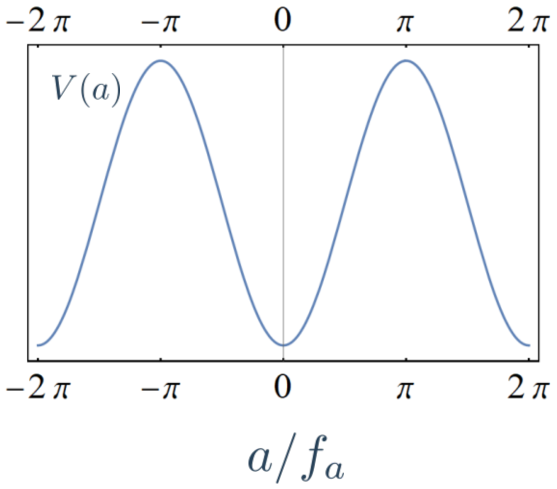
- Goldstone boson of a (new) spontaneously broken U(1) symmetry (at the scale f_a)

Simplest realization: $\phi = \frac{1}{\sqrt{2}}(r + f_a)e^{i\frac{a}{f_a}}$ $\qquad V_\phi = \frac{m_r^2}{2f_a^2}\left(|\phi|^2 - \frac{f_a^2}{2}\right)^2$

- U(1) explicitly broken by the axion potential $V(a)$

$\rightarrow V(a)$ invariant under $a \rightarrow a + 2\pi f_a$

$\rightarrow$ axion mass m_a



Example

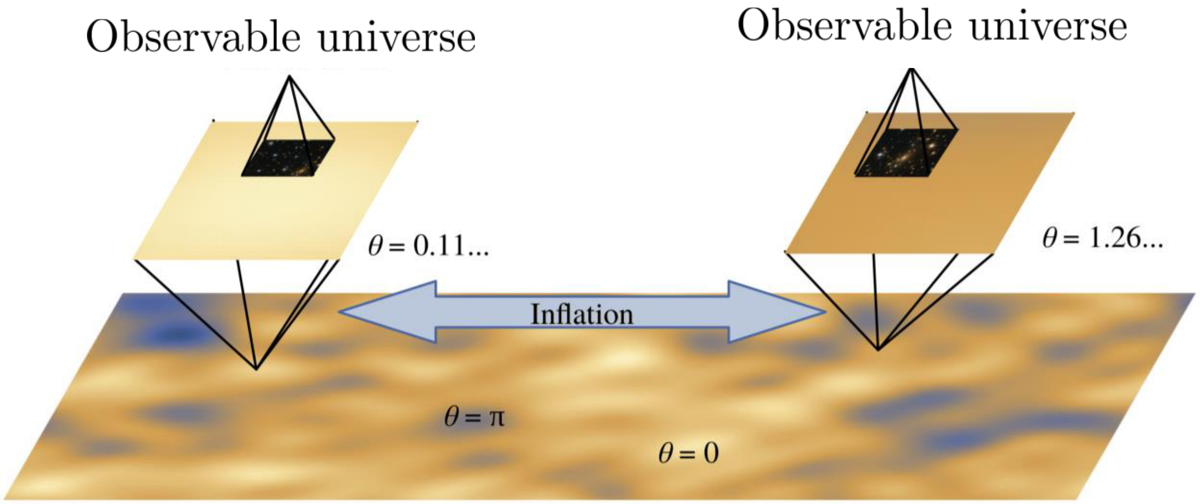
QCD axion: $\mathcal{L} \supset \frac{a}{f_a} G_{\mu\nu}\underbrace{\tilde{G}^{\mu\nu}}_{\epsilon^{\mu\nu\rho\sigma}G_{\rho\sigma}} \longrightarrow m_a \simeq m_\pi f_\pi / f_a$

Strong CP: $\mathcal{L}_{SM} \supset \theta_{\text{QCD}}\, G_{\mu\nu}\tilde{G}^{\mu\nu}$, $\theta_{\text{QCD}} \lesssim 10^{-10}$ $\longrightarrow \theta_{\text{QCD}} = \frac{\langle a \rangle}{f_a} \rightarrow 0$

Cosmological Initial Conditions

Pre-inflationary

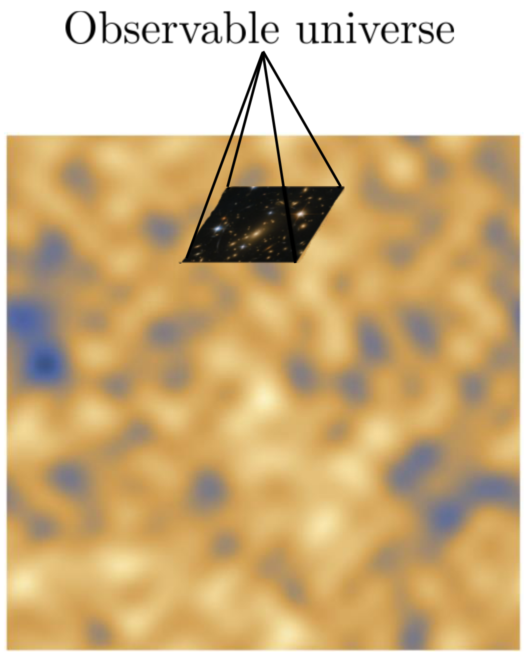
$T_R \lesssim f_a$ and $H_I \lesssim f_a$



$$\theta \equiv \frac{a}{f_a} \in [-\pi, \pi]$$

Post-inflationary

$T_R \gtrsim f_a$ or $H_I \gtrsim f_a$



Origin of the inhomogeneities in the post-inflationary scenario

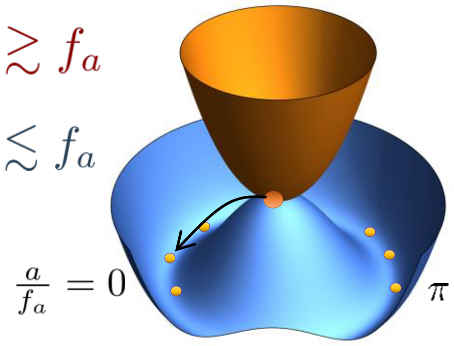
$T_R \gtrsim f_a$ or $H_I \gtrsim f_a$

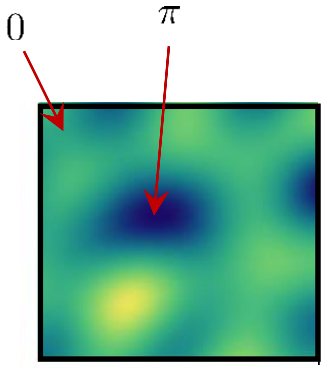
- Finite temperature corrections

$$T_{RH} \lesssim 10^{15}\ \text{GeV}$$

$T \gtrsim f_a$

$T \lesssim f_a$





spatial inhomogeneities

- Quantum inflationary fluctuations:

$$\langle \sigma_a^2 \rangle = \left(\frac{H_I}{2\pi}\right)^2 \quad \rightarrow \quad H_I \gtrsim \frac{f_a}{2\pi}$$

$$\frac{H_I}{2\pi} \lesssim 10^{13}\ \text{GeV}$$

- Direct coupling of ϕ to the inflation φ?

[Kofman,Linde '87]

$$V_\phi \supset g\varphi^2|\phi|^2 \xrightarrow[\text{inflation}]{\text{during}} \underbrace{g\langle\varphi\rangle^2}_{\textbf{effective mass for } \phi}|\phi|^2$$

$$g\langle\varphi\rangle^2 \gtrsim m_r^2$$

Waves

$$\mathcal{L} = |\partial_\mu \phi|^2 - V_\phi \qquad\qquad V_\phi = \frac{m_r^2}{2f_a^2}\left(|\phi|^2 - \frac{f_a^2}{2}\right)^2$$

Trivial inhomogeneous solutions:

$$\begin{cases} r = 0 \text{ (radial mode on its VEV)} \\ \partial_\mu \partial^\mu a = 0 \end{cases}$$

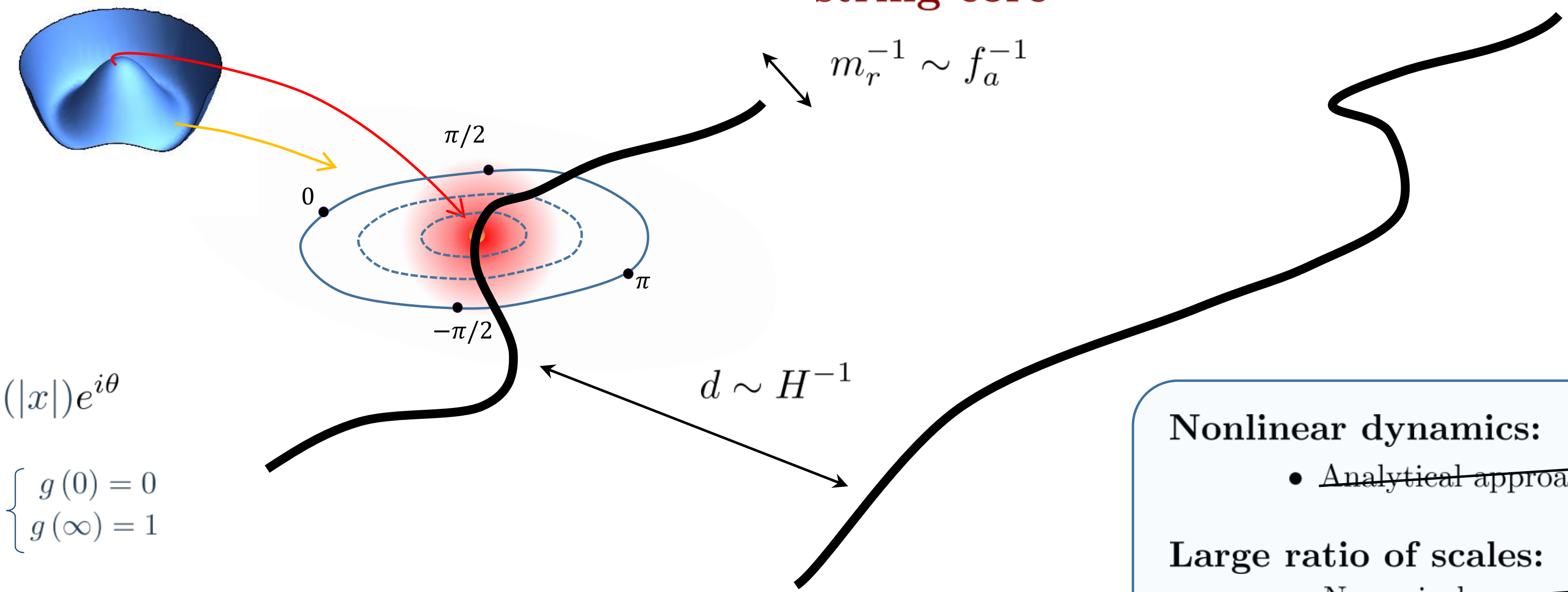
axion waves

$$\phi = \frac{f_a}{\sqrt{2}} e^{ik(t-x)}$$

Strings

axion waves

$$\phi = \frac{f_a}{\sqrt{2}} e^{ik(t-x)}$$



$$\phi = \frac{f_a}{\sqrt{2}} g\left(|x|\right) e^{i\theta}$$

$$\begin{cases} g\left(0\right) = 0 \\ g\left(\infty\right) = 1 \end{cases}$$

Nonlinear dynamics:

- ~~Analytical approach~~ ☹

Large ratio of scales:

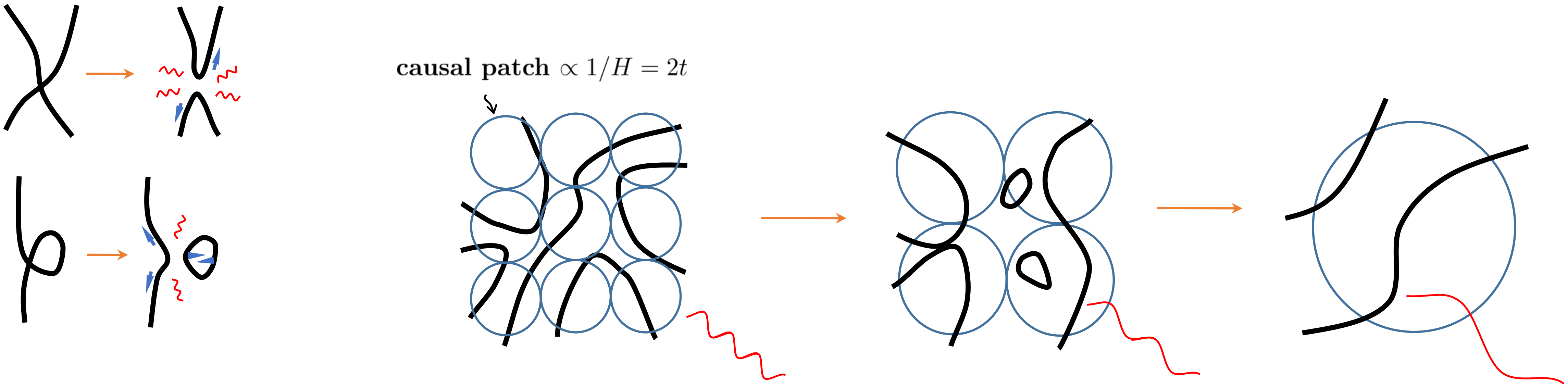
- ~~Numerical approach~~ ☹

string tension

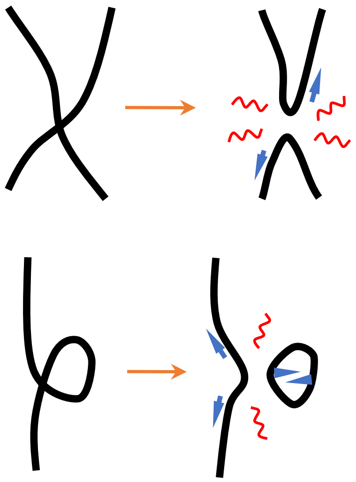
$$\mu = \frac{E}{L} \sim \underbrace{\pi f_a^2}_{\text{core}} \underbrace{\log \frac{d}{m_r^{-1}}}_{\text{axion gradient}} \sim \pi f_a^2 \log \frac{m_r}{H}$$

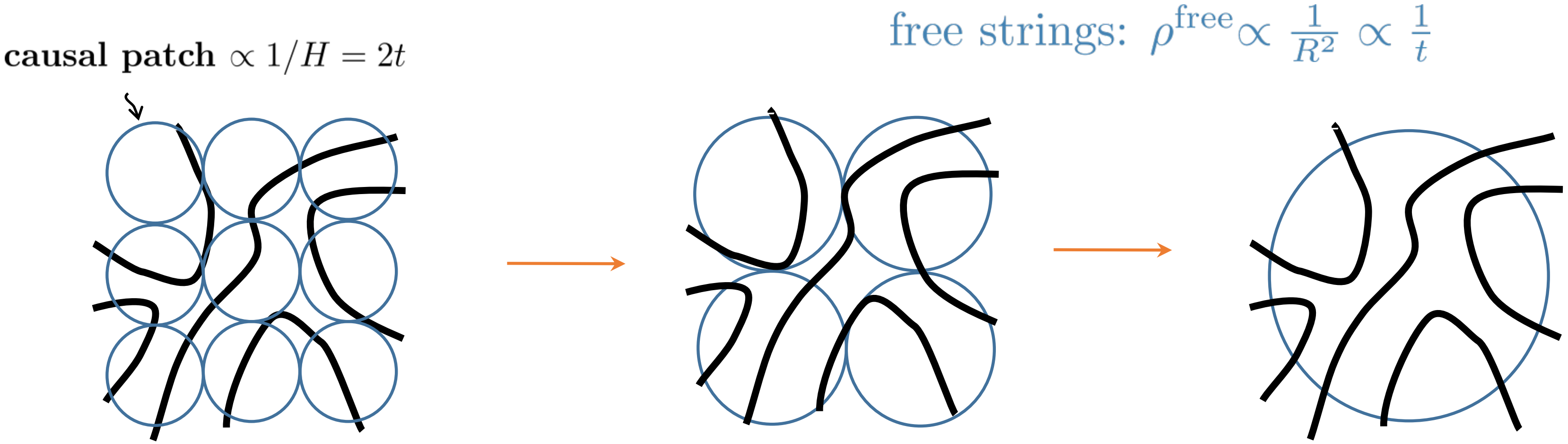
grows logarithmically in time

The Scaling Regime

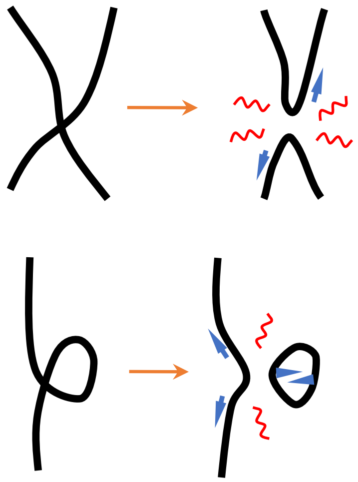


The Scaling Regime

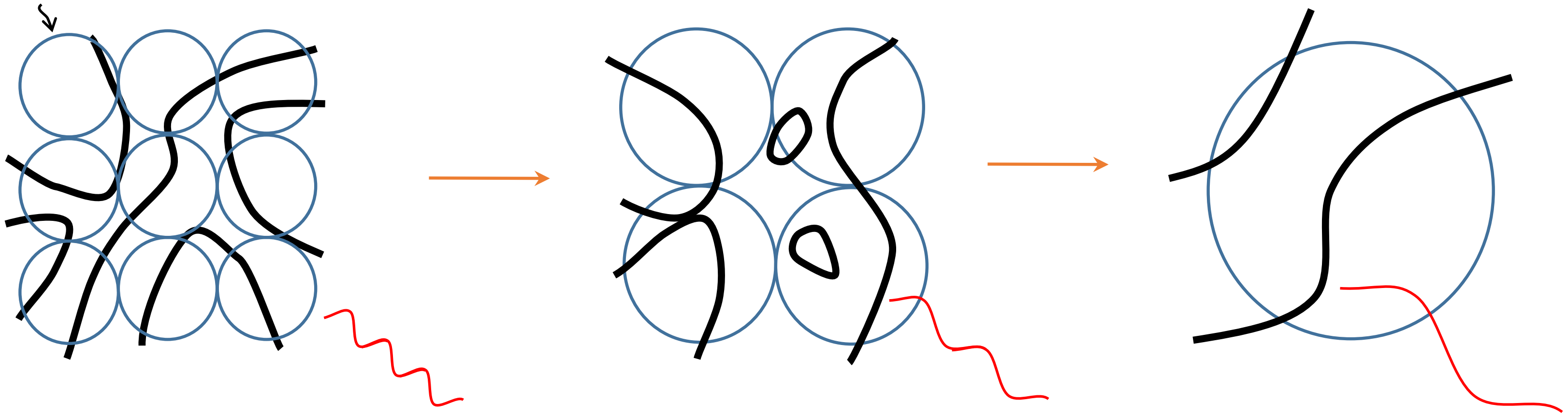




The Scaling Regime



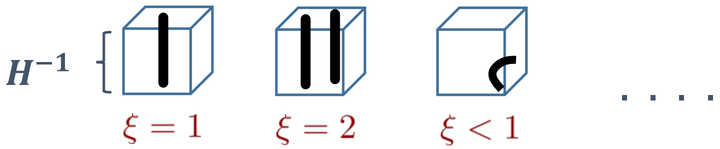
causal patch $\propto 1/H = 2t$

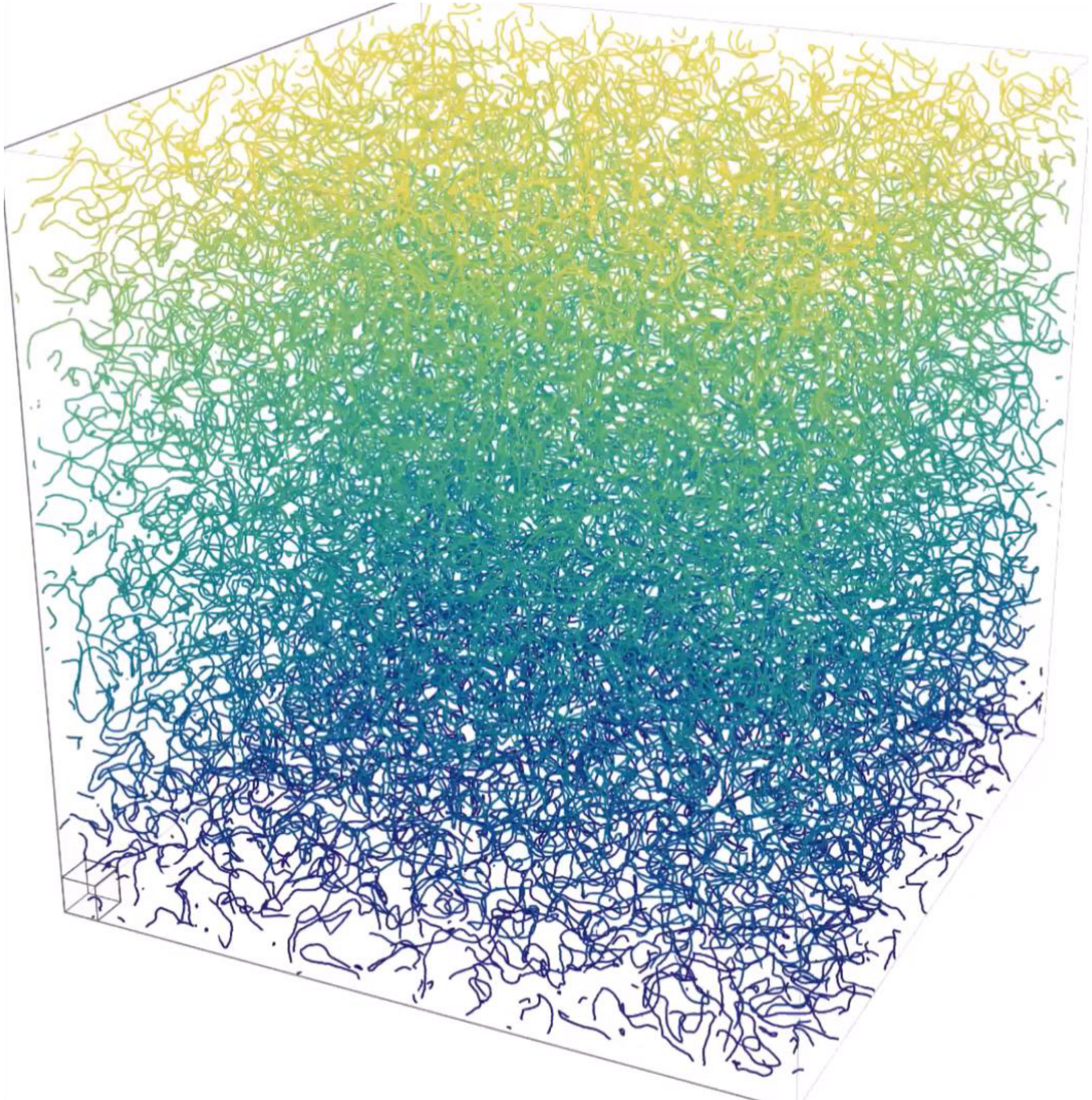


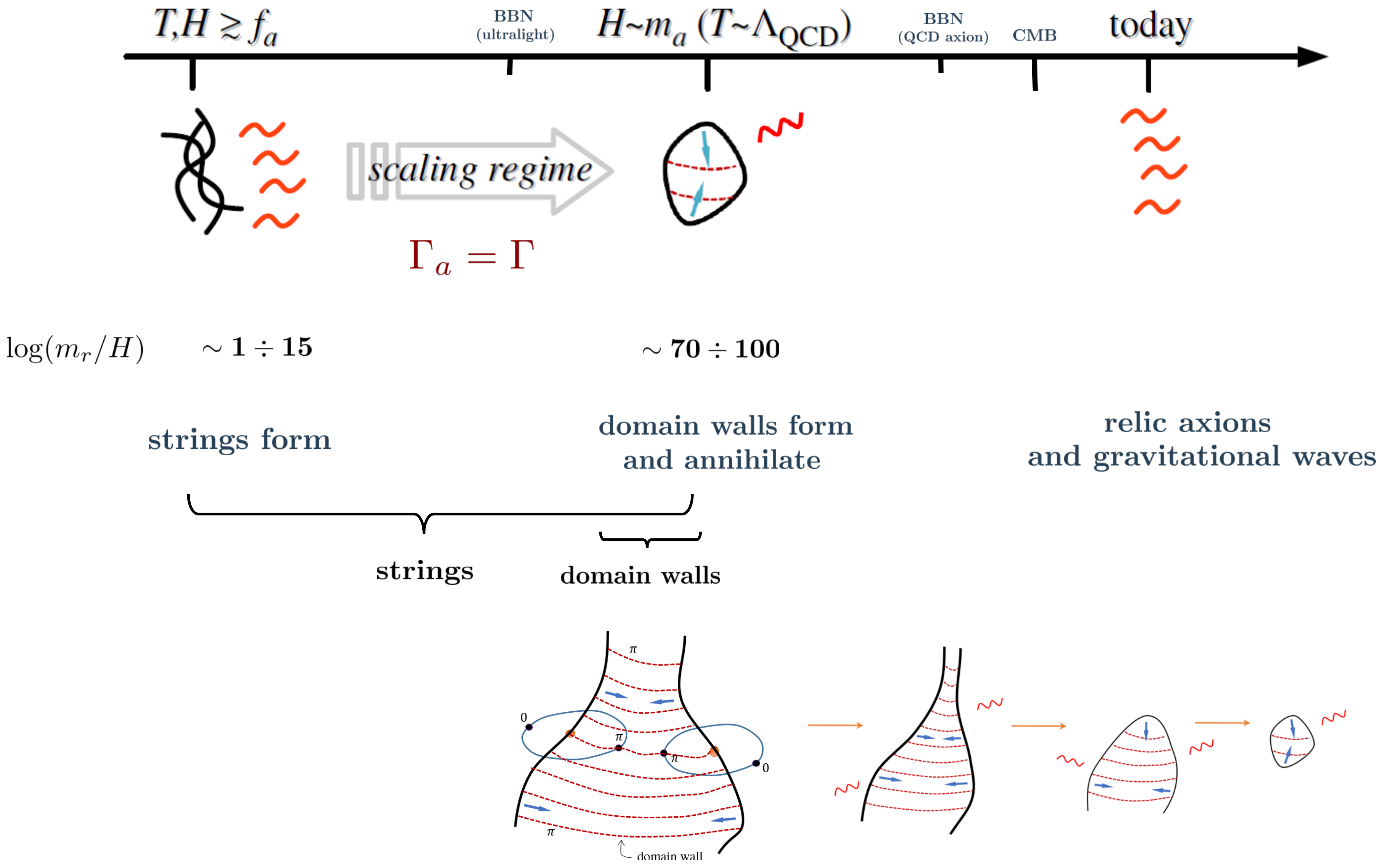
rate of energy loss: $$\Gamma \equiv \frac{d}{dt}\left[\rho^{\text{free}} - \rho^{\text{scal}}\right] \simeq \frac{\xi\mu}{t^3}$$

$$\propto \frac{1}{R^2} \propto \frac{1}{t}$$

$$\frac{\xi\mu}{t^2}$$ **number of strings per Hubble patch**

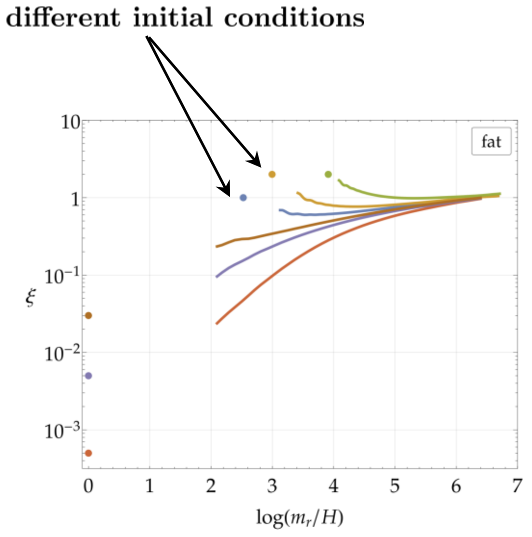


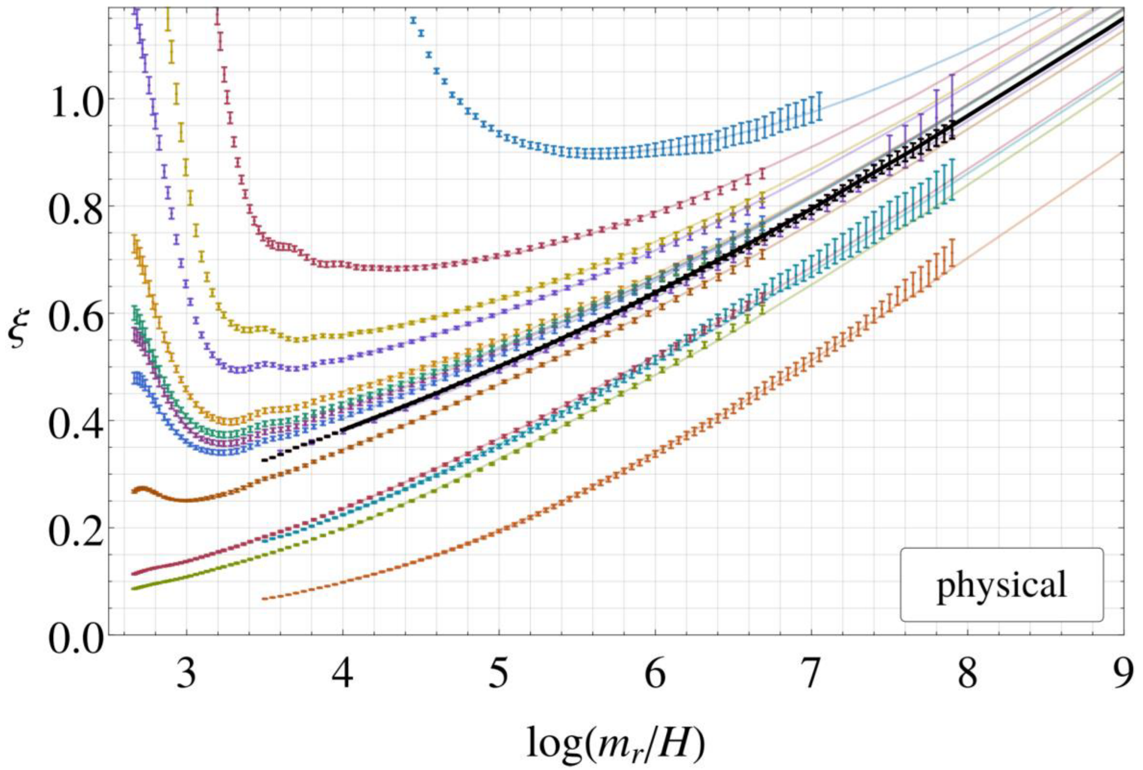


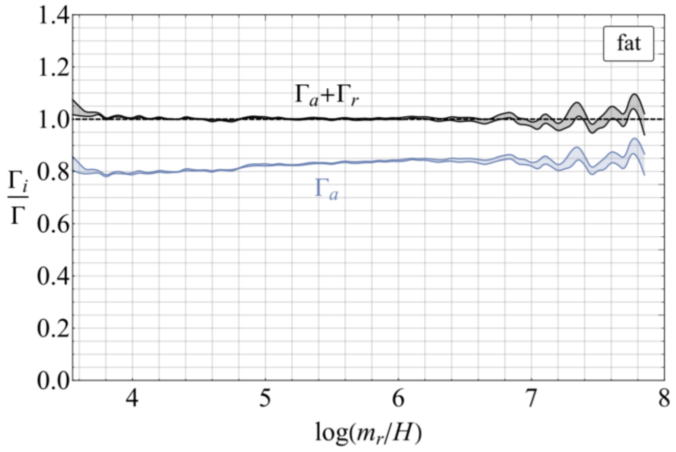


T,H ≳ f_a
BBN
(ultralight)
H~m_a (T~Λ_QCD)
BBN
(QCD axion)
CMB
today
scaling regime
Γ_a = Γ
log(m_r/H)
~ 1 ÷ 15
~ 70 ÷ 100
strings form
domain walls form
and annihilate
relic axions
and gravitational waves
strings
domain walls
π
0
domain wall

Attractor and Logarithmic Running



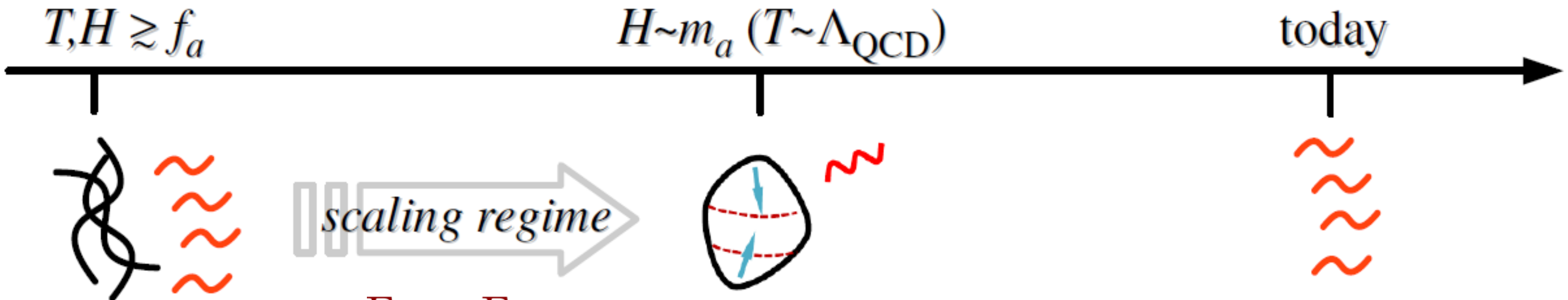




Scaling Violation

$$\boxed{\xi = c_1 \log} + c_0 + \frac{c_{-1}}{\log} + \frac{c_{-2}}{\log^2} \quad \Rightarrow \quad \Gamma_a = \frac{\xi\mu}{t^3} \propto \frac{f_a^2 \log^2}{t^3}$$

Gravitational Waves



$\Gamma_a = \Gamma$

$\Gamma_g = ?$

$\log(m_r/H)$ $\quad \sim \mathbf{1 \div 15}$ $\quad \sim \mathbf{70 \div 100}$

GWs from the scaling regime

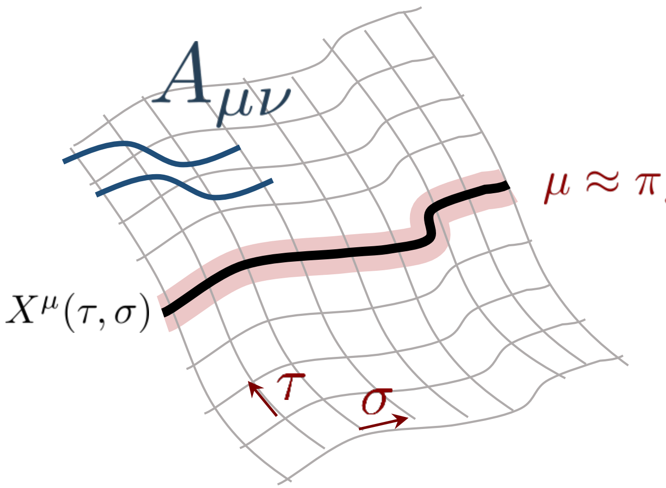
1) GW energy emission rate: $\Gamma_g(t)$ → Nambu–Goto EFT $\Gamma_g \longleftrightarrow \Gamma_a$

2) momentum distribution: $\frac{\partial \Gamma_g}{\partial k}$

String Effective Theory

Degrees of freedom:

- $a \quad \longleftrightarrow \quad A_{\mu\nu}$ $\qquad \partial A \sim F^{\mu\nu\rho} = \epsilon^{\mu\nu\rho\sigma}\partial_\sigma a$
- $X^\mu(\tau, \sigma)$



$S_\phi[\phi]$

axion-to-string coupling

$$S_{\rm EFT}[X, A] = -\mu \int d\tau d\sigma \sqrt{-\gamma} \; - \frac{1}{6}\int d^4x \, (\partial A)^2 \; + 2\pi f_a \int d\tau d\sigma \, \partial_\tau X^\mu \partial_\sigma X^\nu A_{\mu\nu}$$

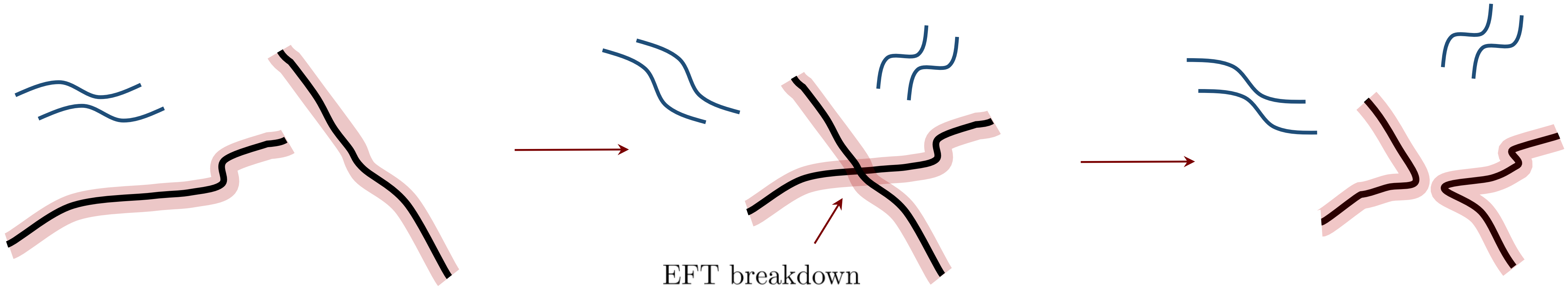
Nambu–Goto action: $\gamma_{ab} = \partial_a X^\mu \partial_b X_\mu$

Axion kinetic term

Axion-string interaction (Kalb–Ramond action)

1) GW energy emission rate: $\Gamma_g(t)$

2) momentum distribution: $\frac{\partial \Gamma_g}{\partial k}$



Gravitational Wave Emission

1) GW energy emission rate: $\Gamma_g(t)$

2) momentum distribution: $\frac{\partial\Gamma_g}{\partial k}$

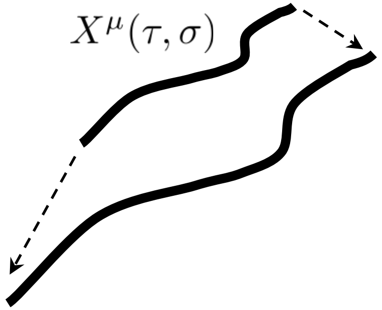
EoM for $A^{\mu\nu}$: $\quad \Box_x A^{\mu\nu} = 2\pi f_a \int d\sigma \dot{X}^{[\mu} X'^{\nu]} \delta^3(\vec{x} - \vec{X})$

Einstein Eq: $\quad \Box_x h^{\mu\nu} = 16\pi G \left(T_s^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu} T_{s\,\lambda}^{\lambda}\right)$

$$T_s^{\mu\nu} = \mu \int d\sigma \left(\dot{X}^\mu \dot{X}^\nu - X'^\mu X'^\nu\right) \delta^3(\vec{x} - \vec{X})$$

$$\frac{dE_a}{dt} = \underbrace{r_a[X]} f_a^2 \qquad \frac{dE_g}{dt} = \underbrace{r_g[X]} G\mu^2$$

dimensionless functionals of the shape of the string trajectory X^μ

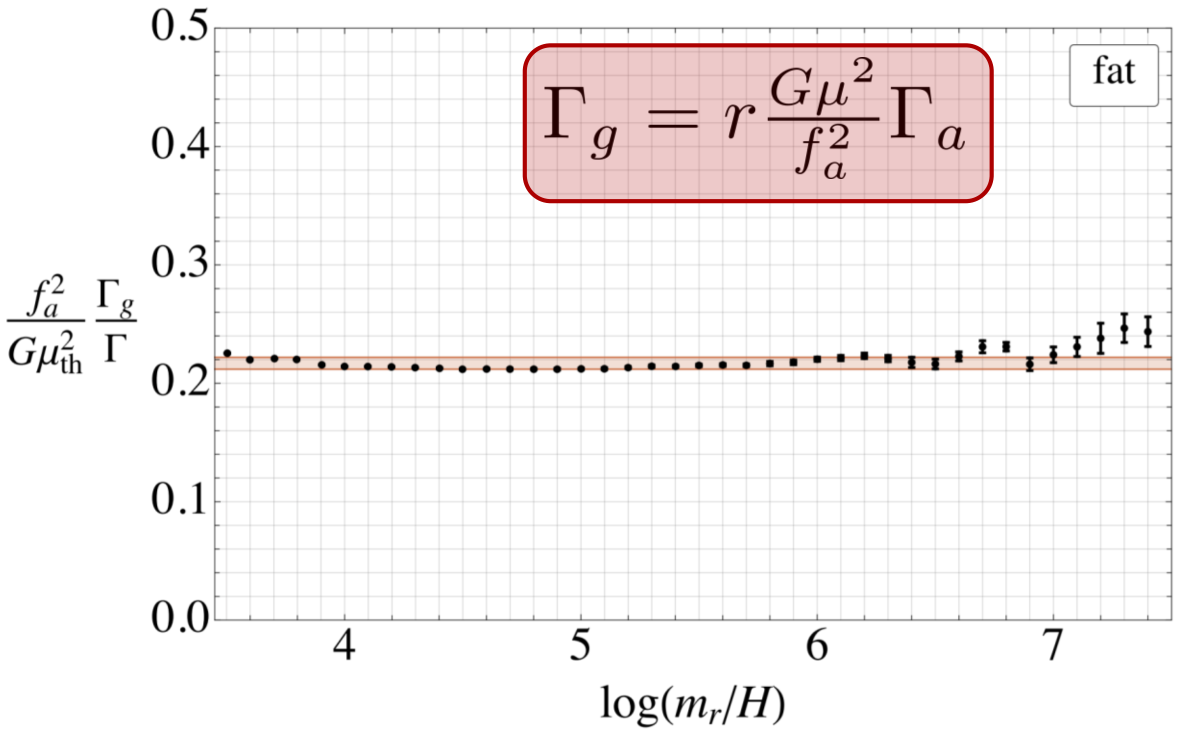


$$\frac{\Gamma_g}{\Gamma_a} = \underbrace{\frac{r_g[X]}{r_a[X]}}_{\equiv r} \frac{G\mu^2}{f_a^2} = \text{const}$$

$$\Gamma_g = r \frac{G\mu^2}{f_a^2} \Gamma_a \propto \frac{\log^4}{t^3}$$

Γ_a: $\frac{\xi\mu}{t^3}$

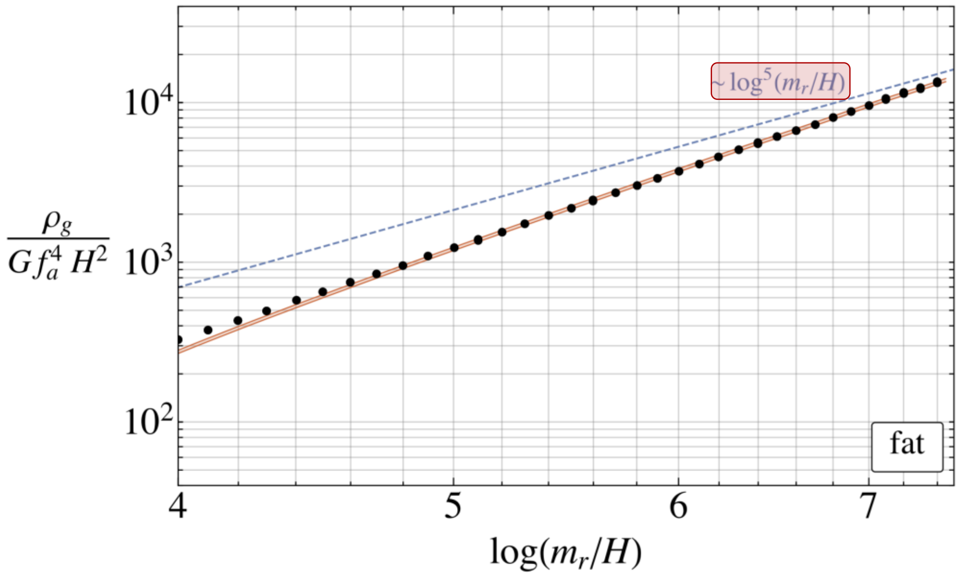
Comparison with the Field Theory Evolution



$$\Gamma_g = r\frac{G\mu^2}{f_a^2}\Gamma_a$$

total GW energy:

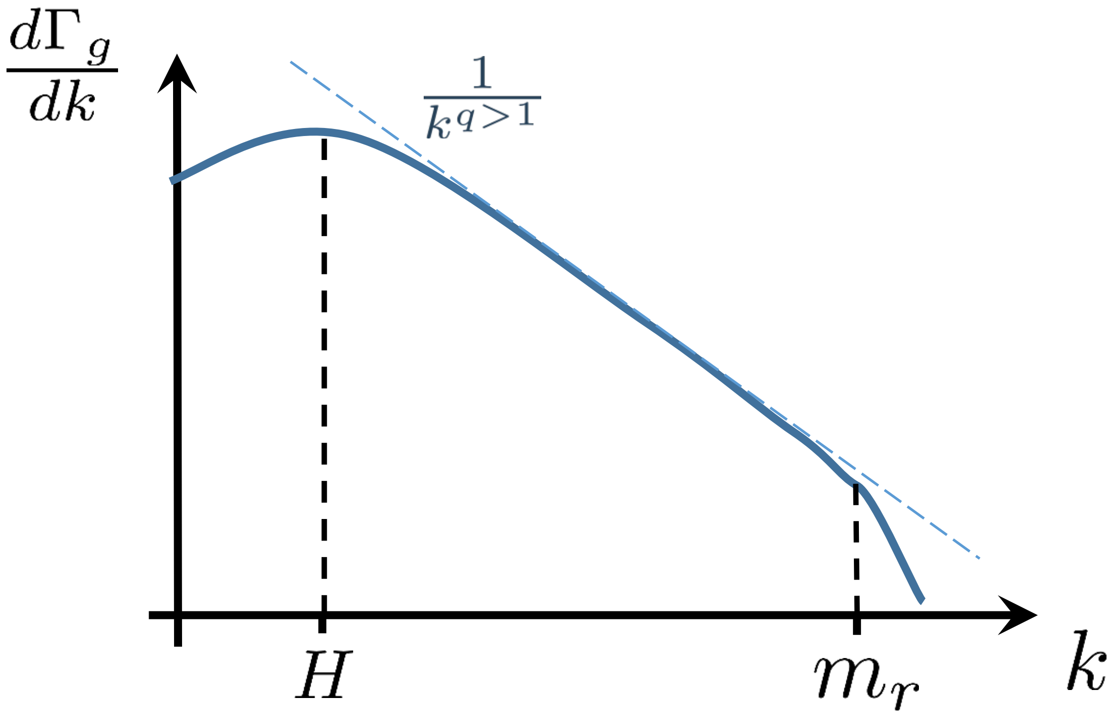
$$\rho_g = \int dt' \left(\frac{R'}{R}\right)^4 \Gamma'_g \propto \frac{\log^5}{t^2}$$



The Gravitational Wave Spectrum

1) GW energy emission rate: $\Gamma_g(t)$

2) momentum distribution: $\frac{\partial \Gamma_g}{\partial k}$



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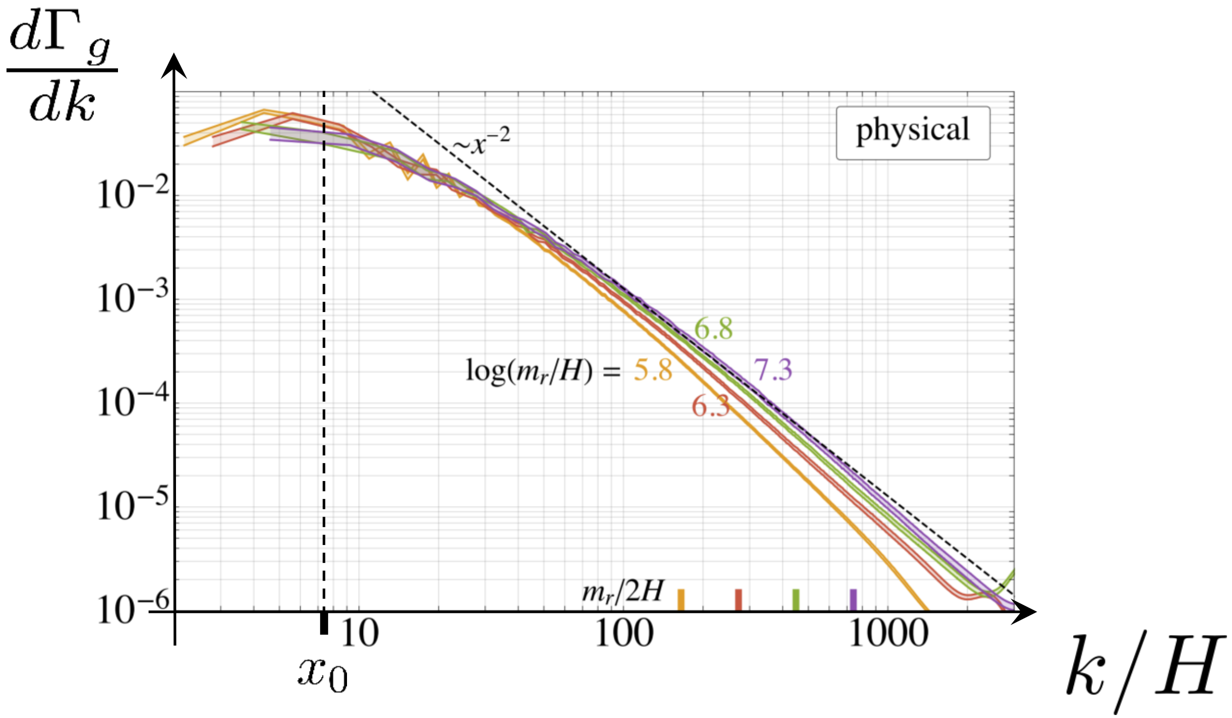
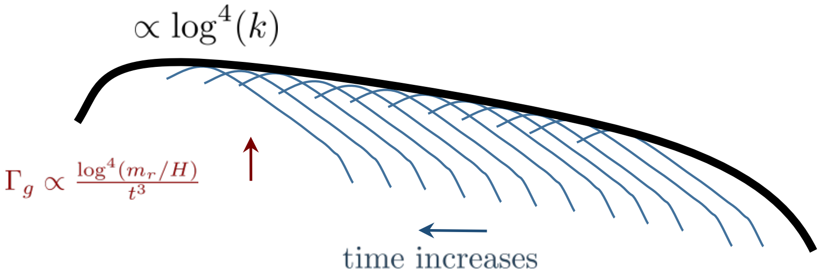


$\propto \frac{\log^4}{t^3}$

$$\frac{\partial \rho_g}{\partial \log k} \equiv \int dt' \, \frac{d\Gamma'_g}{d\log k} \left(\frac{R'}{R}\right)^4 =$$

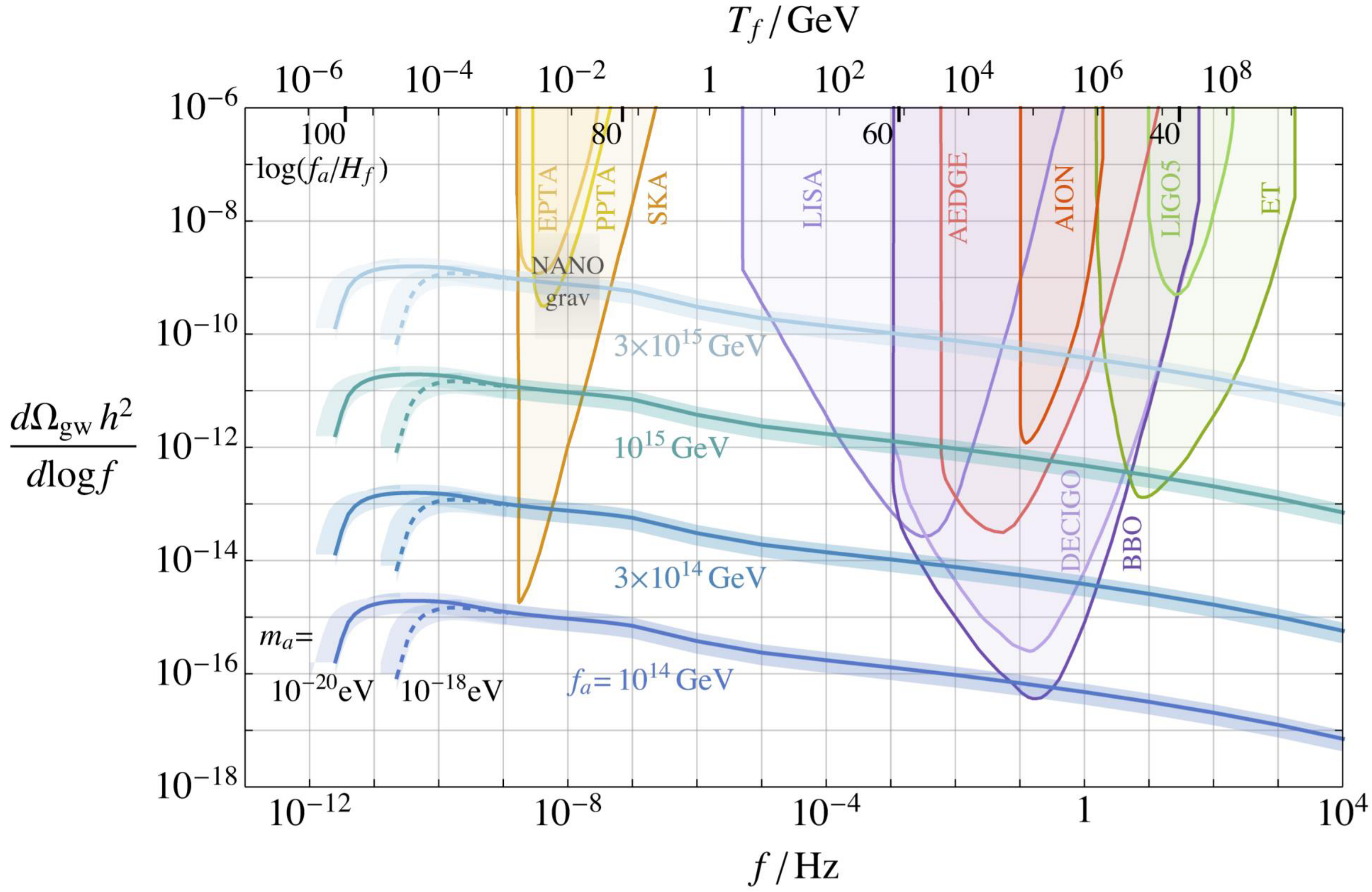
$$\simeq 8\pi^3 r G f_a^4 H^2 \log^4\left[\frac{m_r}{H}\left(\frac{x_0 H}{k}\right)^2\right]$$



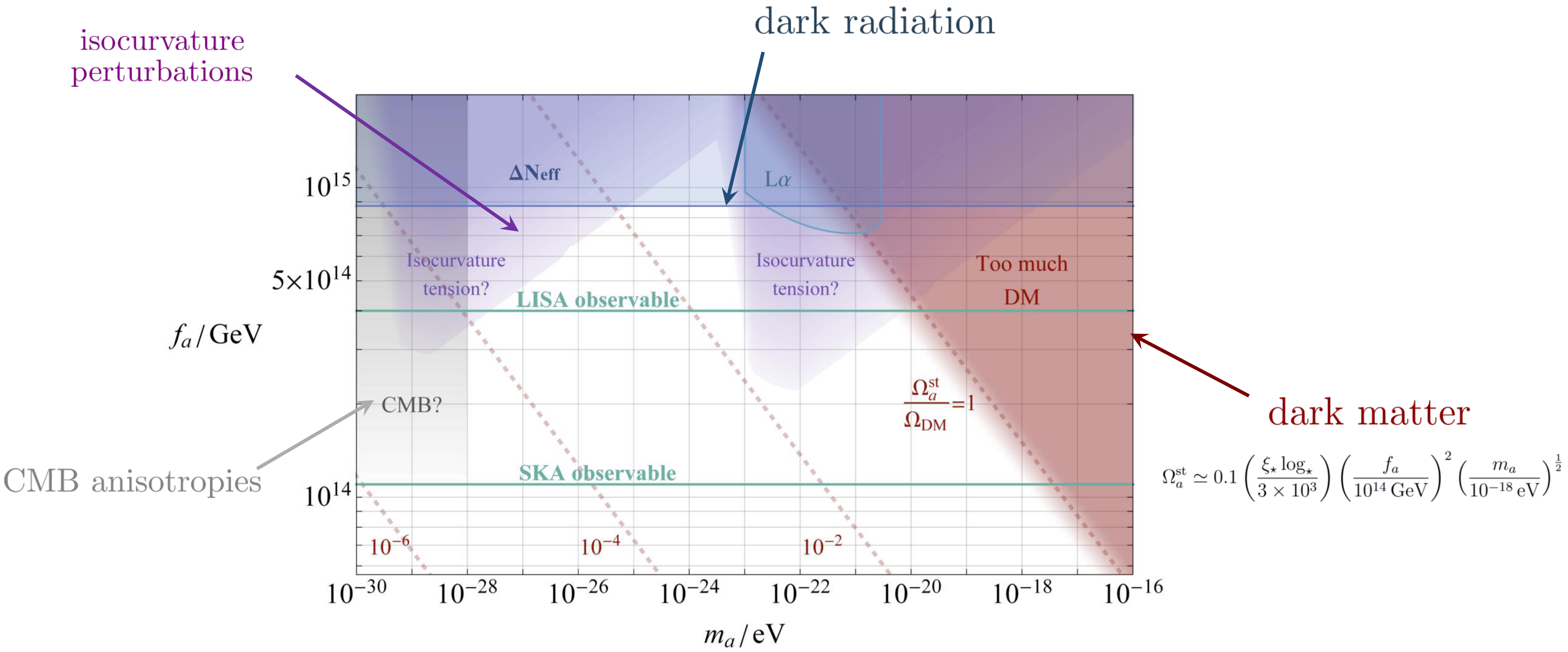
- approximately scale invariant
- $\log^4$ enhancement

$$\frac{d\Omega_{\rm gw}h^2}{d\log f} \simeq 10^{-15}\left(\frac{r}{0.26}\right)\left(\frac{f_a}{10^{14}{\rm GeV}}\right)^4\left(\frac{10}{g_f}\right)^{\frac{1}{3}}\left\{1+0.12\log\left[\left(\frac{m_r}{10^{14}{\rm GeV}}\right)\left(\frac{10^{-8}{\rm Hz}}{f}\right)^2\right]\right\}^4$$

$$\begin{cases} f_a \lesssim 10^{15}\ \text{GeV} \\ m_a \lesssim 10^{-18}\ \text{eV} \end{cases}$$



Bounds on the Post-Inflationary Scenario



Conclusions

- **Axions are motivated BSM candidates**

  → in the post-inflationary scenario, the cosmological evolution is governed by cosmic strings

- **The scaling regime produces an approximately scale invariant GW spectrum**

  → $\Gamma_g \propto \log^4$ (from the increase in $\mu \propto \log$ and $\xi \propto \log$) leads to logarithmic violations of scale invariance

  → enhances the spectrum at low frequencies

- **The spectrum is visible by multiple experiments for $f_a > 10^{14}$ GeV**

  → best prospects in PTAs and LISA

- **Constraints on the post-inflationary scenario**

  → $f_a \lesssim 10^{15}$ GeV and $m_a = 10^{-28} \div 10^{-18}$ eV is viable

Outlook

- Local strings?
- (Initial conditions for the subsequent evolution?)