

Second-order stochastic theory for self-interacting scalar fields in de Sitter spacetime

Archie Cable

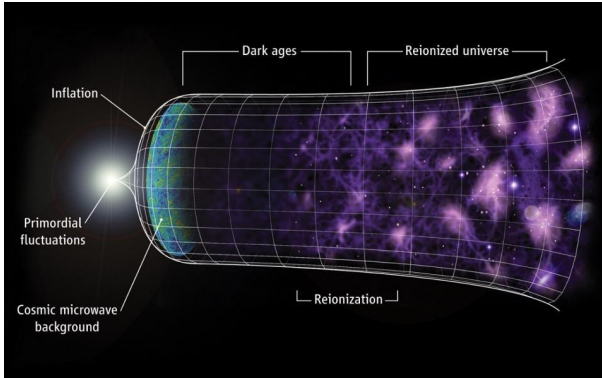
Imperial College London

arXiv:2209.02545

- 1 Introduction
- 2 QFT in de Sitter
- 3 The stochastic approach
- 4 Conclusion

Inflation

The period of accelerated expansion in the early Universe before structure was formed.

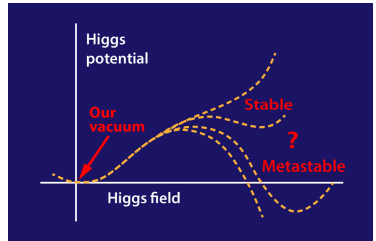
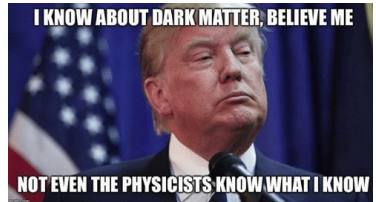


Why study inflation?

- To better understand the early Universe
- To constrain physical parameters

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- To better understand the early Universe
- To constrain physical parameters
- Examples:
 - dark matter
 - curvature/isocurvature perturbations
 - primordial black holes
 - EW vacuum decay

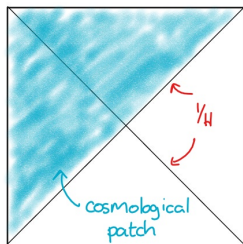


Cosmological de Sitter spacetime

Metric:

$$ds^2 = dt^2 - a(t)^2(dr^2 + r^2 d\Omega_2^2) \quad ; \quad a(t) = e^{Ht}$$

- Horizon at $R_H = 1/H$.
- Subhorizon: scales $< 1/H$
Superhorizon: scales $> 1/H$



Spectator scalar field in de Sitter

- Action:

$$S[\phi] = \int d^4x a(t)^3 \left[\frac{1}{2} \dot{\phi}^2 - \frac{1}{2a(t)^2} (\nabla\phi)^2 - \frac{1}{2} m^2 \phi^2 - \frac{1}{4} \lambda \phi^4 \right]$$

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- Equation of motion:

$$\begin{pmatrix} \dot{\phi} \\ \dot{\pi} \end{pmatrix} = \begin{pmatrix} \pi \\ -3H\pi + \frac{1}{a(t)^2} \nabla^2 \phi - m^2 \phi - \lambda \phi^3 \end{pmatrix}$$

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Long distance behaviour of scalar fields during inflation

Why?

- Modes are amplified by the spacetime expansion, causing them to exit the de Sitter horizon
- These are “frozen”
- Later (today!), they re-enter the de Sitter horizon

The Feynman propagator

The object of interest is

$$i\Delta_F(t, \mathbf{x}; t', \mathbf{x}') = \langle 0_{BD} | \hat{T} \hat{\phi}(t, \mathbf{x}) \hat{\phi}(t', \mathbf{x}') | 0_{BD} \rangle$$

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Perturbative QFT can be used to compute this...

... but results contain infrared (IR) divergences that cannot be renormalised with current techniques

Feynman propagator to one-loop order

Feynman propagator to one-loop order in ϕ^4 theory is given by



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The tadpole diagram contains both infrared and ultraviolet (UV) divergences.

To deal with the UV divergences, we renormalise the mass $m \rightarrow m_R$.

Feynman propagator to $\mathcal{O}(\lambda H^4/m^4)$

The UV-finite part of the Feynman propagator to one-loop is

$$i\Delta_F(t, \mathbf{0}; t, \mathbf{x}) = \left(\frac{H^2}{16\pi^2} \frac{\Gamma(\frac{3}{2} - \nu_R)\Gamma(2\nu_R)4^{\frac{3}{2}-\nu_R}}{\Gamma(\frac{1}{2} + \nu_R)} - \frac{27\lambda H^8}{64\pi^4 m_R^6} + \mathcal{O}\left(\frac{\lambda H^6}{m_R^4}\right) \right) \\ \times |Ha(t)\mathbf{x}|^{-2\left(\frac{3}{2}-\nu_R + \frac{3\lambda H^2}{8\pi^2 m_R^2} + \mathcal{O}(\lambda)\right)}$$

$$\nu_R = \sqrt{\frac{9}{4} - \frac{m_R^2}{H^2}}.$$

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 & \times |Ha(t)\mathbf{x}|^{-2} \left(\frac{3}{2} - \nu_R + \frac{3\lambda H^2}{8\pi^2 m_R^2} + \mathcal{O}(\lambda) \right)
 \end{aligned}$$

IR divergent unless $\lambda \ll m^4/H^4$

UV renormalised (scale-dependent)

The stochastic approach

The idea:

Quantum modes can be approximated by a statistical noise contribution to the classical equations of motion.

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Quantum modes can be approximated by a statistical noise contribution to the classical equations of motion.

We expect it to work if fields are sufficiently light $m \lesssim H$ such that long wavelength modes are stretched by spacetime expansion.

The overdamped (OD) stochastic approach

- In the limits $m \ll H$ and $\lambda \ll m^2/H^2$, we can derive stochastic equations

$$0 = 3H\dot{\phi} + m^2\phi + 3\lambda\phi^2 - \xi_{OD}(t, \mathbf{x})$$

where $\langle \xi(t, \mathbf{x})\xi(t', \mathbf{x}) \rangle = \frac{9H^5}{4\pi^2}\delta(t - t')$.

- This is done by introducing a strict cut-off between sub and superhorizon modes.

OD field correlator

- To one-loop order,

$$\langle \phi(t, \mathbf{0}) \phi(t, \mathbf{x}) \rangle = \left(\frac{3H^4}{8\pi^2 m^2} - \frac{27\lambda H^8}{64\pi^4 m^6} + \mathcal{O}(\lambda^2) \right) |H a(t) \mathbf{x}|^{-2} \left(\frac{m^2}{3H^2} + \frac{3\lambda H^2}{8\pi^2 m^2} + \mathcal{O}(\lambda^2) \right)$$

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same as free term in QFT if $m^2 \ll H^2$

same as $\mathcal{O}(\lambda H^4/m^4)$ term in QFT

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- (a) Doesn't fully reproduce the free Feynman propagator.

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- (b) Never includes the renormalisation scale-dependent terms.

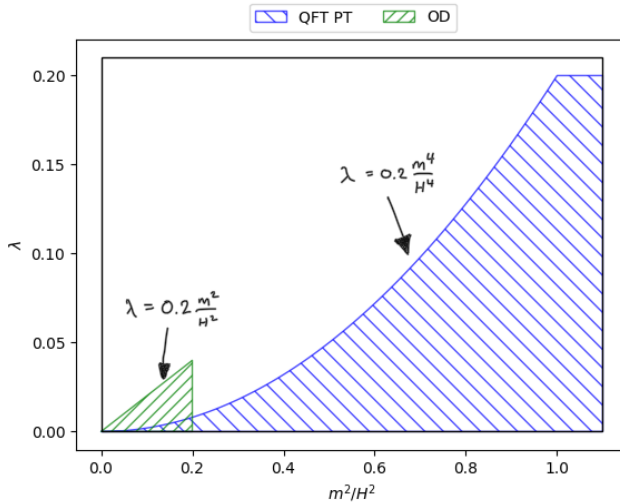
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- (a) Doesn't fully reproduce the free Feynman propagator.
- (b) Never includes the renormalisation scale-dependent terms.
- N.B. Non-perturbative methods are available to compute the OD field correlator [[arXiv:1904.11917](https://arxiv.org/abs/1904.11917)]

The state of play



A second-order stochastic effective theory

Make the ansatz

$$\begin{pmatrix} \dot{\phi} \\ \dot{\pi} \end{pmatrix} = \begin{pmatrix} \pi \\ -3H\pi - m^2\phi - \lambda\phi^3 \end{pmatrix} + \begin{pmatrix} \xi_\phi(t, \mathbf{x}) \\ \xi_\pi(t, \mathbf{x}) \end{pmatrix}$$

with a stochastic white noise contribution

$$\langle \xi_i(t, \mathbf{x}) \xi_j(t', \mathbf{x}) \rangle = \sigma_{ij}^2 \delta(t - t').$$

where $i, j \in \{\phi, \pi\}$.

The Fokker-Planck equation

The time evolution of the probability distribution function (PDF) $P(\phi, \pi; t)$ is given by the Fokker-Planck equation associated with the Langevin equation

$$\begin{aligned}\partial_t P(\phi, \pi; t) &= \left[3H - \pi \partial_\phi + (3H\pi + m^2\phi + \lambda\phi^3) \partial_\pi \right. \\ &\quad \left. + \frac{1}{2} \sigma_{\phi\phi}^2 \partial_\phi^2 + \sigma_{\phi\pi}^2 \partial_\phi \partial_\pi + \frac{1}{2} \sigma_{\pi\pi}^2 \partial_\pi^2 \right] P(\phi, \pi; t) \\ &= \mathcal{L}_{FP} P(\phi, \pi; t).\end{aligned}$$

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- 5 Determine the $\mathcal{O}(\lambda)$ stochastic parameters m, σ_{ij}^2 by matching to the QFT
- 6 Solve the Fokker-Planck equation numerically to obtain non-perturbative results

1. The spectral expansion: 1-PDF

Write the 1-PDF in terms of eigenfunctions $\Psi_N(\phi, \pi)$ and eigenvalues Λ_N

$$P(\phi, \pi; t) = \Psi_0^*(\phi, \pi) \sum_N \Psi_N(\phi, \pi) e^{-\Lambda_N t}$$

where the eigenproblem is

$$\begin{aligned}\mathcal{L}_{FP} \Psi_N(\phi, \pi) &= -\Lambda_N \Psi_N(\phi, \pi), \\ \mathcal{L}_{FP}^* \Psi_N^*(\phi, \pi) &= -\Lambda_N \Psi_N^*(\phi, \pi).\end{aligned}$$

$\{\Psi_N(\phi, \pi)\}$ obey biorthogonality and completeness relations.

1. The spectral expansion: correlators

From this basis, one can find an expression for the spacelike correlation function of two functions $f(\phi, \pi)$ and $g(\phi, \pi)$ composed purely of the eigenfunctions and eigenvalues:

$$\begin{aligned}
 \langle f(\phi, \pi; t, \mathbf{0})g(\phi, \pi; t, \mathbf{x}) \rangle &= \int d\phi_r \int d\pi_r \frac{\Psi_0(\phi_r, \pi_r)}{\Psi_0^*(\phi_r, \pi_r)} \sum_{N'N} \Psi_N^*(\phi_r, \pi_r) \Psi_{N'}^*(\phi_r, \pi_r) \\
 &\times \int d\phi_1 \int d\pi_1 \Psi_0^*(\phi_1, \pi_1) \Psi_N(\phi_1, \pi_1) f(\phi_1, \pi_1) \\
 &\times \int d\phi_2 \int d\pi_2 \Psi_0^*(\phi_2, \pi_2) \Psi_{N'}(\phi_2, \pi_2) g(\phi_2, \pi_2) \\
 &\times |H(a(t)\mathbf{x})|^{-\frac{\Lambda_N + \Lambda_{N'}}{H}}.
 \end{aligned}$$

2. Free field solutions: field correlator

Using these solutions, we evaluate the free field spacelike stochastic correlator to be

$$\begin{aligned} \langle \phi(t, \mathbf{0}) \phi(t, \mathbf{x}) \rangle^{(0)} = & \frac{1}{4\nu^2 H^3} \left[\frac{1}{2\alpha} \left(\sigma_{\pi\pi}^{2(0)} + 2\beta H \sigma_{\phi\pi}^{2(0)} + \beta^2 H^2 \sigma_{\phi\phi}^{2(0)} \right) |Ha(t)\mathbf{x}|^{-3+2\nu} \right. \\ & + \frac{1}{2\beta} \left(\sigma_{\pi\pi}^{2(0)} + 2\alpha H \sigma_{\phi\pi}^{2(0)} + \alpha^2 H^2 \sigma_{\phi\phi}^{2(0)} \right) |Ha(t)\mathbf{x}|^{-3-2\nu} \\ & \left. - \frac{2}{3} \left(\sigma_{\pi\pi}^{2(0)} + 3H \sigma_{\phi\pi}^{2(0)} + m^2 \sigma_{\phi\phi}^{2(0)} \right) |Ha(t)\mathbf{x}|^{-3} \right] \end{aligned}$$

$$\alpha/\beta = \frac{3}{2} + / - \nu \text{ with } \nu = \sqrt{\frac{9}{4} - \frac{m^2}{H^2}}.$$

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3. Free field noise

To reproduce the free Feynman propagator,

$$\sigma_{\phi\phi}^{2(0)} = \frac{H^3 \Gamma(2\nu) \Gamma(\frac{5}{2} - \nu)}{2\pi^{5/2}},$$

$$\sigma_{\phi\pi}^{2(0)} = -\frac{H^4 \alpha \Gamma(2\nu) \Gamma(\frac{5}{2} - \nu)}{2\pi^{5/2}},$$

$$\sigma_{\pi\pi}^{2(0)} = \frac{H^5 \alpha^2 \Gamma(2\nu) \Gamma(\frac{5}{2} - \nu)}{2\pi^{5/2}}.$$

4. The perturbative calculation

We now perform a perturbative expansion around the free eigenspectrum i.e.

$$\Lambda_N = \Lambda_N^{(0)} + \lambda \Lambda_N^{(1)}$$

$$\Psi_N^{(*)}(\phi, \pi) = \Psi_N^{(0)(*)}(\phi, \pi) + \lambda \Psi_N^{(1)(*)}(\phi, \pi)$$

Using standard perturbative techniques, these are written as

$$\Lambda_N^{(1)} = - \int d\phi \int d\pi \Psi_N^{(0)*}(\phi, \pi) \mathcal{L}_{FP}^{(1)} \Psi_N^{(0)}(\phi, \pi)$$

$$\Psi_N^{(1)}(\phi, \pi) = \sum_{N'} \Psi_{N'}^{(0)}(\phi, \pi) \frac{\int d\phi' \int d\pi' \Psi_{N'}^{(0)*}(\phi', \pi') \mathcal{L}_{FP}^{(1)} \Psi_{N'}^{(0)}(\phi', \pi')}{\Lambda_{N'}^{(0)} - \Lambda_N^{(0)}}.$$

4. The perturbative calculation: $\mathcal{O}(\lambda)$ field correlators

To $\mathcal{O}(\lambda)$,

$$\begin{aligned}
 & \langle \phi(t, \mathbf{0}) \phi(t, \mathbf{x}) \rangle \\
 &= \left[\frac{H^2}{16\pi^2} \frac{\Gamma(\frac{3}{2} - \nu) \Gamma(2\nu) 4^{\frac{3}{2} - \nu}}{\Gamma(\frac{1}{2} + \nu)} + \frac{3\lambda(3 - 4\nu) H^4 \Gamma(\nu)^2 \Gamma(\frac{3}{2} - \nu)^2}{32\pi^5 \nu m^2} + \mathcal{O}(\lambda^2) \right] \\
 & \times |Ha(t)\mathbf{x}|^{-3-2\nu - \frac{3\lambda\Gamma(\nu)\Gamma(\frac{3}{2}-\nu)}{8\pi^{5/2}\nu}} + \mathcal{O}(\lambda^2) \\
 & - \left(\frac{\lambda H^4 \Gamma(\nu)^2 \Gamma(\frac{5}{2} - \nu)^2}{8\pi^5 \nu m^2} + \mathcal{O}(\lambda) \right) |Ha(t)\mathbf{x}|^{-3+\mathcal{O}(\lambda^2)}.
 \end{aligned}$$

4. The perturbative calculation: $\mathcal{O}(\lambda H^4/m^4)$ field correlators

To $\mathcal{O}\left(\frac{\lambda H^4}{m^4}\right)$,

$$\begin{aligned} \langle \phi(t, \mathbf{0}) \phi(t, \mathbf{x}) \rangle &= \left[\frac{H^2}{16\pi^2} \frac{\Gamma\left(\frac{3}{2} - \nu\right) \Gamma(2\nu) 4^{\frac{3}{2} - \nu}}{\Gamma\left(\frac{1}{2} + \nu\right)} - \frac{27\lambda H^8}{64\pi^4 m^6} + \mathcal{O}\left(\frac{\lambda H^6}{m^4}\right) \right] \\ &\quad \times |H a(t) \mathbf{x}|^{-3-2\nu + \frac{3\lambda H^2}{8\pi^2 m^2} + \mathcal{O}(\lambda)}. \end{aligned}$$

This has the same form as the Feynman propagator to $\mathcal{O}\left(\frac{\lambda H^4}{m^4}\right)$!

5. Stochastic parameters to $\mathcal{O}(\lambda H^4/m^4)$

Assuming λ is the same in both QFT and stochastic theory,

$$m^2 = m_R^2 \left(1 + \mathcal{O}\left(\frac{\lambda H^2}{m^2}\right) \right)$$

$$\sigma^2 = \frac{H^3 \Gamma(2\nu) \Gamma\left(\frac{5}{2} - \nu\right)}{2\pi^{5/2}} \begin{pmatrix} 1 & -\frac{2m^2}{H(3+2\nu)} \\ -\frac{2m^2}{H(3+2\nu)} & \frac{4m^4}{(3+2\nu)^2 H^2} \end{pmatrix} + \mathcal{O}\left(\frac{\lambda H^2}{m^2}\right).$$

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Crucially, they don't have an IR divergent part $\mathcal{O}(\lambda H^4/m^4)$

6. The numerical calculation

- To solve numerically, we expand about free eigenstates

$\Psi_N(\phi, \pi) = \sum_R c_R^{(N)} \psi_R^{(0)}(\phi, \pi)$ such that the eigenequation becomes

$$\sum_R c_R^{(N)} \mathcal{L}_{FP} \psi_R^{(0)}(\phi, \pi) = \sum_{RR'} c_R^{(N)} \mathcal{M}_{RR'} \psi_{R'}^{(0)} = - \sum_R c_R^{(N)} \Lambda_N \psi_R^{(0)}(\phi, \pi)$$

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- Thus we have the matrix equation

$$\sum_{R'} \mathcal{M}_{RR'}^T c_{R'}^{(N)} = -\Lambda_N c_R^{(N)}$$

such that \mathcal{M}^T can be diagonalised to find $c_R^{(N)}$ and Λ_N .

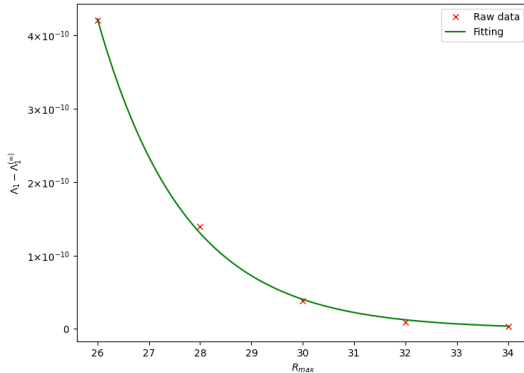
6. The numerical calculation: convergence of solutions

Solutions converge quickly therefore we can truncate our sum

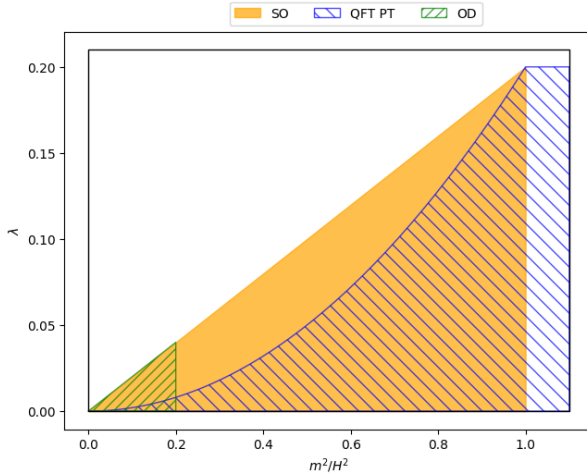
$$\frac{m^2}{H^2} = 0.01$$

$$\lambda = 0.0005$$

$$\Rightarrow \Lambda_1^{(\infty)} = 0.00453$$



New regime of validity plot



Future work

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Thanks for listening! ☺