

# How warm are non-thermal relics? Out-of-equilibrium dark matter production

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Institut d'Astrophysique de Paris – GReCO – Webinar

Based on [[arXiv:2011.13458](https://arxiv.org/abs/2011.13458)]  
in collaboration with **G. Ballesteros & M. A. G. Garcia**

# Outline

## 1. Introduction

1. Motivation
2. Reheating in a nutshell

## 2. Production of out-of-equilibrium dark matter

1. The dark matter phase space distribution
2. Decay of condensate
3. Ultra-violet freeze-in via scattering
4. Freeze-in via decay

## 3. Cosmological imprint

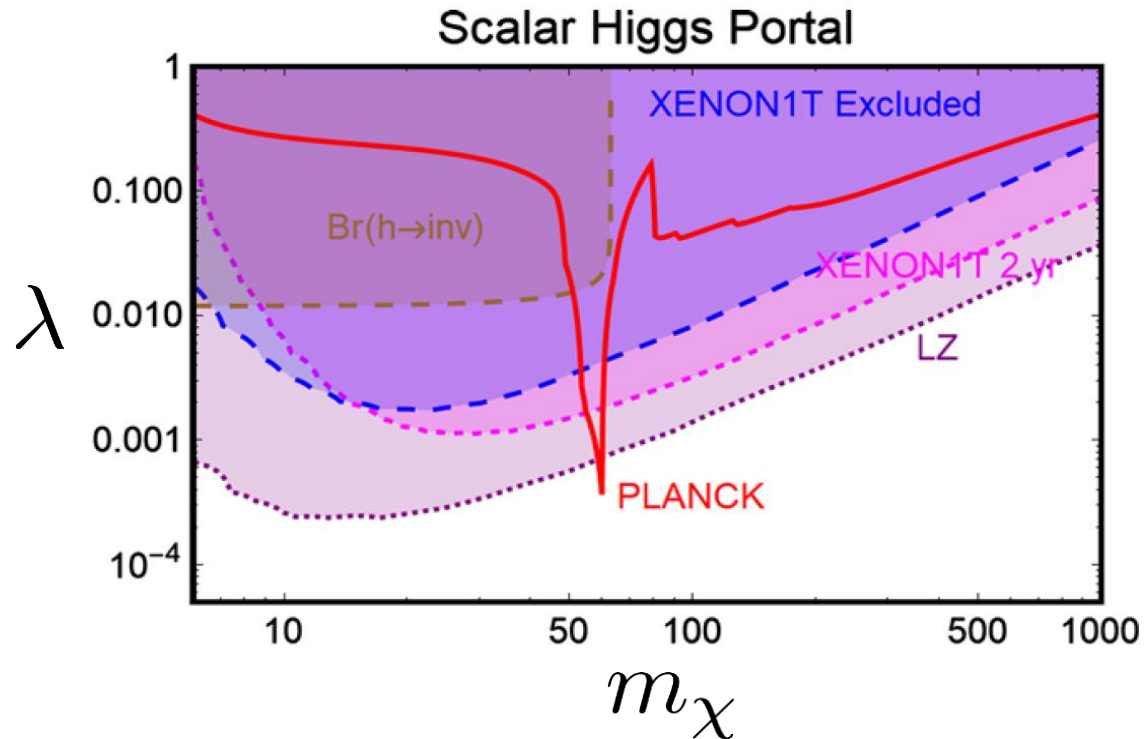
1. Non-Cold Dark Matter : analytical arguments
2. Matter power spectrum and Lyman-alpha forest

# The waning of the WIMP?

- Most minimal Weakly Interacting Massive Particles (**WIMP**) model based on “**freeze-out**” : introduce **dark matter scalar**  $\chi$

$$\mathcal{L} = \lambda |\chi|^2 |H|^2$$

- **DM thermalizes and annihilate to SM**
- Sizable **coupling!**



- **Minimalistic WIMP models under siege!**

[M. Escudero, A. Berlin, D. Hooper, M.-X. Lin - JCAP 12 (2016) 029]

[G. Arcadi, M. Dutra, P. Ghosh, M. Lindner, Y. Mambrini, **MP**, S. Profumo, F. S. Queiroz EPJC 78 (2018) 203]

[G. Arcadi, A. Djouadi, M. Raidal - Phys.Rept. 842 (2020) 1-180]

# The dawn of FIMP?

- Feebly Interacting Massive Particles (**FIMP**) models based on “**freeze-in**” opens up **parameter space**!

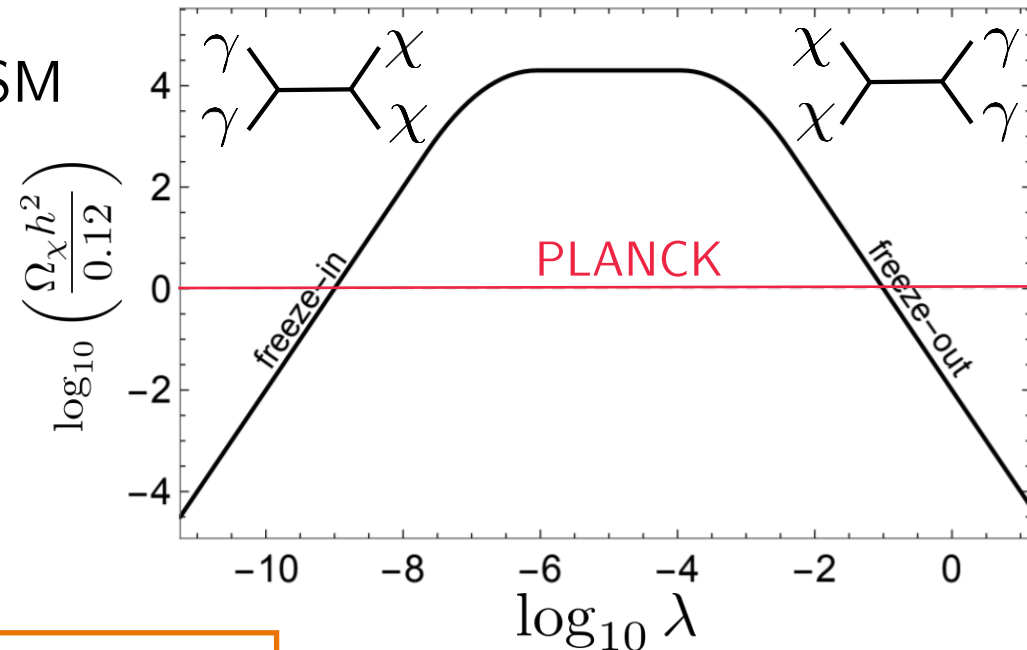
[J. McDonald PRL 88 (2002) 091304 - K.-Y. Choi, L. Roszkowski AIP Conf.Proc. 805 (2005) 1, 30-36

Kusenko PRL 97 (2006) 241301 - K. Petraki, A. Kusenko PRD 77 (2008) 065014

L. J. Hall, K. Jedamzik, J. March-Russell, S. M. West JHEP 03 (2010) 080

N. Bernal, M. Heikinheimo, T. Tenkanen, K. Tuominen and V. Vaskonen – IJMP A 32 (2017) 27, 1730023]

- DM **never thermalizes** with SM



$\chi$  : Dark Matter (DM) particles  
 $\gamma$  : Standard Model (SM) particles

# The dawn of FIMP?

- Feebly Interacting Massive Particles (**FIMP**) models based on “**freeze-in**” opens up **parameter space**!

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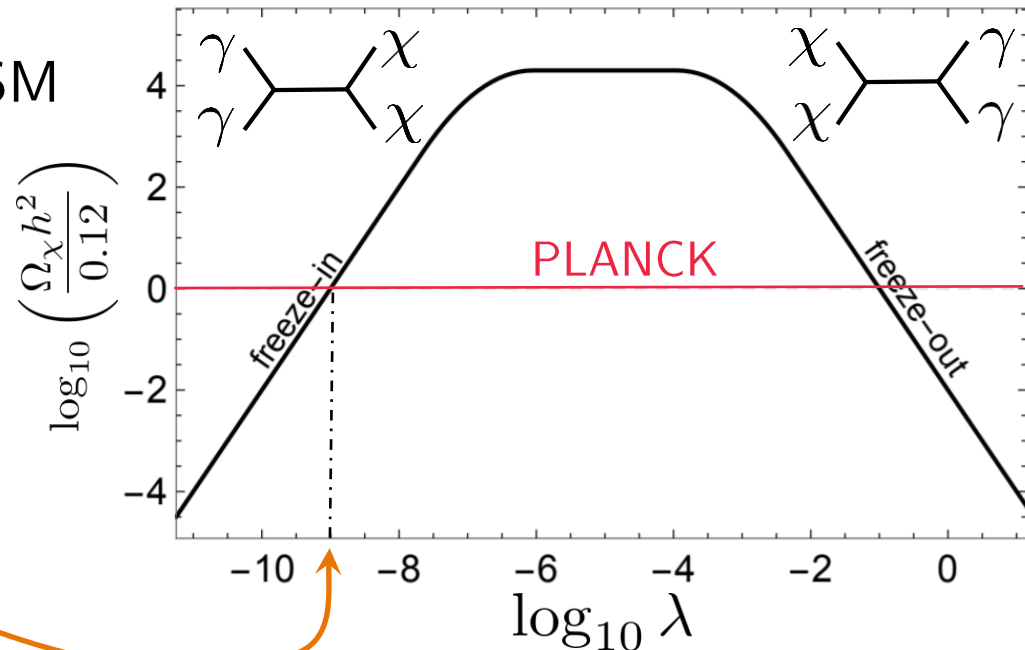
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N. Bernal, M. Heikinheimo, T. Tenkanen, K. Tuominen and V. Vaskonen – IJMP A 32 (2017) 27, 1730023]

- DM **never thermalizes** with SM

- **DM produced from SM annihilations**

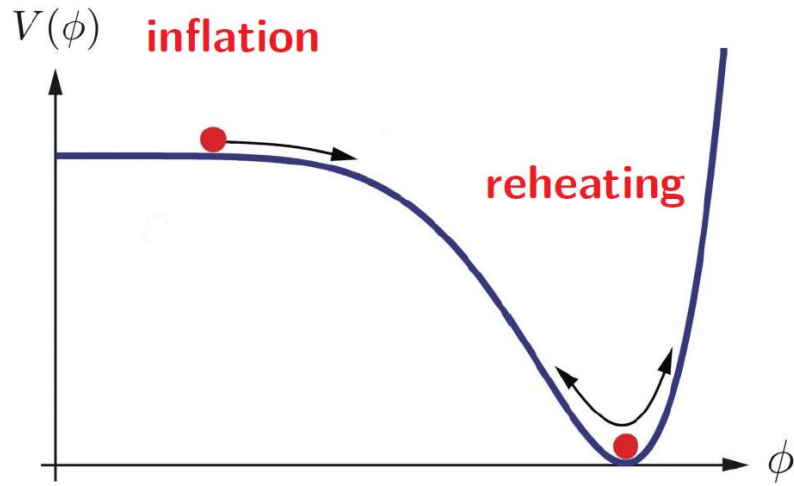
- Requires **small coupling**!



...introduce instead **large seclusion** scale between SM and DM?

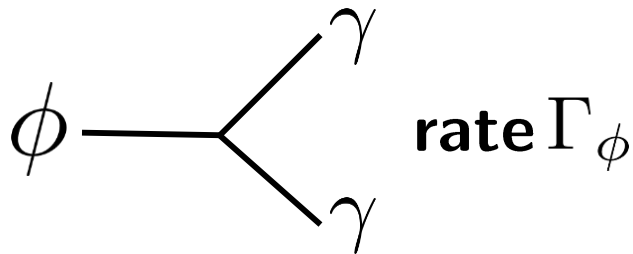
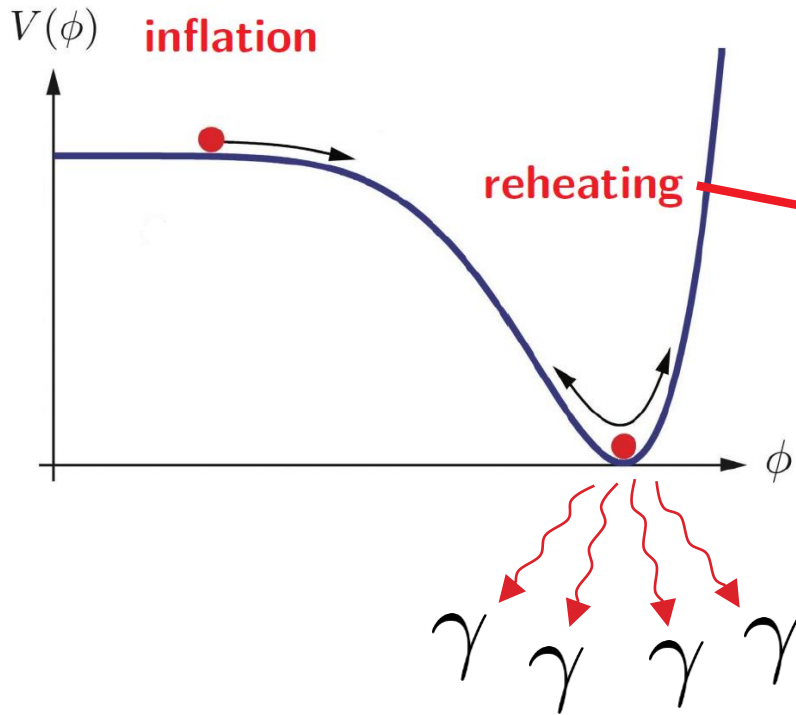
➡ **Ultra-violet (UV) freeze-in** ➡ post **inflation** production!

# Reheating in a nutshell



$\phi$  : Inflaton field

# Reheating in a nutshell



For a **quadratic** potential

$$\frac{d\rho_\phi}{dt} + \textcircled{3}H\rho_\phi = -\Gamma_\phi\rho_\phi$$

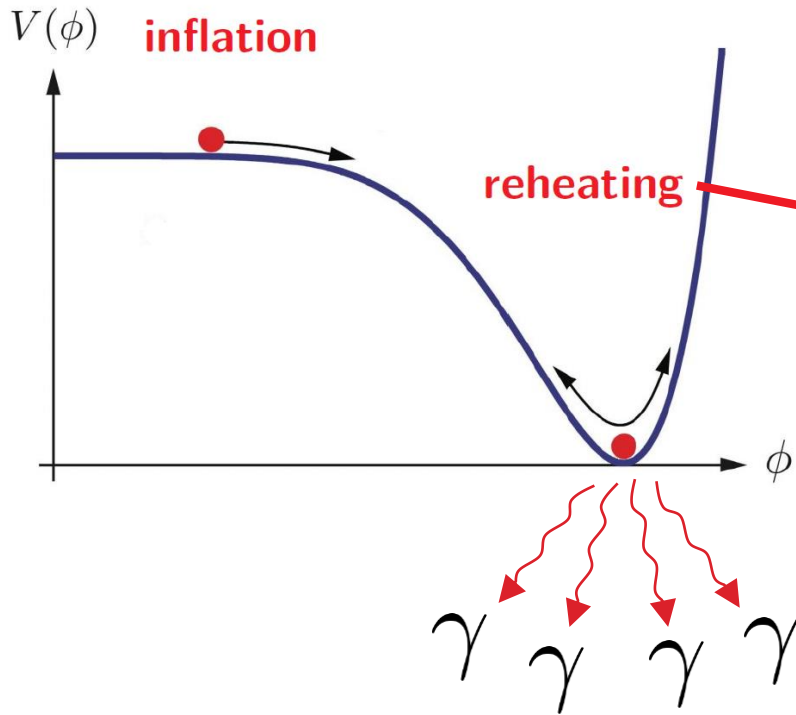
$$\frac{d\rho_\gamma}{dt} + 4H\rho_\gamma = \Gamma_\phi\rho_\phi$$

$$H^2 = \frac{1}{3M_{\text{Pl}}^2}(\rho_\phi + \rho_\gamma)$$

For **non-quadratic** potentials:

[M. A. G. Garcia, K. Kaneta, Y. Mambrini, K. A. Olive PRD 101, 123507 (2020)]

# Reheating in a nutshell



For a **quadratic** potential

$$\frac{d\rho_\phi}{dt} + 3H\rho_\phi = -\Gamma_\phi\rho_\phi$$

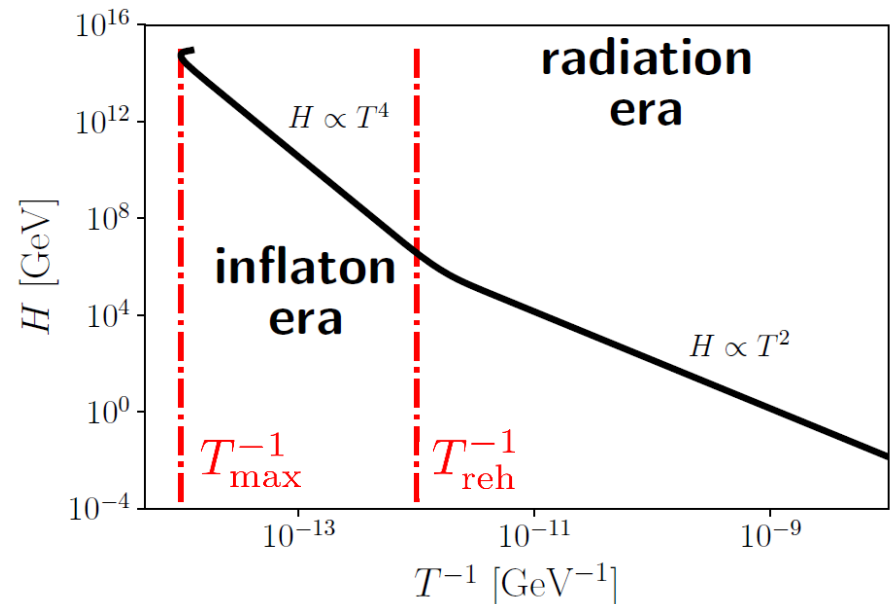
$$\frac{d\rho_\gamma}{dt} + 4H\rho_\gamma = \Gamma_\phi\rho_\phi$$

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$T_{\text{reh}}$  : **Reheating temperature**

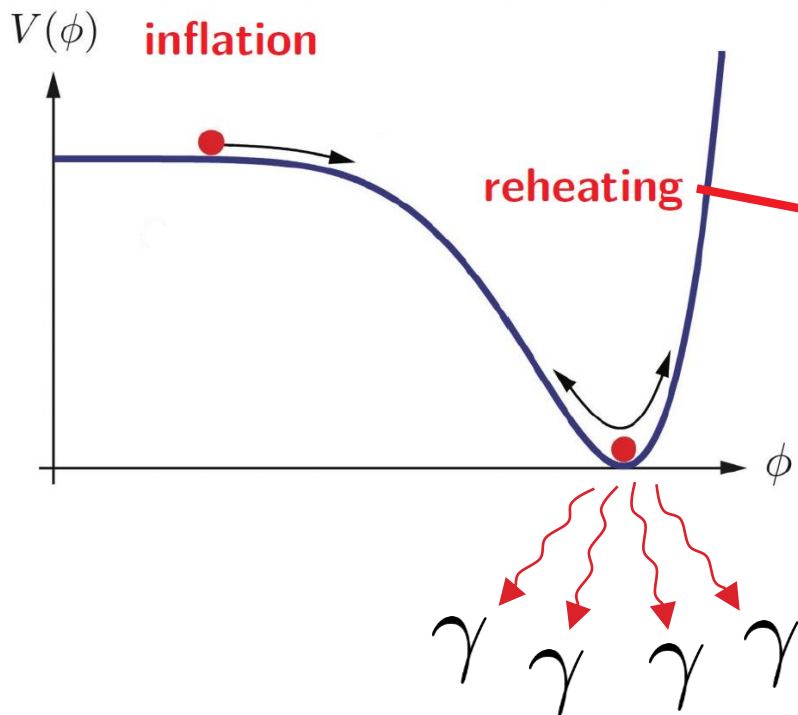
$$\rho_\phi(T_{\text{reh}}) = \rho_\gamma(T_{\text{reh}})$$

$$(\Leftrightarrow t_{\text{reh}} \simeq \Gamma_\phi^{-1})$$





# Reheating in a nutshell



For a **quadratic** potential

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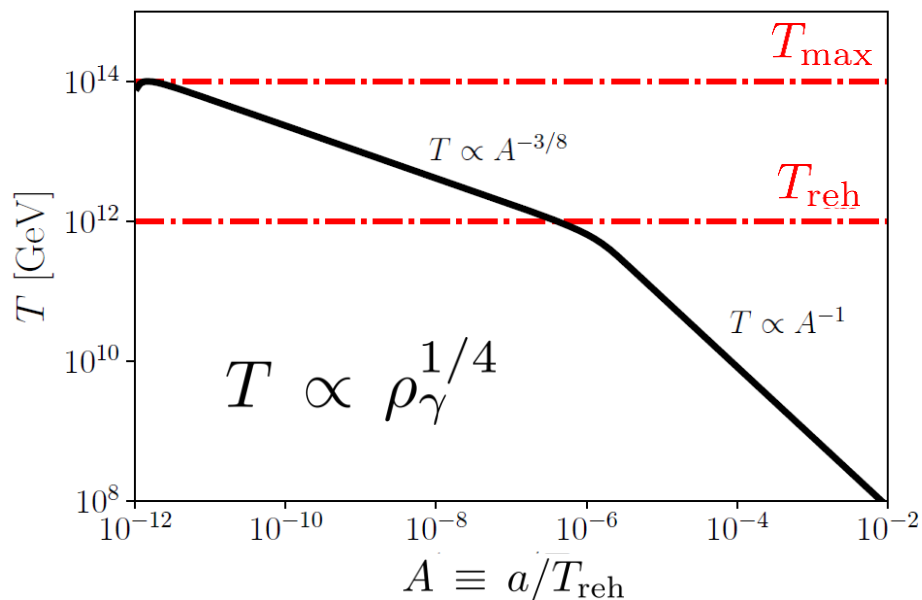
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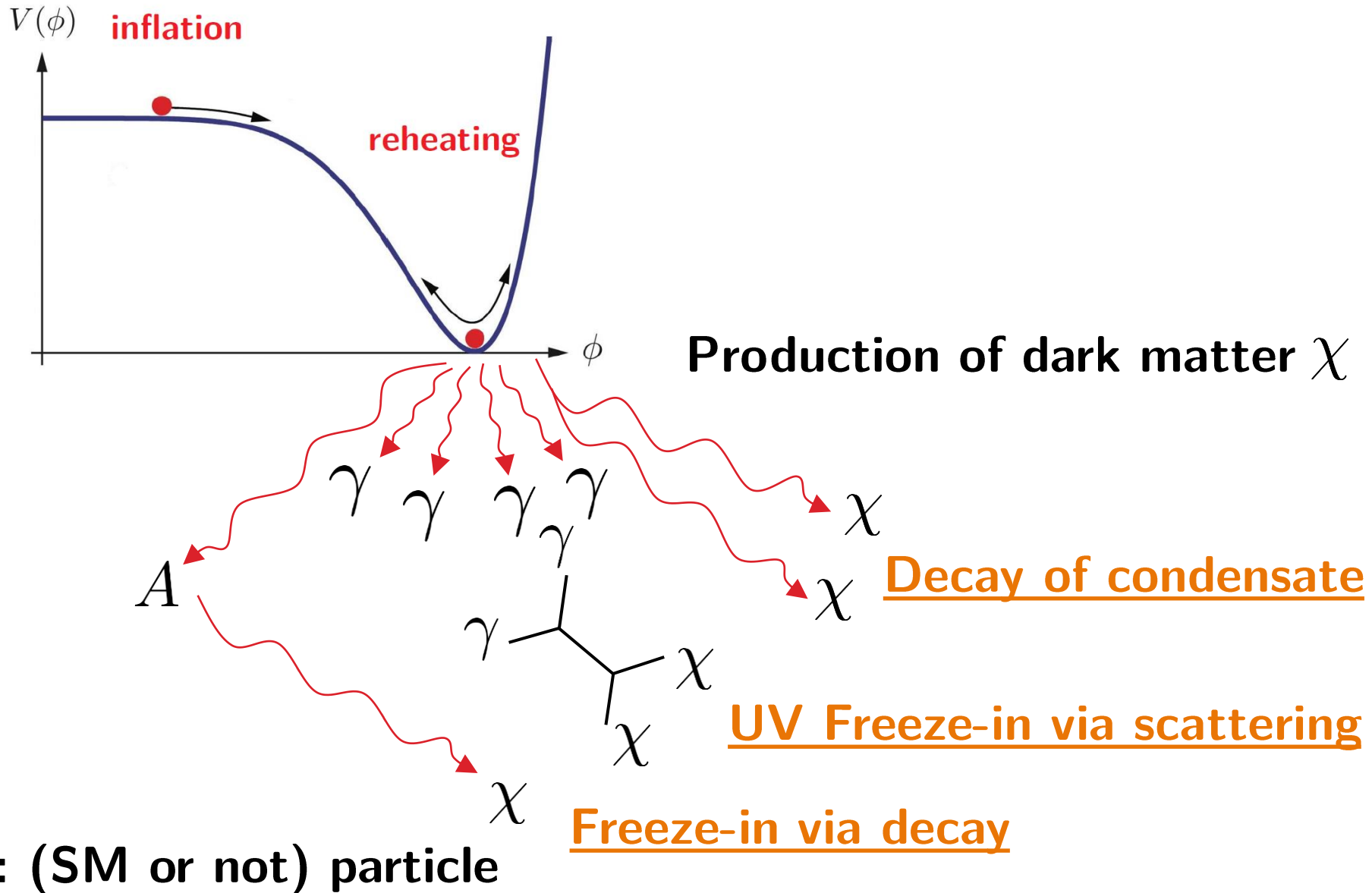
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$$(\Leftrightarrow t_{\text{reh}} \simeq \Gamma_\phi^{-1})$$



# DM production during reheating



# Production of out-of-equilibrium dark matter

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# DM Phase space distribution

- Obtain **phase space distribution** by solving **Boltzmann equation**

$$\frac{\partial f_\chi}{\partial t} - H|\mathbf{p}| \frac{\partial f_\chi}{\partial |\mathbf{p}|} = \mathcal{C}[f_\chi(|\mathbf{p}|, t)]$$

$$n_\chi(t) = \frac{g_\chi}{(2\pi)^3} \int d^3\mathbf{p} f_\chi(p_0, t)$$

**number-density**

$$\rho_\chi(t) = \frac{g_\chi}{(2\pi)^3} \int d^3\mathbf{p} p_0 f_\chi(p_0, t)$$

**energy-density**

- Collision term** for processes  $\chi + a + b + \dots \longleftrightarrow i + j + \dots$

$$\begin{aligned} \mathcal{C}[f_\chi] = & -\frac{1}{2p_0} \int \frac{g_a d^3\mathbf{p}_a}{(2\pi)^3 2p_{a0}} \frac{g_b d^3\mathbf{p}_b}{(2\pi)^3 2p_{b0}} \dots \frac{g_i d^3\mathbf{p}_i}{(2\pi)^3 2p_{i0}} \frac{g_j d^3\mathbf{p}_j}{(2\pi)^3 2p_{j0}} \dots \\ & \times (2\pi)^4 \delta^{(4)}(p_\chi + p_a + p_b + \dots - p_i - p_j - \dots) \\ & \times \left[ |\mathcal{M}|_{\chi+a+b+\dots \rightarrow i+j+\dots}^2 f_a f_b \dots f_\chi (1 \pm f_i)(1 \pm f_j) \dots \right. \\ & \left. - |\mathcal{M}|_{i+j+\dots \rightarrow \chi+a+b+\dots}^2 f_i f_j \dots (1 \pm f_a)(1 \pm f_b) \dots (1 \pm f_\chi) \right] \end{aligned}$$

# DM Phase space distribution

- **Solution** to the Boltzmann equation gives

$$f_\chi(p_0, t) = \int_{t_i}^t \mathcal{C}[f_\chi] \left( \frac{a(t)}{a(t')} |\mathbf{p}|, t' \right) dt'$$

- For  $t > t_{\text{dec}}$  after **decoupling** (when production stops)

$$\frac{\partial f_\chi}{\partial t} - H |\mathbf{p}| \frac{\partial f_\chi}{\partial |\mathbf{p}|} = 0 \quad \longrightarrow \quad f_\chi(|\mathbf{p}|, t) = \bar{f} \left( |\mathbf{p}| \frac{a(t)}{a_{\text{dec}}}, t_{\text{dec}} \right)$$

- After decoupling, distribution function only depends on the **comoving momentum**

$$q \equiv \frac{p a(t)}{T_\star} \quad \longrightarrow \quad n_\chi(t) = \frac{g_\chi}{(2\pi)^3} \frac{T_\star^3}{a^3} \int d^3 \mathbf{q} \bar{f}_\chi(q)$$

$T_\star \equiv T_{\text{NCDM}}$  in **CLASS** [J. Lesgourgues & T. Tram, JCAP 09 (2011) 032]

# Production from inflaton decay

- Pair of **DM** or **SM** produced from **perturbative** inflaton decay

$$\begin{array}{ccc} \gamma & \longleftarrow & \phi & \longrightarrow & \gamma \\ \chi & & & & \chi \end{array}$$

$$\mathcal{C}[f(p, t)] = \frac{8\pi^2}{gm_\phi^2} \Gamma_\phi \text{Br } n_\phi(t) \delta(p - m_\phi/2)$$

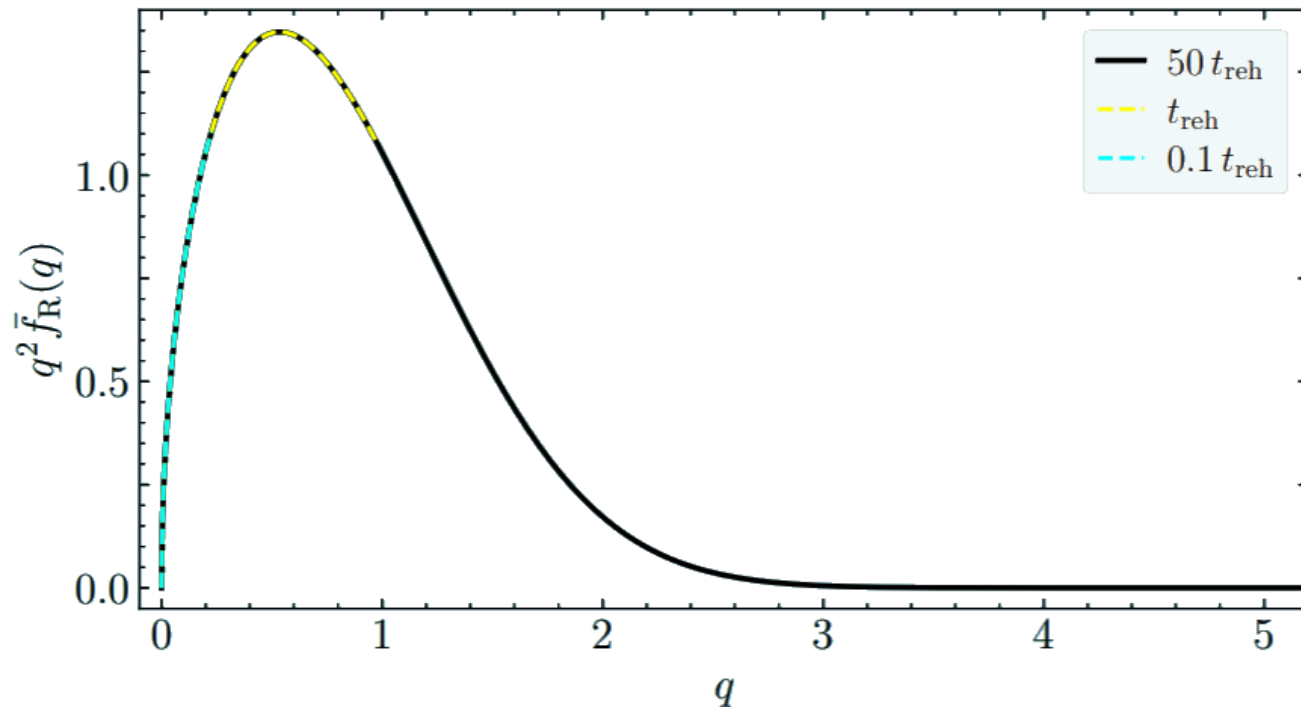
$$f(p, t) = \frac{16\pi^2 \Gamma_\phi \text{Br } n_\phi(\hat{t})}{gm_\phi^3 H(\hat{t})} \theta(t - \hat{t}) \quad \frac{a(t)}{a(\hat{t})} = \frac{m_\phi}{2p}$$

$t \ll t_{\text{reh}}$

$$f(p, t) \simeq \frac{24\pi^2 n(t)}{gm_\phi^3} \left( \frac{m_\phi}{2p} \right)^{3/2} \theta(m_\phi/2 - p)$$

# Inflaton decay : DM production

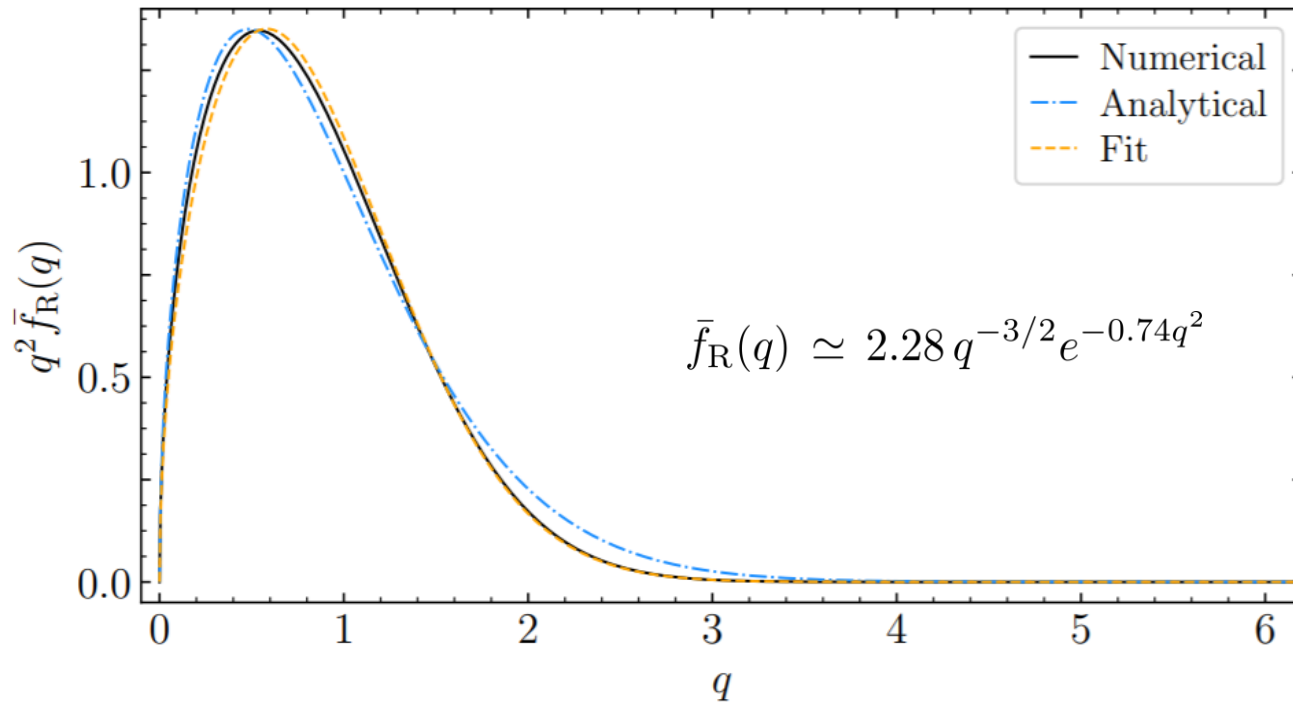
$$f_\chi(p, t) d^3\mathbf{p} = \frac{4\pi^4 \text{Br}_\chi g_{*s}^{\text{reh}}}{5g_\chi} \left(\frac{T_{\text{reh}}}{m_\phi}\right)^4 \left(\frac{a_0}{a(t)}\right)^3 T_\star^3 \bar{f}_R(q) d^3\mathbf{q} \quad T_\star = \left(\frac{g_{*s}^0}{g_{*s}^{\text{reh}}}\right)^{1/3} \frac{m_\phi}{2T_{\text{reh}}} T_0$$



$$\Omega_\chi h^2 \simeq 0.1 \left(\frac{\text{Br}_\chi}{5.5 \times 10^{-4}}\right) \left(\frac{m_\chi}{1 \text{ MeV}}\right) \left(\frac{T_{\text{reh}}}{10^{10} \text{ GeV}}\right) \left(\frac{3 \times 10^{13} \text{ GeV}}{m_\phi}\right)$$

# Inflaton decay : DM production

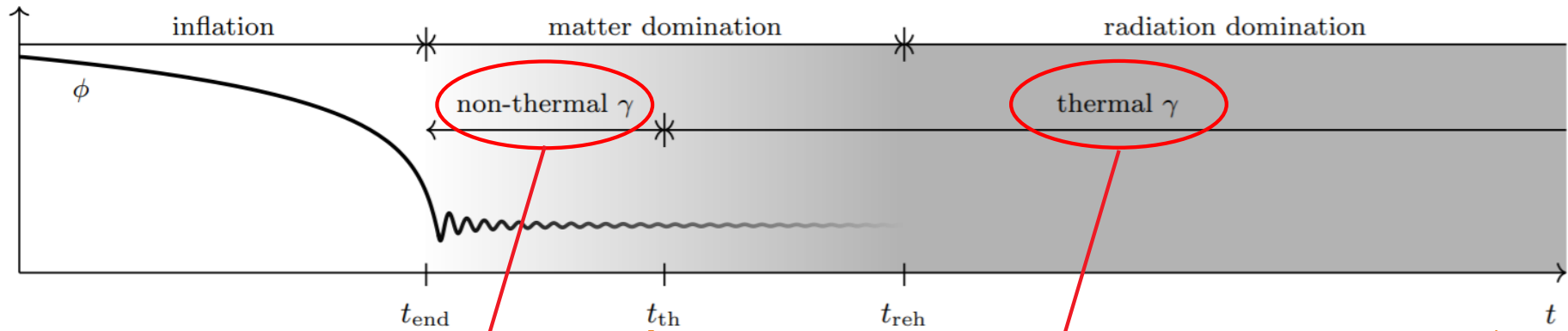
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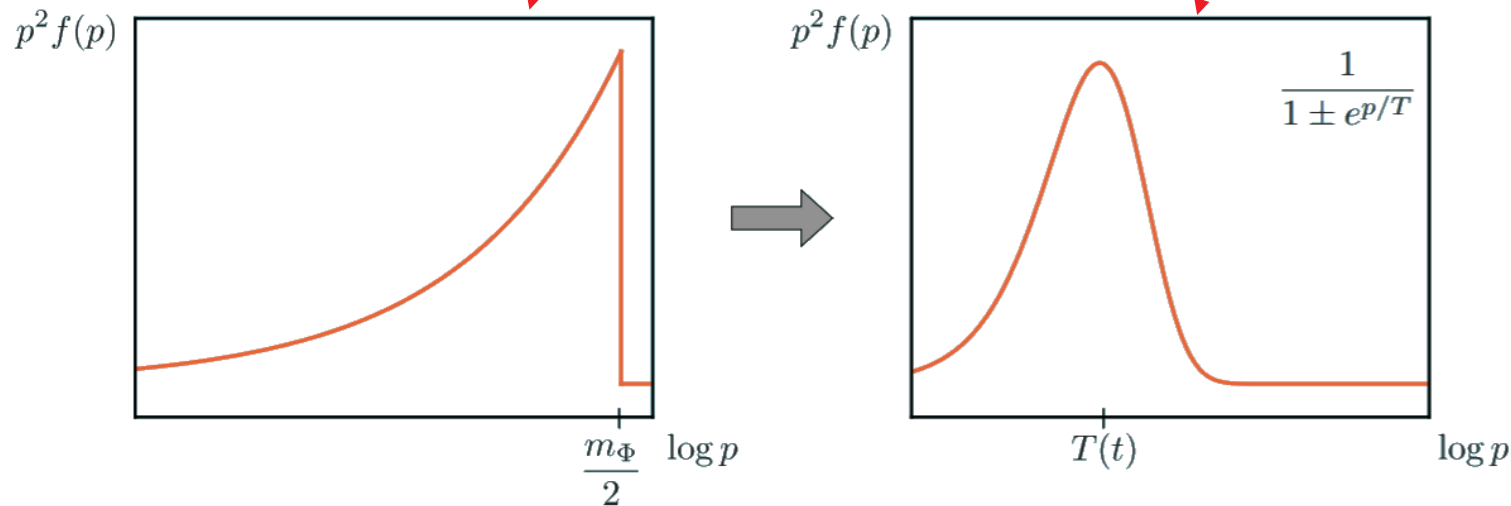
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# Inflaton decay : SM production



[M. A. G. Garcia & M. A. Amin Phys. Rev. D 98, 103504 (2018)]

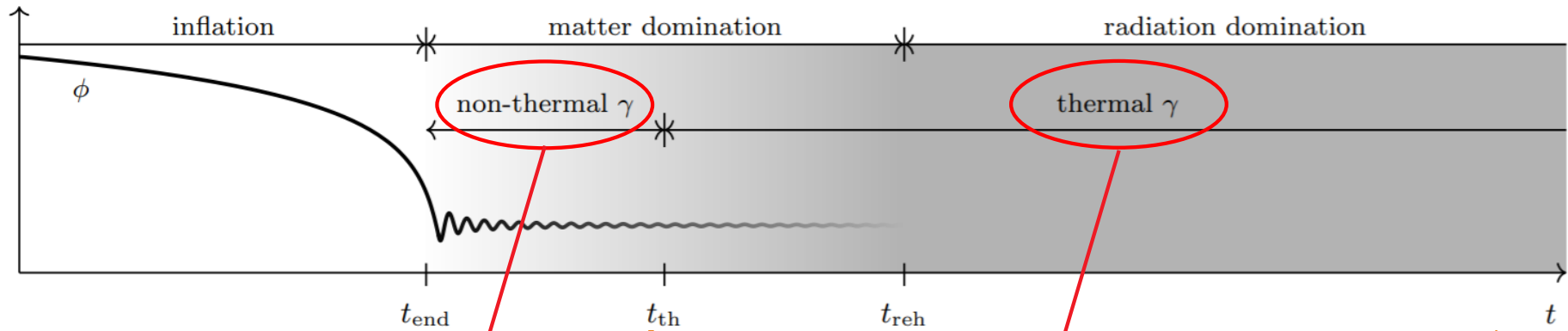


$$\Gamma_\phi t_{\text{th}} \simeq \alpha_{\text{SM}}^{-16/5} \left( \frac{\Gamma_\phi m_\phi^2}{M_{\text{Pl}}^3} \right)^{2/5}$$

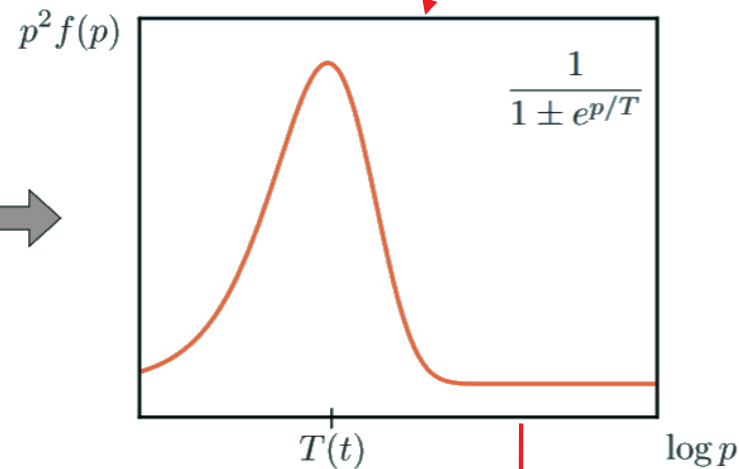
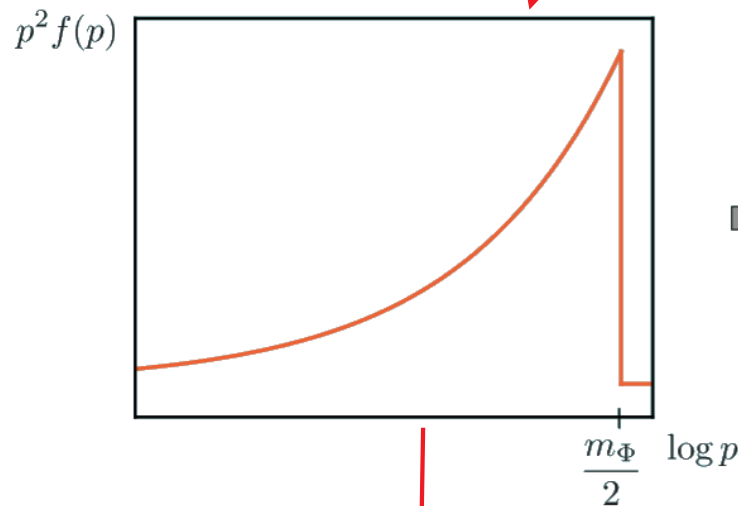
[K. Mukaida & M. Yamada, JCAP 02 (2016) 003]

[K. Harigaya & K. Mukaida, JHEP 05 (2014) 006]

# Inflaton decay : SM production



[M. A. G. Garcia & M. A. Amin Phys. Rev. D 98, 103504 (2018)]



• **Non-thermal UV freeze-in**

• **Thermal UV freeze-in**

# UV freeze-in from scattering

$$\int d^3\mathbf{p} \left( \frac{\partial f_\chi}{\partial t} - H|\mathbf{p}| \frac{\partial f_\chi}{\partial |\mathbf{p}|} = \mathcal{C}[f_\chi(|\mathbf{p}|, t)] \right) \quad \begin{array}{l} \gamma(p_A) \quad \chi \\ \gamma(p_B) \quad \chi \end{array}$$

$$\frac{dn_\chi}{dt} + 3Hn_\chi = R(t)$$

$$R(t) \equiv 2g_A g_B g_\chi^2 \int \frac{d^3\mathbf{p}_A}{(2\pi)^3 2p_1^0} \frac{d^3\mathbf{p}_B}{(2\pi)^3 2p_2^0} s \sigma(s) f_A(p_A) f_B(p_B) : \text{Production rate}$$

- Assume **cross section** for  $\gamma\gamma \rightarrow \chi\chi$  :  $\sigma(s) = \frac{s^{\frac{n}{2}}}{\Lambda^{n+2}}$

- If production from **thermal** SM :  $R(T) \propto \frac{T^{n+6}}{\Lambda^{n+2}}$

- Production occurs around  $t \lesssim t_{\text{reh}}$  for  $n > -1$

$\Lambda$  : High energy scale

$s$  : Center-of-mass energy

# UV freeze-in from scattering

- $n = 0$  **Low-scale SUSY** for **gravitinos**  $\sigma \propto 1/M_{\text{Pl}}^2$  or **axinos**  $\sigma \propto 1/f_a^2$   
[V. Rychkov, A. Strumia, PRD 75 (2007) 075011 - A. Strumia, JHEP 06 (2010) 036]
- $n = 2$  | **Heavy  $Z'$**  from gauge unification  $\sigma \propto s/m_{Z'}^4$   
[Y. Mambrini, K. A. Olive, J. Quevillon, B. Zaldivar- PRL 110, 241306]  
**Gravity** mediated freeze-in  $\sigma \propto s/M_{\text{Pl}}^4$   
[M. Garny, M. Sandora, M. S. Sloth - PRL 116 (2016) 10, 101302  
N. Bernal, M. Dutra, Y. Mambrini, K. Olive, M. Peloso, **MP** - PRD 97 (2018) 11, 115020]
- $n = 4$  **Non-SUSY Spin-3/2 DM** + sterile neutrino  $\sigma \propto s^2/(m_{3/2}m_R M_{\text{Pl}})^2$   
[M A. G. Garcia, Y. Mambrini, K. A. Olive, S. Verner - PRD 102 (2020) 8, 083533]

Derivative coupling  $\Leftrightarrow$  Center-of-mass energy dependence

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- $n = 6$  **High-scale SUSY** for **gravitinos** DM  $\sigma \propto s^3/(m_{3/2}M_{\text{Pl}})^4$   
[K. Benakli, Y. Chen, E. Dudas, Y. Mambrini - PRD 95 (2017) 9, 095002]  
**Heavy Spin-2 mediator**  $\sigma \propto s^3/(m_{\tilde{h}}M_{\text{Pl}})^4$   
[N. Bernal, M. Dutra, Y. Mambrini, K. Olive, M. Peloso, **MP** - PRD 97 (2018) 11, 115020]  
**Energy-momentum tensor portal**  
[P. Anastasopoulos, K. Kaneta, Y. Mambrini, **MP** - PRD 102 (2020) 5, 055019]  
**Disformal** dark matter [P. Brax, K. Kaneta, Y. Mambrini, **MP** - PRD 2021 ?????]
- $n > 6$  **Heavy  $Z'$**  and **non-abelian vector DM**  
[G. Bhattacharyya, M. Dutra, Y. Mambrini, **MP** - PRD (2018) 3, 035038]

# UV freeze-in from scattering ( $n < 6$ )

$$f_\chi(p, t) d^3\mathbf{p} \simeq \left(\frac{6b}{g_{*s}^{\text{reh}}}\right)^{1/2} \frac{3 \cdot 2^{n+6} \Gamma(\frac{n+4}{2}) g_A g_B g_\psi M_P T_{\text{reh}}^{n+1}}{5(2\pi)^3 \Lambda^{n+2}} \left(\frac{a_0}{a(t)}\right)^3 T_\star^3 \bar{f}_{\text{TF}}^{(n)}(q) d^3\mathbf{q}$$

$$T_\star = \left(\frac{g_{*s}^0}{g_{*s}^{\text{reh}}}\right)^{1/3} T_0$$

- **Mild** dependence on progenitor **spin**

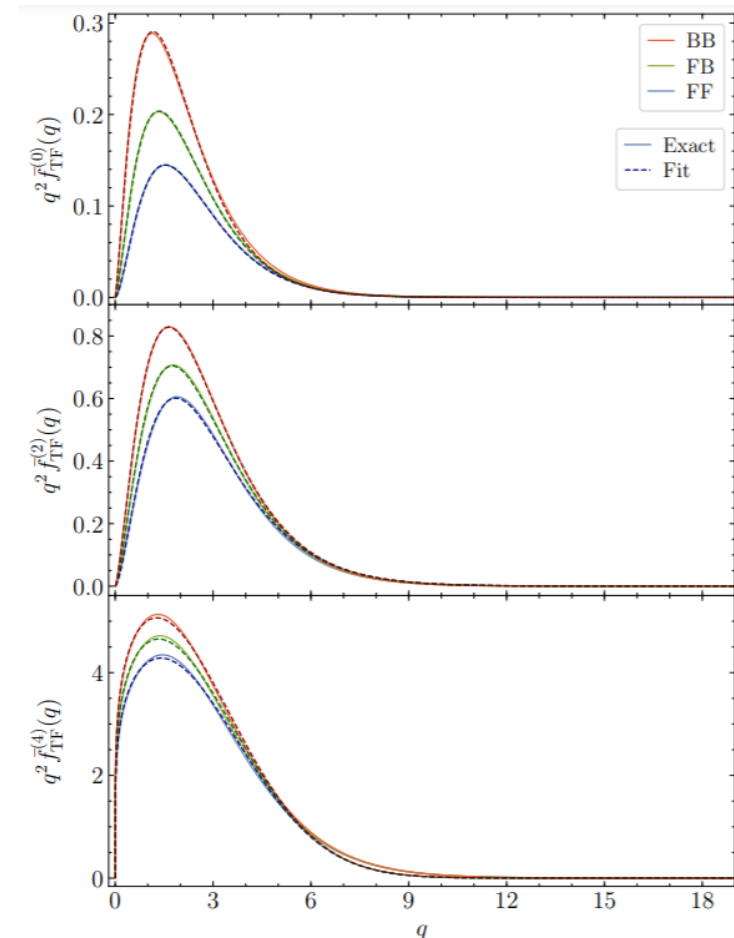
**B** : Bose-Einstein

**F** : Fermi-Dirac

- **Maxwell-Boltzmann**  $\simeq$  **FB**

- Not thermal but well **fitted** by

$$f(q) \propto q^\alpha \exp(-\beta q^\gamma)$$



# UV freeze-in from scattering ( $n < 6$ )

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$$\Omega_\chi^{(n)} h^2 \simeq \frac{g_{AGB} g_\psi g_\chi \sqrt{b} 2^{n+3} \Gamma(\frac{n}{2} + 3)^2 \zeta(\frac{n}{2} + 3)^2 \mathcal{S}(n)}{(6-n)(n+4)} \left(\frac{106.75}{g_{*s}^{\text{reh}}}\right)^{3/2} \\ \times \left(\frac{T_{\text{reh}}}{\Lambda}\right)^{n+1} \left(\frac{10^{16} \text{ GeV}}{\Lambda}\right) \left(\frac{m_{\text{NCDM}}}{1 \text{ keV}}\right)$$

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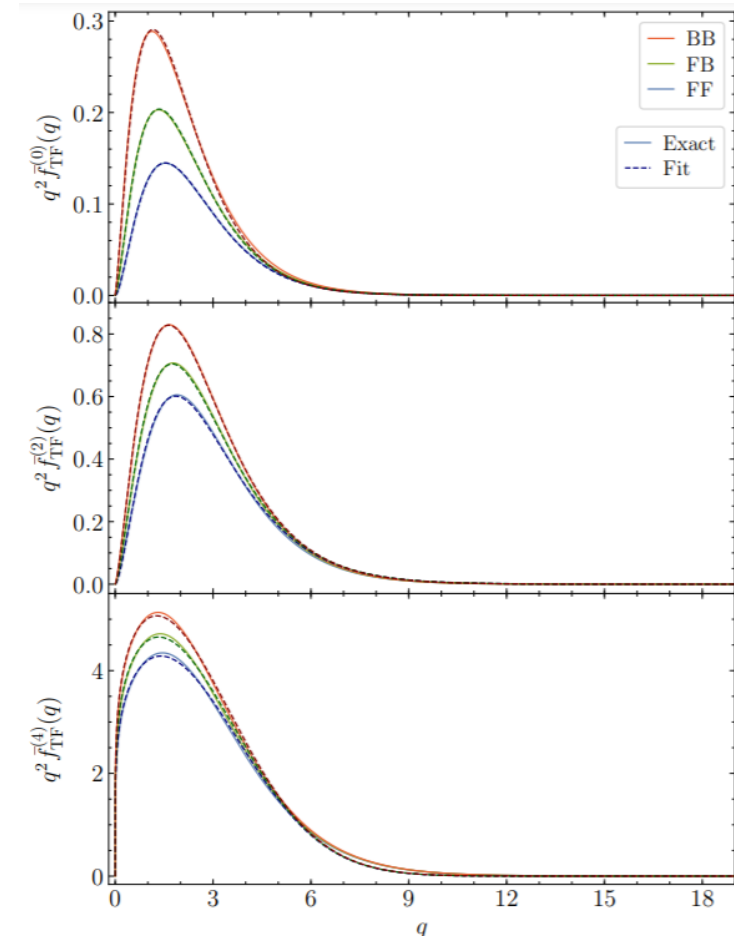
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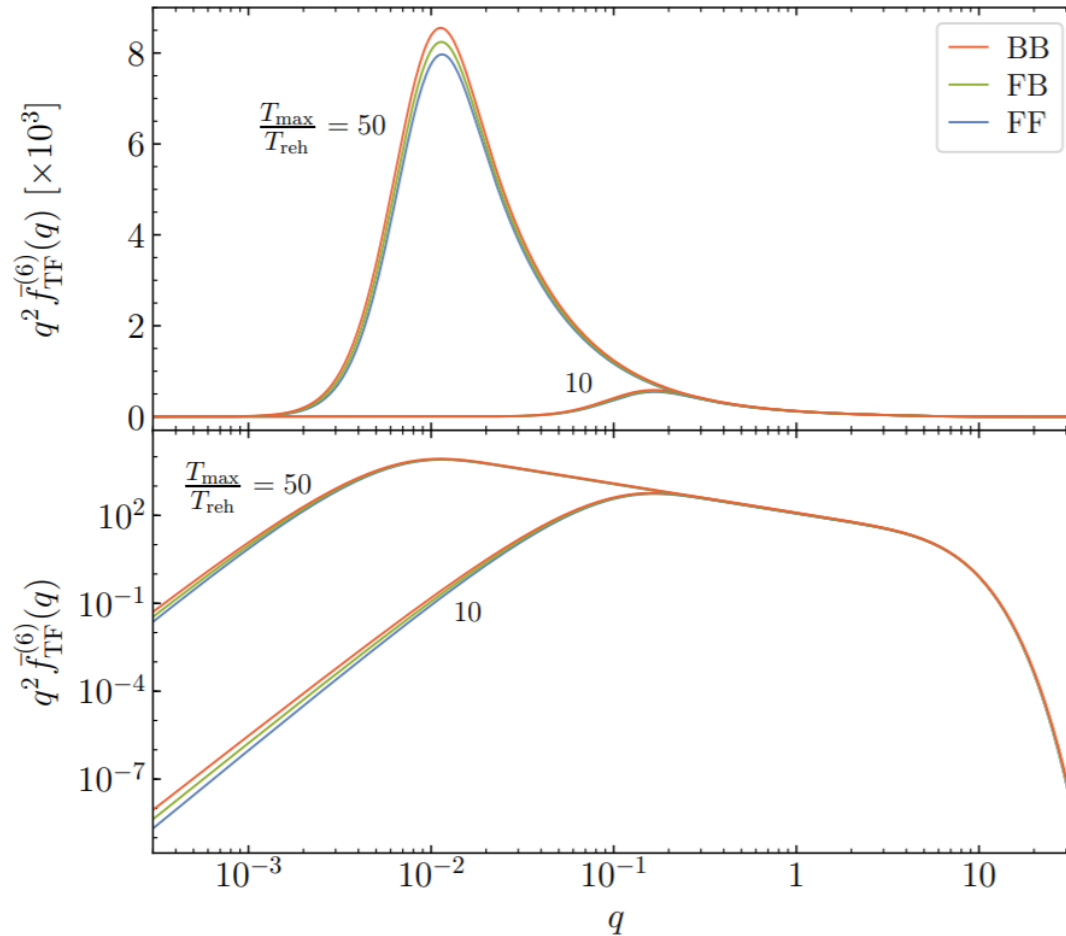
- Not thermal but well **fitted** by

$$f(q) \propto q^\alpha \exp(-\beta q^\gamma)$$



# UV freeze-in from scattering ( $n = 6$ )

$$\Omega_{\chi}^{(6)} h^2 = g_A g_B g_{\psi} g_{\chi} \sqrt{b} \mathcal{S}(6) \left( \frac{106.75}{g_{*s}^{\text{reh}}} \right)^{3/2} \left( \frac{m_{\text{DM}}}{1.2 \text{ keV}} \right) \left( \frac{T_{\text{reh}}}{10^6 \text{ GeV}} \right)^7 \left( \frac{10^8 \text{ GeV}}{\Lambda} \right)^8 \ln \left( \frac{T_{\text{max}}}{T_{\text{reh}}} \right)$$

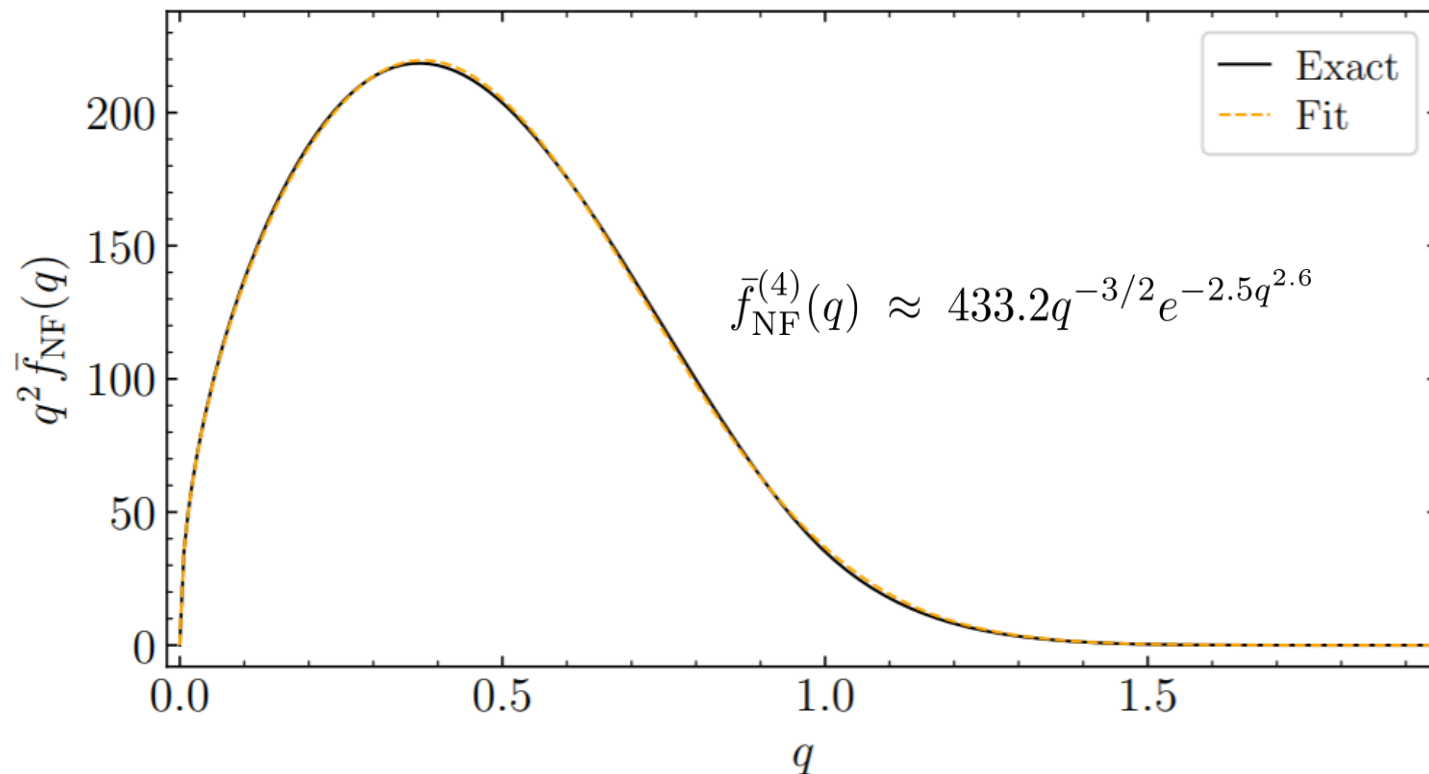




# Non-thermal UV freeze-in ( $n = 4$ )

$$f_\chi(p, t) d^3\mathbf{p} \simeq \frac{256\pi^2 g_\psi}{15015\Lambda^6} \left( \frac{\pi^2 b g_{*s}^{\text{reh}}}{24} \right)^{13/10} \left( \frac{\alpha_{\text{SM}}^{16} T_{\text{reh}}^{26} M_P^{13}}{m_\phi^9} \right)^{1/5} \left( \frac{a_0}{a(t)} \right)^3 T_\star^3 \bar{f}_{\text{NF}}^{(4)}(q) d^3\mathbf{q},$$

$$T_\star = \frac{\alpha_{\text{SM}}^{-32/15}}{2} \left( \frac{g_{*s}^0}{g_{*s}^{\text{reh}}} \right)^{1/3} \left( \frac{\pi^2 b g_{*s}^{\text{reh}}}{24} \right)^{2/15} \left( \frac{m_\phi}{T_{\text{reh}}} \right)^{7/15} \left( \frac{m_\phi}{M_P} \right)^{16/15} T_0,$$



# Freeze-in via decay $\phi \rightarrow A \rightarrow \chi$

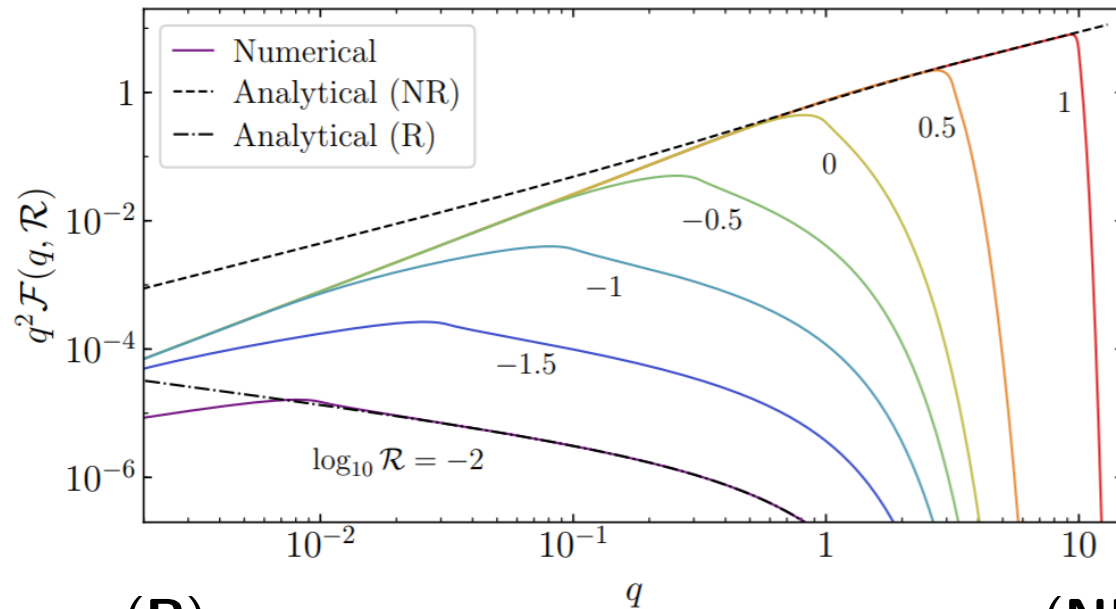
$$f_\chi(p, t) d^3\mathbf{p} = \frac{24\pi^3 \sqrt{10g_{*s}^{\text{reh}}} \text{Br}_\chi \text{Br}_A \Gamma_A M_P}{5g_A m_A^2} \left(\frac{T_{\text{reh}}}{m_\phi}\right)^2 \mathcal{F}(q, \mathcal{R}) \left(\frac{a_0}{a(t)}\right)^3 T_\star^3 d^3\mathbf{q}$$

- **Relativistic factor**  $\mathcal{R} = \left(\frac{g_{*s}^{\text{reh}}}{g_{*s}^{\text{dec}}}\right)^{1/3} \frac{m_A T_{\text{reh}}}{m_\phi T_{\text{dec}}}$  with  $\frac{m_A a(t)}{m_\phi a_{\text{reh}}} \propto \frac{m_A}{\langle p \rangle}$

$$\mathcal{F}(q, \mathcal{R}) = q^{-2} \int_0^{\mathcal{R}} dy y^2 \int_{|q-\frac{y^2}{q}|}^\infty \frac{z dz}{\sqrt{q^2 + 4y^2}} \bar{f}_R(z) \simeq \begin{cases} \bar{f}_{\text{D,NR}}(q), & \mathcal{R} \gg 1, \text{ Non-Relativistic (NR)} \\ \frac{\mathcal{R}^3}{3} \bar{f}_{\text{D,R}}(q), & \mathcal{R} \ll 1. \text{ Relativistic (R)} \end{cases}$$

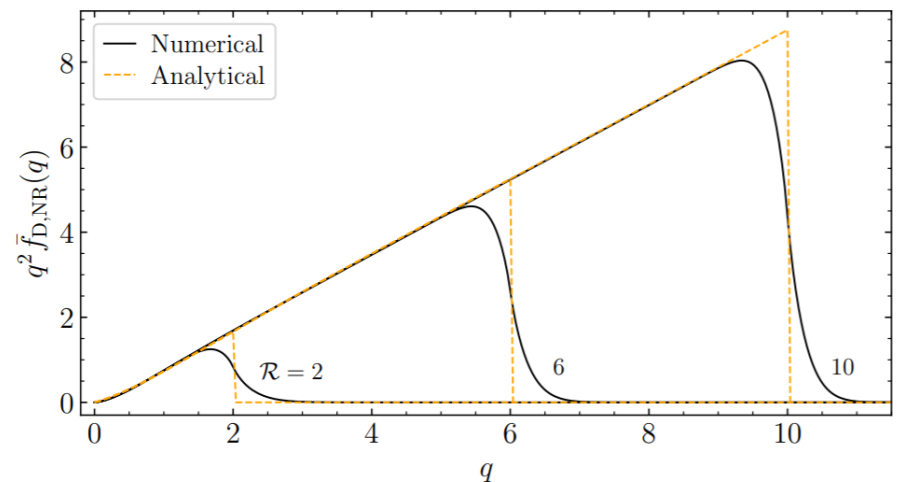
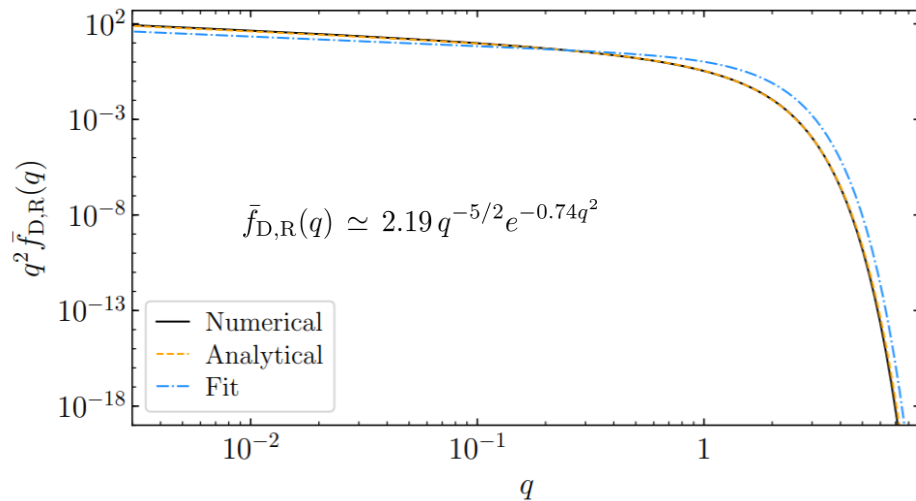
$$n_\chi(t) \simeq g_{*s}^0 \text{Br}_\chi \text{Br}_A \left(\frac{g_\chi}{g_A}\right) \left(\frac{T_{\text{reh}}}{m_\phi}\right) \left(\frac{a_0}{a(t)}\right)^3 T_0^3 \times \begin{cases} \left(\frac{g_{*s}^{\text{reh}}}{g_{*s}^{\text{dec}}}\right)^{1/6}, & \mathcal{R} \gg 1, \\ \left(\frac{g_{*s}^{\text{reh}}}{g_{*s}^{\text{dec}}}\right)^{1/4}, & \mathcal{R} \ll 1. \end{cases}$$

# Freeze-in via decay $\phi \rightarrow A \rightarrow \chi$

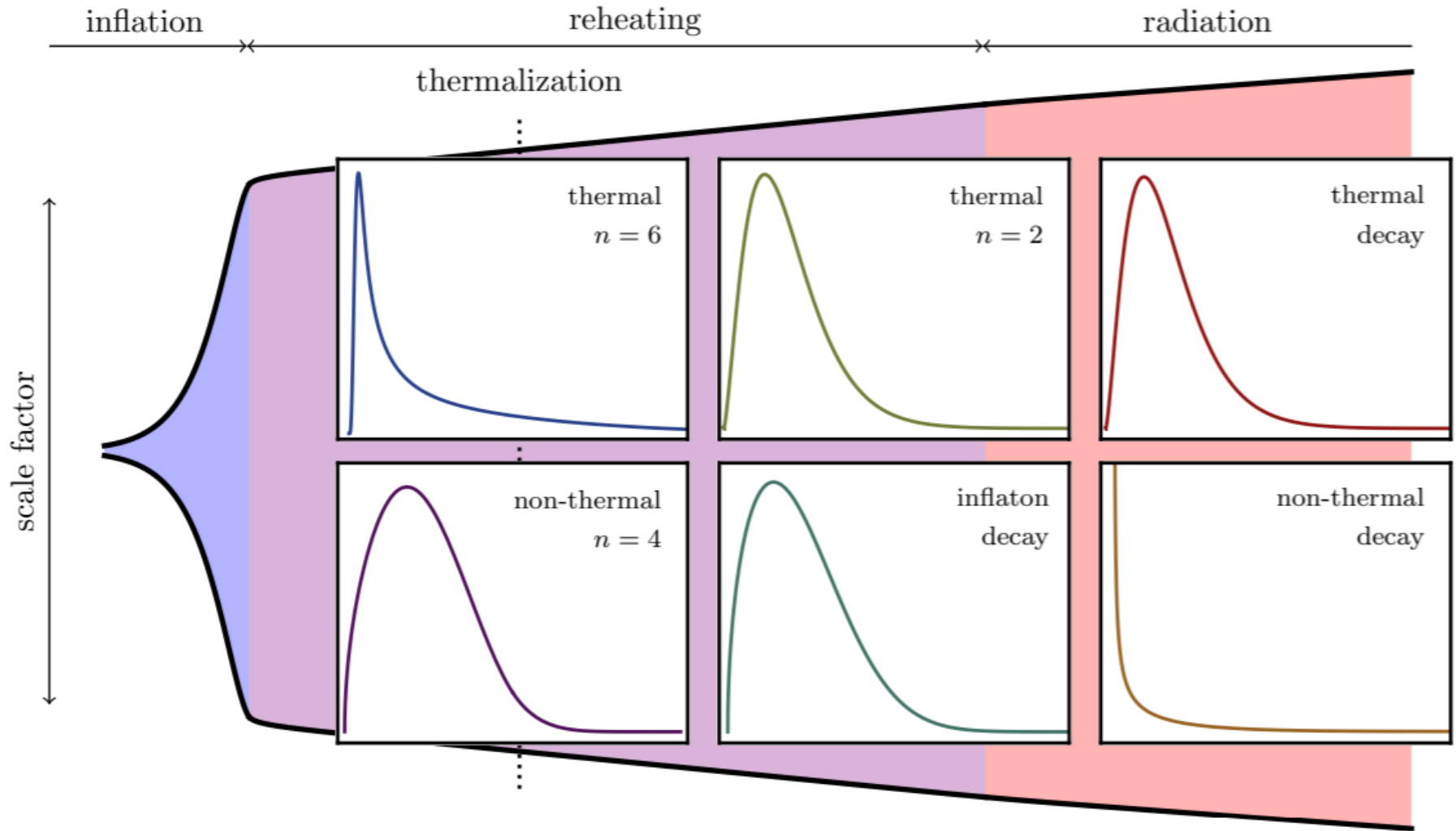


(R)

(NR)



# Summary : out-of-equilibrium DM



# Cosmological imprint

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# Cosmological imprint

- **Cosmological role** of out-of-equilibrium dark matter via


$$\bar{\rho} = 4\pi \left(\frac{T_\star}{a}\right)^4 \int q^2 \epsilon \bar{f}(q) dq$$

**energy-density**

$$\bar{P} = \frac{4\pi}{3} \left(\frac{T_\star}{a}\right)^4 \int q^2 \frac{q^2}{\epsilon} \bar{f}(q) dq$$

**pressure**

$$\epsilon = \sqrt{q^2 + \left(\frac{m_{\text{DM}} a}{T_\star}\right)^2} : \text{“dimensionless energy”}$$

- Define  $w \equiv \bar{P}/\bar{\rho}$  : **equation-of-state parameter**
- In pure  $\Lambda$ CDM :  $w = 0$  precisely (**Cold = pressureless**)
- But  $w \neq 0$  !  **Non-Cold Dark Matter cosmology**

# Non-Cold Dark Matter $w \neq 0$

- Expanding quantities around an homogenous background

$$f(\mathbf{x}, \mathbf{p}, \tau) = \bar{f}(|\mathbf{p}|, \tau)[1 + \Psi(\mathbf{x}, \mathbf{p}, \tau)]$$

- In matter domination, matter **overdensities**  $\delta$  follow

$$\ddot{\delta} + \mathcal{H}\dot{\delta} - \frac{3}{2}\mathcal{H}^2 \left(1 - \frac{k^2}{k_{\text{FS}}^2}\right) \delta = 0 \quad w \ll 1$$

where  $k_{\text{FS}}^2(a) = \frac{9}{10} \frac{\mathcal{H}^2}{w}$  is the **Jeans** or **Free-Streaming scale**

$d\tau \equiv a dt$  : **Conformal time**  $\tau$

$\mathcal{H} \equiv a H$  : **Conformal Hubble rate**

$w \equiv \bar{P}/\bar{\rho}$  : **Equation-of-state parameter**

[C. Ma & E. Bertschinger. ApJ 455 (1995) 7-25]

[J. Lesgourgues & T. Tram, JCAP 09 (2011) 032]

[M. Kunz, S. Nesseris, I. Sawicki, PRD 94, 023510 (2016)]

[G. Ballesteros, M. A. G. Garcia & **MP**, 2011.13458]

# Non-Cold Dark Matter $w \neq 0$

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where  $k_{\text{FS}}(a) = \frac{9}{10} \frac{\mathcal{H}^2}{w}$  is the **Jeans** or **Free-Streaming scale**

- If  $w = 0$  all modes grow “**democratically**” : **CDM** limit

$$w \neq 0 \text{ **cutoff** in the power spectrum at } k_{\text{H}}(a) \equiv \left[ \int_0^a \frac{1}{k_{\text{FS}}(\tilde{a})} \frac{d\tilde{a}}{\tilde{a}} \right]^{-1}$$

- Only  $w$  controls the cutoff scale!**

$d\tau \equiv a dt$  : **Conformal time**  $\tau$

$\mathcal{H} \equiv a H$  : **Conformal Hubble rate**

$w \equiv \bar{P}/\bar{\rho}$  : **Equation-of-state parameter**

[C. Ma & E. Bertschinger. ApJ 455 (1995) 7-25]

[J. Lesgourgues & T. Tram, JCAP 09 (2011) 032]

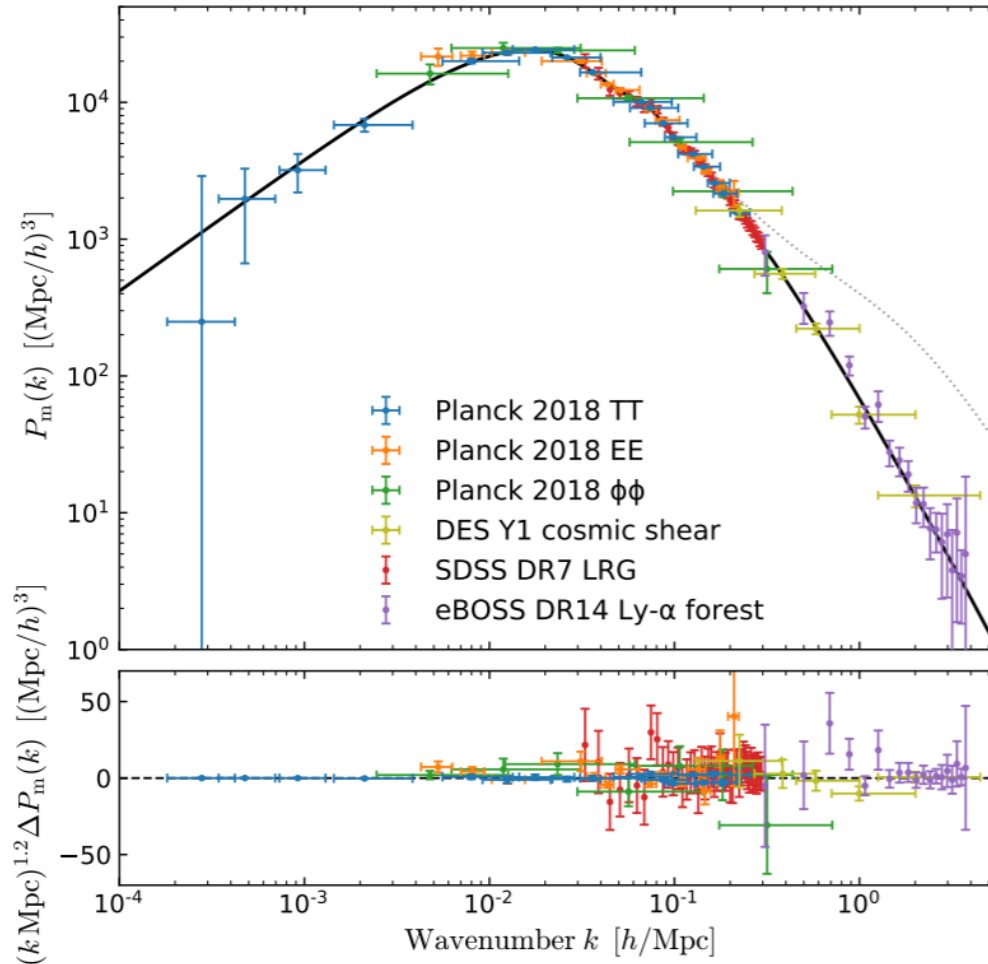
[M. Kunz, S. Nesseris, I. Sawicki, PRD 94, 023510 (2016)]

[G. Ballesteros, M. A. G. Garcia & **MP**, 2011.13458]



# Non-Cold Dark Matter $w \neq 0$

- Small scales of power spectrum probed by **Lyman-alpha forest**

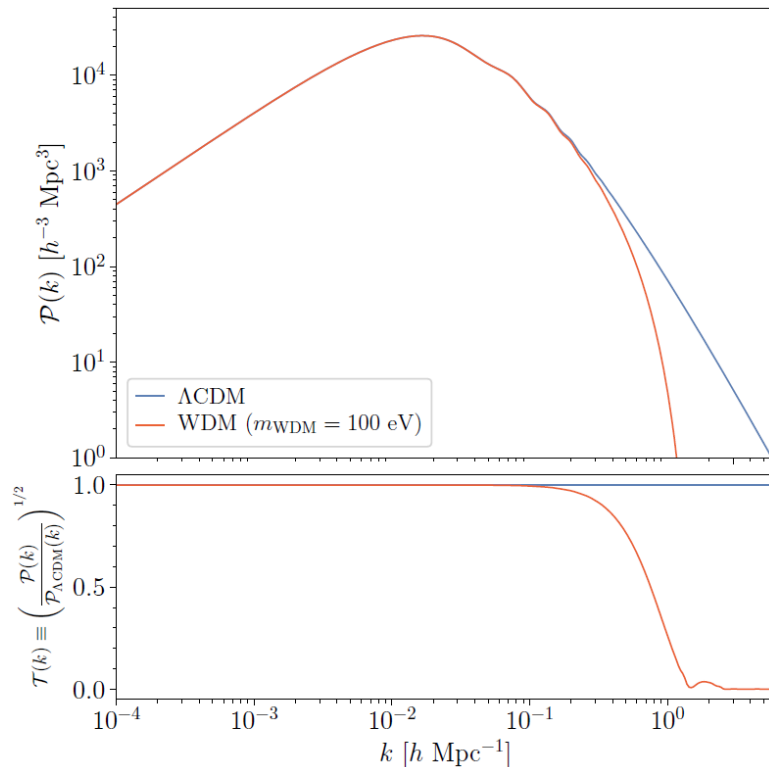


[S. Chabanier, M. Millea, N. Palanque-Desabrouille, MNRAS 489 (2019) 2, 2247-2253]

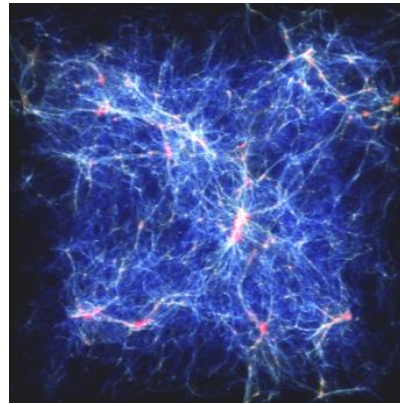
# Non-Cold Dark Matter $w \neq 0$

- Lyman-alpha forest constraints Warm Dark Matter (**WDM**)

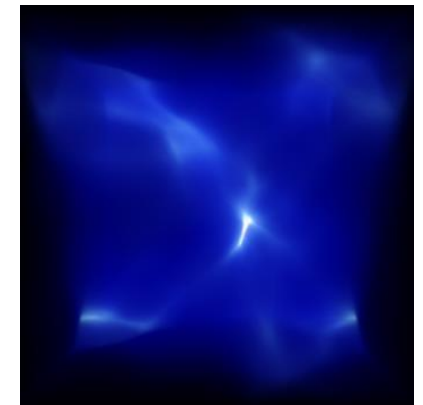
$$\bar{f}_{\text{WDM}}(q) = \frac{1}{1 + e^{q/T_{\text{WDM}}}} \quad \longrightarrow \quad \Omega_{\text{WDM}} h^2 \simeq \left( \frac{m_{\text{WDM}}}{94 \text{ eV}} \right) \left( \frac{T_{\text{WDM}}}{T_\nu} \right)^3$$



$\Lambda$ CDM



WDM



$m_{\text{WDM}} = 100 \text{ eV}$

[J. Baur, N. Palanque-Delabrouille, C. Yèche, C. Magneville, M. Viel, JCAP 08 (2016) 012]

# How warm is Non-Cold Dark Matter?

- From **Lyman-alpha** forest  $m_{\text{WDM}} > (1.9 - 5.3) \text{ keV}$  at 95% C.L.

[Braur et al. JCAP 08 (2016) 012 – Iršič et al. PRD 96 (2017) 2, 023522  
Palanque Delabrouille et al. JCAP 04 (2020) 038 – Viel et al. PRD 88 (2013) 043502  
Viel et al. PRD 71 (2005) 063534 – Narayanan et al. ApJ 543 (2000) L103-L106]

$$w_{\text{WDM}}(a) \simeq 6 \times 10^{-15} a^{-2} \left( \frac{\text{keV}}{m_{\text{WDM}}} \right)^{8/3} \quad \longrightarrow \quad w_{\text{WDM}}(a = 1) < 10^{-15}$$

- Constraints much **stronger** than **CMB!**  $w_{\text{WDM}}(a = 1) < 10^{-10}$   
[M. Kunz, S. Nesseris, I. Sawicki, PRD 94, 023510 (2016)]

- How **cold** are **WIMPs** ?

$$w(a) \simeq 10^{-29} \left( \frac{1}{a^2} \right) \left( \frac{20 T_F}{m_{\text{DM}}} \right) \left( \frac{100 \text{ GeV}}{m_{\text{DM}}} \right)^2 \left( \frac{100}{g_*^F} \right)^{2/3} \quad \longrightarrow$$



- How to translate **Lyman-alpha WDM** bounds on any **DM** ?


$$w(m_{\text{DM}}) = w_{\text{WDM}}(m_{\text{WDM}})$$

[S. Colombi, S. Dodelson, L. M. Widrow ApJ. 458 (1996) 1 - Kamada, N. Yoshida, K. Kohri, T. Takahashi JCAP 03 (2013) 008  
K. J. Bae, R. Jinno, A. Kamada, K. Yanagi JCAP 03 (2020) 042 - A. Kamada & K. Yanagi JCAP 1911 (2019) 029]

# How warm is Non-Cold Dark Matter?

$w$  – matching

$$w \simeq \frac{\delta P}{\delta \rho} = \frac{T_\star^2}{3m_{\text{DM}}^2} \frac{\langle q^2 \rangle}{a^2}$$

$$m_{\text{DM}} = m_{\text{WDM}} \left( \frac{T_\star}{T_{\text{WDM}}} \right) \sqrt{\frac{\langle q^2 \rangle}{\langle q^2 \rangle_{\text{WDM}}}}$$


- Compute **2<sup>nd</sup> moment of distribution** + **determine  $T_\star$**
- If **distribution** can be fitted by  $f(q) \propto q^\alpha \exp(-\beta q^\gamma)$

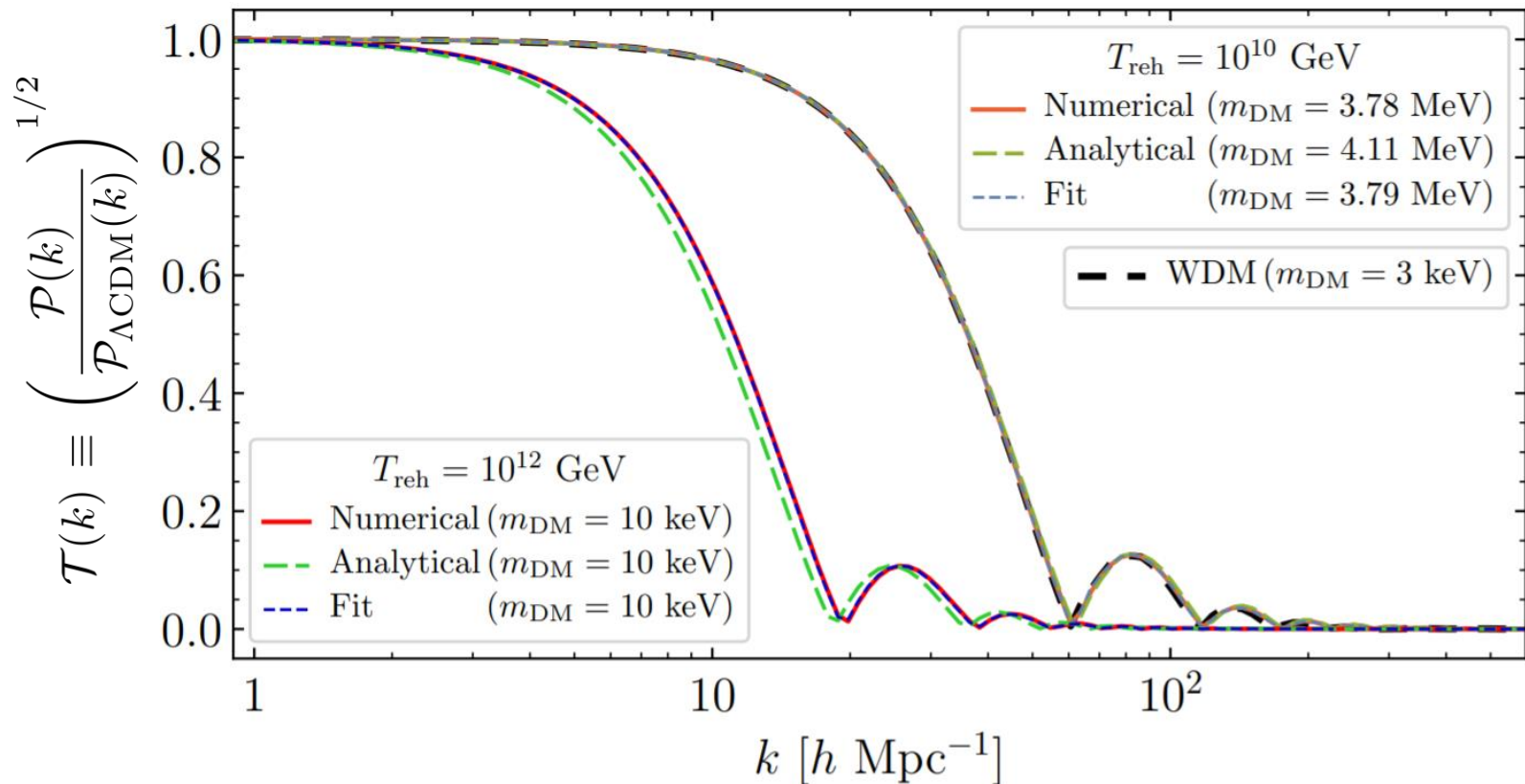
$w$  – matching  $\longrightarrow$

$$m_{\text{DM}} \simeq 7.56 \text{ keV} \left( \frac{m_{\text{WDM}}}{3 \text{ keV}} \right)^{4/3} \left( \frac{\langle p \rangle_0}{T_0} \right) \sqrt{\frac{\Gamma\left(\frac{3+\alpha}{\gamma}\right) \Gamma\left(\frac{5+\alpha}{\gamma}\right)}{\Gamma^2\left(\frac{4+\alpha}{\gamma}\right)}}$$

# How warm is Non-Cold Dark Matter?

- Example: **inflaton decay** case computed using **CLASS**

[D. Blas, J. Lesgourgues & T. Tram JCAP 07 (2011) 034 - J. Lesgourgues & T. Tram, JCAP 09 (2011) 032]



- **Excellent agreement with  $w$  – matching for all distributions!**

# Inflaton decay

- **Lyman-alpha** bounds translate into

$$m_{\text{DM}} \gtrsim \left(\frac{m_{\text{WDM}}}{3 \text{ keV}}\right)^{4/3} \left(\frac{106.75}{g_{*s}^{\text{reh}}}\right)^{1/3} \left(\frac{m_\phi}{3 \times 10^{13} \text{ GeV}}\right) \left(\frac{10^{10} \text{ GeV}}{T_{\text{reh}}}\right) \begin{cases} 3.78 \text{ MeV,} & \text{Numerical,} \\ 4.11 \text{ MeV,} & \text{Analytical,} \\ 3.79 \text{ MeV,} & \text{Fit.} \end{cases}$$

- For **low reheating temperature**  $T_{\text{reh}} \ll m_\phi$

$$m_{\text{DM}} \gtrsim \text{EeV}$$

- **Combining with relic density** condition

$$\text{Br}_\chi < 1.5 \times 10^{-4} \left(\frac{g_{*s}^{\text{reh}}}{106.5}\right)^{1/3} \left(\frac{3 \text{ keV}}{m_{\text{WDM}}}\right)^{4/3}$$

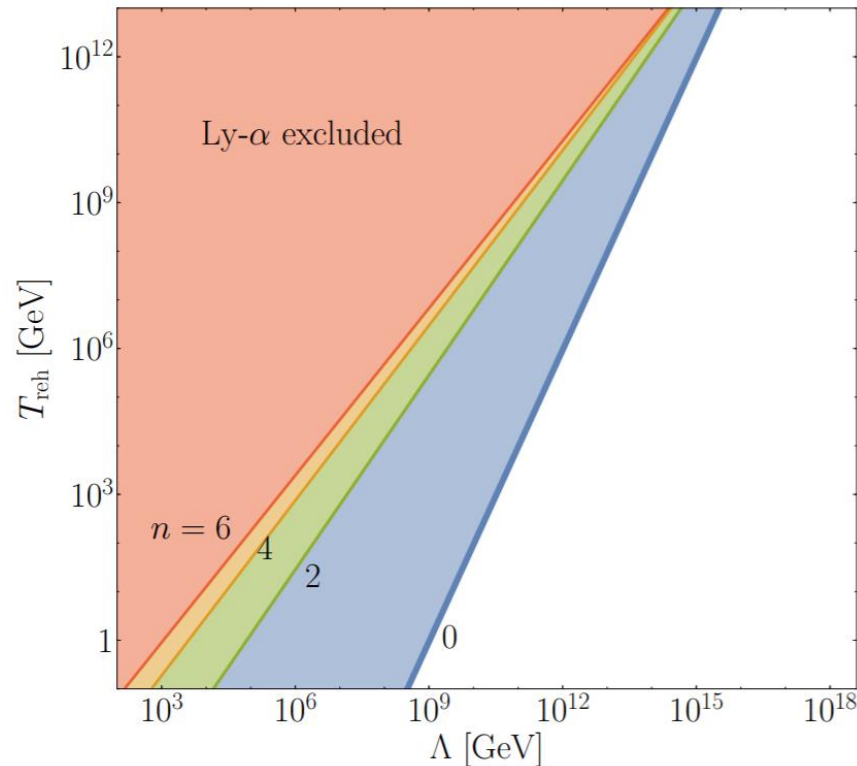
- Even if  $\phi \not\rightarrow \chi\chi$ , since  $\gamma \rightarrow \chi\chi$  then  $\phi \rightarrow \gamma \rightarrow \chi\chi$

[K. Kaneta, Y. Mambrini & Keith A. Olive Phys.Rev.D 99 (2019) 6, 063508]

# UV freeze-in via scattering

$$m_{\text{DM}} \gtrsim \left(\frac{m_{\text{WDM}}}{3 \text{ keV}}\right)^{4/3} \left(\frac{106.75}{g_{*s}^{\text{reh}}}\right)^{1/3} \begin{cases} 7.27 (7.17) \text{ keV}, & \text{FF Numerical (Fit), } n = 0 \\ 8.48 (8.73) \text{ keV}, & \text{FF Numerical (Fit), } n = 2 \\ 8.52 (8.05) \text{ keV}, & \text{FF Numerical (Fit), } n = 4 \end{cases}$$

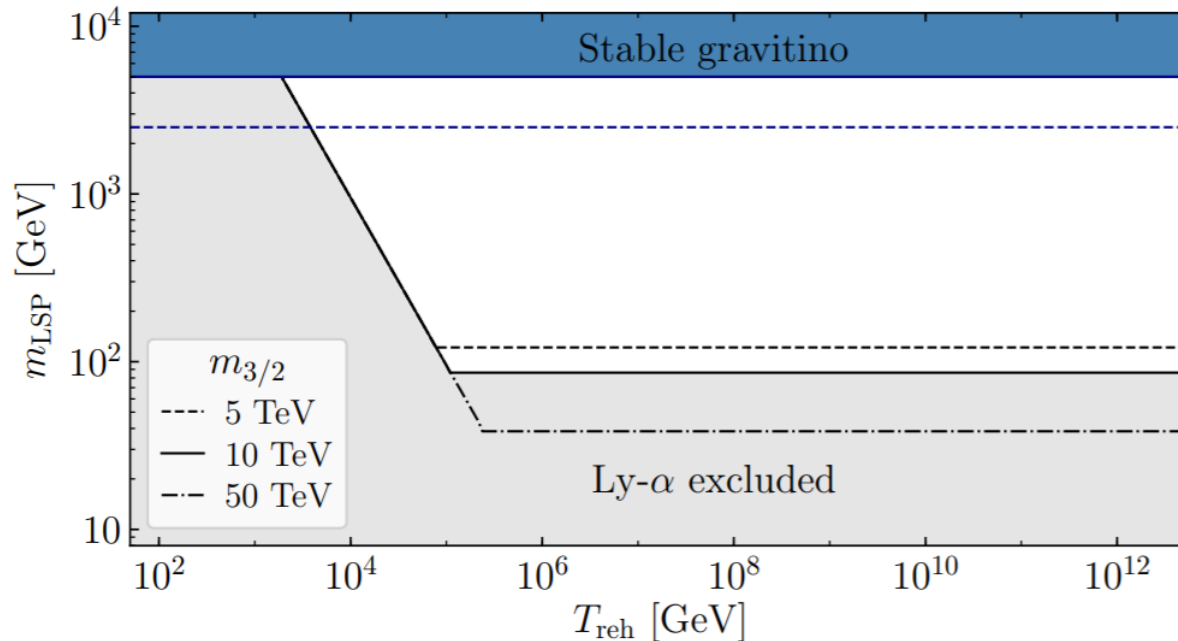
- Combine with **relic density** condition



- Apply to any UV freeze-in model!

# Freeze-in via decay $\phi \rightarrow A \rightarrow \chi$

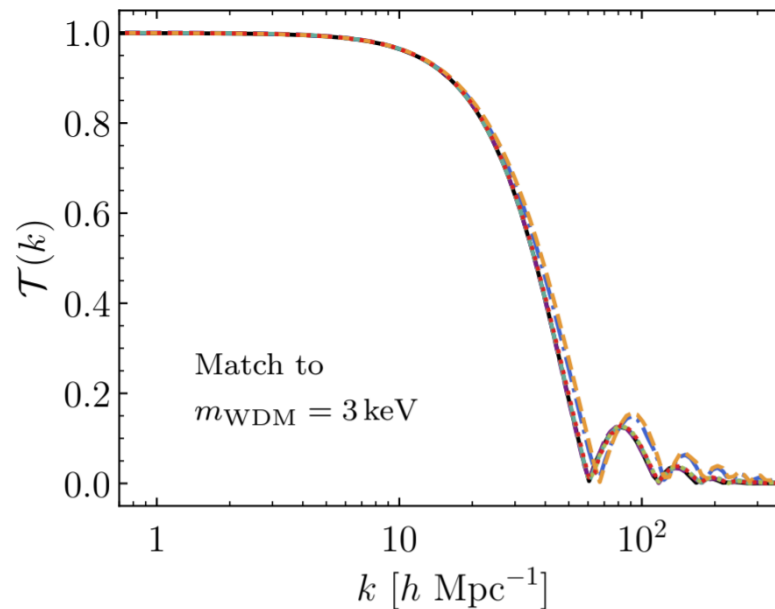
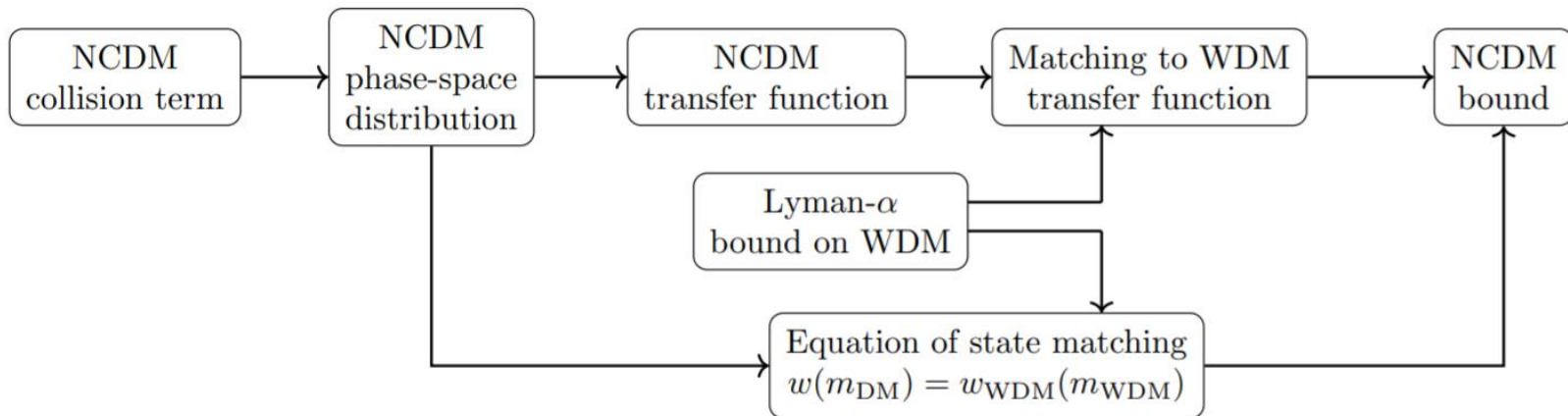
- **Example :**  $\phi \rightarrow \Psi_{3/2} \rightarrow \text{LSP}$   $\Gamma_{3/2} = \frac{193}{384\pi} \frac{m_{3/2}^3}{M_P^2}$



$$m_{\text{LSP}} \gtrsim \begin{cases} 86 \text{ GeV} \left( \frac{m_{\text{WDM}}}{3 \text{ keV}} \right)^{4/3} \left( \frac{10 \text{ TeV}}{m_{3/2}} \right)^{1/2}, & T_{\text{reh}} \gg 10^5 \text{ GeV} \left( \frac{m_{3/2}}{10 \text{ TeV}} \right)^{1/2}, \\ 95 \text{ GeV} \left( \frac{m_{\text{WDM}}}{3 \text{ keV}} \right)^{4/3} \left( \frac{10^5 \text{ GeV}}{T_{\text{reh}}} \right), & T_{\text{reh}} \ll 10^5 \text{ GeV} \left( \frac{m_{3/2}}{10 \text{ TeV}} \right)^{1/2}. \end{cases}$$



# Summary : cosmological imprint



# Take home message

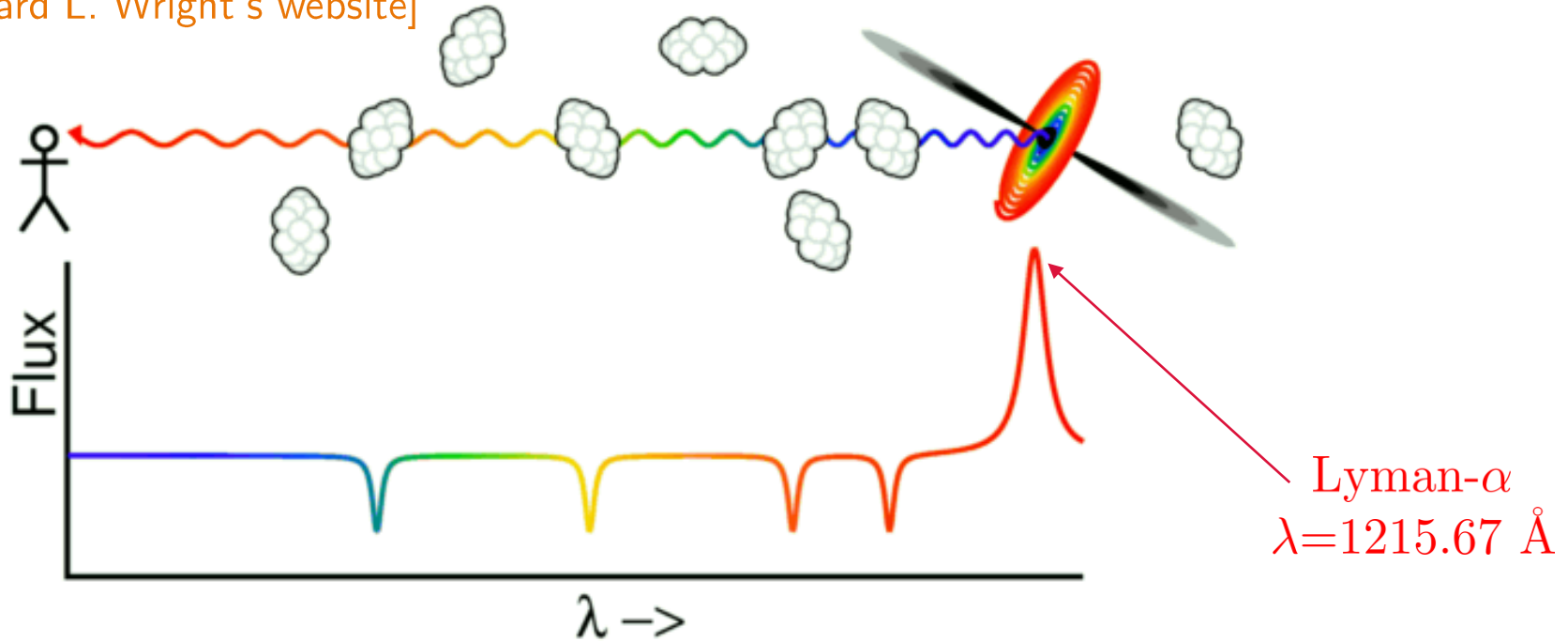
- **Out-of-equilibrium DM** can be produced **after inflation**
- Most **distributions** considered can be **fitted** by
$$f(q) \propto q^\alpha \exp(-\beta q^\gamma)$$
- **Cutoff** in power spectrum can be probed by Lyman-alpha
- DM produced from modulus decay, from decay of thermalized particle + general expressions for collision terms... **in the paper!**
- **Dark matter is cold.**

**Thank you for your attention**

# Back-up Slides

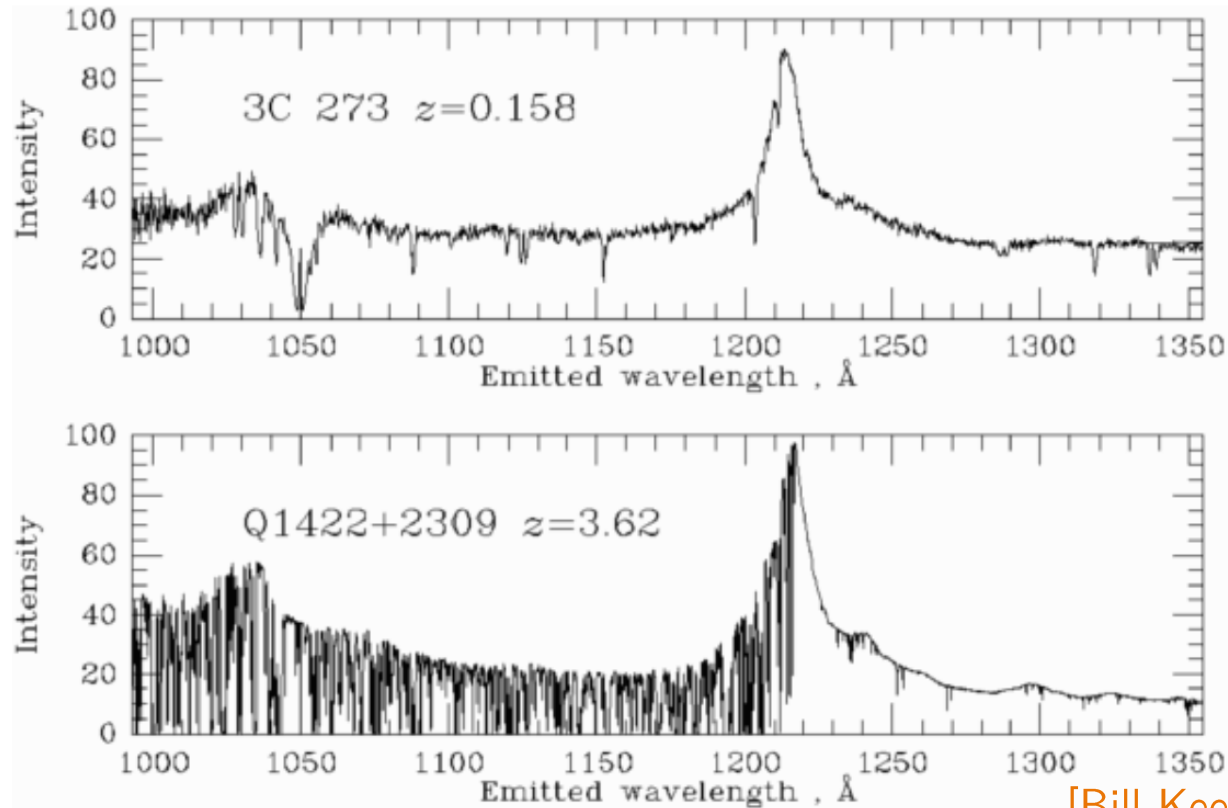
# Lyman-alpha forest

[Edward L. Wright's website]



- Quasi-Stellar Objects (QSO) are luminous astrophysical objects powered by gas spiraling at high velocity into an massive black hole
- Light emitted by distant QSO is absorbed in foreground structures
- Allows for a 1D measure of overdensities along line of sight

# Lyman-alpha forest



[Bill Keel's website]

- Comparison of QSO spectra at low and high redshift in QSO rest frame

# NCDM Cosmology

- Expand around (homogenous) **background quantities**

$$f(\mathbf{x}, \mathbf{p}, \tau) = \bar{f}(|\mathbf{p}|, \tau)[1 + \Psi(\mathbf{x}, \mathbf{p}, \tau)]$$

- Expand **fluctuations** in term of **Legendre polynomials**

$$\Psi(\mathbf{k}, \hat{\mathbf{n}}, q, \tau) = \sum_{\ell=0}^{\infty} (-i)^{\ell} (2\ell + 1) \Psi_{\ell}(\mathbf{k}, q, \tau) P_{\ell}(\hat{\mathbf{k}} \cdot \hat{\mathbf{n}})$$

- Express **fluctuations** in terms of **Legendre coefficients**

$$\delta\bar{\rho} = 4\pi \left(\frac{T_{\star}}{a}\right)^4 \int q^2 \epsilon \bar{f}(q) \Psi_0 dq, \quad \text{energy density fluctuation}$$

$$\delta\bar{P} = \frac{4\pi}{3} \left(\frac{T_{\star}}{a}\right)^4 \int q^2 \frac{q^2}{\epsilon} \bar{f}(q) \Psi_0 dq, \quad \text{pressure (density) fluctuation}$$

$$(\bar{\rho} + \bar{P})\theta = 4\pi k \left(\frac{T_{\star}}{a}\right)^4 \int q^3 \bar{f}(q) \Psi_1 dq, \quad \text{velocity divergence}$$

$$(\bar{\rho} + \bar{P})\sigma = \frac{8\pi k}{3} \left(\frac{T_{\star}}{a}\right)^4 \int q^2 \frac{q^2}{\epsilon} \bar{f}(q) \Psi_2 dq, \quad \text{anisotropic stress.}$$

# NCDM Cosmology

- The phase space distribution satisfies **collisionless Boltzmann equation**

$$\frac{\partial f}{\partial \tau} + \frac{dx^i}{d\tau} \frac{\partial f}{\partial x^i} + \frac{dq}{d\tau} \frac{\partial f}{\partial q} + \frac{dn_i}{d\tau} \frac{\partial f}{\partial n_i} = 0,$$

- Plugging** distribution expansion in **Legendre polynomials** give

$$\dot{\Psi}_0 = -\frac{qk}{\epsilon} \Psi_1 + \frac{1}{6} \dot{h} \frac{d \ln \bar{f}}{d \ln q},$$

$$\dot{\Psi}_1 = \frac{qk}{3\epsilon} (\Psi_0 - 2\Psi_2),$$

$$\dot{\Psi}_2 = \frac{qk}{5\epsilon} (2\Psi_1 - 3\Psi_3) - \left( \frac{1}{15} \dot{h} + \frac{2}{5} \dot{\eta} \right) \frac{d \ln \bar{f}}{d \ln q},$$

$$\dot{\Psi}_\ell = \frac{qk}{(2\ell + 1)\epsilon} (\ell \Psi_{\ell-1} - (\ell + 1) \Psi_{\ell+1}), \quad [\ell \geq 3]$$

$$ds^2 = a(\tau) (-d\tau^2 + (\delta_{ij} + h_{ij}) dx^i dx^j)$$

in **synchronous** gauge  $h \equiv h_{ii}$

For a non-relativistic species, higher multipoles are typically suppressed by (positive) powers of  $q/\epsilon \sim p/m_{\text{DM}}$ , making any  $\Psi_\ell$  with  $\ell \geq 2$  much smaller than  $\Psi_0$  and  $\Psi_1$ . In this case, the Boltzmann hierarchy can be truncated imposing  $\Psi_\ell = 0$  for  $\ell > 1$ . In this (non-relativistic) case  $\Psi_0$  depends only mildly on the variable  $q$ , and the integrals are dominated by the low  $q \ll \epsilon$  regime so that we can identify  $\delta P / \delta \rho \simeq \bar{P} / \bar{\rho} = w$ .

# NCDM Cosmology

- **Neglecting higher multipoles**, for very **non-relativistic DM**, integrating over momenta gives

$$\dot{\delta} = -(1+w)\left(\theta + \frac{\dot{h}}{2}\right) - 3\mathcal{H}(\hat{c}_s^2 - w)\delta + 9\mathcal{H}^2(1+w)(\hat{c}_s^2 - c_a^2)\frac{\theta}{k^2},$$

$$\dot{\theta} = -\mathcal{H}(1 - 3\hat{c}_s^2)\theta + \frac{\hat{c}_s^2}{1+w}k^2\delta,$$

- In **matter domination**, from **Einstein equations**, metric perturbation follow

$$\ddot{h} + \mathcal{H}\dot{h} + 3(1+3w)\mathcal{H}^2\delta = 0,$$

- Which can be translated to evolution of **matter density fluctuations**

$$\ddot{\delta} + \mathcal{H}\dot{\delta} - \frac{3}{2}\mathcal{H}^2\left(1 - w\frac{10}{9}\frac{k^2}{\mathcal{H}^2}\right)\delta = 0.$$

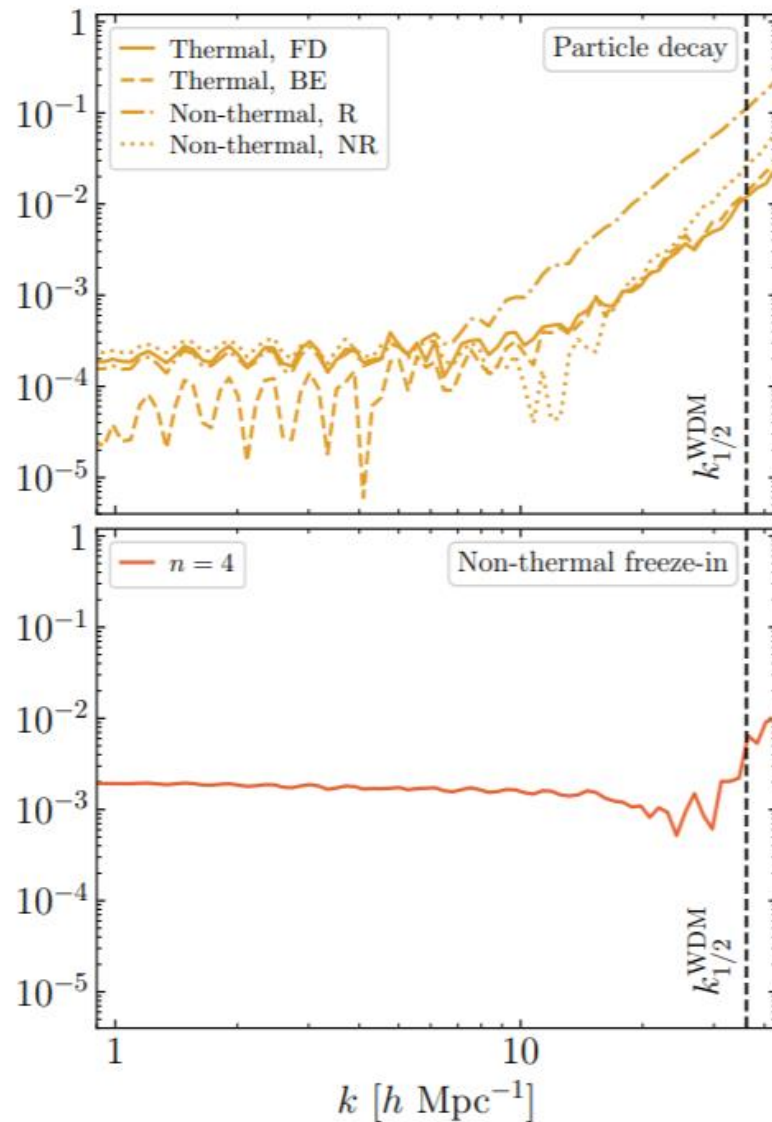
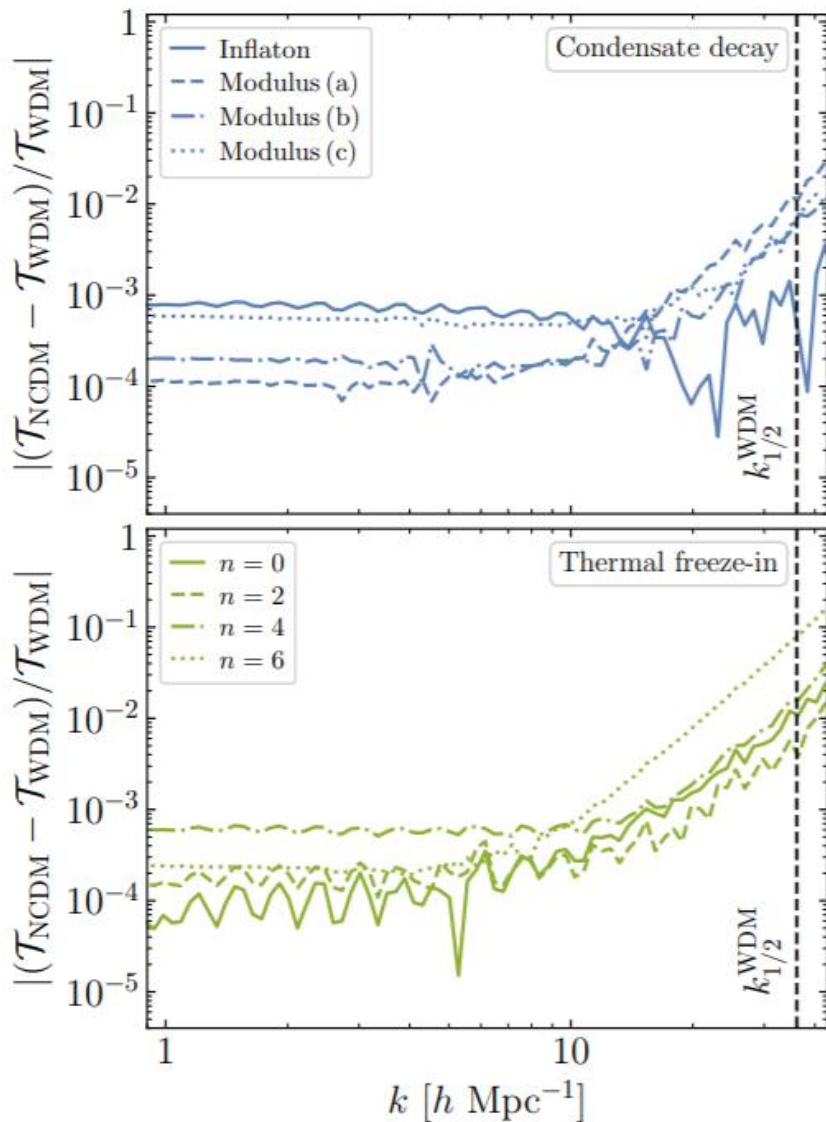


# General phase space distribution

$$f(q) \propto q^\alpha \exp(-\beta q^\gamma)$$

Scenario		$\alpha$	$\beta$	$\gamma$
Inflaton decay		-3/2	0.74	1.00
Moduli decay	during reheating	-3/2	1.00	3/2
	after reheating	-1.00	1.00	2.00
Thermal decay		-1/2	1.00	1.00
Non-thermal decay	non-relativistic	-	-	-
	relativistic	-5/2	0.74	2.00
UV Freeze-in ( $n = 0$ )	BB	0.70	1.13	1.00
	FB	0.51	1.10	1.00
	FF	0.29	1.11	1.00
UV Freeze-in ( $n = 2$ )	BB	0.51	0.91	1.00
	FB	0.42	0.90	1.00
	FF	0.33	0.90	1.00
UV Freeze-in ( $n = 4$ )	BB	0.21	0.06	1.98
	FB	0.21	0.06	2.04
	FF	0.21	0.05	2.10
UV Freeze-in ( $n = 6$ )	BB	-	-	-
	FB	-	-	-
	FF	-	-	-
Non-thermal UV Freeze-in		-3/2	2.5	2.6

# Precision on transfer functions



# Contribution to $N_{\text{eff}}$ ?

$$\begin{aligned} \Delta N_{\text{eff}} &= \frac{8}{7} \left( \frac{T}{T_\nu} \right)^4 \frac{\rho_\chi - m_{\text{DM}} n_\chi}{\rho_\gamma} \\ &= \frac{8\pi\Omega_\chi}{7\Omega_\gamma} \left( \frac{g_{*s}(T)}{g_{*s}^0} \right)^{4/3} \left( \frac{T}{T_\nu} \right)^4 \left( \frac{T_\star}{m_{\text{DM}}} \right) \\ &\quad \times \left[ \left\langle \sqrt{q^2 + \left( \frac{g_{*s}^0}{g_{*s}(T)} \right)^{2/3} \left( \frac{m_{\text{DM}}}{T_\star} \right)^2 \left( \frac{T_0}{T} \right)^2} \right\rangle - \left( \frac{g_{*s}^0}{g_{*s}(T)} \right)^{1/3} \left( \frac{m_{\text{DM}}}{T_\star} \right) \left( \frac{T_0}{T} \right) \right]. \end{aligned}$$

- **Saturating** the Lyman-alpha bound gives

$$\begin{aligned} \Delta N_{\text{eff,max}} &\simeq \frac{1.4 \times 10^{-4}}{\sqrt{\langle q^2 \rangle}} \left( \frac{g_{*s}(T)}{g_{*s}^0} \right)^{4/3} \left( \frac{\Omega_\chi h^2}{0.1} \right) \left( \frac{3 \text{ keV}}{m_{\text{WDM}}} \right)^{4/3} \left( \frac{T}{T_\nu} \right)^4 \\ &\quad \times \left[ \left\langle \sqrt{q^2 + \mu_*(T)^2} \right\rangle - \mu_*(T) \right], \end{aligned}$$

$$\mu_*(T) \equiv \sqrt{\langle q^2 \rangle} \left( \frac{g_{*s}^0}{g_{*s}(T)} \right)^{1/3} \left( \frac{3 \text{ keV}}{m_{\text{WDM}}} \right)^{4/3} \left( \frac{7.56 \text{ keV}}{T} \right).$$

$$\Delta N_{\text{eff}}(T_{\text{BBN}}) \lesssim 5.4 \times 10^{-4} \left( \frac{\langle q \rangle}{\sqrt{\langle q^2 \rangle}} \right) \left( \frac{\Omega_\chi h^2}{0.1} \right) \left( \frac{3 \text{ keV}}{m_{\text{WDM}}} \right)^{4/3},$$