Analogue quantum simulation of scalar fields on Schwarzschild and Kerr black holes

Maxime Jacquet, Kévin Falque, Killian Guerrero, Malte Kroj, Ferdinand Claude, Malo Joly,
Quentin Valnais, Quentin Glorieux, Elisabeth Giacobino, Alberto Bramati

Quantum Optics Group

Laboratoire Kastler Brossel, CNRS and Sorbonne University, Paris

GRECO Seminar 30/01/2023

Today’s talk

The propagation of waves in nonlinear media may be controlled to engineer situations where the waves propagate as though they were on a curved spacetime, like around a black hole or in an inflating universe. This enables the experimental simulation of field theories on curved spacetime.

Today’s talk

The propagation of waves in nonlinear media may be controlled to engineer situations where the waves propagate as though they were on an effectively curved geometry, like around a black hole or in an inflating universe. This enables the experimental study of field theories on curved geometries.

Controlled propagation of waves → effective geometry → linearised excitations
(engineered nonlinearity) (curvature) (quantum field)

Today’s talk

The propagation of waves in nonlinear media may be controlled to engineer situations where the waves propagate as though they were on an effectively curved geometry, like around a black hole or in an inflating universe. This enables the experimental study of field theories on curved geometries.

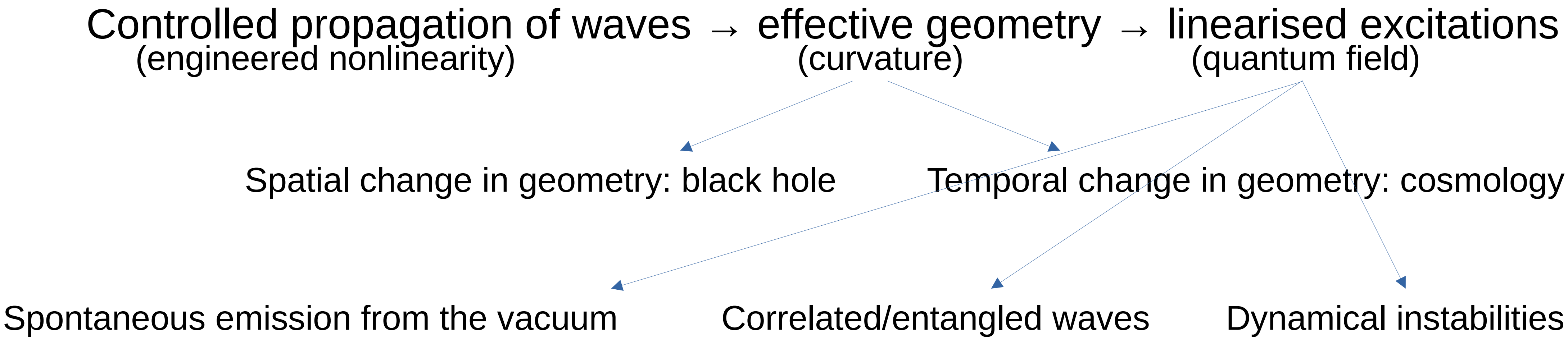
Controlled propagation of waves → effective geometry → linearised excitations
(engineered nonlinearity) (curvature) (quantum field)

Spatial change in geometry: black hole

Temporal change in geometry: cosmology

Today’s talk

The propagation of waves in nonlinear media may be controlled to engineer situations where the waves propagate as though they were on an effectively curved geometry, like around a black hole or in an inflating universe. This enables the experimental study of field theories on curved geometries.



Controlled propagation of waves → effective geometry → linearised excitations

(engineered nonlinearity) (curvature) (quantum field)

Spatial change in geometry: black hole

Temporal change in geometry: cosmology

Spontaneous emission from the vacuum

Correlated/entangled waves

Dynamical instabilities

Today’s talk

Hawking effect on Schwarzschild black hole

Theory of analogue gravity

How to observe the Hawking effect in the laboratory?

Experiments in rotating geometry

Today’s talk

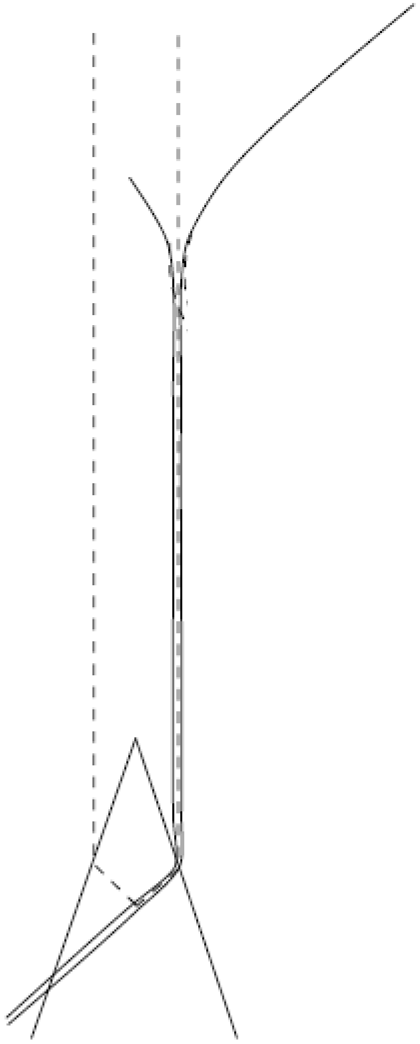
Hawking effect on Schwarzschild black hole

Theory of analogue gravity

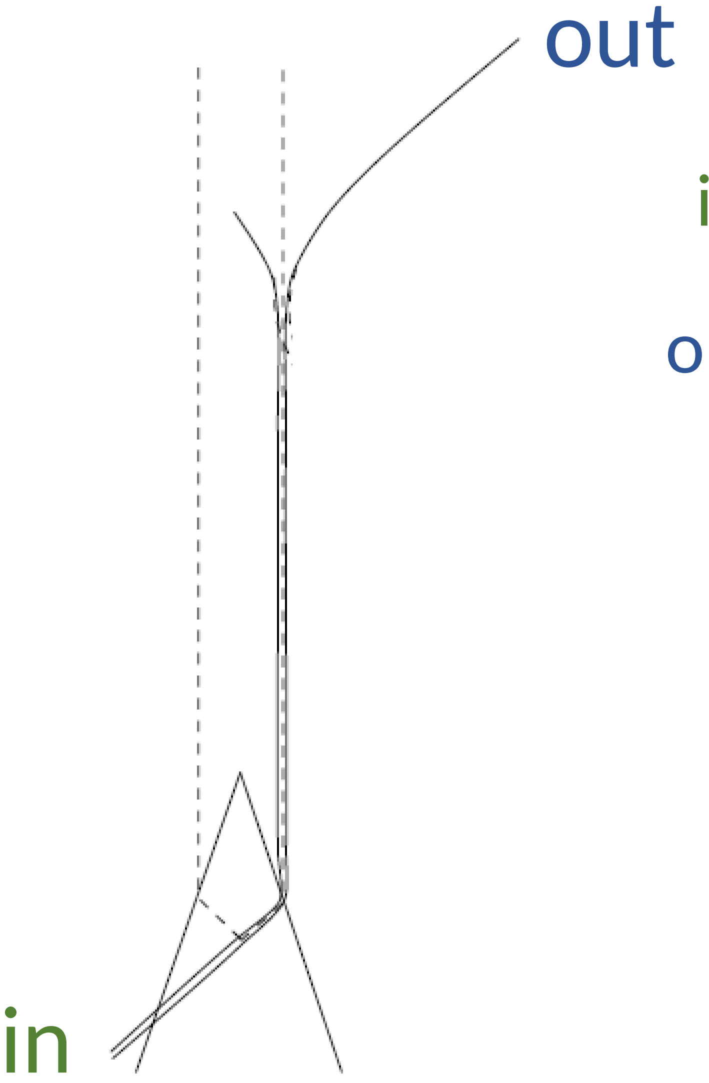
How to observe the Hawking effect in the laboratory?

Experiments in rotating geometry

History of a black hole



Field on the spacetime



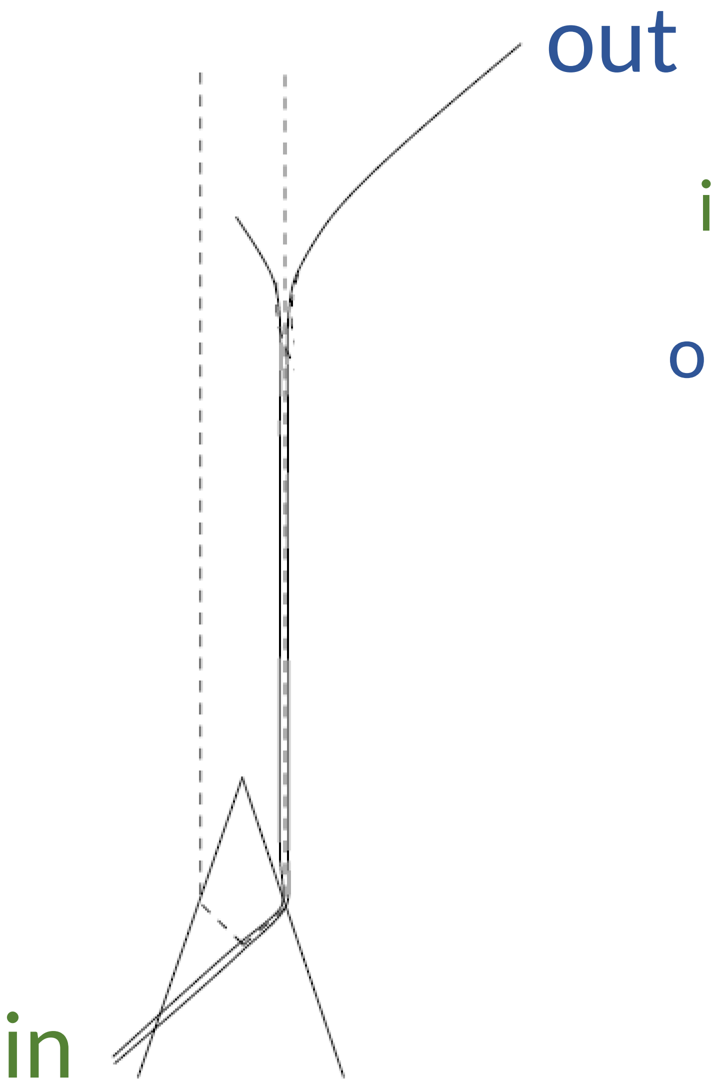
positive frequency wave

negative frequency wave

in: $\phi = \int d\omega \left(a_\omega f_\omega + a^\dagger_\omega f^*_\omega\right) \quad a\left|0\right\rangle = 0$

out: $\phi = \int d\omega \left(\bar{a}_\omega F_\omega + \bar{a}^\dagger_\omega F^*_\omega\right) \quad \bar{a}\left|\bar{0}\right\rangle = 0$

Scattering of positive/negative norm waves



positive
frequency
wave

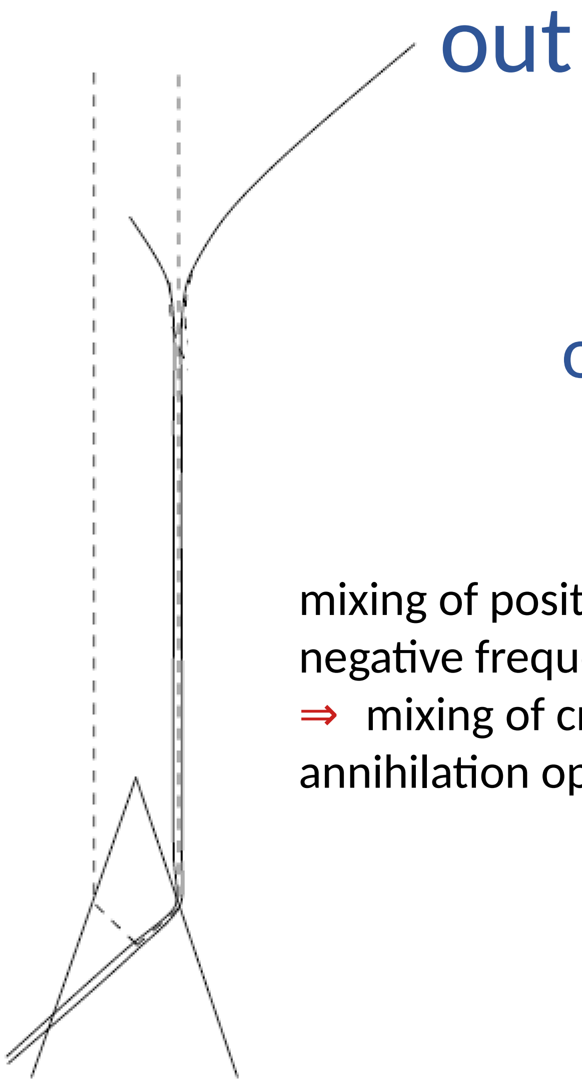
negative
frequency
wave

in: $$\phi = \int d\omega \left(a_\omega f_\omega + a^\dagger_\omega f^*_\omega\right) \quad a\,|0\rangle = 0$$

out: $$\phi = \int d\omega \left(\bar{a}_\omega F_\omega + \bar{a}^\dagger_\omega F^*_\omega\right) \quad \bar{a}\,|\bar{0}\rangle = 0$$

Express out modes in terms of in modes: $F_\omega = \int d\omega' \left(\alpha_{\omega\omega'} f_{\omega'} + \beta_{\omega\omega'} f^*_{\omega'}\right)$

Scattering of positive/negative norm waves



positive frequency wave

negative frequency wave

in: $\phi = \int d\omega \left(a_\omega f_\omega + a_\omega^\dagger f_\omega^*\right) \quad a\,|0\rangle = 0$

out: $\phi = \int d\omega \left(\bar{a}_\omega F_\omega + \bar{a}_\omega^\dagger F_\omega^*\right) \quad \bar{a}\,|\bar{0}\rangle = 0$

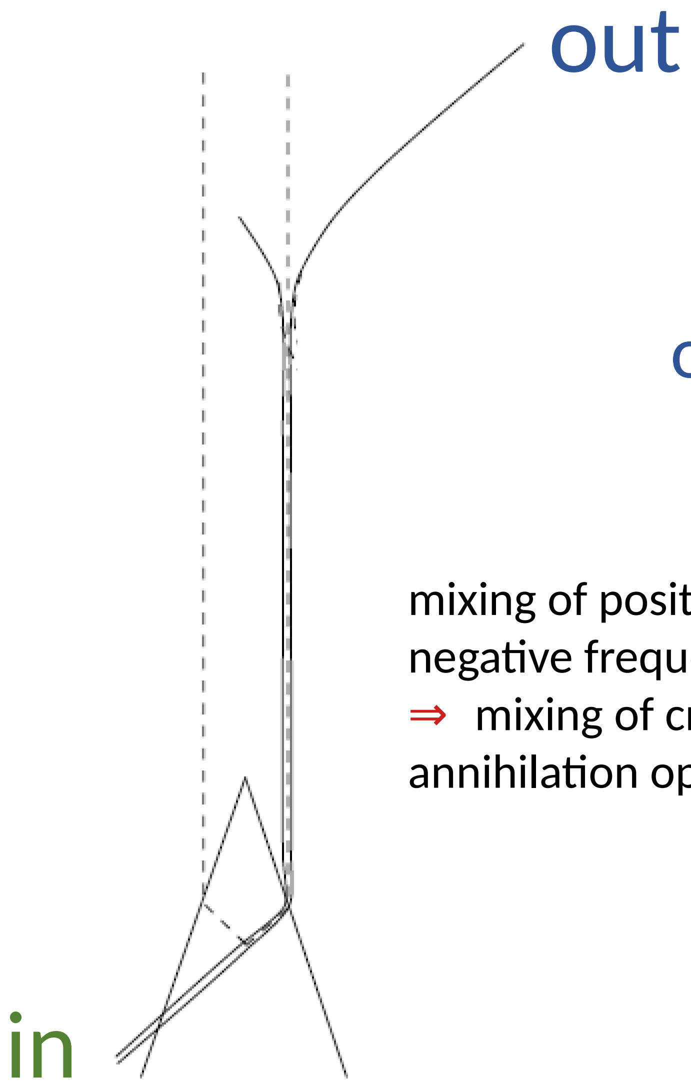
Express out modes in terms of in modes: $F_\omega = \int d\omega' \left(\alpha_{\omega\omega'} f_{\omega'} + \beta_{\omega\omega'} f_{\omega'}^*\right)$

$\Rightarrow |\bar{0}\rangle \neq |0\rangle$

mixing of positive and negative frequency waves

⇒ mixing of creation and annihilation operator

Hawking effect == scattering phenomenon



$\omega_{in} \approx e^{\kappa t} \omega_{out}$

surface gravity of the black hole

positive frequency wave

negative frequency wave

in: $\phi = \int d\omega \left(a_\omega f_\omega + a^\dagger_\omega f^*_\omega \right)$ $\quad a \left|0\right\rangle = 0$

out: $\phi = \int d\omega \left(\bar{a}_\omega F_\omega + \bar{a}^\dagger_\omega F^*_\omega \right)$ $\quad \bar{a} \left|\bar{0}\right\rangle = 0$

Express out modes in terms of in modes: $F_\omega = \int d\omega' \left(\alpha_{\omega\omega'} f_{\omega'} + \beta_{\omega\omega'} f^*_{\omega'} \right)$

$\Rightarrow \left|\bar{0}\right\rangle \neq \left|0\right\rangle$

mixing of positive and negative frequency waves
⇒ mixing of creation and annihilation operator

$$\text{a} \left|\bar{0}\right\rangle = \sum_{\omega'} \beta_{\omega\omega'} \left|\bar{1}\right\rangle > 0$$

Spontaneous emission from the vacuum!

Black hole ⇒ Hawking radiation

Today’s talk

Hawking effect on Schwarzschild black hole

Theory of analogue gravity

How to observe the Hawking effect in the laboratory?

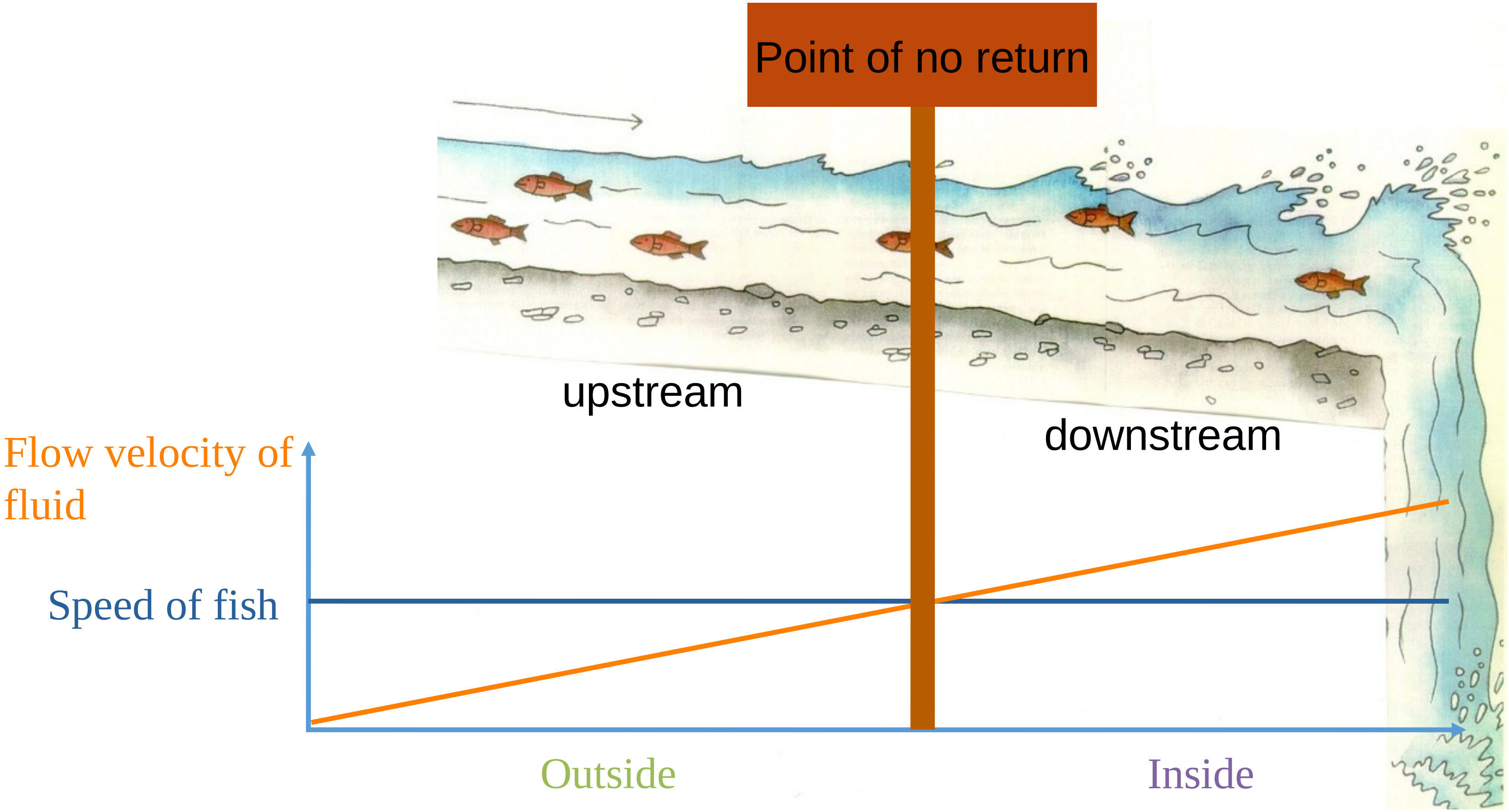
Experiments in rotating geometry

Curvature of spacetime near a Schwarzschild black hole

Schwarzschild black holes are characterised solely by their mass → Schwarzschild black hole = 4-sphere

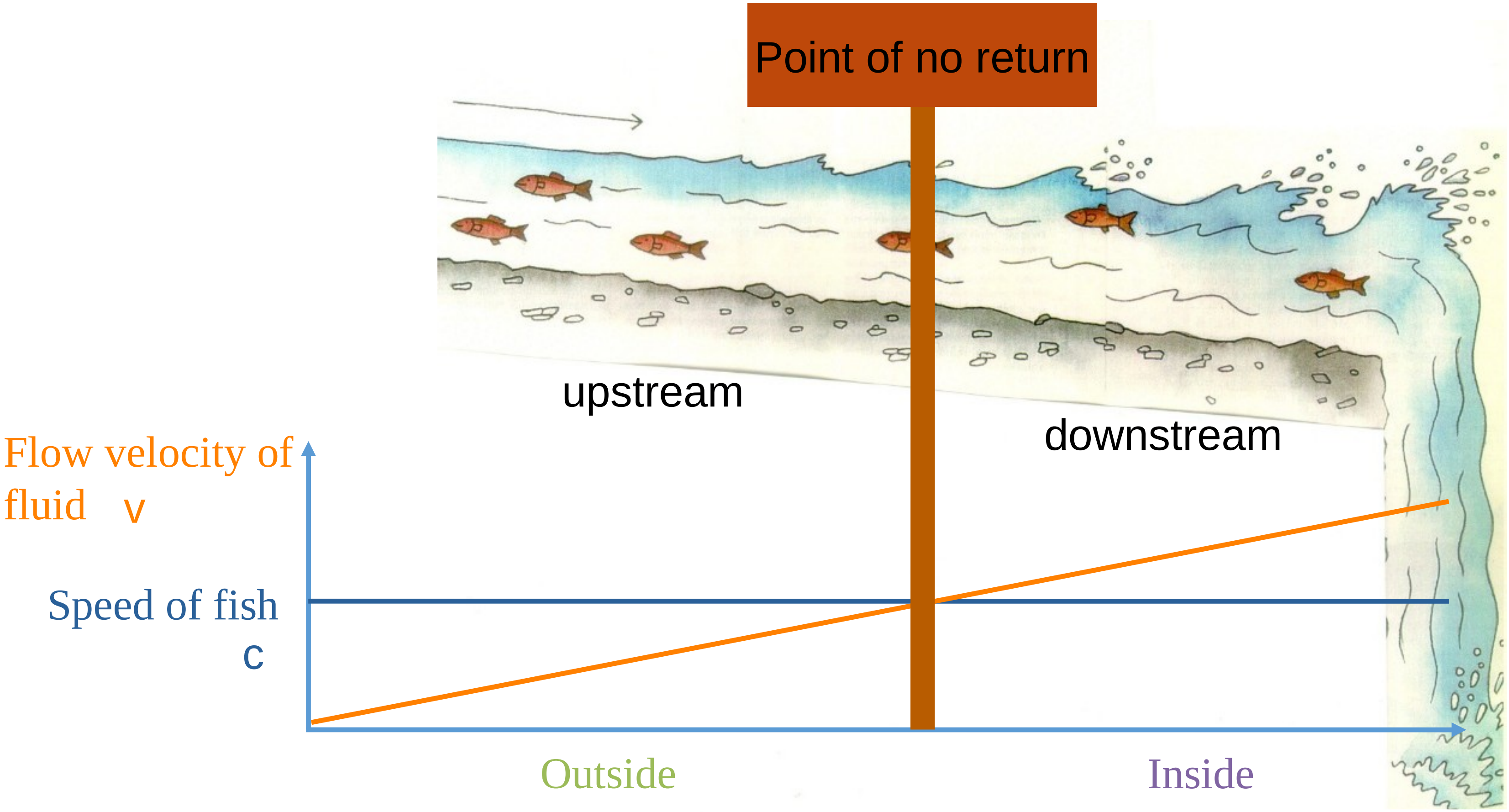
Rotational spatial symmetry

→ full description in 1+1D



Scalar field on Schwarzschild geometry

Schwarzschild geometry ↔ waterfall geometry



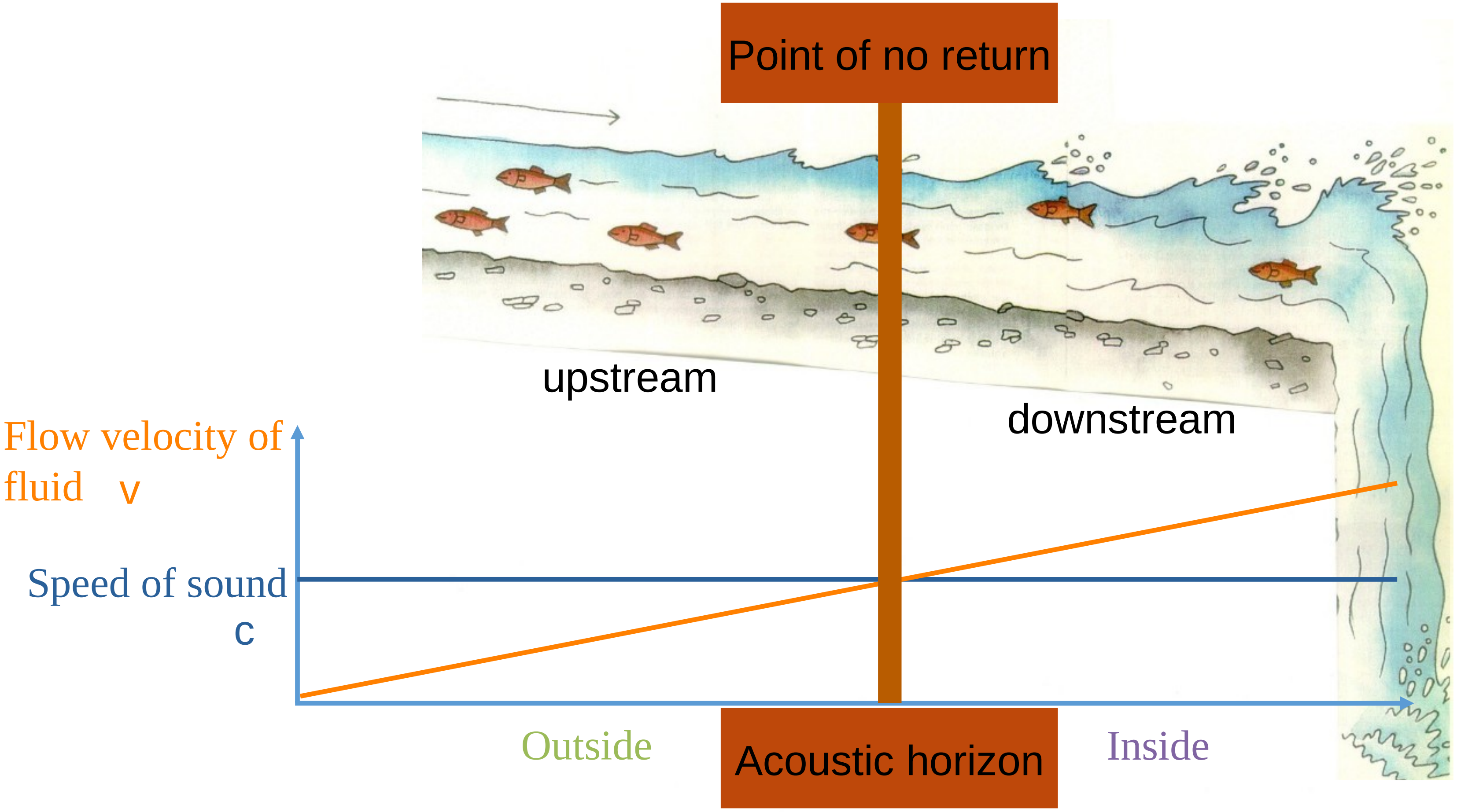
Wave equation for scalar field on Schwarzschild geometry:

$$g^{\mu\nu}_{schw} = \begin{pmatrix} -1 & -v \\ -v & (c^2 - v^2) \end{pmatrix}$$

Inverse metric tensor of Painlevé-Gullstrand metric in 1+1D

Acoustic field in waterfall geometry

Schwarzschild geometry ↔ waterfall geometry



Wave equation for scalar field on Schwarzschild geometry:

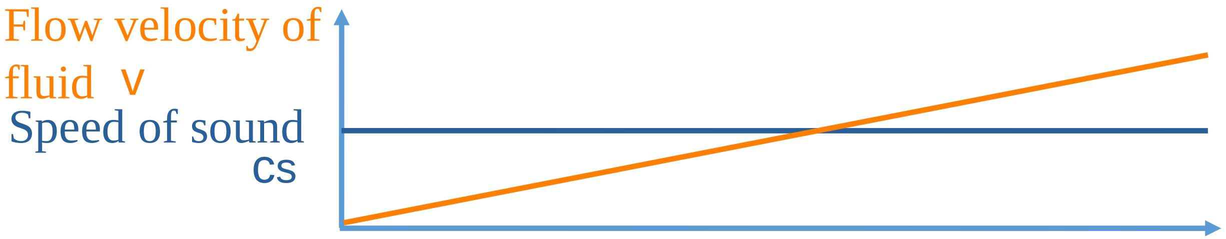
$$g^{\mu\nu}_{schw} = \begin{pmatrix} -1 & -v \\ -v & (c^2 - v^2) \end{pmatrix}$$

$$g^{\mu\nu}_{Unruh} = \begin{pmatrix} -1 & -v \\ -v & (c^2 - v^2) \end{pmatrix}$$

Unruh *PRL* **46** 1351 (1981): wave equations are isomorphic

Derivation of analogy

Schwarzschild geometry ↔ waterfall geometry



Wave equation of fluid (Nonlinear Schrödiner Equation): $\mathrm{i}\partial_t\psi = -\frac{\hbar}{2m}\nabla^2\psi + g|\psi|^2\psi$

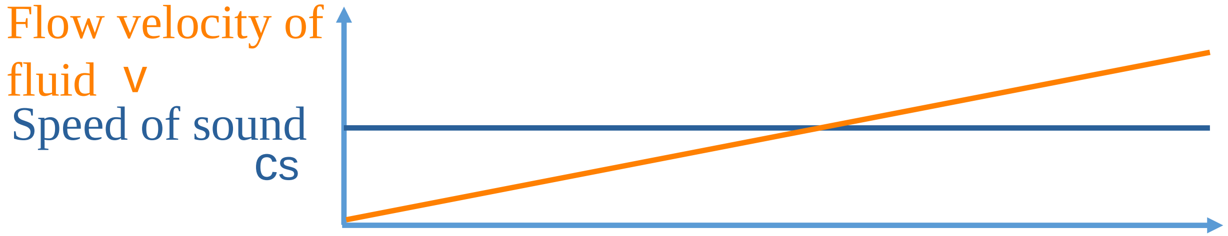
Kinetic energy

Nonlinear interaction

m – mass
g – interaction cst

Derivation of analogy

Schwarzschild geometry ↔ waterfall geometry



Wave equation of fluid (Nonlinear Schrödiner Equation): $\mathrm{i}\partial_t\psi = -\frac{\hbar}{2m}\nabla^2\psi + g|\psi|^2\psi$

m – mass
g – interaction cst

Kinetic energy

Nonlinear interaction

Write complex scalar field in terms of its amplitude and phase (Madelung transform): $\psi = \sqrt{\rho}e^{i\phi}$

→ hydro and Euler eqs:

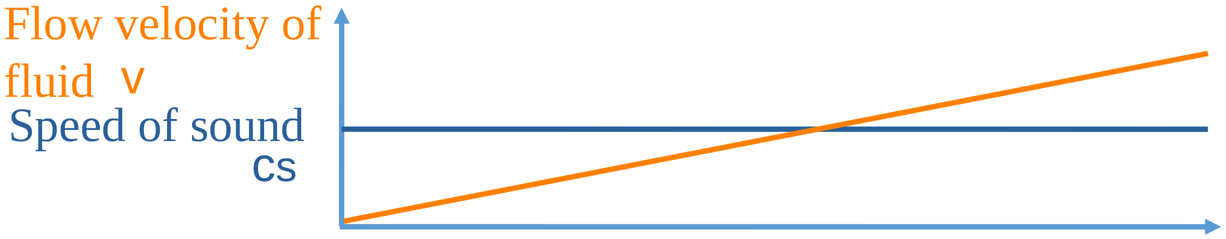
$$\partial_t\rho + \boldsymbol{\nabla}\overset{\cdot}{(}\rho\boldsymbol{v}) = 0$$

$$\partial_t\phi + \frac{1}{2\hbar}m\boldsymbol{v}^2 + g\rho - \cancel{\frac{\hbar}{2m}\frac{\Delta\rho^{1/2}}{\rho^{1/2}}} = 0$$

$\mathrm{v} = (\hbar/m)\boldsymbol{\nabla}\phi$ Fluid velocity

Derivation of analogy

Schwarzschild geometry ↔ waterfall geometry



Wave equation of fluid (Nonlinear Schrödiner Equation): $\mathrm{i}\partial_t\psi = -\frac{\hbar}{2m}\nabla^2\psi + g|\psi|^2\psi$

m – mass
g – interaction cst

Kinetic energy   Nonlinear interaction

Write complex scalar field in terms of its amplitude and phase (Madelung transform): $\psi = \sqrt{\rho}e^{i\phi}$

→ hydro and Euler eqs:

$$\partial_t\rho + \boldsymbol{\nabla}\cdot(\rho\boldsymbol{v}) = 0$$

$$\partial_t\phi + \frac{1}{2\hbar}m\boldsymbol{v}^2 + g\rho - \cancel{\frac{\hbar}{2m}\frac{\Delta\rho^{1/2}}{\rho^{1/2}}} = 0$$

$\mathrm{v}=(\hbar/m)\boldsymbol{\nabla}\phi$ Fluid velocity

Linearise around background: $\rho = \rho_0 + \epsilon\rho_1 + O(\epsilon^2)$

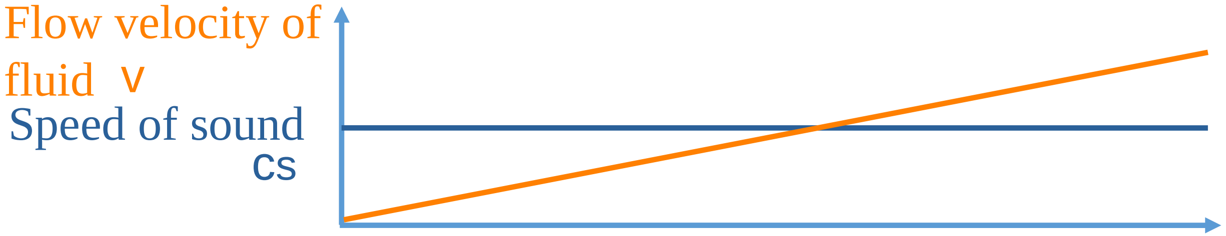
$\mathrm{c}_s = \sqrt{\frac{\hbar g\rho_0}{2m}}$ Speed of sound

→ wave eq for collective excitations (sound waves) of fluid:

$$-\partial_t\left(\frac{\rho_0}{c_s^2}(\partial_t\rho_1 + \boldsymbol{v_0}\boldsymbol{\nabla}\rho_1)\right) + \boldsymbol{\nabla}\left(\rho_0\boldsymbol{\nabla}\rho_1 - \frac{\rho_0\boldsymbol{v_0}}{c_s^2}\ \partial_t\rho_1 + \boldsymbol{v_0}\boldsymbol{\nabla}\rho_1)\right) = 0$$

Derivation of analogy

Schwarzschild geometry ↔ waterfall geometry



$$\mathrm{v}{=}(\hbar/m)\boldsymbol{\nabla}\phi$$ Fluid velocity

$$\mathrm{c}_s = \sqrt{\frac{\hbar g \rho_0}{2m}}$$ Speed of sound

Wave eq for collective excitations (sound waves) of fluid:

$$-\partial_t\left(\frac{\rho_0}{c_s^2}\left(\partial_t\rho_1 + \boldsymbol{v_0}\boldsymbol{\nabla}\rho_1\right)\right) + \boldsymbol{\nabla}\left(\rho_0\boldsymbol{\nabla}\rho_1 - \frac{\rho_0\boldsymbol{v_0}}{c_s^2}\ \partial_t\rho_1 + \boldsymbol{v_0}\boldsymbol{\nabla}\rho_1)\Big)\right. = 0$$

Define metric tensor $$\eta_{\mu\nu} = \begin{pmatrix} -(c_s^2 - \boldsymbol{v_0}^2) & -v_o^x & -v_o^y \\ -v_o^x & 1 & 0 \\ -v_o^y & 0 & 1 \end{pmatrix}$$

Relavistic form of wave eq for collective excitations: $$\Delta\rho_1 = \frac{1}{\sqrt{-\eta}}\partial_\mu(\sqrt{-\eta}\eta^{\mu\nu}\partial_\nu\rho_1) = 0$$

Acoustic metric → motion of sound in inhomogeneous fluid flow == scalar field on curved spacetime

Unruh *PRL* **46** 1351 (1981)

Controlling the effective spacetime

$$\eta_{\mu\nu} = \begin{pmatrix} -(c_s^2 - \boldsymbol{v_0}^2) & -v_o^x & -v_o^y \\ -v_o^x & 1 & 0 \\ -v_o^y & 0 & 1 \end{pmatrix}$$

$\mathrm{v}=(\hbar/m)\boldsymbol{\nabla}\phi$ Fluid velocity

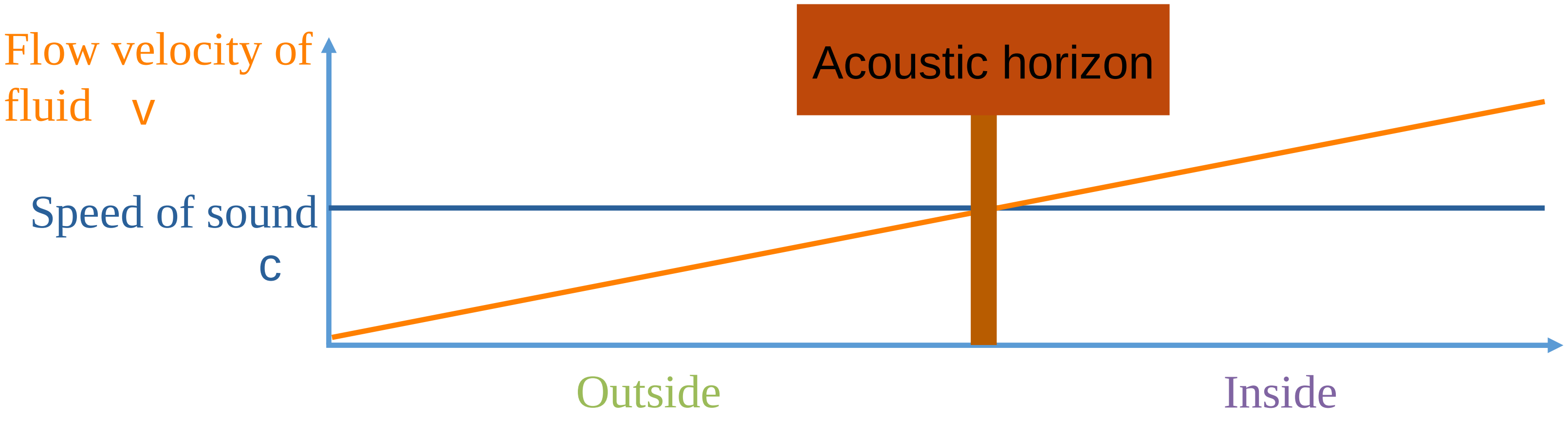
$\mathrm{c}_s = \sqrt{\frac{\hbar g \rho_0}{2m}}$ Speed of sound

(i) accelerating flow along 1 spatial dimension → Schwarschild
(ii) radially accelerating flow in 2 spatial dimensions → Schwarzschild
Horizon where $\mathrm{v}_0 = c_s$
(iii) radially and azimuthally accelerating flow in 2 dimensions → Kerr
Horizon where $\mathrm{v}_r = c_s$
Ergosurface where $|\boldsymbol{v_0}| = c_s$

Visser *Class Quant Grav* **15** 1767 (1998)

Quantised acoustic field in waterfall geometry

Schwarzschild geometry ↔ waterfall geometry



Quantised acoustic field:

in: $\phi = \int d\omega \left(a_\omega f_\omega + a^\dagger_\omega f^*_\omega \right) \quad a\,|0\rangle = 0$

out: $\phi = \int d\omega \left(\bar{a}_\omega F_\omega + \bar{a}^\dagger_\omega F^*_\omega \right) \quad \bar{a}\,|\bar{0}\rangle = 0$

Express out modes in terms of in modes:

$$F_\omega = \int d\omega' \left(\alpha_{\omega\omega'} f_{\omega'} + \beta_{\omega\omega'} f^*_{\omega'} \right)$$

Different speeds on either side of the horizon $\Rightarrow |\bar{0}\rangle \neq |0\rangle \Rightarrow \beta_{\omega\omega'} \neq 0$

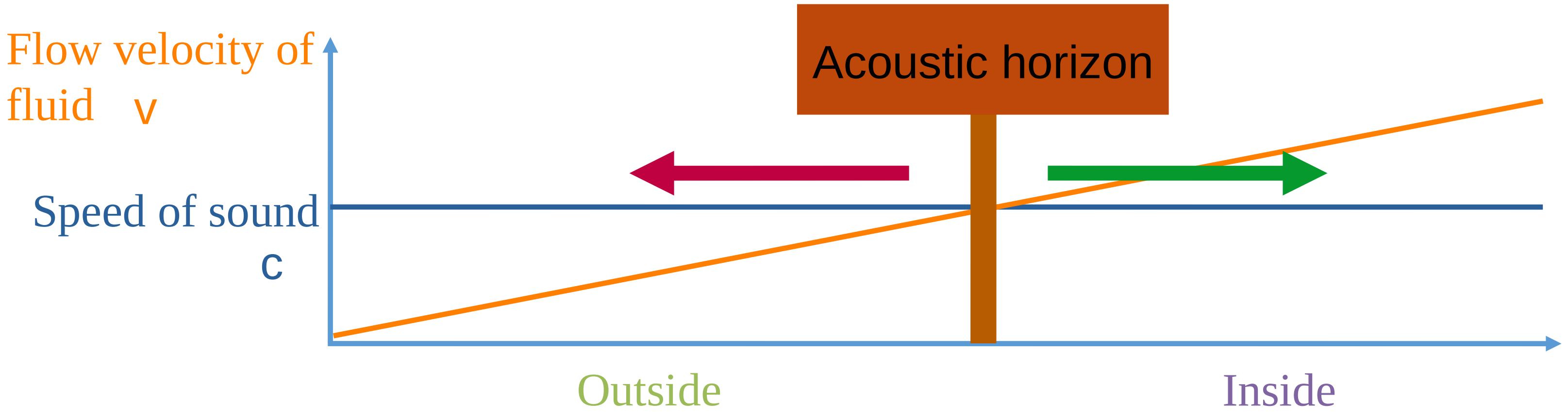
mixing of positive and negative frequency waves
⇒ mixing of creation and annihilation operators

$$\mathrm{a}\,|\bar{0}\rangle = \sum_{\omega'} \beta_{\omega\omega'} |\bar{1}\rangle > 0$$

Quantised acoustic field in waterfall geometry

Unruh *PRL* **46** 1351 (1981)

Schwarzschild geometry ↔ waterfall geometry



Quantised acoustic field:

$$\text{in:}\quad \phi = \int d\omega \left(a_\omega f_\omega + a_\omega^\dagger f_\omega^*\right) \qquad a\left|0\right\rangle = 0$$

$$\text{out:}\ \phi = \int d\omega \left(\bar{a}_\omega F_\omega + \bar{a}_\omega^\dagger F_\omega^*\right) \qquad \bar{a}\left|\bar{0}\right\rangle = 0$$

Express out modes in terms of in modes:

$$F_\omega = \int d\omega' \left(\alpha_{\omega\omega'} f_{\omega'} + \beta_{\omega\omega'} f_{\omega'}^*\right)$$

Different speeds on either side of the horizon $\Rightarrow \left|\bar{0}\right\rangle \neq \left|0\right\rangle \Rightarrow \beta_{\omega\omega'} \neq 0$

mixing of positive and negative frequency waves
⇒ mixing of creation and annihilation operators

Spontaneous emission from the vacuum!

Hawking effect

‘Black hole’ experiments in analogue gravity

Sound waves

In BEC

Hawking correlations
Steinhauer 2019

Black hole laser? → no
Steinhauer 2014
Steinhauer 2022

In fluid of light
(microcavity polaritons)
Proof of principle by Amo and Bloch 2015
New experiments in Paris 2022

Gravity/Capilary waves

Scattering at the white hole
Rousseaux and Leonhardt 2008
Weinfurtner and Unruh 2010
Correlations across the WH horizon
Rousseaux and Parentani 2016
Correlations across the BH horizon
Rousseaux 2020

Rotating black hole - superradiance
Weinfurtner 2016
Rotating black hole – oscillation of light rings (QNMs)
Weinfurtner 2020

Light waves

Scatteringt at the BH/WH horizon
König and Leonhardt (Fibre) 2008
Faccio (Bulk) 2010
König (Fibre) 2012
Wang (Fibre) 2013
Murdoch (Fibre) 2015?
Bose (Fibre) 2015
Ciret (waveguide) 2016
Kanakis (Fibre) 2016
Gaafar (waveguide) 2017
König and Jacquet (Fibre) 2018
Leonhardt (Fibre) 2019

Negative frequency waves
König and Faccio 2012
König 2014, 2015

Universality of the Hawking effect, Unruh and Schützhold PRD **71** 024028 (2005)?

Today’s talk

Hawking effect on Schwarzschild black hole

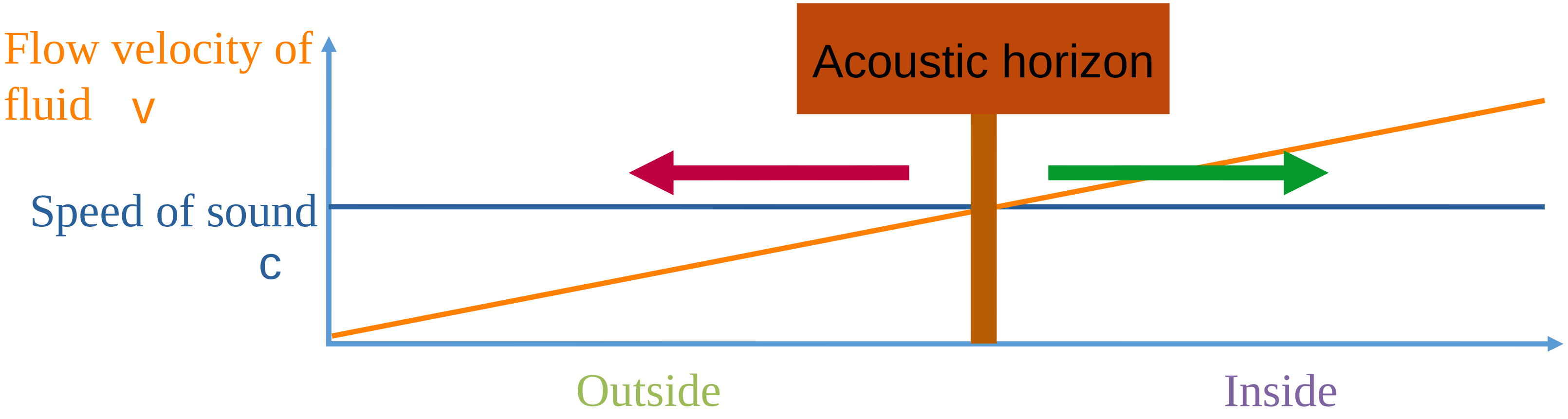
Theory of analogue gravity

How to observe the Hawking effect in the laboratory?

Experiments in rotating geometry

How to observe the Hawking effect in the laboratory?

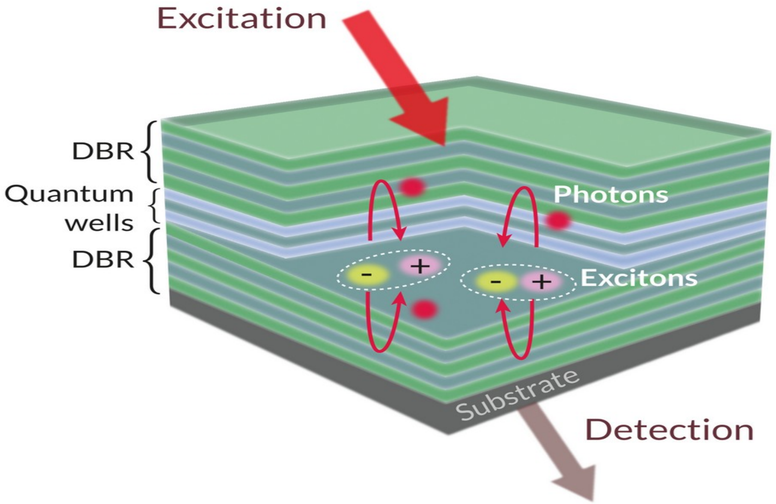
1) create a transsonic fluid → acoustic horizon where v=c
different speed on either side of acoustic horizon → mixing of positive and negative frequency waves
→ spontaneous emission of phonon pairs from the vacuum
2) observe Hawking spectrum
Unruh *PRL* **46** 1351 (1981)
3) observe correlations across the horizon

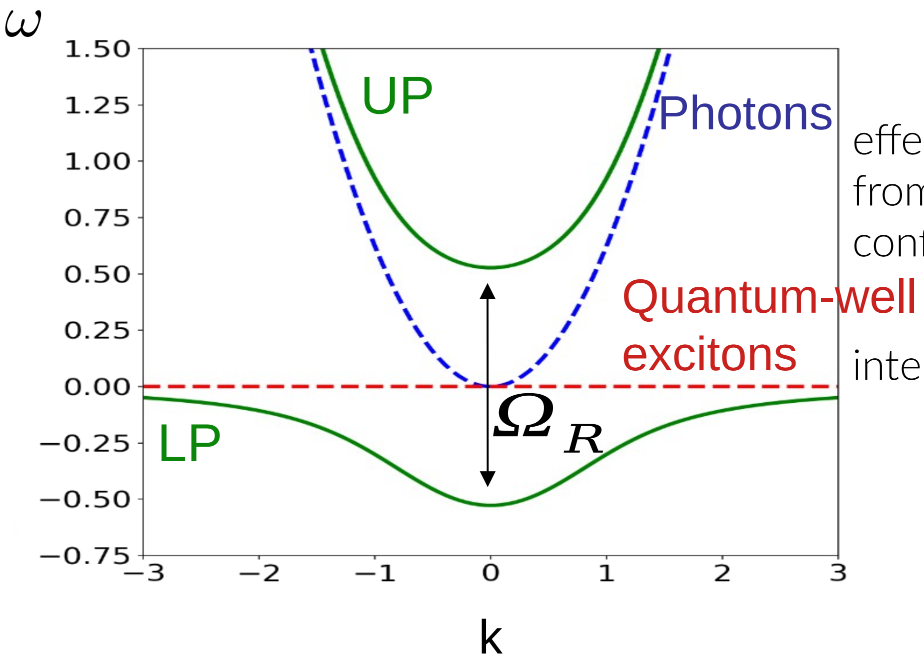


Microcavity polaritons

Carusotto and Ciutti RMP **85** 299 (2013)

Polaritons= photons dressed with material excitations that live in the cavity plane





effective mass from confinement

interactions

Dynamics in the cavity plane described by Gross-Pitaevskii (Nonlinear Schrödinger) equation:

$$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2\nabla^2}{2m^*_{LP}} + g\overset{|\psi|^2}{n}\right)\psi - \frac{i\hbar\gamma}{2}\psi + P(r,t)$$

g polariton-polariton interaction constant

γ losses

P pump

Driven-dissipative dynamics → Out-of-equilibrium system

Resonantly excited microcavity polaritons

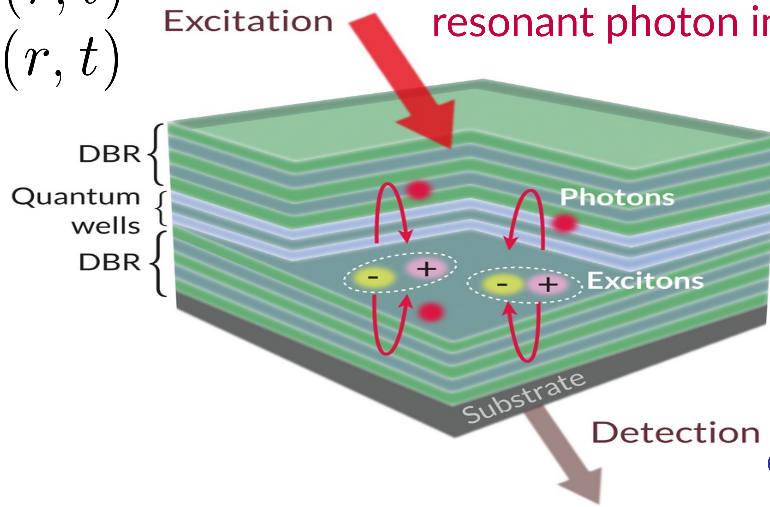
Maître *et al* PRX **10** 041028 (2020)

Polaritons= photons dressed with material excitations that live in the cavity plane

$$I(r,t) \rightarrow n(r,t)$$
$$\phi(r,t) \rightarrow v(r,t)$$

Phase + intensity profile of driving field

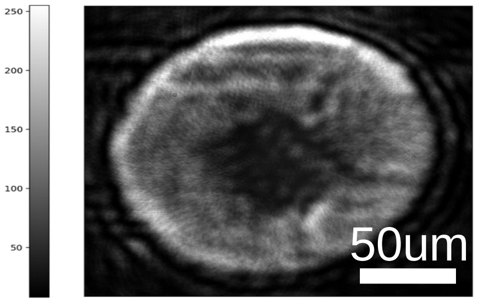
→ Spatial Light Modulator (SLM)



resonant photon injection

Imaging photons leaking out of the cavity

$$n(r,t) \rightarrow I(r,t)$$



$$v(r,t) \rightarrow \phi(r,t)$$

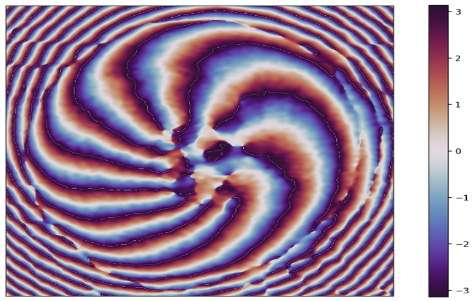
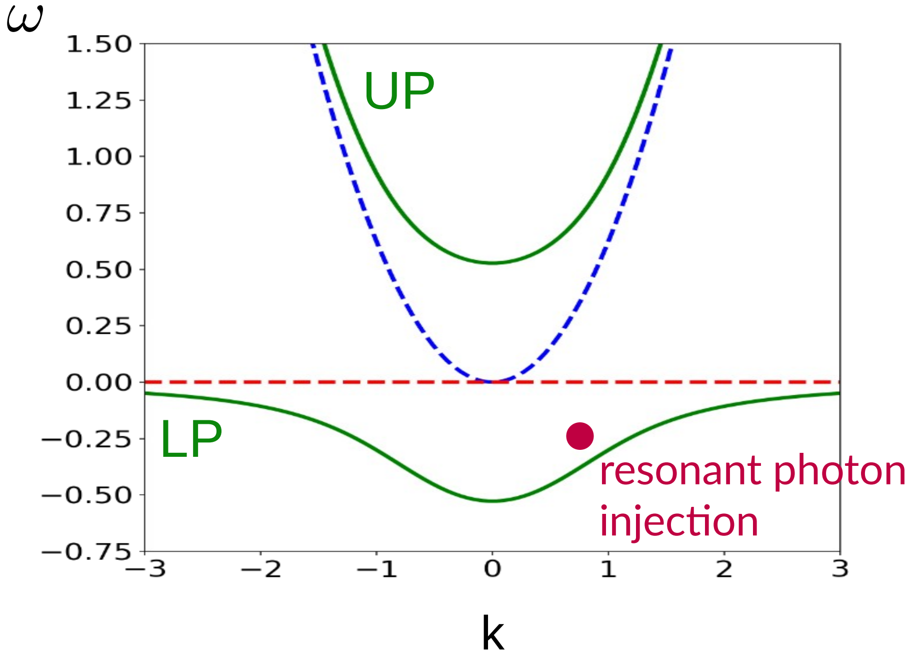


Image of the **cavity plane**

→ density map: $c \propto \sqrt{n}$

→ velocity map: $v \propto \nabla \phi$



resonant photon injection

Full optical experiment

(2D planar sample, no microstructure)

Collective excitations of polariton fluid

F Claude, M Jacquet *et al* PRL **129** 103601 (2022)
F Claude, M Jacquet *et al* arxiv:2211.12373

GPE: $$\mathrm{i}\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2\nabla^2}{2m^*_{LP}} + gn\right)\psi - \frac{i\hbar\gamma}{2}\psi + P(r,t)$$

interaction cst $|\psi|^2$ losses pump

Bogoliubov theory:

1. Linearise GPE around steady-state solution $\psi(r,t) = \psi_0(r,t) + \delta\psi(r,t)$

Collective excitations of polariton fluid

F Claude, M Jacquet *et al* PRL **129** 103601 (2022)
F Claude, M Jacquet *et al* arxiv:2211.12373

GPE: $$\mathrm{i}\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2\nabla^2}{2m^*_{LP}} + gn\right)\psi - \frac{i\hbar\gamma}{2}\psi + P(r,t)$$

interaction cst $|\psi|^2$ losses pump

Bogoliubov theory:

1. Linearise GPE around steady-state solution $\psi(r,t) = \psi_0(r,t) + \delta\psi(r,t)$
2. Equation of motion of weak perturbations $$\mathrm{i}\hbar\frac{\partial}{\partial t}\begin{pmatrix} \delta\psi(r,t) \\ \delta\psi^*(r,t) \end{pmatrix} = L_{\mathrm{Bog}}\begin{pmatrix} \delta\psi(r,t) \\ \delta\psi^*(r,t) \end{pmatrix}$$

Bogo operator

Collective excitations of polariton fluid

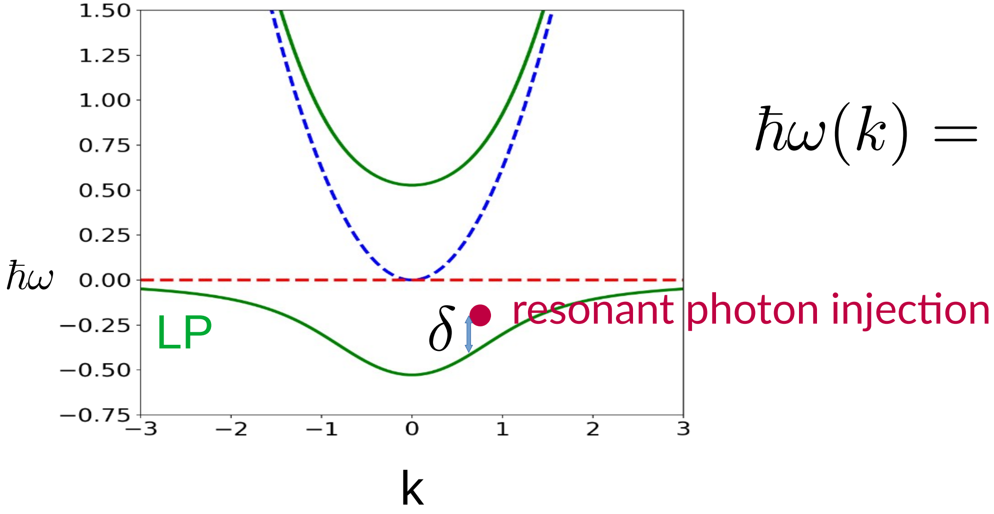
F Claude, M Jacquet *et al* PRL **129** 103601 (2022)
F Claude, M Jacquet *et al* arxiv:2211.12373

GPE: $$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2\nabla^2}{2m^*_{LP}} + gn\right)\psi - \frac{i\hbar\gamma}{2}\psi + P(r,t)$$

interaction cst $|\psi|^2$ losses pump

Bogoliubov theory:

1. Linearise GPE around steady-state solution $$\psi(r,t) = \psi_0(r,t) + \delta\psi(r,t)$$
2. Equation of motion of weak perturbations $$i\hbar\frac{\partial}{\partial t}\begin{pmatrix}\delta\psi(r,t)\\ \delta\psi^*(r,t)\end{pmatrix} = L_{\mathrm{Bog}}\begin{pmatrix}\delta\psi(r,t)\\ \delta\psi^*(r,t)\end{pmatrix}$$
Bogo operator
3. Eigenvalues of Bogoliubov operator == dispersion relation



$$\hbar\omega(k) = \pm\sqrt{\left(\frac{\hbar^2k^2}{2m} - \delta + 2gn\right)^2 - (gn)^2} - i\gamma$$

Collective excitations of polariton fluid

F Claude, M Jacquet *et al* PRL **129** 103601 (2022)
F Claude, M Jacquet *et al* arxiv:2211.12373

GPE: $$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2\nabla^2}{2m^*_{LP}} + gn\right)\psi - \frac{i\hbar\gamma}{2}\psi + P(r,t)$$

interaction cst $|\psi|^2$ losses pump

Bogoliubov theory:

1. Linearise GPE around steady-state solution $$\psi(r,t) = \psi_0(r,t) + \delta\psi(r,t)$$
2. Equation of motion of weak perturbations $$i\hbar\frac{\partial}{\partial t}\begin{pmatrix}\delta\psi(r,t)\\ \delta\psi^*(r,t)\end{pmatrix} = L_{\text{Bog}}\begin{pmatrix}\delta\psi(r,t)\\ \delta\psi^*(r,t)\end{pmatrix}$$
3. Eigenvalues of Bogoliubov operator == dispersion relation

Bogo operator

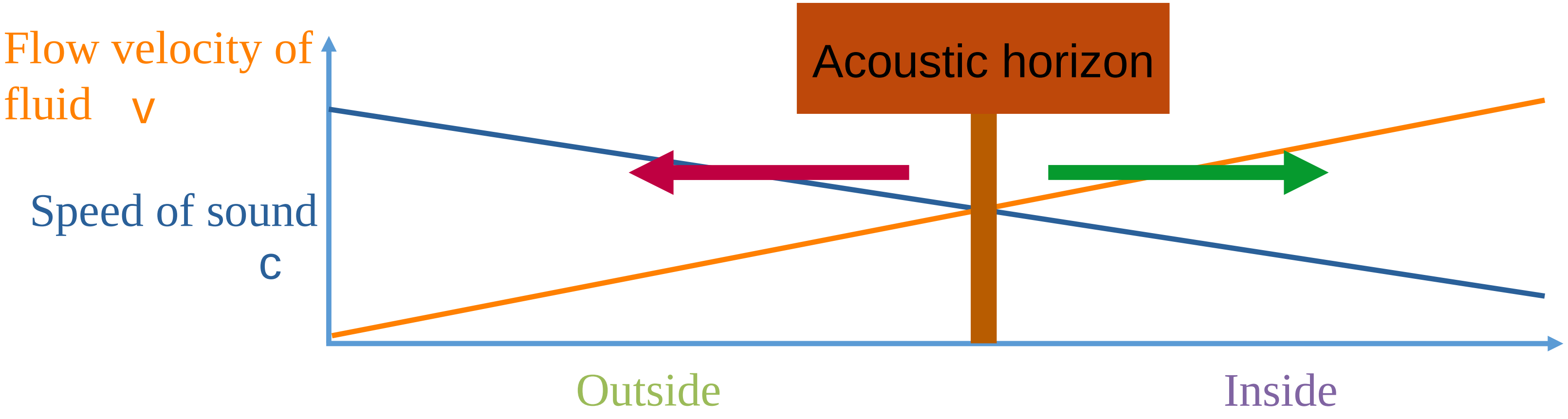
$$\omega(k) = \pm\sqrt{\frac{\hbar k^2}{2m}\left(\frac{\hbar k^2}{2m} + 2gn\right)}$$

At low *k*, dispersion is linear → excitations are phononic with “speed of sound” $\mathrm{c}_s = \sqrt{\hbar gn/m}$

Acoustic horizon and Hawking effect in polaritons

1) create a transsonic fluid → acoustic horizon where v=c
2) observe Hawking spectrum
3) observe correlations across the horizon

Unruh *PRL* **46** 1351 (1981)
Visser *Class Quant Grav* **15** 1767 (1998)



First proposal by Solnyshkov *et al. PRB* **84** 233405 (2011)
Numerical studies in Gerace and Carusotto *PRB* **86** 144505 (2012)
Grisins *et al*. *PRB* **94** 144518 (2016)
Jacquet *et al*. *EPJD* **76** 152 (2022)
Proof of principle experiments for acoustic horizon by Nguyen *et al. PRL* **114** 036402 (2015)
Jacquet *et al.* PTRSA **378** 201190225 (2020)

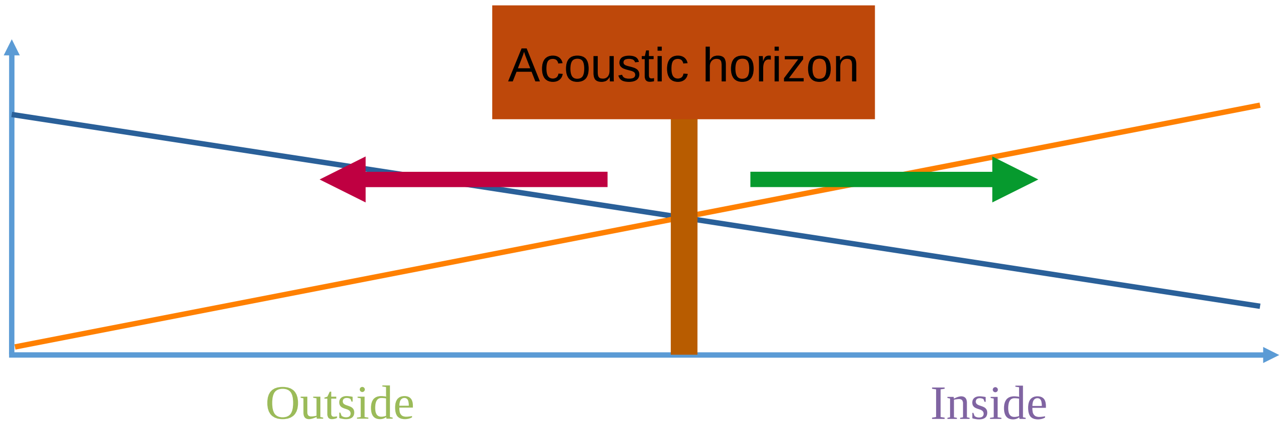
Hawking effect has not been seen in polaritons to date

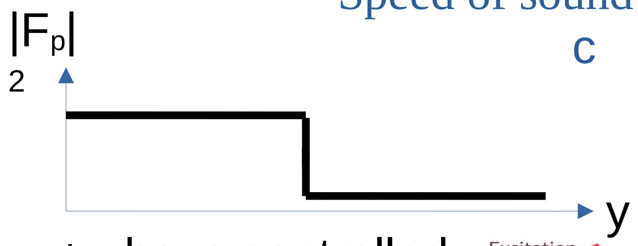
Acoustic horizon

Preliminary data by PhD student Kévin Falque

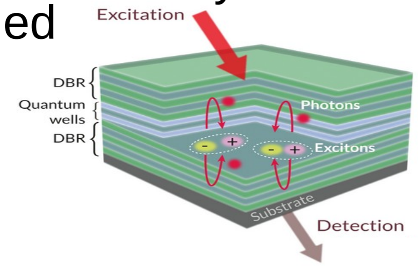
Flow velocity of fluid v

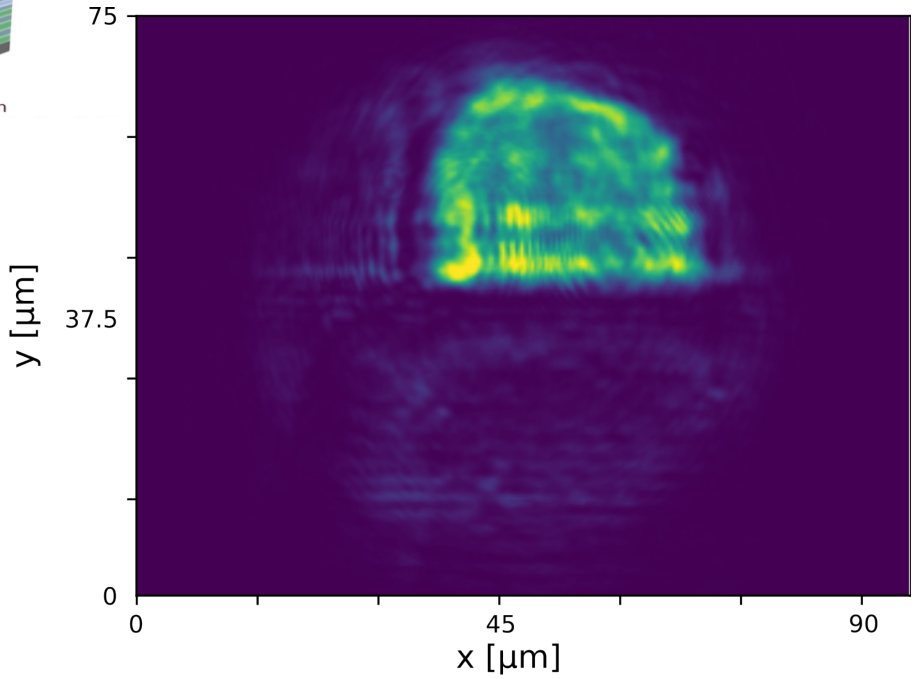
Speed of sound c



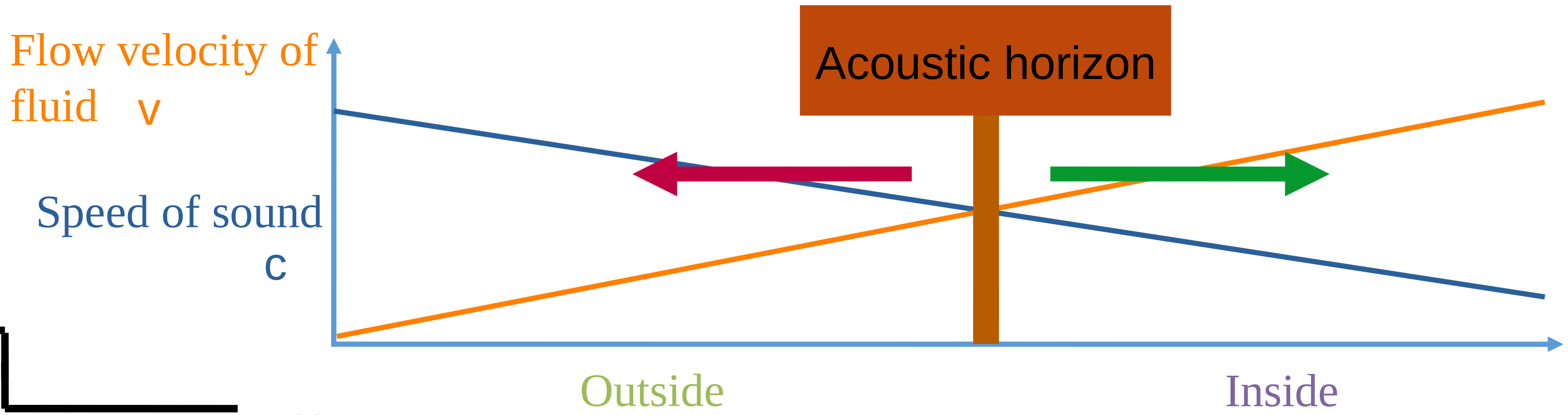


+ phase controlled with SLM





Acoustic horizon



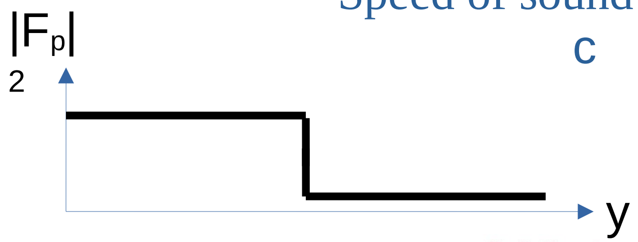
Flow velocity of fluid v

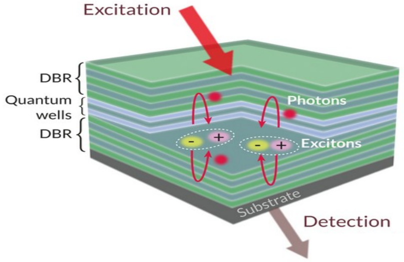
Speed of sound c

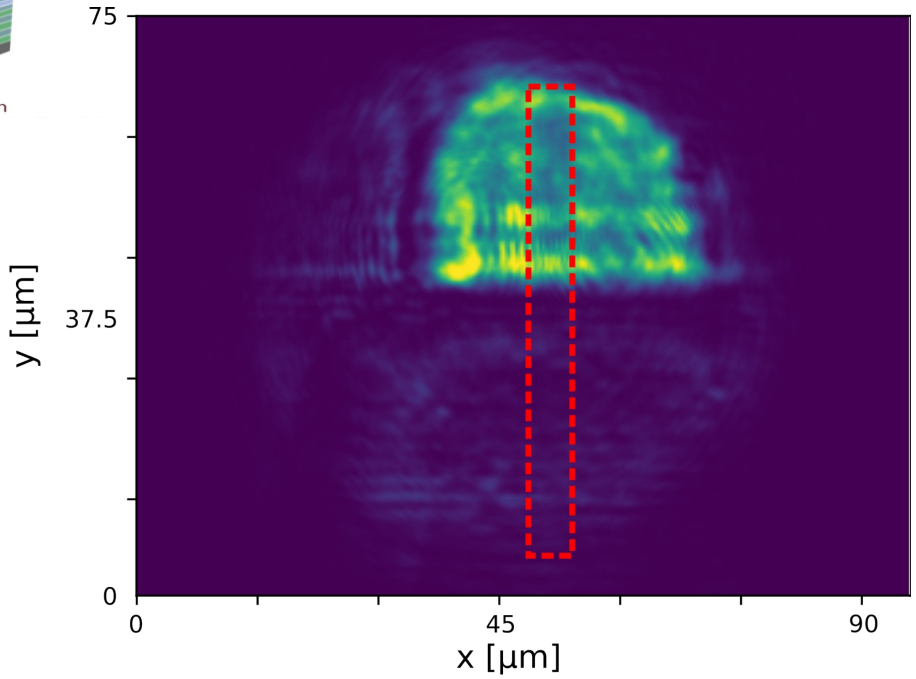
Acoustic horizon

Outside

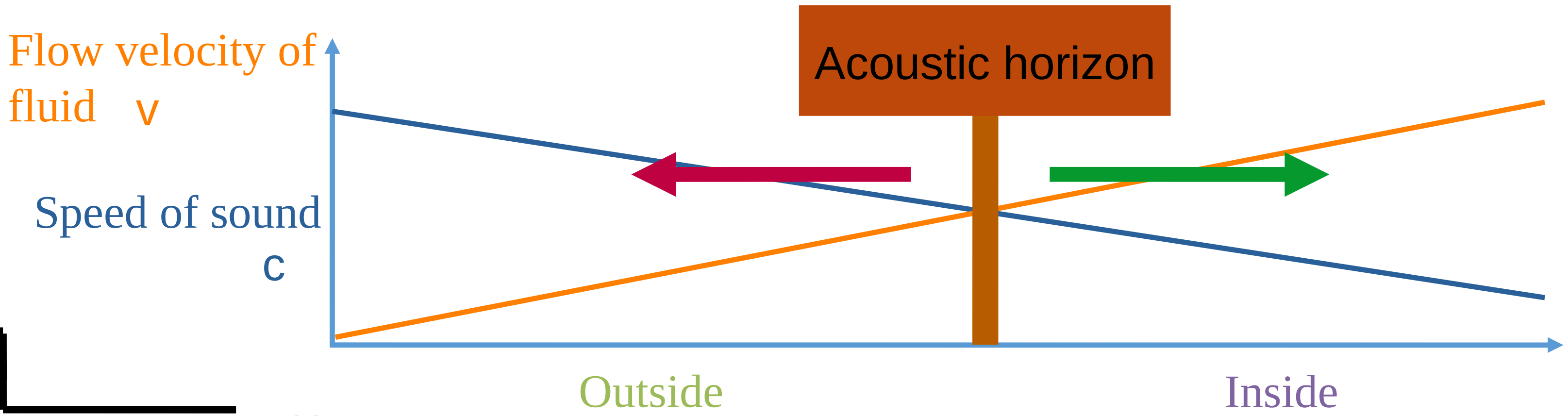
Inside

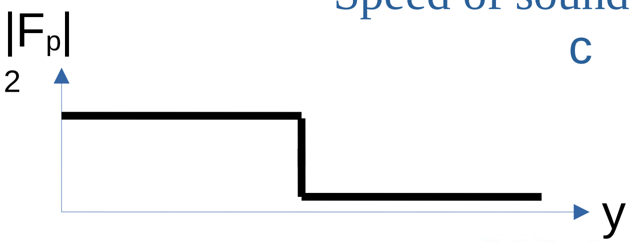


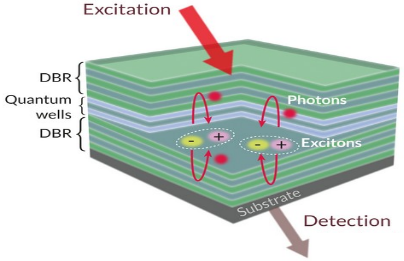


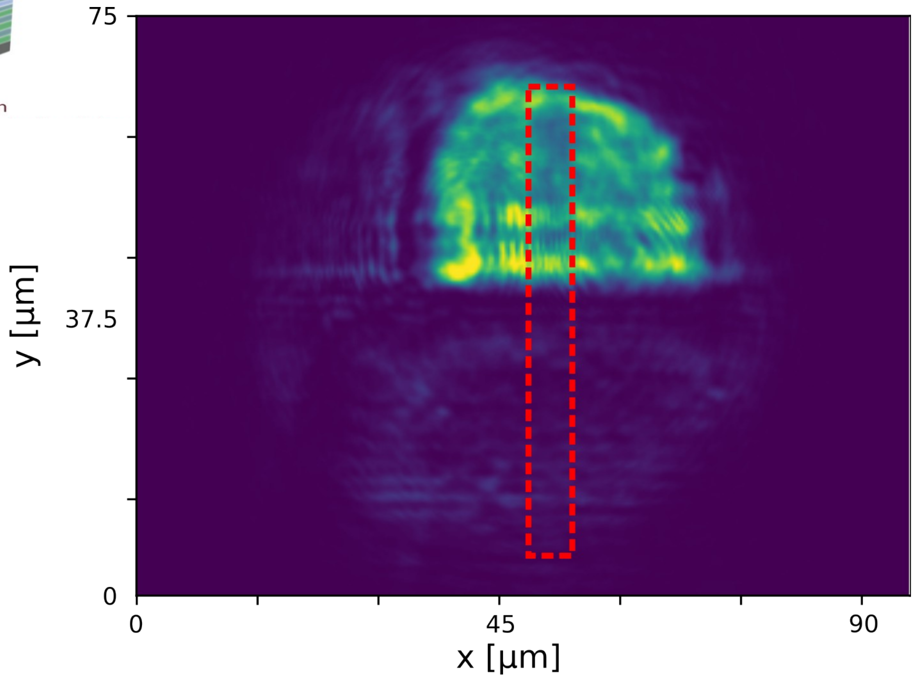


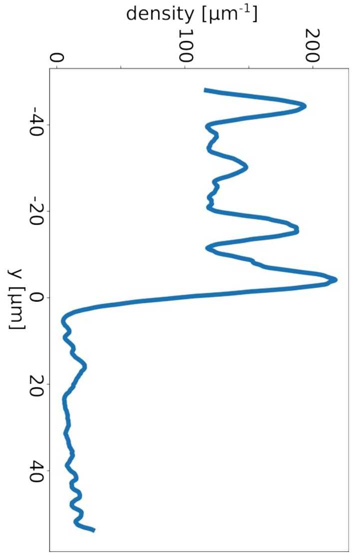
Acoustic horizon







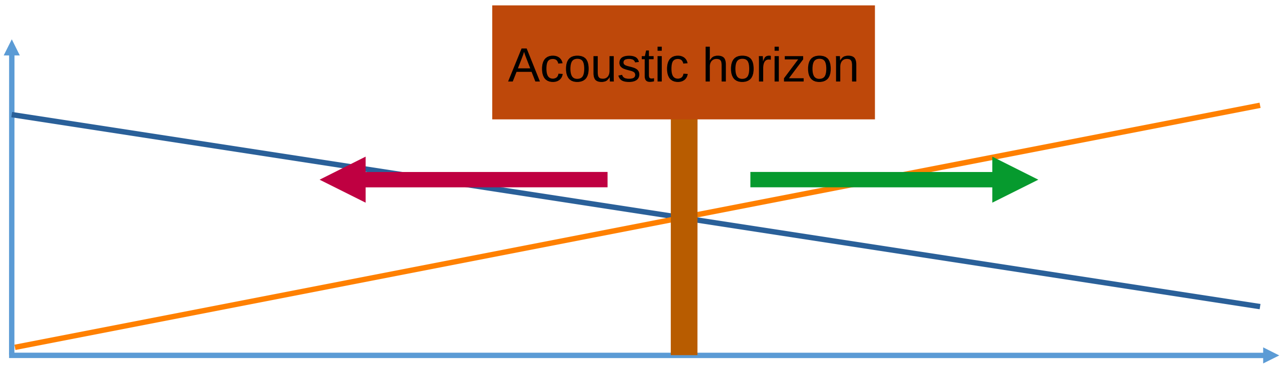




Acoustic horizon

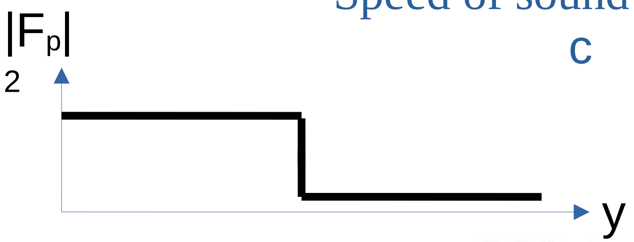
Flow velocity of fluid v

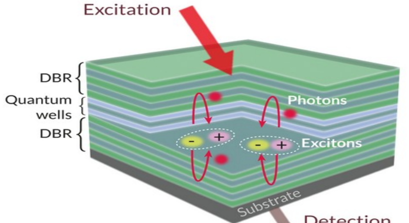
Speed of sound c



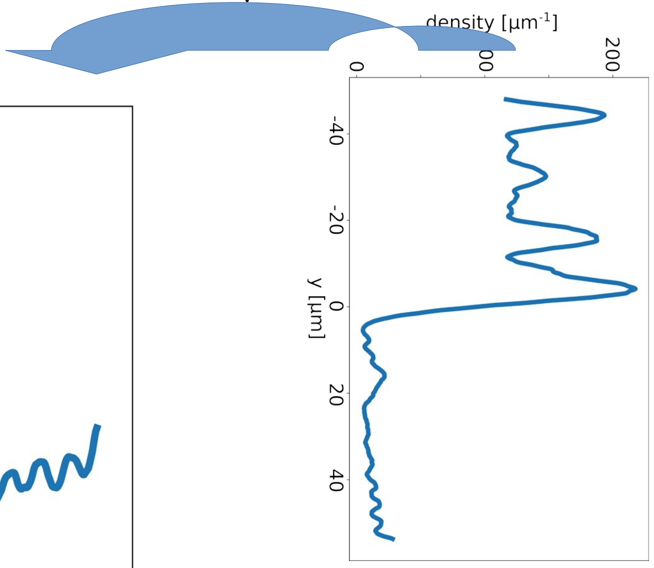
Outside

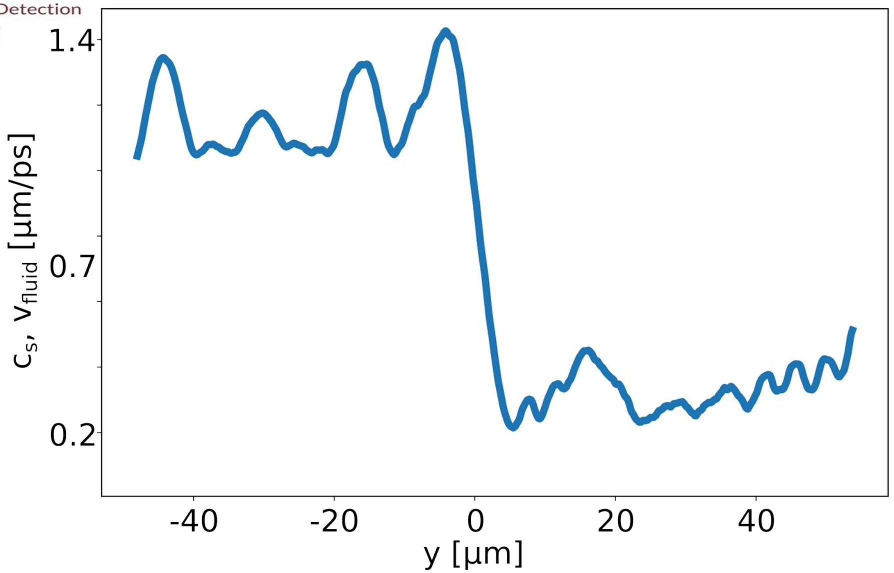
Inside





$$c \propto \sqrt{n}$$



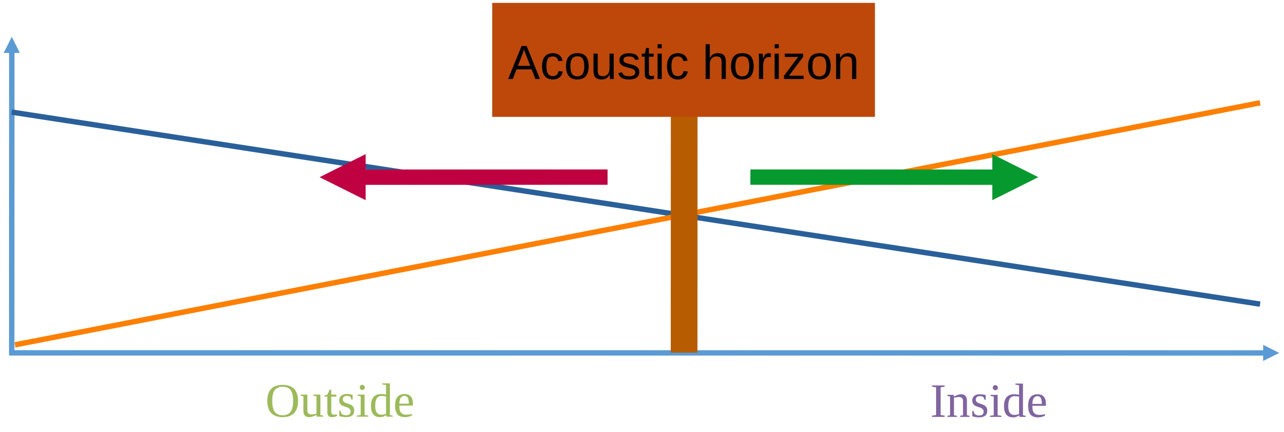


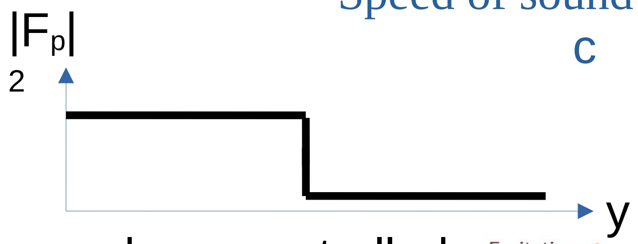
Acoustic horizon

Preliminary data by PhD student Kévin Falque

Flow velocity of fluid v

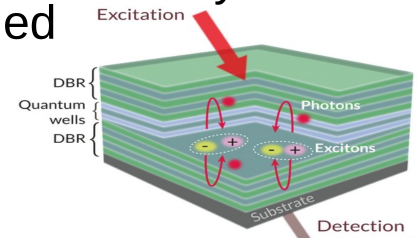
Speed of sound c



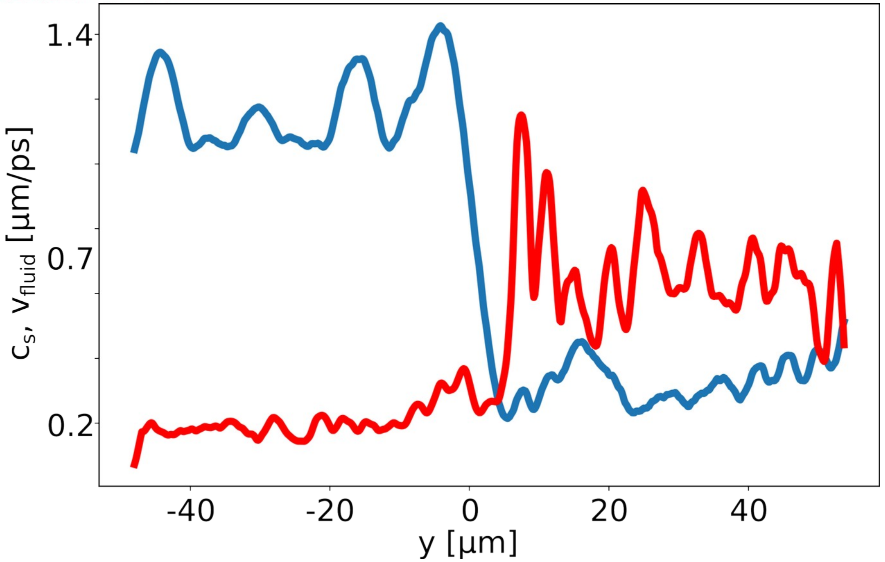


+ phase controlled with SLM

$v \propto \nabla\phi$

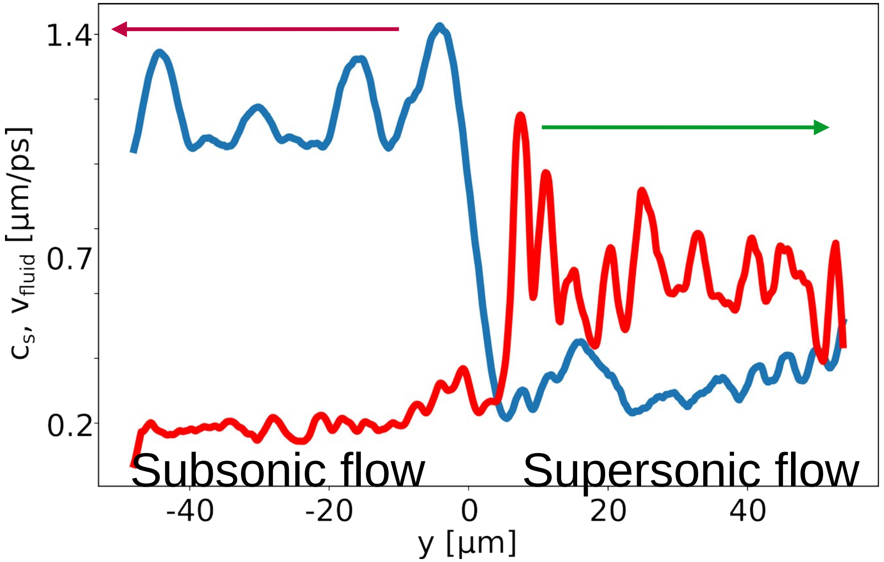


$c \propto \sqrt{n}$



Hawking effect

Hawking effect at the horizon: emission of acoustic waves on either side of the horizon



Hawking effect stimulated

Hawking effect at the horizon: scattering of acoustic waves at the horizon

Stimulate emission with **coherent probe at input** → create acoustic wave that impinges on horizon and scatters

→ reflection = Hawking radiation

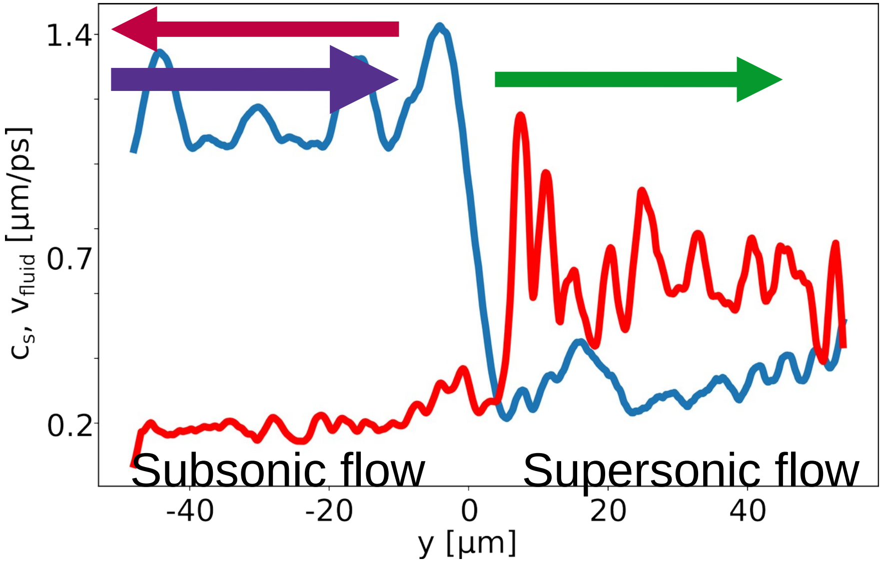
transmission = partner

$$|\bar{0}\rangle \neq |0\rangle \;\Rightarrow\; \beta_{\omega\omega'} \neq 0$$

Scattering matrix elements

Can be applied on any ket in Fock basis: $|in\rangle = |\eta\rangle \otimes |0\rangle$

$$\left\langle \hat{N}^{out} \right\rangle = \left|\beta^{in,out}\right|^2 |\eta|^2 + \left|\beta^{vac,out}\right|^2$$

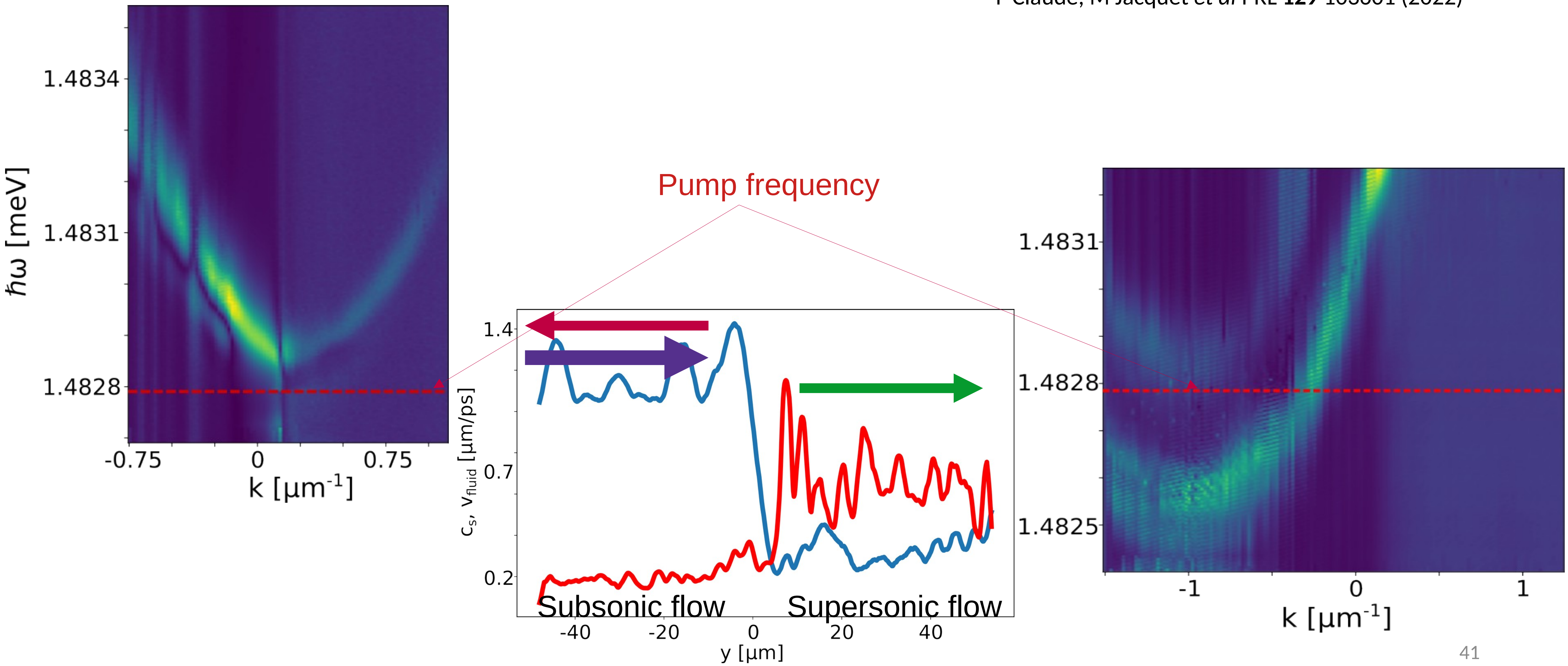


Hawking effect : proof of principle

Sound waves on either side of the horizon: dispersion relation $\omega(k) = kv_0 \pm \sqrt{\frac{\hbar k^2}{2m}\left(\frac{\hbar k^2}{2m} + 2gn\right)}$

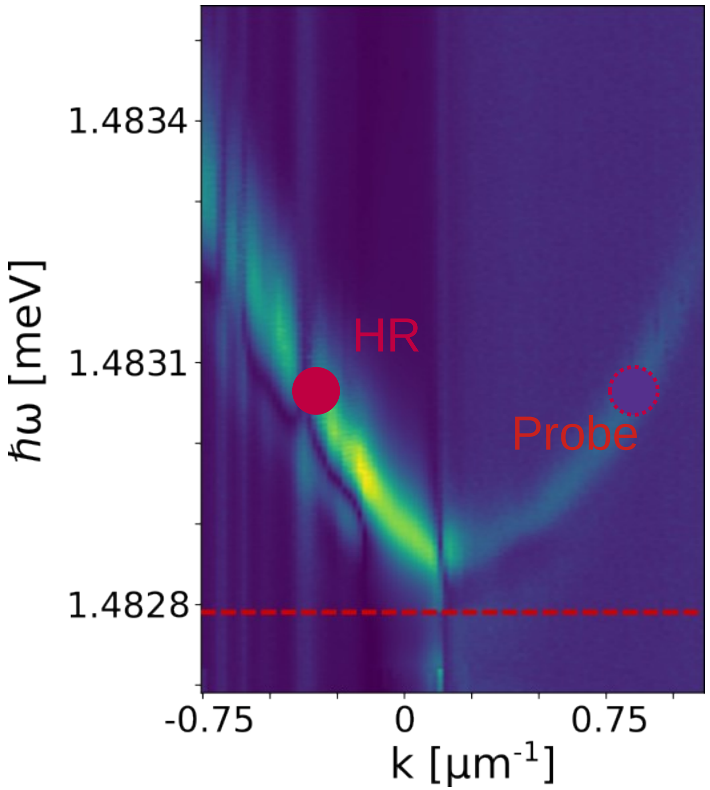
measured with coherent probe spectroscopy

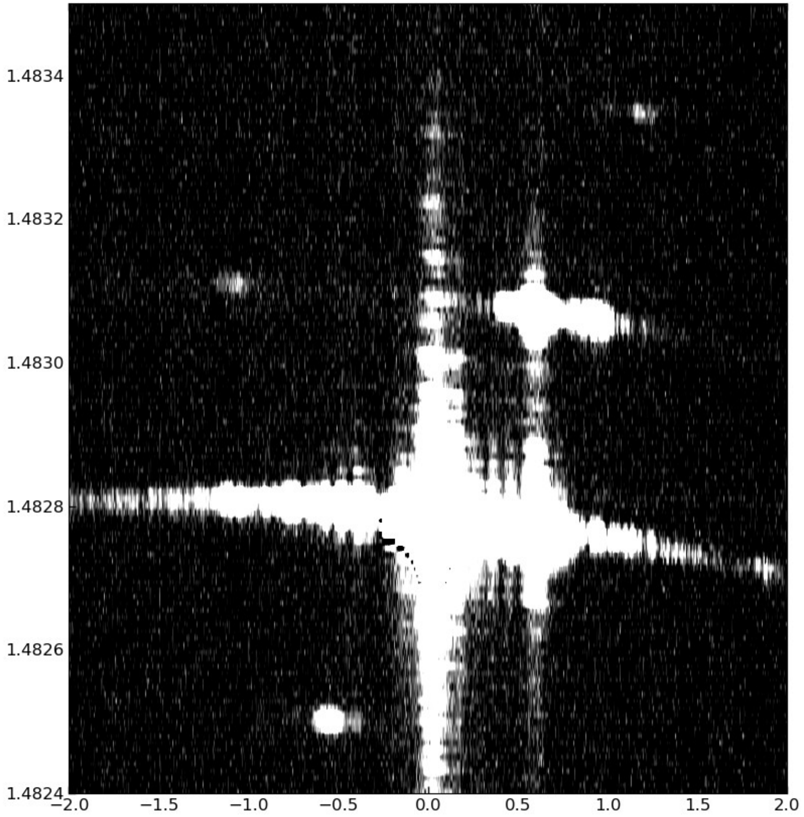
F Claude, M Jacquet *et al* PRL **129** 103601 (2022)

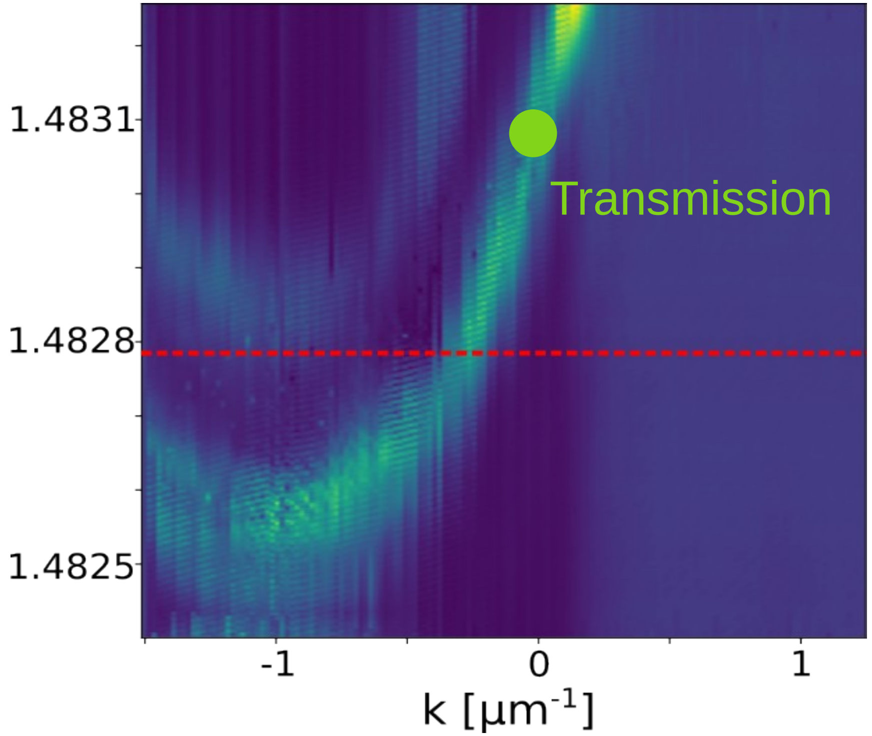


Hawking effect : proof of principle

Stimulate emission with **coherent probe at input** → create acoustic wave that impinges on horizon and scatters
→ reflection = Hawking radiation
transmission = partner

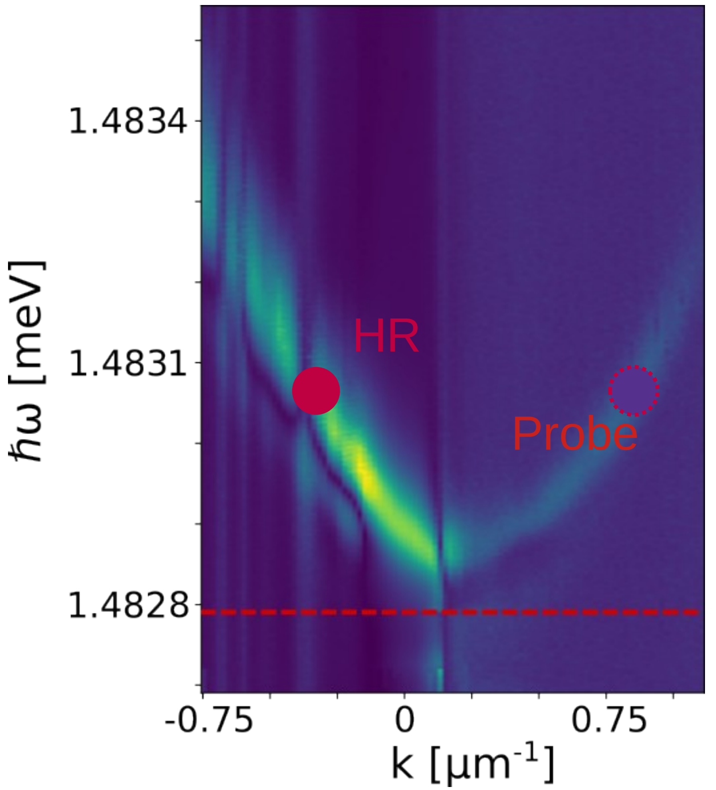


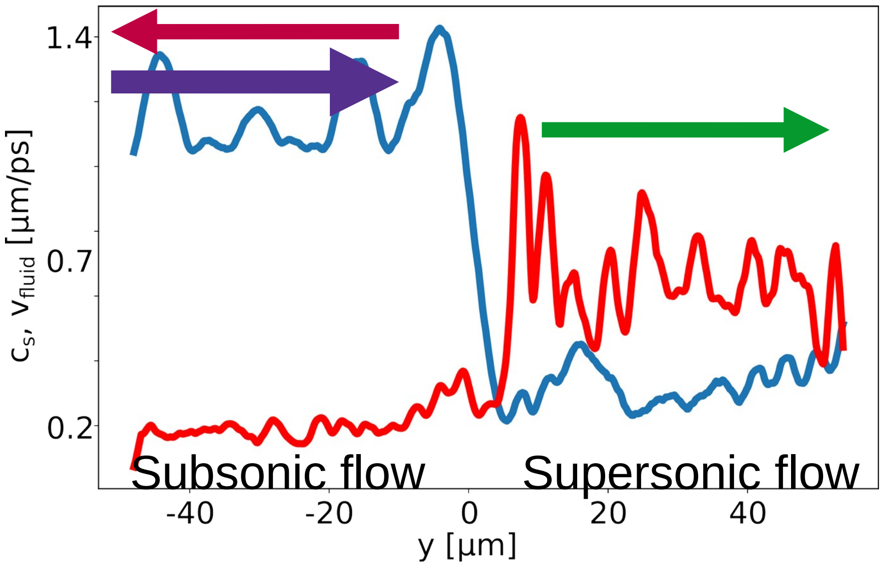


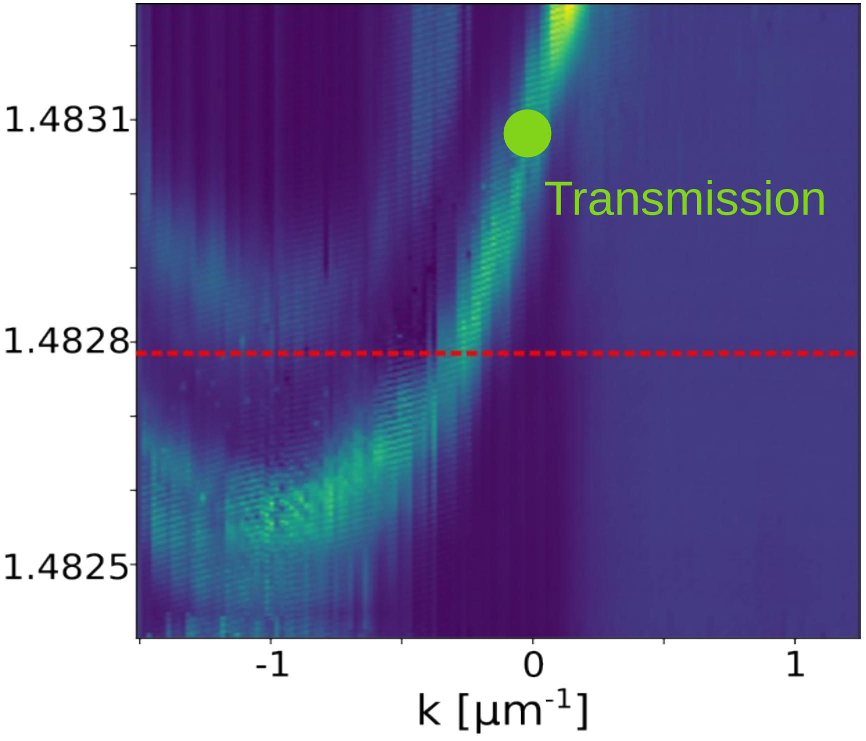


Hawking effect : proof of principle

Scattering of probe at acoustic horizon = observation of Hawking effect







Today’s talk

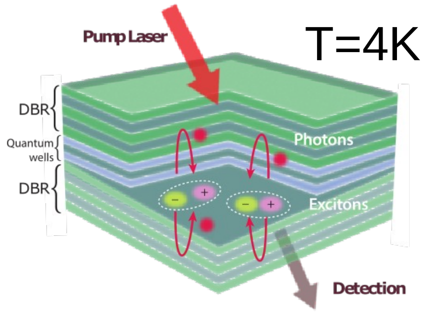
Hawking effect on Schwarzschild black hole

Theory of analogue gravity

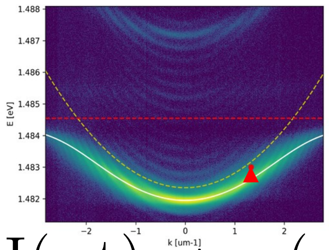
How to observe the Hawking effect in the laboratory?

Experiments in rotating geometry

Vortex flow of polaritons: experiment



T=4K



$$\mathrm{I(r,t)} \rightarrow n(r,t)$$

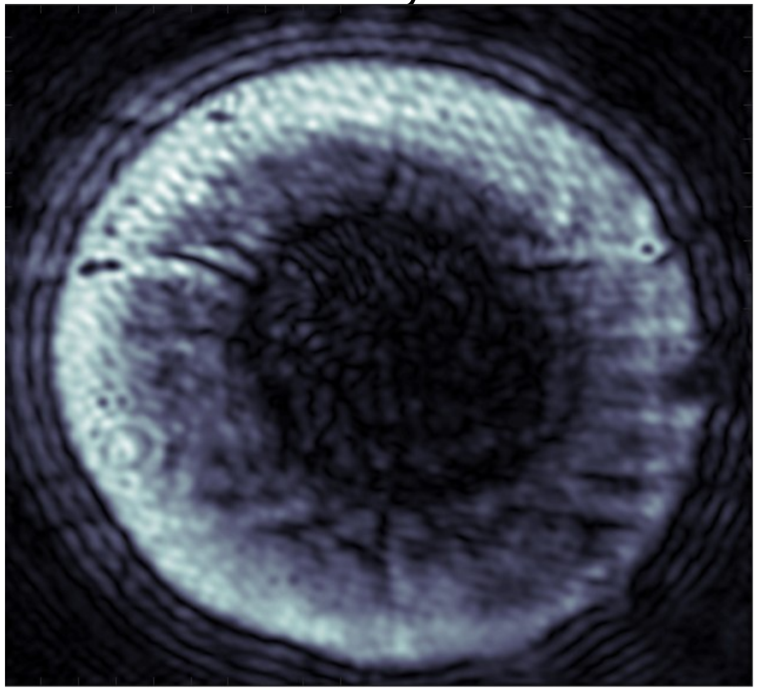
$$\phi(r,t) \rightarrow v(r,t)$$

Phase + intensity profile of driving field

→ Spatial Light Modulator (SLM)

Pump with a Laguerre-Gauss beam (LG=20) → induce rotation in flow

Density



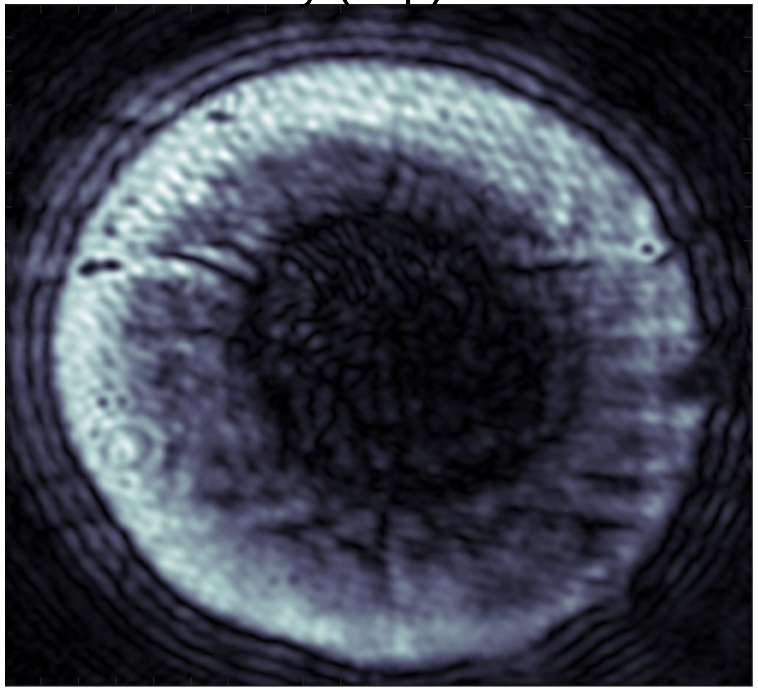
Phase

Quasi-resonant, continuous excitation.

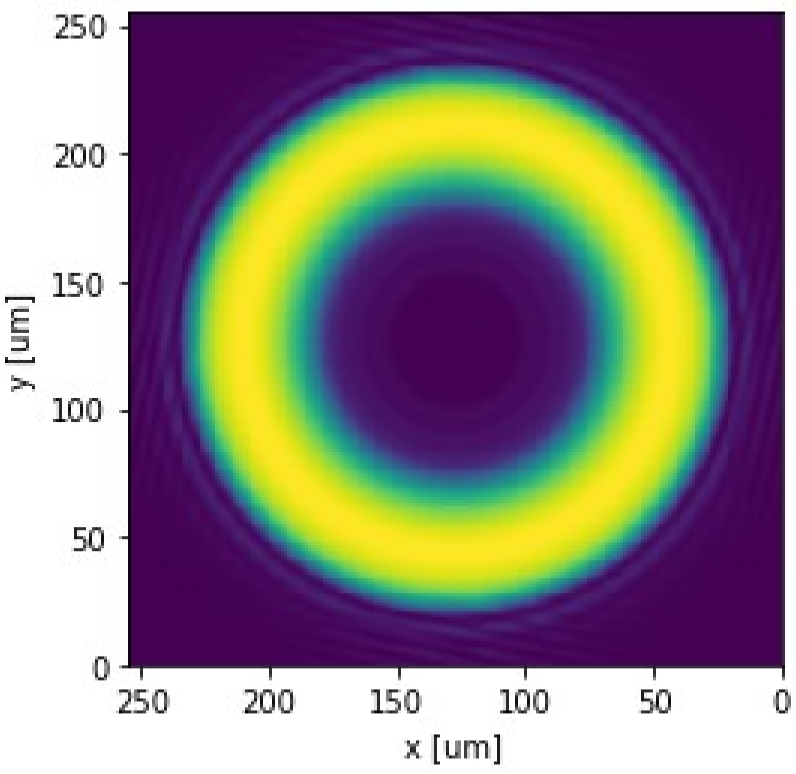
Vortex flow of polaritons: simulations

Pump with a Laguerre-Gauss beam (LG=20) → induce rotation in flow

Density (exp)



Density (simu)



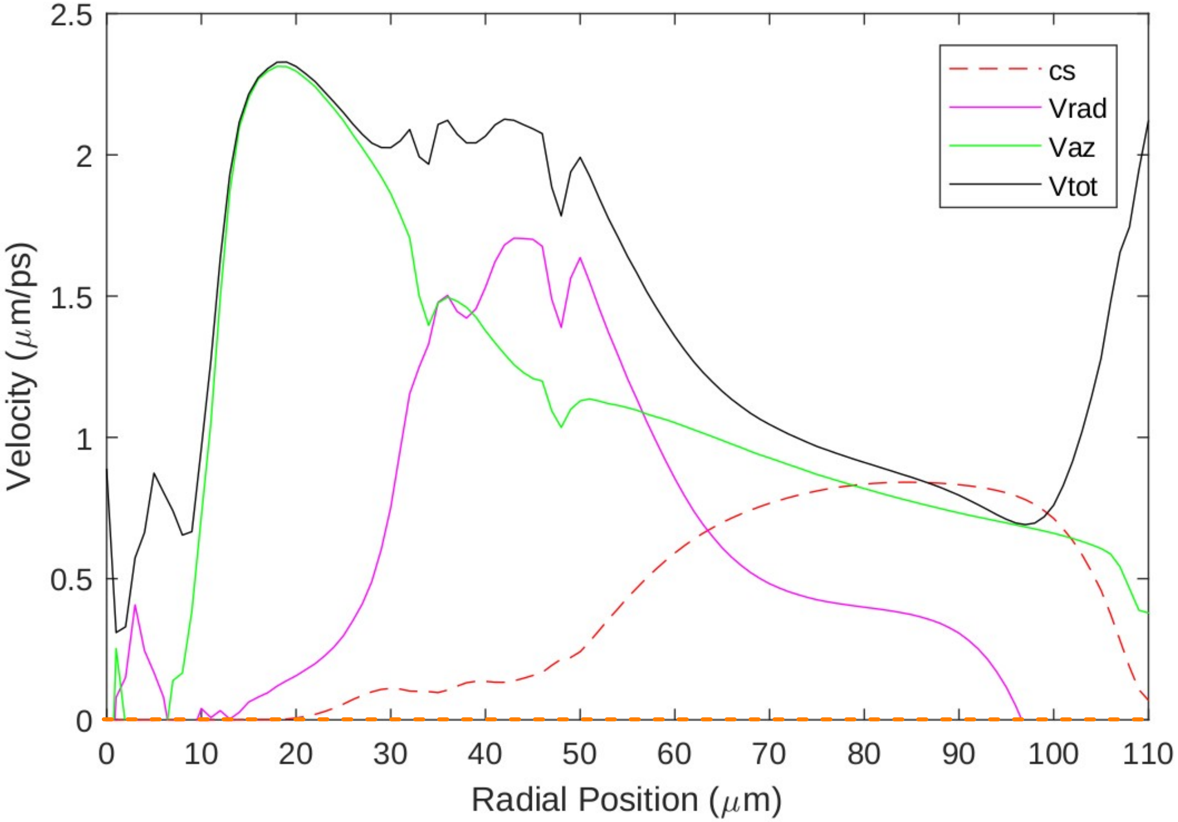
Vortex flow of polaritons: simulations

Pump with a Laguerre-Gauss beam (LG=20) → induce rotation in flow

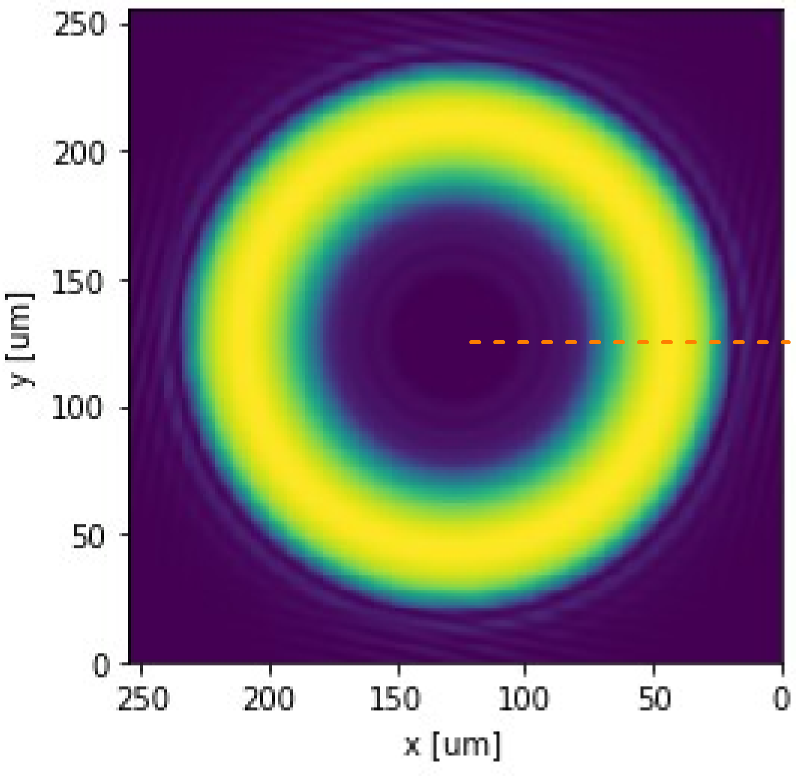
$$v_x \propto \partial_x \phi$$

$$cs_x = \sqrt{\frac{g\,n(x)}{m}}$$

Velocities along radial cut



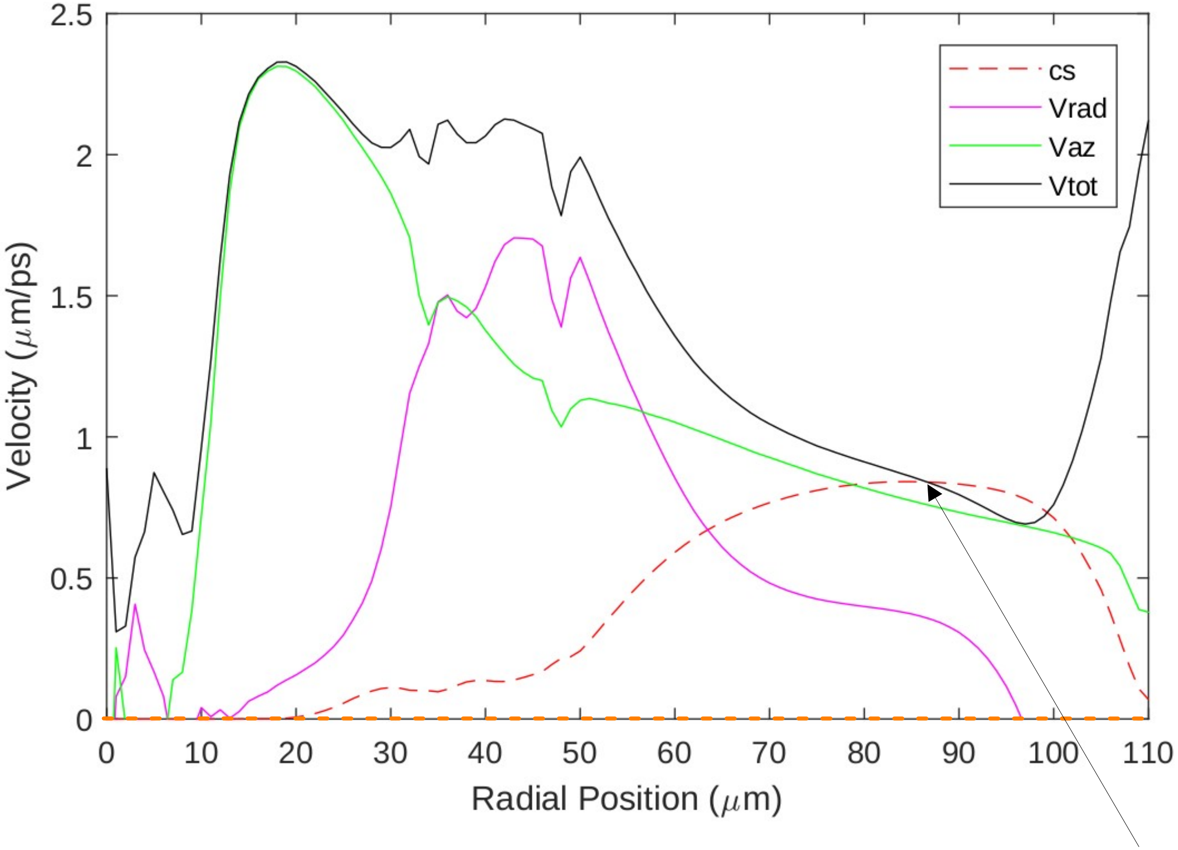
Density (simu)



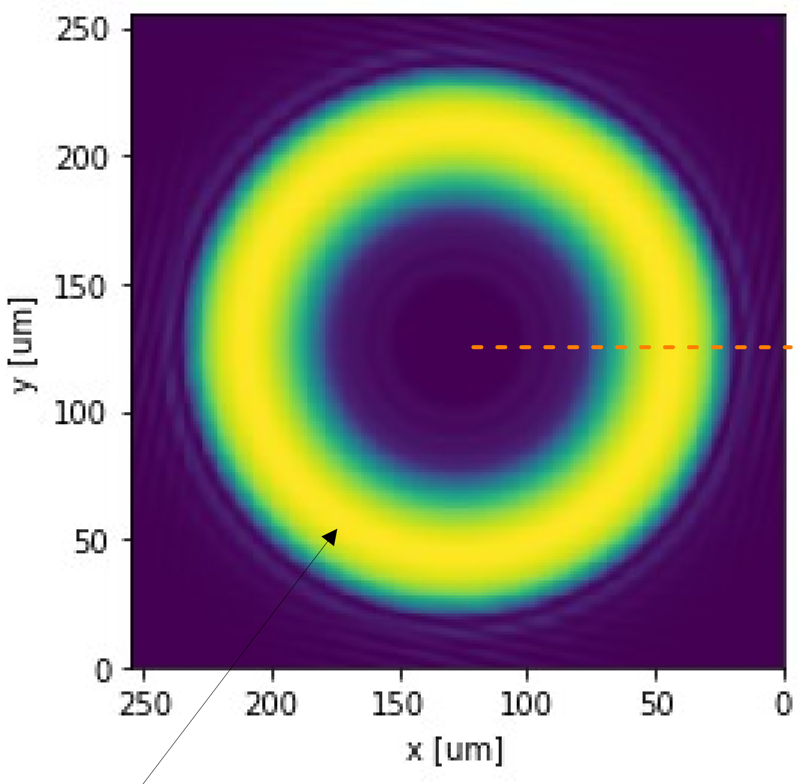
Vortex flow of polaritons: ergosurface and horizon

Pump with a Laguerre-Gauss beam (LG=20) → induce rotation in flow

Velocities along radial cut



Density (simu)



$$v_x \propto \partial_x \phi$$

$$cs_x = \sqrt{\frac{g n(x)}{m}}$$

ergosurface where $v_{total} = c_s$

Analogue gravity:

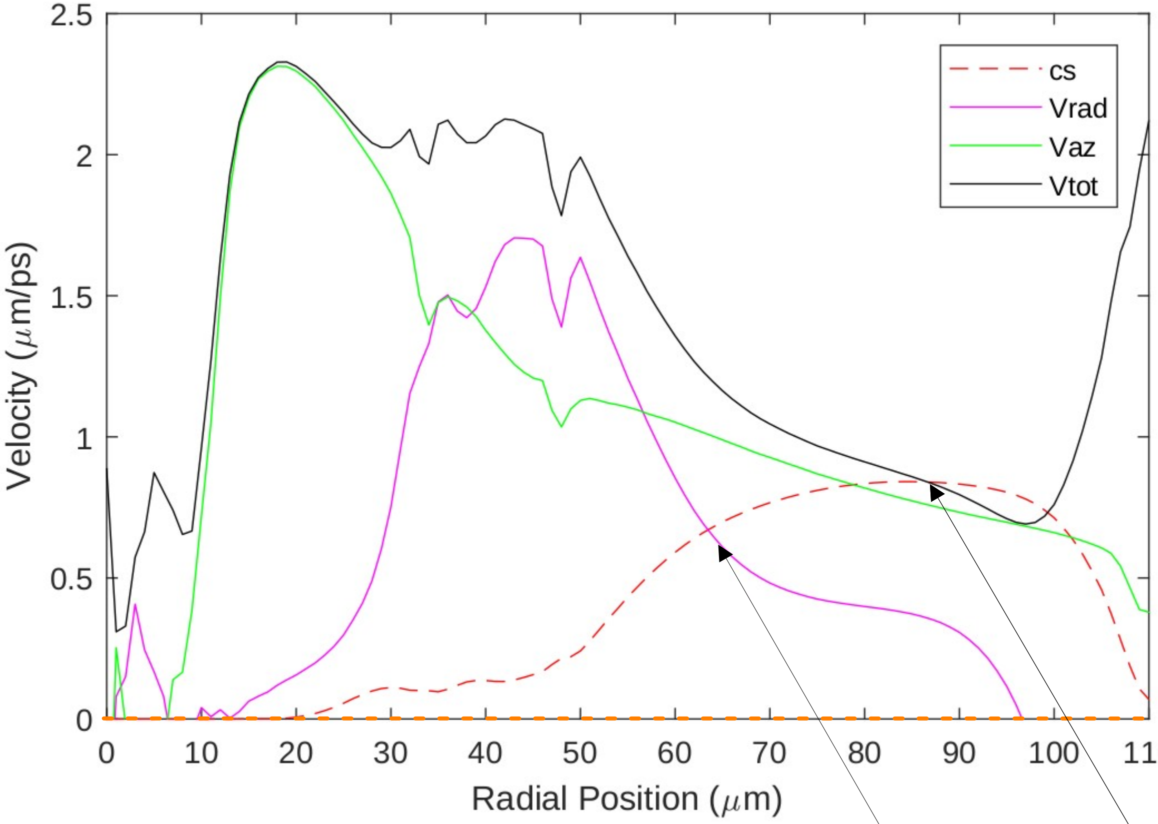
Vortex flow of polaritons: ergosurface and horizon

Pump with a Laguerre-Gauss beam (LG=20) → induce rotation in flow

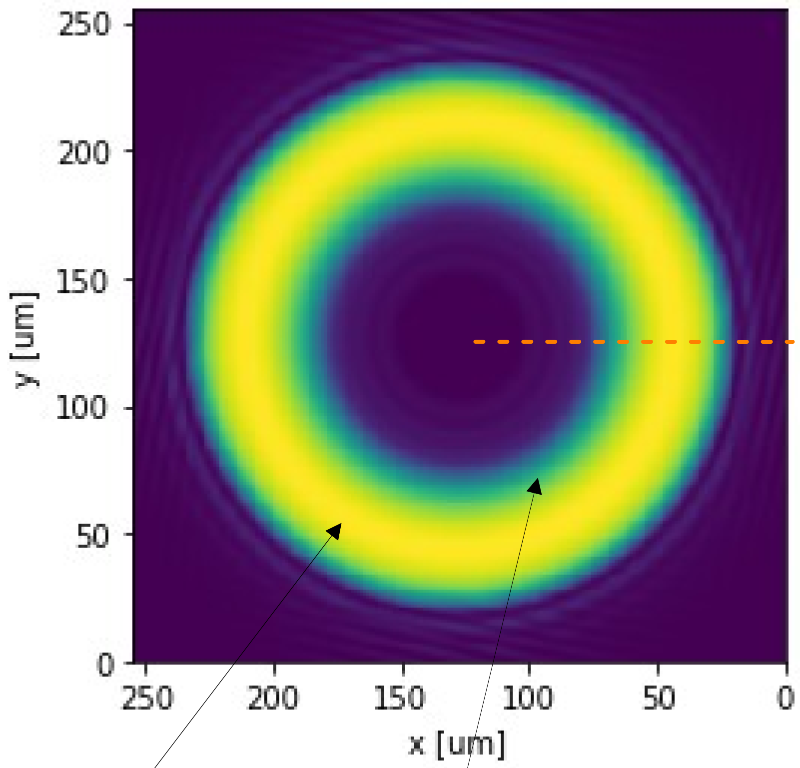
$$v_x \propto \partial_x \phi$$

$$cs_x = \sqrt{\frac{g\,n(x)}{m}}$$

Velocities along radial cut



Density (simu)



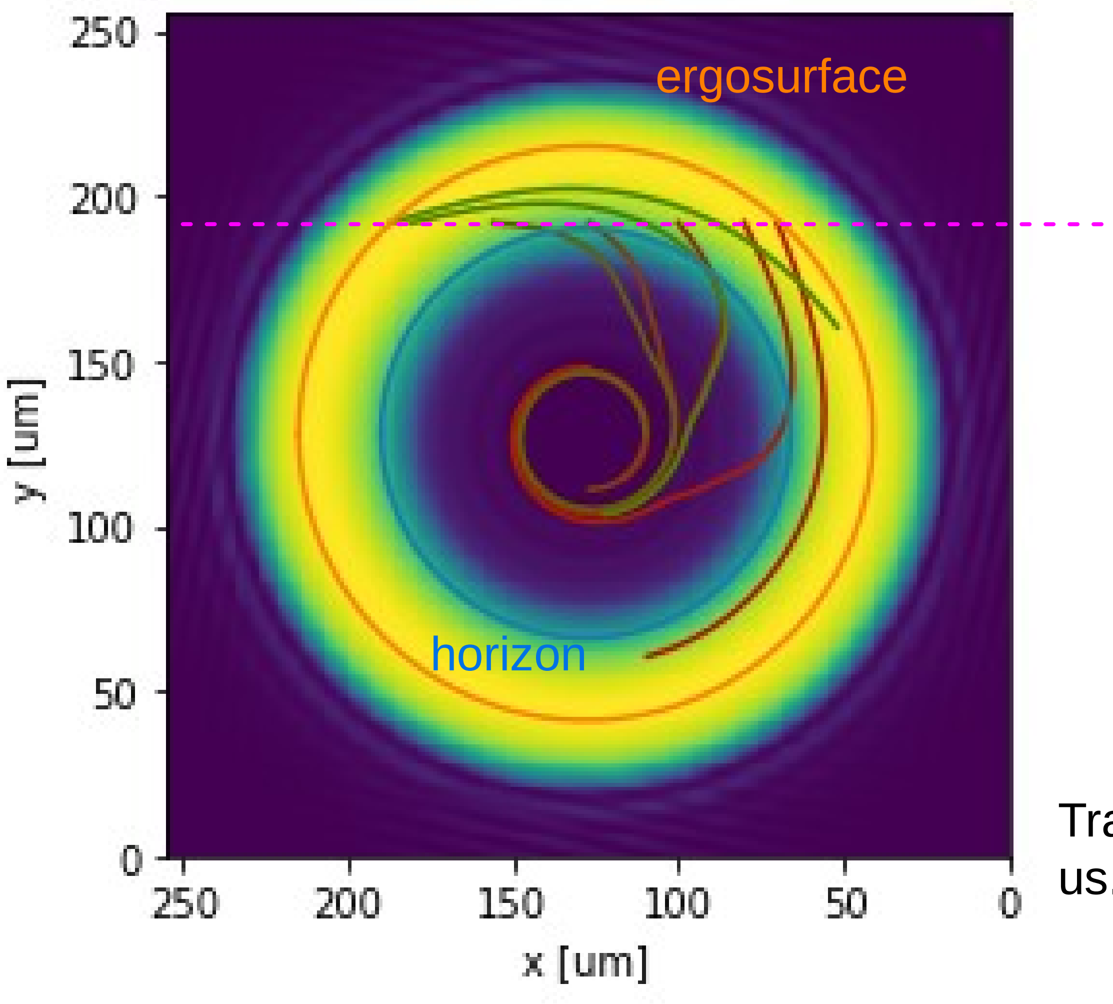
Analogue gravity:

ergosurface where $v_{total}=c_s$

event horizon where $v_{rad}=c_s$

Trajectory of ‘phonons’ on analogue black hole

Congruence of rays on the vortex flow



All rays originate from the same horizontal line 2um ‘above’ the horizon, with k_x=v_x

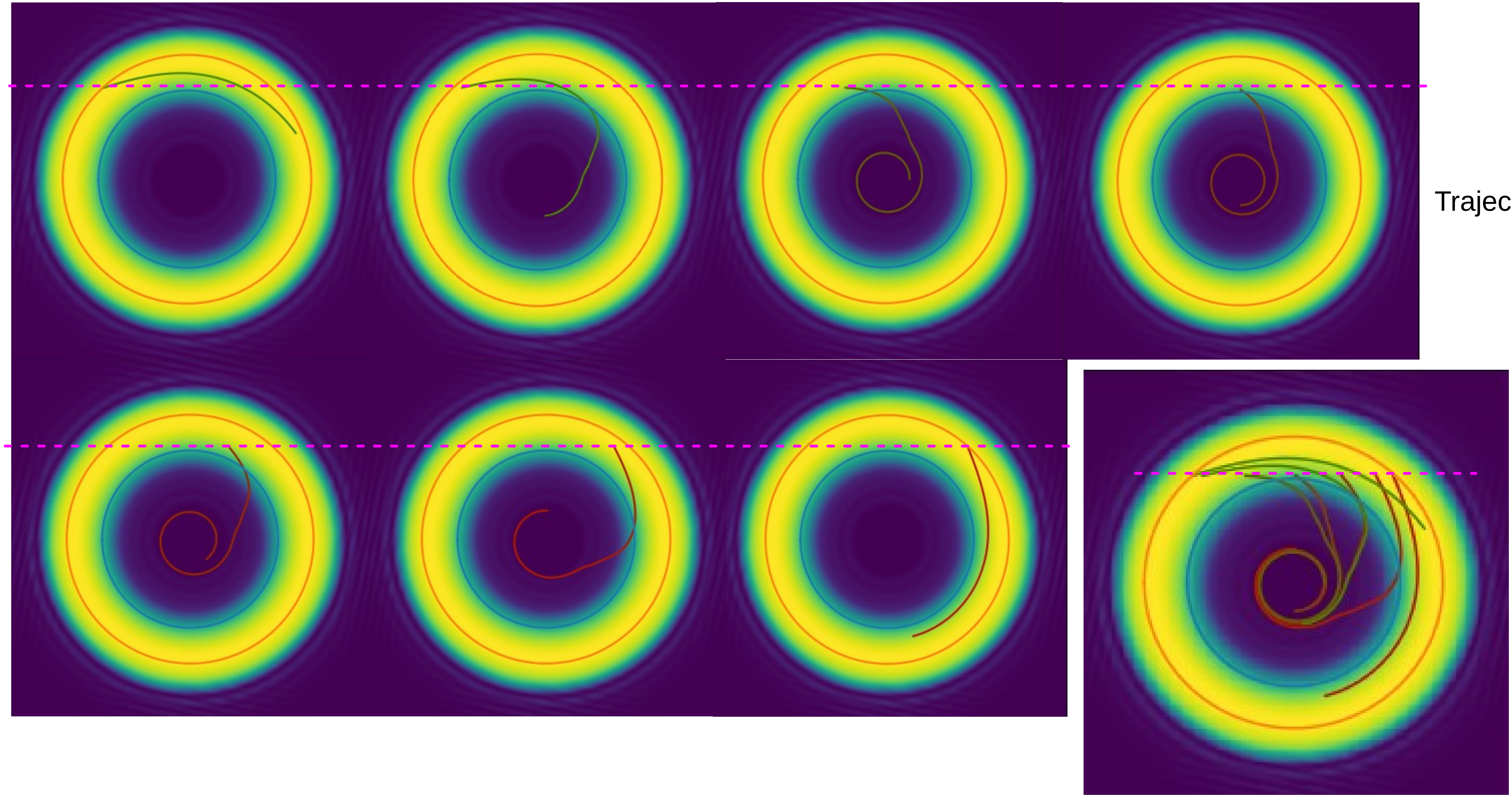
Analytically solve Hamilton Jacobi equations and then numerically integrate with odeint (python)

$$H(x,k)=\omega-v_0.k+c_s|k|$$

Trajectories over 105 us.

Trajectory of ‘phonons’ on analogue black hole

Congruence of rays on the vortex flow



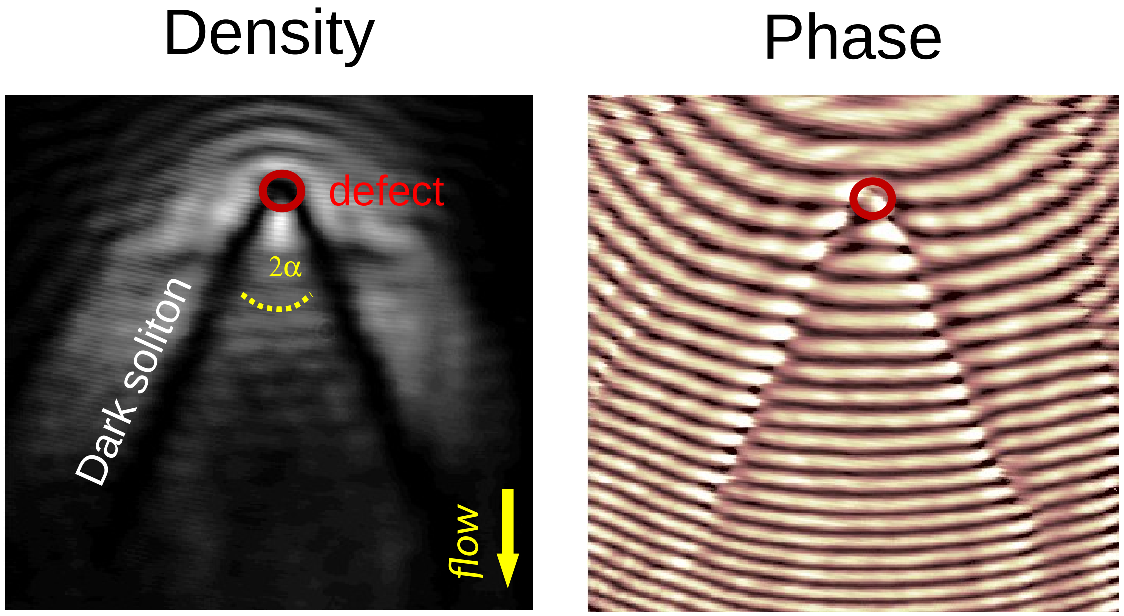
Trajectories over 105 us.

Dark solitons

Dark soliton = localized & stable collective excitation in nonlinear medium

Solitons are spontaneously generated **in pairs** in the wake of a defect

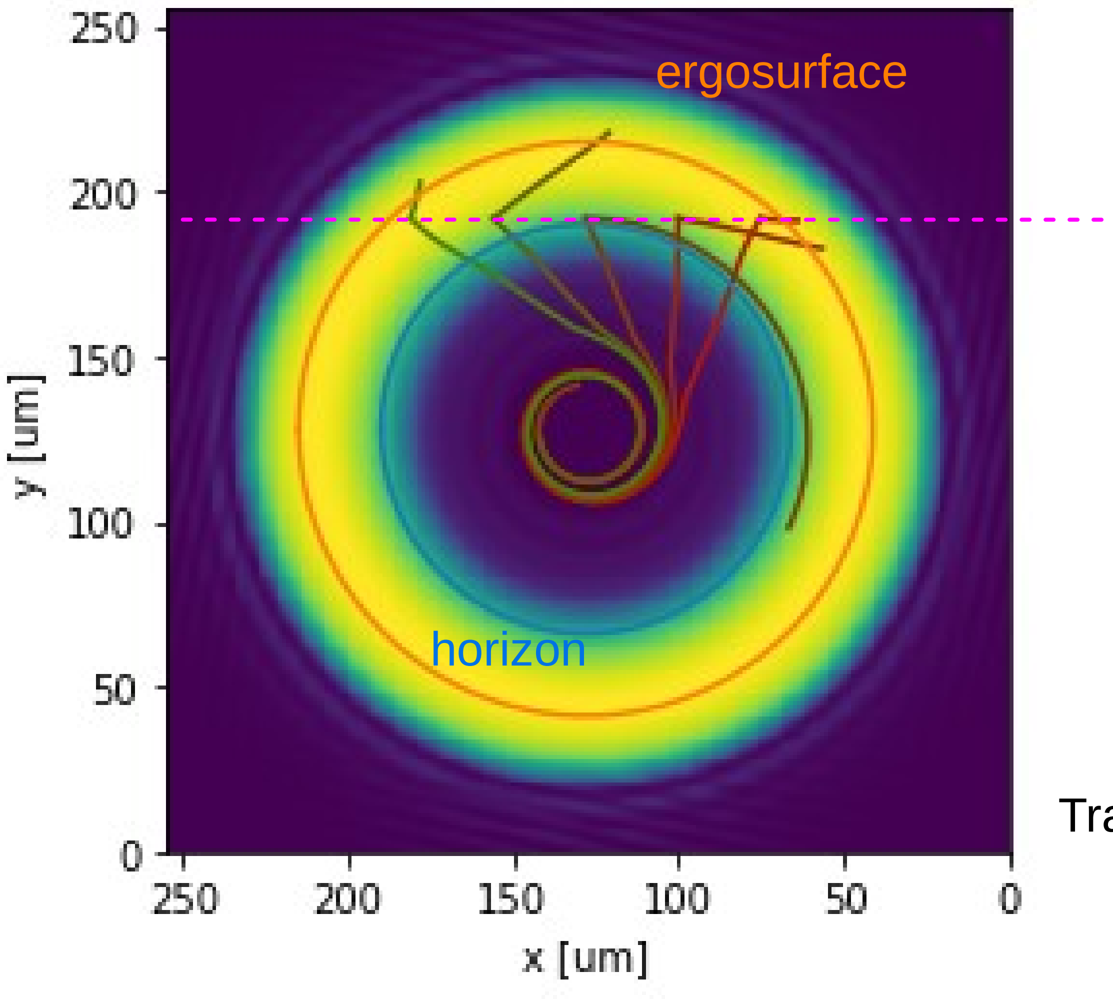
Propagate with initial angle inside Cerenkov cone



Amo *et al*, Science 2011

Analytical calculation of trajectories:

Congruence of rays on the vortex flow



Consider that solitons start off at Cerenkov angle. Trajectory treated with eikonal optics.

All rays originate from the same horizontal line 2um ‘above’ the horizon.

Analytically solve Hamilton Jacobi equations and then numerically integrate with odeint (python)

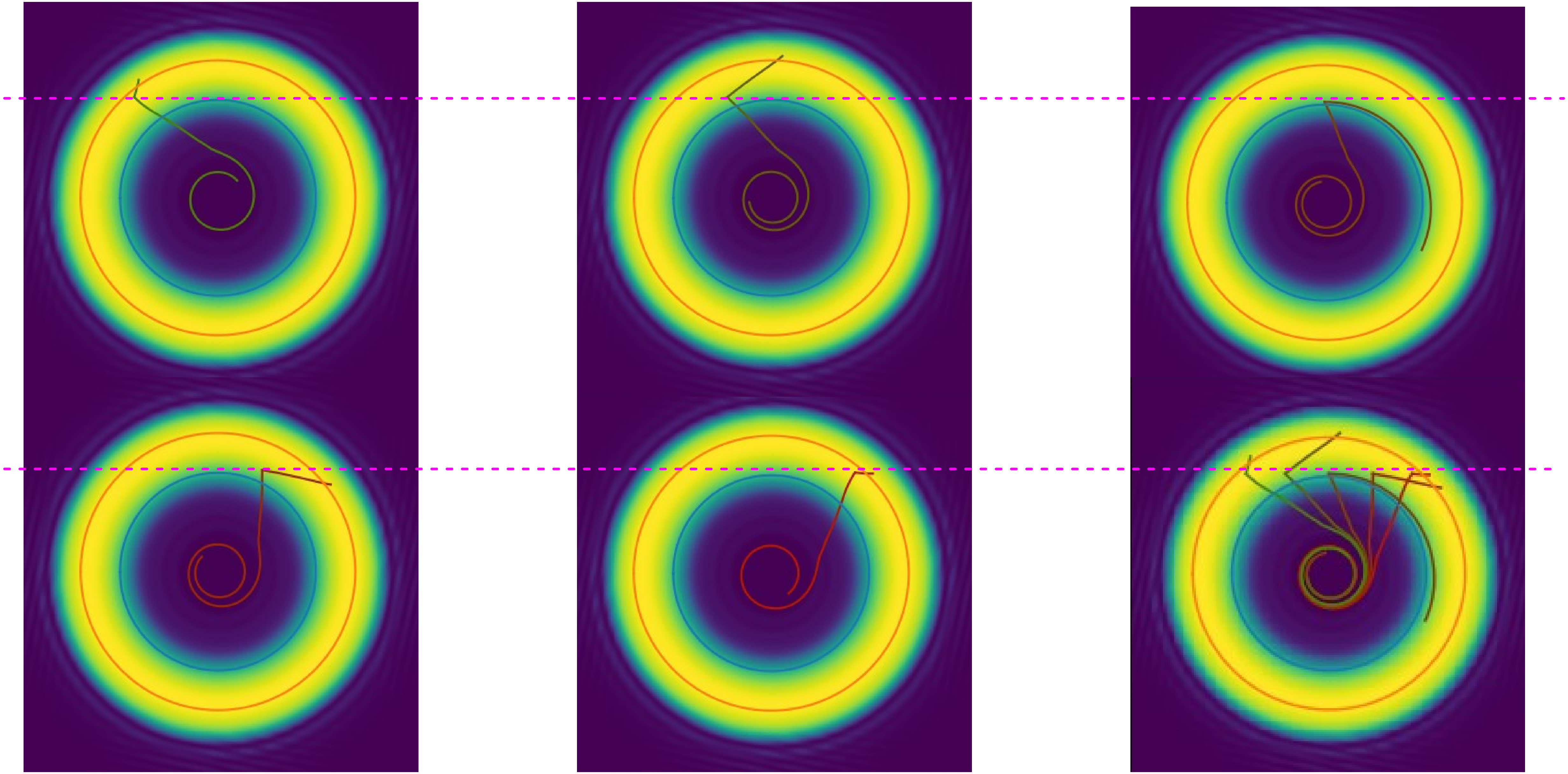
$$H(x,k)=\omega - v_0 . k + c_s|k|$$

Trajectories over 120 us.

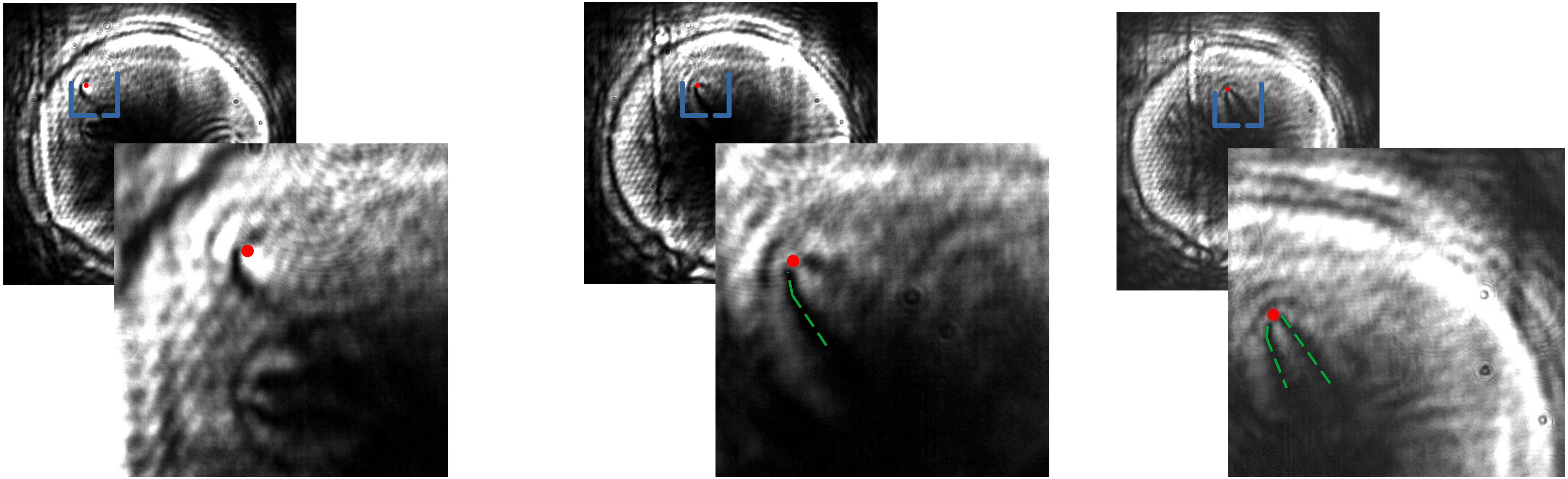
Analytical calculation of trajectories:

Congruence of rays on the vortex flow

Trajectories over 120 us.



Solitons on rotating black hole: experiment

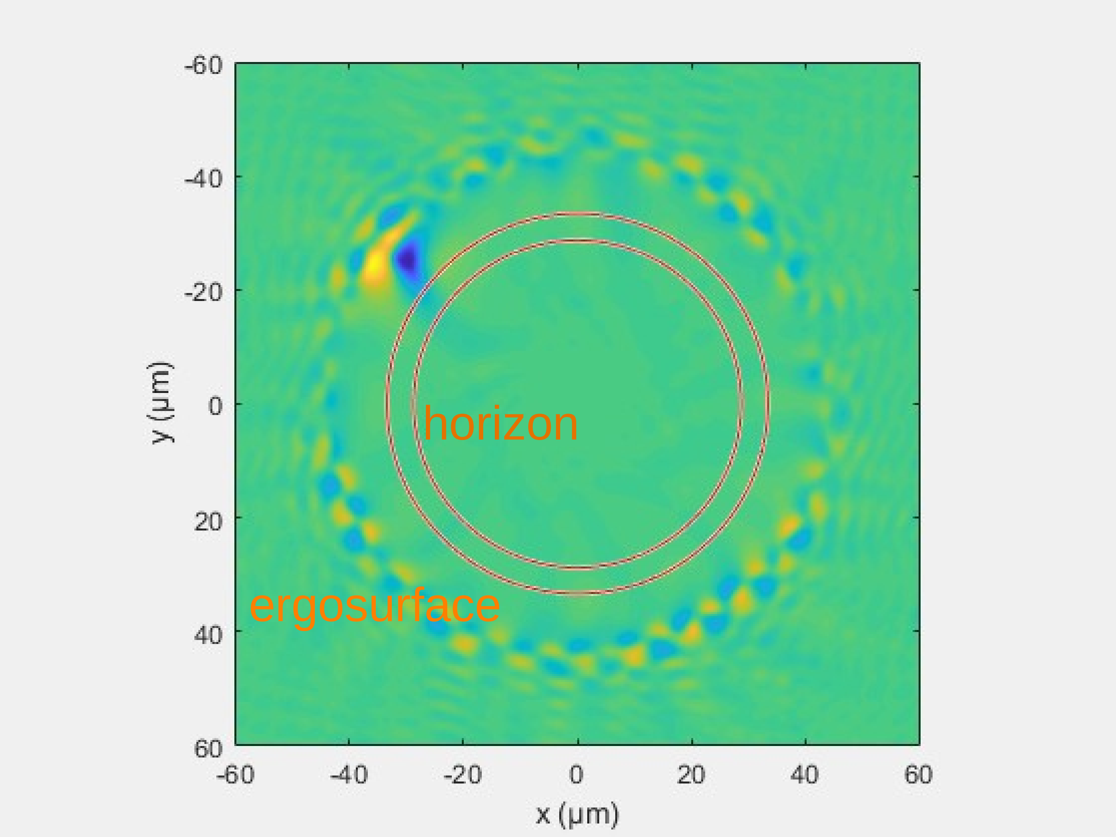


Experimental observation

- 3 different regimes of soliton propagation on a vortex flow: 0, 1 and 2 solitons
- Depends on defect position along the vortex flow

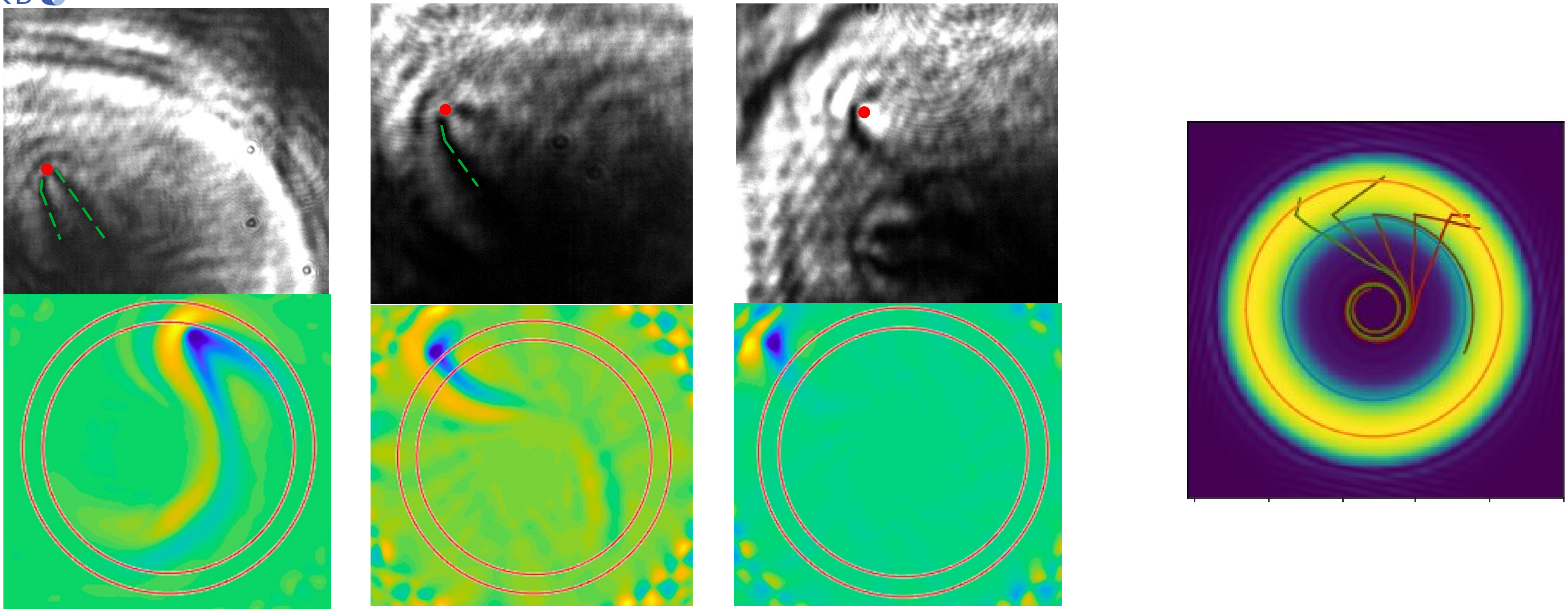
Solitons on rotating black hole: simulations

Animation created from steady-state images of GPE



- 3 different regimes of soliton propagation on a vortex flow: 2, 1 and 0 solitons
- Agrees with analytical trajectories

Solitons on rotating black hole: comparison



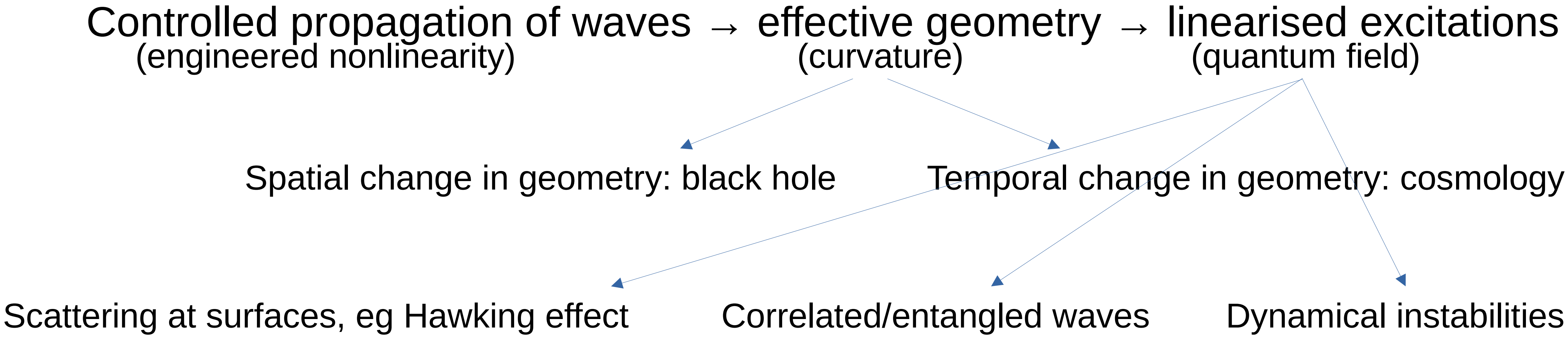
Experimental and numerical observation of 3 different regimes of soliton propagation on a vortex flow:

- 0 solitons when the defect is outside the ergosurface (phase set by the pump)
- 1 soliton in the ergoregion (other soliton does not exist because phase is set by the pump)
- 2 solitons inside the horizon, with curling around the vortex core

Behaviour corresponds to trajectories of ‘phonons’ setting off along the Cerenkov cone.

Today’s talk

The propagation of waves in nonlinear media may be controlled to engineer situations where the waves propagate as though they were on an effectively curved geometry, like around a black hole or in an inflating universe. This enables the experimental study of field theories on curved geometries.



Controlled propagation of waves → effective geometry → linearised excitations
(engineered nonlinearity) (curvature) (quantum field)

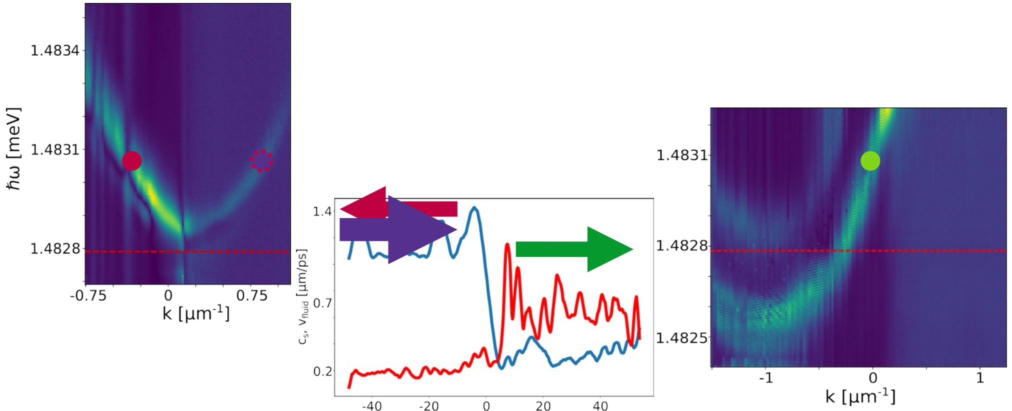
Spatial change in geometry: black hole

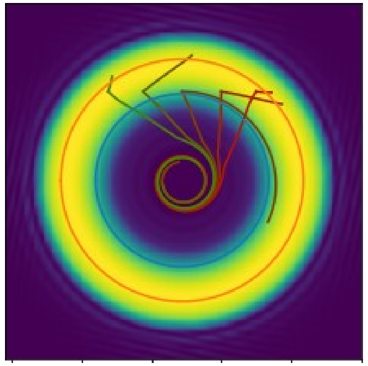
Temporal change in geometry: cosmology

Scattering at surfaces, eg Hawking effect

Correlated/entangled waves

Dynamical instabilities





The next generation of analogue gravity experiments

9 – 10 December 2019

Organised by Dr Maxime Jacquet, Dr Silke Weinfurtner and Dr Friedrich König.

THE
ROYAL
SOCIETY

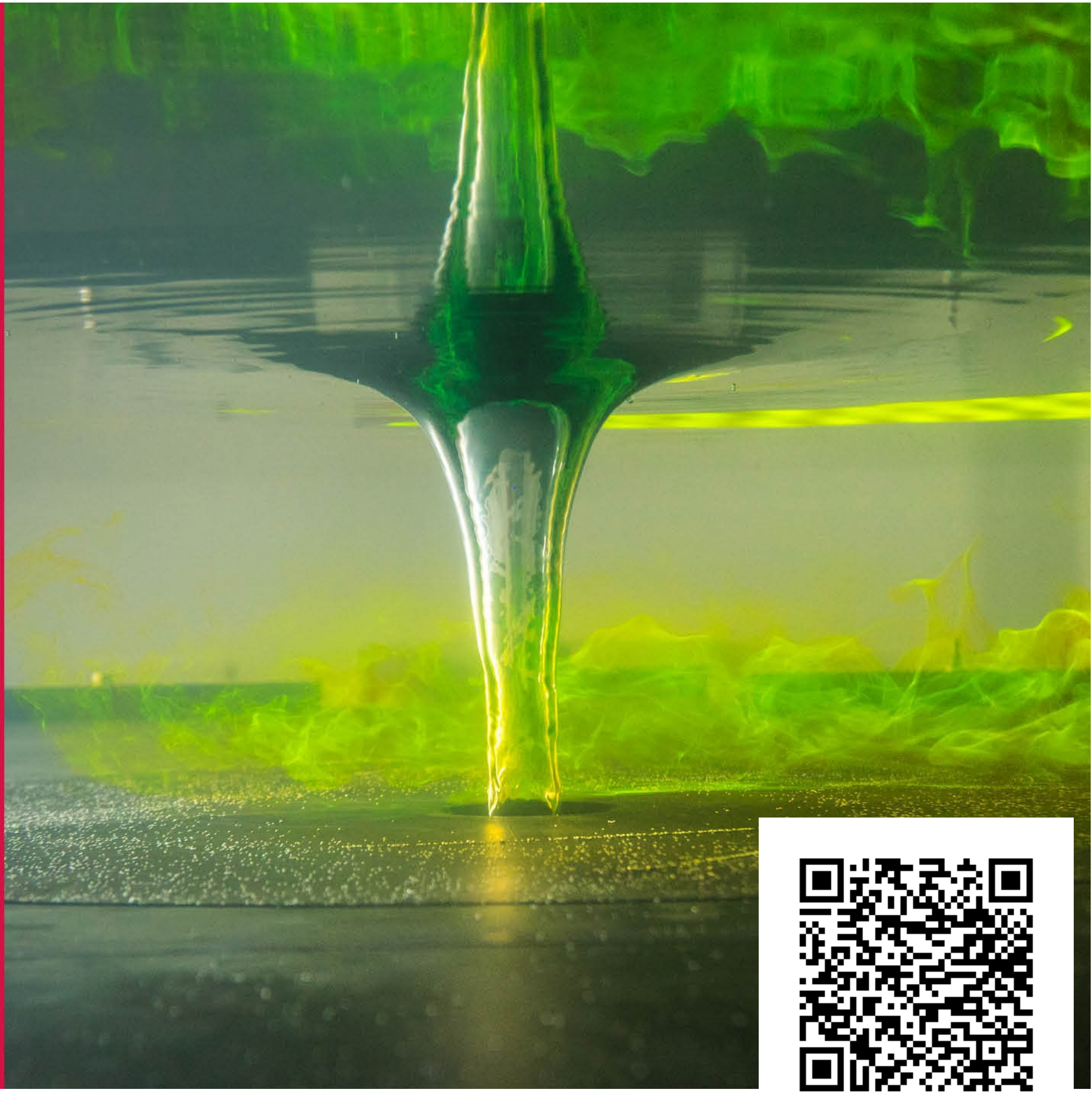
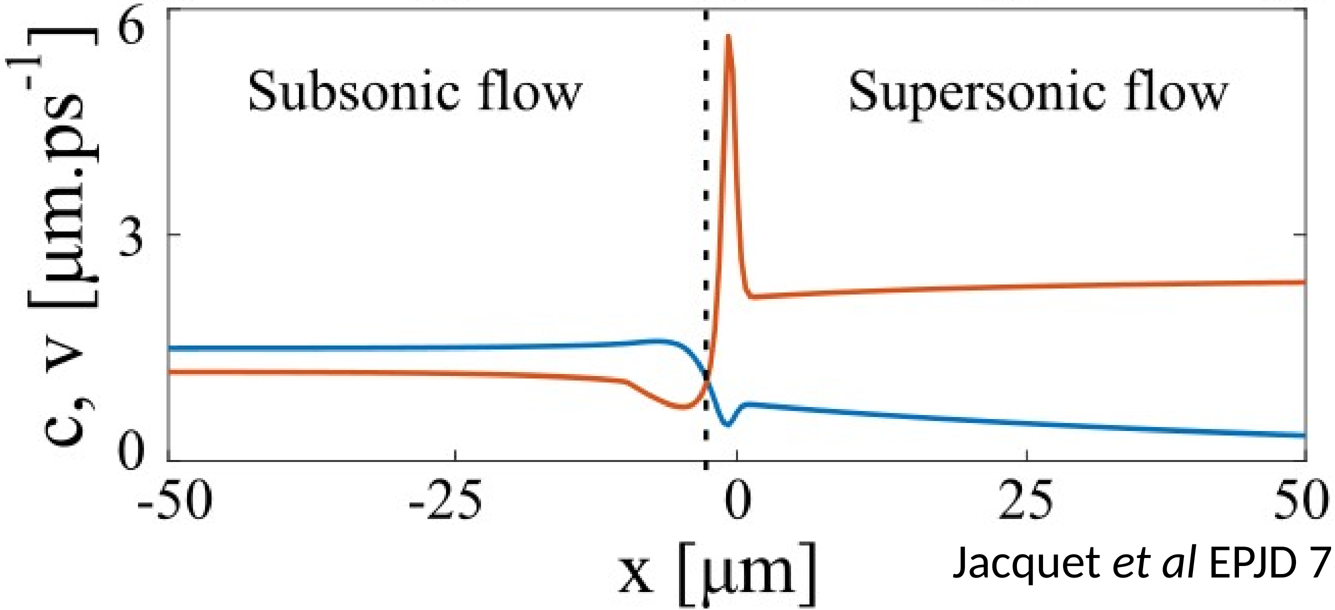
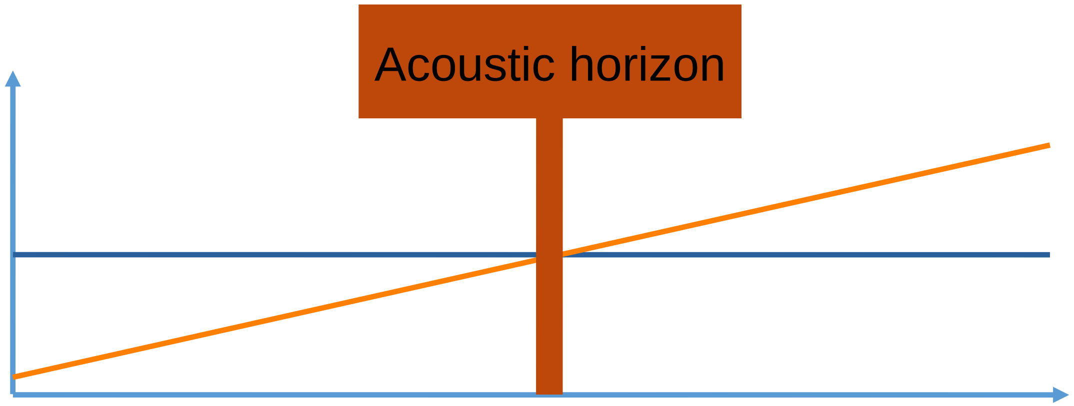


Image: © Alex Wilkinson Media.

Modelling a BH spacetime with a fluid of microcavity polaritons

Acoustic horizon

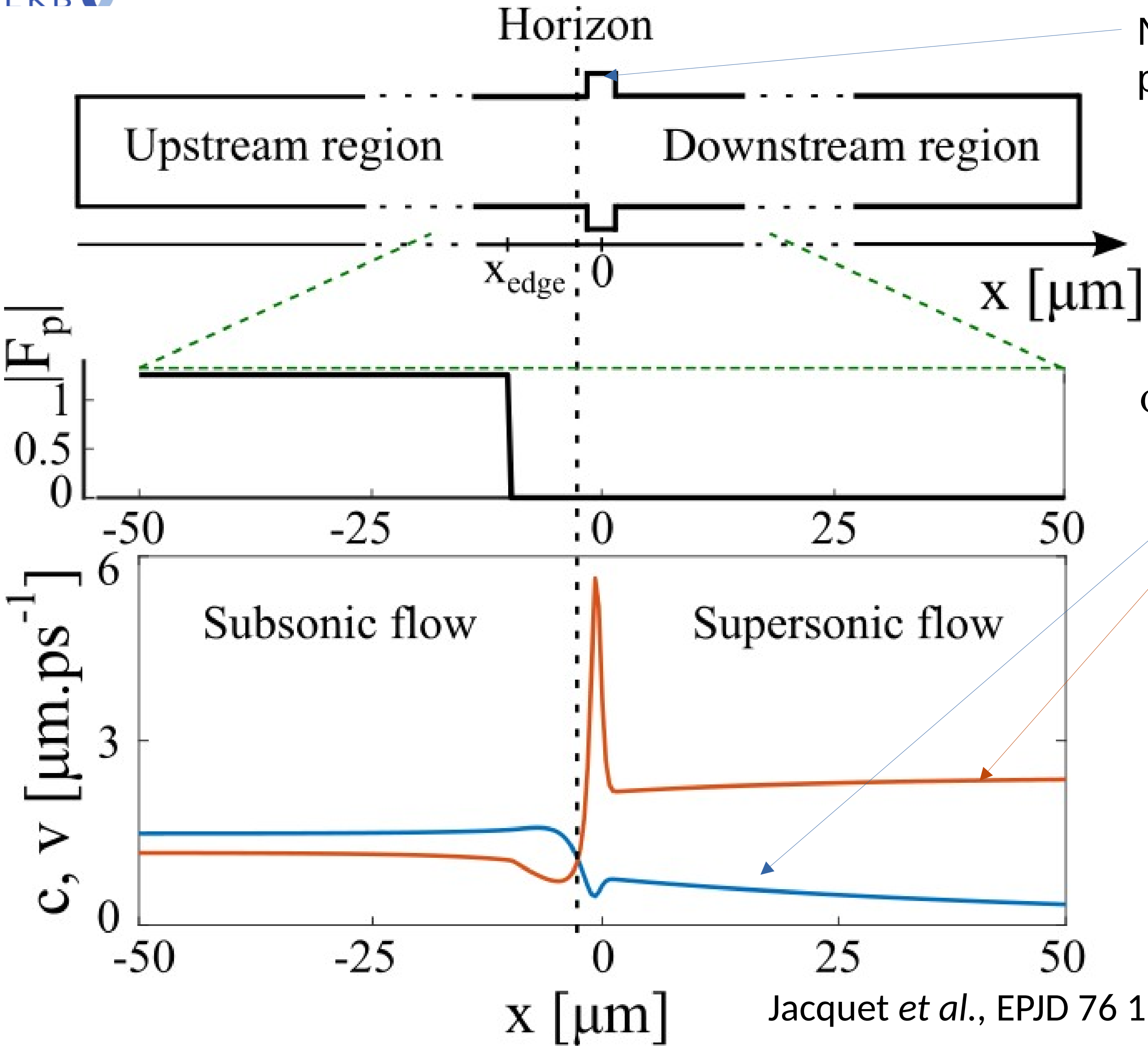




Acoustic horizon

Jacquet *et al* EPJD 76 152 (2022)

Acoustic horizon in polaritons



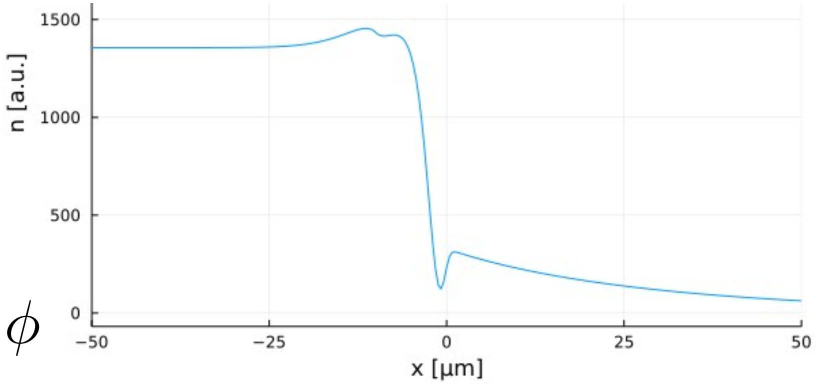
Jacquet *et al.*, EPJD 76 152 (2022)

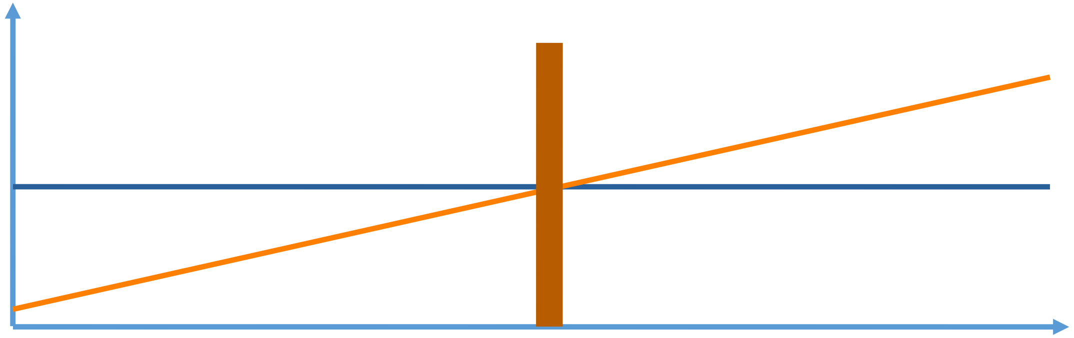
Negative potential

Simulate sample of Nguyen HS *et al.*,
PRL 114 036402 (2015)

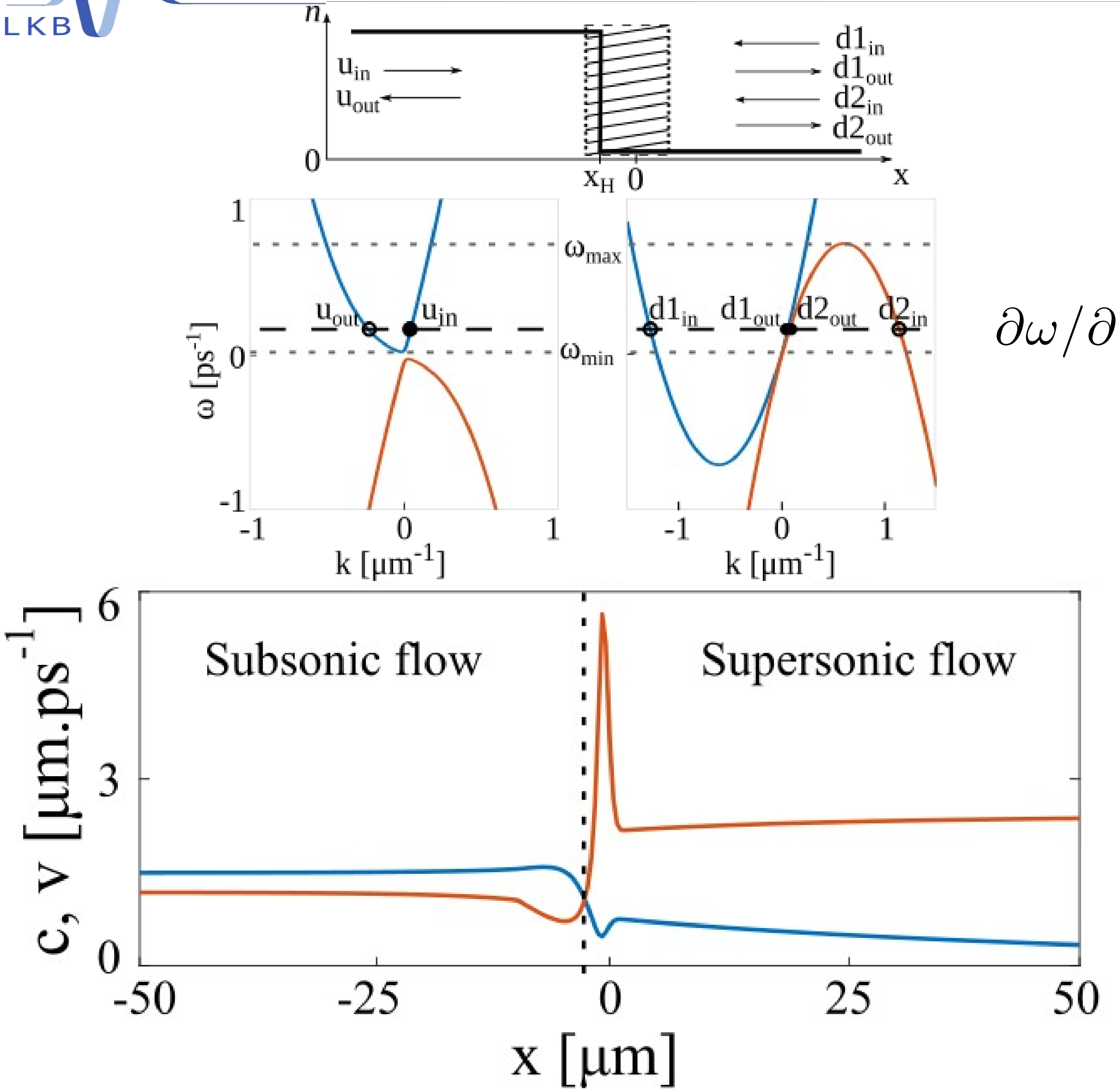
c$\propto \sqrt{n}$

v$\propto \partial_x \phi$



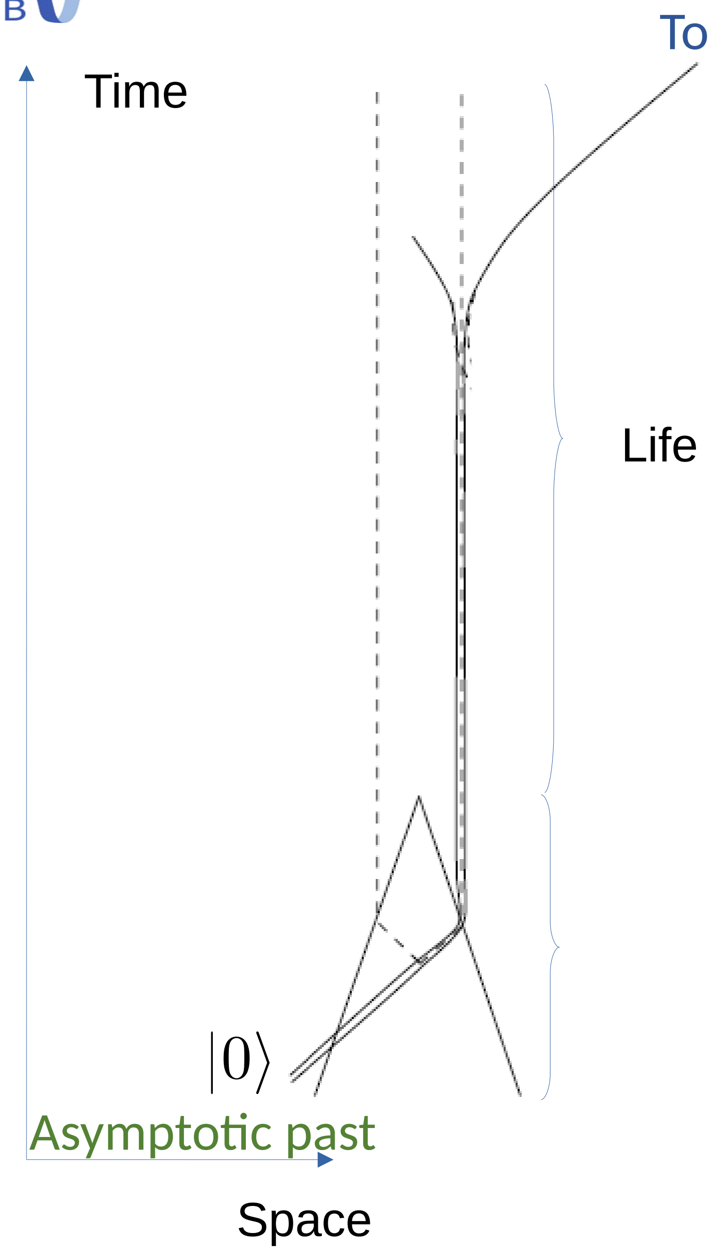


Acoustic horizon in polaritons: the modes



$\partial\omega/\partial k$ Group velocity of modes → propagation w.r.t horizon

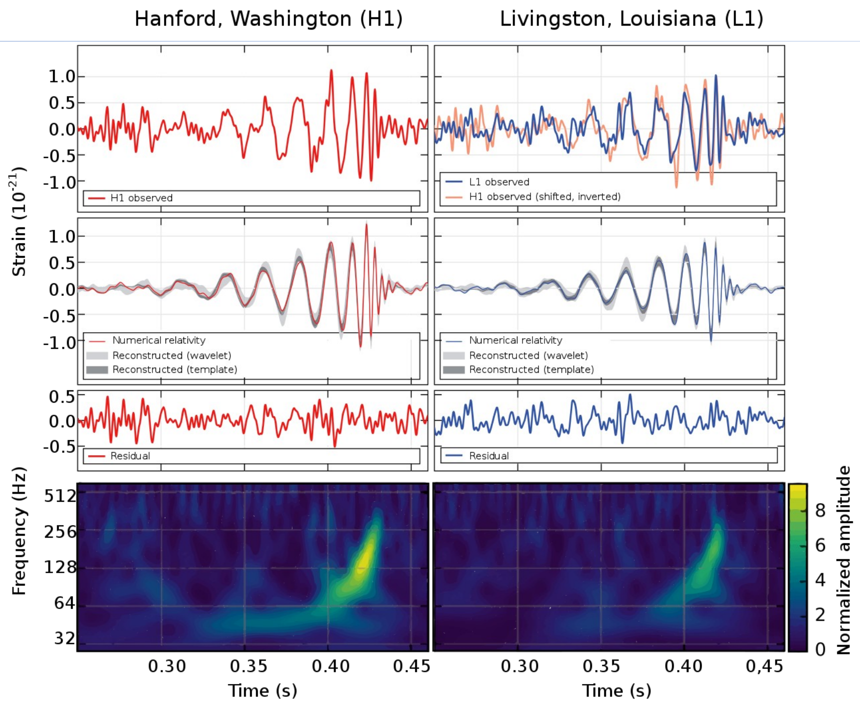
Perturbation of black hole



Life of a Schwarzschild black hole

if perturbed by large amplitude object, relaxation by gravitational waves

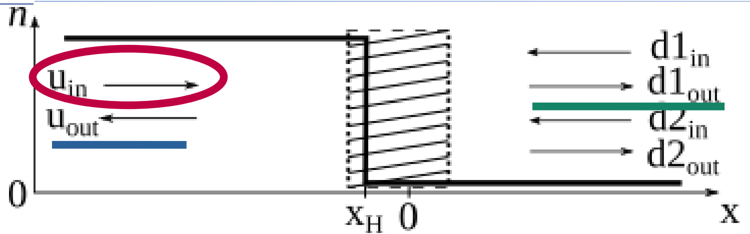
Quasi-normal mode of the gravitational field



Ligo Virgo 2016

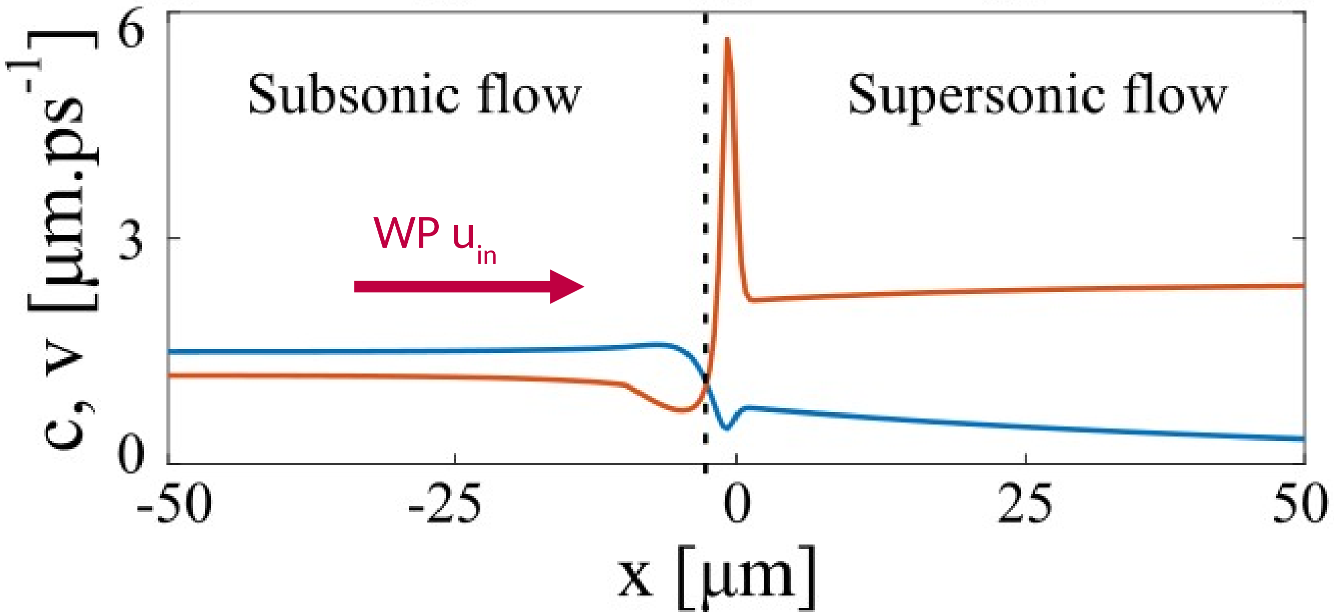
Jacquet *et al.*, arxiv:2110.14452

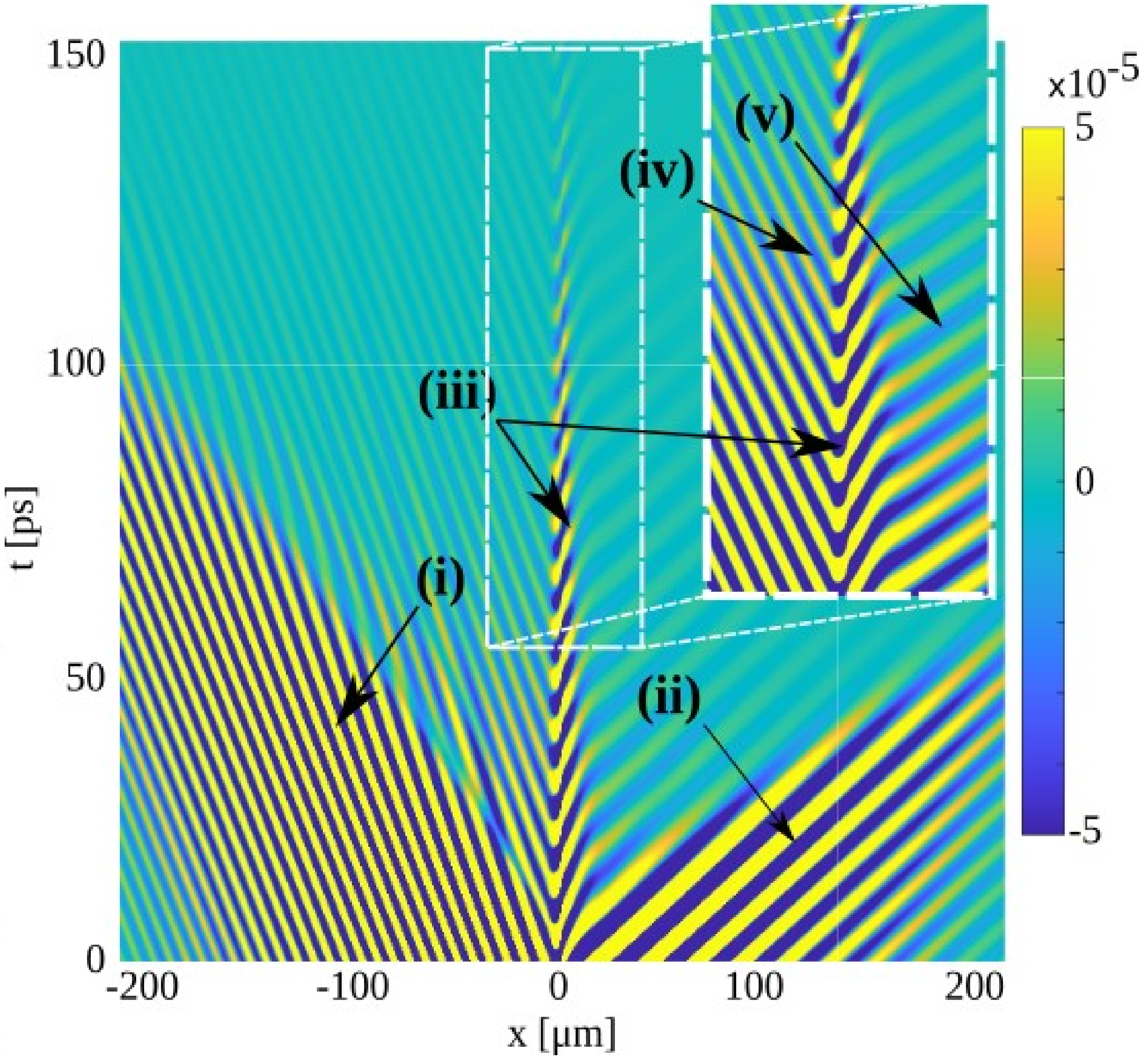
Perturb horizon with classical wavepacket



Send wavepacket u_{in} toward horizon:

(i) reflection
(ii) transmission
(iii) density @horizon oscillates and dampens
(iv) density @horizon couples with mode propagating outward
(v) density @horizon couples with mode propagating inward

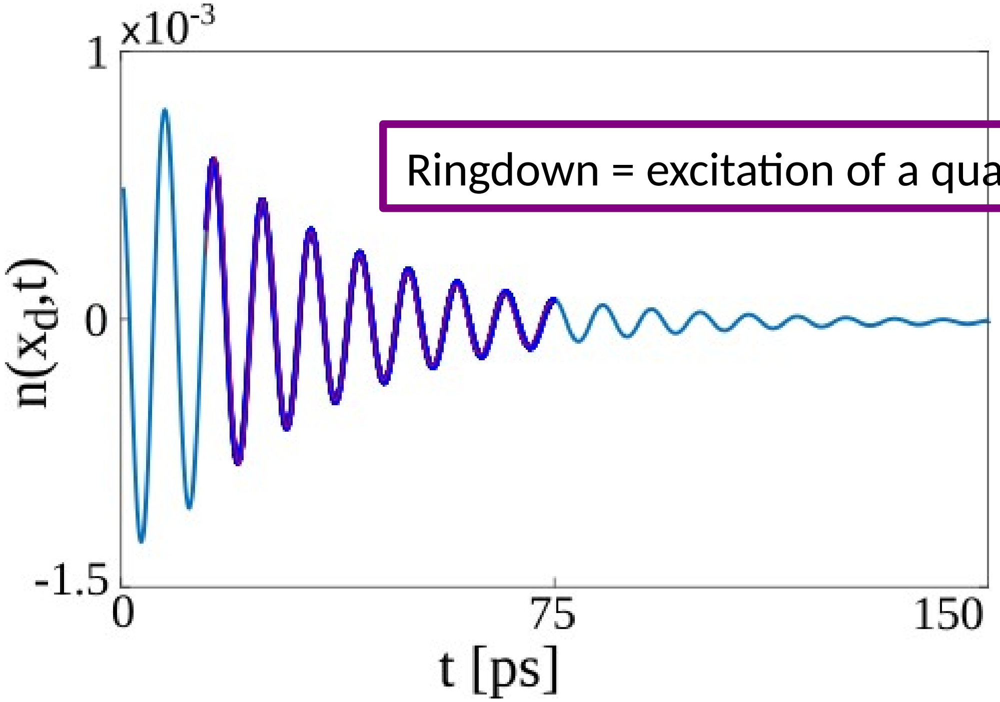




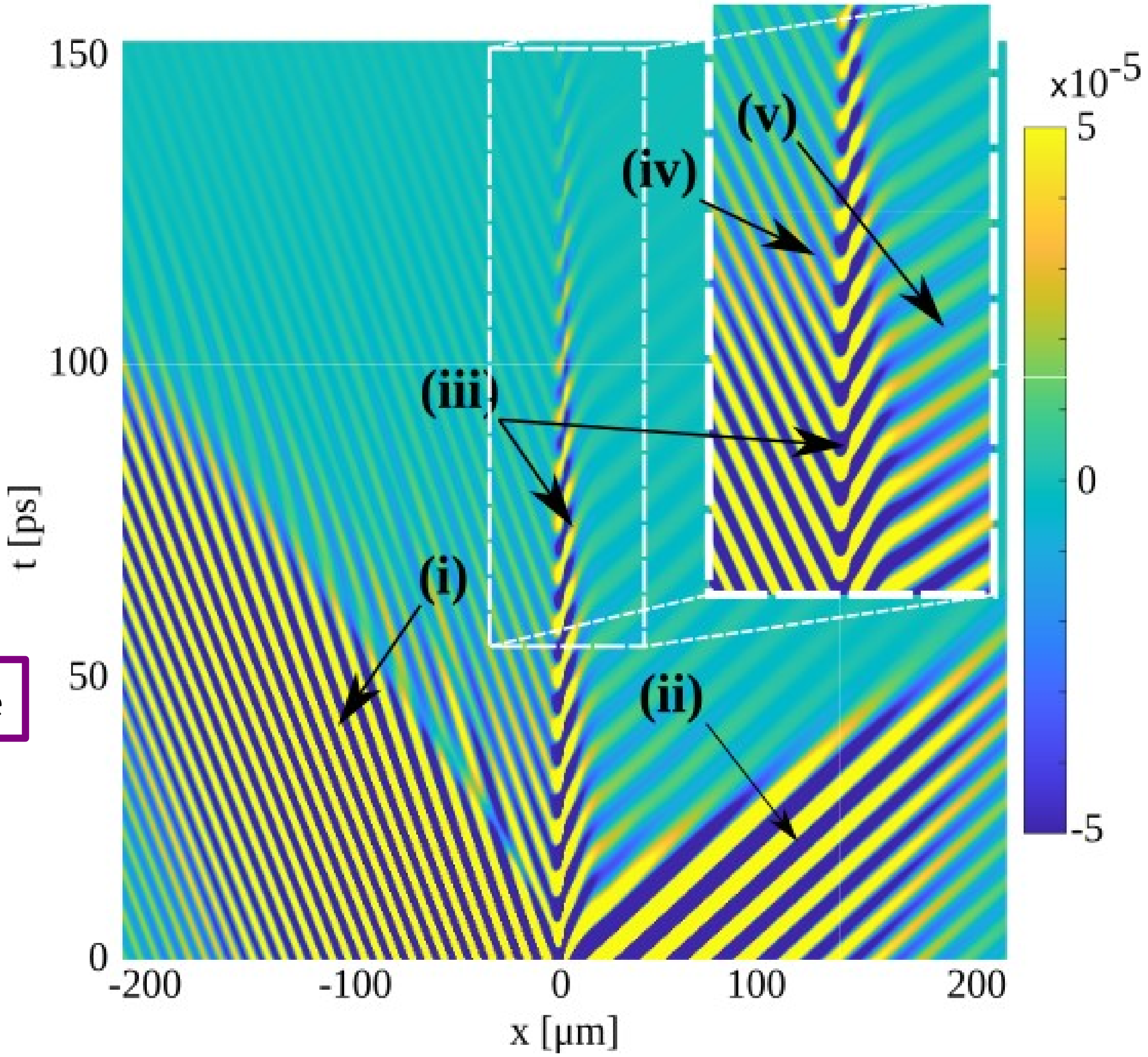
Ringdown upon perturbation!

Send wavepacket u_{in} toward horizon:

(i) reflection
(ii) transmission
(iii) density @horizon oscillates and dampens
(iv) density @horizon couples with mode propagating outward
(v) density @horizon couples with mode propagating inward



Ringdown = excitation of a quasi-normal mode



$\mathrm{Re}(\omega)$ Frequency of oscillation
$\mathrm{Im}(\omega)$ Decay rate < γ (bare polariton lifetime)

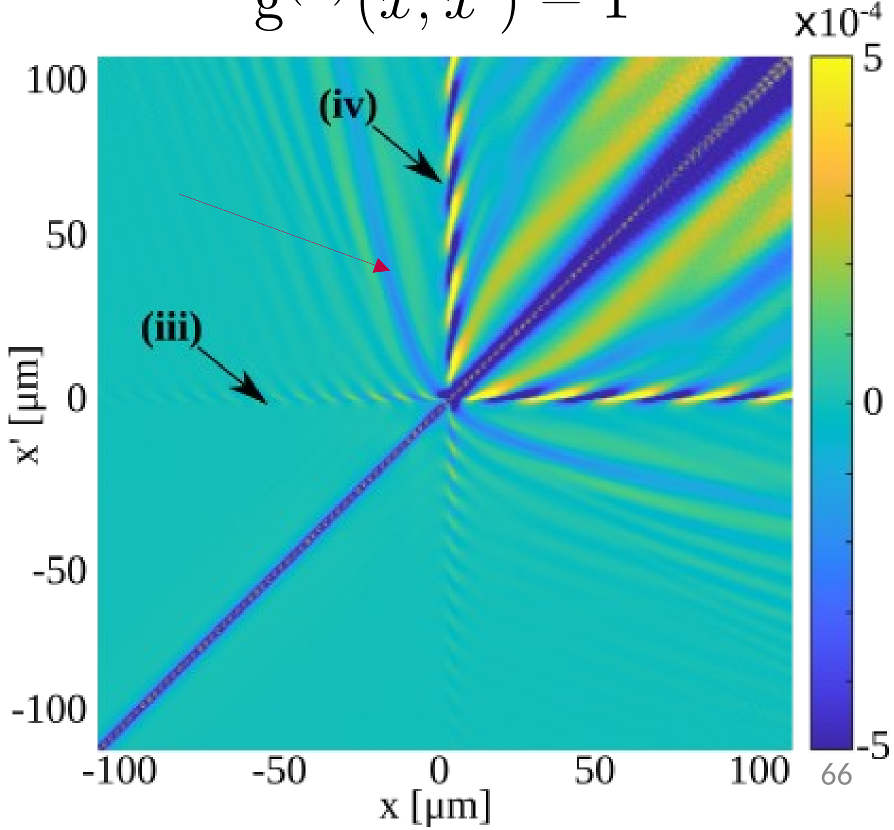
Scattering of vacuum fluctuations: long and strong Hawking correlations

Numerical simulation: Truncated Wigner Approximation
(1 billion realisations)

Measure equal time correlations

$$g^{(2)}(x,x') = \frac{\left\langle \Psi(x)^\dagger \Psi(x')^\dagger \Psi(x) \Psi(x') \right\rangle}{\langle \Psi(x)^\dagger \Psi(x) \rangle \langle \Psi(x')^\dagger \Psi(x') \rangle}$$

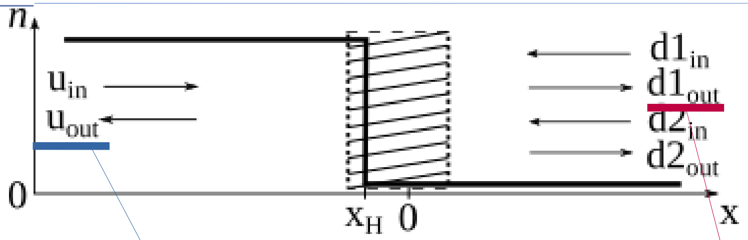
$$g^{(2)}(x,x') - 1$$



Hawking
correlations

(iii) horizon - outside
(iv) horizon - inside

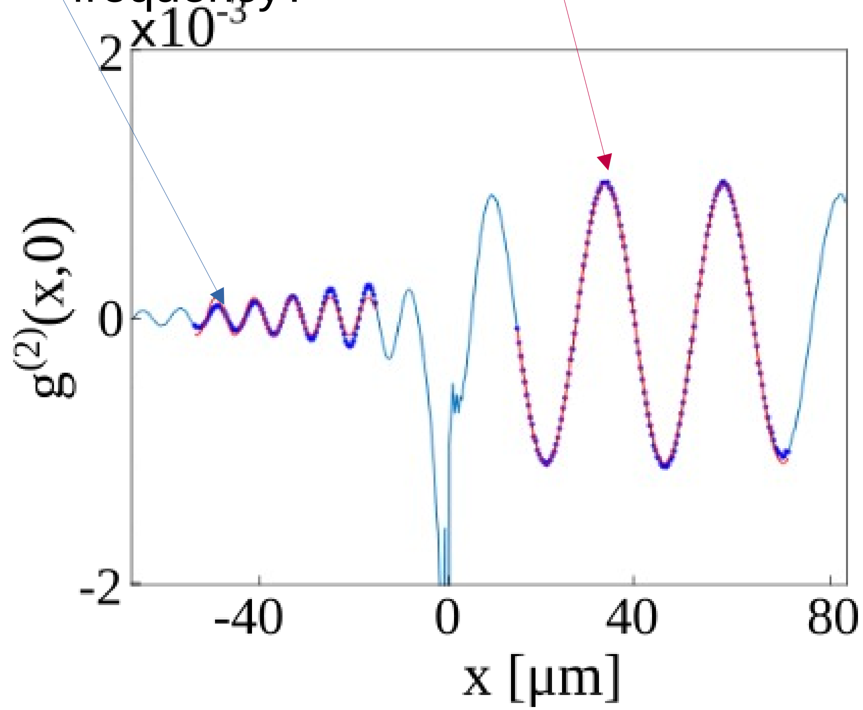
Scattering of vacuum fluctuations: horizon correlations

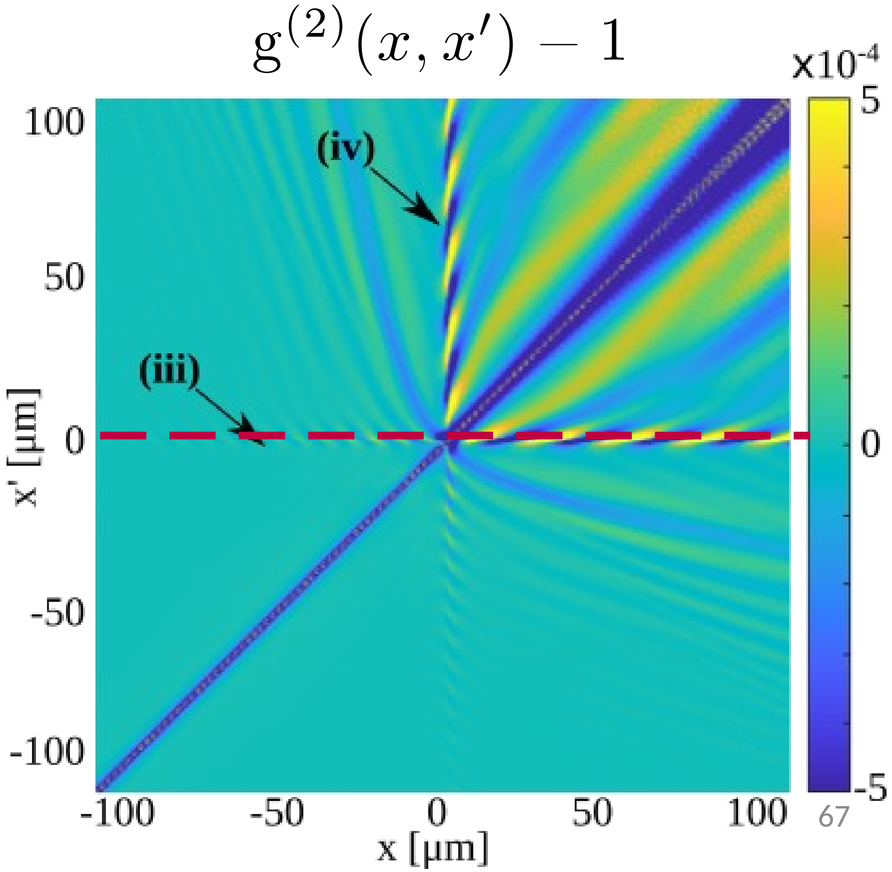


(iii) horizon - outside

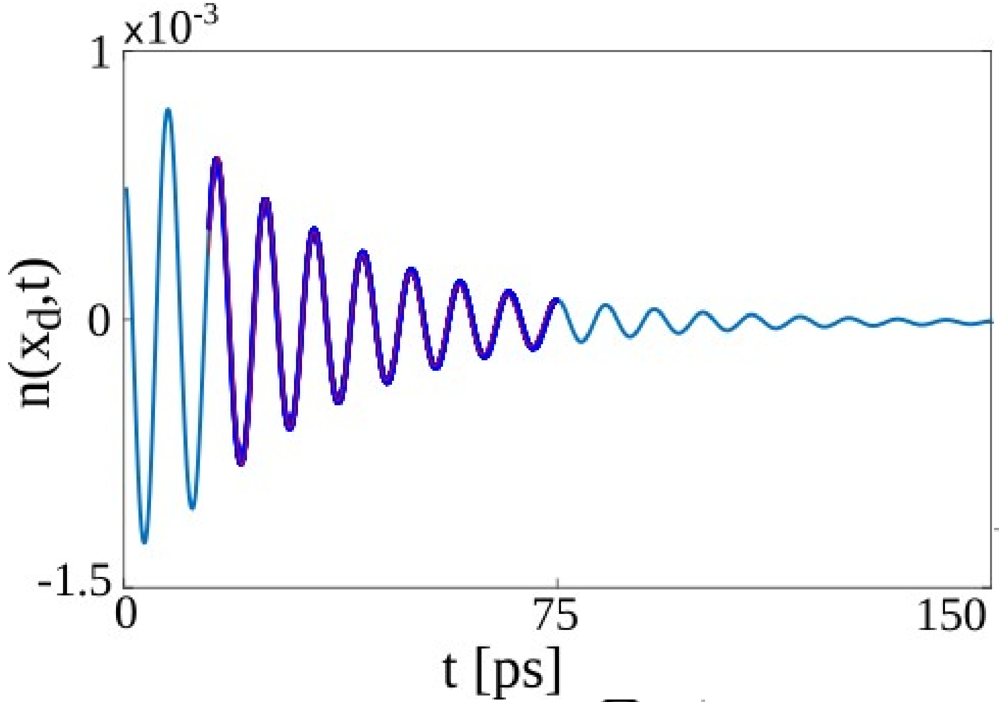
(iv) horizon - inside

Spatial frequency?



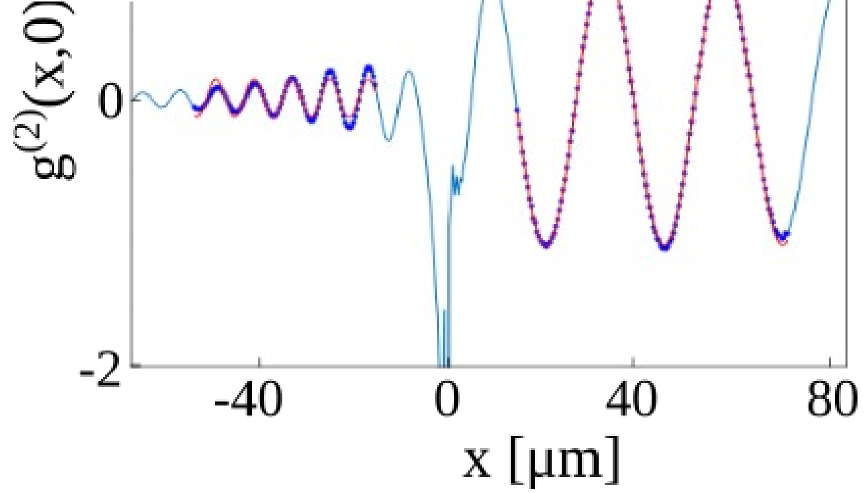


Scattering of vacuum fluctuations: excitation of a quasi normal mode



$\mathrm{Re}(\omega)$ Frequency of oscillation

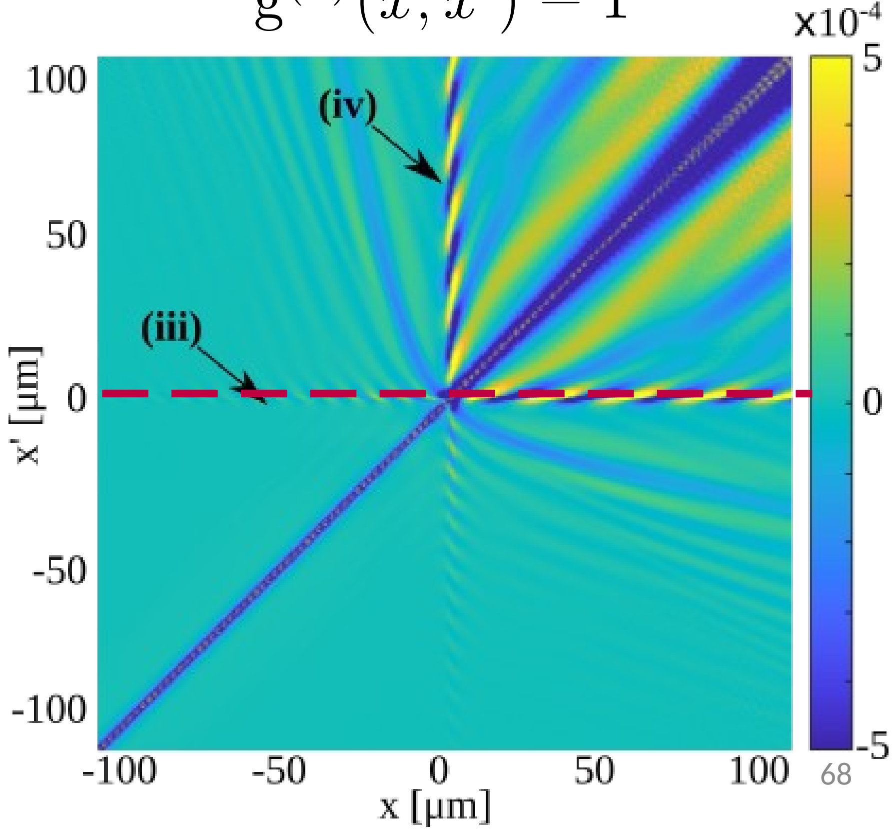
$\mathrm{Im}(\omega)$ Decay rate < γ



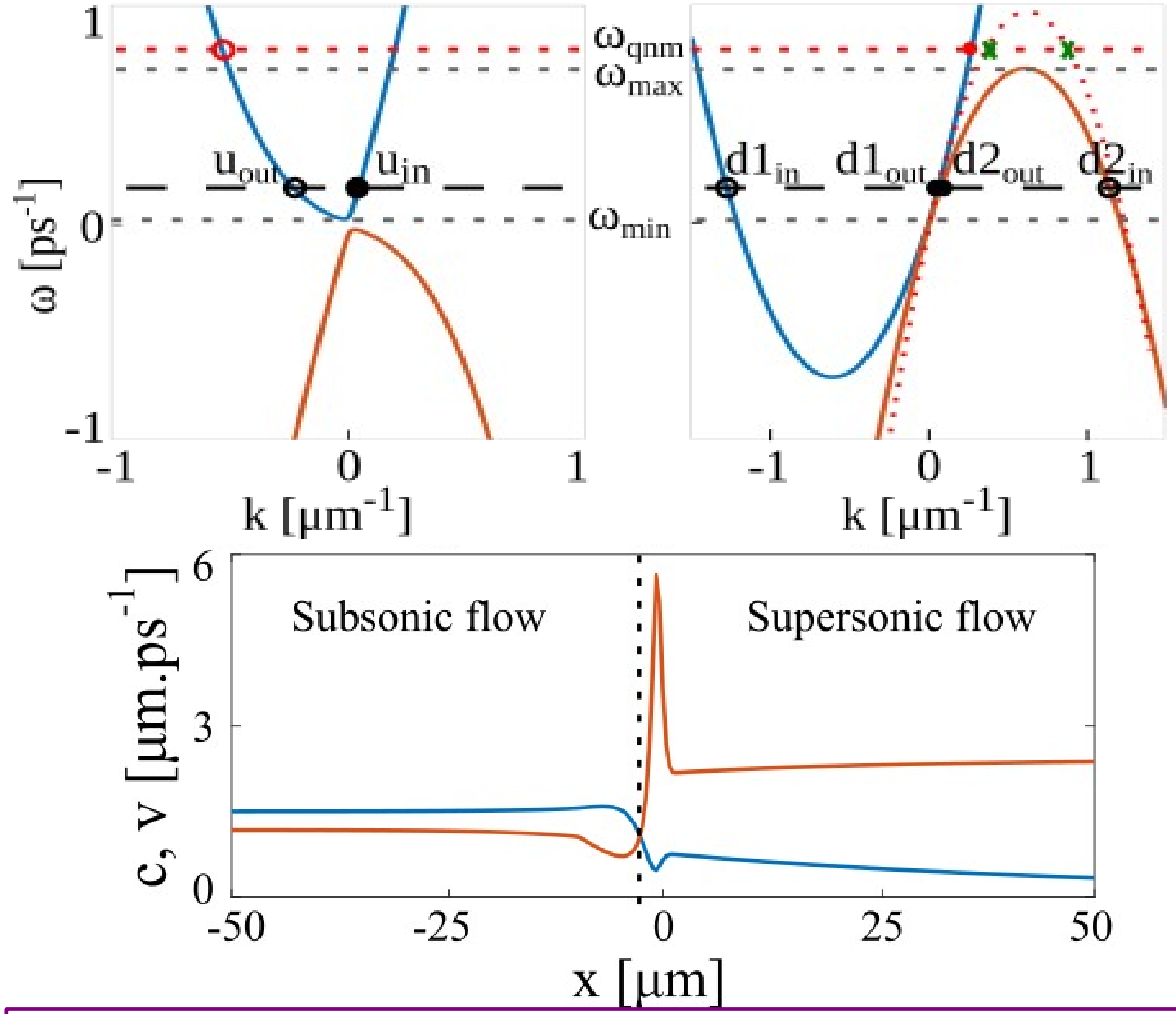
(iii) horizon - outside
(iv) horizon - inside

→ excitation of a quasi-normal mode

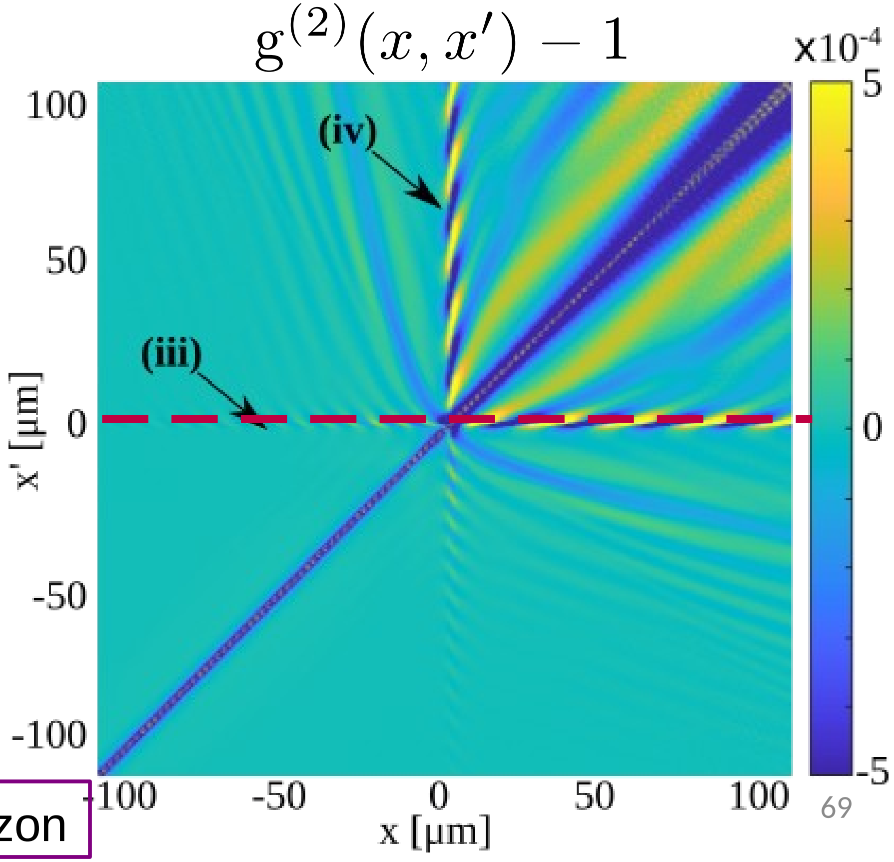
$g^{(2)}(x,x') - 1$



Scattering of vacuum fluctuations: effective potential

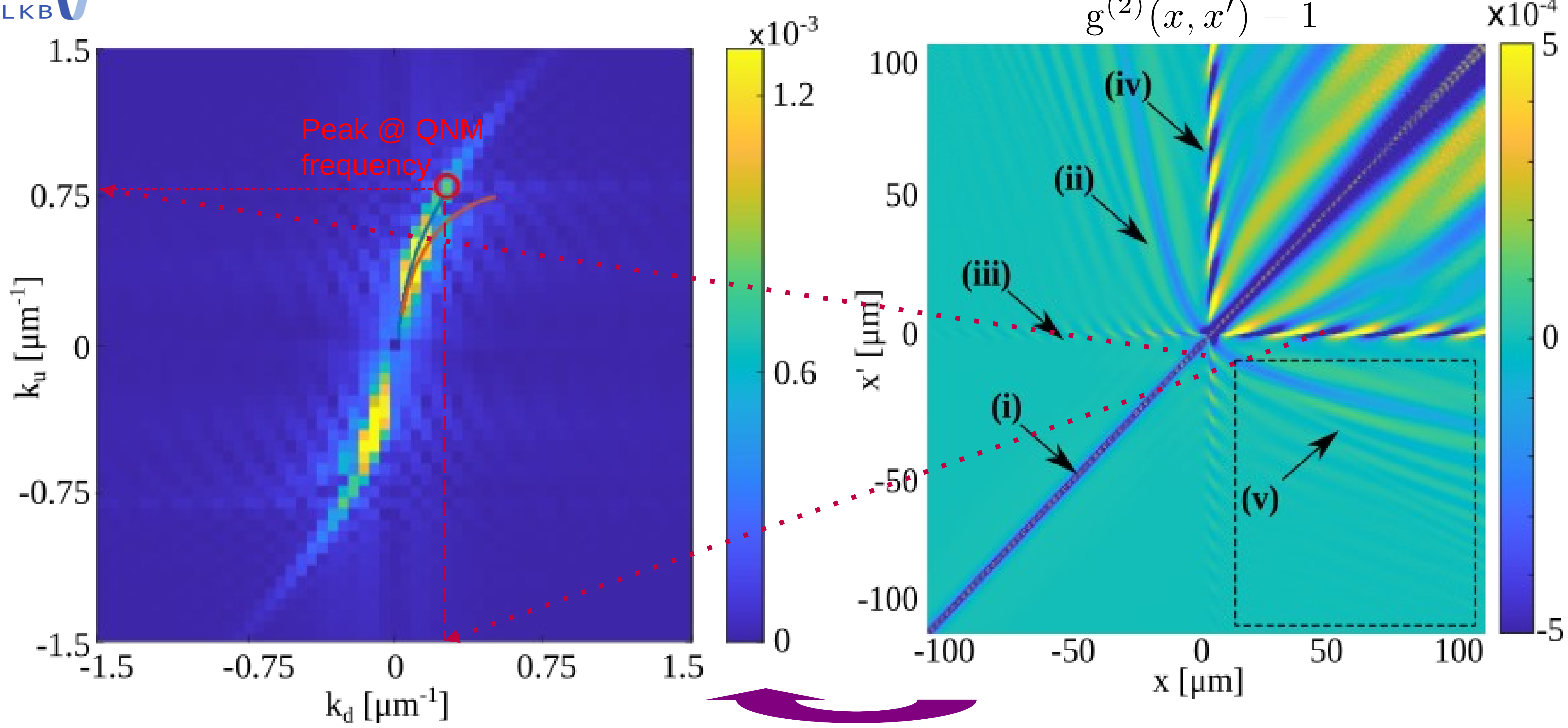


(iii) horizon - outside
(iv) horizon - inside



Velocity spike creates negative potential for negative modes near horizon

Spectral modulation



2D Fourier Transform